VARIABEL RANDOM KONTINU DAN DISTRIBUSI PROBABILITASNYA

Oleh : Tim Dosen Pengantar Statistika

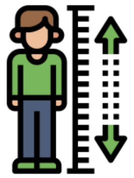
VARIABEL ACAK KONTINU

Variabel tidak dapat dihitung (***uncountable***) atau nilainya adalah **semua nilai dalam suatu interval** (*measurement*)

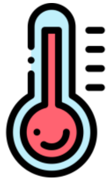
Contoh :

Coba sebutkan yang lainnya

Berat badan



tinggi badan

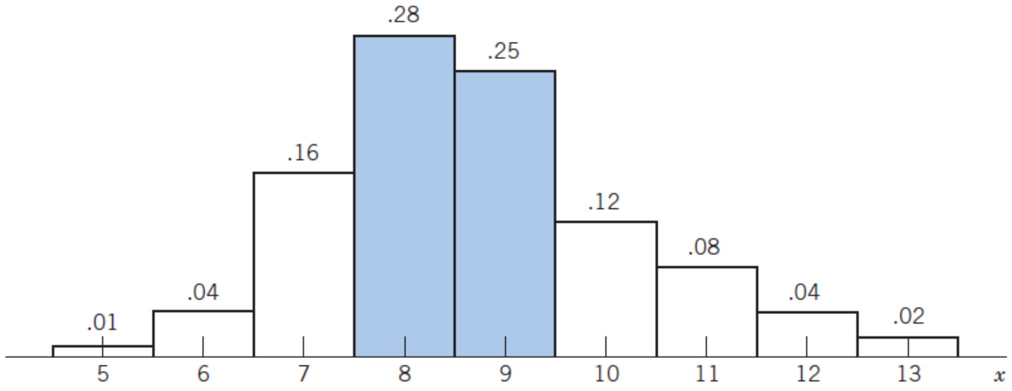


Suhu

Ciri histogram

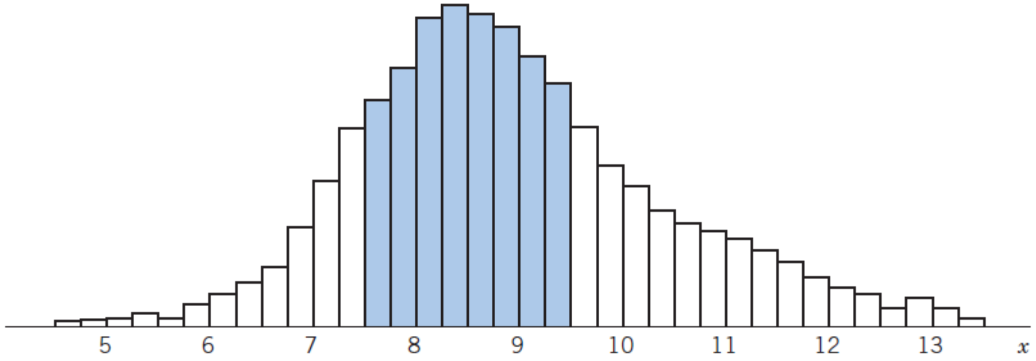
- Total area di bawah kurva adalah 1
- Relative frequency dari interval a dan b adalah area di bawah histogram antara titik a dan b

100 pengukuran



(a)

5000 pengukuran



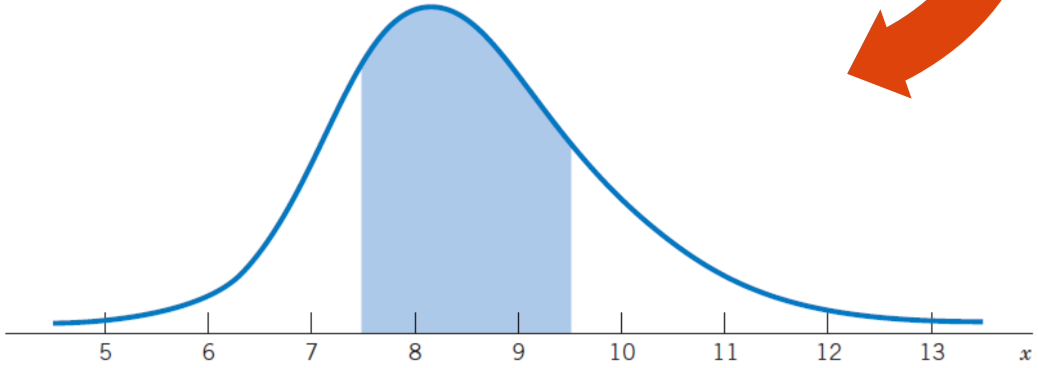
(b)

Kelas interval lebih sempit dan jumlah observasi lebih besar

Frekuensi relative jangka Panjang

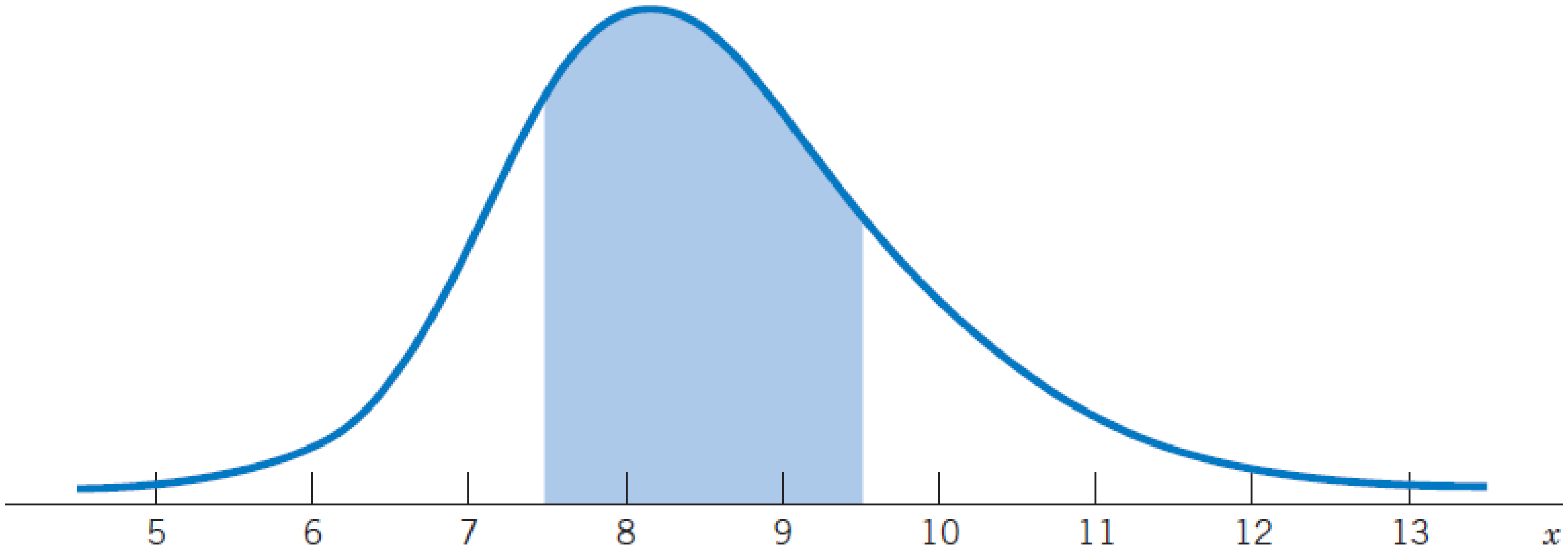
Kelas interval semakin sempit dengan jumlah observasi yang semakin besar

