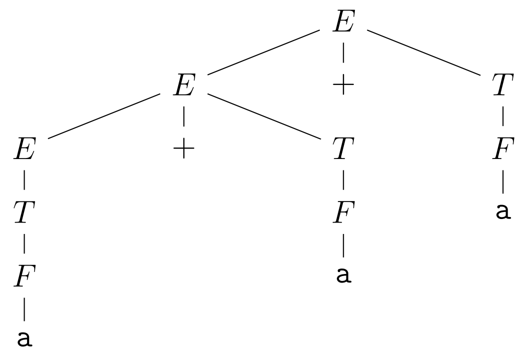
B62005Y-02 理论计算机科学基础 2017 年 3 月 29 日

Solutions to Homework # 2

主讲教师：杨光 张远航 2015K8009929045

Sipser 2.1 (c) Derivations: $E \Rightarrow E{+}T \Rightarrow E{+}T{+}T \Rightarrow E{+}T{+}T \Rightarrow F{+}T{+}T \Rightarrow \ldots \Rightarrow \mathtt{a{+}a{+}a}$.

The parse tree:



Sipser 2.2 (a) First, we show that both A and B are CFLs. The following grammars derive them:

$$
\begin{aligned}
S &\to RT \\
R &\to \mathtt{a}R \mid \varepsilon \\
T &\to \mathtt{b}T\mathtt{c} \mid \varepsilon
\end{aligned}
$$

and

$$
\begin{aligned}
S &\to TR \\
R &\to \mathtt{a}T\mathtt{b} \mid \varepsilon \\
T &\to \mathtt{c}R \mid \varepsilon
\end{aligned}
$$

Note that $A \cap B = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$ is not context-free by Example 2.20. We have found a counterexample and therefore the CFLs are not closed under intersection.

(b) *Proof.* We prove this in two parts:

- *CFLs are closed under the union operation.* Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ be two arbitrary CFGs. We can construct a grammar

$G = (V, \Sigma, R, S)$ that recognized their union:let $V = V_1 \cup V_2$ and $R = R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\}$. (W.L.O.G. assume that $R_1 \cap R_2 = \emptyset$; otherwise rename the variables.)

- *CFGs are not closed under complementation.* Assume to the contrary that the CFGs are closed under complementation; then for two CFGs G_1 and G_2, $\overline{L(G_1)}$ and $\overline{L(G_2)}$ are context-free. It follows that $\overline{L(G_1)} \cup \overline{L(G_2)}$ is context free. Our assumption implies that $\overline{\overline{L(G_1)} \cup \overline{L(G_2)}} = L(G_1) \cap L(G_2)$ (by de Morgan's Law) is also context-free; however, following part (a) and setting $G_1 = A$ and $G_2 = B$ leads us to a contradiction. Therefore the context-free languages are not closed under complementation. $\square$

Sipser 2.4 (b) The following grammar generates it:

$$S \to \mathtt{0}T\mathtt{0} \mid \mathtt{1}T\mathtt{1}$$
$$T \to \mathtt{0}T\mathtt{0} \mid \mathtt{1}T \mid \varepsilon$$

Sipser 2.6 (b) We can write the complement of the language $\{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$ as the union of two languages:

- $\{\mathtt{a}^i\mathtt{b}^j \mid i, j \geq 0, i \neq j\}$, and
- arbitrary strings of $\mathtt{a}$ and $\mathtt{b}$ with "$\mathtt{ba}$" in between: $(\mathtt{a} \cup \mathtt{b})^*\mathtt{ba}(\mathtt{a} \cup \mathtt{b})^*$.

