B62005Y-02 理论计算机科学基础	2017 年 3 月 15 日

Solutions to Homework # 1

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Sipser 1.14 (a) *Proof.* Let M' be the DFA M with the accept and nonaccept states swapped. Suppose M' accepts x; then running M' on x, we end in an accept state of M'. On the other hand, if we run M on x, we would end in a nonaccept state. Therefore $x \notin B$. Similarly, if x is not accepted by M', then it would be accepted by M. So M' accepts exactly those strings not accepted by M, i.e. M' recognizes the complement of B. By the arbitrariness of B we conclude that the class of regular languages is closed under complement. □

(b) Let us consider the example N_1 in the textbook (Fig. 1.27). We can check that the string `101` can be simultaneously accepted by N_1 and N_1'; hence swapping the accept/nonaccept states does not necessarily yield an NFA recognizing the complement of C.

However, the class of languages recognized by NFAs is closed under complement. This follows from the conclusion in part (a) and Theorem 1.39, which implies that the class of languages recognized by NFAs is precisely the class of languages recognized by DFAs.

Sipser 1.16 (b) To construct an equivalent DFA D, we first determine D's state set:

$$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}.$$

Next, calculate the ε-closures. For each $R \subseteq Q$,

$$E(R) = \{q \mid q \text{ can be reached from } R \text{ by travelling along 0 or more } \varepsilon \text{ arrows}\},$$

$$E(\emptyset) = \emptyset, E(\{1\}) = \{1,2\}, E(\{2\}) = \{2\}, E(\{3\}) = \{3\},$$

$$E(\{1,2\}) = \{1,2\}, E(\{1,3\}) = \{1,2,3\}, E(\{2,3\}) = \{2,3\}, E(\{1,2,3\}) = \{1,2,3\}.$$

For each $R \in Q'$ and $a \in \Sigma$, $\delta'(R,a) = \bigcup_{r\in R} E(\delta(r,a))$:

$$\delta'(\emptyset, a) = \delta'(\emptyset, b) = \emptyset,$$

$$\delta'(\{1\}, a) = \{3\},\ \delta'(\{1\}, b) = \emptyset,$$

$$\delta'(\{2\}, a) = \{1, 2\},\ \delta'(\{2\}, b) = \emptyset,$$

$$\delta'(\{3\}, a) = \{2\},\ \delta'(\{3\}, b) = \{2, 3\},$$

$$\delta'(\{1, 2\}, a) = \{1, 2, 3\},\ \delta'(\{1, 2\}, b) = \emptyset,$$

$$\delta'(\{1, 3\}, a) = \{2, 3\},\ \delta'(\{1, 3\}, b) = \{2, 3\},$$

$$\delta'(\{2, 3\}, a) = \{1, 2\},\ \delta'(\{2, 3\}, b) = \{2, 3\},$$

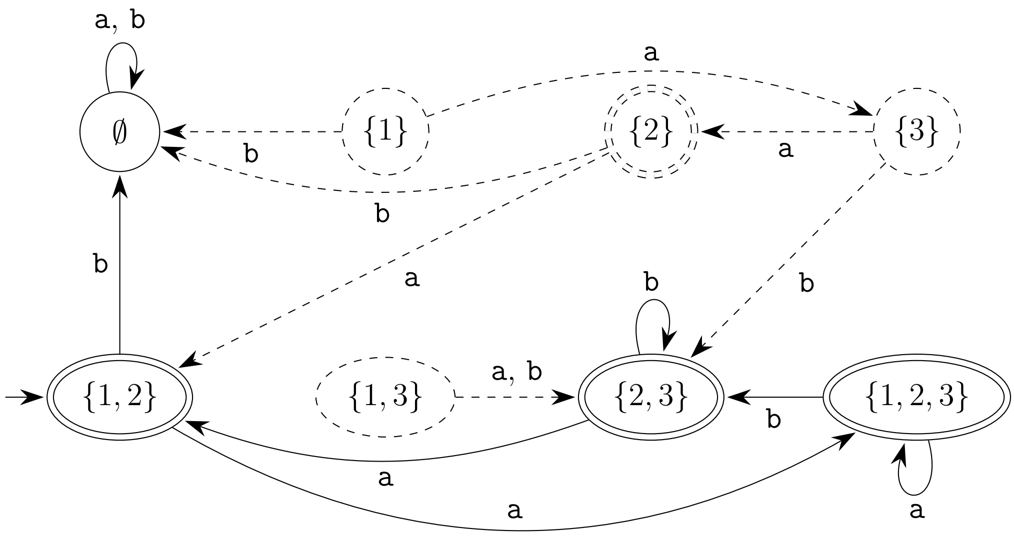
$$\delta'(\{1, 2, 3\}, a) = \{1, 2, 3\},\ \delta'(\{1, 2, 3\}, b) = \{2, 3\}.$$

The new start and accept states are:

$$q_0' = E(q_0) = E(\{1\}) = \{1, 2\},$$

$$F' = \{R \in Q' \mid R \text{ contains an accept state of } N\} = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}.$$

The DFA D we obtain is shown in the following figure. (The minimal DFA $D_{\min}$ is D with the dashed states and transitions removed.)

