Programming Assignment

Comp 3270

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I certify that I wrote the code I am submitting. I did not copy whole or parts of it from another student or have another person write the code for me. Any code I am reusing in my program is clearly marked as such with its source clearly identified in comments.

For the coding portion of this assignment I selected to solve the program using the java language. I used the JGRASP IDE to write and compile my code. As the instructor suggested, I used nanoseconds to calculate my runtimes for enhanced precision. This meant that my time values would need to instead be Long type instead of Int type, so I hope that will not affect my grade since my matrix is not of type Int.

Before starting the charts, the max functions must be analyzed. Since the **Math.max** function does the following: finds the first value, finds the second value, compares them, then returns one of the two, we will just assume it has a **cost of 4**. For the three-input **max** function I implemented below the algorithms, we will assume it has the best-case **cost of 7**.

Alorithm-1

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	n + 1
3	1	$\Sigma_{i=1 \text{ to } n}(i + 1)$
4	1	$\Sigma_{i=1 \text{ to } n}(i)$
5	1	$\Sigma_{j=1 \text{ to } i}\ \Sigma_{i=1 \text{ to } n}(j + 1)$
6	6	$\Sigma_{j=1 \text{ to } i}\ \Sigma_{i=1 \text{ to } n}(j)$
7	7	$\Sigma_{i=1 \text{ to } n}(i)$
8	2	1

Multiply col.1 with col.2, add across rows and simplify

$T_1(n) = 1 + n + 1 + \Sigma_{i=1 \text{ to } n}(i + 1) + \Sigma_{i=1 \text{ to } n}(i) + \Sigma_{j=1 \text{ to } i}\ \Sigma_{i=1 \text{ to } n}(j + 1) +$

$6[\Sigma_{j=1 \text{ to } i}\ \Sigma_{i=1 \text{ to } n}(j)] + 7[\Sigma_{i=1 \text{ to } n}(i)] + 2$

$T_1(n) = 4 + n + n(n+1)/2 + n + n(n+1)/2 + [n(n+1)/2] * n + [n(n+1)/2] * n +$

$6[n(n+1)/2] * n + 7[n(n+1)/2]$

$T_1(n) = 4 + 2n + (n^2+n)/2 + (n^2+n)/2 + n^3 \ldots.$

$T_1(n) = O(n^3)$

Algorithm-2

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	n + 1
3	1	n
4	1	$\Sigma_{i=1 \text{ to } n} (i + 1)$
5	6	$\Sigma_{i=1 \text{ to } n} (i)$
6	7	$\Sigma_{i=1 \text{ to } n} (i)$
7	2	1

Multiply col.1 with col.2, add across rows and simplify

$T_2(n) = 1 + n + 1 + n + \Sigma_{i=1 \text{ to } n} (i + 1) + 6[\Sigma_{i=1 \text{ to } n} (i + 1)] + 7[\Sigma_{i=1 \text{ to } n} (i + 1)] + 2$

$T_2(n) = 4 + 2n + n(n+1)/2 + n + 6[n(n+1)/2] + 7[n(n+1)/2]$

$T_2(n) = 4 + 3n + (n^2+n)/2 + 6(n^2+n)/2 + 7(n^2+n)/2$

$T_2(n) = O(n^2)$

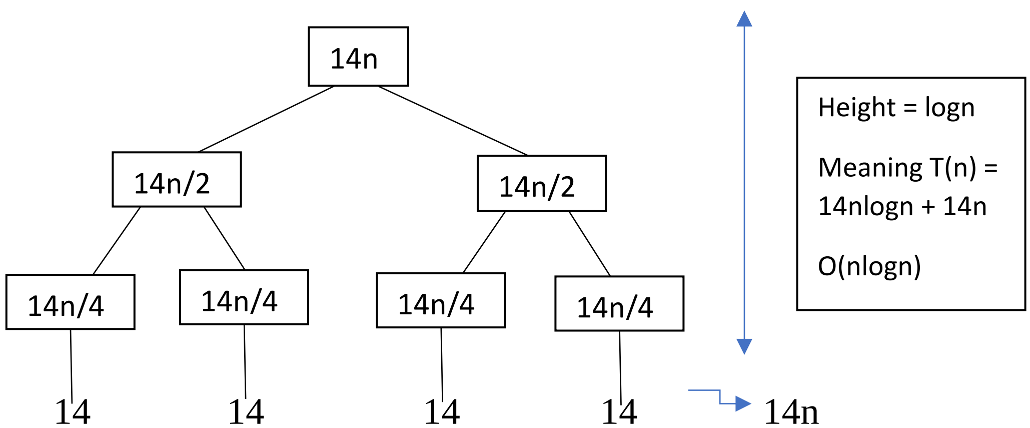
Algorithm-3

Step	Cost of each execution	Total # of times executed
1	4	1
2	11	1
Steps executed when the input is a base case: 1 or 2		
First recurrence relation: T(n=1 or n=0) = T(0) = 4, T(1) = 11		
3	5	1
4	2	1
5	1	n/2 + 1
6	6	n/2
7	7	n/2
8	2	1

