Plot the probability density function of the random variables and determine the mean and variance. The pdfs should be as follows:

§ Exponential

§ Bernoulli

§ Binomial

§ Geometric

§ Normal

§ Rayleigh

§ Lognormal

Step-1: Random Number Generation[0 to 1].

```
clc
clear all
L_limit=0
```

```
L_limit = 0
```

```
U_limit=1
```

```
U_limit = 1
```

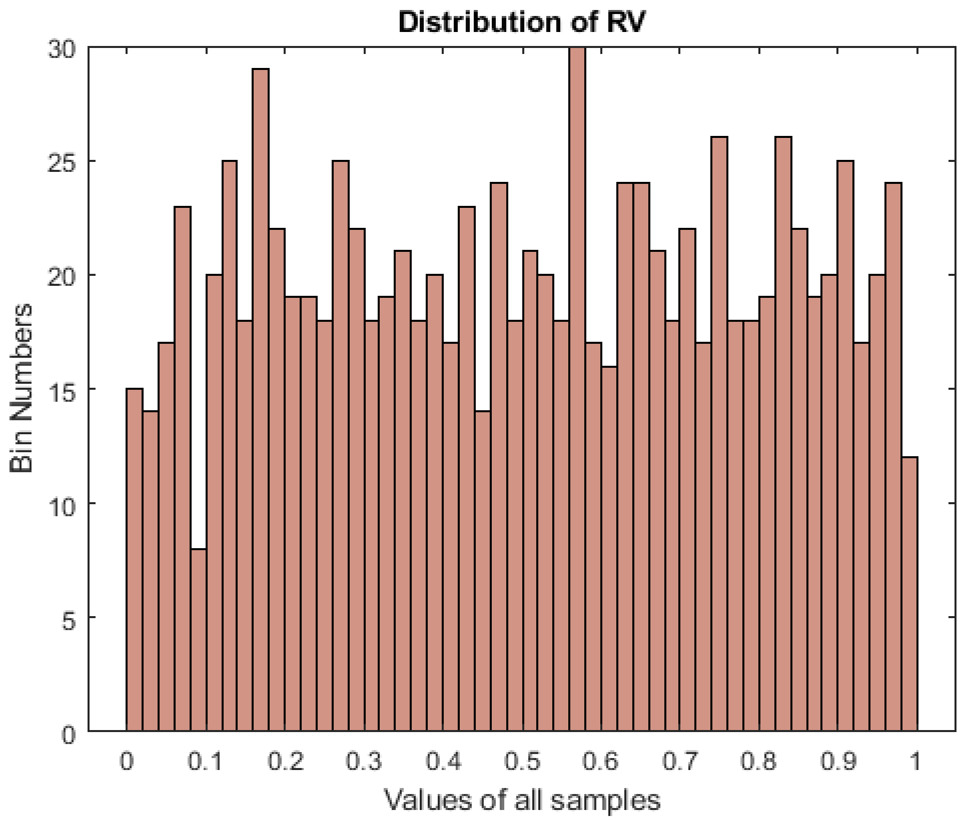
```
total=1000
```

```
total = 1000
```

```
R=L_limit+(U_limit-L_limit)*rand(total,1)  %Raw function to generate RV
```

```
R = 1000×1
    0.1612
    0.8227
    0.4010
    0.0447
    0.7845
    0.3258
    0.3312
    0.0756
    0.5072
    0.2685
      ⋮
```

```
Bin=50;
X=histogram(R,Bin);
xlabel('Values of all samples');
ylabel('Bin Numbers');
title('Distribution of RV')
X.FaceColor = [.7 .3 .2]
```




```
X =
  Histogram with properties:

             Data: [1000×1 double]
           Values: [15 14 17 23 8 20 25 18 29 22 19 19 18 25 22 18 19 21 18 20 17 23 14 24 18 21 20 18 30 17 16 24
          NumBins: 50
         BinEdges: [1×51 double]
         BinWidth: 0.0200
        BinLimits: [0 1]
    Normalization: 'count'
        FaceColor: [0.7000 0.3000 0.2000]
        EdgeColor: [0 0 0]

  Show all properties
```

```
m_1 = 0;
for count_1=1:total
    m_1=m_1+R(count_1);
end

Mean_R=(m_1/total)  %Raw code of Mean
```

```
Mean_R = 0.5077
```

```
Var_R=sum((R(1:end)-sum(R(1:end))./length(R)).^2)/(length(R)-1) %Raw code of Variance
```

```
Var_R = 0.0801
```

```
Mean_R1 =mean(R)  %Using Matlab function
```

```
Mean_R1 = 0.5077
```

```
Var_R1 =var(R)    %Using Matlab function
```

```
Var_R1 = 0.0801
```

Above function can generate 1000 random numbers between [0] to [1] with their Mean along with Variance and plot histogram with all samples and number of Bins.

Step-2.1: Exponential conversion from RV(Random Variable).

Now, we have to obtain an exponential random variable with mean 5 and representing the inter-arrival times of packets to a buffer.

From the formula of Exponential RV, we know:

FX(x) = 1-e^{-λ*x*}...(i)

As we have taken random variables from 0 to 1 and PDF also operates between 0 to 1. We can consider to use our RV as a function of PDF(Fx).

FX(x) = R

Now, if we put this value to equation (i), it will become:

FX(x) = 1-*e*^{-λ*x*}

>   R = 1 - *e*^{-λ*x*}

>   *e*^{-λ*x*} = 1 - R

>   -λ**x* = log(1- R)   % 'log' in both sides.

x = -(1/λ)*log(1-R)

Here, (1-R) = R   %Because it varies from 1 to 0.

Now, here mean is 5. So, 1/Lamda =5.

