

Data Structures and Algorithms

CSE 2101

Class time:

Wednesday and Thursday – 11.10am

Google classroom code: **mpsx2hl**

Course Teacher:

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Lectures	Topics
1	Complexity analysis
2	Searching <ul style="list-style-type: none"> Linear search, binary Search, application of Binary Search- finding element in a sorted array finding nth root of a real number, solving equations.
3-6	Recursion <ul style="list-style-type: none"> Basic idea of recursion (3 laws-base case, call itself, move towards base case by state change) tracing output of a recursive function applications- merge sort, permutation, combination. Memoization
	Sorting <ul style="list-style-type: none"> Insertion sort, selection sort, bubble sort, merge sort, quick sort (randomized quick sort) distribution sort (counting sort, radix sort, bucket sort) lower bounds for sorting, external sort
7	Linked List <ul style="list-style-type: none"> Singly/doubly/circular linked lists, basic operations on linked list (insertion, deletion, and traverse), dynamic array and its application.
8-9	Stack Basic <ul style="list-style-type: none"> stack operations (push/pop/peek), stack-class implementation using Array and linked list, in-fix to post-fix expressions conversion and evaluation, balancing parentheses using stack,
10-11	Queue <ul style="list-style-type: none"> basic queue operations (Enqueue, dequeue), circular queue/ dequeue, queue-class implementation using array and linked list application- Josephus problem, palindrome checker using stack and queue.

12	Binary tree <ul style="list-style-type: none"> Binary tree representation using array and Pointer traversal of Binary Tree (in-order, pre-order and post-order).
13-14	Binary Search Tree <ul style="list-style-type: none"> BST representation basic operations on BST (creation, insertion, deletion, querying and traversing), application- searching, sets.
15-16	Heap <ul style="list-style-type: none"> Min-heap, max-heap, Fibonacci-heap applications-priority queue heap sort.
17	General Tree Implementation, application of general tree- file system
18	Disjoint Set Union finds, path compression.
20	Huffman Coding: application- Compression.
21 -22	Graph representation (adjacency matrix/adjacency list), basic operations on graph (node/edge insertion and deletion), traversing a graph: breadth-first search (BFS), depth-first search (DFS), graph-bi-colouring.
23	Self-balancing Binary Search Tree: AVL tree (Rotation, insertion).
24-25	Set Operations: Set representation using bitmask, set/clear bit, querying the status of a bit, toggling bit values, LSB, application of set operations.
26-27	String ADT: The concatenation of two strings, the extraction of substrings, searching a string for a matching substring, parsing.

Data Structures ?

- A data structure is a way of **organizing** and **storing** data in a **computer** so that it can be **accessed** and **modified efficiently**.
- Different types of data structures are suited to different kinds of applications:
- One of the first recorded uses of a data structure was the Jacquard loom in 1801, which used a punched card to control the pattern of a woven textile.
- some are highly specialized to specific tasks
- Some common examples of data structures include
 - Arrays
 - Linked lists
 - Stacks
 - Queues
 - Trees
 - graphs.

Algorithms?

- Algorithms are a **set of instructions** for carrying out a specific task or solving a specific problem
- A key part of an algorithm is the **use** of one or more data structures in order to store and organize the data that **the algorithm operates on**.

Relationship –

- data structures provide a way to store and organize data, and
- algorithms use the data structures to accomplish a specific task or solve a specific problem.

Searching Algorithms: Linear and Binary Search

Linear Search

- **Sequential search**
 - It traverses the array sequentially to locate the required element.
 - It searches for an element by comparing it with each element of the array one by one.
- **Applicability**
 - No information is given about the array.
 - The given array is unsorted or the elements are unordered.
 - The list of data items is smaller.

Linear Search

Best Case

- The element being searched may be found at the first position.
- In this case, the search terminates in success with just one comparison.
- Thus in best case, linear search algorithm takes $O(1)$ operations.