Kurva Densitas Probabilitas



(c)

Kurva Densitas Probabilitas



***Probability Density Function* (PDF)**
$f(x)$

Nilai Variabel Acak X

Fungsi Densitas Probabilitas (PDF)

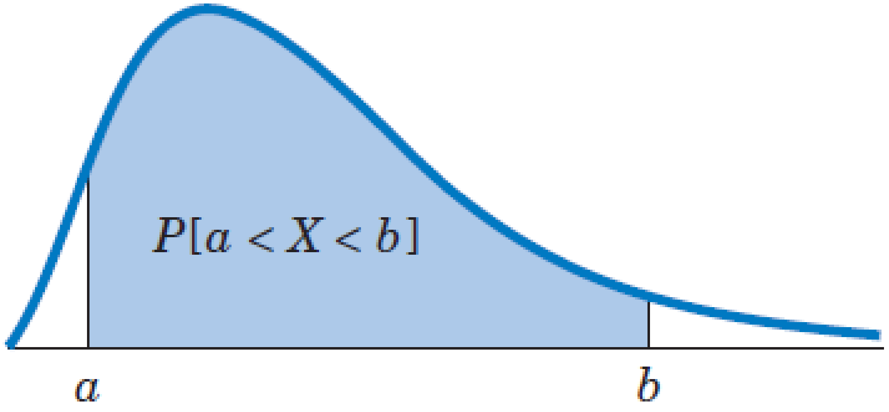
Menggambarkan distribusi probabilitas untuk suatu variabel acak kontinu

Sifatnya adalah:

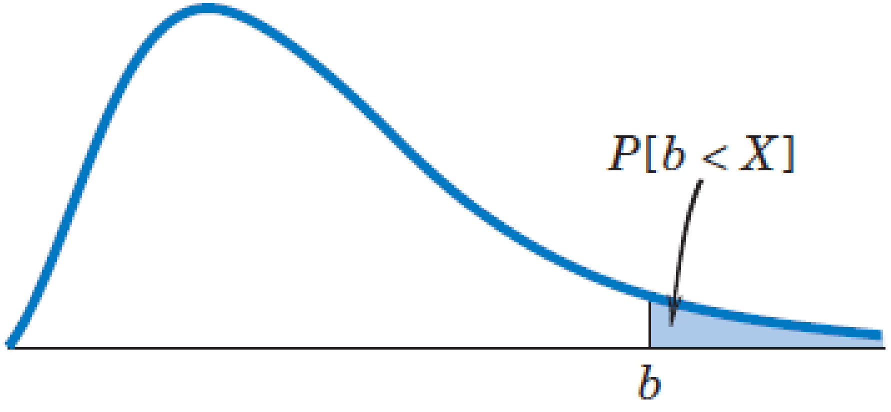
1. Total area di bawah kurva probabilitas distribusi adalah 1.
2. $P[a \leq X \leq b]$ adalah area di bawah kurva probabilitas antara a dan b
3. $f(x) \geq 0$ untuk semua x

- Pada variabel acak kontinu,
$$P(X = x) = 0$$
- PDF menunjukkan probabilitas suatu interval $[a, b]$ yang adalah area di bawah kurva batas a dan b

$$P[a \leq X \leq b] = P[a < X \leq b] = P[a \leq X < b] = P[a < X < b]$$



$$P[a < X < b] = (\text{area kiri } b) - (\text{area kiri } a)$$
$$= P(X < b) - P(X < a)$$
$$= F(b) - F(a)$$



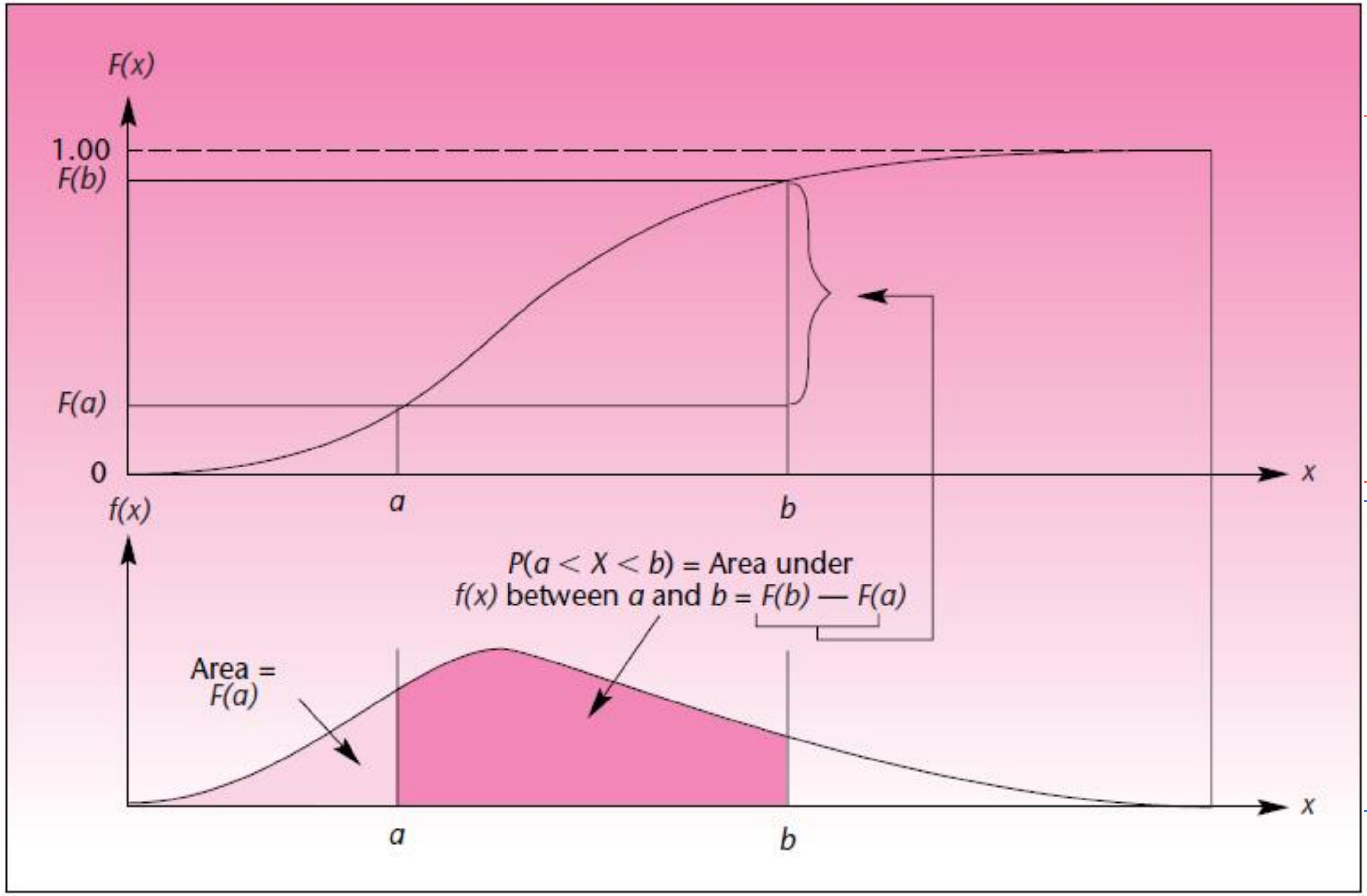
$$P[b < X] = P[X > b] = 1 - (\text{area kiri } b)$$
$$= 1 - P(X < b)$$
$$= 1 - F(b)$$

