Homography Estimation - Theoretical Q&A

1. What mathematical technique is used to solve the system Ah = 0?

The technique used is Singular Value Decomposition (SVD). According to the slides, SVD decomposes the matrix A into three matrices: A = UDV^T, where D is a diagonal matrix of singular values. The solution h is the last column of V, which corresponds to the smallest singular value of A.

2. What is the significance of the smallest singular value in SVD for solving homography?

The smallest singular value represents the direction in which the data matrix A loses the least amount of information. The corresponding singular vector (last column of V) provides the optimal solution h for the homogeneous system Ah = 0.

3. Why must we avoid the trivial solution h = 0 when solving Ah = 0?

The trivial solution h = 0 leads to a zero homography matrix, which is not useful in practice. Applying this matrix results in x' = [0, 0, 0]^T, which is undefined in projective geometry. Thus, it must be avoided.

4. What constraint is applied to avoid the trivial solution in homography estimation?

To avoid the trivial solution, we apply a constraint on the norm of vector h, such that ||h|| = 1. This constrained minimization ensures that the solution is non-zero and meaningful.

5. What is the optimization goal in solving Ah approximately 0?

The optimization goal is to find a vector h such that ||Ah|| is minimized. This minimizes the error vector and provides an approximate solution to the homogeneous equation.

6. How is ||Ah|| minimized in practice under the constraint ||h|| = 1?

Minimization is performed using SVD, where the optimal solution h is the eigenvector of A^TA corresponding to the smallest eigenvalue. This provides the best approximation to the solution of Ah ~ 0 under the constraint ||h|| = 1.

7. What is the shape of the covariance matrix ATA in homography estimation?

If A is a 2n x 9 matrix, then the covariance matrix A^T A is of size 9 x 9. It summarizes the squared

projections of the data points in each dimension.

8. Why is matrix ATA full rank when more than 4 correspondences are used?

With more than 4 point correspondences and measurement noise present, the system becomes over-constrained. This ensures that A^T A has full rank (rank 9), as all columns provide linearly independent information.

9. What does the eigenvector corresponding to the smallest eigenvalue of ATA represent?

It represents the optimal solution h for the homography matrix. This eigenvector points in the direction where the system loses the least amount of information, satisfying Ah ~ 0.

10. How are the matrices U, D, and V interpreted in the SVD of matrix A?

U and V are orthogonal matrices, and D is a diagonal matrix containing singular values. The last column of V, corresponding to the smallest singular value, is the solution h to the homography problem.