# Database Management Systems INFO 210

Summary - Part 2 (SQL,Relational Algebra, NFs)

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#### **Relational Query Languages**

- Query languages (QLs) allow manipulating and retrieving data from databases
- The relational model supports simple and powerful QLs:
  - Strong formal foundation based on logic
  - High amenability for effective optimizations
- Query Languages != programming languages!
  - QLs are not expected to be "Turing complete"
  - QLs are not intended to be used for complex calculations
  - QLs support easy and efficient access to large datasets

**SQL** 

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#### DDL and DML

- The SQL language has two main aspects (there are other aspects which we will discuss in next classes)
  - Data Definition Language (DDL)
    - Allows creating, modifying, and deleting relations and views
    - Allows specifying constraints
    - Allows administering users, security, etc.
  - Data Manipulation Language (DML)
    - Allows posing queries to find tuples that satisfy criteria
    - Allows adding, modifying, and removing tuples

#### Creating Relations in SQL

- S1 can be used to create the "Students" relation
- S2 can be used to create the "Enrolled" relation

CREATE TABLE Students (sid: CHAR(20), name: CHAR(20), login: CHAR(10), age: INTEGER, gpa: REAL) CREATE TABLE Enrolled (sid: CHAR(20), cid: CHAR(20), grade: CHAR(2))

**S2** 

**S1** 

The DBMS enforces domain constraints whenever tuples are added or modified

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#### Adding and Deleting Tuples

• We can insert a single tuple to the "Students" relation using:

INSERT INTO Students (sid, name, login, age, gpa) VALUES (53688, 'Smith', 'smith@ee', 18, 3.2)

 We can delete all tuples from the "Students" relation which satisfy some condition (e.g., name = Smith):

> DELETE FROM Students S WHERE S.name = 'Smith'

Powerful variants of these commands are available!

## Querying a Relation

■ How can we find all 18-year old students?

sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@eecs	18	3.2
53650	Smith	smith@math	19	3.8

SELECT \*
FROM Students S
WHERE S.age=18



sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@ee	18	3.2

How can we find just names and logins?

SELECT S.name, S.login FROM Students S WHERE S.age=18

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## Querying Multiple Relations

What does the following query compute assuming S and E?

SELECT S.name, E.cid FROM Students S, Enrolled E WHERE S.sid=E.sid AND E.grade="A"

sid	name	login	age	gpa
53666	Jones	jones@cs	18	3.4
53688	Smith	smith@eecs	18	3.2
53650	Smith	smith@math	19	3.8

sid	cid	grade
53831	Carnatic101	С
53831	Reggae203	В
53650	Topology112	A
53666	History105	В

S

We get:

S.name E.cid
Smith Topology112

#### **Destroying and Altering Relations**

How to destroy the relation "Students"?

**DROP TABLE** Students

The schema information and the tuples are deleted

How to alter the schema of "Students" in order to add a new field?

ALTER TABLE Students
ADD COLUMN firstYear: integer

Every tuple in the current instance is extended with a *null* value in the new field!

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#### **Integrity Constraints (ICs)**

- An IC is a condition that must be true for any instance of the database (e.g., domain constraints)
  - ICs are specified when schemas are defined
  - ICs are checked when relations are modified
- A legal instance of a relation is one that satisfies all specified ICs
  - DBMS should not allow illegal instances
- If the DBMS checks ICs, stored data is more faithful to real-world meaning
  - Avoids data entry errors, too!

#### Keys

- Keys help associate tuples in different relations
- Keys are one form of integrity constraints (ICs)

#### Enrolled

#### Students

sid	cid	grade						
53666	15-101	C		sid	name	login	age	gpa
53666	18-203	В -	$\rightarrow$	53666	Jones	jones@cs	18	3.4
53650	15-112	A		53688	Smith	smith@cs	18	3.2
53666	15-105	B		53650	Smith	smith@math	19	3.8
			l					

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#### Keys

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- Keys are one form of integrity constraints (ICs)

sid	Enrolled	grade	I	$\bigcirc$		Students		
53666	15-101	С		sid	name	login	age	gpa
53666	18-203	В -		53666	Jones	jones@cs	18	3.4
53650	15-112	A		53688	Smith	smith@cs	18	3.2
53666	15-105	В		53650	Smith	smith@math	19	3.8
F	FOREIGN Key				PRIM	1ARY Key	<b>,</b>	

