Compsci 367 Assignment 1

Hasnain Cheena
190411106
hche737

Question 1: Propositional Logic

a) If A is a base, it is either a purine or a pyrimidine, but not both, similarly for T, C and G.

$$base_A \Rightarrow (purine_A \vee pyrimidine_A) \wedge \neg(purine_A \wedge pyrimidine_A)$$
$$base_T \Rightarrow (purine_T \vee pyrimidine_T) \wedge \neg(purine_T \wedge pyrimidine_T)$$
$$base_G \Rightarrow (purine_G \vee pyrimidine_G) \wedge \neg(purine_G \wedge pyrimidine_G)$$
$$base_C \Rightarrow (purine_C \vee pyrimidine_C) \wedge \neg(purine_C \wedge pyrimidine_C)$$

$purine_A$	$pyrimidine_A$	$base_A$	$purine_A \wedge pyrimidine_A$	$\neg(purine_A \wedge pyrimidine_A)$	$(purine_A \vee pyrimidine_A) \wedge \neg(purine_A \wedge pyrimidine_A)$	$base_A \Rightarrow purine_A \vee pyrimidine_A \wedge \neg(purine_A \wedge pyrimidine_A)$
T	F	T	F	T	T	T

$purine_G$	$pyrimidine_G$	$base_G$	$purine_G \wedge pyrimidine_G$	$\neg(purine_G \wedge pyrimidine_G)$	$(purine_G \vee pyrimidine_G) \wedge \neg(purine_G \wedge pyrimidine_G)$	$base_G \Rightarrow purine_G \vee pyrimidine_G \wedge \neg(purine_G \wedge pyrimidine_G)$
T	F	T	F	T	T	T

$purine_C$	$pyrimidine_C$	$base_C$	$purine_C \wedge pyrimidine_C$	$\neg(purine_G \wedge pyrimidine_G)$	$(purine_C \vee pyrimidine_C) \wedge \neg(purine_C \wedge pyrimidine_C)$	$base_C \Rightarrow purine_C \vee pyrimidine_C \wedge \neg(purine_C \wedge pyrimidine_C)$
F	T	T	F	T	T	T

$purine_T$	$pyrimidine_T$	$base_T$	$purine_T \wedge pyrimidine_T$	$\neg(purine_T \wedge pyrimidine_T)$	$(purine_T \vee pyrimidine_T) \wedge \neg(purine_T \wedge pyrimidine_T)$	$base_T \Rightarrow purine_T \vee pyrimidine_T \wedge \neg(purine_T \wedge pyrimidine_T)$
F	T	T	F	T	T	T

b) If A bonds with C, it does not bond with T or G. Similarly, for the other bases.

$$A\_bondswith\_T \Rightarrow \neg(A\_bondswith\_G) \vee \neg(A\_bondswith\_C)$$
$$\text{A\_bondswith\_G} \Rightarrow \neg(\text{A\_bondswith\_T}) \vee \neg(\text{A\_bondswith\_C})$$
$$A\_bondswith\_C \Rightarrow \neg(A\_bondswith\_T) \vee \neg(A\_bondswith\_G)$$
$$C\_bondswith\_G \Rightarrow \neg(C\_bondswith\_A) \vee \neg(C\_bondswith\_T)$$

$$C\_bondswith\_T \Rightarrow \neg(C\_bondswith\_A) \vee \neg(C\_bondswith\_G)$$

$$G\_bondswith\_T \Rightarrow \neg(G\_bondswith\_A) \vee \neg(G\_bondswith\_C)$$

$A\_bondswith\_G$	$A\_bondswith\_C$	$A\_bondswith\_T$	$\neg A\_bondswith\_G$	$\neg A\_bondswith\_C$	$\neg A\_bondswith\_G \vee \neg A\_bondswith\_C$	$A\_bondswith\_T \Rightarrow \neg A\_bondswith\_G \vee \neg A\_bondswith\_C$
F	F	T	T	T	T	T

$A\_bondswith\_T$	$A\_bondswith\_C$	$A\_bondswith\_G$	$\neg A\_bondswith\_T$	$\neg A\_bondswith\_C$	$\neg A\_bondswith\_T \vee \neg A\_bondswith\_C$	$A\_bondswith\_G \Rightarrow \neg A\_bondswith\_T \vee \neg A\_bondswith\_C$
T	F	F	F	T	T	T

$A\_bondswith\_T$	$A\_bondswith\_G$	$A\_bondswith\_C$	$\neg A\_bondswith\_T$	$\neg A\_bondswith\_G$	$\neg A\_bondswith\_T \vee \neg A\_bondswith\_G$	$A\_bondswith\_C \Rightarrow \neg A\_bondswith\_T \vee \neg A\_bondswith\_G$
T	F	F	F	T	T	T