The CFGs for them are

$$S_1 \to \mathtt{a}S_1\mathtt{b} \mid A \mid B$$
$$A \to \mathtt{a}A \mid \mathtt{a}$$
$$B \to B\mathtt{b} \mid \mathtt{b}$$

and

$$S_2 \to E\mathtt{ba}E$$
$$E \to EE \mid a \mid b \mid \varepsilon$$

Combining them we get a CFG for the language asked:

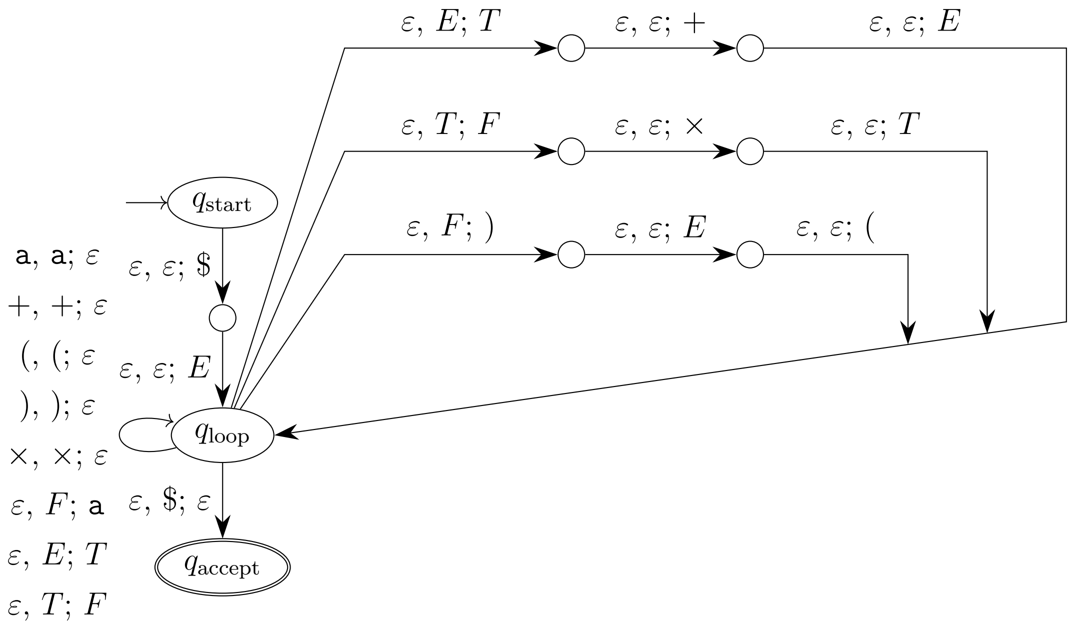
$$S \to S_1 \mid S_2$$
$$S_1 \to \mathtt{a}S_1\mathtt{b} \mid A \mid B$$
$$A \to \mathtt{a}A \mid \mathtt{a}$$

$$B \to B\texttt{b} \mid \texttt{b}$$

$$S_2 \to E\texttt{ba}E$$

$$E \to EE \mid a \mid b \mid \varepsilon$$

Sipser 2.11 Here is the constructed automata:



Sipser 2.14 First, add a start variable:

$$S \to A$$

$$A \to BAB \mid B \mid \varepsilon$$

$$B \to \texttt{00} \mid \varepsilon$$

Second, remove all ε-rules. We remove $B \to \varepsilon$:

$$S \to A$$

$$A \to BAB \mid B \mid AB \mid BA \mid A \mid \varepsilon$$

$$B \to \texttt{00}$$

Also remove $A \to \varepsilon$:

$$S \to A \mid \varepsilon$$

$$A \to BAB \mid B \mid AB \mid BA \mid BB \mid A$$

$$B \to \texttt{00}$$

Third, we handle all unit rules. Remove $A \to A$:

$$
\begin{aligned}
S &\to BAB \mid B \mid AB \mid BA \mid BB \mid \varepsilon \\
A &\to BAB \mid B \mid AB \mid BA \mid BB \\
B &\to \mathsf{00}
\end{aligned}
$$

Also remove $A \to B$:

$$
\begin{aligned}
S &\to BAB \mid B \mid AB \mid BA \mid BB \mid \mathsf{00} \mid \varepsilon \\
A &\to BAB \mid B \mid AB \mid BA \mid BB \mid \mathsf{00} \\
B &\to \mathsf{00}
\end{aligned}
$$

Finally, convert it into Chomsky normal form:

$$
\begin{aligned}
S &\to BC \mid AB \mid BA \mid BB \mid UU \mid \varepsilon \\
A &\to BC \mid AB \mid BA \mid BB \mid UU \\
C &\to AB \\
B &\to UU \\
U &\to \mathsf{0}
\end{aligned}
$$