Stats 326: Assignment 4

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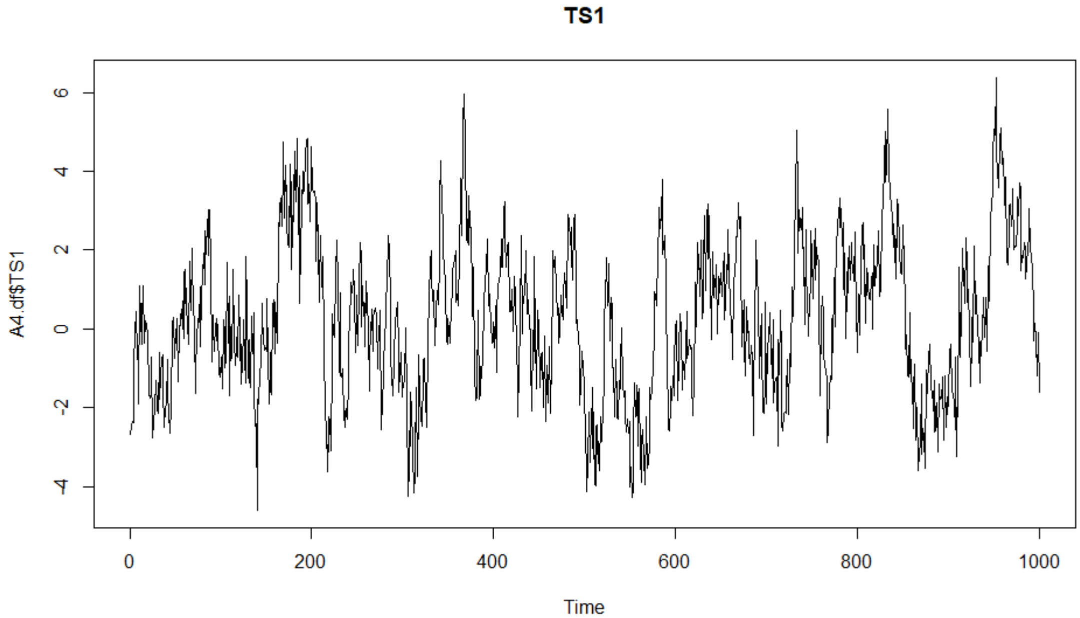
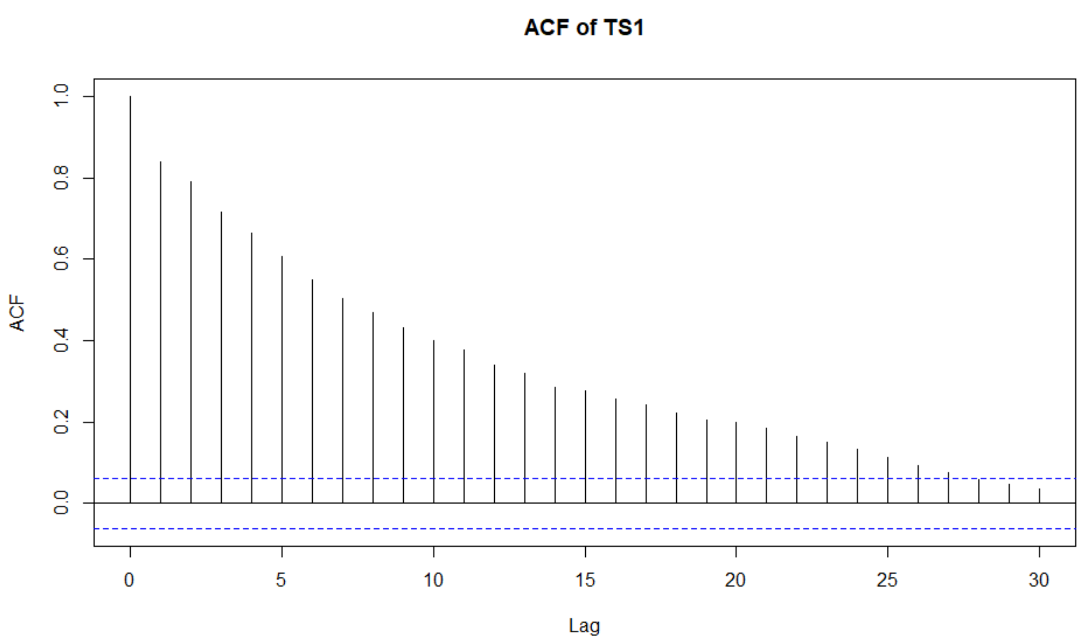
Question 1

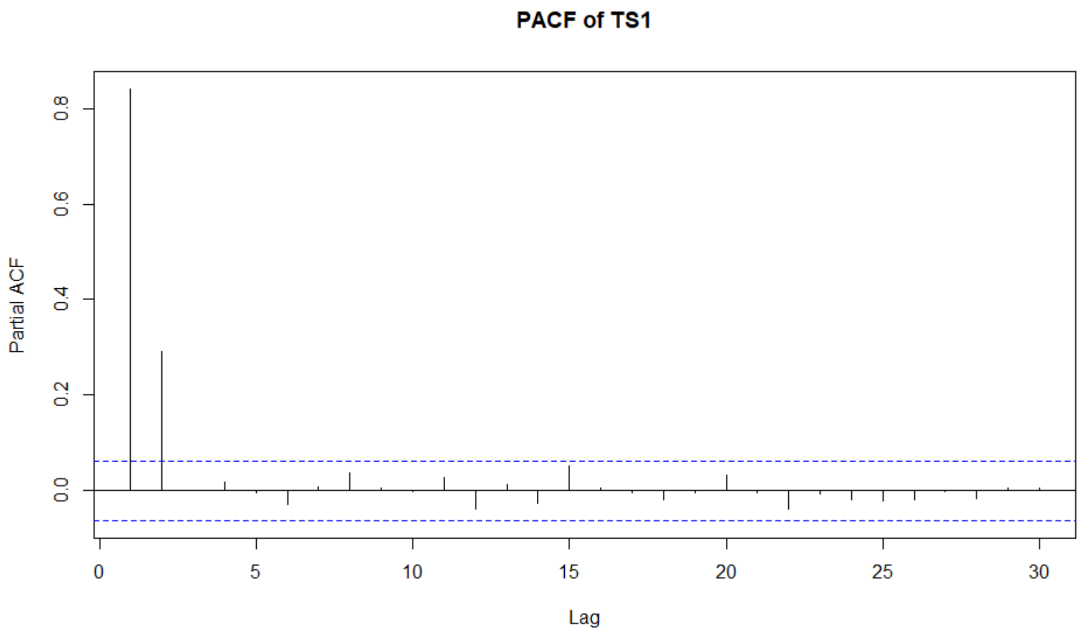
```r
plot.ts(A4.df$TS1, main="TS1")
```




```r
acf(A4.df$TS1, main="ACF of TS1")
```



```r
pacf(A4.df$TS1, main="PACF of TS1")
```



The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay while the pacf shows cut-off at lag 2. This suggests AR(2) is the most suitable model. The general form of the model is shown below:

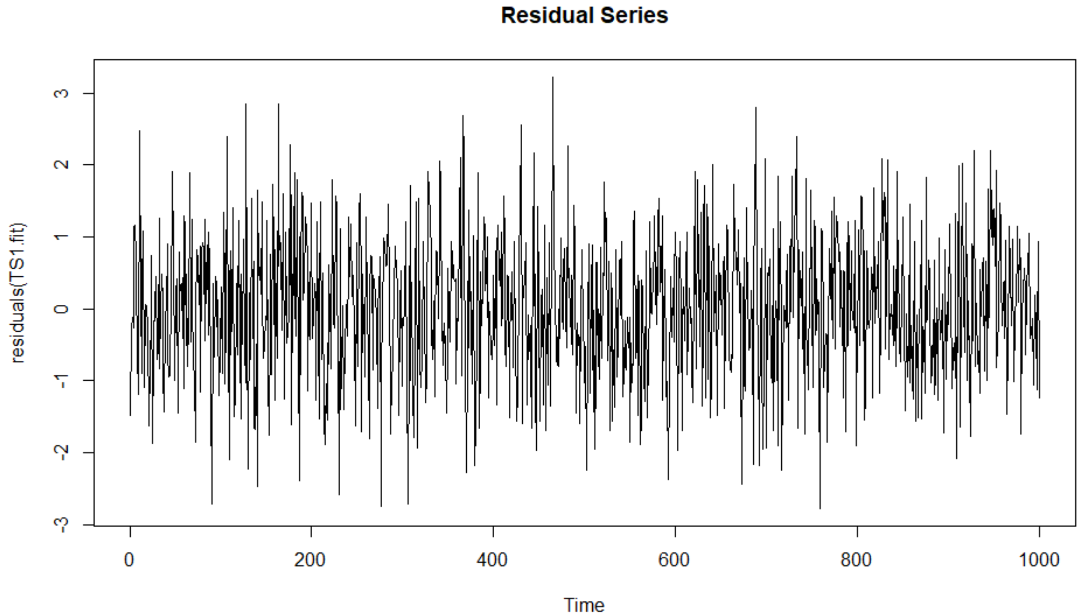
$$y_t = \rho_1 y_{t-1} \times \rho_2 y_{t-2} + \varepsilon_t$$

```r
TS1.fit = arima(A4.df$TS1, order=c(2,0,0))
TS1.fit

##
## Call:
## arima(x = A4.df$TS1, order = c(2, 0, 0))
##
## Coefficients:
##          ar1     ar2  intercept
##       0.5958  0.2928     0.2106
## s.e.  0.0302  0.0303     0.2821
##
## sigma^2 estimated as 1.008:  log likelihood = -1423.72,  aic = 2855.44
```