$C\_bondswith\_A$	$C\_bondswith\_T$	$C\_bondswith\_G$	$\neg C\_bondswith\_A$	$\neg C\_bondswith\_T$	$\neg C\_bondswith\_A \vee \neg C\_bondswith\_T$	$C\_bondswith\_G \Rightarrow \neg C\_bondswith\_A \vee \neg C\_bondswith\_T$
F	F	T	T	T	T	T

$C\_bondswith\_A$	$C\_bondswith\_G$	$C\_bondswith\_T$	$\neg C\_bondswith\_A$	$\neg C\_bondswith\_G$	$\neg C\_bondswith\_A \vee \neg C\_bondswith\_G$	$C\_bondswith\_T \Rightarrow \neg C\_bondswith\_A \vee \neg C\_bondswith\_G$
F	T	F	T	F	T	T

$G\_bondswith\_A$	$G\_bondswith\_C$	$G\_bondswith\_T$	$\neg G\_bondswith\_A$	$\neg G\_bondswith\_C$	$\neg G\_bondswith\_A \vee \neg G\_bondswith\_C$	$G\_bondswith\_T \Rightarrow \neg G\_bondswith\_A \vee \neg G\_bondswith\_C$
F	T	F	T	F	T	T

c) A does not bond with A; similarly C, T or G do not bond with themselves.

$$\neg A\_bondswith\_A$$

$$\neg G\_bondswith\_G$$

$$\neg T\_bondswith\_T$$

$$\neg C\_bondswith\_C$$

$A\_bondswith\_A$	$\neg A\_bondswith\_A$
F	T

$T\_bondswith\_T$	$\neg T\_bondswith\_T$
F	T

$C\_bondswith\_C$	$\neg C\_bondswith\_C$
F	T

$G\_bondswith\_G$	$\neg G\_bondswith\_G$
F	T

d) If A bonds with T, it is the case that A is a base and that T is a base, and it is either the case that A is a purine and T a pyrimidine, or vice versa. Similarly with other pairs of bases.

$$\text{A\_bondswith\_T} \Rightarrow \text{base}_\text{A} \wedge \text{base}_\text{T} \wedge \left((\text{purine}_\text{A} \wedge \text{pyrimidine}_\text{T}) \vee (\text{purine}_\text{T} \wedge \text{pyrimidine}_\text{A})\right)$$

$$A\_bondswith\_G \Rightarrow base_A \wedge base_G \wedge \left((purine_A \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_A)\right)$$

$$A\_bondswith\_C \Rightarrow base_A \wedge base_C \wedge \left((purine_A \wedge pyrimidine_C) \vee (purine_C \wedge pyrimidine_A)\right)$$

$$C\_bondswith\_T \Rightarrow base_C \wedge base_T \wedge \left((purine_C \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_C)\right)$$

$$C\_bondswith\_G \Rightarrow base_C \wedge base_G \wedge \left((purine_C \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_C)\right)$$

$$G\_bondswith\_T \Rightarrow base_G \wedge base_T \wedge \left((purine_G \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_G)\right)$$

$A\_bondswith\_T$	$base_A$	$base_T$	$pyrimidine_A$	$purine_A$	$pyrimidine_T$	$purine_T$	$purine_A \wedge pyrimidine_T$	$purine_T \wedge pyrimidine_A$	$\left((purine_A \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_A)\right)$	$base_A \wedge base_T \wedge \left((purine_A \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_A)\right)$	$A\_bondswith\_T \Rightarrow base_A \wedge base_T \wedge \left((purine_A \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_A)\right)$
T	T	T	F	T	T	F	T	F	T	T	T

$A\_bondswith\_G$	$base_A$	$base_G$	$pyrimidine_A$	$purine_A$	$pyrimidine_G$	$purine_G$	$purine_G \wedge pyrimidine_A$	$purine_A \wedge pyrimidine_G$	$((purine_A \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_A))$	$base_A \wedge base_G \wedge ((purine_A \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_A))$	$A\_bondswith\_G \Rightarrow base_A \wedge base_G \wedge ((purine_A \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_A))$
F	T	T	F	T	F	T	F	F	F	F	T

