Why can't we use the trivial solution h=0 when estimating a homography, and how do we avoid it?

Problem:
h=0 (the zero homography) maps all points to (0,0,0), which is meaningless (no valid transformation) and loses all structure.

Solution:
- Constraint: Force : ||h // =1 (unit norm) to avoid the zero solution.
- SVD automatically handles this by returning the smallest singular vector, which is non-zero and normalized.

SVD (Singular Value Decomposition) helps by:

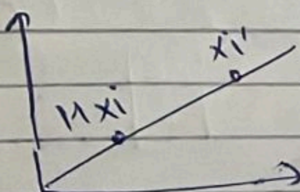
- Decomposing A into UΣVT
- The solution
- h is the last column of V (corresponding to the smallest singular value in Σ).
- This gives the least-squares solution, minimizing the error // Ah // while satisfying // h // =1.

$x_i' = H x_i$

$\vec{x}_i' = k_i H \vec{x}_i$

where x_i and x_i' are homogenous equ. (vectors)
H = homography matrix and k is
the scalar to normalize $H x_i$

x_i' and $H\vec{x}_i$ are collinear, proved by
cross product $\rightarrow$ are of parallelogram $= \underline{\underline{0}}$



It shows x_i' & $H x_i'$
are scaled version of
each other & collinear.

$$x_i' = \begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix}, \quad H x_i = \begin{bmatrix} h^{1T} x_i \\ h^{2T} x_i \\ h^{3T} x_i \end{bmatrix}$$

$$\vec{x}_i' \times H\vec{x}_i = \begin{array}{ccc} i & j & k \end{array} \begin{bmatrix} x_i' & y_i' & w_i' \\ h^{1T} x_i & h^{2T} x_i & h^{3T} x_i \end{bmatrix} = [0]_{3\times 1}$$

$$\vec{x}_i' \times H\vec{x}_i' = \begin{bmatrix} y_i' h^{3T} x_i - w_i' h^{2T} x_i \\ w_i' h^{1T} x_i - x_i' h^{3T} x_i \\ x_i' h^{2T} x_i - y_i' h^{1T} x_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[y_i' h^{3T} x_i - w_i' h^{2T} x_i] = [0 \quad 0 \quad 0 \quad - w_i' x_i \; - w_i' y_i \; - w_i' w_i \quad + y_i' x_i + y_i' y_i + y_i' w_i]$$

$$= [\, 0^T \quad -w_i' x_i^T \quad y_i' x_i^T \,]$$

$$\begin{bmatrix} 0^T & -w_i' x_i^T & y_i' x_i^T \end{bmatrix}_{1\times 9} \begin{bmatrix} \vec{h_1} \\ \vec{h_2} \\ \vec{h_3} \end{bmatrix}_{9\times 1} = \begin{bmatrix} 0 \end{bmatrix}_{1\times 1} \text{ — } (A)$$

$$(w_i' h^{1T} x_i - x_i' h^{3T} x_i) = \begin{bmatrix} w_i' x_i & w_i y_1 & w_i w_1 \\ 0 & 0 & 0 \\ -x_i' x_1 & -x_i' y_1 & -x_i' w_1 \end{bmatrix}$$

$$= \begin{bmatrix} w_i' x_i^T & 0^T & -x_i' x_i^T \end{bmatrix}$$

$$\begin{bmatrix} w_i' x_i^T & 0^T & -x_i' x_1 \end{bmatrix}_{1\times 9} \begin{bmatrix} \vec{h_1} \\ \vec{h_2} \\ \vec{h_3} \end{bmatrix}_{9\times 1} = \begin{bmatrix} 0 \end{bmatrix}_{1\times 1} \text{ — } (B)$$

$$(x_i' h^{2T} x_i - y_i' h^{1T} x_i) = \begin{bmatrix} -y_i x_i & -y_i y_i & -y w_i \\ x_i' x_i & x_i y_1 & x_i w_i \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -y' x_i^T & x_i x_i^T & 0^T \end{bmatrix}$$

$$\begin{bmatrix} -y_i' x_i^T & x_i x_1^T & 0^T \end{bmatrix} \begin{bmatrix} h_1^T \\ h_2^T \\ h^{3T} \end{bmatrix} = 0 \text{ — } (C).$$

$$\begin{bmatrix} 0^T & -w_i' x_i^T & y_i x_1^T \\ w_i' x_i & 0^T & -x_1' x_1 \\ -y_i' x_i^T & x_i^T x_i & 0^T \end{bmatrix}_{3\times 9} \begin{bmatrix} \vec{h^1} \\ \vec{h^2} \\ \vec{h^3} \end{bmatrix}_{9\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

$$\begin{array}{ccc} & P1 & P2 \\ img1 = & (1,2), & (2,3) \\ img2 = & (2,3), & (4,5) \end{array}$$

solve for point1 (P1).

$x=1, y=2, \quad x'=2, y'=3$

$$\begin{bmatrix} 0 & 0 & 0 & -x & -y & -1 & +y'x & y'y & y' \\ x & y & 1 & 0 & 0 & 0 & -x'x & -x'y & -x' \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & -2 & -1 & 3 & 6 & 3 \\ 1 & 2 & 1 & 0 & 0 & 0 & -2 & -4 & -2 \end{bmatrix}$$

Solve for point 2:

$x=2, y=3, \quad x'=4, y'=5.$

$$\begin{bmatrix} 0 & 0 & 0 & -2 & -3 & 1 & 10 & 15 & 5 \\ 2 & 3 & 1 & 0 & 0 & 0 & -8 & -12 & -4 \end{bmatrix}$$

$2n \times 9$ $\quad$ $8\times 9 \quad 9\times 1 \quad 8\times 1$

$$A = \begin{bmatrix} 0 & 0 & 0 & -1 & -2 & -1 & 3 & 6 & 3 \\ 1 & 2 & 1 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & -2 & -3 & -1 & 12 & 18 & 6 \\ 2 & 3 & 1 & 0 & 0 & 0 & -8 & -12 & -4 \end{bmatrix}$$

Solve $Ah = 0$.

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & -1 & -2 & -1 & 3 & 6 & 3 \\ 1 & 2 & 1 & 0 & 0 & 0 & -2 & -4 & -2 \\ 0 & 0 & 0 & -2 & -3 & -1 & 12 & 18 & 6 \\ 2 & 3 & 1 & 0 & 0 & 0 & -8 & -12 & -4 \end{bmatrix}}_{4\times 9} \underbrace{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_9 \end{bmatrix}}_{9\times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{4\times 1}$$