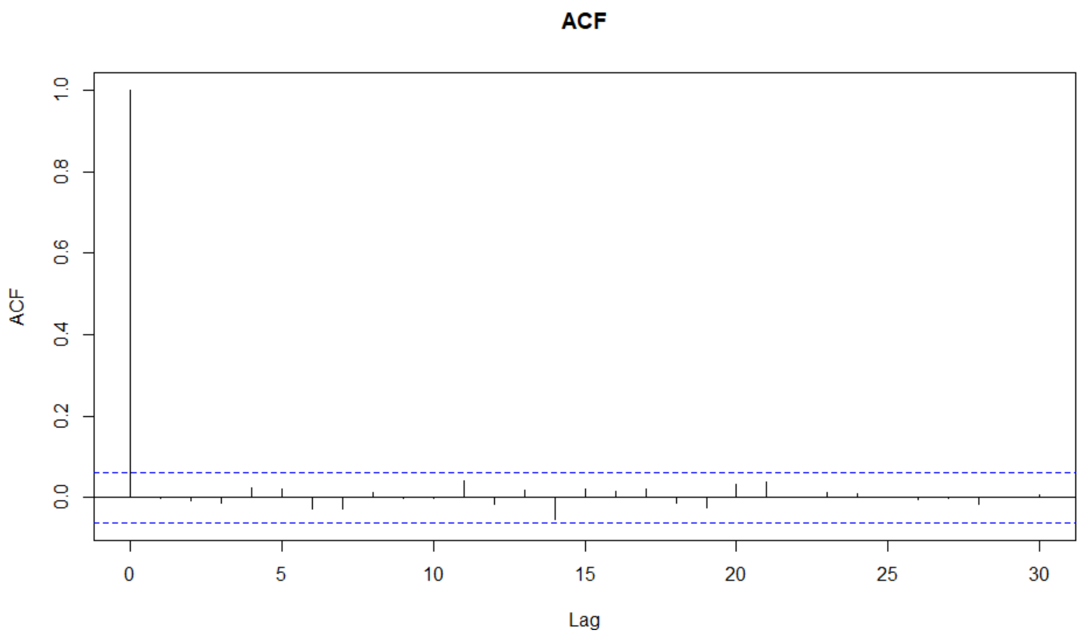
Estimated equation of model:

$$y_t = 0.5958 y_{t-1} \times 0.2928 y_{t-2} + \varepsilon_t$$

```r
plot(residuals(TS1.fit), main="Residual Series")
```




```r
acf(residuals(TS1.fit), main="ACF")
```



The Residual Series appear to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags. Therefore, AR(2) is appropriate.

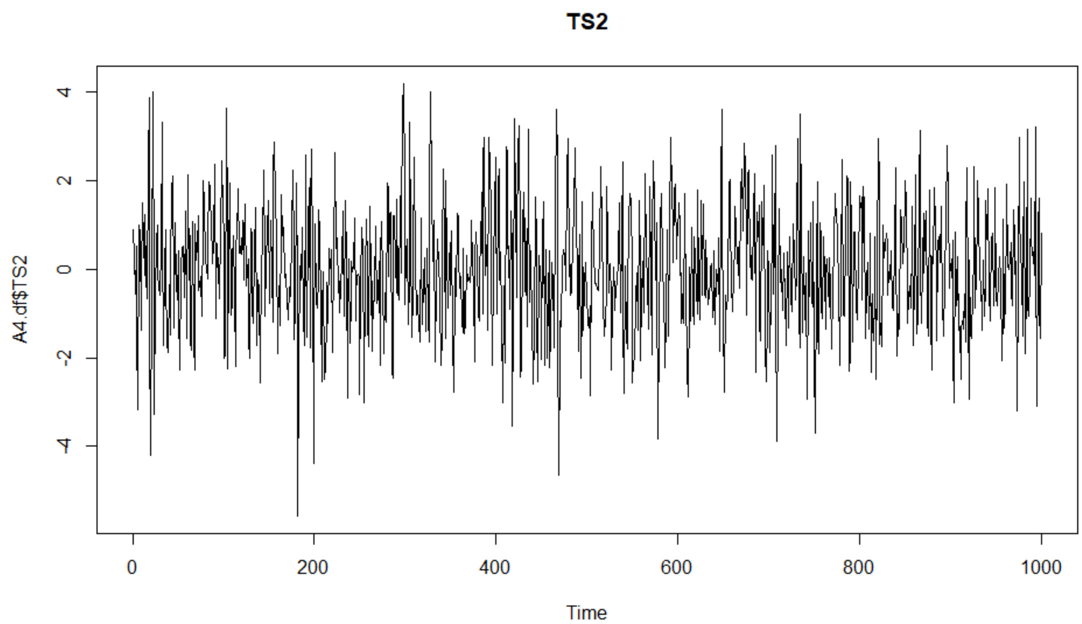
Other models tried:

AR(3) AIC = 2857.86
ARMA(2,1) AIC = 2889.29

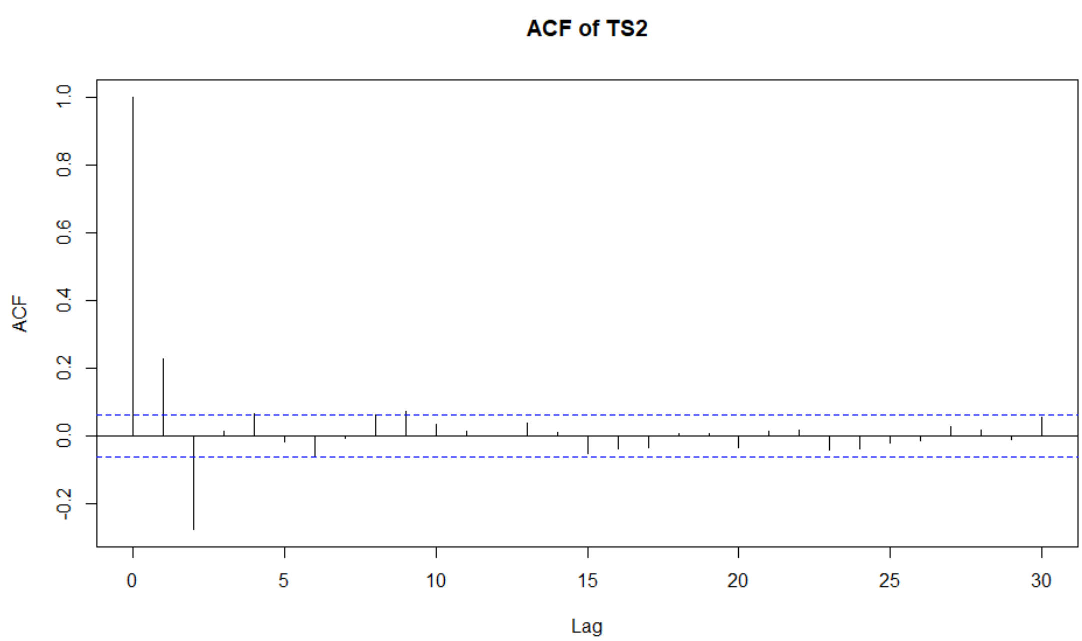
AR(2) is the best model. This is because AR(2) had the lowest AIC score relative to other models tried and all terms were significant.

