Neural Network Architectures I

Beyond CS 181

Matthew Nazari

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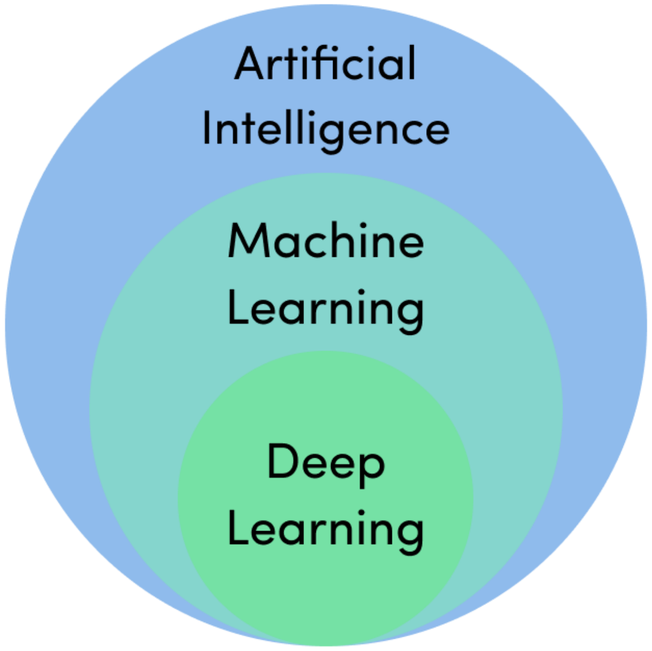
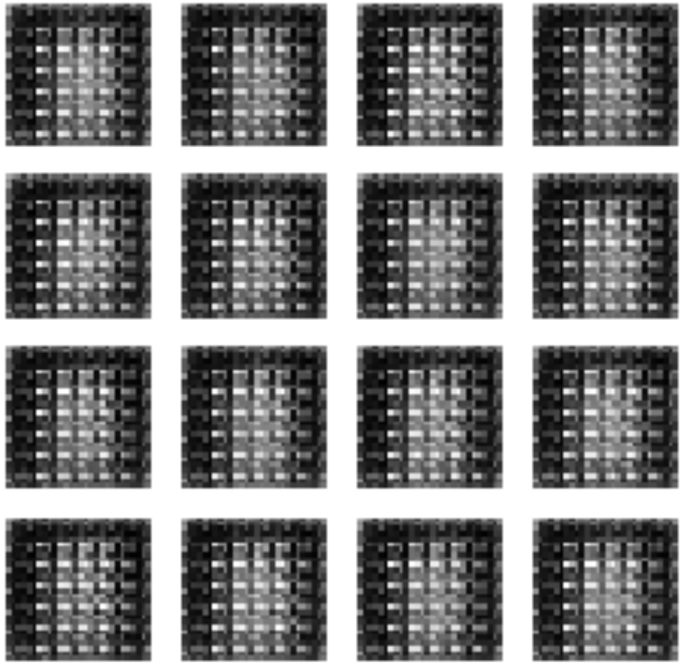
Deep learning

- Machine learning with artificial neural networks
- We will consider feed-forward and convolutional architectures





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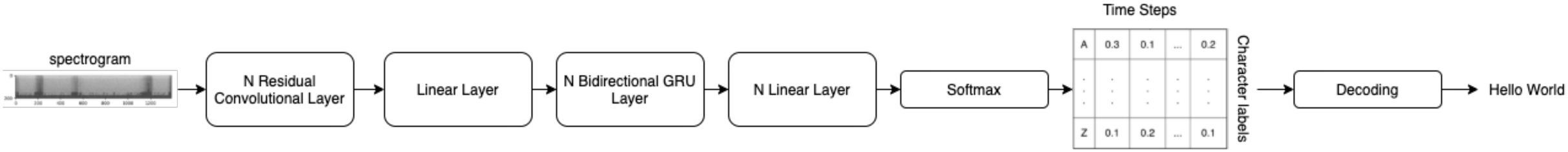
Deep learning



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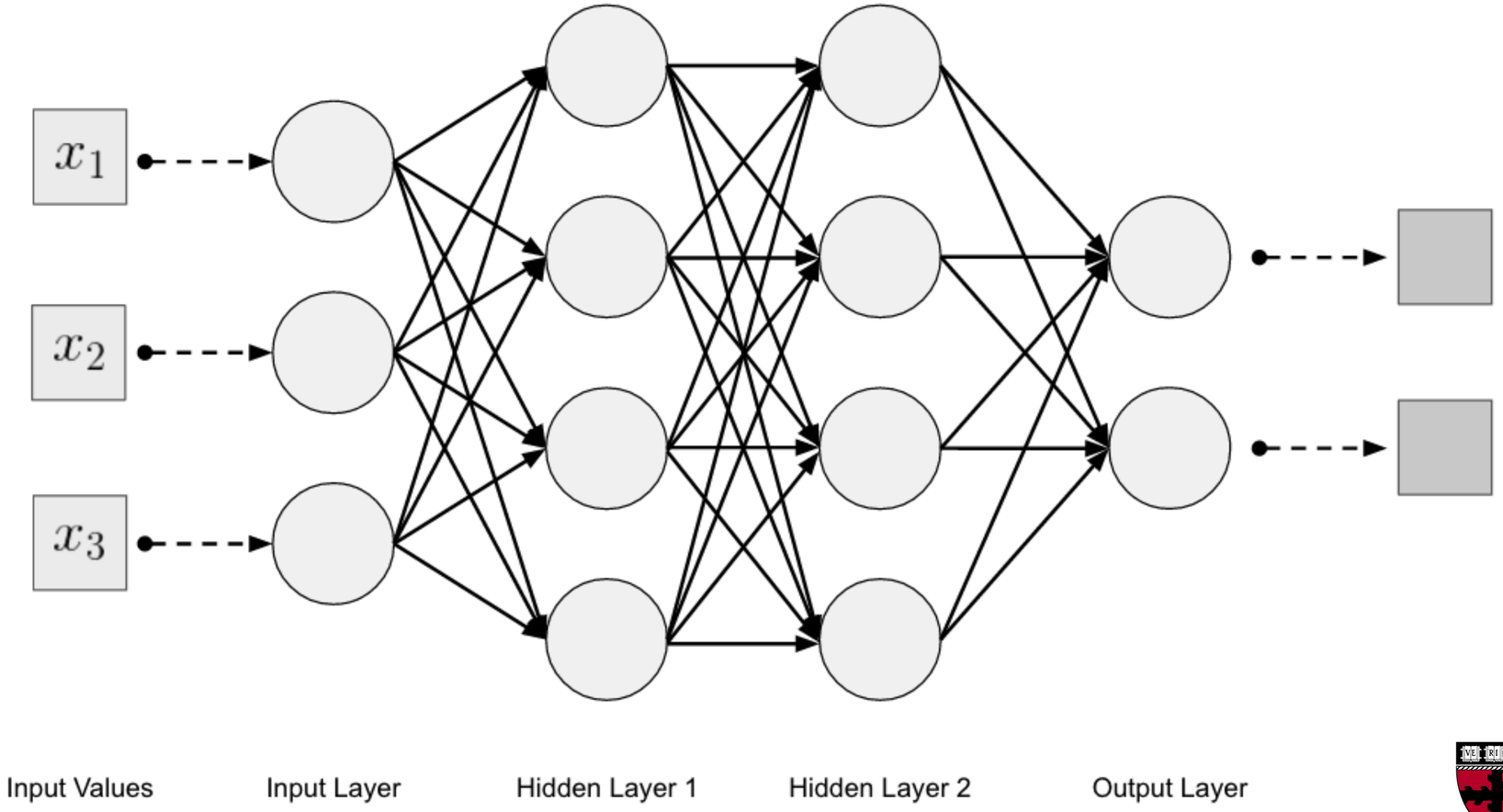
Deep learning