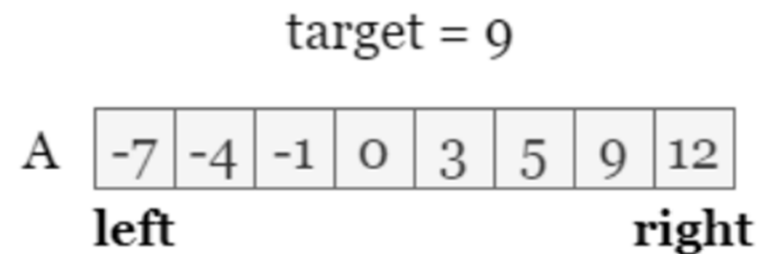
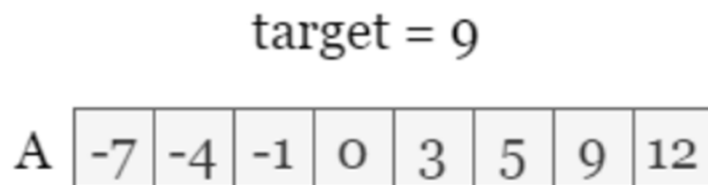
Worst Case

- The element being searched may be present at the last position or not present in the array at all.
- In the former case, the search terminates in success with n comparisons.
- In the later case, the search terminates in failure with n comparisons.
- Thus in worst case, linear search algorithm takes $O(n)$ operations.

Binary Search

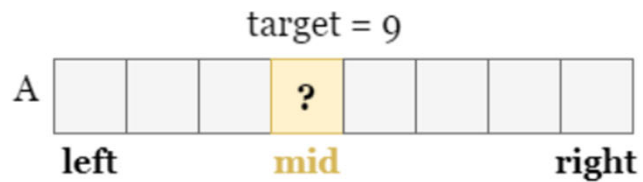
First, we define the search space using two boundary indexes, `left` and `right`

- We shall continue searching over the search space as long as it is not empty.
- A while loop with a condition: `left <= right`



Determine the left and right boundaries

Binary Search



Get the middle index:
 $\text{mid} = (\text{left} + \text{right}) / 2$

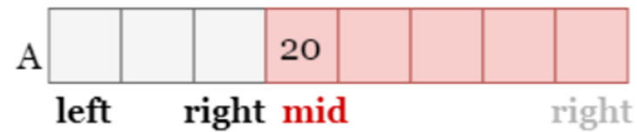
compare the mid
value $A[\text{mid}]$ with **target**



If $A[\text{mid}] = \text{target}$
we find target in the array!

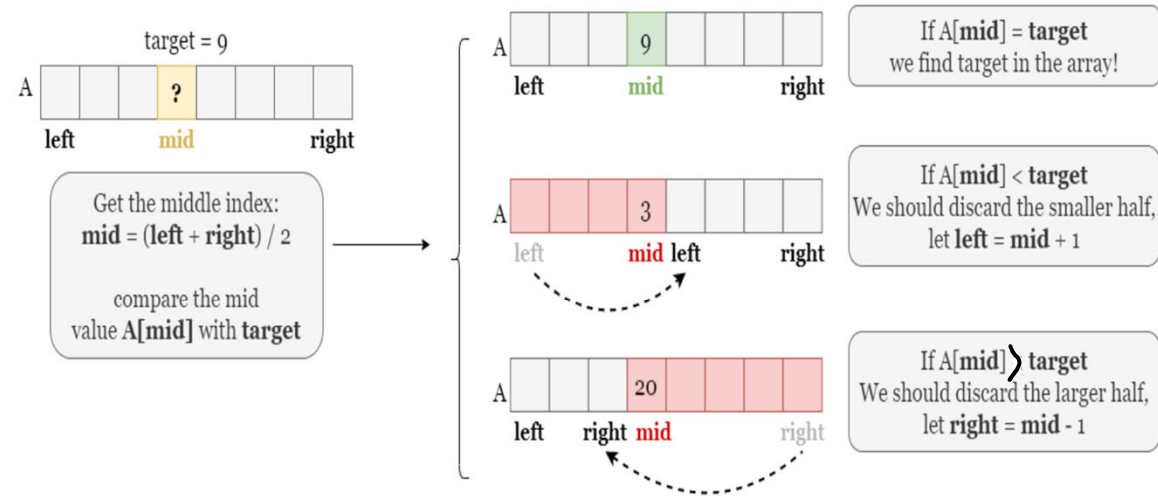


If $A[\text{mid}] < \text{target}$
We should discard the smaller half,
let $\text{left} = \text{mid} + 1$



If $A[\text{mid}] > \text{target}$
We should discard the larger half,
let $\text{right} = \text{mid} - 1$

