Computer Vision

Dr. Syed Faisal Bukhari

Associate Professor

Department of Data Science

Faculty of Computing and Information Technology

University of the Punjab

Textbook

Multiple View Geometry in Computer Vision, Hartley, R., and Zisserman

Richard Szeliski, **Computer Vision: Algorithms and Applications**, 1st edition, 2010

Reference books

Readings for these lecture notes:

- Hartley, R., and Zisserman, A. **Multiple View Geometry in Computer Vision**, Cambridge University Press, 2004, Chapters 1-3.

- Forsyth, D., and Ponce, J. **Computer Vision: A Modern Approach**, Prentice-Hall, 2003, Chapter 2.

- **Linear Algebra and its application**

by David C Lay

These notes contain material c Hartley and Zisserman (2004), Forsyth and Ponce (2003), an Linear Algebra and its application

by David C Lay

References

These notes are based

- Dr. Matthew N. Dailey's course: AT70.20: Machine Vision for Robotics and HCI

- Dr. Sohaib Ahmad Khan CS436 / CS5310 Computer Vision Fundamentals at LUMS

Inverse Transformations

- Inverse transformation should 'undo' the effect of the original transformation
- Simply taking the matrix inverse will work $AA^{-1} = I$
- Inverse Transforms
- $$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} \frac{1}{S_x} & 0 & 0 \\ 0 & \frac{1}{S_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -e_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -e_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
- Remember that when inverting concatenation of transforms, their order reverses
- $(ABC)^{-1} = C^{-1}\, B^{-1}\, A^{-1}$

Groups

- A group is a **set** ***G*** together with an **operation** $\bullet$ that combines **two elements** of ***a*** and ***b*** to form **another element** $a \bullet b$. To form a group, $(G, \bullet)$ must satisfy the following axioms
- **Closure**
  - For all a, b in G, $a \bullet b$ is also in G
- **Associativity**
  - For all a, b, c in G, $(a \bullet b) \bullet c = a \bullet (b \bullet c)$
- **Identity**
  - There exists an element e in G s.t. $a \bullet e = e \bullet a = a$
- **Inverse**
  - For each a in G, there exists an element b in G s.t. $a \bullet b = e$

Example 1: Integers Under Addition $(\mathbb{Z}, +)$

1. **Closure:** If a, b $\in \mathbb{Z}$, then a + b $\in \mathbb{Z}$.

   Example: $4 + (-7) = -3 \in \mathbb{Z}$.

2. **Associativity:** $(a + b) + c = a + (b + c)$.

   Example: $(2 + 3) + 4 = 2 + (3 + 4) = 9$.

3. **Identity:** The identity element is 0.

   Example: $6 + 0 = 0 + 6 = 6$.

4. **Inverse:** Every a $\in \mathbb{Z}$ has an inverse -a such that $a + (-a) = (-a) + a = 0$.

   Example: $5 + (-5) = 0$, $(-7) + 7 = 0$.

Conclusion: Since all properties hold, **$(\mathbb{Z}, +)$ is a group.**

Example 2: Nonzero Rational Numbers under Multiplication ($\mathbb{Q}^*$, ×)

1. Closure: If a, b $\in \mathbb{Q}^*$, then a × b $\in \mathbb{Q}^*$.

Example: $\frac{3}{4} \times \frac{2}{5} = \frac{3}{10} \in \mathbb{Q}^*$.

2. Associativity: (a × b) × c = a × (b × c).

Example: $(2 \times 3) \times 5 = 2 \times (3 \times 5) = 30$.

3. Identity: The identity element is 1.

Example: $\frac{5}{7} \times 1 = \frac{5}{7}$.

4. Inverse: The inverse of a is $\frac{1}{a}$.

Example: The inverse of $\frac{3}{4}$ is $\frac{4}{3}$, since $(\frac{3}{4}) \times (\frac{4}{3}) = 1$.

$\mathbb{Q}^*$ with **multiplication is a group**.

Hierarchy of Transformation Groups

Translation $\mathbf{x}' = [\, \mathbf{I}_{2\times2} \mid t \,]_{2\times3}\, \mathbf{x}$

$$x' = x + t_x$$
$$y' = y + t_y$$

Rigid Body Transformation (or Euclidean Transformation) (or rotation plus translation)

Rigid Body Transformation: **No stretching happening or no scaling**

$\mathbf{x}' = [\, \mathbf{R}_{2\times2} \mid t \,]_{2\times3}\, \mathbf{x}$

$$x' = x\cos\theta - y\sin\theta + t_x$$
$$y' = x\sin\theta + y\cos\theta + t_y$$

Similarity

$\mathbf{x}' = [\, s\mathbf{R}_{2\times2} \mid t \,]_{2\times3}\, \mathbf{x}$

$$x' = sx\cos\theta - sy\sin\theta + t_x$$
$$y' = sx\sin\theta + sy\cos\theta + t_y$$