Question 2

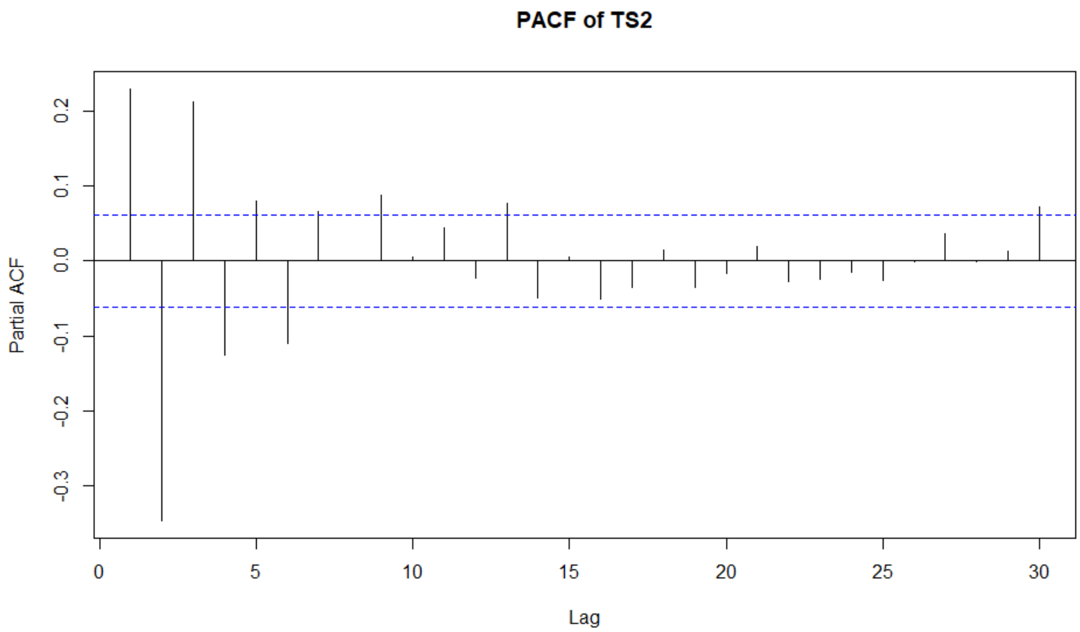
```r
plot.ts(A4.df$TS2, main="TS2")
```




```r
acf(A4.df$TS2, main="ACF of TS2")
```



```r
pacf(A4.df$TS2, main="PACF of TS2")
```



The plot of the series shows no discernable pattern. The acf shows cut-off at lag 2 and the pacf shows decay (or persistence). This suggests MA(2) is the most suitable model.
The general form of the model is shown below:

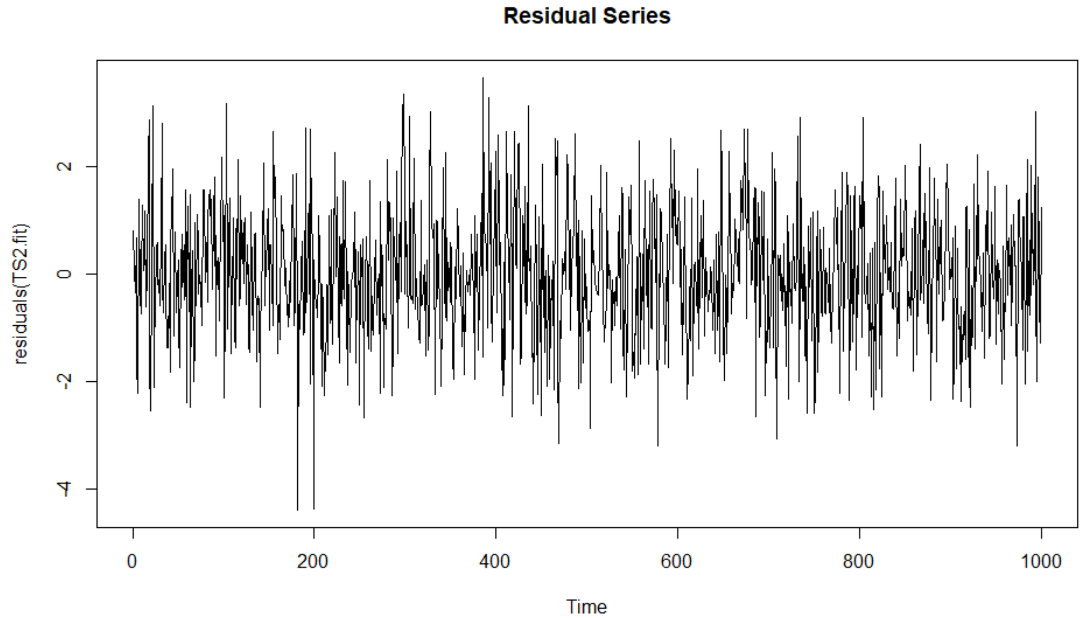
$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2}$$

```r
TS2.fit = arima(A4.df$TS2, order=c(0,0,2))
TS2.fit

##
## Call:
## arima(x = A4.df$TS2, order = c(0, 0, 2))
##
## Coefficients:
##          ma1     ma2  intercept
##       0.4377  -0.311    -0.0086
## s.e.  0.0302   0.030     0.0433
##
## sigma^2 estimated as 1.475:  log likelihood = -1613.7,  aic = 3235.4
```

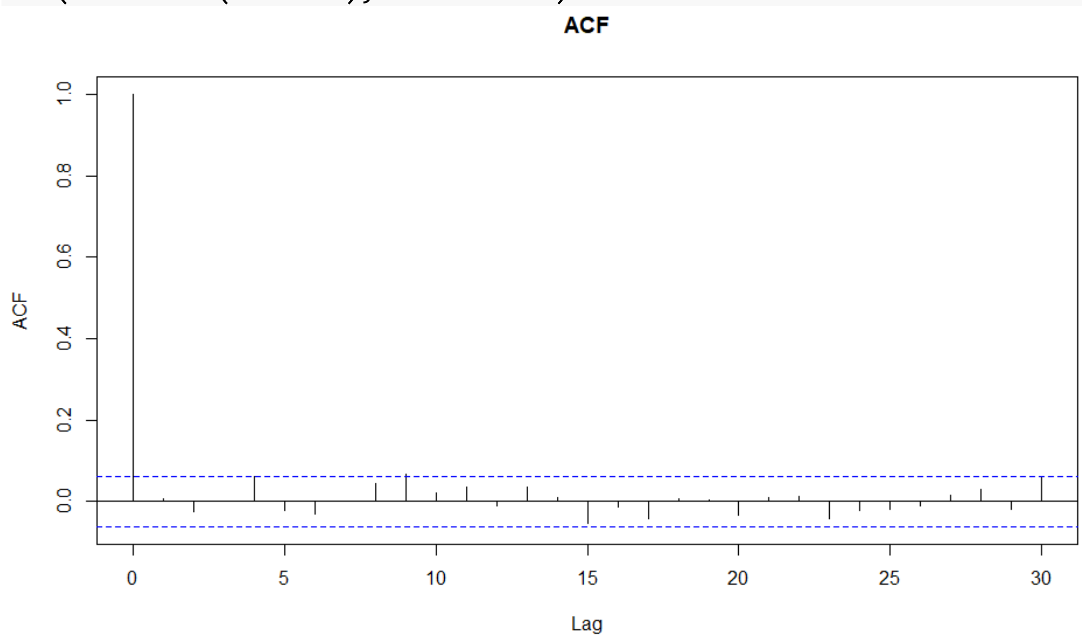
Estimated equation of model:

$$y_t = \varepsilon_t + 0.4377\varepsilon_{t-1} - 0.311\varepsilon_{t-2}$$

```r
plot(residuals(TS2.fit), main="Residual Series")
```




```r
acf(residuals(TS2.fit), main="ACF")
```



The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags. Therefore, MA(2) is appropriate.