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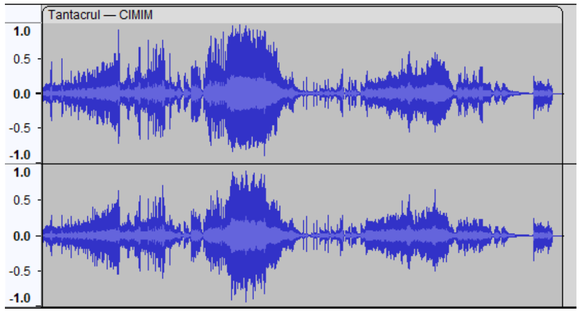
Data

- A numerical representation of a real world ✨ thing ✨
- We pick how to represent it! We pick the features
- Hopefully these features are “useful”

The quick brown fox jumps over the lazy dog

Text



Sound

Images

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Finding "useful representations"

- Some representations (or bases) are more helpful than others

- In linear regression, features $\mathbf{x}$ are useful if there exists a model in the family $f(\mathbf{x}; \mathbf{w}) := \mathbf{w}^\top \mathbf{x}$ that does well

Bad features: $\begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix}$ Useful features! : $\begin{bmatrix} 1 \\ x_1 x_2 \end{bmatrix}$

Finding “useful representations”

- Domain experts can tell us which bases are useful

$$\phi(\mathbf{x}) := \begin{bmatrix} 1 \\ x_1 x_2 \end{bmatrix} \implies \phi(\mathbf{x}) = \begin{bmatrix} \phi_1(\mathbf{x}) \\ \phi_2(\mathbf{x}) \end{bmatrix}$$

- What if we… oh idk… use linear regression to find useful $\{\phi_i\}$!!!!!

Finding “useful representations”

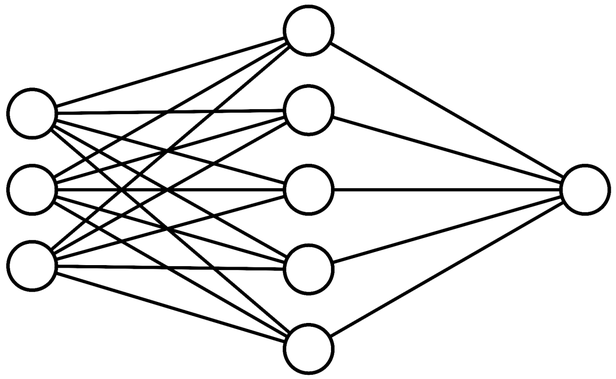
- Hence, we have a “adaptive basis”, once that we can toggle and change with by toggling or LEARNING $\{\mathbf{w}_i\}$

$$f(\mathbf{x}; \mathbf{w}, \{\mathbf{w}_i\}) := \mathbf{w}^\top \phi(\mathbf{x})$$

$$\phi_i = \mathbf{w}_i^\top \mathbf{x}$$

$$\mathbf{h} = \mathbf{W}_1 \mathbf{x}$$

$$y = \mathbf{w}_2 \mathbf{h}$$



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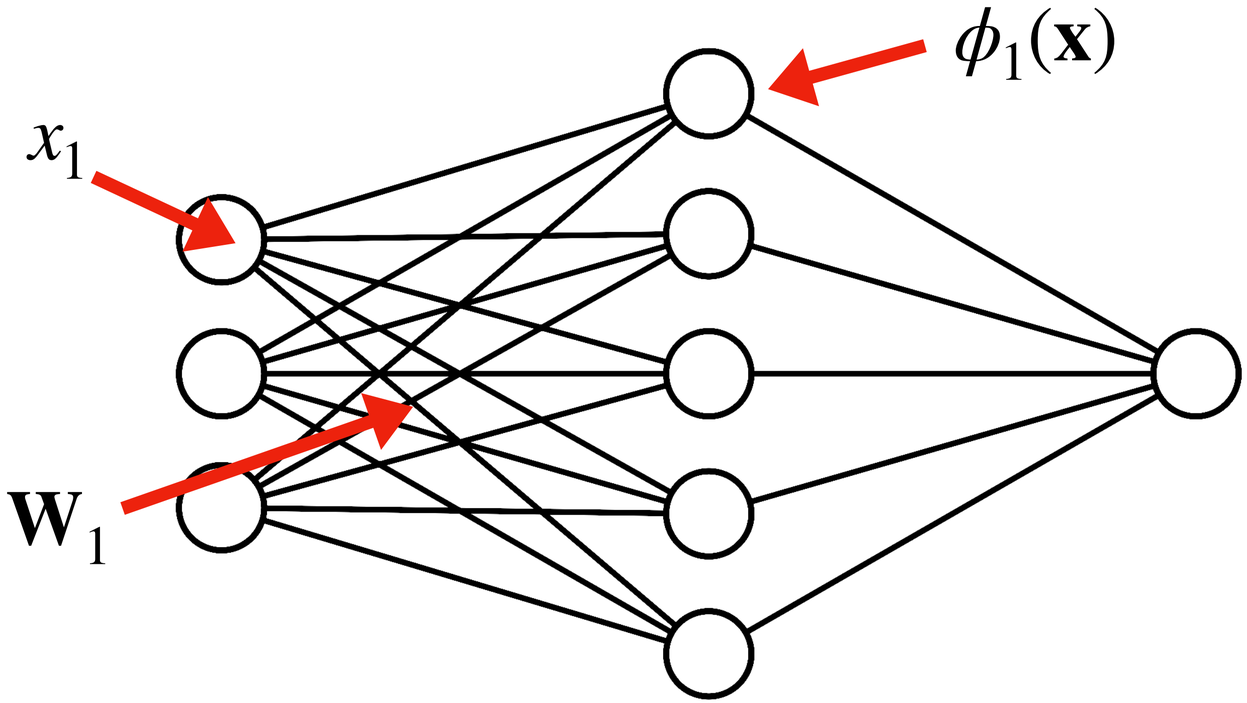
Neural networks



