Computer Vision

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Textbook

Multiple View Geometry in Computer Vision, Hartley, R., and Zisserman

Richard Szeliski, **Computer Vision: Algorithms and Applications**, 1st edition, 2010

Reference books

Readings for these lecture notes:

- Hartley, R., and Zisserman, A. **Multiple View Geometry in Computer Vision**, Cambridge University Press, 2004, Chapters 1-3.
- Forsyth, D., and Ponce, J. **Computer Vision: A Modern Approach**, Prentice-Hall, 2003, Chapter 2.
- **Linear Algebra and its application**

by David C Lay

These notes contain material c Hartley and Zisserman (2004), Forsyth and Ponce (2003), an Linear Algebra and its application by David C Lay

References

These notes are based

- ❑Dr. Matthew N. Dailey's course: AT70.20: Machine Vision for Robotics and HCI

- ❑Dr. Sohaib Ahmad Khan CS436 / CS5310 Computer Vision Fundamentals at LUMS

2D Transformation

- **Definition:** A mapping from one **2D coordinate** system to **another**
- Also called
  - spatial transformation,
  - geometric transformation,
  - warp

Image Registration

- **Image Registration:** Process of transforming two images so that **same features overlap**

- **Image registration** aims to **geometrically align one image with another** and is a prerequisite for **all brain imaging applications** that compare images across subjects, across imaging modalities, or across time (Toga & Thompson, 2001).

What is Image Registration?

- Process of aligning **two or more images** of the same scene.
- Used **when images are captured** at different times, **viewpoints, or sensors**.
- **Transforms one image** to match another using **spatial transformation models**.

Steps in Image Registration

1. Feature Detection: Identify key points (e.g., edges, corners).

2. Feature Matching: Pair corresponding features between images.

3. Transformation Estimation: Compute a mathematical model to align images.

4. Resampling & Interpolation: Transform and interpolate the image to match.

Applications of Image Registration

- **Medical Imaging:** Aligning MRI, CT, and PET scans for diagnosis.
- **Remote Sensing:** Change detection, disaster assessment, and multi-sensor fusion.
- **Computer Vision & AR:** Object tracking, video stabilization, and augmented reality.
- **Robotics & Autonomous Systems:** SLAM for robotics, self-driving car navigation.
- **Forensics & Security:** Fingerprint recognition, facial authentication.
- **Astronomy & Space:** Aligning celestial images for deep space observations.

Example: Medical Imaging

- **Image registration** helps align **MRI and CT scans** for accurate diagnosis.

- Detects tumor growth by comparing **past and current scans**.

- Used in PET-CT fusion to combine anatomical and functional imaging.

Example: Remote Sensing

- Satellite images are aligned for change detection.
- Used for tracking deforestation, urban expansion, and disaster damage.
- Helps in merging optical and radar images for better analysis.

Conclusion

- **Image registration** is crucial for various fields like healthcare, AI, and space.

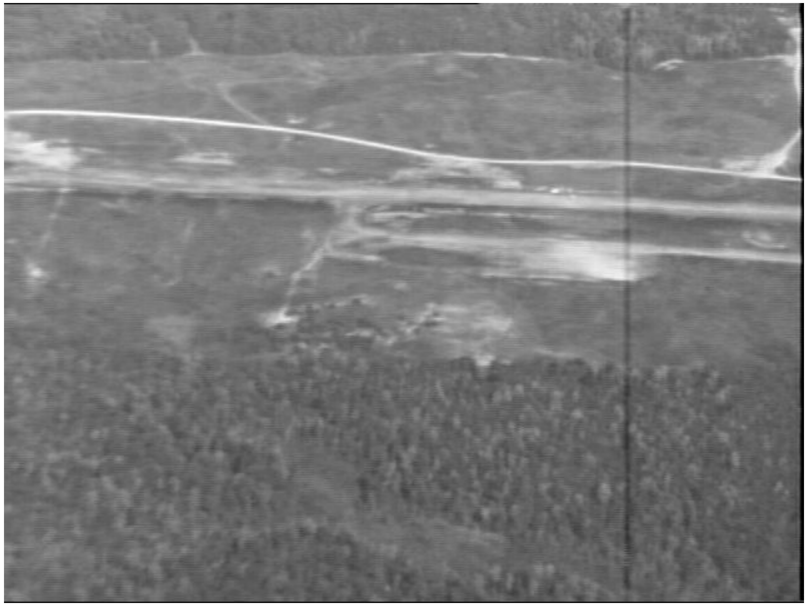
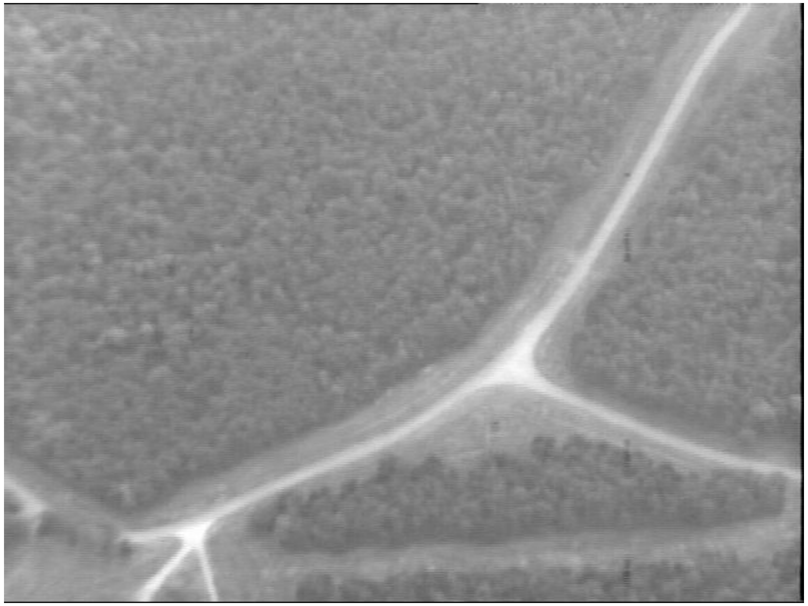
- **Aligns images** from different sources for improved analysis.