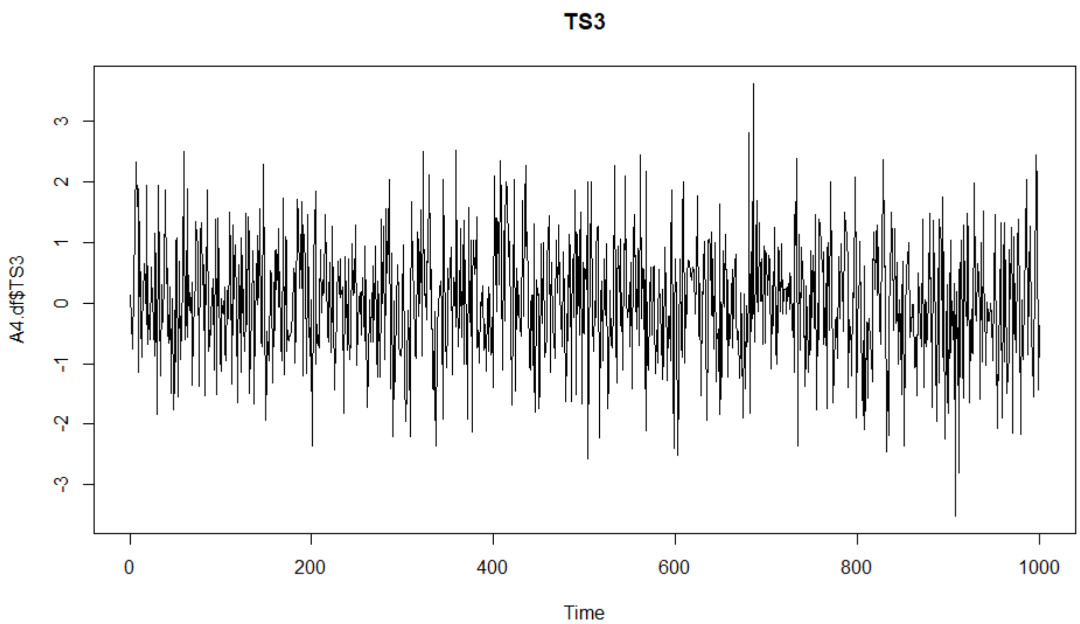
Other models tried:

MA(3) AIC = 3236.86
ARMA(1,2) AIC = 3236.98

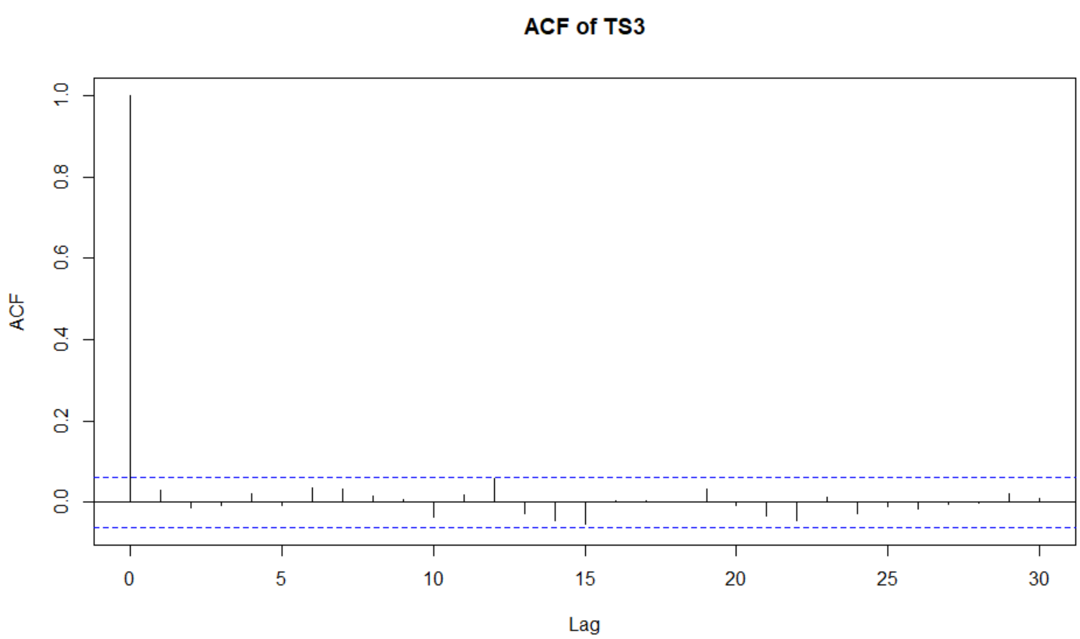
MA(2) is the best model. This is because MA(2) had the lowest AIC score relative to other models and all terms were significant.

Question 3

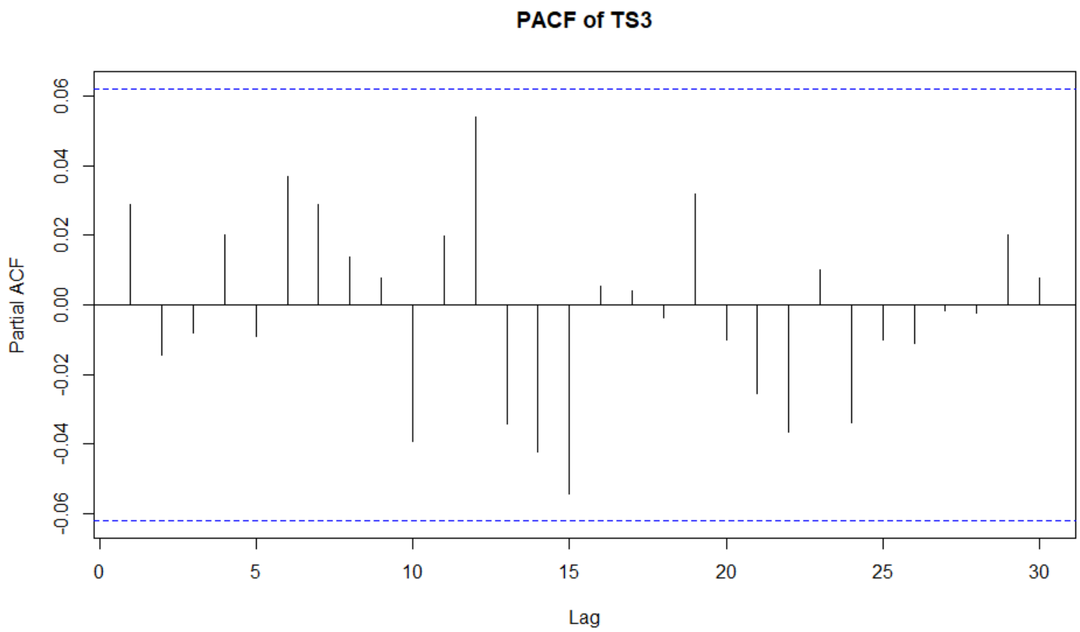
```
plot.ts(A4.df$TS3, main="TS3")
```




```
acf(A4.df$TS3, main="ACF of TS3")
```



```r
pacf(A4.df$TS3, main="PACF of TS3")
```



The plot of the series shows no discernible pattern. The acf and pacf show no significant lags. This suggests the series is White Noise. The general form of the model is shown below:

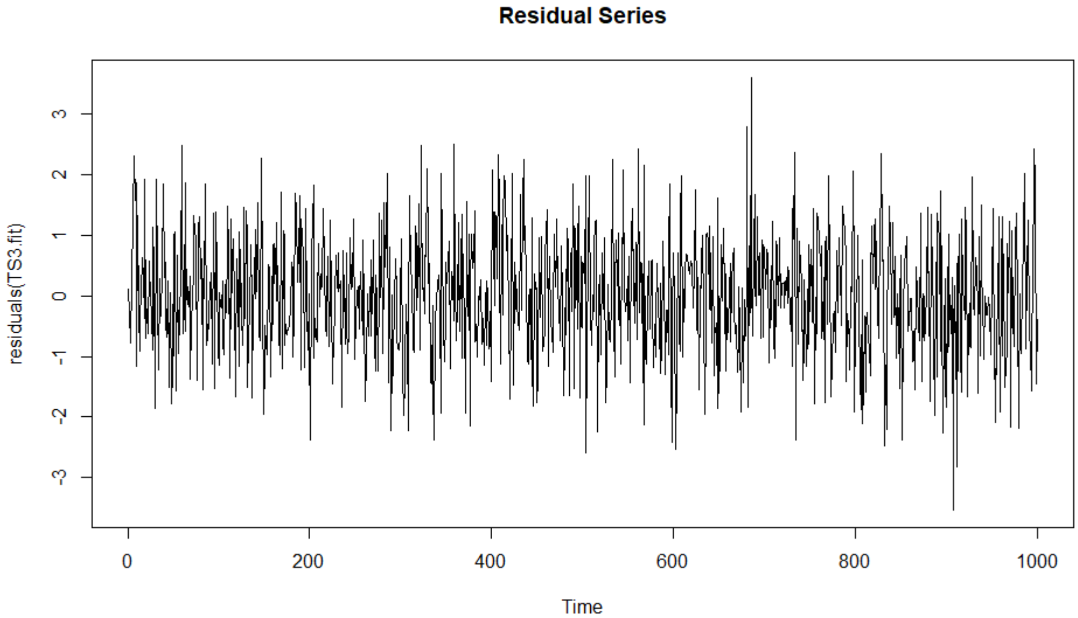
$$y_t = \varepsilon_t$$

```r
TS3.fit = arima(A4.df$TS3, order=c(0,0,0))
TS3.fit

##
## Call:
## arima(x = A4.df$TS3, order = c(0, 0, 0))
##
## Coefficients:
##       intercept
##          0.0211
## s.e.     0.0316
##
## sigma^2 estimated as 0.9961:  log likelihood = -1417,  aic = 2838
```