#### Superkey, Primary and Candidate Keys

- A set of fields is a *superkey* if:
  - No two distinct tuples can have same values in *all* key fields
- A set of fields is a *primary key* for a relation if:
  - It is a *minimal* superkey
- What if there is more than one key for a relation?
  - One of the keys is chosen (by DBA) to be the primary key
  - Other keys are called candidate keys
- Examples:
  - sid is a key for Students (what about name?)
  - The set {sid, name} is a superkey (or a set of fields that contains a key)

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#### Primary and Candidate Keys in SQL

- Many candidate keys (specified using UNIQUE) can be designated and one is chosen as a primary key
- Keys must be used carefully!
- "For a given student and course, there is a single grade"

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```
CREATE TABLE Enrolled
(sid CHAR(20)
cid CHAR(20),
grade CHAR(2),
PRIMARY KEY (sid,cid))
```

VS.

```
CREATE TABLE Enrolled

(sid CHAR(20)

cid CHAR(20),

grade CHAR(2),

PRIMARY KEY (sid),

UNIQUE (cid, grade))
```

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#### Primary and Candidate Keys in SQL

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CREATE TABLE Enrolled
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vs.

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CREATE TABLE Enrolled
(sid CHAR(20)
cid CHAR(20),
grade CHAR(2),
PRIMARY KEY (sid),
UNIQUE (cid, grade))
```

Q: What does this mean?

#### Primary and Candidate Keys in SQL

- Many candidate keys (specified using UNIQUE) can be designated and one is chosen as a primary key
- Keys must be used carefully!
- "For a given student and course, there is a single grade"

```
CREATE TABLE Enrolled
(sid CHAR(20)
cid CHAR(20),
grade CHAR(2),
PRIMARY KEY (sid,cid))
```

CREATE TABLE Enrolled
(sid CHAR(20)
cid CHAR(20),
grade CHAR(2),
PRIMARY KEY (sid),
UNIQUE (cid, grade))

"A student can take only one course, and no two students in a course receive the same grade"

VS.

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#### Foreign Keys and Referential Integrity

- A foreign key is a set of fields referring to a tuple in another relation
  - It must correspond to the primary key of the other relation
  - It acts like a `logical pointer'
- If all foreign key constraints are enforced, referential integrity is said to be achieved (i.e., no dangling references)

#### Foreign Keys in SQL

- Example: Only existing students may enroll for courses
  - sid is a foreign key referring to Students
  - How can we write this in SQL?

	Enrolled					G. 1 .		
sid	cid	grade				Students		
53666	15-101	C		sid	name	login	age	gpa
53666	18-203	в –		53666	Jones	jones@cs	18	3.4
53650	15-112	Α _		53688	Smith	smith@cs	18	3.2
53666	15-105	В /	<b>*</b>	53650	Smith	smith@math	19	3.8
			•			,		,

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## Foreign Keys in SQL

Example: Only existing students may enroll for courses

CREATE TABLE Enrolled
(sid CHAR(20),cid CHAR(20),grade CHAR(2),
PRIMARY KEY (sid,cid),
FOREIGN KEY (sid) REFERENCES Students)

Enrolled

sid	cid	grade	Students					
53666	15-101	C		sid	name	login	age	gpa
53666	18-203	В -	$\rightarrow$	53666	Jones	jones@cs	18	3.4
53650	15-112	Α _		53688	Smith	smith@cs	18	3.2
53666	15-105	В /		53650	Smith	smith@math	19	3.8

#### **Enforcing Referential Integrity**

- What should be done if an "Enrolled" tuple with a nonexistent student id is inserted? (Reject it!)
- What should be done if a "Students" tuple is deleted?
  - Disallow its deletion
  - Delete all Enrolled tuples that refer to it
  - Set sid in Enrolled tuples that refer to it to a default sid
  - Set sid in Enrolled tuples that refer to it to a special value null, denoting `unknown' or `inapplicable'
- What if a "Students" tuple is <u>updated</u>?