- Enables better decision-making through accurate visual integration.

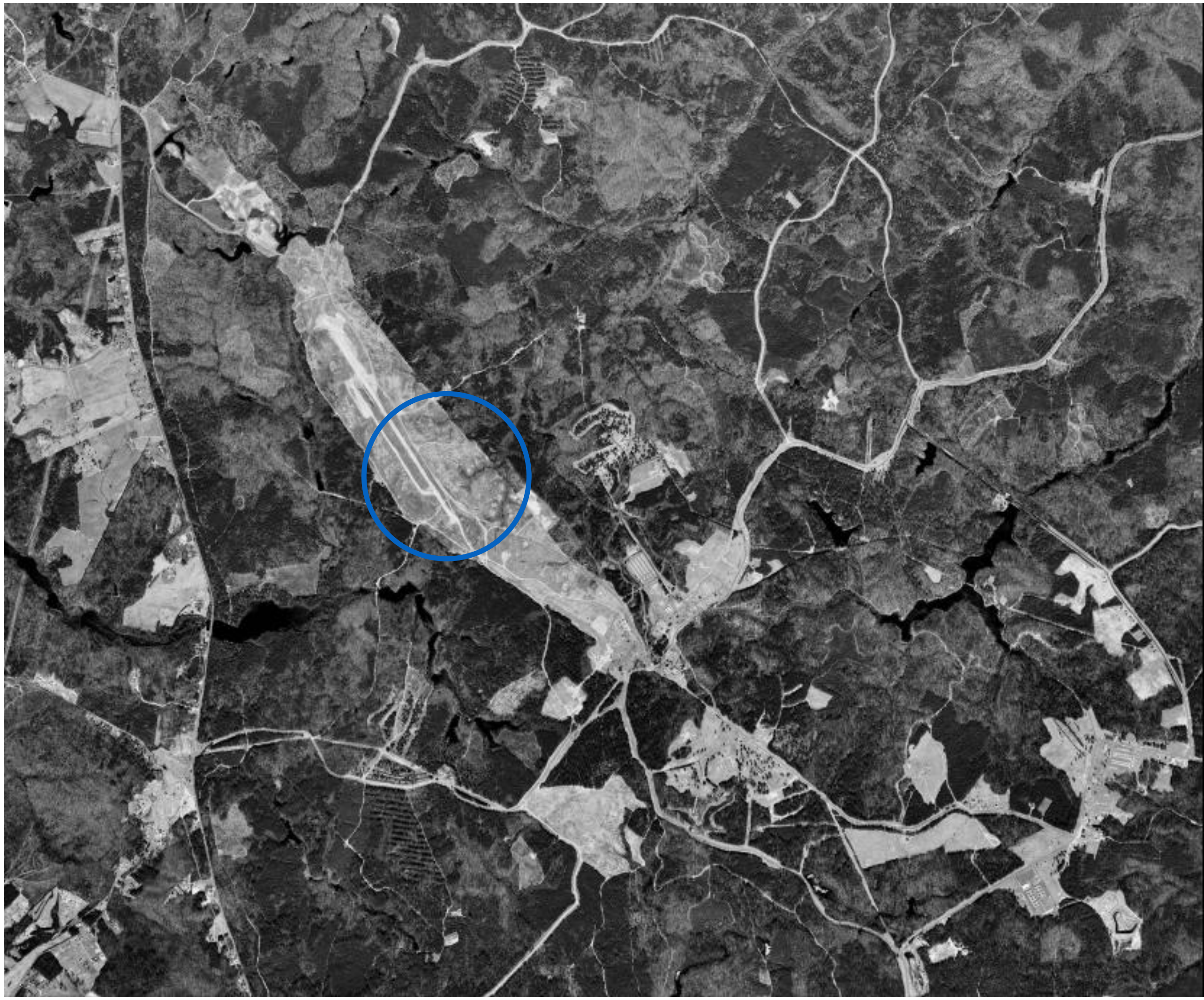
Example Application: Image Registration

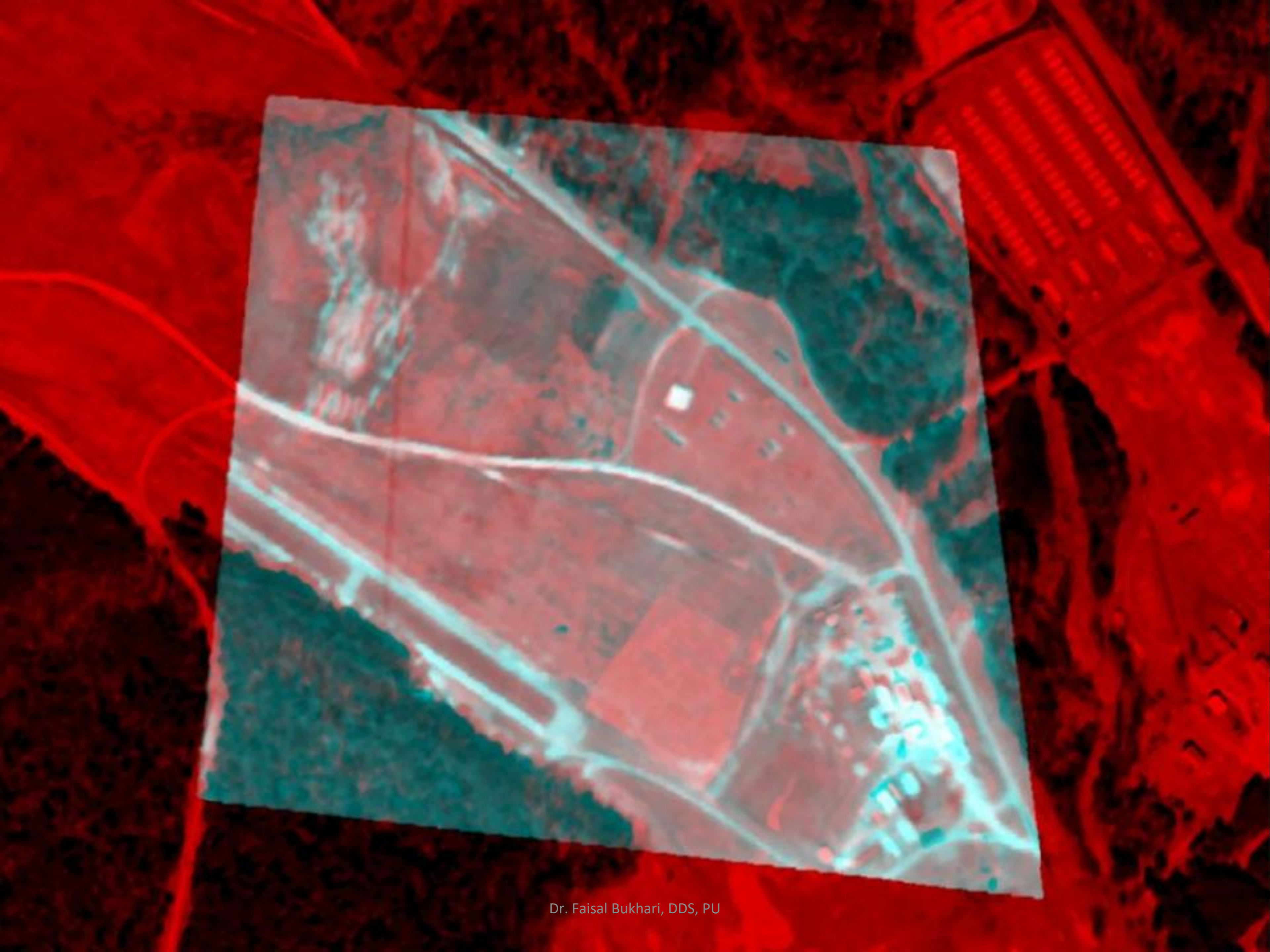
Reference
