ICCS200: Assignment 5

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2: Mathematical truths

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(1)

Predicate:

p(i) $\equiv$ The binary tree on n node's where each has either zero or two children has precisely $\frac{n+1}{2}$ leaves.

Base case:

p(1) $\equiv$ The binary tree has 1 node has 1 leaf, which is true.

Inductive Step:

Assume k node's has $\frac{k+1}{2}$ leaves, Where k is 1,2,3,..,n. (Inductive Hypothesis)

W.T.S.: k+2 node's has $\frac{m+1}{2}$ leaves, where m=k+2.

Since each node has either zero or two children. By removing the node we will be left with two trees with p nodes and q nodes. By inductive hypothesis both p and q will also have $\frac{p+1}{2}$ and $\frac{q+1}{2}$ leaves respectively, since the maximum that either p or q can be is k.

p+q node's=k+1 node's.

$\frac{p+1}{2} + \frac{q+1}{2} = \frac{p+q+2}{2}$

$= \frac{k+1+2}{2}$

$= \frac{m+1}{2}$ where m = k+2

Therefore by mathematical induction n node's where each has either zero or two children has $\frac{n+1}{2}$.

(2)

Assume we have a binary heap tree of n node's, if we divide it in half we will be left with two trees. Since it is a binary heap tree where, the binary tree is complete, it will complete the left side first, therefore the left half will be $h(\frac{n-1}{2}) + 1$ where it(+1) is the root node. T(n)=T(n/2) solves to O(logn). Therefore n nodes has height at most O(log n).