$A\_bondswith\_C$	$base_A$	$base_C$	$pyrimidine_A$	$purine_A$	$pyrimidine_C$	$purine_C$	$purine_A \wedge pyrimidine_C$	$purine_C \wedge pyrimidine_A$	$((purine_A \wedge pyrimidine_C) \vee (purine_C \wedge pyrimidine_A))$	$base_A \wedge base_C \wedge ((purine_A \wedge pyrimidine_C) \vee (purine_C \wedge pyrimidine_A))$	$A\_bondswith\_C \Rightarrow base_A \wedge base_C \wedge ((purine_A \wedge pyrimidine_C) \vee (purine_C \wedge pyrimidine_A))$
F	T	T	F	T	T	F	T	F	T	T	T

$C\_bondswith\_T$	$base_T$	$base_C$	$pyrimidine_T$	$purine_T$	$pyrimidine_C$	$purine_C$	$purine_T \wedge pyrimidine_C$	$purine_C \wedge pyrimidine_T$	$((purine_C \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_C))$	$base_C \wedge base_T \wedge ((purine_C \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_C))$	$C\_bondswith\_T \Rightarrow base_C \wedge base_T \wedge ((purine_C \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_C))$
F	T	T	T	F	T	F	F	F	F	F	T

$C\_bondswith\_G$	$base_G$	$base_C$	$pyrimidine_G$	$purine_G$	$pyrimidine_C$	$purine_C$	$purine_G \wedge pyrimidine_C$	$purine_C \wedge pyrimidine_G$	$((purine_C \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_C))$	$base_C \wedge base_G \wedge ((purine_C \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_C))$	$C\_bondswith\_G \Rightarrow base_C \wedge base_G \wedge ((purine_C \wedge pyrimidine_G) \vee (purine_G \wedge pyrimidine_C))$
T	T	T	F	T	T	F	T	F	T	T	T

$G\_bondswith\_T$	$base_G$	$base_C$	$pyrimidine_G$	$purine_G$	$pyrimidine_T$	$purine_T$	$purine_T \wedge pyrimidine_G$	$purine_G \wedge pyrimidine_T$	$((purine_G \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_G))$	$base_G \wedge base_T \wedge ((purine_G \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_G))$	$G\_bondswith\_T \Rightarrow base_G \wedge base_T \wedge ((purine_G \wedge pyrimidine_T) \vee (purine_T \wedge pyrimidine_G))$
F	T	T	F	T	T	F	F	T	T	T	T

e) If A bonds with T, then T bonds with A, and similarly for all other pairs.

$$A\_bondswith\_T \iff T\_bondswith\_A$$

$$A\_bondswith\_C \iff C\_bondswith\_A$$

$$A\_bondswith\_G \iff G\_bondswith\_A$$

$$C\_bondswith\_G \iff G\_bondswith\_C$$

$$C\_bondswith\_T \iff T\_bondswith\_C$$

$$G\_bondswith\_T \iff T\_bondswith\_G$$

$T\_bondswith\_A$	$A\_bondswith\_T$	$A\_bondswith\_T \iff T\_bondswith\_A$
T	T	T

$C\_bondswith\_A$	$A\_bondswith\_C$	$A\_bondswith\_C \iff C\_bondswith\_A$
F	F	T

$G\_bondswith\_A$	$A\_bondswith\_G$	$A\_bondswith\_G \iff G\_bondswith\_A$
F	F	T

$C\_bondswith\_G$	$G\_bondswith\_C$	$G\_bondswith\_C \iff C\_bondswith\_G$
T	T	T

$C\_bondswith\_T$	$T\_bondswith\_C$	$T\_bondswith\_C \iff C\_bondswith\_T$
F	F	T

$G\_bondswith\_T$	$T\_bondswith\_G$	$T\_bondswith\_G \iff G\_bondswith\_T$
F	F	T

Question 2 follows on the next page.

Question 2: Predicate Logic

$\forall a$
$base(a) \Rightarrow \Big(\big(purine(a) \vee pyrimidine(a)\big) \wedge \neg\big(purine(a) \wedge pyrimidine(a)\big)\Big)$

$\forall a \exists b$
$\text{bondswith(a, b)} \;\Rightarrow\; \neg\, \exists z \,\big(\text{bondswith(a, c)} \wedge \neg(\text{b} = \text{c})\big)$

$\forall a$
$\neg\, bondswith(a, a)$

$\forall a \exists b$
$\text{bondswith}(a, \text{b}) \;\Rightarrow\; \text{base(a)} \wedge \text{base(b)} \wedge \Big(\big(\text{purine(a)} \wedge \text{pyrimidine(b)}\big) \vee \big(\text{purine(b)} \wedge \text{pyrimidine(a)}\big)\Big)$

$\forall a \exists b$
$bondswith(a, b) \Leftrightarrow bondswith(b, a)$