$$\mathbf{h} = \mathbf{W}_1 \mathbf{x}$$

$$y = \mathbf{w}_2 \mathbf{h}$$

Input layer

$\mathbf{x} \in \mathbb{R}^D$

Hidden layer

$\mathbf{h} \in \mathbb{R}^J$

Output

$y \in \mathbb{R}$

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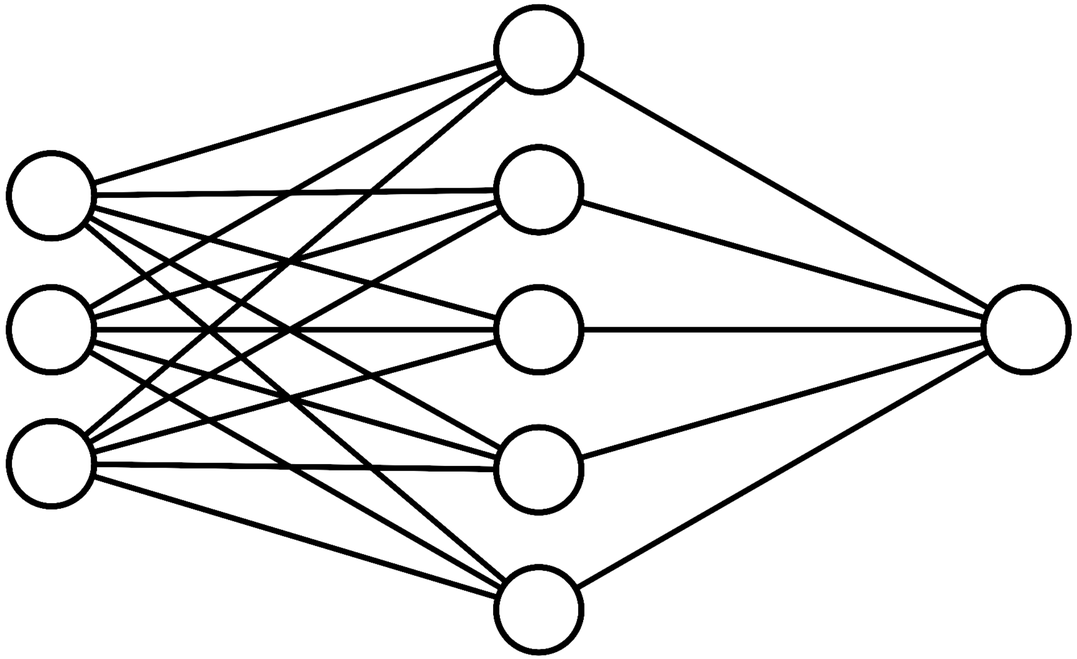
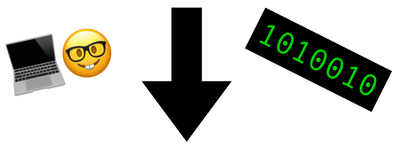
Neural networks

$$\mathbf{h} = \mathbf{W}_1 \mathbf{x}$$

$$y = \mathbf{w}_2 \mathbf{h}$$






```python
from torch import nn  # Torch's neural network library

D, J = 3, 5  # Define our dimensions
net = nn.Sequential(
    nn.Linear(in_features=D, out_features=J),  # Hidden layer
    nn.Linear(in_features=J, out_features=1)   # Output layer
)
```

Input layer

$\mathbf{x} \in \mathbb{R}^D$

Hidden layer

$\mathbf{h} \in \mathbb{R}^J$

Output

$y \in \mathbb{R}$

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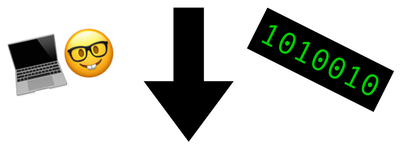
Neural networks

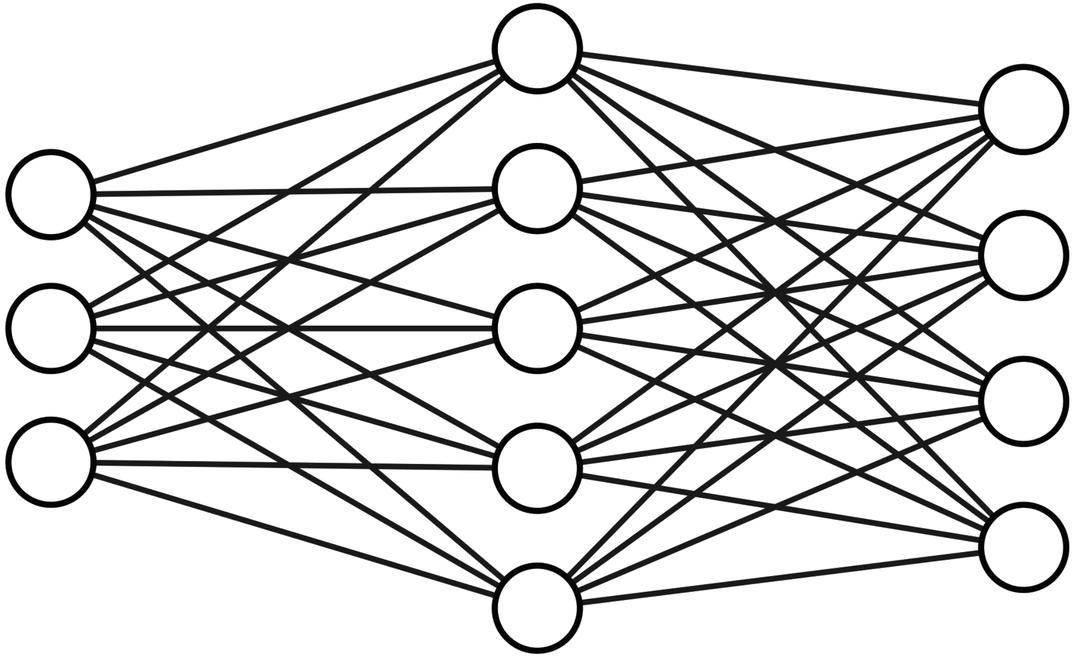
$$\mathbf{h} = \mathbf{W}_1 \mathbf{x}$$

$$\mathbf{y} = \mathbf{W}_2 \mathbf{h}$$




```
from torch import nn

D, J, K = 3, 5, 4
net = nn.Sequential(
    nn.Linear(D, J),
    nn.Linear(J, K)
)
```