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#### Referential Integrity in SQL

- SQL/92 and SQL:1999 support all 4 options on deletes and updates
  - Default is NO ACTION (i.e., delete/update is rejected)
  - CASCADE (also delete all tuples that refer to the deleted tuple)
  - SET NULL / SET DEFAULT (sets foreign key value of referencing tuple)

```
CREATE TABLE Enrolled
(sid CHAR(20),
cid CHAR(20),
grade CHAR(2),
PRIMARY KEY (sid,cid),
FOREIGN KEY (sid)
REFERENCES Students
ON DELETE CASCADE
ON UPDATE SET DEFAULT)
```

What does this mean?

#### Where do ICs Come From?

- ICs are based upon the semantics of the real-world enterprise that is being described in the database relations
- We can check a database instance to see if an IC is violated, but we can NEVER infer that an IC is true by looking at an instance
  - An IC is a statement about *all possible* instances!
  - From the "Students" relation, we know *name* is not a key, but the assertion that *sid* is a key is given to us
- Key and foreign key ICs are the most common; more general ICs are supported too

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#### **Views**

 A view is a table whose rows are not explicitly stored but computed as needed

CREATE VIEW YoungActiveStudents (name, grade)
AS SELECT S.name, E.grade
FROM Students S, Enrolled E
WHERE S.sid = E.sid and S.age<21

- Views can be queried
  - Querying YoungActiveStudents would necessitate computing it first then applying the query on the result as being like any other relation
- Views can be dropped using the DROP VIEW command
  - How to handle DROP TABLE if there's a view on the table?
    - DROP TABLE command has options to let the user specify this

#### Formal Relational Query Languages

- There are two mathematical Query Languages which form the basis for commercial languages (e.g. SQL)
  - Relational Algebra
    - Queries are composed of operators
    - Each query describes a step-by-step procedure for computing the desired answer
    - Very useful for representing execution plans
  - Relational Calculus
    - Queries are subsets of first-order logic
    - Queries describe desired answers without specifying how they will be computed
    - A type of *non-procedural* (or *declarative*) formal query language

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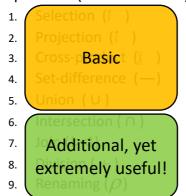
#### Relational Algebra

- Operators (with notations):
  - 1. Selection (σ)
  - 2. Projection (π)
  - 3. Cross-product (X)
  - 4. Set-difference (-)
  - 5. Union (U)
  - Intersection (∩)

  - 8. Division  $(\div)$
  - 9. Renaming  $(\rho)$
- Each operation returns a relation, hence, operations can be *composed*! (i.e., Algebra is "closed")

#### Relational Algebra

Operators (with notations):



Each operation returns a relation, hence, operations can be composed! (i.e., Algebra is "closed")

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## The Projection Operation

- $\begin{tabular}{ll} \blacksquare & {\it Projection:} & $\pi_{\it att-list}(R)$ \\ & \blacksquare & "{\it Project out"} & {\it attributes that are NOT in att-list} \end{tabular}$ 

  - The schema of the output relation contains ONLY the fields in att-list, with the same names that they had in the input relation
- $\pi_{sname,rating}(S2)$ Example 1:

#### Input Relation:

#### sid sname rating age 28 9 35.0 yuppy 8 31 55.5 lubber 5 44 35.0 guppy 58 rusty 10 35.0 **S2**

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

**Output Relation:** 

## The Selection Operation

- (R)Selection:  $\sigma_{\scriptscriptstyle condition}$ 
  - Selects rows that satisfy the selection condition
  - The schema of the output relation is identical to the schema of the input relation
- $\sigma_{rating>8}(S2)$ Example:

**Input Relation:** 

#### **Output Relation:**

sid	sname	rating	age		
28	yuppy	9	35.0		
31	lubber	8	55.5		
44	guppy	5	35.0		
58	rusty	10	35.0		

sname rating age 9 35.0 yuppy 10 35.0 rusty

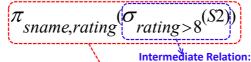
S2

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#### **Operator Composition**

The output relation can be the input for another relational algebra operation! (Operator composition)

Example:



sid

Input Relation:

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	ructs	10	35.0

yuppy 58 10 rusty

sname

Final Output Relation:				
sname	rating			

9

rating

age

35.0

35.0

9 yuppy 10 rusty

#### The Union Operation

- Union: RUS
  - The two input relations must be union-compatible
    - Same number of fields
    - 'Corresponding' fields have the same type
  - The output relation includes all tuples that occur "in either" R or S "or both"
  - The schema of the output relation is identical to the schema of R