Keterangan

$F(x) = P(X \leq x)$ adalah area di bawah kurva $f(x)$ yang berada di antara nilai x terkecil yang mungkin (pada umumnya $-\infty$) dan titik x

Cumulative Distribution Function (CDF)

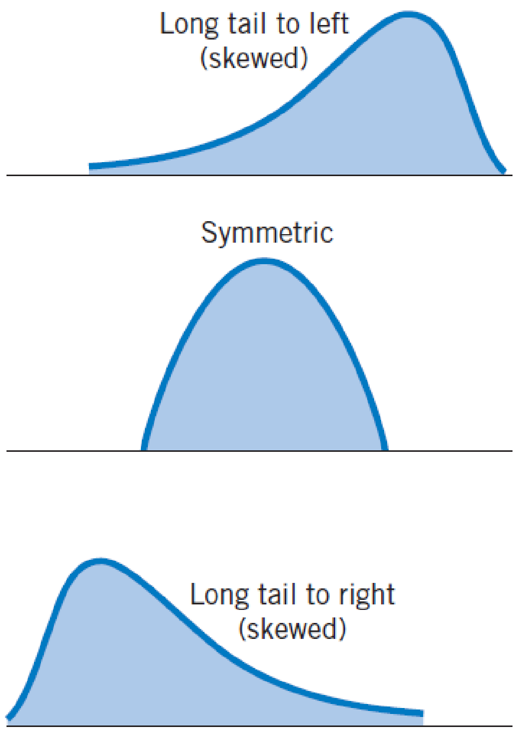
Hubungan Antara PDF dengan CDF



CDF

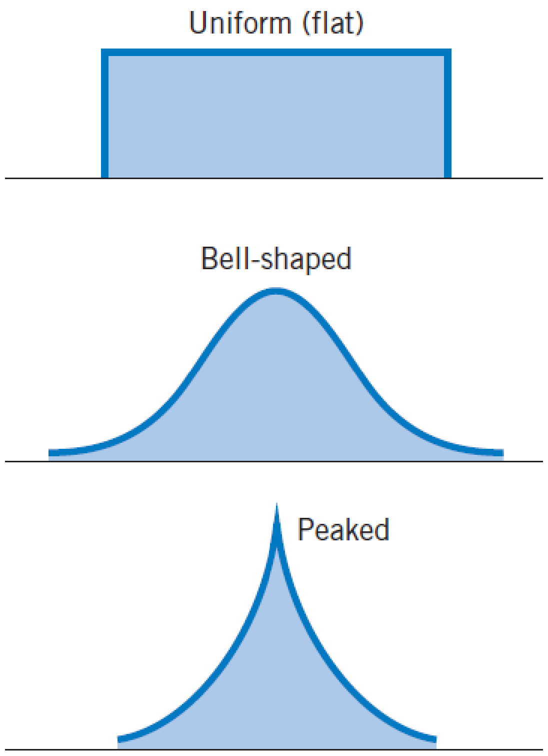
PDF

Bentuk Kurva Densitas Probabilitas



A

Symmetry and Deviations from symmetry



B

Different Peakedness

Ukuran Pemusatan dan Penyebaran Distribusi Probabilitas

Mean = Expected Value = $E(X) = \mu$
Standard Deviation = σ

Nilai μ dan σ tergantung dari distribusi probabilitasnya

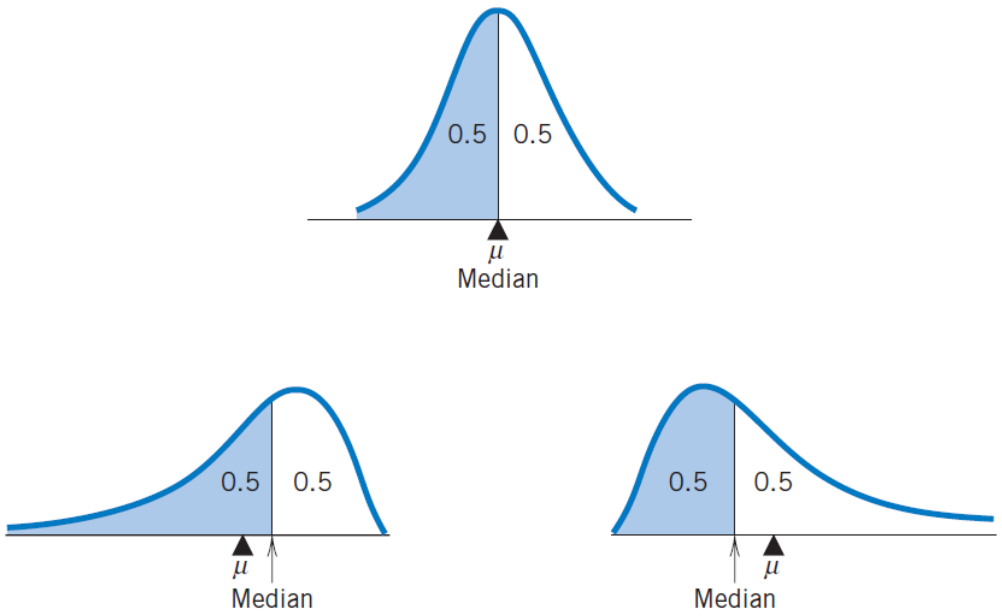


Figure 3 Mean as the balance point and median as the point of equal division of the probability mass.

