ICCS200: Assignment 3

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The date

1: Tail sum of Squares

Given Program :

int sumHelper(int n , int a) {

if (n==0) return a;

else return sumHelper(n-1, a + n*n); }

int sumSqr(int n) {

return sumHelper(n, 0); }

Prove for $n \geq 1$, sumSqr(n) $\hookrightarrow 1^2 + 2^2 + 3^2 + ... + n^2$.

Predicate:

P(n) $\equiv \forall$ a, sumHelper(n,a) $\hookrightarrow$ a+ $\frac{n(2n+1)(n+1)}{6}$

Base Case:

P(1) $\equiv$ sumHelper(1,0) $\hookrightarrow$ 1 = 1^2

Inductive Step:

Assume $\forall$ k <n P(k) is true.

sumHelper(n-1,a) $\hookrightarrow a + 1^2 + 2^2 + 3^2 + ... + (n-1)^2$.

sumHelper(n,a) $\hookrightarrow a + 1^2 + 2^2 + 3^2 + ... + (n-1)^2 + n^2$.

We know that a+$1^2 + 2^2 + 3^2 + ... + (n-1)^2$ = a+$\frac{n(n-1)(2n-1)}{6}$.

Therefore a+$1^2 + 2^2 + 3^2 + ... + (n-1)^2 + n^2$ = a+$\frac{n(n-1)(2n-1)}{6}$ + n^2 . By Inductive Hypothesis.

$\Rightarrow$ a+$1^2 + 2^2 + 3^2 + ... + (n-1)^2 + n^2$

=a+ $\frac{n(n-1)(2n-1)+6n^2}{6}$

= a+$\frac{2n^3+3n^2+n}{6}$

= a+$\frac{n(2n^2+3n+1)}{6}$

= a+$\frac{n(2n+1)(n+1)}{6}$

Therefore proved

2: Mysterious Function

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Predicate :

P(n) $\equiv \forall n \geq 1, foo(n) \hookrightarrow (p, q),\ \frac{p}{q} = 1 - \frac{1}{n+1}$

Base case :

P(1) $\equiv foo(1) \hookrightarrow (1, 2)$ such that $\frac{1}{2} = 1 - \frac{1}{2}$ which is true

Inductive Step:

Assume P(n-1) is true.

P(n-1) $\equiv foo(n-1) \hookrightarrow (p, q), \frac{p}{q} = 1 - \frac{1}{n} = \frac{n-1}{n}$

Therefore p = n-1 and q = n

By Inductive Hypothesis

$foo(n) \hookrightarrow (q + p * n * (n+1), q * n * (n+1))$

$= (n + (n-1) * n * (n+1), n * n * (n+1))$

$= n + (n^2 - n) * (n+1), n^3 + n^2$

$= n + n^3 - n^2 + n^2 - n, n^3 + n^2$

$= n^3, n^3 + n^2$

for P(n),$p = n^3$ and $q = n^3 + n^2$

$foo(n) \hookrightarrow \frac{n^3}{n^3+n^2}$

$= \frac{n^2(n)}{n^2(n+1)}$

$= \frac{n}{n+1}$

$= \frac{n+1-1}{n+1}$

$= \frac{n+1}{n+1} - \frac{1}{n+1}$

$= 1 - \frac{1}{n+1}$

since $foo(n) \hookrightarrow 1 - \frac{1}{n+1}$

Therefore P(n) is true.

3: Missing Tile

(task 1) proof by Induction

Predicate:

P(i) $\equiv$ grid of size 2^nx2^n with a painted cell is L-tilable.

Base Case:

P(1) $\equiv$ grid of size 2^1x2^1 is L-tilable wiht a painted cell is true.

Inductive Step:

Assume P(k) $\equiv$ grid size 2^kx2^k wiht a painted cell is L-tilable.

We break the grid into four sections of size 2^kx2^k each.

If the painted cell is in the middle and we add an L tile facing away from the painted cell, we will be left with 4 grids of size 2^kx2^k each with with an already occupied cell from the painted cell and the L-tile we placed.

By Inductive Hypothesis,

each of the 2^kx2^k grid is L-tilable, therefore the grid with a painted cell is tilable.

5: Midway Tower of Hanoi

(task 1)

T(n)=2T(n-1)+1

Predicate: P(i) $\equiv$ T(i)=2^i-1=1

Base Case:

T(i)=2^1-1=1

Inductive Step:

Assume T(n)=$2^n - 1$

T(n+1)= 2T(n)+1 (By Inductive Hypothesis)

=2($2^n - 1$) +1

=(2^{n+1}) $- 2 + 1$

=(2^{n+1}) $- 1$

Therefore T(n+1)=(2^{n+1}) $- 1$

Which is what we have to show

Therefore proved.

(task 3)

(1)

$g(n) = 2g(n-1) + n$

$g(n) = af(n) + bn + c$

$f(n) = 2T(n-1) + 1$

$g(0) = af(0) + b.0 + c = 0$

Therefore c=0

(2)

$g(n) = 2g(n-1) + n$

$g(n) = af(n) + bn$

$af(n) + bn = 2(af(n-1) + b(n-1)) + n$

$a(f(n) + 2f(n-1)) + bn = 2b(n-1) + n$

$a(1) + bn + 2b - 2bn - n = 0$

$a(1) + 2b - n(b+1) = 0$

(P)n + (Q)

By this P and Q are both 0

Solving for a and b:

$b + 1 = 0$

$b = -1$

$a + 2b = 0$

$a = -2b$

$a = 2$

Therefore $g(n) = 2f(n) - n$

(3)

Since f(n)=$2^n - 1$

and g(n)=2f(n)-n

Therefore closed form of $g(n) = 2(2^n - 1) - n$

(4)

Predicate:

P(i)≡ g(i)=2f(i)-i

Base Case:

g(0)≡ 2f(0)-0=0 is true

Inductive Step:

Assume g(k)=2f(k)-k $\forall k > 0$

Given:

g(n)=2g(n-1)+n

g(k+1)=2g(k)+k+1 By IH:

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g(k+1)=2(2f(k)-k)+(n+1)

= 4f(k)-k+1+1-1

=2(f(k)+1)-(k-1)

=2(f(k+1))-(k-1)