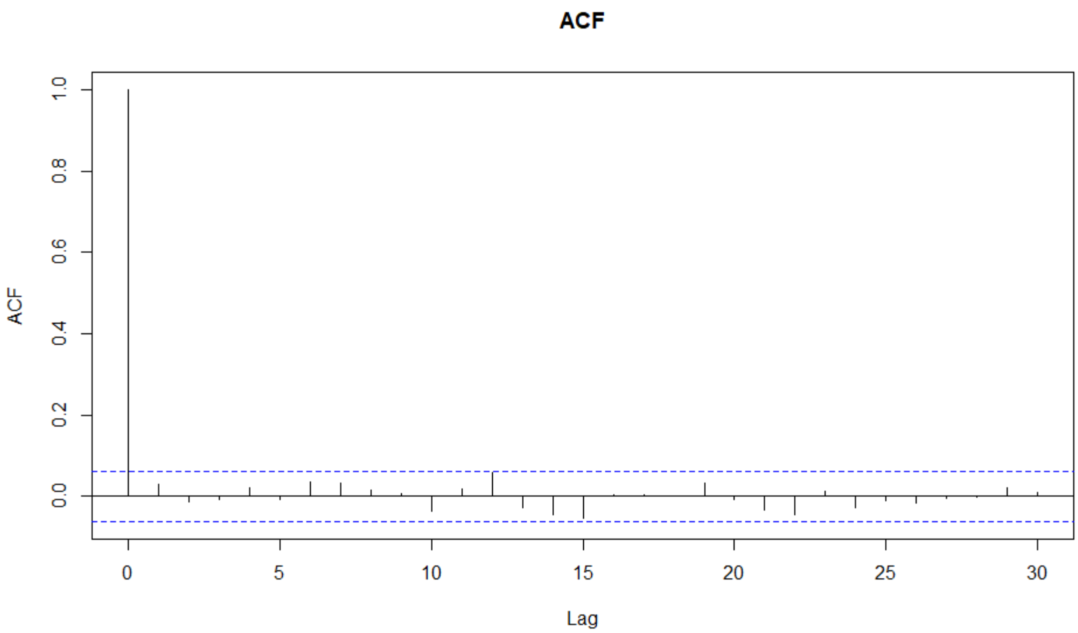
Estimated model:

$$y_t = \varepsilon_t$$

```r
plot(residuals(TS3.fit), main="Residual Series")
```




```r
acf(residuals(TS3.fit), main="ACF")
```



The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags. Therefore, the white noise model is appropriate.

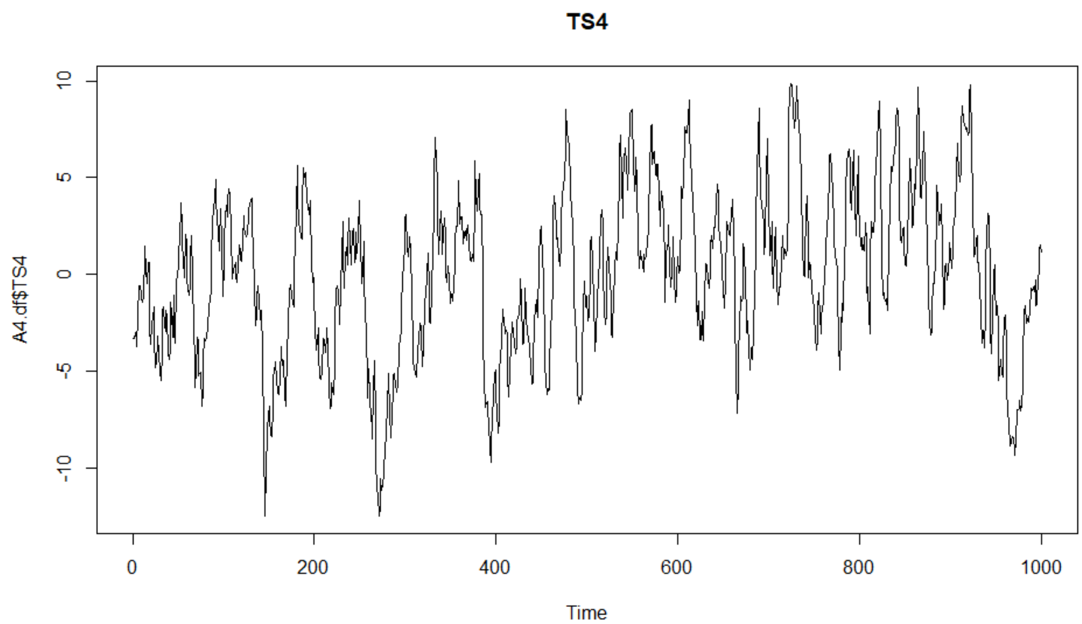
Other models tried:

AR(1)	AIC: 2839.17
MA(1)	AIC: 2839.14

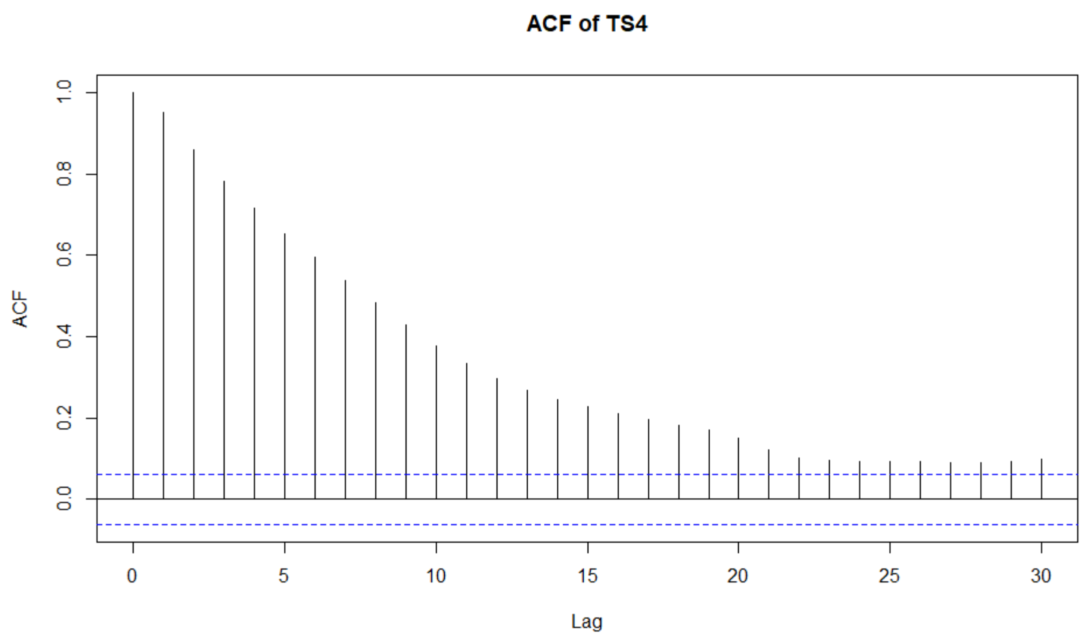
The white noise model is the best model as it has the lowest AIC score relative to the other models tried.