Input layer

$\mathbf{x} \in \mathbb{R}^D$

Hidden layer

$\mathbf{h} \in \mathbb{R}^J$

Output

$\mathbf{y} \in \mathbb{R}^K$

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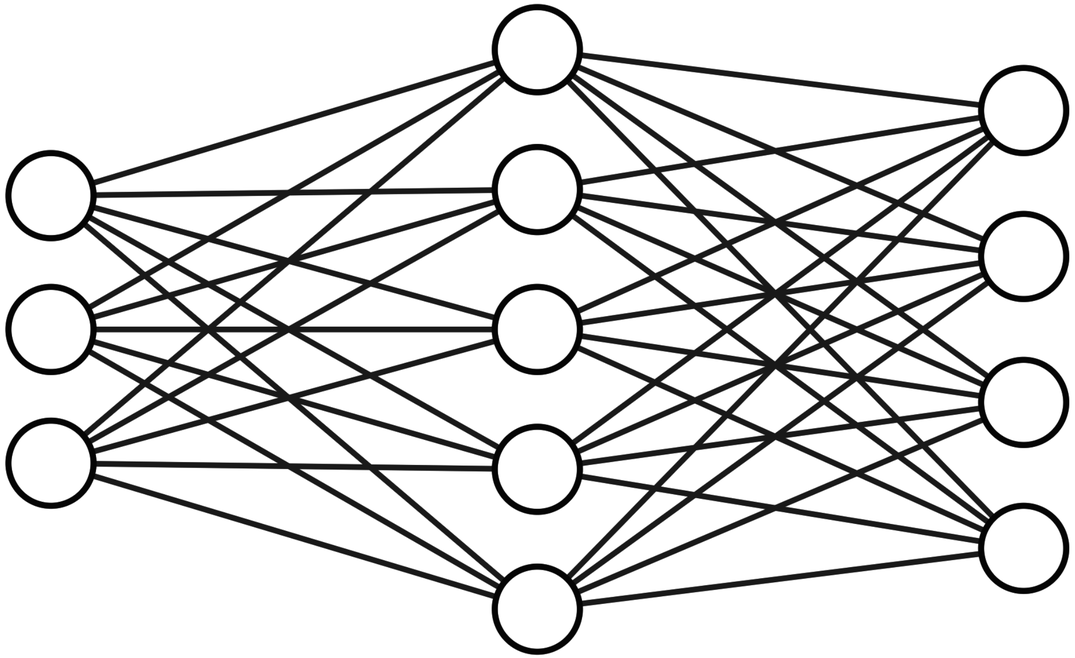
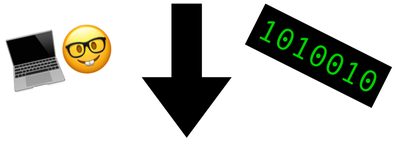
Neural networks

$$\mathbf{h} = \sigma(\mathbf{W}_1 \mathbf{x})$$

$$\mathbf{y} = \mathbf{W}_2 \mathbf{h}$$






```python
from torch import nn

D, J, K = 3, 5, 4
net = nn.Sequential(
    nn.Linear(D, J),
    nn.ReLU(),
    nn.Linear(J, K)
)
```

Input layer

$\mathbf{x} \in \mathbb{R}^D$

Hidden layer

$\mathbf{h} \in \mathbb{R}^J$

Output

$\mathbf{y} \in \mathbb{R}^K$

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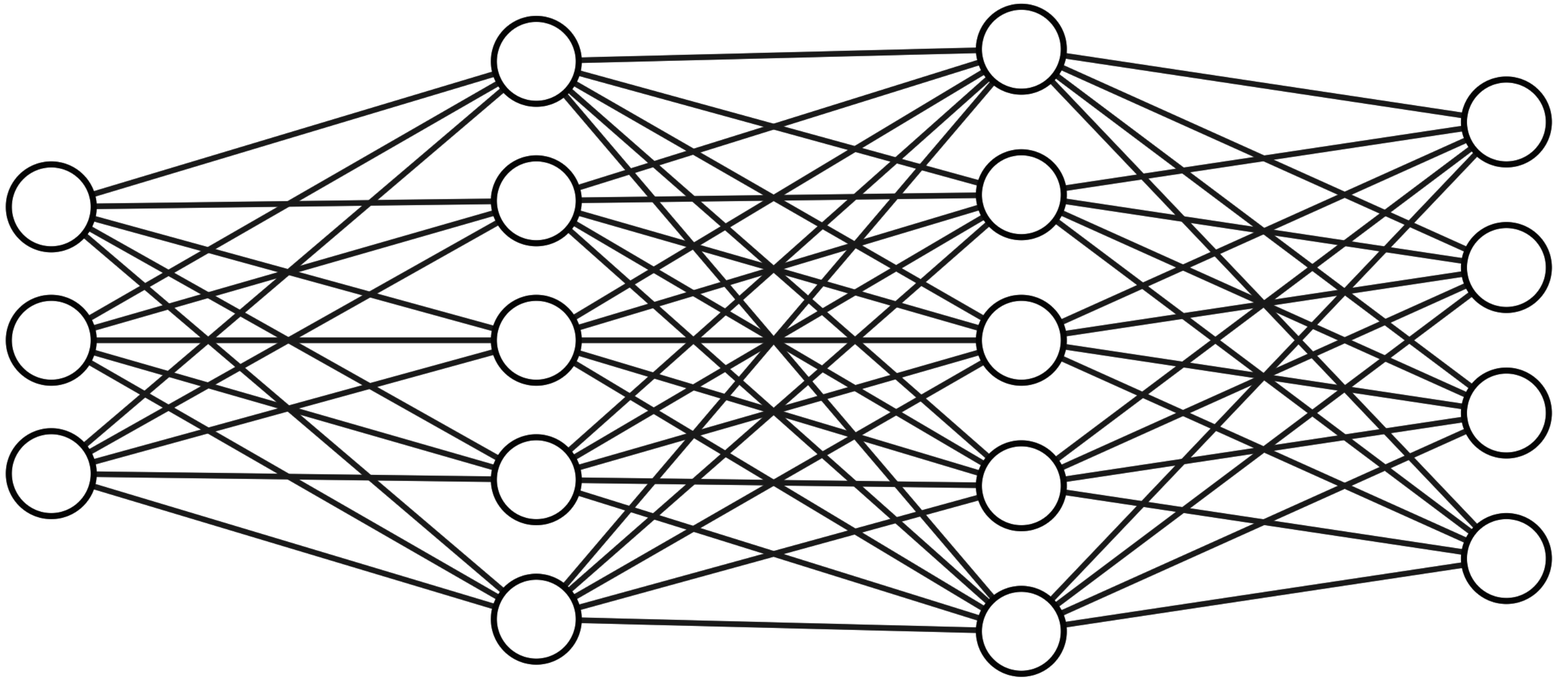
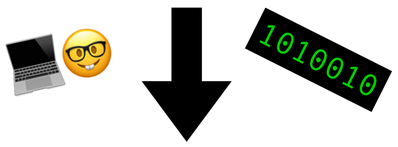
Neural networks

$$\mathbf{h}_1 = \sigma_1(\mathbf{W}_1\,\mathbf{x})$$
$$\mathbf{h}_2 = \sigma_2(\mathbf{W}_2\,\mathbf{x})$$
$$\mathbf{y} = \mathbf{W}_3\,\mathbf{h}_2$$






```
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D, J, K = 3, 5, 4
net = nn.Sequential(
    nn.Linear(D, J),
    nn.ReLU(),
    nn.Linear(J, J),
    nn.ReLU(),
    nn.Linear(J, K)
)
```