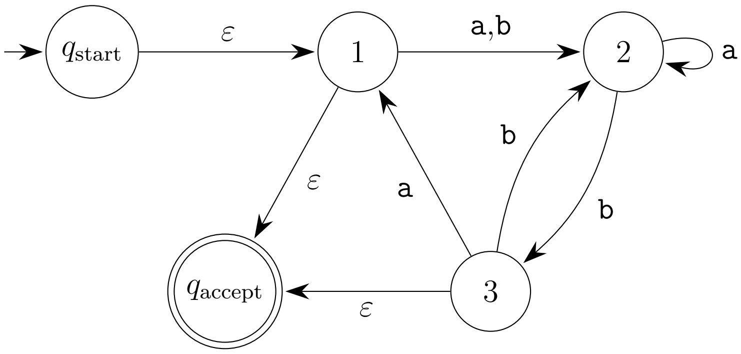


Sipser 1.21 (b) The answer is not unique; for example, these expressions are equally valid:

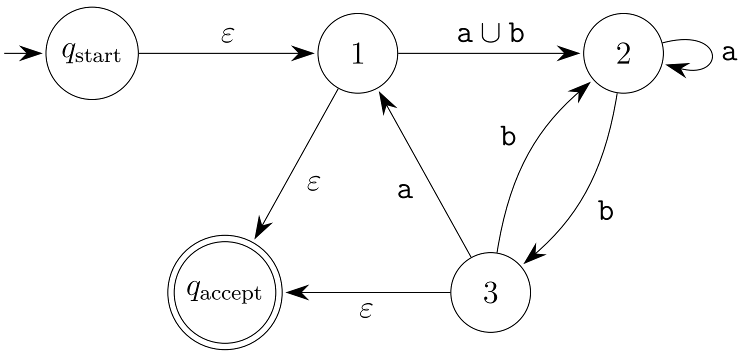
- $((\texttt{a} \cup \texttt{b})\texttt{a}^*\texttt{b}(\texttt{ba}^*\texttt{b})^*\texttt{a})^*(\varepsilon \cup ((\texttt{a} \cup \texttt{b})\texttt{a}^*\texttt{b}(\texttt{ba}^*\texttt{b})^*))$,
- $((\texttt{a} \cup \texttt{b})\texttt{a}^*\texttt{bba}^*\texttt{a})^*((\texttt{a} \cup \texttt{b})\texttt{ba}^*\texttt{b} \cup \varepsilon)$,
- $\varepsilon \cup ((\texttt{a} \cup \texttt{b})\texttt{a}^*\texttt{b}((\texttt{a}(\texttt{a} \cup \texttt{b}) \cup \texttt{b})\texttt{a}^*\texttt{b})^*(\varepsilon \cup \texttt{a}))$,
- $((\texttt{a} \cup \texttt{b})(\texttt{a} \cup \texttt{bb})^*\texttt{ab})^*(\varepsilon \cup (\texttt{a} \cup \texttt{b})(\texttt{a} \cup \texttt{bb})^*(\texttt{b} \cup \texttt{ab}))$.

We give a detailed construction for the third representation:

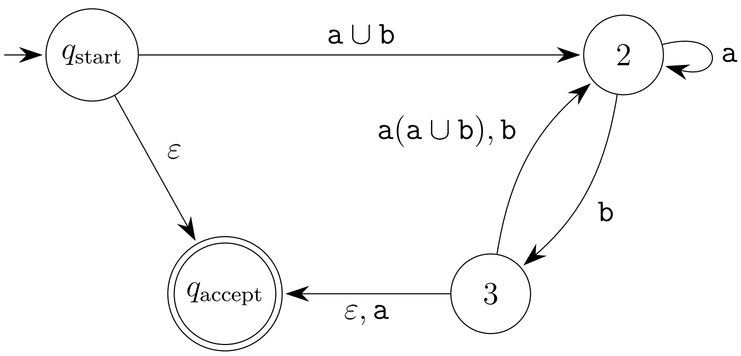
1. Add a new start state and accept state and necessary ε-transitions; make original final states non-final.



