

ICCS200: Assignment 3

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The date

2: Stuttering Substring

(2)

```
For n=a.length
and m=b.length
public static boolean isSubstr(String a, String b){
    int c=0;
    boolean ans=false;
    if(a.length()==0){
        return true;
    }
    for(int i=0;i<b.length();i++){           //O(m)
        if(a.charAt(c)==b.charAt(i)){        //O(1) n times = O(n)
            c++;
        }

        if(a.length()==c) {
            ans = true;
            break;
        }
    }
    return ans;
}
```

While the other lines take $O(1)$, therefore
Run time = $O(n+m)$

(5)

```
For n=a.length
and m=b.length
public static int maxStutter(String a, String b){
    int low=0;
    int high=(b.length()/a.length()+1;
    if((a.length()==0)){
        return 0;
    }
    int i=(b.length()/a.length())/2;
    while(high-low>1){ //It runs log n time as the value of either high
                        //or low is being halved everytime = O(log n)

        if(isSubstr(stutter(a,i),b)){ //Each time it is called it takes O(m+n)
            low=i;
            i=(high+low)/2;
        }
    }
}
```

```

    }
    else if(!isSubstr(stutter(a,i),b)){ //O(m+n)
        high=i;
        i=(high+low)/2;
    }
}
if(isSubstr(stutter(a,i),b)){ //O(m+n)
    return i;
}
else{
    return i-1;
}
}

```

The other lines take $O(1)$, therefore

Run time = $O((m+n) \log n)$

4: Quick Sort Recurrence

(2)

$$g(n) = \frac{f(n)}{n+1}$$

(Given)

$$n \cdot f(n) = 2n + (n+1) \cdot f(n-1)$$

Divide both sides by $n(n+1)$

$$\frac{n \cdot f(n)}{n \cdot (n+1)} = \frac{2n}{n \cdot (n+1)} + \frac{(n+1) \cdot f(n-1)}{n \cdot (n+1)}$$

$$\frac{f(n)}{n+1} = \frac{2}{n+1} + \frac{f(n-1)}{n}$$

$$g(n) = \frac{2}{n+1} + g(n-1)$$

(3)

$$g(n) = \frac{2}{n+1} + g(n-1)$$

$$g(n) = \frac{2}{n+1} + \frac{2}{n} + g(n-2)$$

$$g(n) = \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + g(n-3)$$

$$g(n) = \frac{2}{n+1} + \frac{2}{n} + \frac{2}{n-1} + \dots + 2$$

$$g(n) = 2\left(\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \dots + 1\right)$$

$$\text{Since } H_n = \frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \dots + 1$$

$$g(n) = H_n + \frac{1}{n+1}$$

(4)

$$g(n) = \frac{f(n)}{n+1}$$

$$(H_n + \frac{1}{n+1})(n+1) = f(n)$$

$$H_n (n+1) = f(n)$$

$$f(n) \leq (1 + \ln(n))(n+1)$$

$$f(n) \leq n \log n$$

Therefore $f(n) = O(n \log n)$