Input layer	Hidden layer	Hidden layer	Output
$\mathbf{x} \in \mathbb{R}^D$	$\mathbf{h}_1 \in \mathbb{R}^J$	$\mathbf{h}_2 \in \mathbb{R}^J$	$\mathbf{y} \in \mathbb{R}^K$

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Neural networks

- There's an issue… we can't pick up on SPATIAL CORRELATION!

$$\phi_i(\mathbf{x}) = \sigma(\mathbf{w}_i^\top \mathbf{x})$$

- (There's also SO many parameters….)

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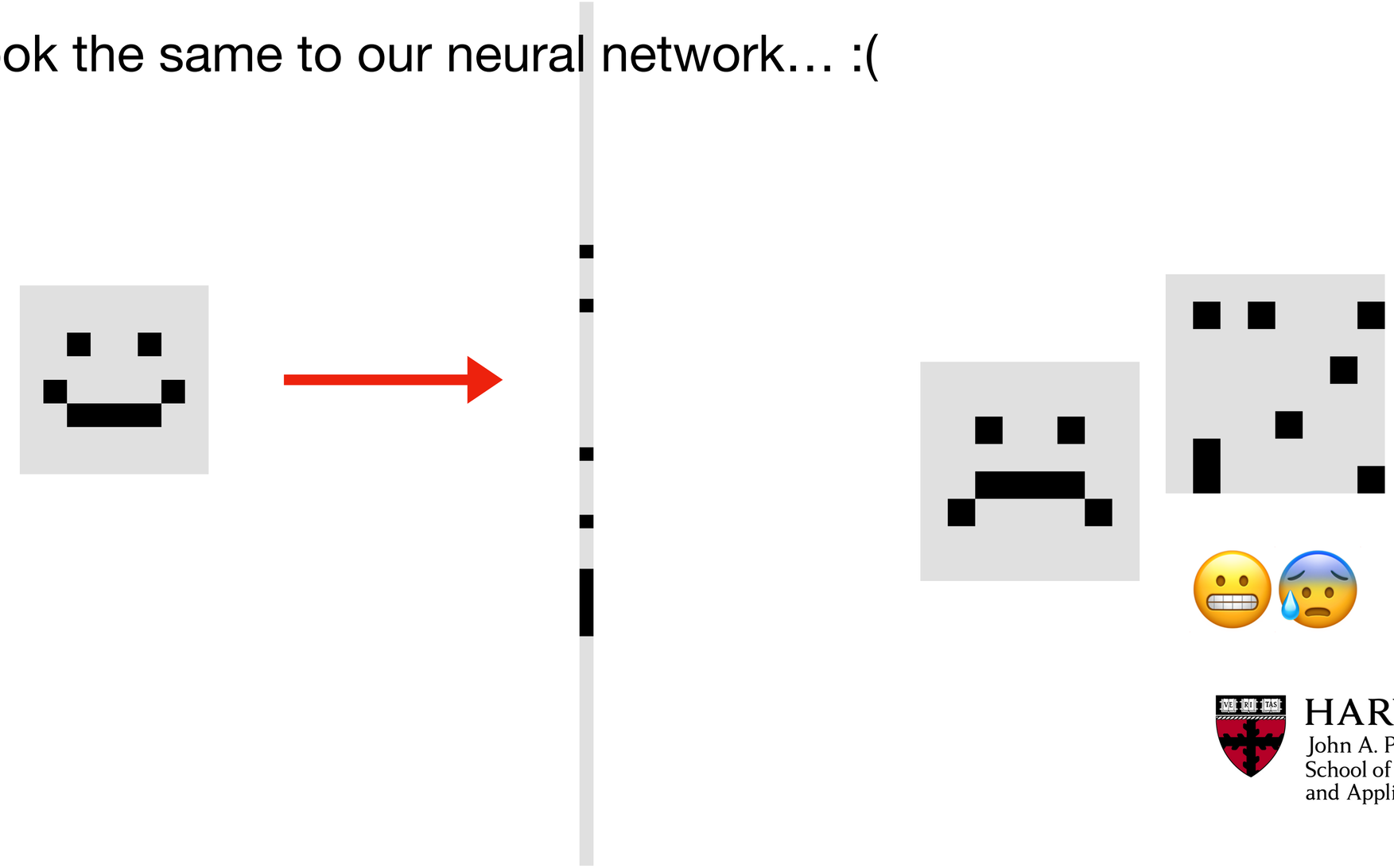
Spatial correlation

- These all look the same to our neural network… :(



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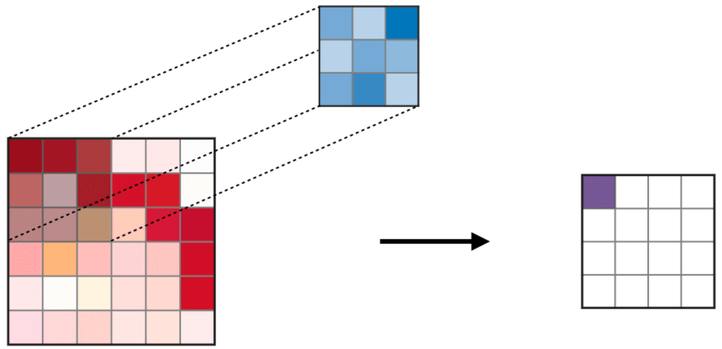
Convolutional Neural Networks (CNNs)

- Neural networks that exploit spatial locality by use of convolutional layers
- We are still trying to find a useful representation, i.e. column vector $\phi(\mathbf{x})$
- Now, though, each feature $\phi_i(\mathbf{x})$ we discover are functions over not all of $\mathbf{x}$ but regions



