Homework 2
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Computer Structures

1) let $r = 2$
let $n = 4$

2^n 16
$16 - 1 = 15$
$\underbrace{1111}_{4\ \text{numbers}} = 15$

let r = a radix
let N = maximum digits
assume $\forall r$, the maximum number that can be represented with N digits is r^{N-1}
I have shown that $r = 2$ $N = 4$ holds
Lets assume r^{N+1} holds

$$r^N = \text{true}$$

2) for the addition algorithm we use to preform addition by hand, the carry bit will always be a 1 or a 0 because in every step we only ever add single digits, and the highest two single digits can ever amount to is $r + r$ which will never result in a leftmost digit > 1. for proof see P.6

3) The maximum two's complement numbers that can be represented with N bits is $(2^{N-1}) - 1$
minimum is $-2^{(N-1)}$

4) $B = 0011_2 = 3_{10}$

$\overline{B} = 1100$
$+1$

$1101 + 1 = 1111$

$-B = 1111$

5) For signed positive numbers it's easy to see that no amount of added 0's will change the number.

e.g. 4 bits to 8 bits

4 bits 0101 = 5

8 bits 0000 0101 = 5

But when you consider a negative number, say 1101 = -5 then you see that if you were to append 1s then the number changes. Except it doesn't because we're representing the negative number in 2s complement so any extension does not change the value

6) Decimal to binary

a) 121

Q	121	60	30	15	7
R	1	0	0	1	1

... Q	3	1
R	1	1

a) 1111001

Hex	Dec	Bin
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

B)

Q	1537	768	384	192	96	48	24	12	6
R	1	0	0	0	0	0	0	0	0

Ans= 1100 0000 001

c)

Q:	31333	15666	7833	3916	1958	979	489	244	122	61	30
R:	1	0	1	0	0	1	1	0	0	1	0

...Q:	15	7	3	1
R:	1	1	1	1

Ans= 1111 0100 1100 101

D)

Q:	97	48	24	12	6	3	1
R:	1	0	0	0	0	1	1

Ans= 1100 001

7)

a) $\overset{Y}{121} - \overset{X}{41}$

$X = 041$

$\overline{X} = 958$

$$\begin{array}{r} 121 \\ 958 \\ +\ 1 \\ \hline 080 \end{array}$$

b) $\overset{Y}{1022} - \overset{X}{35}$

$X = 0035$

$\overline{X} = 9964$

$$\begin{array}{r} \boxed{1} \\ 1022 \\ 9964 \\ +\ 1 \\ \hline 0987 \end{array}$$

c) $\overset{y}{151} - \overset{x}{90}$

$X = 090$ $\quad$ $\overset{1}{151}$

$\bar{X} = 909$ $\quad$ 909

$+ \quad 1$

061

d) $\overset{y}{2120} - \overset{x}{101}$

$X = 0101$ $\quad$ $\overset{1\,1}{2120}$

$\bar{X} = 9898$ $\quad$ 9898

$+ \quad 1$

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8)

a) -121

Q:	-121	60	30	15	7	3	1
R:	1	0	0	1	1	1	1

1111 001

= 0111 1001 (unsigned binary 121)

= 1000 0110

+ 1

1000 0111

b) -51

Q:	51	25	12	6	3	1
R:	1	1	0	0	1	1

110011 (unsigned binary 51)
0011 0011
1100 1100 (ones complement)
\+ 1
1100 1101 (signed twos complement)

c) -104

Q	104	52	26	13	6	3	1
R	0	0	0	1	0	1	1

1101 000
= 0110 1000
= 1001 0111
\+ 1
1001 1000

d) 115

Q:	115	57	28	14	7	3	1
R:	1	1	0	0	1	1	1

1110 011
= 0111 0011

e) 127

Q:	127	63	31	15	7	3	1
R:	1	1	1	1	1	1	1

= 1111 111

= 0111 1111

4.2) let us assume $\forall r \geq 1.\ P(n) \leq n0-1$
Where r is a radix and n is
the largest single digit in the number system r
and $P(n) = n + n$
base $r = 1$

$P(r) = r + r$
$P(1) = 2$
$2 < n0-1$

hence $P(1)$ holds
let $k \geq 1$ be an arbitrary number
let us assume the inductive hypothesis
$\forall 1 \geq i \geq k : P(i)$
we will prove $P(k)$ that is $\forall k \geq 1\ n + n \leq n0-1$
now, $k + k = k-1 + k$
since $1 < k-1 < k$ using the inductive hypothesis
we know this is true which establishes $P(k)$
Hence we have proved by induction on n that
For any $n \geq 1$, $n + n \leq n0-1$