Binary Search



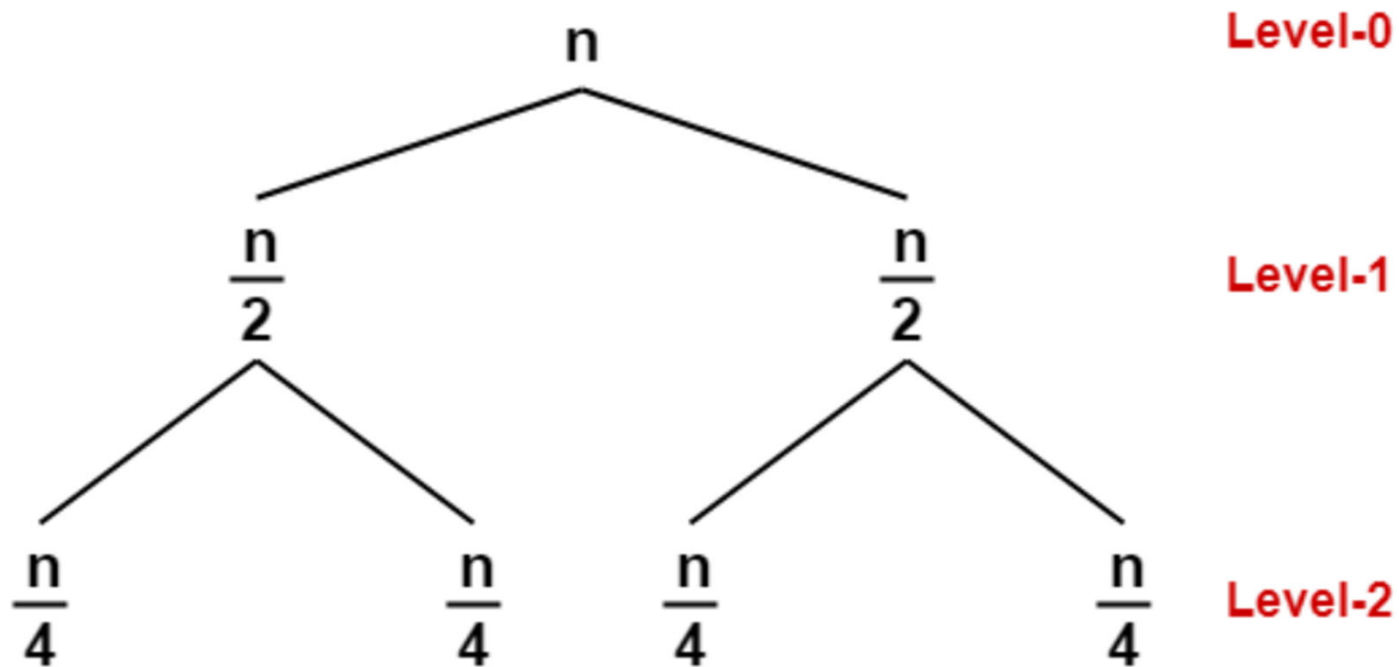
Algorithm

1. Initialize the boundaries of the search space as $left = 0$ and $right = \text{nums.size} - 1$.
2. If there are elements in the range $[left, right]$, we find the middle index $mid = (left + right) / 2$ and compare the middle value $\text{nums}[mid]$ with $target$:
 - If $\text{nums}[mid] = target$, return mid .
 - If $\text{nums}[mid] < target$, let $left = mid + 1$ and repeat step 2.
 - If $\text{nums}[mid] > target$, let $right = mid - 1$ and repeat step 2.
3. We finish the loop without finding $target$, return -1 .

Binary Search

```
1 class Solution:
2     def search(self, nums: List[int], target: int) -> int:
3         # Set the left and right boundaries
4         left = 0
5         right = len(nums) - 1
6
7         # Under this condition
8         while left <= right:
9             # Get the middle index and the middle value.
10            mid = (left + right) // 2
11
12            # Case 1, return the middle index.
13            if nums[mid] == target:
14                return mid
15            # Case 2, discard the smaller half.
16            elif nums[mid] < target:
17                left = mid + 1
18            # Case 3, discard the larger half.
19            else:
20                right = mid - 1
21
22            # If we finish the search without finding target, return -1.
23            return -1
```

Binary Search



$n \rightarrow n/2 \rightarrow n/4 \rightarrow \dots \rightarrow 1$

$$n/2^k = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

Assuming our search space is exhausted after k level

Binary Search

Complexity Analysis

Let n be the size of the input array `nums` .