- Each **higher group** completely contains the **lower group**

Affine Group

General 2 x 3 linear transform

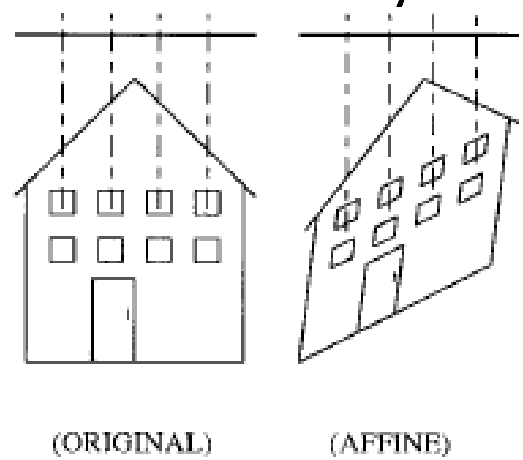
$\mathbf{x}' = [\,A\,]_{2\times 3}\,\mathbf{x}$

$\mathbf{x}' = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}\mathbf{x}$

$x' = a_{11}x + a_{12}y + a_{13}$

$y' = a_{21}x + a_{22}y + a_{23}$

- Contains **rotation, scaling, shear, translation** and any combination thereof
- **Preserves Parallel lines**



Ref: Steve Mann & Rosalind W. Picard, "Video Orbits of the Projective Group: A simple approach to featureless estimation of parameters", IEEE Trans. on Image Processing, Vol. 6, No. 9, September 1997

Projective Group (Homography)

3 x 3 transform defined in **Homogeneous coordinates**

$\mathbf{x}' = [\,H]_{3\times3}\,\mathbf{x}$

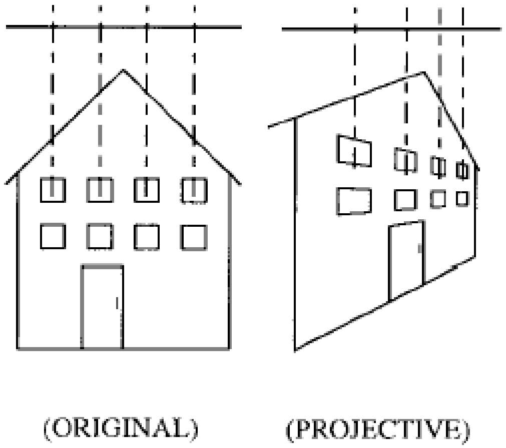
$$\mathbf{x}' = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \mathbf{x}$$

Inhomogeneous representation

$$\mathbf{x}' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{31}}$$

$$\mathbf{y}' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{31}}$$

- Simulates out of plane rotations
- **Preserves straight lines**
- Physical Interpretation: Plane + Camera



Translation, Rigid Body Transformation, Similarity, Affine

1. $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$

Translation Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 & 0 & \mathrm{t_x} \\ 0 & 1 & \mathrm{t_y} \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times1}$$

$$\Longrightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} \mathrm{x} + \mathrm{t_x} \\ \mathrm{y} + \mathrm{t_y} \\ 1 \end{bmatrix}_{3\times1}$$

Rigid Body Transformation(Euclidean Transformation)

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times1}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} x\cos\theta - y\sin\theta + t_x \\ x\sin\theta + y\cos\theta + t_y \\ 1 \end{bmatrix}_{3\times1}$$

Similarity Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} \mathrm{s}\cos\theta & -\mathrm{s}\sin\theta & \mathrm{t_x} \\ \mathrm{s}\sin\theta & \mathrm{s}\cos\theta & \mathrm{t_y} \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times 1}$$

$$\Longrightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} \mathrm{sx}\cos\theta - \mathrm{sy}\sin\theta + \mathrm{t_x} \\ \mathrm{sx}\sin\theta + \mathrm{sy}\cos\theta + \mathrm{t_y} \\ 1 \end{bmatrix}_{3\times 1}$$

Affine Group

3 x 3 transform defined in Homogeneous coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times 1}$$

$$\Longrightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} a_{11}\mathrm{x} + a_{12}\mathrm{y} + a_{13} \\ a_{21}\mathrm{x} + a_{22}\mathrm{y} + a_{23} \\ 1 \end{bmatrix}_{3\times 1}$$

- Contains **rotation, scaling, shear, translation** and any combination thereof
- **Preserves Parallel lines**