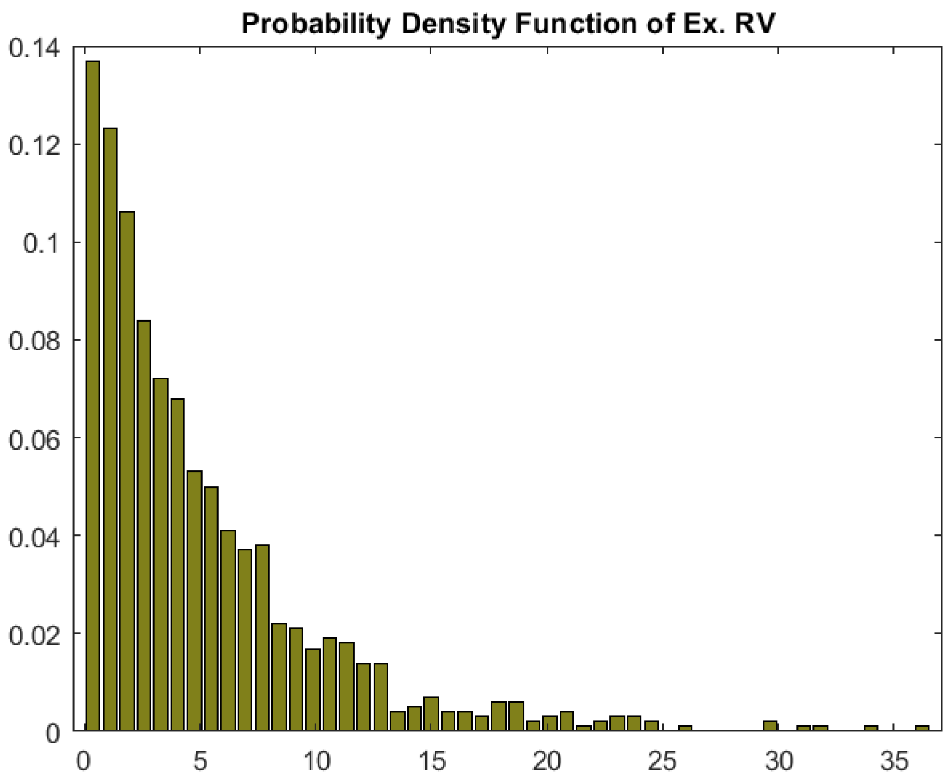
x = -(1/0.2)*log(R)

```
mean_x=5;
x = -((mean_x)*log(R))
```

```
x = 1000×1
    0.5047
    0.3043
    1.1000
    7.0932
    0.1232
    5.7406
    1.0364
```

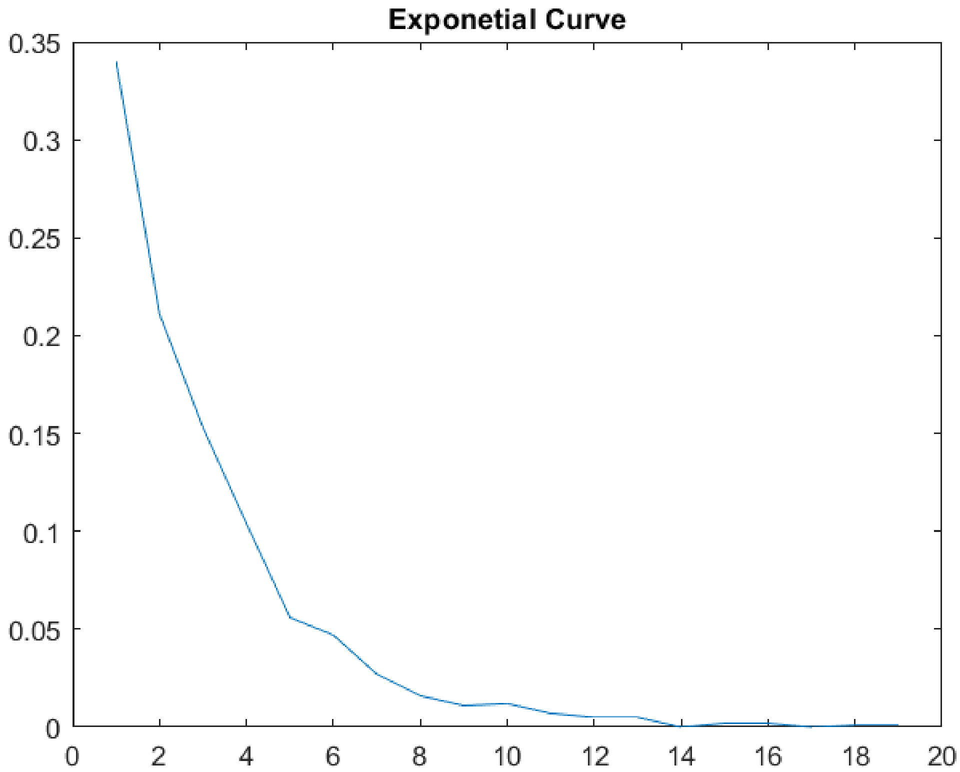
```
    1.8018
    6.5568
    2.6417
      ⋮
```

```
Bin=50;
%x1=x/1000;
[y1,y2]=hist(x,Bin);     %Histogram plot of PDF.
l_1=bar(y2,y1/1000);    %Normalizing y axis.
%x2=histogram(x1,Bin)
title('Probability Density Function of Ex. RV')
l_1.FaceColor = [.5 0.5 .1]
```




```
l_1 = 
  Bar with properties:

    BarLayout: 'grouped'
     BarWidth: 0.8000
    FaceColor: [0.5000 0.5000 0.1000]
    EdgeColor: [0 0 0]
    BaseValue: 0
        XData: [1×50 double]
        YData: [1×50 double]

  Show all properties
```

```
E=(histcounts(x))/total;
plot(E)
title('Exponetial Curve')
```




```
m= 0;
for count=1:total
    m=m+x(count);
end
Mean_X=(m/total)  %Raw code of Mean
```

```
Mean_X = 5.0056
```

```
Var_X=sum((x(1:end)-sum(x(1:end))./length(x)).^2)/(length(x)-1) %Raw code of Variance
```

```
Var_X = 26.6879
```

```
Mean_X1=mean(x)    %%Mean of the function
```

```
Mean_X1 = 5.0056
```

```
Var_X1=var(x)     %%Variance of the function
```

```
Var_X1 = 26.6879
```

Now, we have calculated Mean and Variance of 1000 exponential random variables.

We got Mean ≈ 5

From the formula, **E[X] =1/λ= 5**

So, Both are almost simiar.

And , We got Variance ≈25

From the formula, **E[X]^2 =1/(λ)^2= 25**

So, It's also simiar to our calculated value.