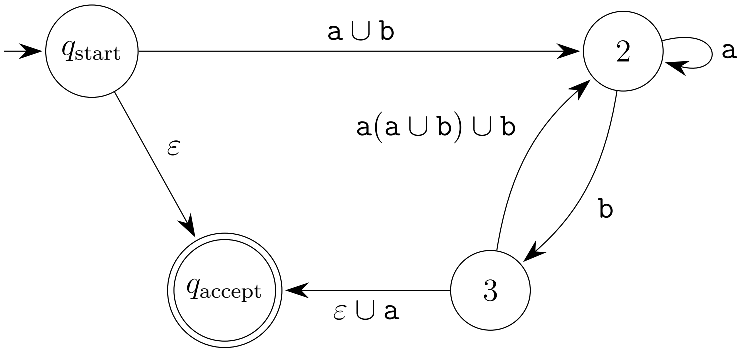
2. Perform union on the edge from state 1 to state 2.



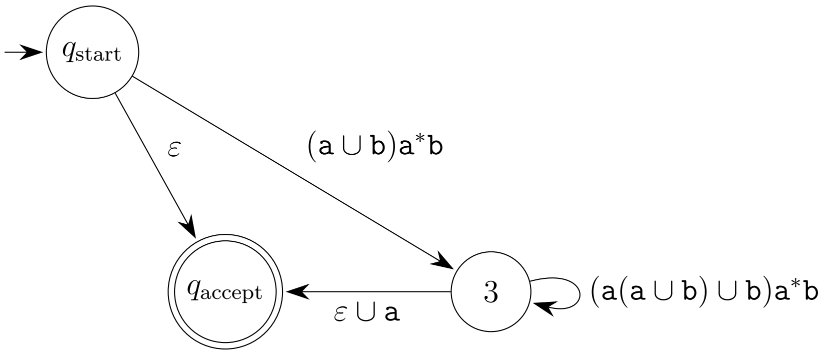
3. Remove state 1.



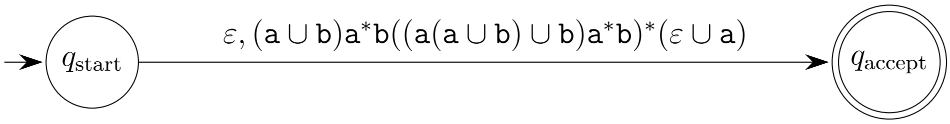
4. Perform union on edges.



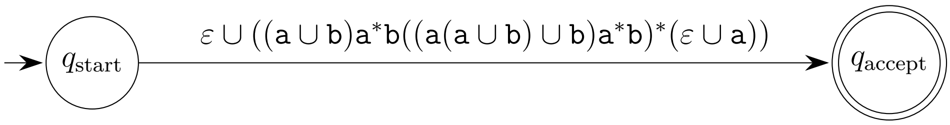
5. Remove state 2.



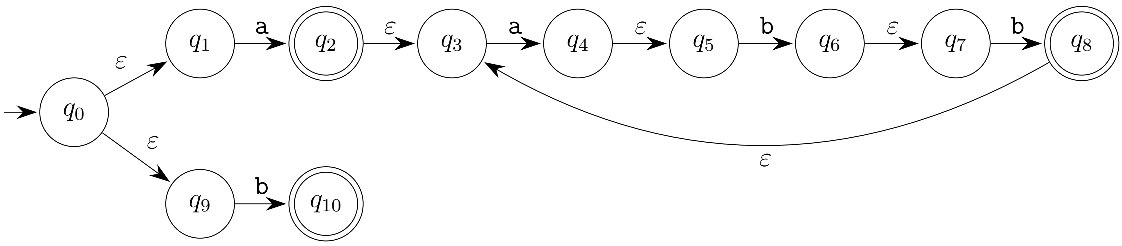
6. Remove state 3.