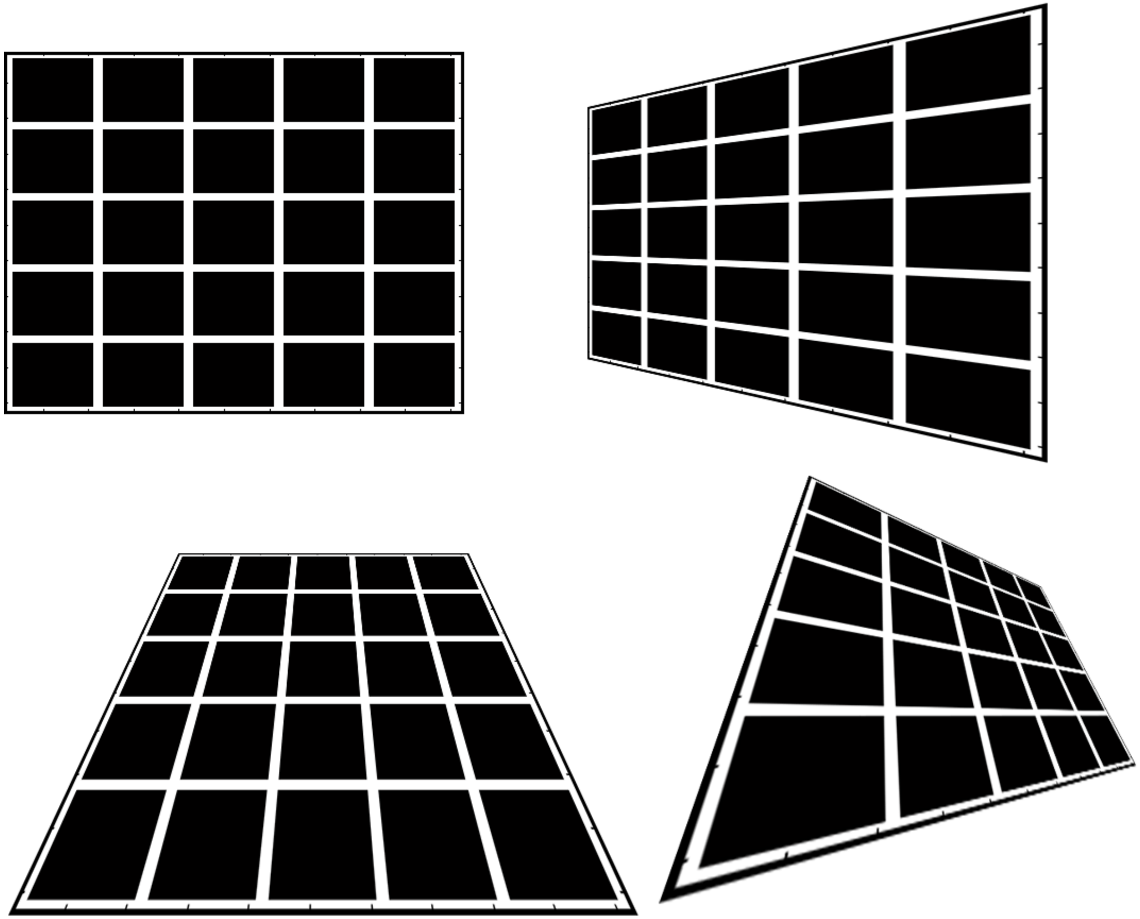
Projective Group (Homography)

3 x 3 transform defined in Homogeneous coordinates

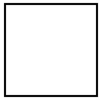
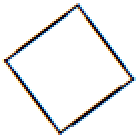
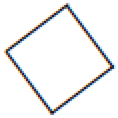
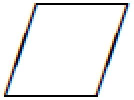
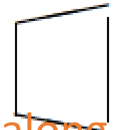
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}_{3\times3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times1}$$

- Simulates out of plane rotations
- **Preserves straight lines**
- Physical Interpretation: Plane + Camera

Examples of Projective Transformations

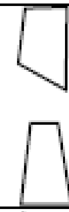
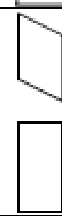
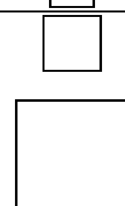
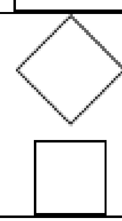


Hierarchy of 2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles Length Ratios	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism Length Ratios along a line	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines Length Cross-Ratios along a line	

- Note: In the previous slides representation of translation, rigid body transformation, similarity and affine group represent non homogenous system. But homography represents a **homogeneous system**.

2D projective geometry
A hierarchy of transforms

Group	Matrix	Distortion	Invariant properties
Projective 8 dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, order of contact, tangent discontinuities and cusps, cross ratios
Affine 6 dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallelism, ratio of areas, ratio of lengths on collinear or parallel lines, linear combinations of vectors, the line at infinity $\mathbf{l}_\infty$
Similarity 4 dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratio of lengths, angle
Euclidean 3 dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Length, area