Question 4

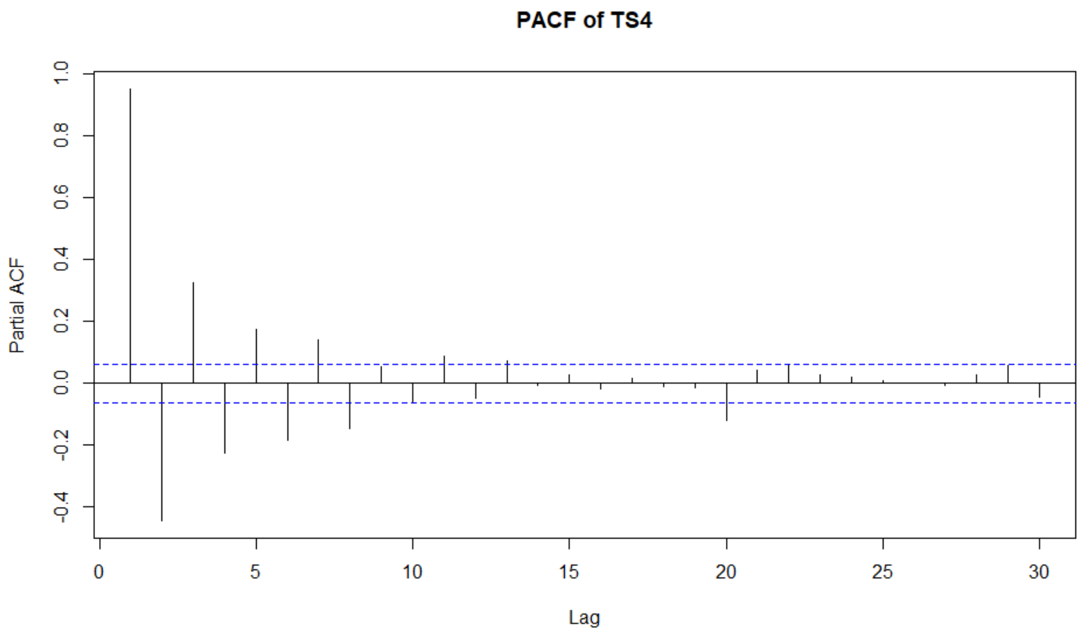
```
plot.ts(A4.df$TS4, main="TS4")
```




```
acf(A4.df$TS4, main="ACF of TS4")
```



```r
pacf(A4.df$TS4, main="PACF of TS4")
```



The plot of the series shows clustering indicating positive autocorrelation. Both the acf and pacf show decay. This suggests ARMA(p,q) is an appropriate model. However, from the plots we have no indication of what order ARMA model must be used. Therefore, I began with ARMA(1,1). The general form of ARMA(1,1) is shown below:

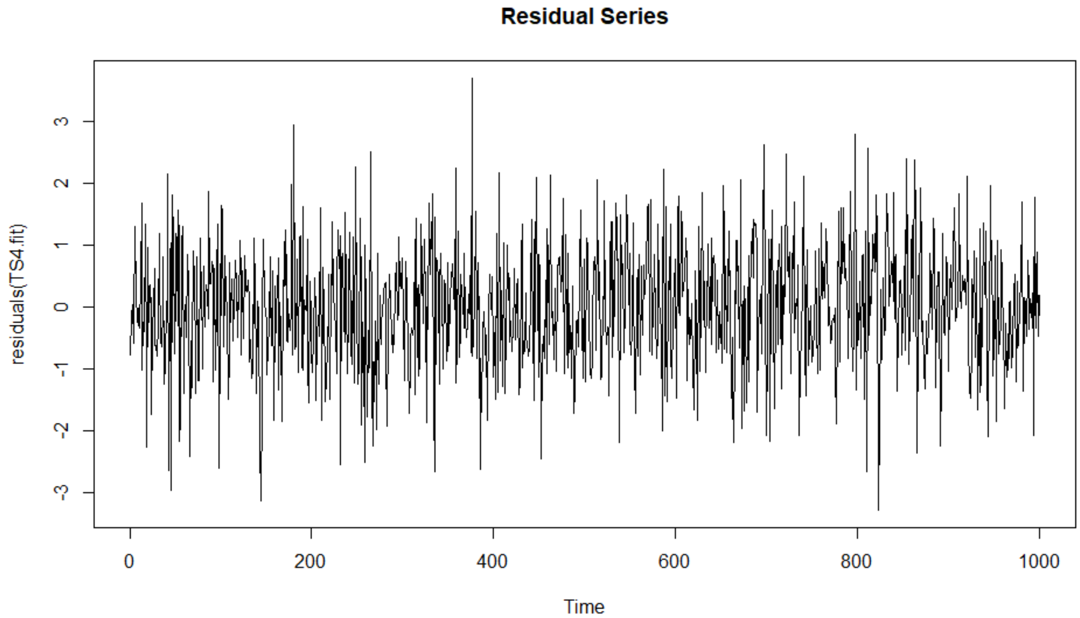
$$y_t = \rho_1 y_{t-1} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t$$

```r
TS4.fit = arima(A4.df$TS4, order=c(1,0,1))
TS4.fit

## 
## Call:
## arima(x = A4.df$TS4, order = c(1, 0, 1))
## 
## Coefficients:
##          ar1     ma1  intercept
##       0.8974  0.9121    -0.0147
## s.e.  0.0139  0.0128     0.5786
## 
## sigma^2 estimated as 0.9828:  log likelihood = -1412.55,  aic = 2833.11
```

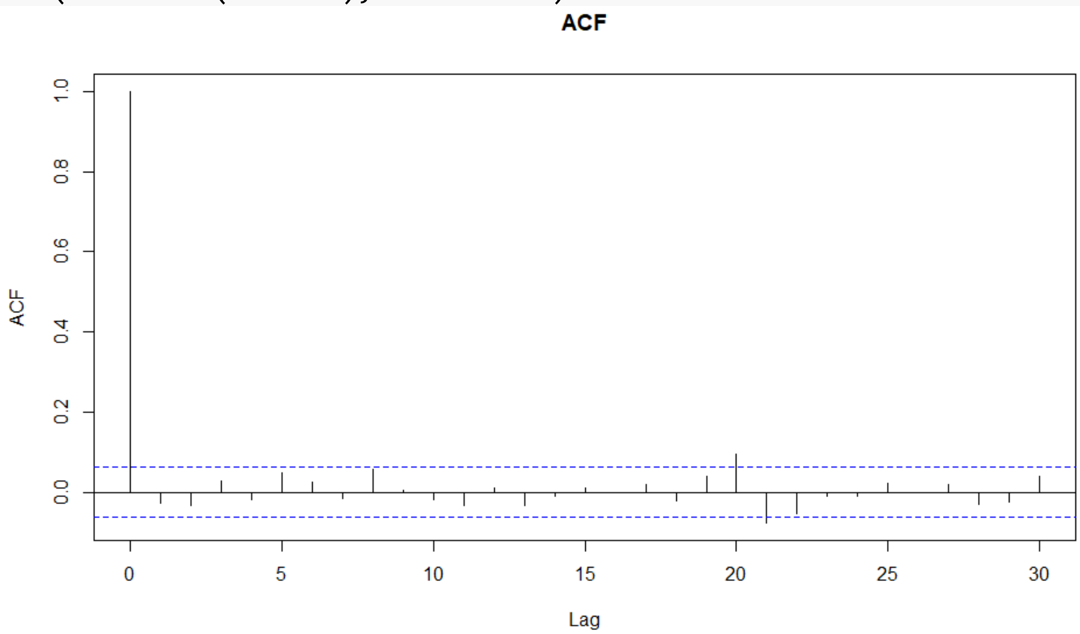
Estimate equation of the model:

$$y_t = 0.8974 y_{t-1} + 0.9121 \varepsilon_{t-1} + \varepsilon_t$$

```r
plot(residuals(TS4.fit), main="Residual Series")
```




```r
acf(residuals(TS4.fit), main="ACF")
```



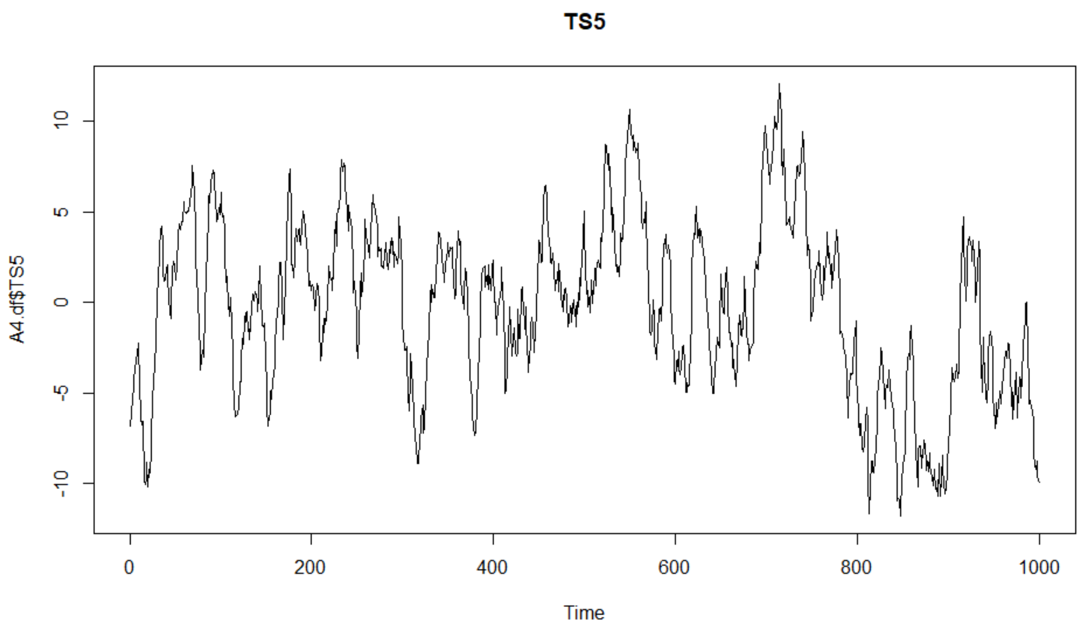
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows 2 weakly significant lags at lags 20 and 21. As they are weakly significant, they are not a concern. Therefore, ARMA(1,1) is appropriate.