Image

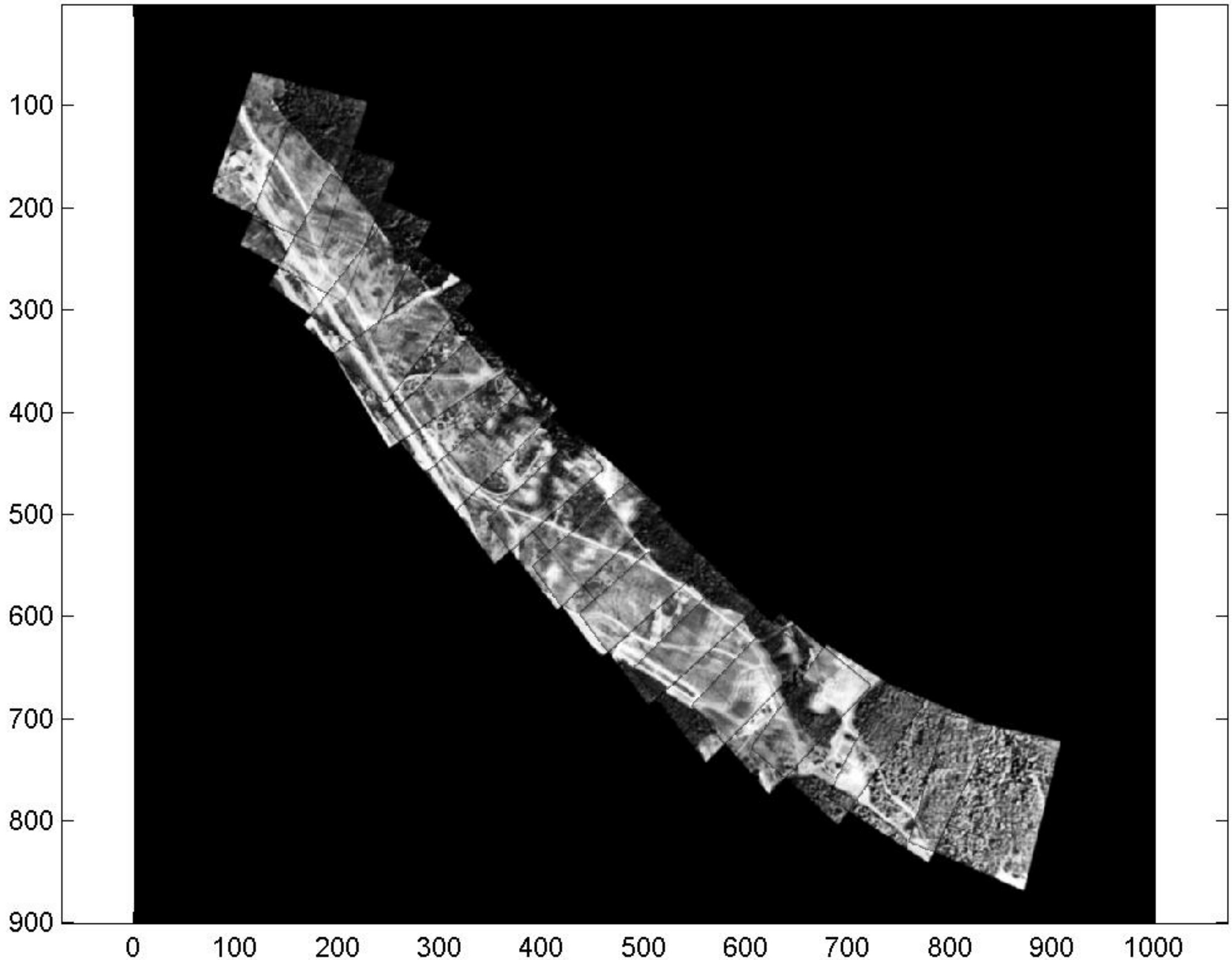
Mission Images


Registration = Computing Transformation



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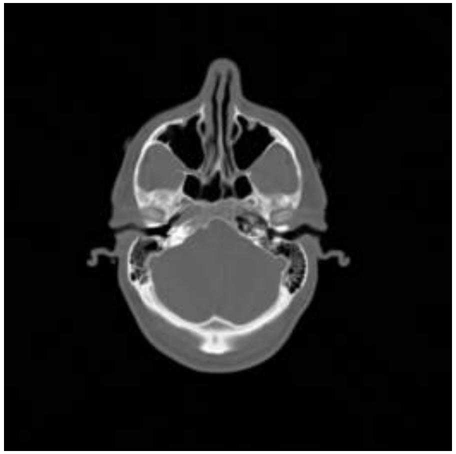