Example:  $S1 \cup S2$ 

#### **Input Relations:**

#### sname sid age 45.0 31 lubber 55.5 58 10 35.0 rusty

sname 28 31 35.0 55.5 yuppy 8 lubber 44 35.0 guppy rusty

**Output Relation:** 

	sid	sname	rating	age
	22	dustin	7	45.0
	31	lubber	8	55.5
)	58	rusty	10	35.0
•	44	guppy	5	35.0
	28	yuppy	9	35.0

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#### The Intersection Operation

- Intersection:  $R \cap S$ 
  - The two input relations must be *union-compatible*
  - The output relation includes all tuples that occur "in both" R and S
  - The schema of the output relation is identical to the schema of R

Example:  $S1 \cap S2$ 

#### **Input Relations:**

sid	sname	rating	age	sid	sname	rating	age
22	dustin	7	45.0	28	yuppy	9	35.0
31	lubber	8	55.5	31 44	lubber guppy	8 5	55.5 35.0
58	rusty	10	35.0	58	rusty	10	35.0
	S	1	1			S2	



sname rating lubber 55.5 35.0 rusty

**Output Relation:** 



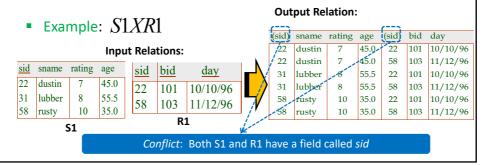
- Set-Difference: R S
  - The two input relations must be *union-compatible*
  - The output relation includes all tuples that occur in R "but not" in S
  - The schema of the output relation is identical to the schema of R
- Example: *S*1–*S*2

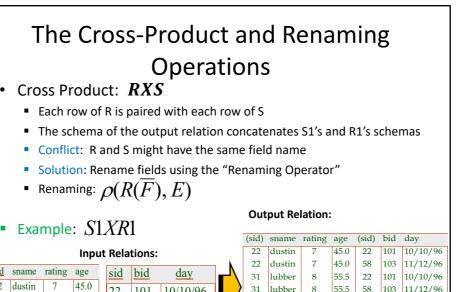


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## The Cross-Product and Renaming Operations

- Cross Product: RXS
  - Each row of R is paired with each row of S
    - The schema of the output relation concatenates S1's and R1's schemas
    - Conflict: R and S might have the same field name
    - Solution: Rename fields using the "Renaming Operator"
    - Renaming:  $\rho(R(F), E)$





lubber

 $\rho(C(1 \rightarrow sid1, 5 \rightarrow sid2), S1 \times R1)$ 

58 103

101

11/12/96

10/10/96

22

58

55.5

10 35.0

S1

101

103

10/10/96

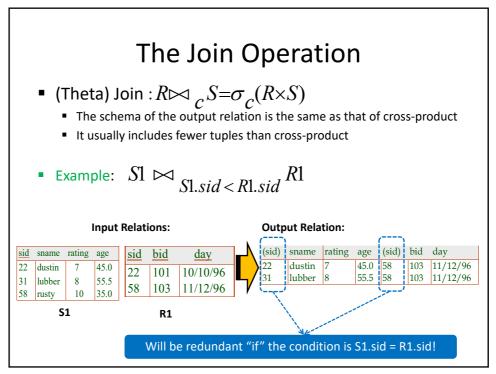
11/12/96

35

31

58 rusty

lubber



#### The Join Operation

- Equi-Join:  $R \bowtie_{\mathcal{C}} S = \sigma_{\mathcal{C}}(R \times S)$ 
  - A special case of theta join where the condition c contains only **equalities**
  - The schema of the output relation is the same as that of cross-product, "but only one copy of the fields for which equality is specified"
- Natural Join:  $R \bowtie S$ 
  - Equijoin on "all" common fields



 sid
 sname
 rating
 age

 22
 dustin
 7
 45.0

 31
 lubber
 8
 55.5

 58
 rusty
 10
 35.0

 S1
 R1

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#### The Division Operation

- Division:  $R \div S$ 
  - Not supported as a primitive operator, but useful for expressing queries like:

Find sailors who have reserved all boats

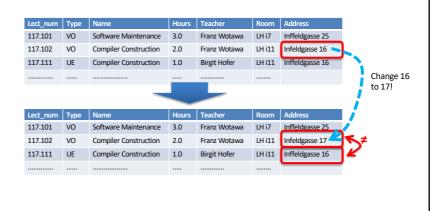
- Let A have 2 fields, x and y; B have only field y:
  - A/B contains all x tuples (sailors) such that for <u>every</u> y tuple (boat) in B, there is an xy tuple in A
  - Or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B, then x value is in A/B
  - $\bullet \text{ Formally: A/B} = \left\{ \left\langle x \right\rangle | \ \exists \left\langle x,y \right\rangle \in A \ \forall \left\langle y \right\rangle \in B \right\}$
- In general, x and y can be any lists of fields; y is the list of fields in B, and x y is the list of fields in A

#### **NORMAL FORMS**

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## Example of data inconsistencies

• Change of the address of a lecture hall!



## 1. Normal form (1NF)

- "A relation is in first normal form if and only if the domain of each attribute contains only atomic (indivisible) values, and the value of each attribute contains only a single value from that domain."
- The 1NF enables queries and sorting!
- There is one extension: "... and each relation must have a primary key."

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# Lect\_num Type Name Hours Teacher Room Address 117.101 VO Software Maintenance 3.0 Franz Wotawa, Bright Hofer 117.111 UE Compiler Construction 1.0 Birgit Hofer LH i11 Infeldgasse 16 117.111 UE compiler Construction 1.0 Birgit Hofer LH i11 Infeldgasse 16 117.111 VO Software Maintenance 3.0 Franz Wotawa LH i11 Infeldgasse 16 117.111 VO Software Maintenance 3.0 Franz Wotawa LH i7 Infeldgasse 25 117.101 VO Software Maintenance 3.0 Franz Wotawa LH i7 Infeldgasse 25 117.102 VO Compiler Construction 2.0 Franz Wotawa LH i7 Infeldgasse 25 117.102 VO Compiler Construction 2.0 Franz Wotawa LH i11 Infeldgasse 16 117.111 UE Compiler Construction 1.0 Birgit Hofer LH i11 Infeldgasse 16 117.111 UE Compiler Construction 1.0 Birgit Hofer LH i11 Infeldgasse 16

#### 2. Normal form (2NF)

• "A relation in 1NF is in 2NF if and only if no **non-prime attribute** is **dependent** on any **proper subset** of any (candidate) key of the relation.

A non-prime attribute of a relation is an attribute that is not part of any (candidate) key of the relation."

Every non-prime attribute has to be dependent on the key only

A database that is not in 2NF comprises redundancies!

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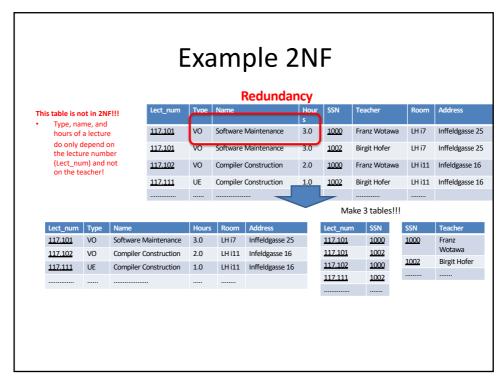
#### Example 2NF

• Let us consider the following table with primary key {Lect\_num,SSN}:



- Dependencies:
  - $\quad \{ \text{Lect\_num} \} \rightarrow \! \{ \textit{Type, Name, Hours, Room} \}$
  - {SSN} →{Teacher}
  - {Room} →{Address}

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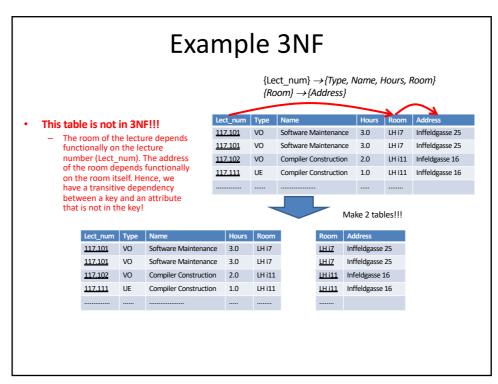
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#### 3. Normal form (3NF)

 "A relation in 2NF is in 3NF if and only if all the attributes in a table are determined only by the candidate keys of that relation and not by any non-prime attributes.