Other models tried:
ARMA(2,1) AIC: 2833.86
ARMA(1,2) AIC: 2833.73

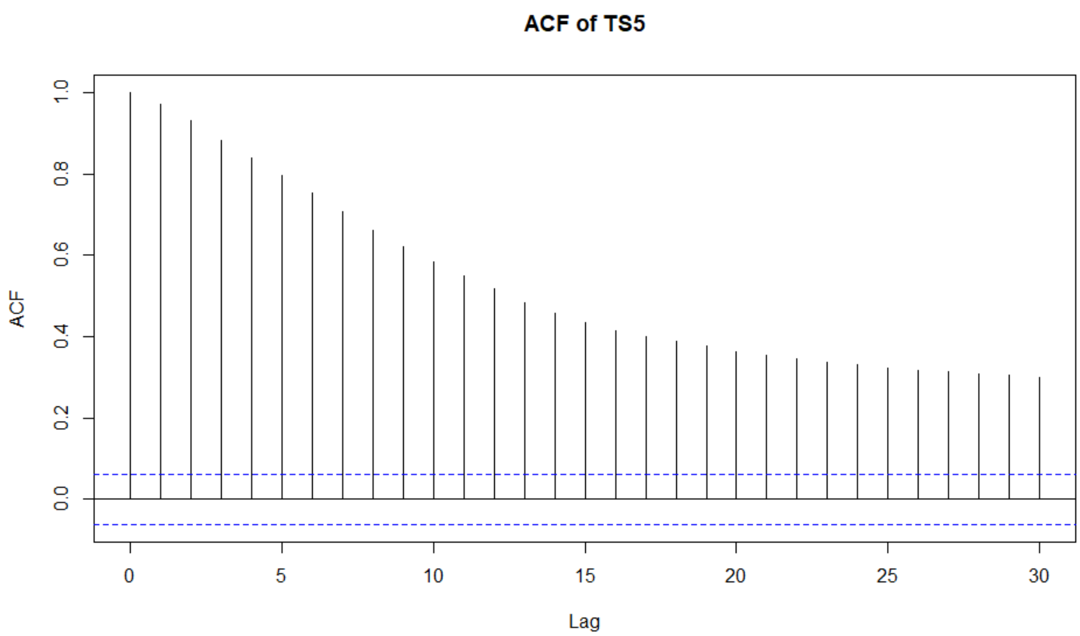
ARMA(1,1) is the best model because all terms are significant and it has the lowest AIC score relative to the other models tried.

Question 5

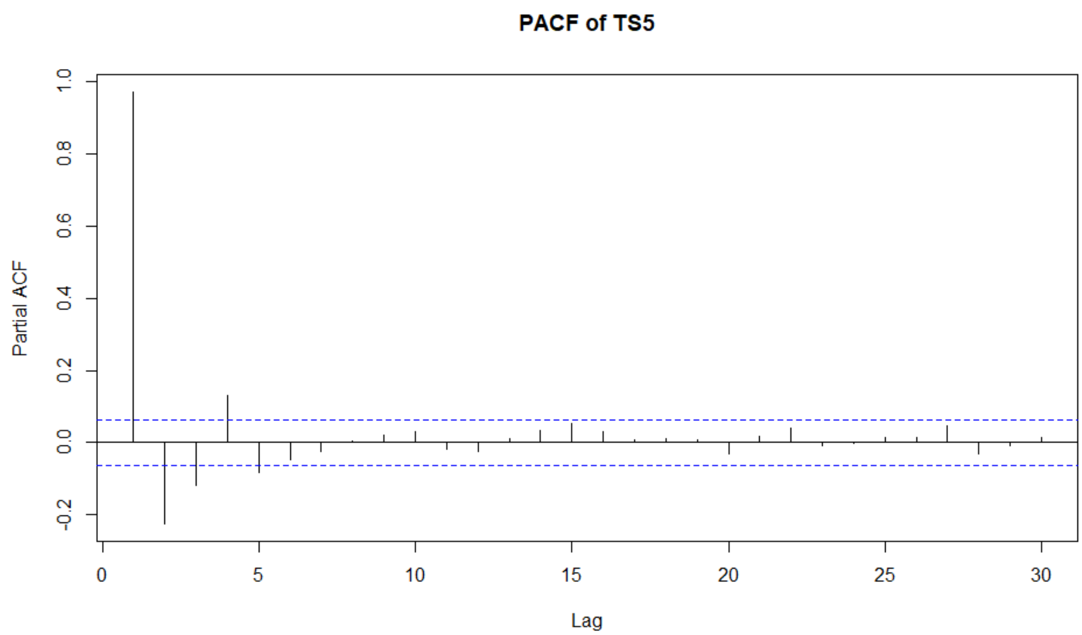
```
plot.ts(A4.df$TS5, main="TS5")
```




```
acf(A4.df$TS5, main="ACF of TS5")
```



```r
pacf(A4.df$TS5, main="PACF of TS5")
```



The plot of the series shows clustering indicating positive autocorrelation. Both the acf and pacf show decay/persistence. This suggests ARMA(p,q) is an appropriate model. However, from the plots we have no indication of what order ARMA model must be used. Therefore, I began with ARMA(1,1). The general form of ARMA(1,1) is shown below:

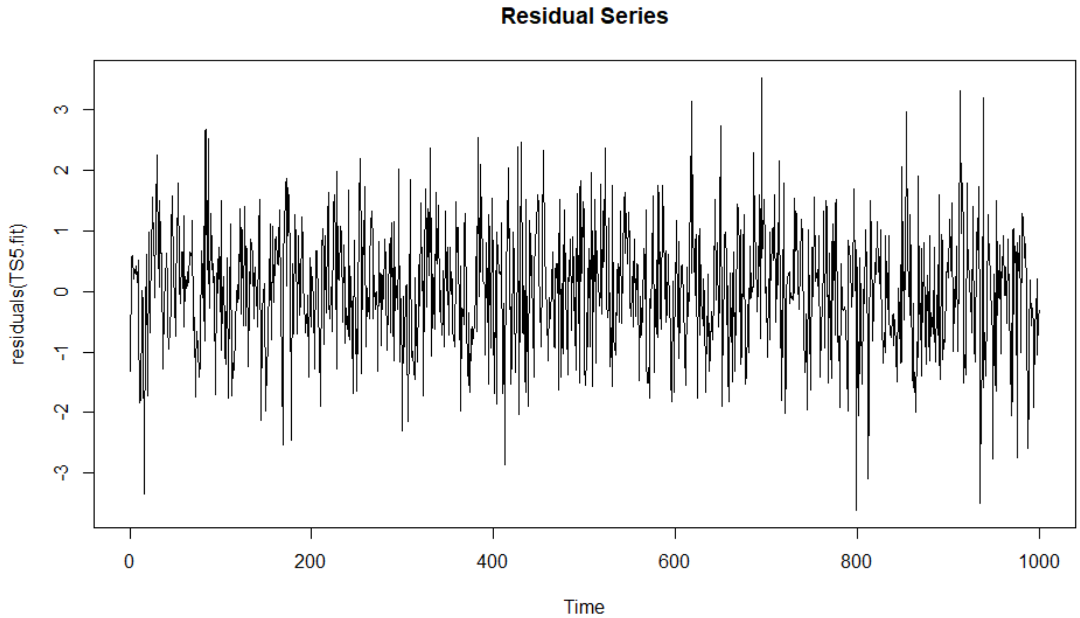
$$y_t = \rho_1 y_{t-1} + \alpha_1 \varepsilon_{t-1} + \varepsilon_t$$

```r
TS5.fit = arima(A4.df$TS5, order=c(1,0,1))

## Warning in arima(A4.df$TS5, order = c(1, 0, 1)): possible convergence prob
lem:
## optim gave code = 1

TS5.fit

##
## Call:
## arima(x = A4.df$TS5, order = c(1, 0, 1))
##
## Coefficients:
##          ar1     ma1  intercept
##       0.9674  0.1876    -0.6895
## s.e.  0.0082  0.0260     1.1571
##
## sigma^2 estimated as 1.063:  log likelihood = -1450.83,  aic = 2909.67
```