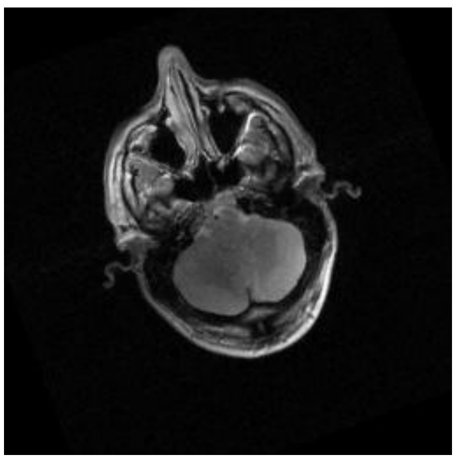

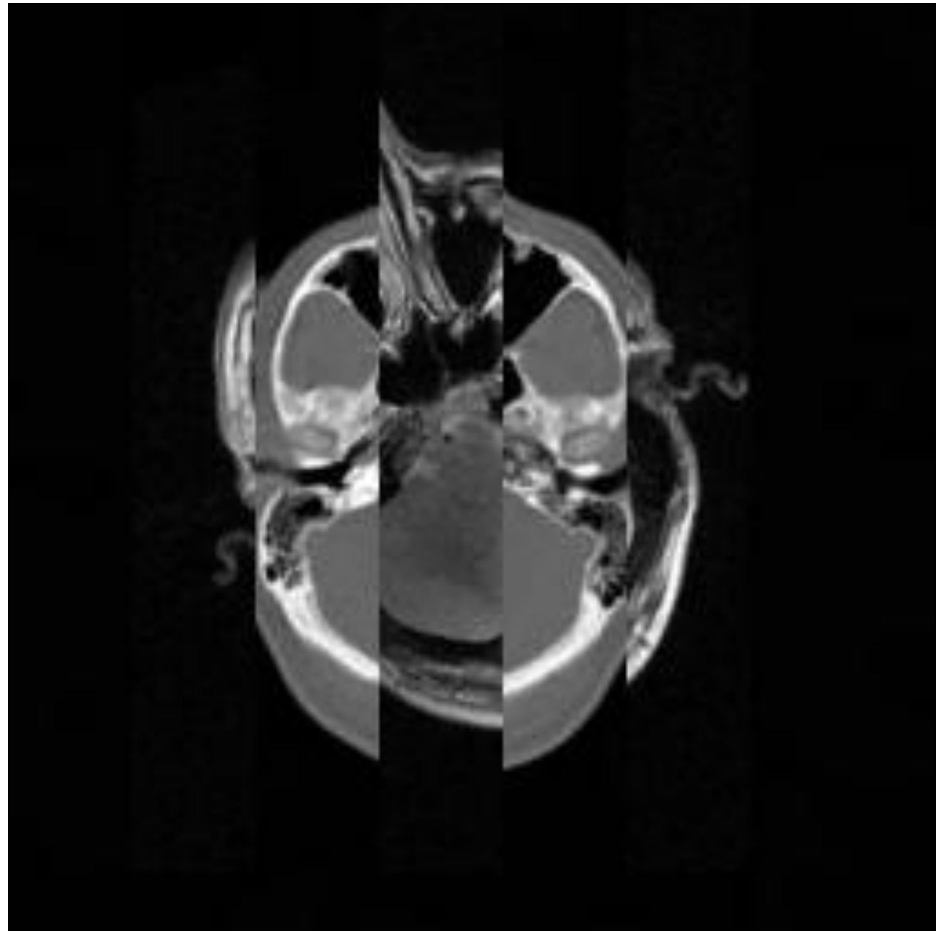

Applications of 2D Image Registration

Panoramas
Multiple Images Stitched Together

Panoramas
Multiple Images Stitched Together

Applications of 2D Image Registration

Image by Sergey Semenov (http://www.sergesemenov.com/) - Winner of Epson International Photographic Pano Award 2013
http://www.dailymail.co.uk/sciencetech/article-2260276/New-York-youve-seen-Incredible-interactive-panorama-lets-zoom.html

Applications of 2D Image Registration

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Spherical 360^{o} Imaging

Lady Bug Camera, by Point Grey
6 0.8MP cameras image 75% of a full sphere
http://www.ptgrey.com/products/ladybug2/ladybug2_360_video_camera.asp

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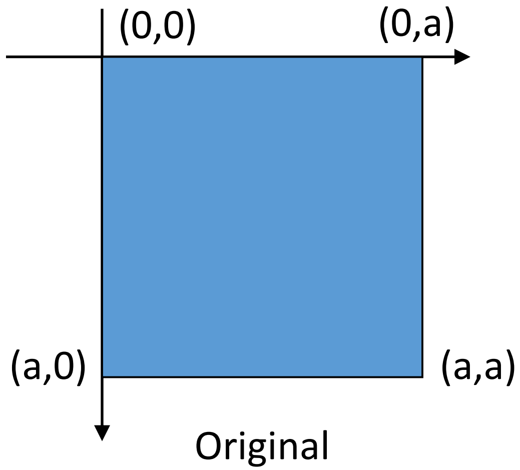
2D Transformations

- Basic operation of all 2D transformations is matrix multiplication

  - Point to be transformed: $(x, y)^{\mathsf{T}}$
  - Point after transformation: $(x', y')^{\mathsf{T}}$

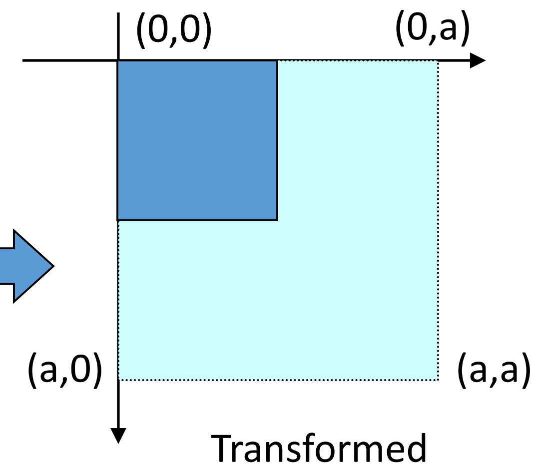