9	1	n/2 + 1
10	6	n/2
11	7	n/2
12	4	1
13	4	(cost excluding the recursive call) 1
14	5	(cost excluding the recursive call) 1
15	11	1
Steps executed when input is NOT a base case: 1 to 15		
Second recurrence relation: T(n>1) = 50 + 14n		
Simplified second recurrence relation (ignore the constant term): T(n>1) = 14n		

4 + 11 + 5 + 2 + n/2 + 1 + 3n + 7n/2 + 2 + n/2 + 1 + 3n + 7n/2 + 4 + 4 + 5 + 11
= 50 + 6n + 8n = 50 + 14n

Solve the two recurrence relations using any method (recommended method is the Recursion Tree). Show your work below:



$T_3(n) = 14n\log n + 14n = O(n\log n)$

Algorithm-4

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	1
3	1	n + 1
4	10	n

5	7	n
6	2	1

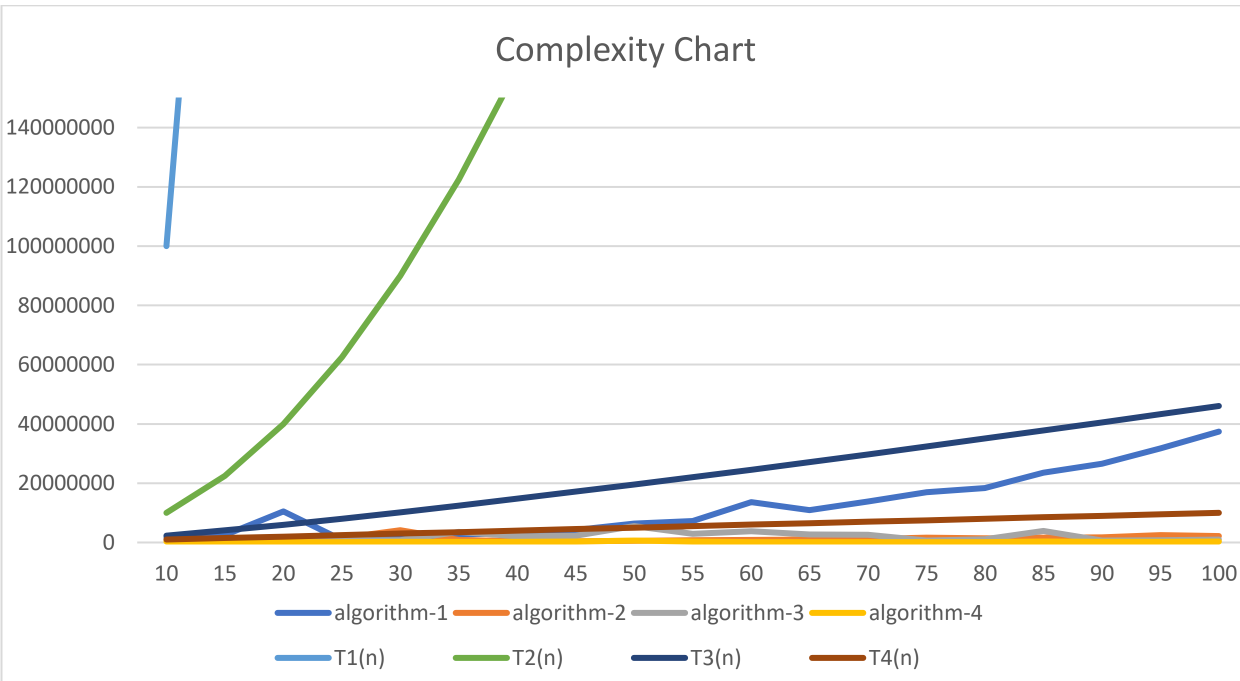
Multiply col.1 with col.2, add across rows and simplify

$T_4(n) = 1 + 1 + n + 1 + 10n + 7n + 2$

$T_4(n) = 5 + 18n$

$T_4(n) = O(n)$

Data Analysis



The algorithms for this assignment were calculated to have the following complexities:

- ⇨ Algorithm-1 = $O(n^3)$
- ⇨ Algorithm-2 = $O(n^2)$
- ⇨ Algorithm-3 = O(nlogn)
- ⇨ Algorithm-4 = O(n)

From the data provided above in the graph, we can see that the $T_1(n)$ curve has the quickest rising pattern, followed by the $T_2(n)$ curve, then $T_3(n)$, and finally $T_4(n)$. From the calculated complexities we would assume the time data from each of the algorithms would follow this pattern with the first algorithm's complexity (aka

$T_1(n)$) having the most prominent curve of the bunch since it is $O(n^3)$. The second curve also stands out as having a steep curve since the second algorithm is $O(n^2)$. Overall, these curves appear to follow the theoretical data for the times it would take to run each of the provided algorithms.