Step-2.2: Random numbers to Bernoulli conversion

A Bernoulli experiment includes a test once and noting while a experiment is conducting.

"Success" or "Failure" these are two possible outcome for a bernoulli random variable experiment.

So, for an event , let's declare 's' as probability of success.

if an event's probability is more than p then it is success , else failure.

if Event > s then 0 otherwise 1

Bernoulli PMF,

P(X=x)=p^k(1-p)^(1-k)   Here: p = s

Now, if s=0.5,

Random numbers to Bernoulli conversion given below

```
s=0.5;
L_limit=0
```

```
L_limit = 0
```

```
U_limit=1
```

```
U_limit = 1
```

```
total=1000
```

```
total = 1000
```

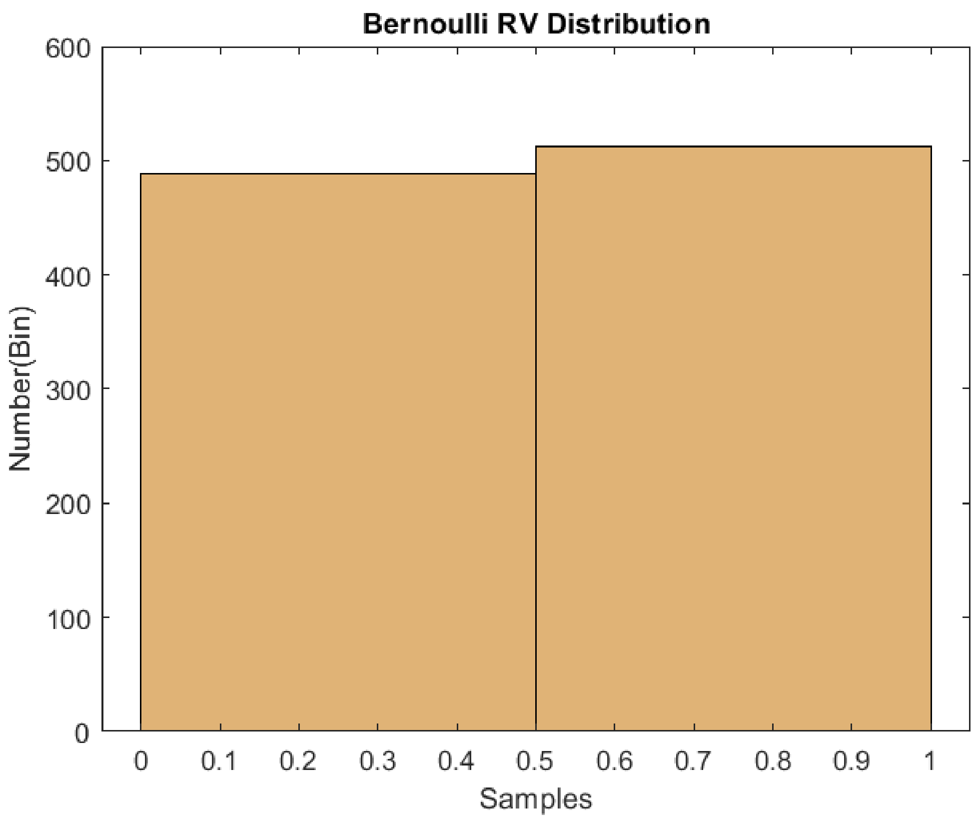
```
R=L_limit+(U_limit-L_limit)*rand(total,1)  %Raw function to generate RV
```

```
R = 1000×1
    0.8147
    0.9058
    0.1270
    0.9134
    0.6324
    0.0975
    0.2785
    0.5469
    0.9575
    0.9649
      ⋮
```

```
Bin=2
```

```
Bin = 2
```

```
Brv=(R<=s);
Brv;
B=histogram(Brv,Bin);
title('Bernoulli RV Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
B.FaceColor = [0.8 .5 .1];
```



Mean and Variance

```
m= 0;
for count=1:total
    m=m+Brv(count);
end
Mean_RawBrv=(m/total)  %Raw code of Mean
```

```
Mean_RawBrv = 0.5120
```

```
Var_RawBrv=sum((Brv(1:end)-sum(Brv(1:end))./length(Brv)).^2)/(length(Brv)-1) %Raw code of Varia
```

```
Var_RawBrv = 0.2501
```

```
Mean_Brv =mean(Brv)  %Mean Using Matlab function
```

```
Mean_Brv = 0.5120
```

```
Var_Brv =var(Brv)    %Variance Using Matlab function
```

```
Var_Brv = 0.2501
```

We got Mean ≈ 0.5 from the MATLAB funcion. Let's calculate using formula. 's' as probability of success.

E[X]= s =0.5

Furthermore, We got Variance approx ≈ 0.25, Let's calculate using formula

VAR[X] = s(1-s)= 0.25

Step-2.3: Random numbers to Binomial conversion

we know from the formula,

Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where

n = the number of trials (or the number being sampled)

x = the number of successes desired

p = probability of getting a success in one trial

q = 1 − *p* = the probability of getting a failure in one trial

We can convert Bernoulli it into Binomial just by getting summations of all the bernoulli which are repeated(n Bernoulli). Now, let's declare 'p' as probability of success.

