University of Tripoli / Faculty of engineering / EEE department

EE303 / Fall 2023

Special assignment

Design of longitudinal gradient coil for MRI system

1 - Introduction:

Gradient coils are required in MRI system to produce a linearly varying magnetic fields within some region. This will vary the resonance frequency linearly within that region and this can be used to excite a required slice of the patient's body with the suitable rf pulse frequency. Hydrogen atoms within this slice will be excited, and when they relaxed they generate rf signal that can be processed to plot an image.

Usually, three different coils are required to generate gradients in the three orthogonal directions *x, y,* and *z*. The basic of one of the designs of the longitudinal (*z* direction) gradient coil is the Maxwell's coil shown in figure (1). It can be shown that the magnetic field along the *z* axis produced by this pair of loops is given by:

$$B_z = \frac{\mu_0 I a^2}{2[(d/2 - z)^2 + a^2]^{\frac{3}{2}}} - \frac{\mu_0 I a^2}{2[(d/2 + z)^2 + a^2]^{\frac{3}{2}}} \tag{1}$$

It can also be shown that all even derivatives of B_z with respect to *z* at $z = 0$ equal zero. Choosing $d = \sqrt{3}a$ will cancel the third derivative and leads to the following expansion of B_z around $z = 0$:

$$B_z(z) = B_z'(0)z + \mathcal{O}[(z/d)^5] \tag{2}$$

That is, B_z around $z = 0$ is linear up through the fourth power of z. To obtain a better gradient we can add more pairs of loops as shown in figure (2). In this case we have for N pairs:

$$B_z(z) = \frac{\mu_0 a^2}{2} \sum_{n=1}^{N} I_n \left\{ [(z - z_n)^2 + a^2]^{\frac{-3}{2}} - [(z + z_n)^2 + a^2]^{\frac{-3}{2}} \right\} \tag{3}$$

Where I_n is the current of the pair *n* which is located at $\pm z_n$. By properly selecting I_n and z_n we can achieve a better *z* gradient.

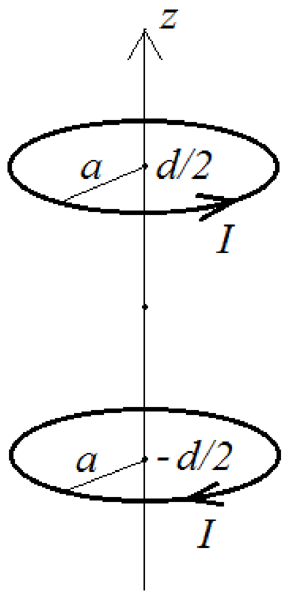

Figure (1) Maxwell coil

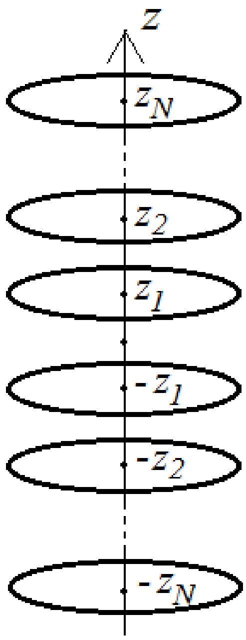

Figure (2) Multiple pairs used to improve the gradient.

2 - Design method:

One approach to design such coil is to set all I_n to a fixed value I, then searching for the values of z_n such that the following integration is minimized:

$$\varepsilon = \int_{-L}^{L} \left[B_z(z) - B_z^{desired}(z)\right]^2 dz \qquad (4)$$

Where $B_z^{desired}(z)$ is the ideal desired field distribution which is a linear function of z. ε defined by equation (4) is the error between the obtained field distribution and the ideal field distribution in the region of interest defined by $-L < z < L$.

To minimize the error defined by (4) an optimization method is required. One of the most powerful methods is the Particle Swarm Optimization (PSO).

