Date:

2D homography:

transformation b/w two 2D-views.
using $H_{3\times3}$.

3D homography:

transformation b/w two 3D planes
using $H_{4\times4}$.

Projective space $P3$

is an extension of Eucledian space R^3
which includes set of point at infinity
which lie on plane at infinity π_∞

$R_3 \Leftarrow$ 3D point ——— (x, y, z)
$P_3 \Leftarrow$ homogenous form — $(x, y, z, 1)$
$\pi_\infty \Leftarrow$ Point at infinity — $(x, y, z, 0)$

Point in 3D space:

$X \in$ ~~B~~ P^3 is represented as:

$$X^T = (X_1, X_2, X_3, X_4) \Rightarrow P^3$$
$$X = \left(\frac{X_1}{X_4}, \frac{X_2}{X_4}, \frac{X_3}{X_4}\right)^T \Rightarrow R^3.$$

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$$\boxed{X'_{4\times1} \doteq H_{4\times4} \cdot X_{4\times1}}$$

$H \rightarrow$ 16 element

↳ homogenous → defined upto scale

↳ $DOF = 16 - 1 = 15$

2D Projective Geometry	3D ~~Geometri~~ Projective Geometry
duality b/w point ⟷ line	duality b/w point ⟷ plane.
	line difficult to deal.

⇒ why are 3D lines difficult?

↳ not simply explain by one equation as in 2D.

↳ need two constrains to describe line in 3D.

↳ no matrix representation as of point & planes exist.

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Q) whats line ?

↳ line segment •—•

↳ infinite line ⟷

- define by 2 points.
- 1D object embedded in R^3

↳ Each point ⇒ 3 ~~cartes pap~~ coordinates

↳ Two points ⇒ 6 coordinates

⇓

$\boxed{6 \text{ numbers}}$

Still DOF = 4

Reason:

↳ Redundancy exist

↳ only 4 are independent.

So we need:

4 points minimal.

- Line: 1D object in 2D (R^2)
- plane: 2D object in 3D (R^3)

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(A) $\Rightarrow$ $X' = H X$.

$\Rightarrow x'_{(4\times1)} = H_{(4\times4)} X_{(4\times1)}$

$H_{4\times4} \rightarrow$ transforms shape/scene

$H_{3\times3} \rightarrow$ maps b/w view.

Plane Equation: $\boxed{\pi_1 X + \pi_2 Y + \pi_3 Z + \pi_4 = 0}$ — (A)

↳ $\pi_1, \pi_2, \pi_3, \pi_4 \rightarrow$ constants.

↳ X, Y, Z — point on plane.

$\boxed{X, \pi \in P^3}$

2D) line: $ax + by + c = 0$

3D) plane: $\pi_1 X + \pi_2 Y + \pi_3 Z + \pi_4 = 0$.

Using plane equation:

$\pi = (\pi_1, \pi_2, \pi_3, \pi_4)^T$

Homogenous form: $\boxed{\pi^T X = 0}$

DOF = 3

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$$\Pi_1 X + \Pi_2 Y + \Pi_3 Z + \Pi_4 = 0. \quad — (A)$$

$$X = \frac{x_1}{x_4}, \quad Y = \frac{x_2}{x_4}, \quad Z = \frac{x_3}{x_4}$$

$$\Pi_1\left(\frac{x_1}{x_4}\right) + \left(\frac{x_2}{x_4}\right)\Pi_2 + \left(\frac{x_3}{x_4}\right)\Pi_3 + \Pi_4 = 0.$$

$$\Pi_1 x_1 + \Pi_2 x_2 + \Pi_3 x_3 + \Pi_4 x_4 = 0$$

$$\hookrightarrow \boxed{\Pi^T X = 0.}$$

Determine Plane Π contains points?

$$\begin{bmatrix} X_1^T \\ X_2^T \\ X_3^T \end{bmatrix}_{3\times 4} \vec{\Pi}_{4\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

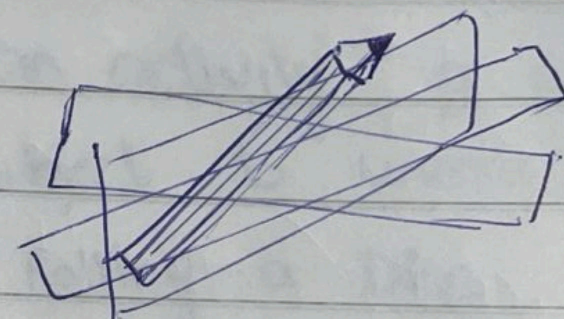
$$\boxed{A\Pi = 0}$$

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & X_2 & Z_2 & 1 \\ X_3 & Y_3 & Z_3 & 1 \end{bmatrix} \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If matrix is Full Rank
- nullspace — 1D
- unique solution
-

~~Week~~

Pencil of Plane



collection of planes passing by same line.

Lef Null Space	$\in \mathbb{R}^m$	$A\vec{x} = 0$
Right Null Space	$\in \mathbb{R}^n$	$A^T y$

~~Right~~ Nu