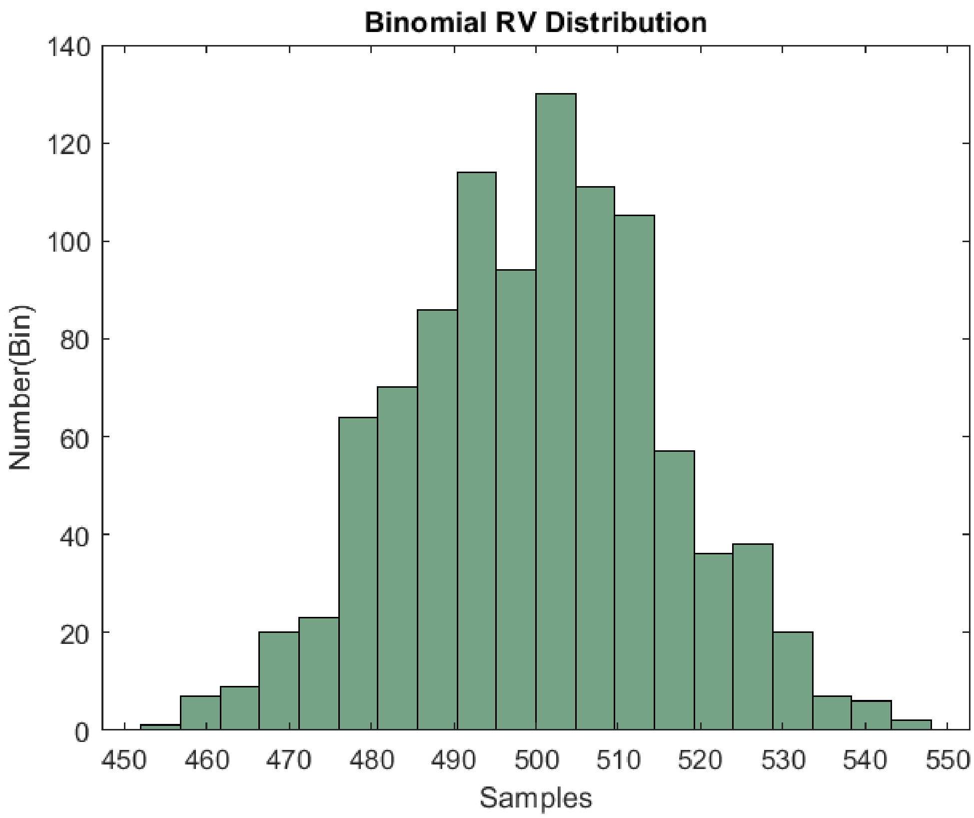
Let's consider p=0.5 and Number of experiment 1000 times.

```
n=1000;
p=0.5;
i=1;
Bi_rv = zeros(1,n);
while i<=n
    L_limit=0;
    U_limit=1;
    R=L_limit+(U_limit-L_limit)*rand(n,1);
    Brv=(R<=p);
    Bi_rv(i) = sum(Brv);
    i=i+1;
end
Bi_rv
```

```
Bi_rv = 1×1000
```

```
   500    519    521    480    509    498    498    500    481    517    523    523    487 ⋯
```

```
Bi=histogram(Bi_rv,20);
title('Binomial RV Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
Bi.FaceColor = [0.1 0.4 0.2];
```



Mean and Variance

```
m= 0
```

```
m = 0
```

```
for count=1:total
    m=m+Bi_rv(count);
end
Mean_RawBirv=(m/total)  %Raw code of Mean
```

```
Mean_RawBirv = 499.6610
```

```
Var_RawBirv=sum((Bi_rv(1:end)-sum(Bi_rv(1:end))./length(Bi_rv)).^2)/(length(Bi_rv)-1) %Raw code
```

```
Var_RawBirv = 251.5136
```

```

Mean_Bi_rv =mean(Bi_rv)  %Mean Using Matlab function
```

```
Mean_Bi_rv = 499.6610
```

```
Var_Bi_rv =var(Bi_rv)    %Variance Using Matlab function
```

```
Var_Bi_rv = 251.5136
```

We got Mean ≈ 500 from the MATLAB funcion. Let's calculate using formula

E[X]= np =500

Furthermore, We got Variance approx ≈ 250, Let's calculate using formula

VAR[X] = np(1-p)= 250

Step-2.4: Random numbers to Geometric Distribution

In Geometric Distribution before first success the experiment is repeated.

Number of failure variable 'F' and 'p' as probability of success.

The probability of **F** is ,

Before 1st success : (1-p)^F-1

Let R be the uniform random variable from 0 to 1 representing probability of F,

Now, our Random variable 'R' in respect to failure.

R= (1-p)^F-1

> log(R)= log((1-p)^F-1)

> (F-1)*log(1-p)= log(R)

Hence, F = 1+ log(R)/log(1-p)

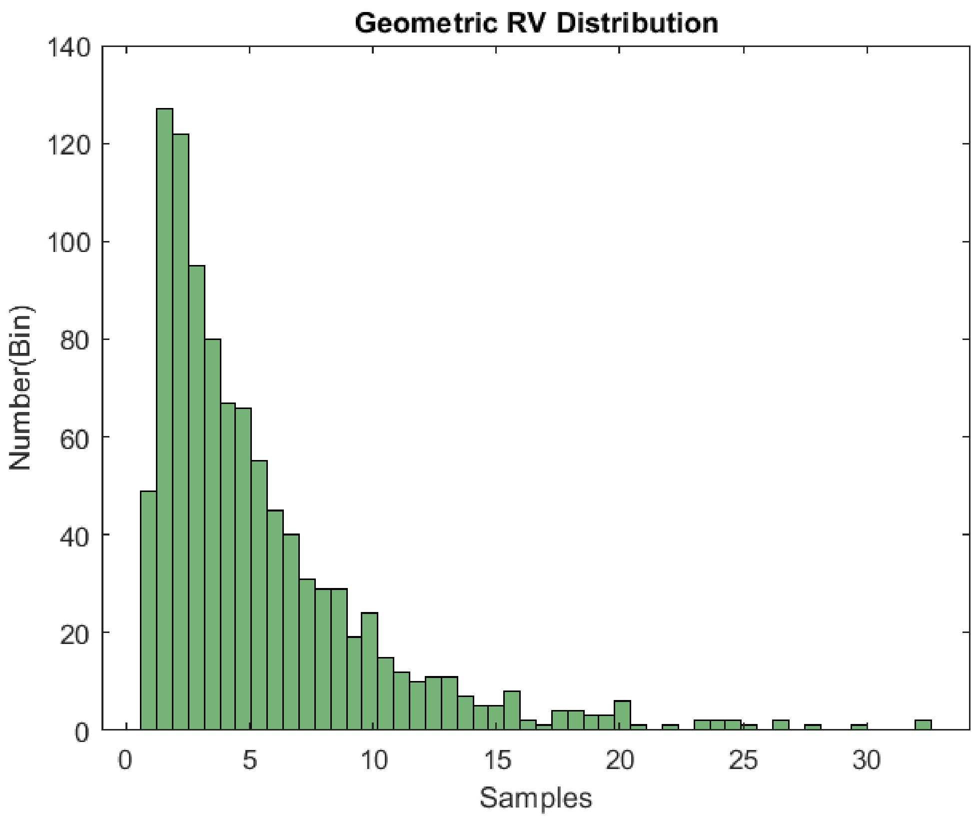
Calculating PDF untill 1st case:

```
n=1000;
L_limit=0;
U_limit=1;
R=L_limit+(U_limit-L_limit)*rand(n,1);
p=0.2;

Grv=1+(log(R)/log(1-p));
Grv
```

```
Grv = 1000×1
    2.3204
   12.5981
    1.1923
    2.1960
    9.6150
   10.2260
    1.3486
    4.2673
    2.6646
    5.0250
```