A general procedure for PSO is summarized in the flow chart of figure (3). The searching space is N dimensional with z_n's as dimensions. Each agent has a location defined by $(z_1, z_2, z_3, \dots, z_N)$ and moves with a velocity vector of N components. First, these values are initialized randomly, then they are updated systematically based on the fitness evaluations. Whenever an agent scores a better fitness than its own best fitness, the personal best location is updated, and whenever some agent scores a better fitness than any other agent scored, the global best location is updated.

The fitness defined by equation (4) can be obtained by using one of the integration algorithms such as Trapezoidal or Simpson's methods.

Other issues regarding the boundary conditions, the maximum velocity limiting, and the inertial weight reduction not illustrated in figure (3) should be taken into consideration.

3 - Problem to be solved

Specifications:

- Loop radius $a = 0.4$m.
- Region of interest $-0.25 < z < 0.25$m (where B_z is required to be a linear function of z).
- Number of loop pairs $N = 6$.
- Current $I = 25$A.
- The desired field $B_z^{desired}(z) = 0.001z$.

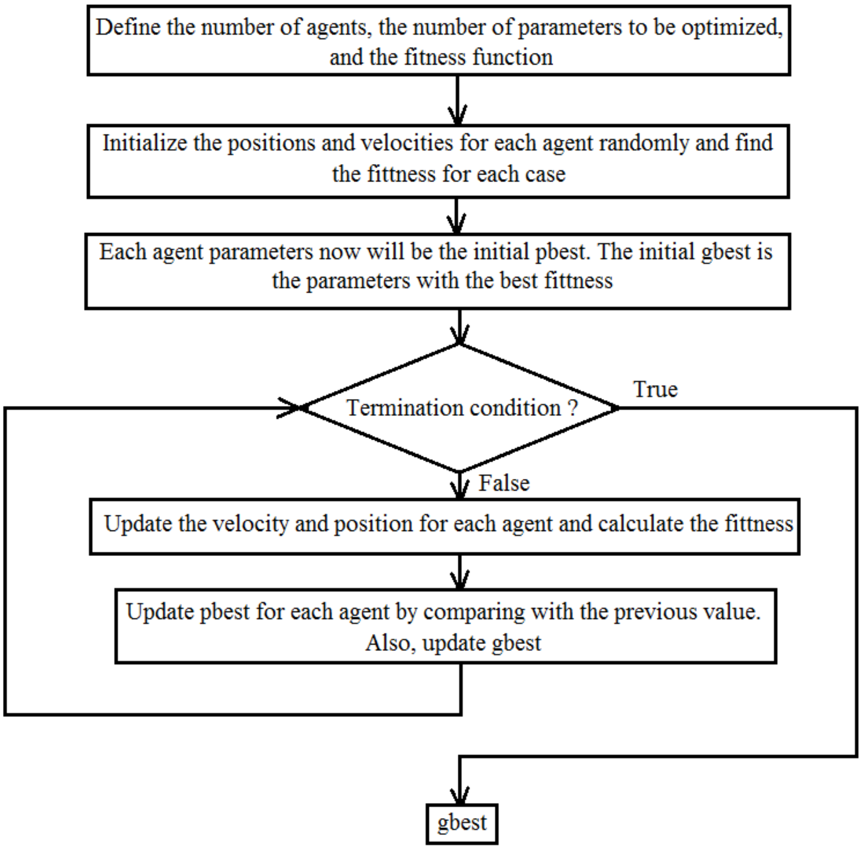

Figure (3) A flow chart showing a simplified procedure of the PSO algorithm.

PSO assumptions:

- Use a suitable number of agents (Choose a number greater than 8).
- Choose a suitable criteria to terminate the optimization process.
- Choose a suitable boundaries for the optimization space.
- Choose a suitable type of boundary conditions (absorbing, reflecting, or transparent).

Program requirements:

Write MATLAB program to design the coil according to the provided specifications by using PSO algorithm. The program should give an optimized pairs positions $(z_n's)$ and plots the obtained field together with the desired field within the region of interest.