7. Perform union on edges.



Sipser 1.28 (a) The NFA is as follows:



Sipser 1.46 [1](c) Suppose that the language $L = \{w \mid w \in \{\mathtt{0},\mathtt{1}\}^* \text{ is not a palindrome}\}$ is regular, then $\bar{L} = \{w \mid w \in \{\mathtt{0},\mathtt{1}\}^* \text{ is a palindrome}\}$ must be regular. Let p be the pumping length for $\bar{L}$; we choose the string s to be $\mathtt{0}^p\mathtt{1}\mathtt{0}^p$, then the pumping lemma implies that $s = xyz$ with $|y| > 0$ and $|xy| \leq p$, which means that $y = \mathtt{0}^k$, where $1 \leq k \leq p$. Since $p - k < p$, $xy^0z = \mathtt{0}^{p-k}\mathtt{1}\mathtt{0}^p$ cannot be in the language $\bar{L}$, a contradiction. Therefore L is not a regular language. □

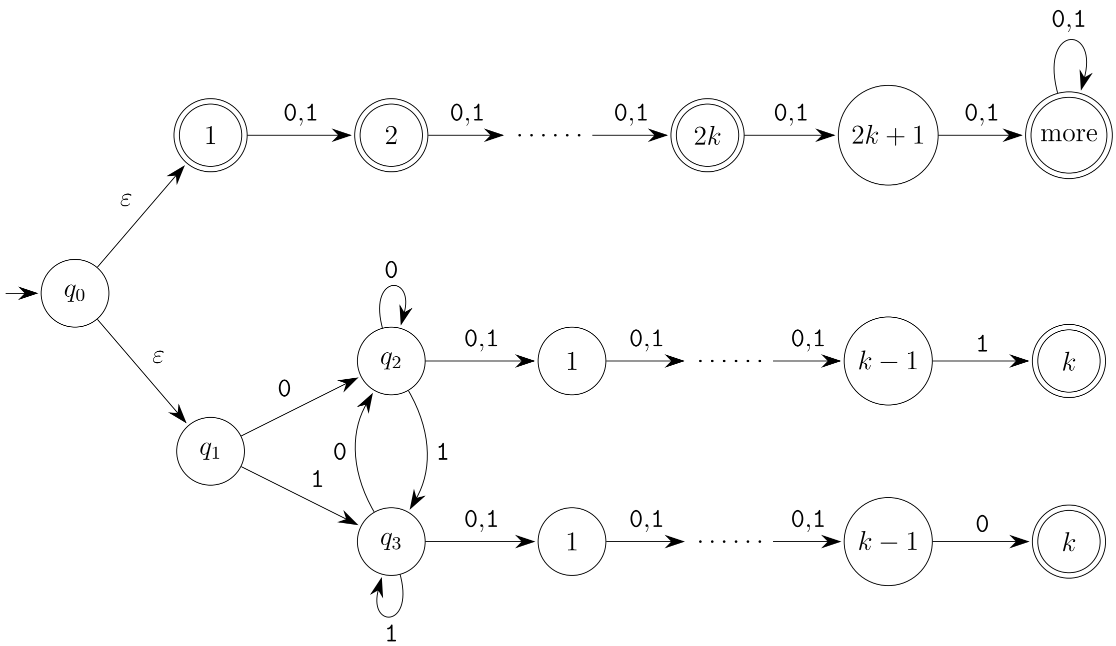
Sisper 1.71 (a) Indeed, the language A can be written as the regular expression $\mathtt{0}(\mathtt{0} \cup \mathtt{1})^*\mathtt{0}$. First, for each $k \geq 1$, $\mathtt{0}^k u \mathtt{0}^k$ is in $\mathtt{0}(\mathtt{0} \cup \mathtt{1})^*\mathtt{0}$, so $A \subseteq \mathtt{0}(\mathtt{0} \cup \mathtt{1})^*\mathtt{0}$; moreover, for any string w in $\mathtt{0}(\mathtt{0}\cup\mathtt{1})^*\mathtt{0}$, $w = \mathtt{0}^1 u \mathtt{0}^1$, so $\mathtt{0}(\mathtt{0}\cup\mathtt{1})^*\mathtt{0} \subseteq A$. Therefore $A = \mathtt{0}(\mathtt{0}\cup\mathtt{1})^*\mathtt{0}$. □

(b) Assume that B is a regular language and let p be its pumping length. We choose the string s to be $\mathtt{0}^p\mathtt{1}\mathtt{1}\mathtt{0}^p$, which is in B and of length $2p + 2 \geq p$. Thus $s = xyz$ where $|y| > 0$ and $|xy| \leq p$, which implies that $y = \mathtt{0}^k$, where $k \geq 1$. Then $xy^2z = \mathtt{0}^{p+k}\mathtt{1}\mathtt{1}\mathtt{0}^p$ is not in the language B, a contradiction. Therefore B is not a regular language. □

Sisper 1.69 (b) A string in the language $\overline{WW_k}$ satisfies one of the following conditions:

- either $|s| \neq 2k$, or
- $s = xy$, where $|x| = |y| = k$ but $x \neq y$.

Therefore we can construct the following NFA:



[1] These questions are labeled as Problem 1.51, 1.68, and 1.73 in the Chinese version of the book.