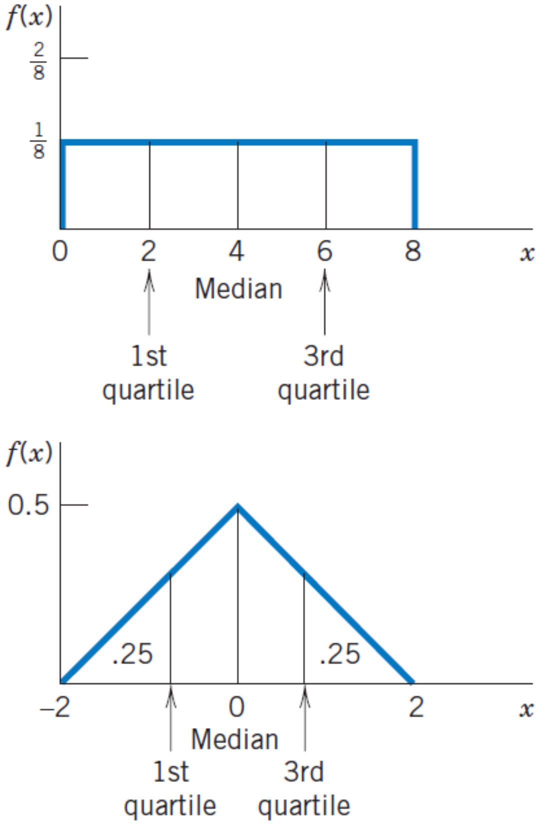


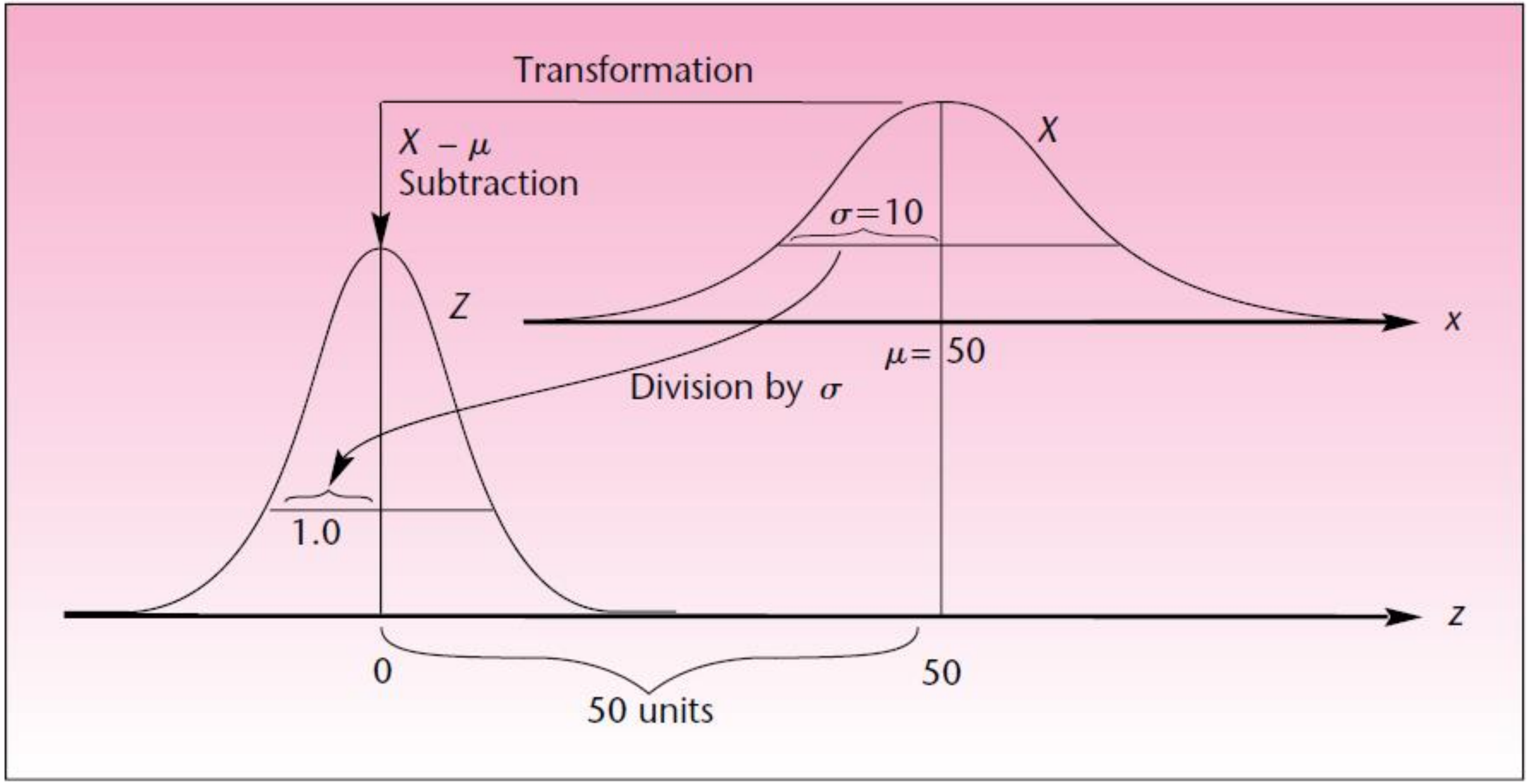
Figure 4 Quartiles of two continuous distributions.

Variabel yang distandarisasi

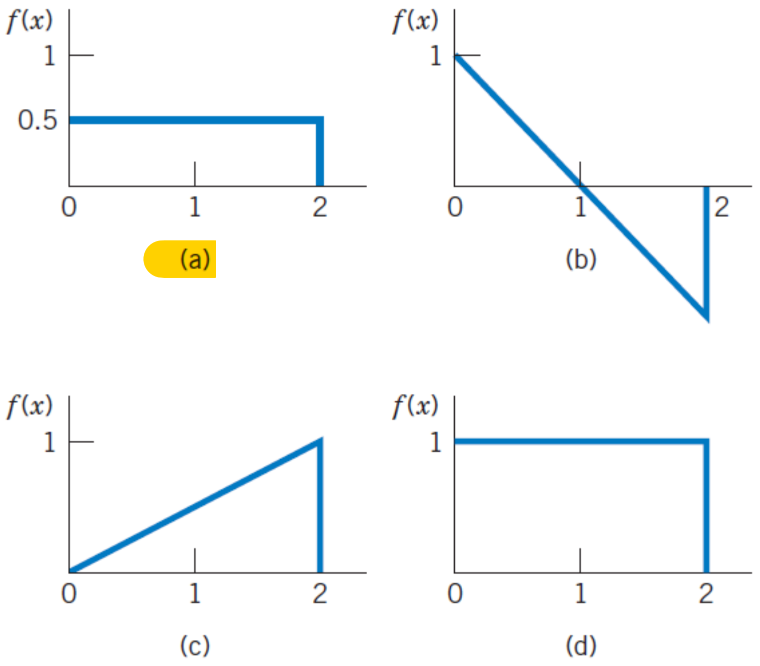
(Standardized Variable)

$$Z = \frac{X - \mu}{\sigma}$$

Mempunyai mean 0 dan standard deviation 1



Latihan Soal



1

Which of the functions sketched above could be a probability density function for a continuous random variable? Why or why not? a c d

2

Determine the following probabilities from the curve f (x) diagrammed in Exercise 1(a).

a. $P[\,0<X<0.5]$
b. $P[\,0.5<X<1]$
c. $P[1.5<X<2]$
d. $P[\,X<1\,]$

3

For the curve f (x) graphed in Exercise 1(c), which of the two intervals $[0<X<0.5]$ or $[1.5<X<2]$ is assigned a higher probability?