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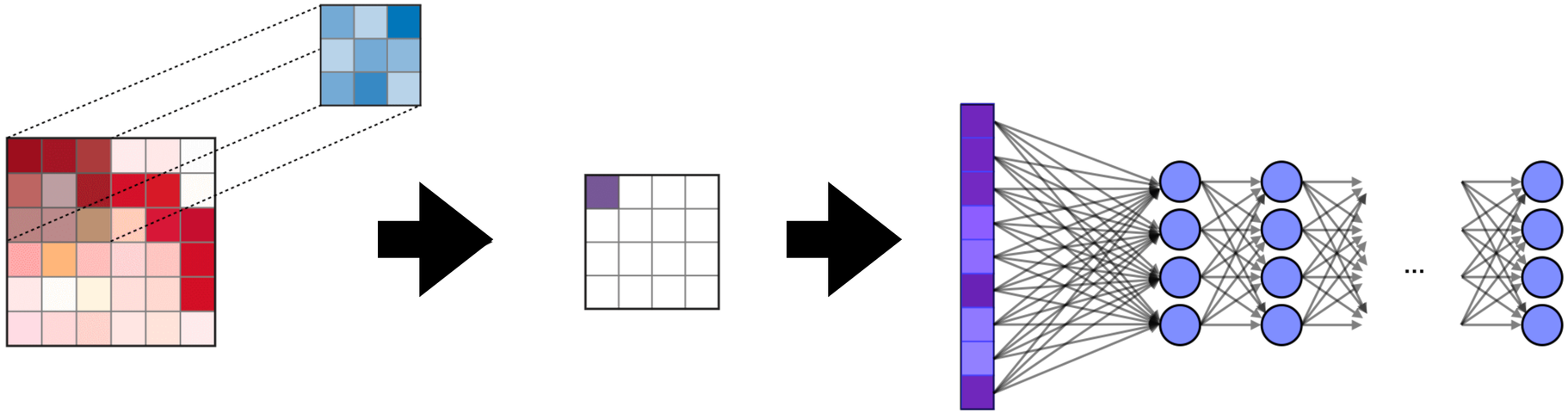
Credit for all images from here on out:
https://stanford.edu/~shervine/teaching/cs-230/cheatsheet-convolutional-neural-networks

Convolutional Neural Networks (CNNs)

- Each filter has the theoretical capacity to detect any feature, like “ear” or “nose” (those are too abstract, really)

- The final features in purple take into account spatial information in an image



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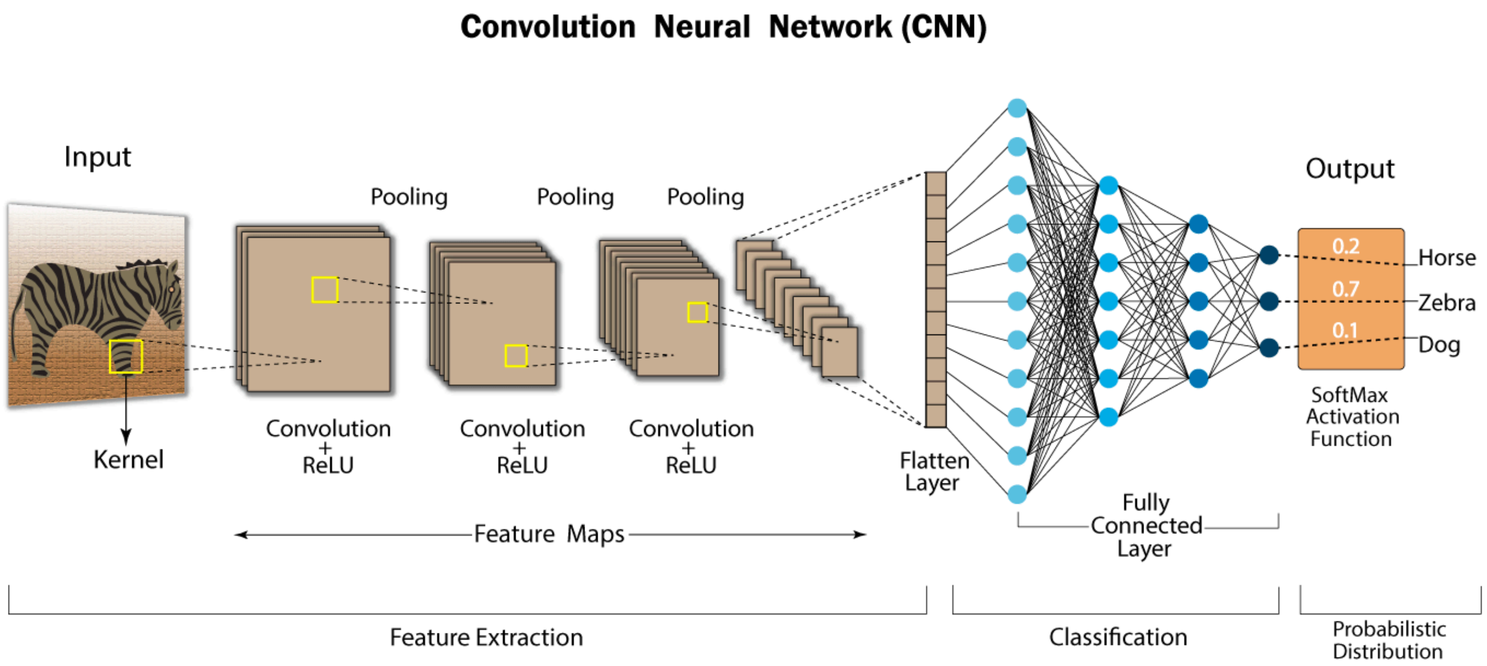
https://stanford.edu/~shervine/teaching/cs-230/cheatsheet-convolutional-neural-networks