⋮

```
g=histogram(Grv,50);
g.FaceColor = [1 0.2 0];
title('Geometric RV Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
g.FaceColor = [0.1 0.5 0.1];
```



Mean and Variance

```
m= 0;
for count=1:n
    m=m+Grv(count);
end
Mean_RawGrv=(m/n)  %Raw code of Mean
```

```
Mean_RawGrv = 5.4810
```

```
Var_RawGrv=sum((Grv(1:end)-sum(Grv(1:end))./length(Grv)).^2)/(length(Grv)-1) %Raw code of Varia
```

```
Var_RawGrv = 21.6032
```

```
Mean_Grv =mean(Grv)  %Mean Using Matlab function
```

```
Mean_Grv = 5.4810
```

```
Var_Grv =var(Grv)     %Variance Using Matlab function
```

```
Var_Grv = 21.6032
```

We got Mean ≈ 5 from the MATLAB funcion. Let's calculate using formula

E[X]= 1/p =5

Furthermore, We got Variance ≈ 20, Let's calculate using formula

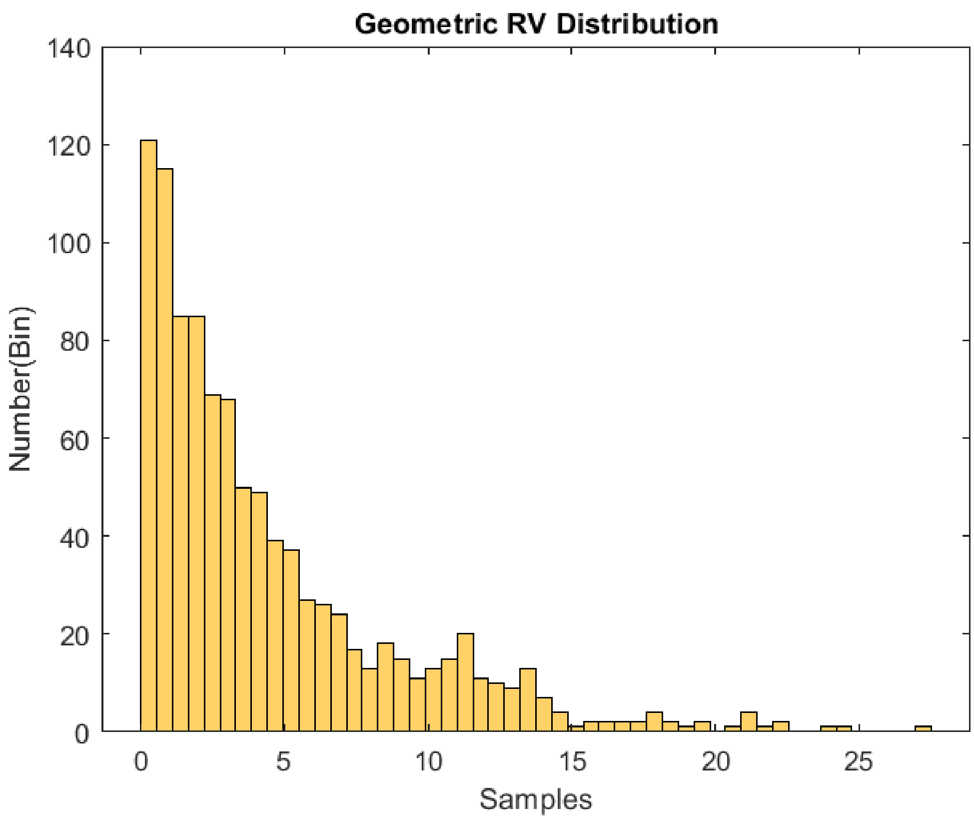
VAR[X] = (1-p)/p^2= 20

Calculating PDF before 1st case:

```
n=1000;
L_limit=0;
U_limit=1;
R=L_limit+(U_limit-L_limit)*rand(n,1);
p=0.2;

Grv_1=log(R)/log(1-p);
Grv_1
```

```
Grv_1 = 1000×1
   10.2614
    0.7058
   17.8996
    0.5537
    4.3888
    0.6463
   10.0208
    2.0057
   19.5087
    4.9621
      ⋮
```

```
g1=histogram(Grv_1,50);
title('Geometric RV Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
g1.FaceColor = [1 0.7 0];
```



Mean and Variance

```
m= 0;
for count=1:n
    m=m+Grv_1(count);
end
Mean_RawGrv_1=(m/n)  %Raw code of Mean
```

```
Mean_RawGrv_1 = 4.4299
```

```
Var_RawGrv_1=sum((Grv_1(1:end)-sum(Grv_1(1:end))./length(Grv_1)).^2)/(length(Grv_1)-1) %Raw coc
```

```
Var_RawGrv_1 = 19.6366
```

```
Mean_Grv_1 =mean(Grv_1)  %Mean Using Matlab function
```

```
Mean_Grv_1 = 4.4299
```

```
Var_Grv_1 =var(Grv_1)    %Variance Using Matlab function
```

```
Var_Grv_1 = 19.6366
```

We got Mean ≈ 4 from the MATLAB funcion. Let's calculate using formula

E[X]= (1-p)/p =4

Furthermore, We got Variance ≈ 20, Let's calculate using formula

VAR[X] = (1-p)/p^2= 20

Step-2.5: Random numbers to Normal Distribution

In this case, we will take two random variables from a uniform distribution of [0,1]. Let the name of the variables R1 and R2. We will convert them to random normal distribution using below formula,

N1_rv= sqrt(-2*ln(R1)* Cos(2*pi*R1)

and

N2_rv= sqrt(-2*ln(R2)* Cos(2*pi*R2)

