**Theorem:**
Let A be an $n \times n$ matrix. Then A is invertible if and only if:

- The number 0 is not an eigenvalue of A.
- The determinant of A is not zero

*" A matrix must be **non-invertible** (i.e., rank-deficient) to have a **non-trivial** solution to $Ax=0$."*

Imagine the matrix turns a square into another shape.

- If the shape becomes flat (like a line), the determinant = 0 ⇒ **not invertible.**
- If the shape stays 2D (stretched or rotated), the determinant ≠ 0 ⇒ **invertible.**

**Theorem:**
The eigenvalues of a triangular matrix are the entries on its main diagonal

**Eigenvectors for different eigenvalues are Linearly Independent.**

**Theorem:**
If $v_1, \ldots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \ldots, \lambda_r$ of an matrix A, then the set $\{v_1, \ldots, v_r\}$ is linearly independent.

**Theorem:**
If A is a triangular matrix, then det A is the product of the entries on the main diagonal of A

The scalar $\lambda$ is an eigenvalue of A if and only if the equation $(A- \lambda I)x = 0$ has a nontrivial solution, that is, iff equation has free variable.

**Theorem:**
Let A and B be $n \times m$ matrices.
a. A is invertible if and only if det ≠ 0

b. det AB=(det A)(det B)
c. det (A)^T =det A

**Orthogonal Matrix:**

A square matrix having the same inverse and transpose.
Cols are Orthonormal.
A^(-1) = A^(T)
A^(T) * A = I = A * A^(T)

Orthogonal matrices have these properties:

- All **columns (and rows)** are:
    - **Orthogonal to each other** (dot product = 0)
    - **Unit length** (magnitude = 1)