Convolutional Architecture

- Channels
- Filters
- Padding and strides
- Pooling
- Data augmentation



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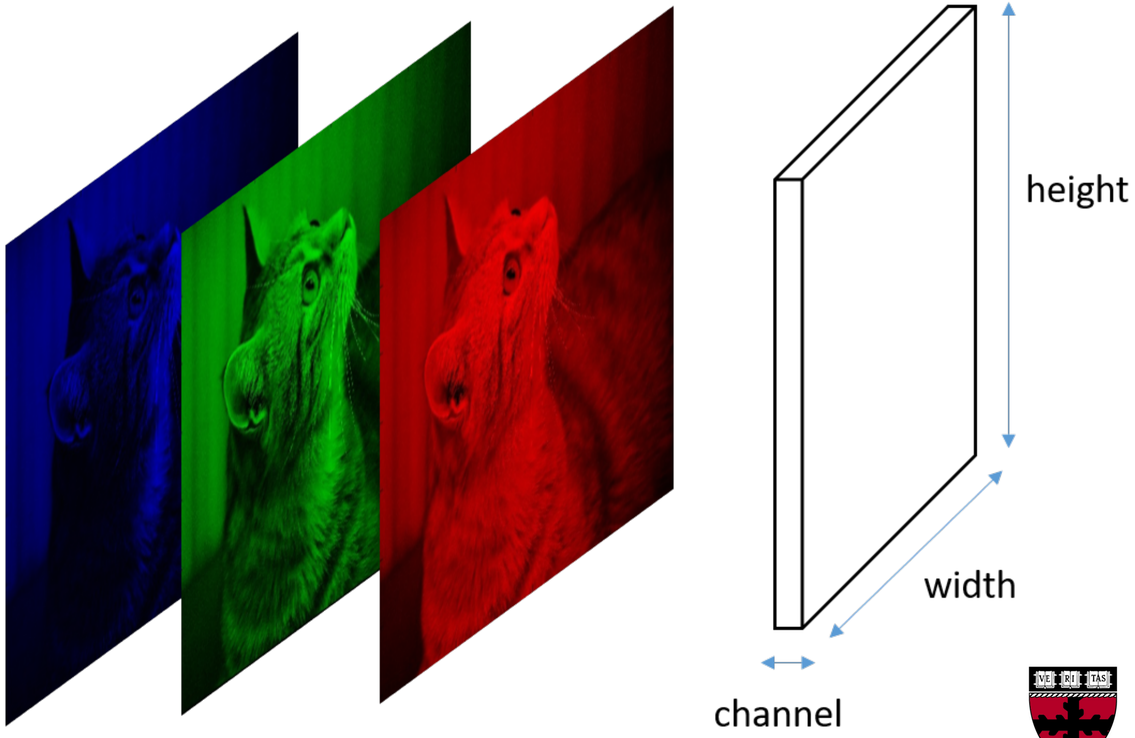
Channels

- Each raw datapoint is an image, which translates to a tensor! $\mathbf{x} \in \mathbb{R}^{C \times H \times W}$
- $C =$ Channels (like colors)
- $H =$ Height
- $W =$ Width



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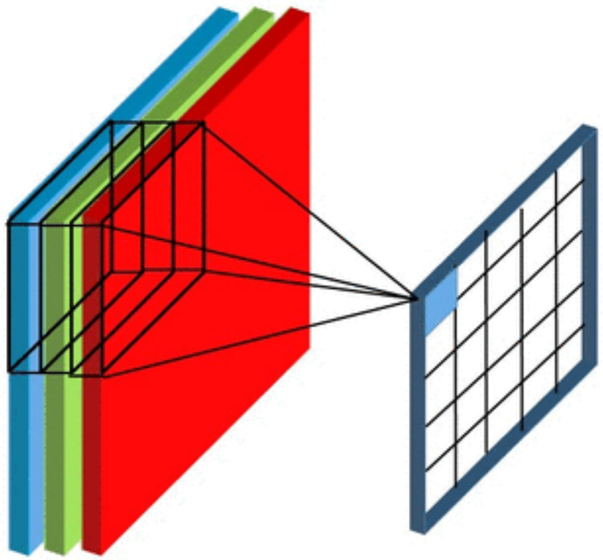
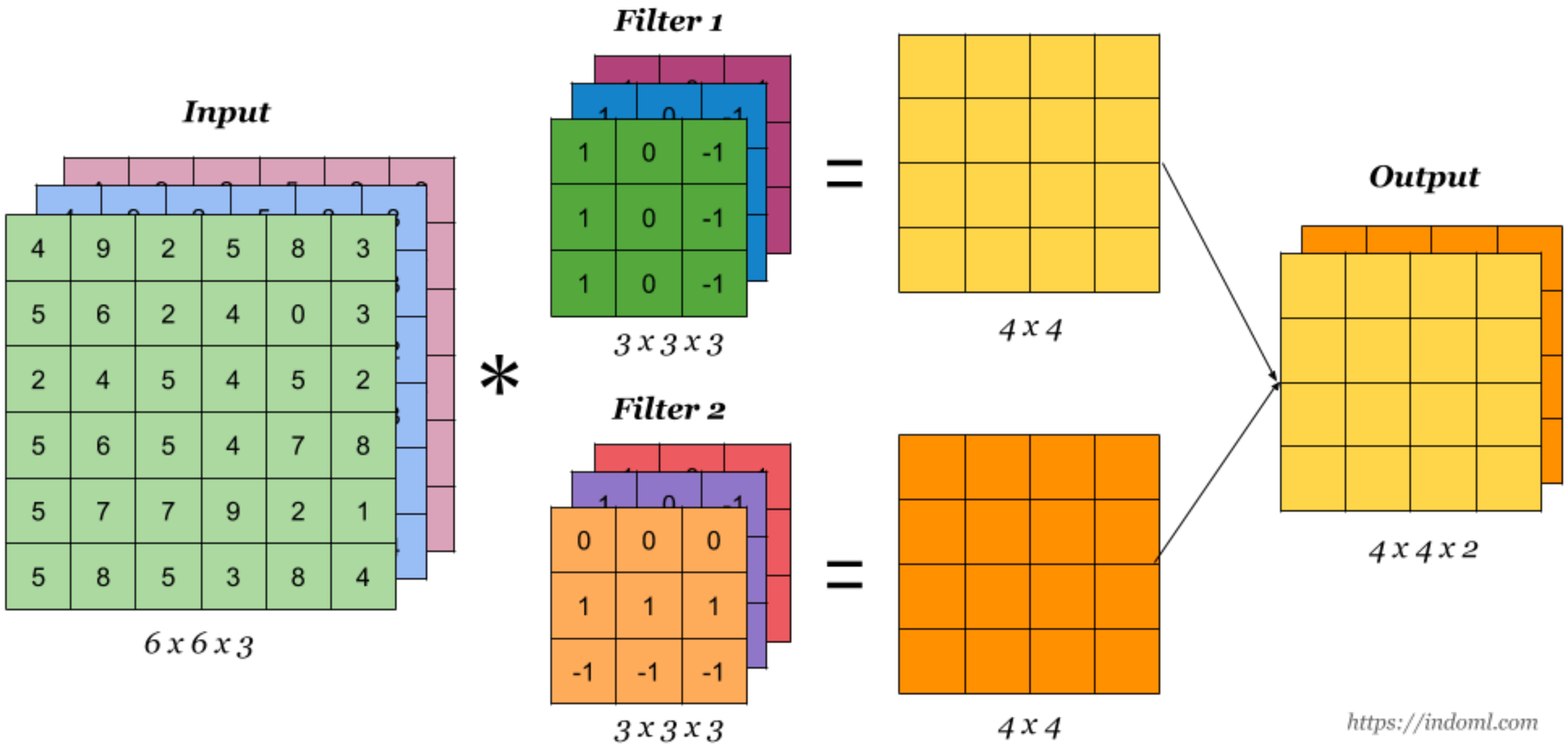
Filters

- Usually square windows that stretch across all channels
- More filters than there are channels