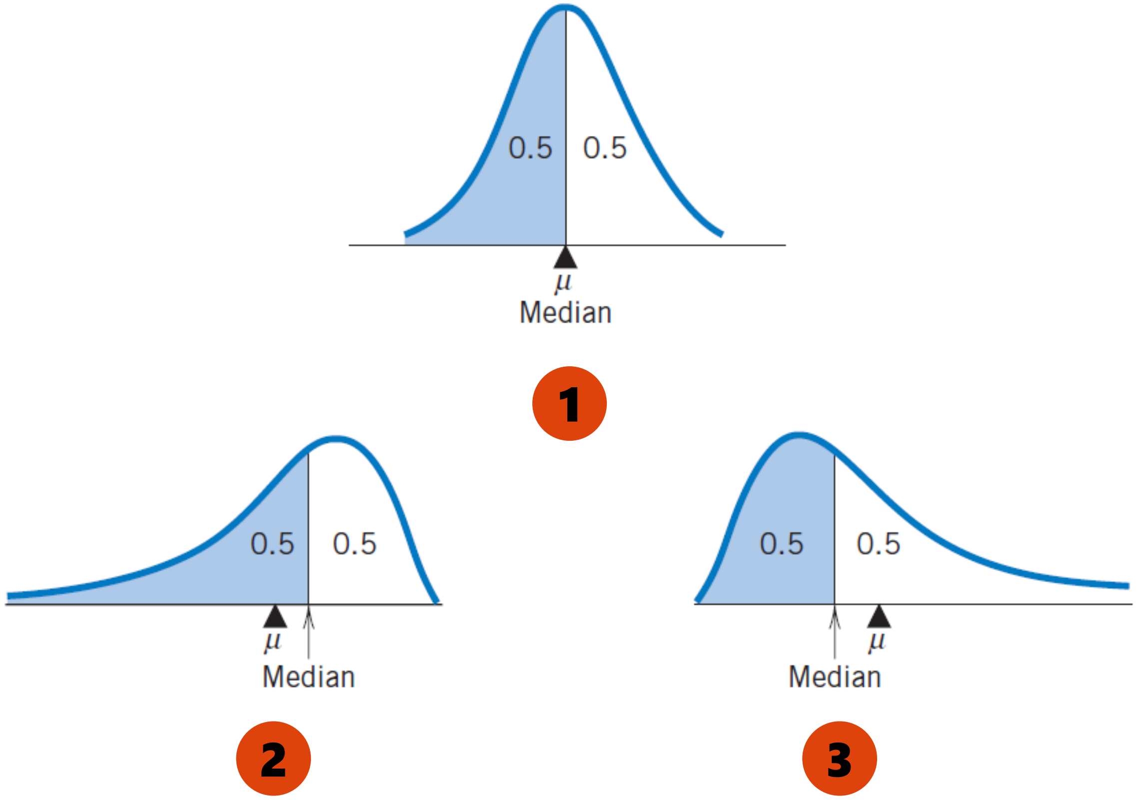
4

Determine the median and the quartiles for the probability distribution depicted in Exercise 1(a)

5

Determine the 15th percentile of the curve in Exercise 1(a).

Latihan Soal

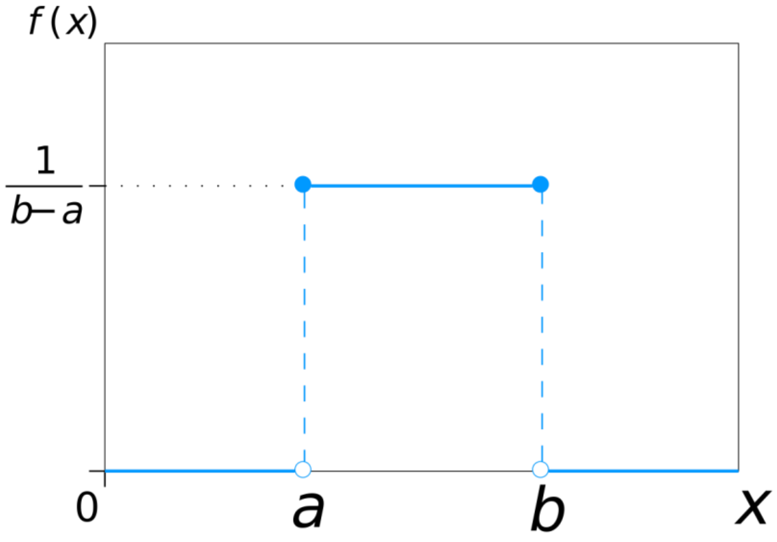


Which of the distributions in Figure are compatible with the following statements?

A The first test was too easy because over half the class scored above the mean. 2/3

B In spite of recent large increases in salary, half of the professional football players still make less than the average salary 1

Distribusi Uniform



$X \sim U(a, b)$

PDF

$$f(x) = \frac{1}{(b-a)} \quad \text{untuk } a \leq x \leq b$$

$$= 0 \quad \text{untuk } x \text{ lainnya}$$

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

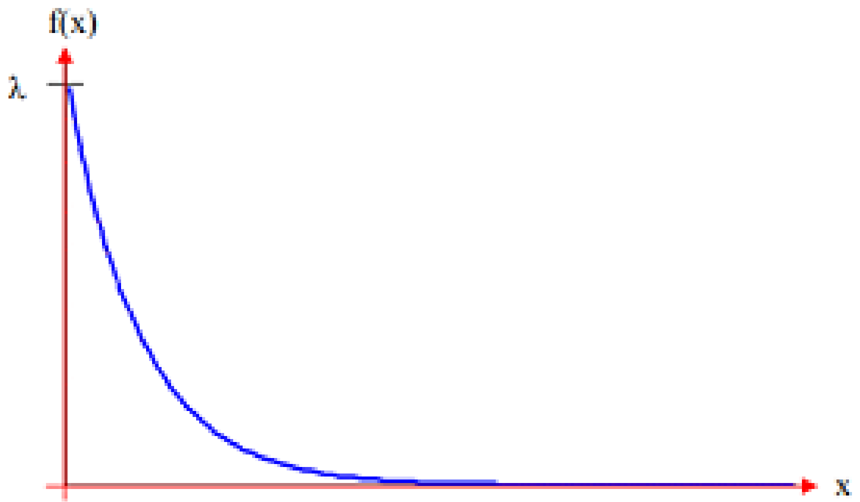
dimana $a \leq x_1 \leq x_2 \leq b$

Expected Value & Variance

$$E(X) = \frac{a+b}{2}$$

$$V(X) = \frac{(b-a)^2}{12}$$

Distribusi Eksponensial



X~E(λ)

PDF:

$f(x) = \lambda e^{-\lambda x};\ untuk\ x \geq 0$

$= 0$; untuk lainnya

- $P(X \leq x) = 1 - e^{-\lambda x}\ \ untuk\ x \geq 0$
- $P(X \geq x) = e^{-\lambda x}\ \ untuk\ x \geq 0$
- $P(x_1 \leq X \leq x_2) = e^{-\lambda x_1} - e^{-\lambda x_2}$
  untuk $0 \leq x_1 < x_2$

Expected Value & Variance

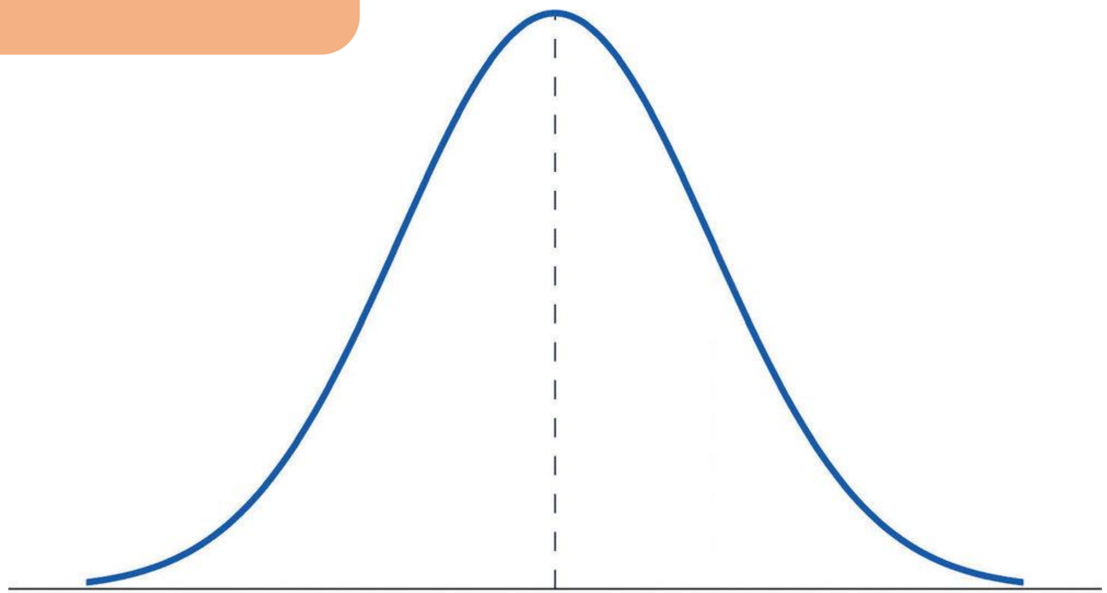
$$E(X) = \frac{1}{\lambda}$$

$$V(X) = \frac{1}{\lambda^2}$$

Distribusi Normal

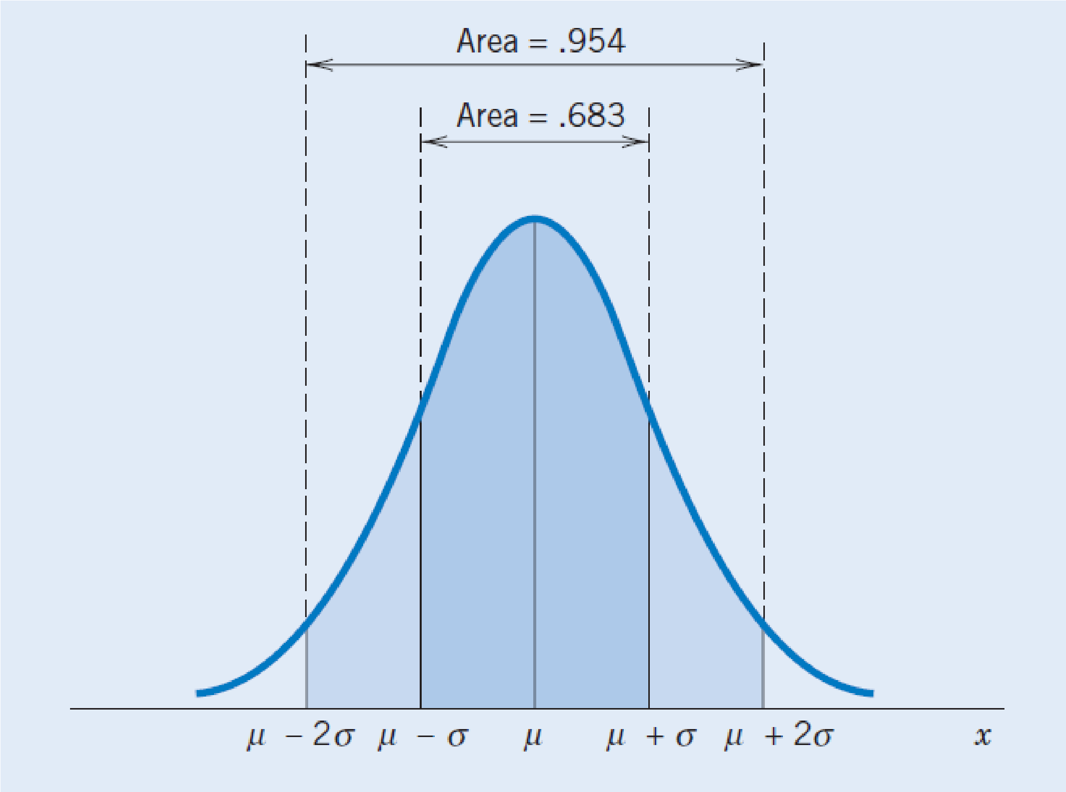
- Pada umumnya, data diharapkan mempunyai distribusi normal. Jika tidak, maka data tersebut diragukan.
- Tetapi pada kenyataannya, banyak data yang tidak berdistribusi normal.
- Meskipun begitu, distribusi normal sangat diperlukan dalam statistika terutama dalam prosedur inferensia

- Berbentuk seperti lonceng
- Simetris



$$X \sim N(\mu, \sigma)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \text{ untuk } -\infty < x < \infty$$



The probability of the interval extending

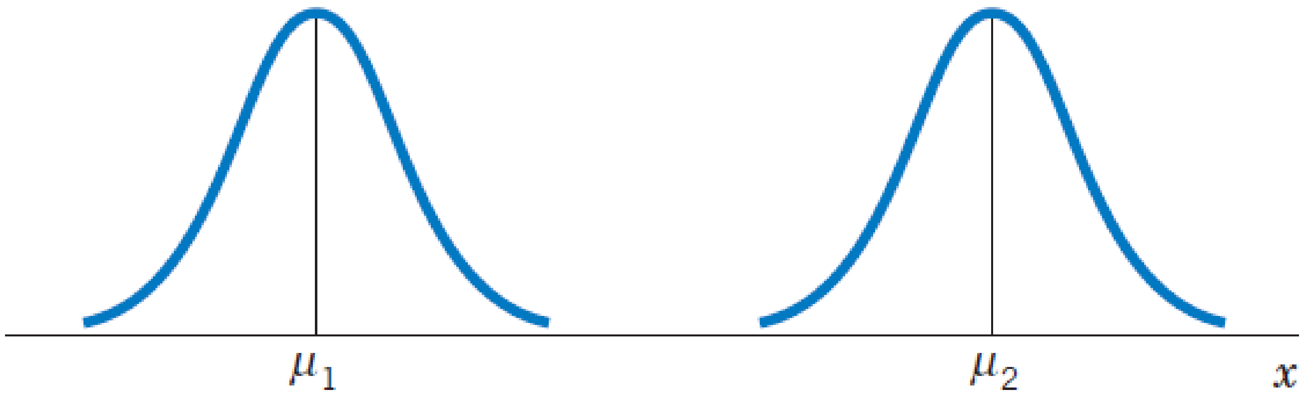
One sd on each side of the mean: $P[\mu - \sigma < X < \mu + \sigma] = .683$