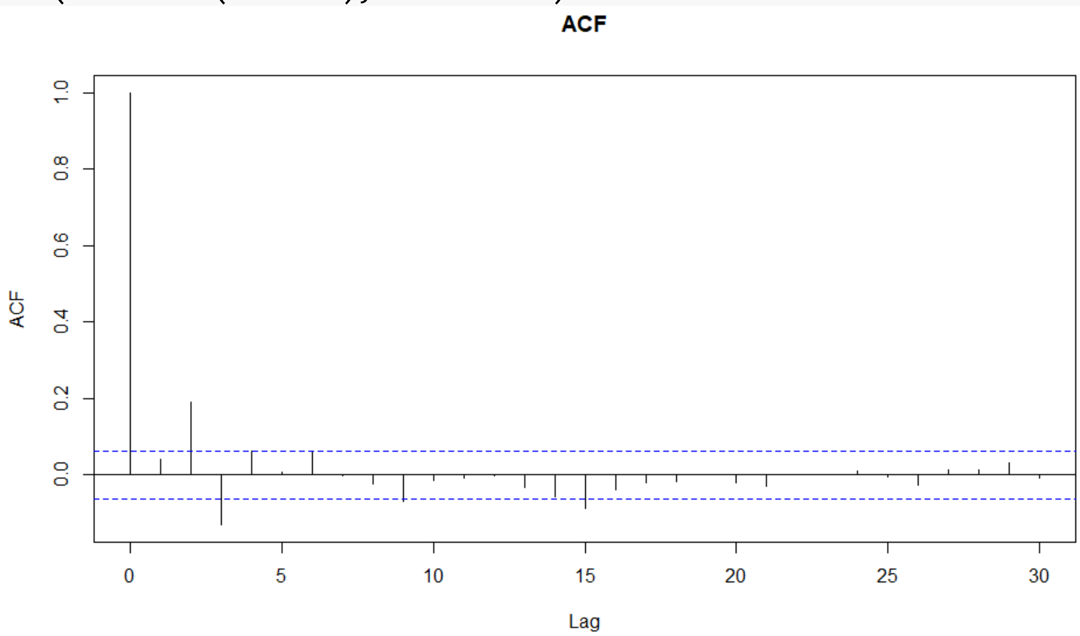
Estimated equation:

$$y_t = 0.9674 y_{t-1} + 0.1876 \varepsilon_{t-1} + \varepsilon_t$$

```
plot(residuals(TS5.fit), main="Residual Series")
```




```
acf(residuals(TS5.fit), main="ACF")
```



The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows 4 significant lags at lags 2, 3, 9 and 15. A better model is outlined on the next page.

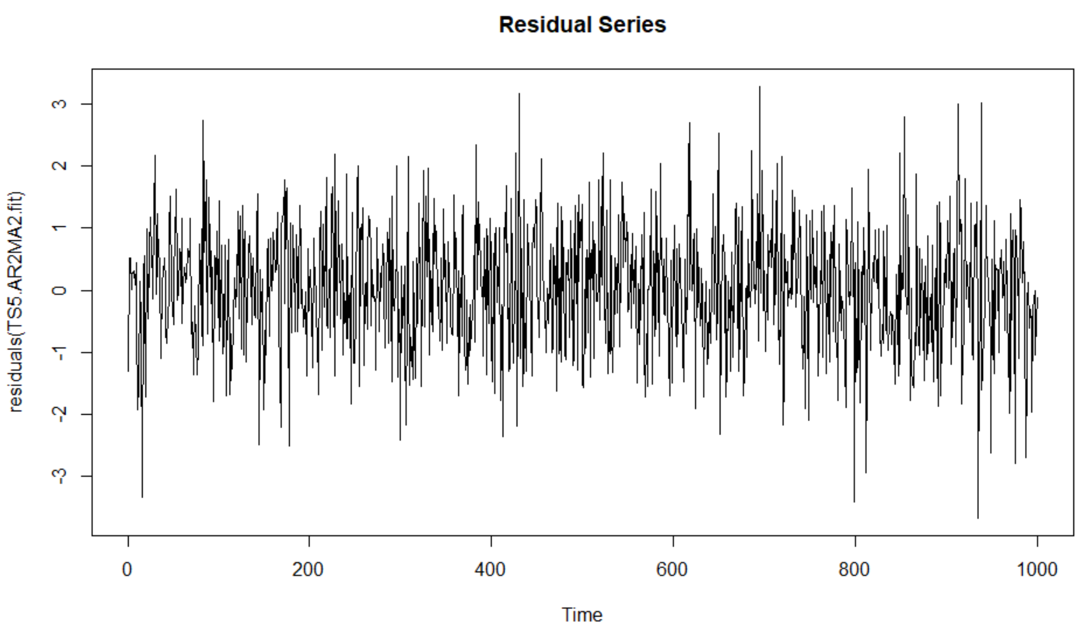
Better Model - ARMA(2,2):

```
TS5.AR2MA2.fit = arima(A4.df$TS5, order=c(2,0,2))
TS5.AR2MA2.fit

##
## Call:
## arima(x = A4.df$TS5, order = c(2, 0, 2))
##
## Coefficients:
##          ar1     ar2     ma1     ma2  intercept
##       0.5840  0.3552  0.6361  0.3235    -0.5989
## s.e.  0.1063  0.1040  0.1014  0.0325     0.9989
##
## sigma^2 estimated as 0.9954:  log likelihood = -1418.31,  aic = 2848.63
```

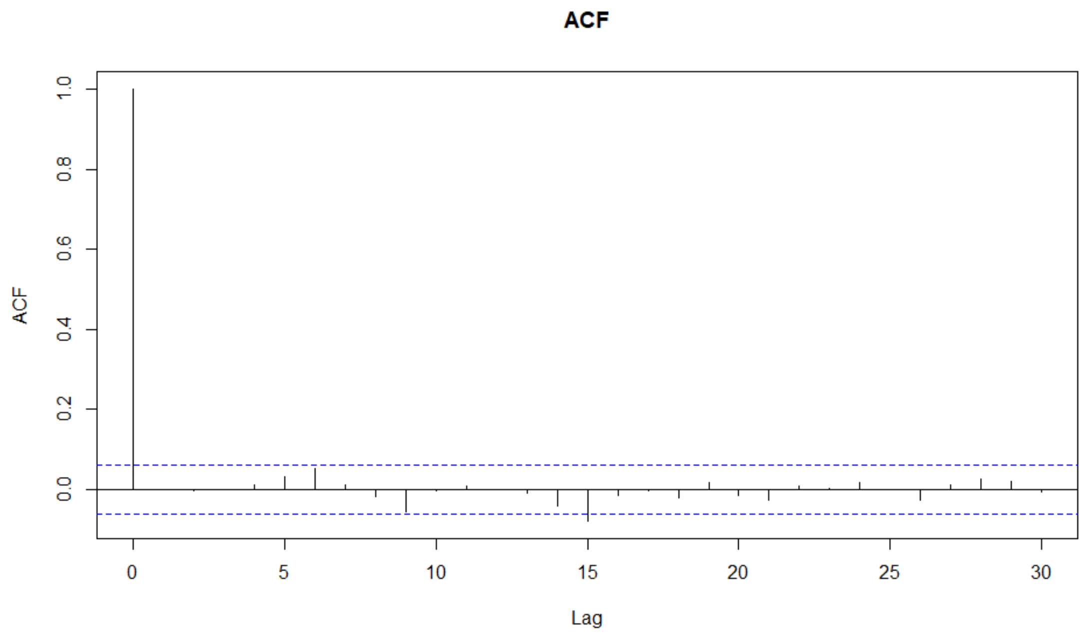
Estimated equation:

$$y_t = 0.5840y_{t-1} + 0.3552y_{t-2} + 0.6361\varepsilon_{t-1} + 0.3235\varepsilon_{t-2} + \varepsilon_t$$

```
plot(residuals(TS5.AR2MA2.fit), main="Residual Series")
```



```r
acf(residuals(TS5.AR2MA2.fit), main="ACF")
```



The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows a significant lag at lag 15. However, as this lag is weakly significant it is not of concern. Therefore, ARMA(2,2) is appropriate.

Therefore, ARMA (2,2) is the best model for this series as all estimates are significant and it has the lowest AIC score relative to the other models tried.