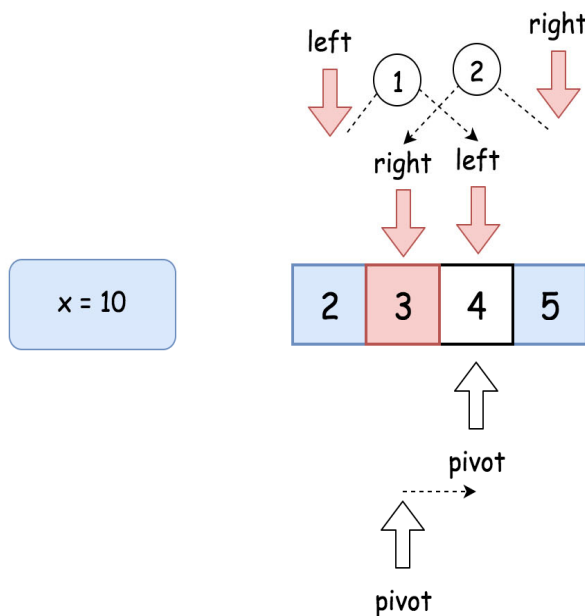
- Time complexity: $O(\log n)$
 - `nums` is divided into half each time. In the worst-case scenario, we need to cut `nums` until the range has no element, and it takes logarithmic time to reach this break condition.
- Space complexity: $O(1)$
 - During the loop, we only need to record three indexes, `left` , `right` , and `mid` , they take constant space.

Binary Search – finding the square root of a real number x

Let's go back to the interview context. For $x \geq 2$ the square root is always smaller than $x/2$ and larger than 0 : $0 < a < x/2$.

Since a is an integer, the problem goes down to the iteration over the sorted set of integer numbers. Here the binary search enters the scene.

- If $x < 2$, return x .
- Set the left boundary to 2, and the right boundary to $x / 2$.
- While $\text{left} \leq \text{right}$:
 - Take $\text{num} = (\text{left} + \text{right}) / 2$ as a guess. Compute $\text{num} * \text{num}$ and compare it with x :
 - If $\text{num} * \text{num} > x$, move the right boundary $\text{right} = \text{pivot} - 1$
 - Else, if $\text{num} * \text{num} < x$, move the left boundary $\text{left} = \text{pivot} + 1$
 - Otherwise $\text{num} * \text{num} == x$, the integer square root is here, let's return it
- Return right



Binary Search – finding the square root of a real number x

```
1 class Solution:
2     def search(self, nums: List[int], target: int) -> int:
3         # Set the left and right boundaries
4         left = 0
5         right = len(nums) - 1
6
7         # Under this condition
8         while left <= right:
9             # Get the middle index and the middle value.
10            mid = (left + right) // 2
11
12            # Case 1, return the middle index.
13            if nums[mid] == target:
14                return mid
15
16            # Case 2, discard the smaller half.
17            elif nums[mid] < target:
18                left = mid + 1
19
20            # Case 3, discard the larger half.
21            else:
22                right = mid - 1
23
24            # If we finish the search without finding target, return -1.
25            return -1
```

```
1 class Solution:
2     def mySqrt(self, x):
3         if x < 2:
4             return x
5
6         left, right = 2, x // 2
7
8         while left <= right:
9             pivot = left + (right - left) // 2
10            print("This is the pivot value:", pivot)
11            num = pivot * pivot
12            if num > x:
13                right = pivot - 1
14            elif num < x:
15                left = pivot + 1
16            else:
17                return pivot
18
19        return right
20
21 sol = Solution()
22 print(sol.mySqrt(23))
```

```
This is the pivot value: 6
This is the pivot value: 3
This is the pivot value: 4
This is the pivot value: 5
```


Binary Search – Recursive Implementation

```
if __name__ == '__main__':  
  
    nums = [2, 5, 6, 8, 9, 10]  
    target = 5  
  
    (left, right) = (0, len(nums) - 1)  
    index = binarySearch(nums, left, right, target)  
  
    if index != -1:  
        print('Element found at index', index)  
    else:  
        print('Element found not in the list')
```

```
# Recursive implementation of the binary search algorithm to return  
# the position of `target` in subarray nums[left...right]  
def binarySearch(nums, left, right, target):  
  
    # Base condition (search space is exhausted)  
    if left > right:  
        return -1  
  
    # find the mid-value in the search space and  
    # compares it with the target  
  
    mid = (left + right) // 2  
  
    # overflow can happen. Use below  
    # mid = left + (right - left) / 2  
  
    # Base condition (a target is found)  
    if target == nums[mid]:  
        return mid  
  
    # discard all elements in the right search space,  
    # including the middle element  
    elif target < nums[mid]:  
        return binarySearch(nums, left, mid - 1, target)  
  
    # discard all elements in the left search space,  
    # including the middle element  
    else:  
        return binarySearch(nums, mid + 1, right, target)
```