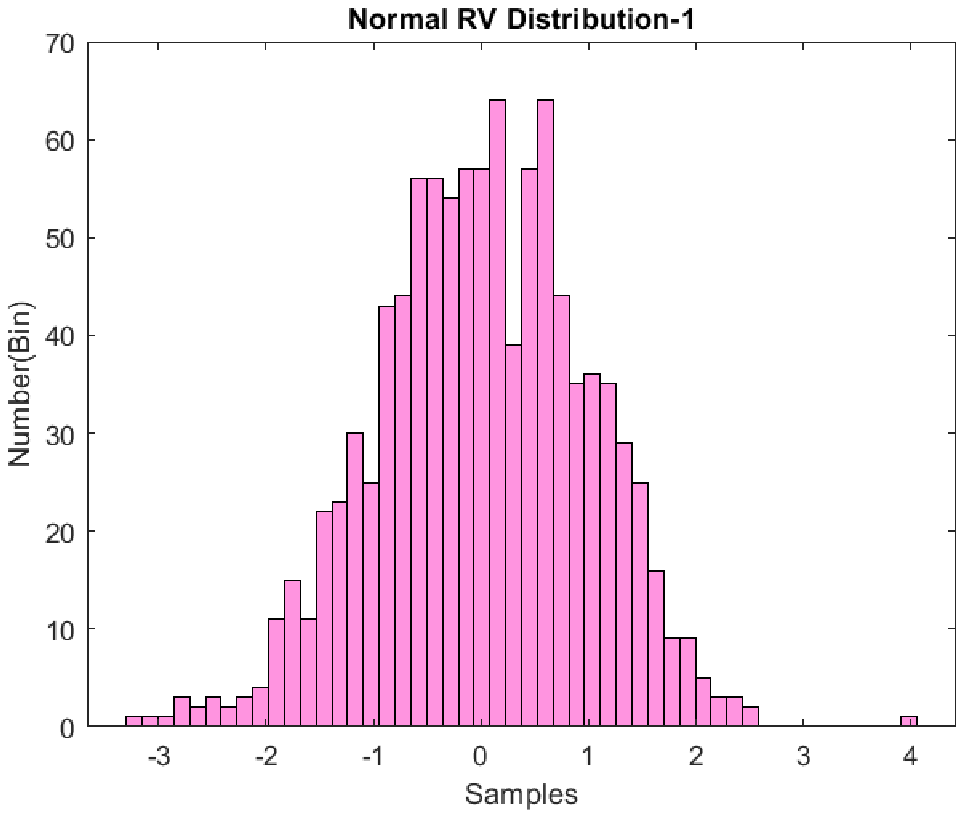
As, Normal distribution N(0,1), N1_rv and N2_rv zero mean and variance 1.

Now, we will creat two random variables with 100 samples.

```
n=1000;
L_limit=0;
U_limit=1;
R1=L_limit+(U_limit-L_limit)*rand(n,1);
R2=L_limit+(U_limit-L_limit)*rand(n,1);

S=1; %small Sigma
M=0; %Mu
Bin=50;

N1 = sqrt(-2*log(R1)) .* cos(2*pi*R2);
Normal_rv1=N1*(S+M)
```

```
Normal_rv1 = 1000×1
   -0.0180
    0.5865
    1.0011
    1.3327
    1.9768
   -0.2879
    2.1827
    0.5299
    0.6141
    1.4873
      ⋮
```

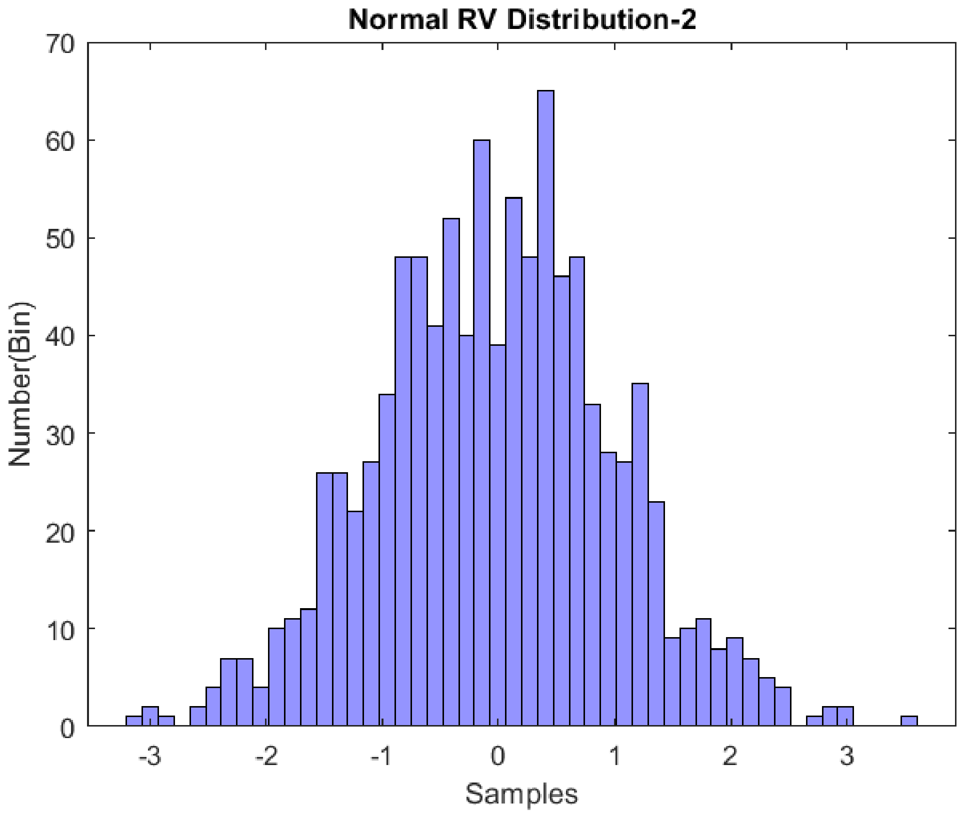
```
n1=histogram(Normal_rv1,Bin);
title('Normal RV Distribution-1');
xlabel('Samples');
ylabel('Number(Bin)');
n1.FaceColor = [1 0.3 0.8];
```




```
N2= sqrt(-2*log(R2)) .* sin(2*pi*R1);
Normal_rv2=N2*(S+M)
```

```
Normal_rv2 = 1000×1
   -1.3211
   -0.4086
   -1.7400
    1.9779
    0.1833
   -0.9422
    1.1088
   -0.5322
    1.3449
    1.3629
      ⋮
```

```
n2=histogram(Normal_rv2,Bin);
title('Normal RV Distribution-2');
xlabel('Samples');
ylabel('Number(Bin)');
n2.FaceColor = [0.3 0.3 1];
```