No non-prime attributes are allowed to be transitive dependent on a prime attribute!"

Eliminates problems occurring when changing information!



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#### **Boyce-Codd Normal Form**

A relation R is in Boyce-Codd Normal Form (BCNF) if whenever  $X \rightarrow A$  is a nontrivial FD that holds in R, then X is a superkey

#### Remember:

- nontrivial means A ∉ X
- a <u>superkey</u> is any superset of a key (not necessarily a strict superset)

"Each attribute must describe the key, the whole key, and nothing but the key"

## All two-attribute relations are in BCNF

Case 1: Suppose there is no dependency

Nothing is being violated, so fine

Case 2: Suppose A  $\rightarrow$  B is the only dependency

 $A^+=AB$ , so left side of  $A \rightarrow B$  is a key

Case 3: Suppose B  $\rightarrow$  A is the only dependency

 $B^+=AB$ , so left side of  $B \rightarrow A$  is a key

Case 4: Suppose  $A \rightarrow B$  and  $B \rightarrow A$  are two dependencies

 $A^{+} = AB, B^{+} = AB,$ 

so left side of B  $\rightarrow$  A is a key and left side of A  $\rightarrow$  B is also a key

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#### What 3NF and BCNF Give You

There are two important properties of a decomposition:

- ◆ Losslessness: It should be possible
  - to project the original relation onto the decomposed schema
  - ♦and then reconstruct the original by a natural join
- Dependency Preservation: It should be possible to check in the projected relations whether all the given FDs are satisfied

# Functional Dependencies

- A form of constraint
  - hence, part of the schema
- Finding them is part of the database design
- Also used in normalizing the relations

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# Functional Dependencies

Definition:

If two tuples agree on the attributes

$$A_1, A_2, ..., A_n$$

then they must also agree on the attributes

$$B_1, B_2, ..., B_m$$

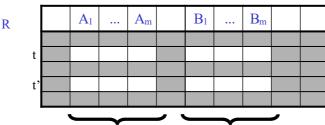
Formally:

 $A_1, A_2, ..., A_n \rightarrow B_1, B_2, ..., B_m$ 

## When Does a FD Hold

Definition:  $A_1, ..., A_m \rightarrow B_1, ..., B_n$  holds in R if:

 $\forall t, t' \in R, (t.A_1 = t'.A_1 \land ... \land t.A_m = t'.A_m \Rightarrow t.B_1 = t'.B_1 \land ... \land t.B_n = t'.B_n)$ 



if t, t' agree here then t, t' agree here

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## Example: Movie table

Title	Year	Length	Genre	StudioName	StarName
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	Drama	MGM	Vivien Leigh
Wayne's World	1992	95	Comedy	Paramount	Dana Carvey
Wayne's World	1992	95	Comedy	Paramount	Mike Meyers

title, year → length

title, year → genre

title, year → length, genre, studioName

title, year → studioName

# Example: Movie table

Title	Year	Length	Genre	StudioName	StarName
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
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## **Examples**

An FD holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E11 1	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

EmpID → Name, Phone, Position

but not Name → EmpID

or Name → Phone

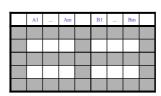
# Armstrong's Rules (1/3)

$$A_1, A_2, ..., A_n \rightarrow B_1, B_2, ..., B_m$$

Is equivalent to

$$\begin{array}{c} A_{1}, A_{2}, ..., A_{n} \xrightarrow{} B_{1} \\ A_{1}, A_{2}, ..., A_{n} \xrightarrow{} B_{2} \\ ..... \\ A_{1}, A_{2}, ..., A_{n} \xrightarrow{} B_{m} \end{array}$$

Splitting rule and Combing rule



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# Armstrong's Rules (2/3)

$$A_1, A_2, ..., A_n \rightarrow A_i$$

**Trivial Rule** 

where i = 1, 2, ..., n

Why?



# Armstrong's Rules (3/3)

#### **Transitive Closure Rule**

If  $A_1, A_2, ..., A_n \rightarrow B_1, B_2, ..., B_m$ 

and  $B_1, B_2, ..., B_m \rightarrow C_1, C_2, ..., C_p$ 

then  $A_1, A_2, ..., A_n \rightarrow C_1, C_2, ..., C_p$ 

Why?