Two sd on each side of the mean: $P[\mu - 2\sigma < X < \mu + 2\sigma] = .954$

Three sd on each side of the mean: $P[\mu - 3\sigma < X < \mu + 3\sigma] = .997$

Parameter Distribusi Normal

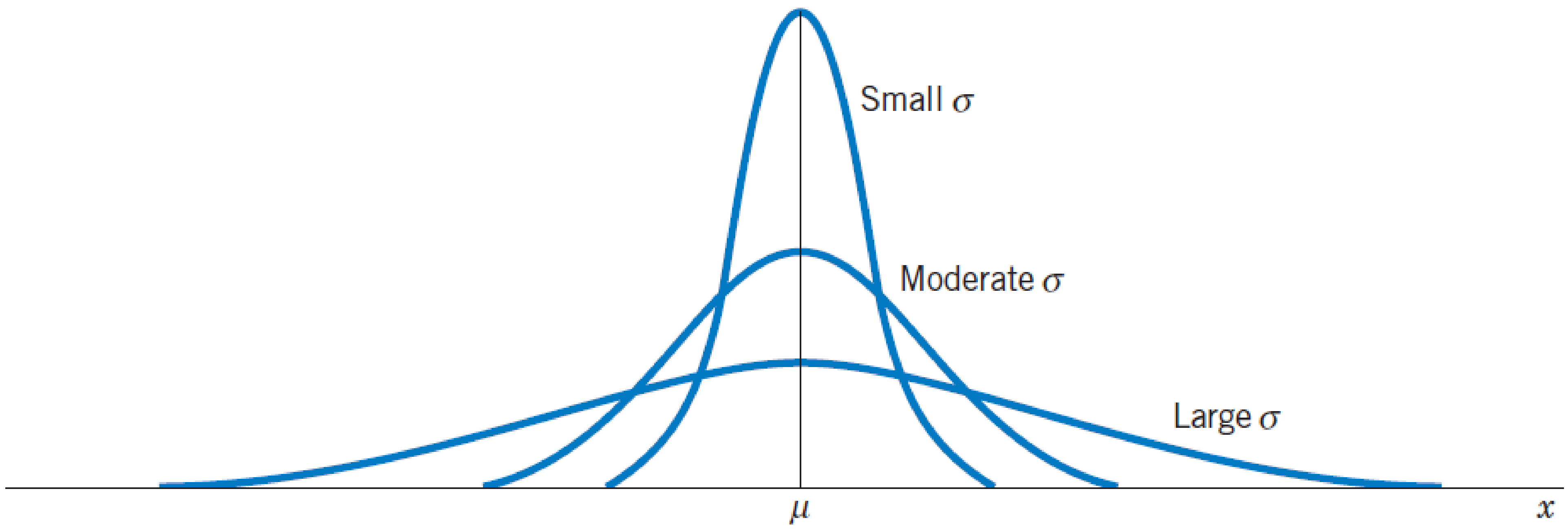
Parameter distribusi Normal adalah μ dan σ.
Mean μ menyatakan lokasi.



μ_1 lebih kecil dibandingkan dengan μ_2 tetapi mempunyai deviasi standar yang sama