Mean and Variance

```
m= 0;
for count=1:n
    m=m+Normal_rv1(count);
end
Mean_RawNormal_rv1=(m/n)  %Raw code of Mean
```

```
Mean_RawNormal_rv1 = 0.0079
```

```
Var_RawNormal_rv1=sum((Normal_rv1(1:end)-sum(Normal_rv1(1:end))./length(Normal_rv1)).^2)/(lengt
```

```
Var_RawNormal_rv1 = 0.9589
```

```
Mean_Normal_rv1 =mean(Normal_rv1)  %Mean Using Matlab function
```

```
Mean_Normal_rv1 = 0.0079
```

```
Var_Normal_rv1 =var(Normal_rv1)    %Variance Using Matlab function
```

```
Var_Normal_rv1 = 0.9589
```

```
m= 0;
for count=1:n
    m=m+Normal_rv2(count);
end
Mean_RawNormal_rv2=(m/n)  %Raw code of Mean
```

```
Mean_RawNormal_rv2 = -0.0254
```

```
Var_RawNormal_rv2=sum((Normal_rv2(1:end)-sum(Normal_rv2(1:end))./length(Normal_rv2)).^2)/(lengt
```

```
Var_RawNormal_rv2 = 1.0559
```

```
Mean_Normal_rv2 =mean(Normal_rv2)  %Mean Using Matlab function
```

```
Mean_Normal_rv2 = -0.0254
```

```
Var_Normal_rv2 =var(Normal_rv2)    %Variance Using Matlab function
```

```
Var_Normal_rv2 = 1.0559
```

We got Mean ≈ 0 from the MATLAB funcion. Let's calculate using formula

E[X]= Mu =0

Furthermore, We got Variance ≈ 1, Let's calculate using formula

VAR[X] = Sigma^2= 1

Step-2.6: Random numbers to Rayleigh

$R = \sqrt{X^2 + Y^2}$, this is Rayleigh distribution.

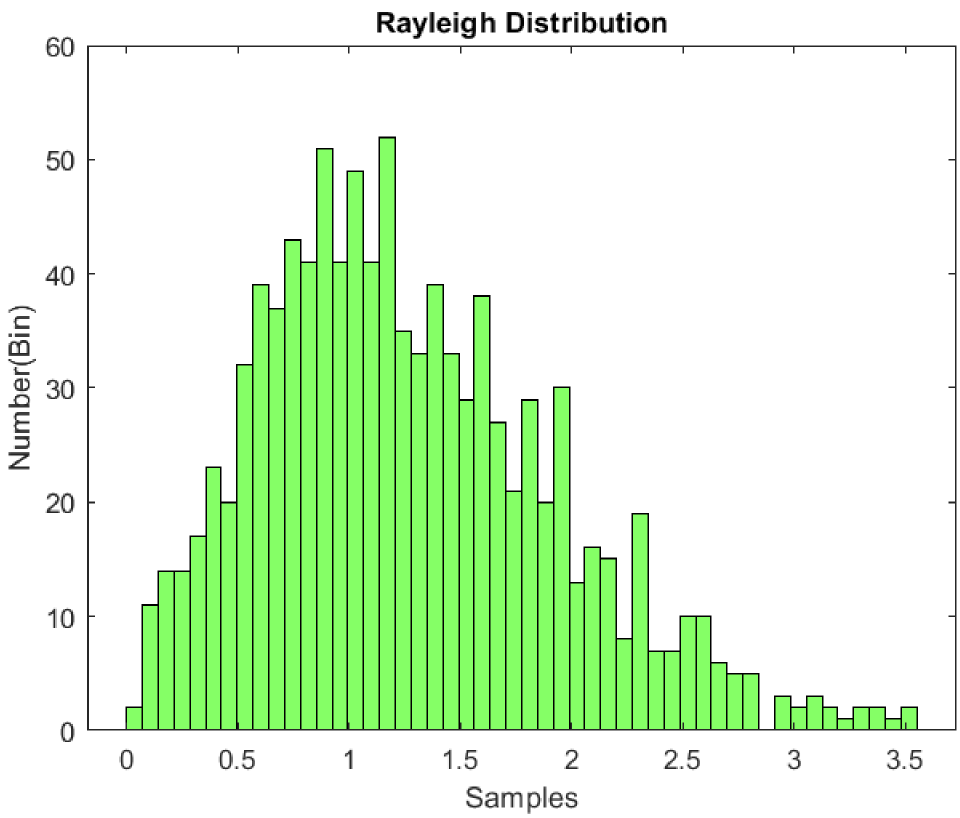
Here, X,Y tends to N(**Mu,Sigma**) . Bith are independent normal RV.

Let's consider Mu=0 and Sigma=1.

```
n=1000;
L_limit=0;
U_limit=1;
R1=L_limit+(U_limit-L_limit)*rand(n,1);
R2=L_limit+(U_limit-L_limit)*rand(n,1);

Bin=50;

N1 = sqrt(-2*log(R1)) .* cos(2*pi*R2);
N2= sqrt(-2*log(R1)) .* sin(2*pi*R2);

R_rv=sqrt(N1.^2 + N2.^2)
```

```
R_rv = 1000×1
    0.9182
    0.7339
    1.1992
    0.5848
    1.5897
    1.0043
    1.7645
    1.2699
    1.3980
    0.3460
      ⋮
```

```
r=histogram(R_rv,Bin);
title('Rayleigh Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
r.FaceColor = [.2 1 0];
```