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 Aı		Am	Bı		Bm	C <sub>1</sub>		Cp	
Ai	•••	Alli	Di	•••	Dill	CI	***	Ср	

## Closure of a set of Attributes

Given a set of attributes  $A_1, ..., A_n$  and a set of FDs

The **closure**,  $\{A_1, ..., A_n\}^+$  = the set of attributes B s.t.  $A_1, ..., A_n \rightarrow B$ 

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## Closures Example

name	category	color	department	price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office-supp.	59

Example: name → color

category → department color, category → price

Closures:

 $name^+ = \{name, color\}$ 

{name, category}+= {name, category, color, department, price}

 $color^+ = \{color\}$ 

## Closure Algorithm

```
Algorithm

X={A1, ..., An}.

Repeat until X doesn't change do:

if B_1, ..., B_n \rightarrow C is a FD and
B_1, ..., B_n are all in X

then add C to X.

{name, category}+=
{ name, category, color, department, price }

Hence: name, category \rightarrow color, department, price
```

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# Does a new FD logically follows from a set of FDs?

```
R(A_1, A_2, \dots, A_m) Set of FDs A \text{ new FD}
```

- To check if  $X \rightarrow A$  (new FD)
  - Using given set of FDs Compute X<sup>+</sup> i.e., {left side}<sup>+</sup>
  - Check if  $A \in X^+$



## Using Closure to Infer ALL FDs

Example:

$$\begin{array}{c}
A, B \to C \\
A, D \to B \\
B \to D
\end{array}$$

Step 1: Compute  $X^+$ , for every  $X \subseteq \{A,B,C,D\}$ :

```
A+=A, B+=BD, C+=C, D+=D

AB+=ABCD, AC+=AC, AD+=ABCD,

BC+=BCD, BD+=BD, CD+=CD

ABC+=ABD+=ACD+=ABCD (no need to compute-why?)

BCD+=BCD, ABCD+=ABCD
```

Step 2: Enumerate all FD's  $X \rightarrow Y$ , s.t.  $Y \subseteq X^+$  and  $X \cap Y = \emptyset$ AB  $\rightarrow$  CD, AD $\rightarrow$ BC, BC  $\rightarrow$ D, ABC  $\rightarrow$  D, ABD  $\rightarrow$  C, ACD  $\rightarrow$ B

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# Application of Closure: Finding Keys

- A superkey is a set of attributes  $A_1, ..., A_n$  s.t. for any other attribute B, we have  $A_1, ..., A_n \rightarrow B$
- A key is a minimal superkey
  - i.e. set of attributes which is a superkey and for which no subset is a superkey

# Computing (Super)Keys

- Compute  $X^+$  for all sets X
- If  $X^+$  = all attributes, then X is a key
- List only the minimal X's

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#### **Decomposition into BCNF**

Given: relation R with FDs F

Goal: decompose R into relations R<sub>1</sub>,...,R<sub>m</sub> such that

- each R<sub>i</sub> is a projection of R
- each R<sub>i</sub> is in BCNF

(wrt the projection of F)

• R is the *natural join* of R<sub>1</sub>,...,R<sub>m</sub>

Intuition: R is broken into pieces

- that contain the same information as R,
- · but are free of redundancy

#### The Algorithm: Divide and Conquer

- Look in F for an FD X → B that violates BCNF
   (If any FD following from F violates BCNF, then there is surely an FD in F itself that violates BCNF)
- Compute X<sup>+</sup>
   (X<sup>+</sup> does not contain all attributes of R,
   otherwise X would be superkey)
- Decompose R using X → B, i.e., replace R by relations with schemas

$$R_1 = X^+$$

$$R_2 = (R - X^+) \cup X$$

- Compute the projections  $F_1$ ,  $F_2$  of F on  $R_1$ ,  $R_2$
- Continue with R<sub>1</sub>, F<sub>1</sub> and R<sub>2</sub>, F<sub>2</sub>

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## BCNF decomposition example

#### R(A,B,C,D,E)

 $A \rightarrow C$   $B \rightarrow D$  $C, B \rightarrow E$  Is this relation in BCNF?
If not, convert it into BCNF.
Is the conversion dependency preserving?