

# ICCS200: Assignment 6

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The date

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## 1: Facts About Graphs

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(a) Since each edge connects two nodes, which means for each edge there is are two nodes where each node has a degree of 1, which sums up to 2 degree. Since an edge always has two nodes the number of degrees of nodes for each edge is always 2. Therefore The sum of the vertex degrees is exactly  $2 \times (\text{number of edges})$ .

(b) By proving contraposition, if  $G$  contains no cycles, then  $G$  has a vertex with degree less than 2.  $G$  is a forest if it has no cycle if it has no cycle. If  $N$  a component of  $G$  is trivial then  $G$  has a vertex of degree 0. If  $N$  is nontrivial then  $N$  is a nontrivial tree. Which implies that  $N$  has at least two end-vertices of degree 1. Therefore if  $G$  has no cycle then  $G$  has at least one vertex with degree less than 2. Then if this is true therefore if  $G$  has degree at least 2, then  $G$  contains a cycle.

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## 2: Lets Grow a Tree

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(2) Since the runtime for it's recursion is  $n$  and each time we divide it into left and right subtree therefore it's depth is  $\log n$  since we do it  $\log n$  time. Worst case we may be left with an extra node left which is the leaf of the tree which cannot be divided anymore. Therefore it's depth  $\log n + 1$ .

```
import java.util.Arrays;

public class MakeTree {
    public static BinaryTreeNode buildBST(int[] keys){
        Arrays.sort(keys); //Takes  $n \log n$ 
        int medium = keys.length / 2;
        return new BinaryTreeNode(tree(keys, low: 0, high: medium - 1), keys[medium], tree(keys, low: medium + 1, high: keys.length - 1))
        //2T(n/2) (From helper)

        //Therefore it's run time is  $2 \times \log n + n \log n = O(n \log n)$ 
    }

    public static BinaryTreeNode tree(int[] keys, int low, int high) {
        if(low==high){
            return new BinaryTreeNode(keys[(low+high)/2]);
        }
        else{
            int mid=(low+high)/2;
            if(mid-1<low){
                return new BinaryTreeNode( left: null,keys[mid],tree(keys, low: mid+1,high));
            }
            if(mid+1>high){
                return new BinaryTreeNode(tree(keys,low, high: mid-1),keys[mid], right: null);
            }
            return new BinaryTreeNode(tree(keys,low, high: mid-1),keys[mid],tree(keys, low: mid+1,high));
        }
    }
}
```