$$\underbrace{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}_{2\times 2}}_{\text{Transformation Matrix}} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1}}_{\text{Position before transformation}} = \begin{bmatrix} a_1 x + a_2 y \\ a_3 x + a_4 y \end{bmatrix}_{2\times 1} = \underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix}_{2\times 1}}_{\text{Position after transformation}}$$

Example



Apply

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$



$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5a \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5a \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} a \\ a \end{bmatrix} = \begin{bmatrix} 0.5a \\ 0.5a \end{bmatrix}$$

2D Transformations

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 0.5 \end{bmatrix} = ?$$

In general, scaling (zoom / unzoom) transformation is given by

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

2D Transformations

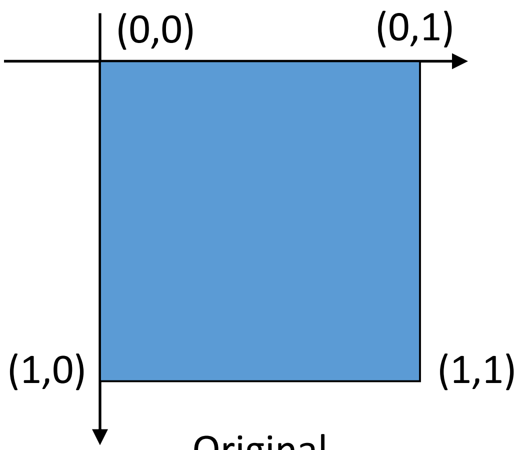
$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = ?$$

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

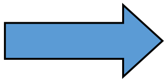
$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$