Mean and Variance

```
m= 0;
for count=1:n
    m=m+R_rv(count);
end
Mean_RawR_rv=(m/n)  %Raw code of Mean
```

```
Mean_RawR_rv = 1.2664
```

```
Var_RawR_rv=sum((R_rv(1:end)-sum(R_rv(1:end))./length(R_rv)).^2)/(length(R_rv)-1) %Raw code of
```

```
Var_RawR_rv = 0.4372
```

```
Mean_R_rv =mean(R_rv)  %Mean Using Matlab function
```

```
Mean_R_rv = 1.2664
```

```
Var_R_rv =var(R_rv)    %Variance Using Matlab function
```

```
Var_R_rv = 0.4372
```

We got Mean ≈ 1.2 from the MATLAB funcion. Let's calculate using formula

E[X]= sigma*sqrt(pi/2) =1*sqrt(pi/2)=1.2

Furthermore, We got Variance ≈ 0.4, Let's calculate using formula

VAR[X] =(2- *pi/2)*Sigma^2*= (2- *pi/2)*1^2=0.4*

Step-2.7: Random numbers to Lognormal

Lognormal RV has similarities with Normal distribution. Normal distribution is like bell curve and in Lognormal distribution there's right skew with bell curve where tail never goes to '0'.

we can get lognormal distribution if we apply log function to normal distribution

Let, lognormal distribution = Ln

if 'N' is the normal distribution, then

N=log(Ln)

If we take exponential on both sides,

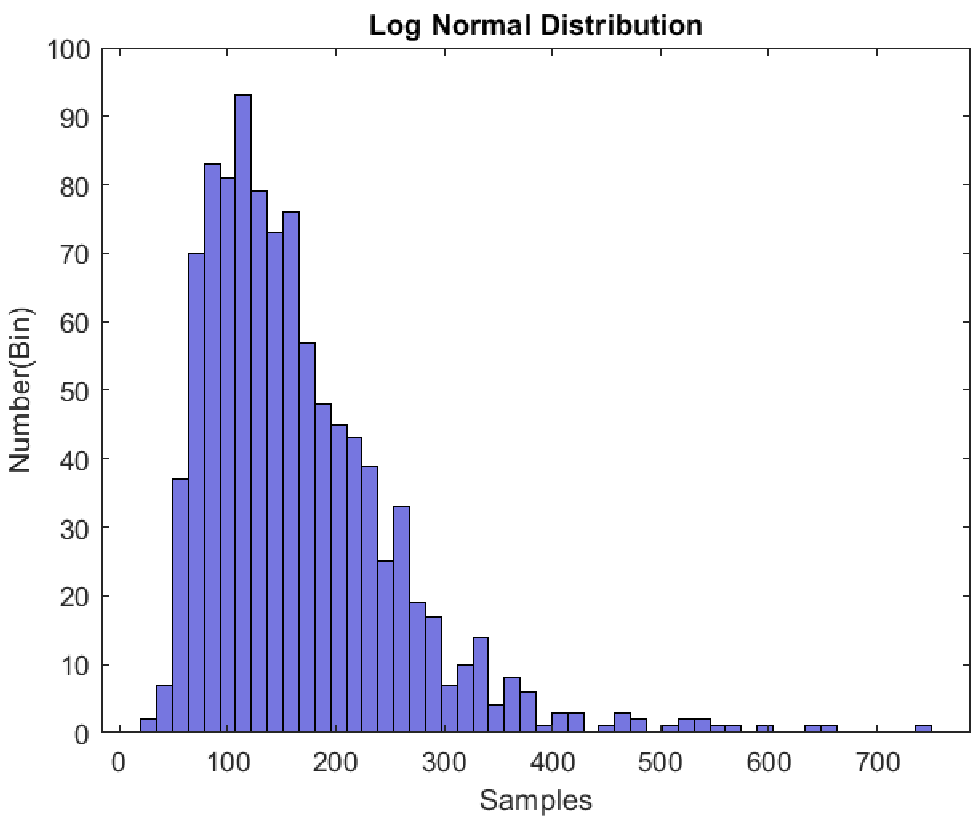
Ln=exp^(N)

Let's consider Mu=0 and Sigma=1.

```
n=1000;
L_limit=0;
U_limit=1;
R1=L_limit+(U_limit-L_limit)*rand(n,1);
R2=L_limit+(U_limit-L_limit)*rand(n,1);

S=0.5; %small Sigma
M=5; %Mu
Bin=50;

L = sqrt(-2*log(R1)) .* cos(2*pi*R2);
Normal_rv=L*S+M;

Lognormal_RV=exp(Normal_rv)
```

```
Lognormal_RV = 1000×1
  119.4188
  129.4886
  409.7770
  153.6059
  112.6885
   63.2798
   80.8746
  209.4124
  171.6596
  131.9476
      ⋮
```

```
l=histogram(Lognormal_RV,Bin);
title('Log Normal Distribution');
xlabel('Samples');
ylabel('Number(Bin)');
l.FaceColor = [0.1 0.1 0.8];
```



<u>Mean and Variance</u>

```
m= 0;
for count=1:n
    m=m+Lognormal_RV(count);
end
Mean_RawLognormal_RV=(m/n)  %Raw code of Mean
```

```
Mean_RawLognormal_RV = 166.1797
```

```
Var_RawLognormal_RV=sum((Lognormal_RV(1:end)-sum(Lognormal_RV(1:end))./length(Lognormal_RV)).^
```

```
Var_RawLognormal_RV = 8.2223e+03
```

```
Mean_Lognormal_RV =mean(Lognormal_RV)  %Mean Using Matlab function
```

```
Mean_Lognormal_RV = 166.1797
```

```
Var_Lognormal_RV =var(Lognormal_RV)    %Variance Using Matlab function
```

```
Var_Lognormal_RV = 8.2223e+03
```

We got Mean ≈ 166 from the MATLAB funcion. Let's calculate using formula

E[X]= e^(Mu+.5*Sigma^2) =168

Furthermore, We got Variance ≈ 8223, Let's calculate using formula

VAR[X] =e^(2*Mu+Sigma^2)= 8033

Discussion:

The main objective of this project was converting random variables to different types of distribution such as Exponential, Bernoulli, Binomial, Geometric, Normal, Rayleigh, Lognormal. I have checked our calculated values with formulated values by comparing Mean and Variance to prove formulas of Textbook. Another property I notiched while doing this project, all RV(Random Variables) are deeply connected with one another. We can convert any of them using mathemetics and get the help from MATLAB.