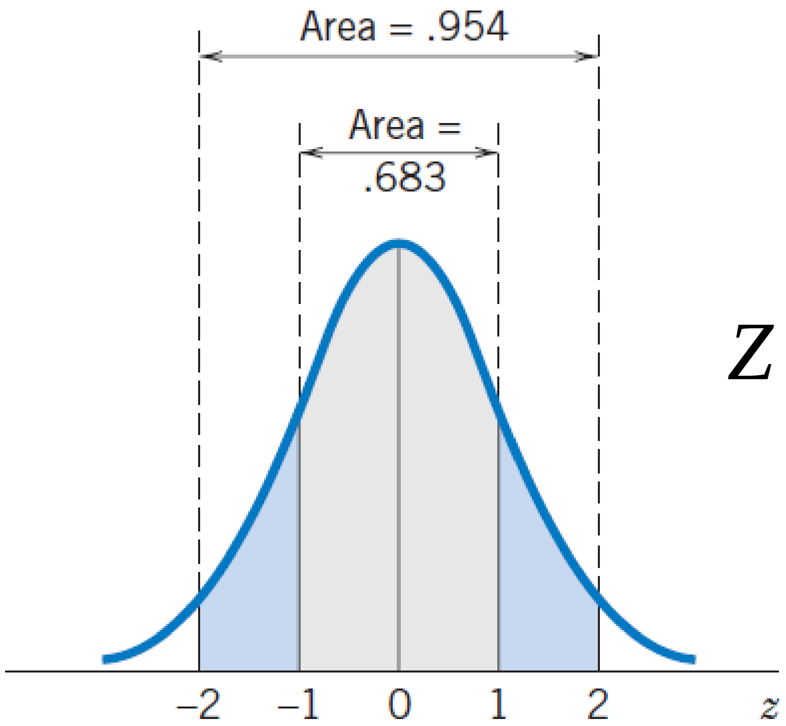
Parameter Distribusi Normal

Deviasi Standar menyatakan sebaran data

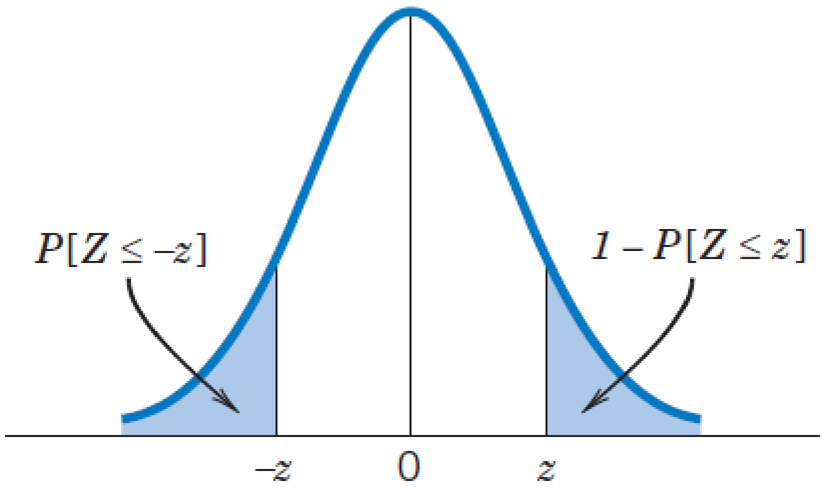


Distribusi Normal Standar

Distribusi normal yang mempunyai mean 0 dan deviasi standar 1



$$Z \sim N(0,1)$$



$P[Z \leq z]$ = area di bawah kurva dari titik z ke kiri

$P[a \leq Z \leq b]$ = area kiri b – area kiri a

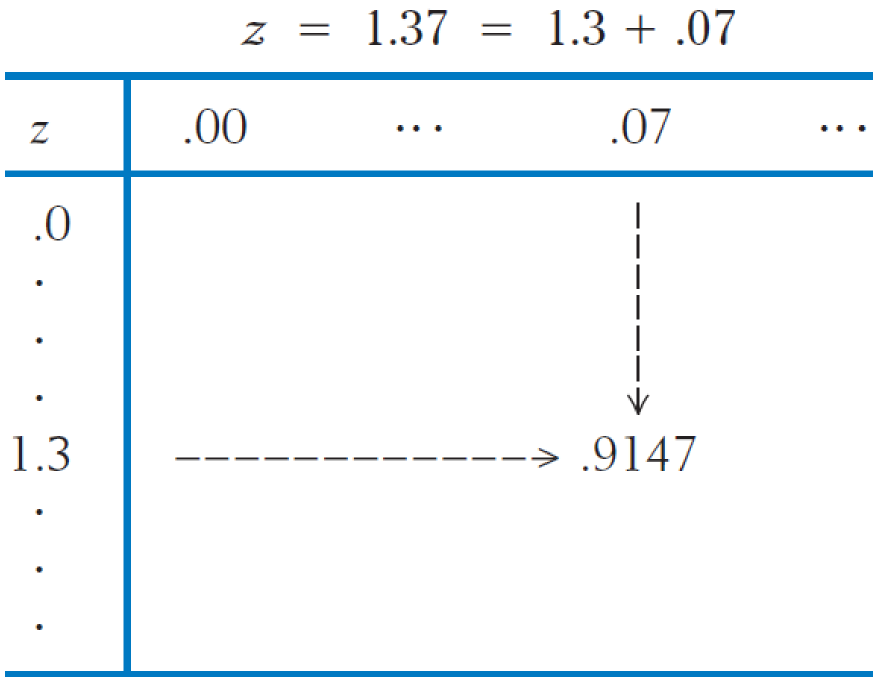
$P[Z \leq 0] = 0{,}5$

$P[Z \leq -z] = 1 - P[Z \leq z] = P[Z \geq z]$

https://www.mathsisfun.com/data/standard-normal-distribution-table.html

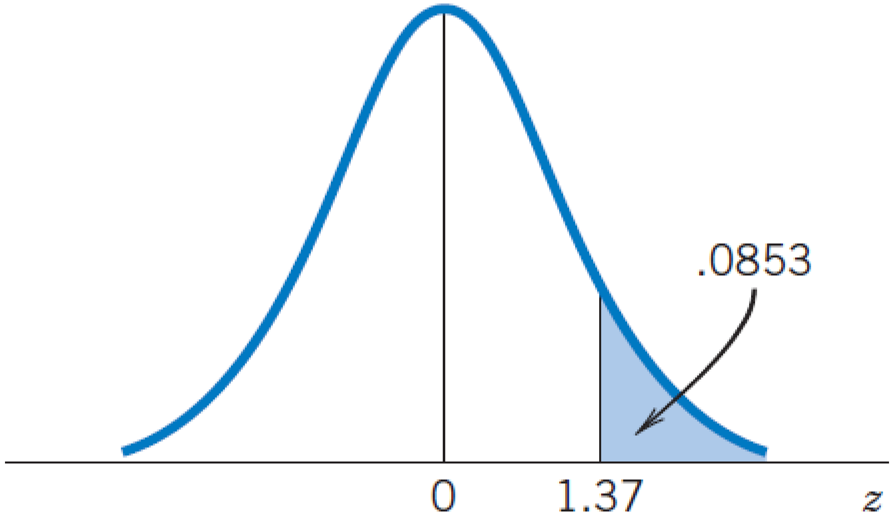
CONTOH

a. Temukan $P[Z \leq 1{,}37]$



Dengan tabel distribusi Normal:
$P[Z \leq 1{,}37] = 0{,}9147$

b. Temukan $P[Z > 1{,}37]$



$$\begin{aligned} P[Z > 1{,}37] &= 1 - P[Z \leq 1{,}37] \\ &= 1 - 0{,}9147 \\ &= 0{,}0853 \end{aligned}$$

LATIHAN

1. Hitung $P[-0{,}155 < Z < 1{,}60]$
2. Hitung $P[\mathrm{Z} < -1{,}9\ atau\ Z > 2{,}1]$
3. Jika $P[\ Z > z] = 0{,}025$, maka berapakah z?
4. Jika $P[-z\ \leq Z \leq z] = 0{,}90$, maka berapakah z?

Penghitungan Probabilitas dengan Distribusi Normal

Jika X berdistribusi $N(\mu, \sigma)$, maka variabel tersebut dapat distandarisasi dengan cara

Transformasi X ke Z

$$Z = \frac{X - \mu}{\sigma}$$

sehingga Z mempunyai distribusi **normal standar**

CONTOH

Seseorang mencatat kalori pada makan siangnya setiap hari selama satu bulan. Kemudian diketahui bahwa data kalori tersebut berdistribusi normal dengan rata-rata 200 dan deviasi standar 5. Berapakah probabilitas makan siang tersebut mempunyai kalori sebanyak:

a. Lebih dari 208 kalori
b. Antara 190 dengan 200 kalori

JAWAB

Misal X adalah banyak kalori pada makan siang tersebut, maka variabel yang terstandarisasi adalah

$$Z = \frac{X - 200}{5}$$

a. Nilai z dari $x = 208$ adalah

$$z = \frac{208 - 200}{5} = 1,6$$

maka,

$$\begin{aligned} P(X > 208) &= P(Z > 1,6) \\ &= 1 - P(Z \leq 1,6) \\ &= 1 - 0,9452 = 0,0548 \end{aligned}$$

b. Nilai z masing-masing untuk $x =$ 190 dan $x = 200$ adalah

$$\frac{190 - 200}{5} = -2 \quad \text{dan} \quad \frac{200 - 200}{5} = 0$$

maka,

$$\begin{aligned} P(190 \leq X \leq 200) &= P(-2 \leq Z \leq 0) \\ &= 0,5 - 0,0228 \\ &= 0,4772 \end{aligned}$$

DAFTAR PUSTAKA

Aczel, A. and Sounderpandian, J., 2008, *Complete Business Statistics*, 7th *Edition*, McGraw-Hill/Irwin, USA.

Richard, A.J. and Bhattacharyya, G.K., 2010, *Statistics: Principles and Methods*, 6th *Edition*, John Wiley and Sons, USA.

THANK YOU