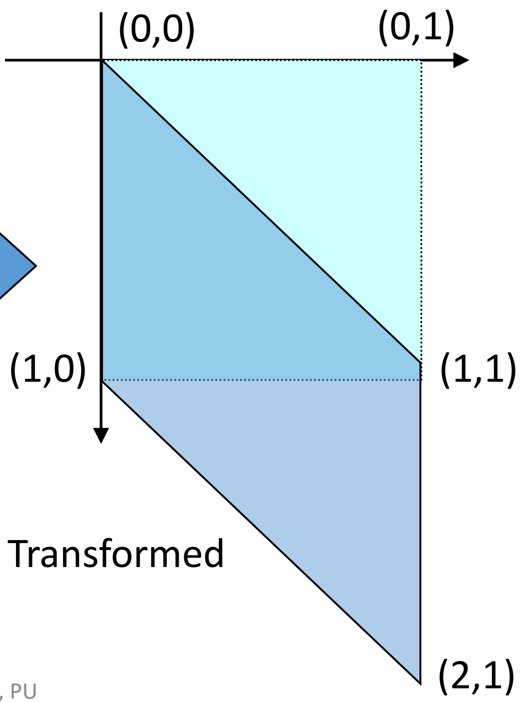
$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$



Original





Transformed

Shear in x-direction

$$\begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ey \\ y \end{bmatrix}$$

x-coordinate moves with an amount proportional to the y-coordinate

For example:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + 1 \times 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

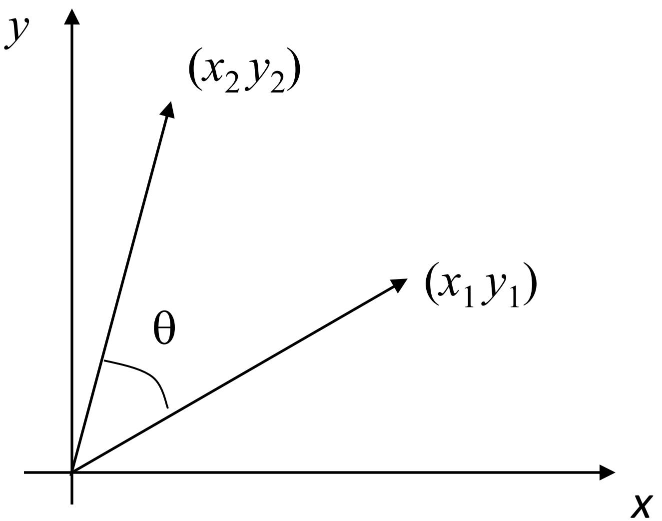
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 + 1 \times 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Shear in y-direction

$$\begin{bmatrix} 1 & 0 \\ e & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ ex + y \end{bmatrix}$$

y-coordinate moves with an amount proportional to the x-coordinate

Rotation

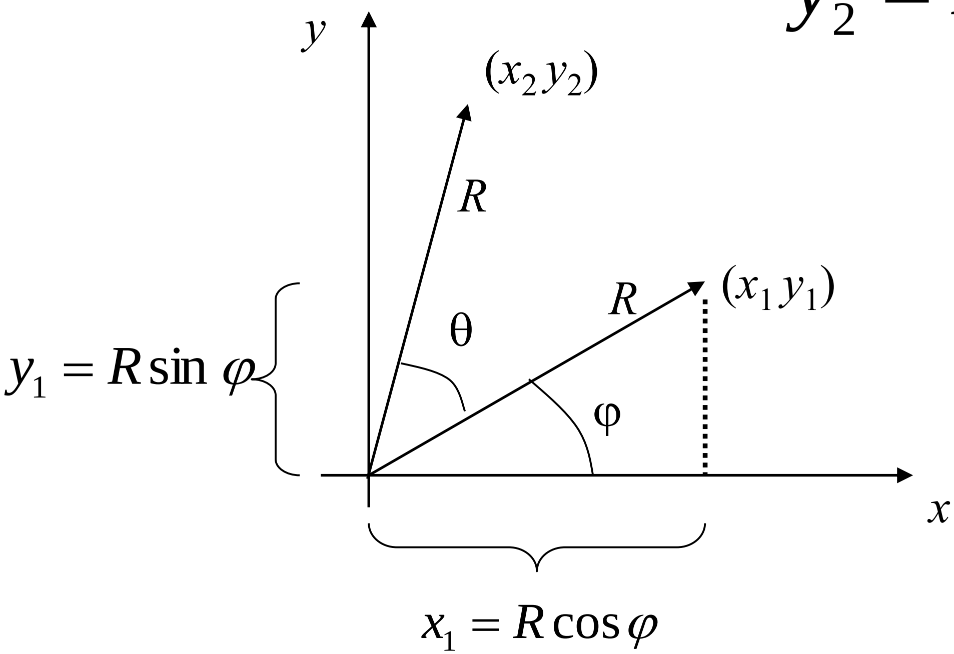


- Task: Relate (x_2, y_2) to (x_1, y_1)

Rotation

$$x_2 = R\cos(\theta + \varphi)$$
$$y_2 = R\sin(\theta + \varphi)$$

$$x_2 = R\cos\theta\cos\varphi - R\sin\theta\sin\varphi$$
$$y_2 = R\sin\theta\cos\varphi + R\cos\theta\sin\varphi$$



$$x_2 = x_1\cos\theta - y_1\sin\theta$$
$$y_2 = x_1\sin\theta + y_1\cos\theta$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