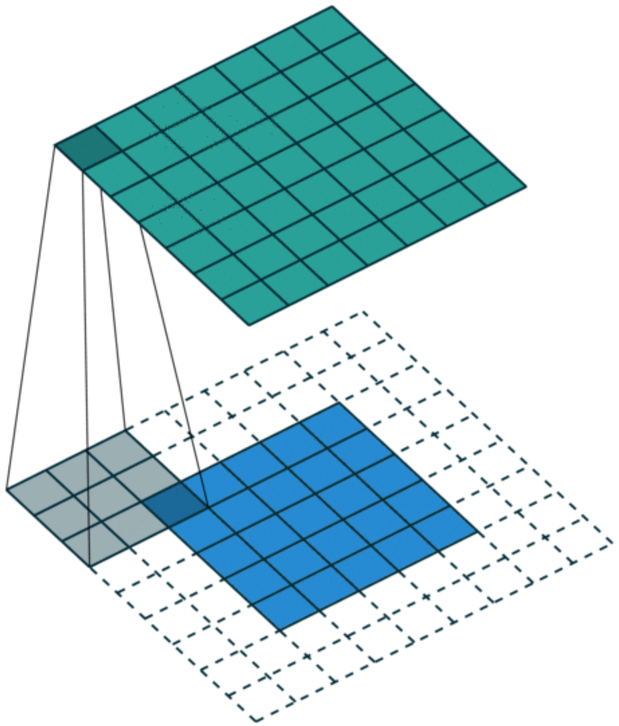


Filters


Input
4 9 2 5 8 3
5 6 2 4 0 3
2 4 5 4 5 2
5 6 5 4 7 8
5 7 7 9 2 1
5 8 5 3 8 4
6 x 6 x 3
*
Filter 1
1 0 -1
1 0 -1
1 0 -1
3 x 3 x 3
=
4 x 4
Filter 2
0 0 0
1 1 1
-1 -1 -1
3 x 3 x 3
=
4 x 4
Output
4 x 4 x 2
https://indoml.com

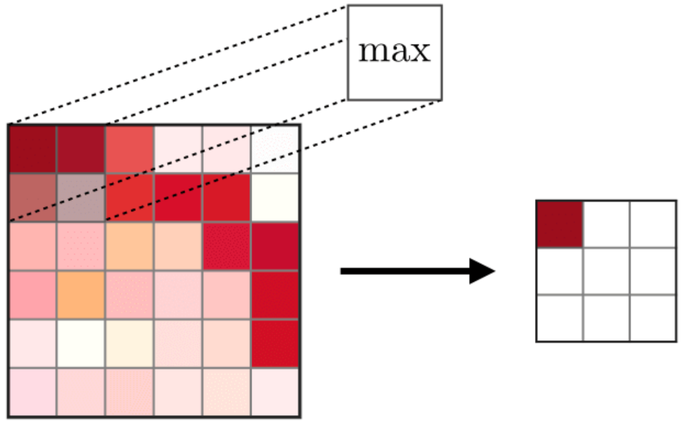
Padding and Stride

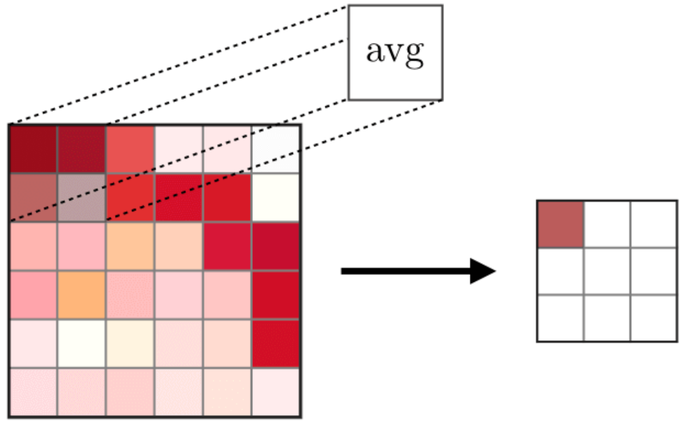
- These determine the pattern the filter slides over the image



Pooling

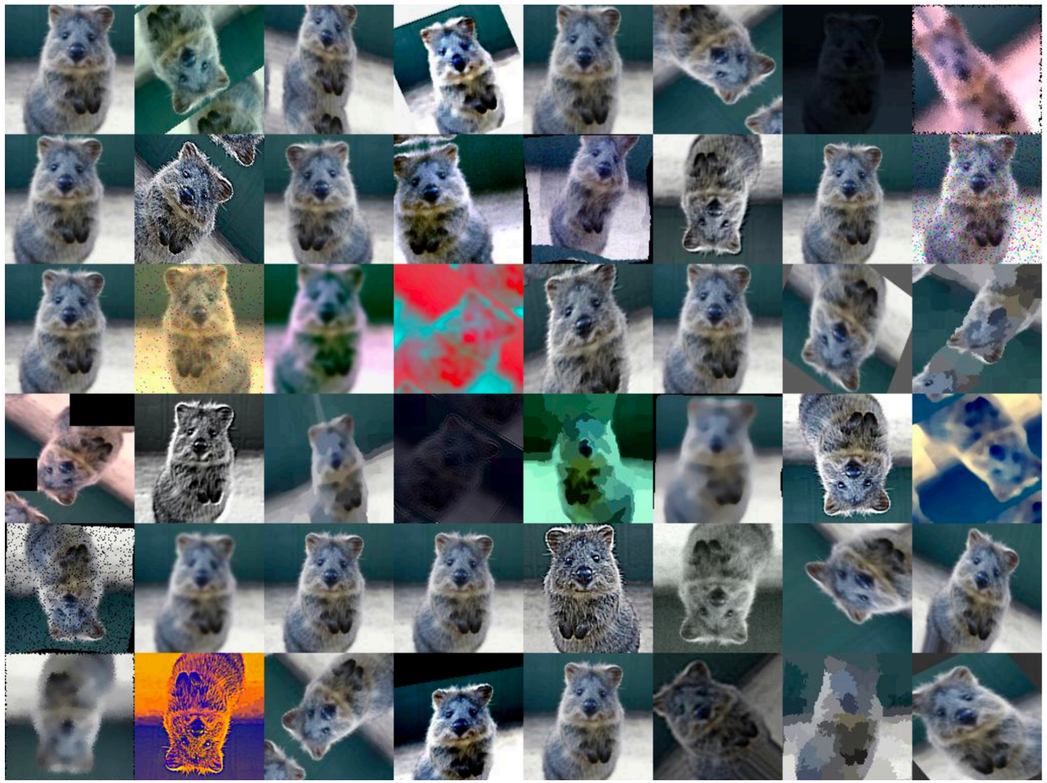
- Reduce the dimensionality
- Preserve detected features (Max) or downsize feature map (Avg)





Data Augmentation

- Literally just perturb the image
- Adds robustness!



Implementing in PyTorch