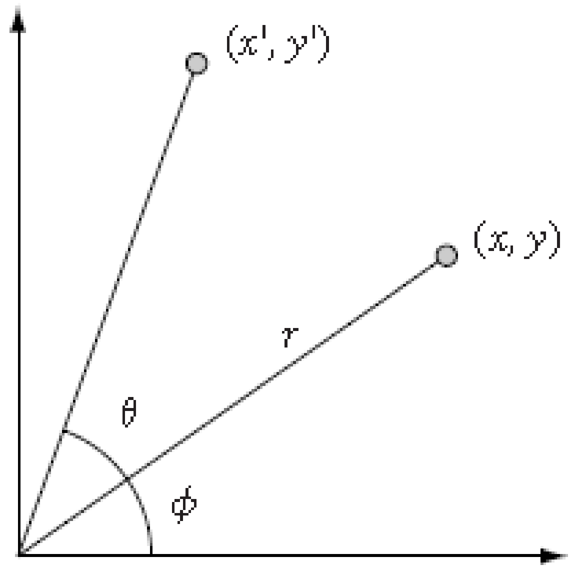
R is rotation by θ **counterclockwise about origin**

Alternative Method for Derivation

Example Let **T:** $\mathbb{R}^2 \to \mathbb{R}^2$ be the **transformation** that **rotates each point** in $\mathbb{R}^2$ **about the origin** through an **angle** $\boldsymbol{\varphi}$ with **counterclockwise** rotation for a **positive angle**.

We could show geometrically that such a transformation is linear. Find the **standard matrix A** of this **transformation**.

Derivation of Rotation matrix:



x = r $cos\varphi$ -----------(1)

y = r $sin\varphi$ ------------(2)

Squaring (1) and (2) and then adding, we get

r = $\sqrt{x^2+y^2}$

x'= r $cos(\varphi + \theta)$ -------(3)

$\because\ cos(\varphi + \theta) = \cos\varphi\cos\theta - \sin\varphi\sin\theta$

Substitute the value of cos$(\varphi + \theta)$ in (3), we get

x'= r cosφcos θ - r sinφ $sin\ \theta$ -----(4)

y' = r $sin(\varphi + \theta)$-------(5)

$\because$ $sin(A + B) =$ **sinAcos B – cosAsin B**

Substitute the value of $\sin(\varphi + \theta)$ in (5), we get

y' **= r sinφcos θ + r cosφsin θ ------(6)**

Substitute the values of x and y from (1) and (2) in (4), we get

x' **= xcosθ - ysin θ ---------------------(7)**

Substitute the values of x and y from (1) and (2) in (6), we get

y' **= ycos θ + xsin θ**

y' **= xsin θ + ycosθ ------------------(8)**

Using (7) and (8), we get

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation

- **R(θ) = $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$**
- Rotation Matrix has some special properties
  - Each row/column has **norm of 1** [prove]
  - Each row/column is **orthogonal** to the other [prove]
  - So Rotation matrix is an **orthonormal** matrix
- Inverse of an **orthonormal matrix** is its transpose [prove]

To prove that each row (or column) is **orthogonal** to the other, we compute their dot product.

Step 1: Define Rows and Compute Dot Product

The first row of **R($\boldsymbol{\theta}$) is**

r_1 = (cosθ,−sinθ)

The second row of **R($\boldsymbol{\theta}$) is**

r_2 = (sinθ,−cosθ)

The dot product of these two row vectors is:

$r_1 \cdot r_2$ = (cosθ)(sinθ) + (−sinθ)(cosθ)

= cosθsinθ − sinθcosθ

= 0

Since their dot product is **zero**, the rows are orthogonal.

Step 2: Define Columns and Compute Dot Product

The first col of **R(θ) is**

$$c_1 = \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix}$$

The second col of **R(θ) is**

$$c_2 = \begin{bmatrix} -sin\theta \\ cos\theta \end{bmatrix}$$

The dot product of these two row vectors is:

$c_1 \cdot c_2$ = (cosθ)(-sinθ) + (sinθ)(cosθ)

=-cosθsinθ +sinθcosθ

= 0

Since their dot product is **zero**, the columns are orthogonal.

Orthonormal Matrix

An **orthonormal matrix** is a **square matrix** whose **columns and rows are both orthogonal and unit vectors** (having a magnitude of 1).

- Each row/column of **R(θ)** has **norm of 1**
- Each row/column of **R(θ)** is **orthogonal** to the other
- So Rotation matrix **R(θ)** is an **orthonormal** matrix
- Inverse of an **orthonormal matrix** is its transpose

Inverse of an orthonormal matrix is its transpose

R(θ) = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Inverse of **R(θ),** an **orthonormal matrix** is its transpose.

Transpose of **R(θ) is**

R(θ)$^{-1}$ = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$\mathrm{R}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{aligned} \mathbf{det(R)} &= (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \end{aligned}$$

$$\mathbf{adj(R)} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\mathrm{R}^{-1} = \frac{1}{\det(\mathrm{R})}\ \mathrm{adj(R)}$$

$$= \frac{1}{1}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\mathrm{R}(\theta)^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Identifying Rotation and Reflection Matrices

$\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$ = (-1)(-1) – 0 =1

It is a rotation matrix

$\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ = -1

It is a reflection matrix.

$\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$ = -1

It is a reflection matrix.

Homogeneous system

In general, a matrix multiplication lets us linearly combine components of a vector

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

- This is sufficient for **scale**, **rotate**, **skew transformations**.
- But notice, we **can't add a constant**, within the **same format.**

Homogeneous system

- Solution is to use **homogeneous coordinates** for vectors

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + & by + c \\ dx + & ey + f \\ & 1 \end{bmatrix}$$

- Now we can **rotate**, **scale**, and **skew** like before, AND **translate** (note how the multiplication works out, above)

Translation

- In matrix form

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

- We could not have written **T** **multiplicatively** without using **homogeneous coordinates**
- Compact way to write
- $\boldsymbol{x}' = \begin{bmatrix} \mathbf{I}_{2\times 2} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix} \boldsymbol{x}$

Basic 2D Transformations

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & e_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ e_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Concatenation or Composition of Transformations

- Suppose we first want to scale, then rotate
  - **x' = Sx ------------(1)**
  - **x''= Rx'------------(2)**

  Substitute value of x' from (1) into (2), we get

  = R(Sx)
  = (RS) x (Using associate property of matrices)
- So two transformations can be represented by a **single** transformation matrix
  - **M = RS**
- Important: **read** from **right**-side to **get order of application of transformations**

Order of Transformations

Example 1:

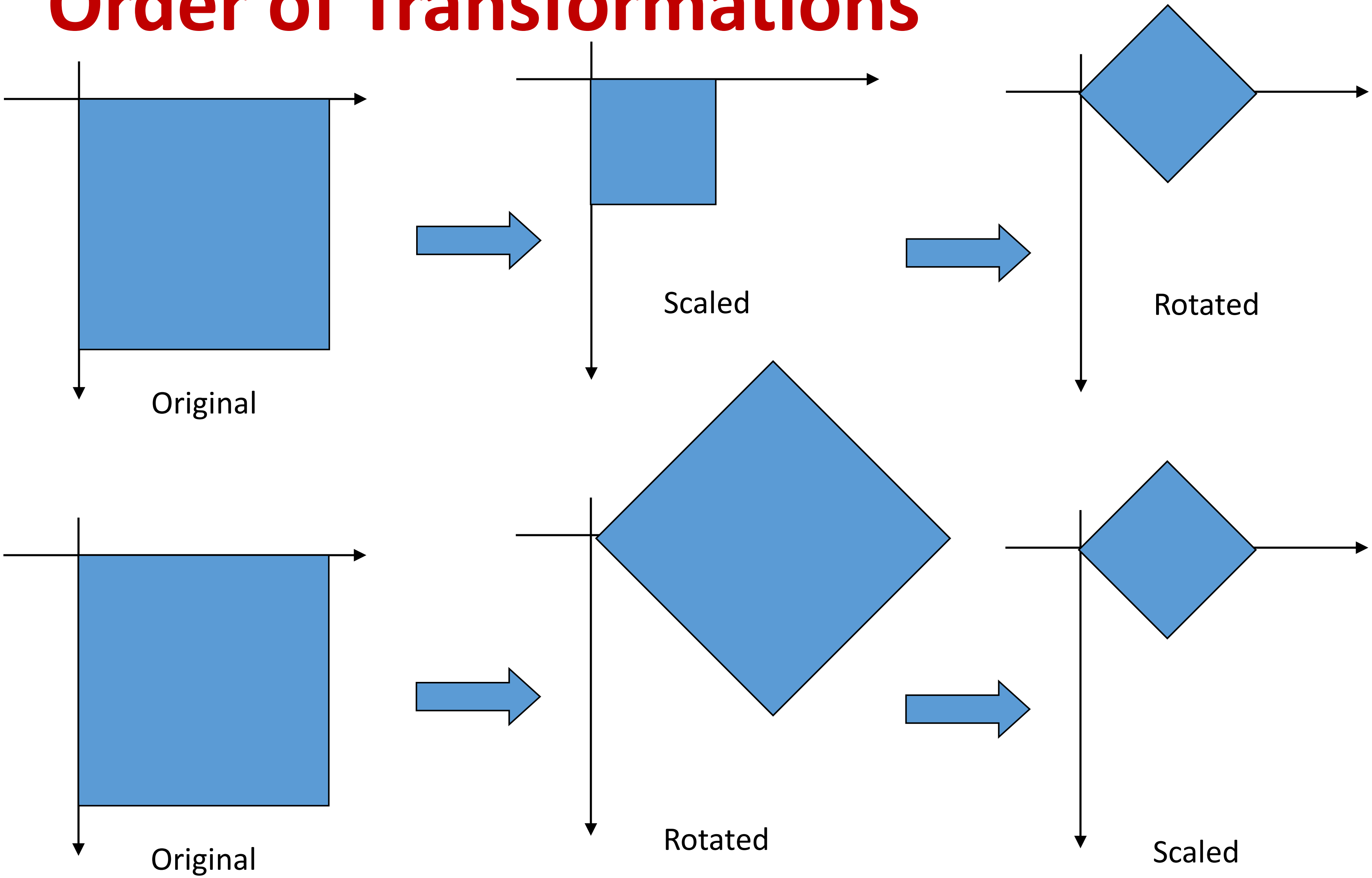
Scaling of x and y coordinates by 0.5

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation by 45^0 in counter clock direction

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos 45^0 & -\sin 45^0 & 0 \\ \sin 45^0 & \cos 45^0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Order of Transformations



Order of Transformations

Example 2: Scaling of x coordinates by 0.5

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation by 45^0 in counter clock direction

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos45^0 & -\sin45^0 & 0 \\ \sin45^0 & \cos45^0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Order of Transformations