Section A - Answer ALL questions

1. Briefly explain the tradeoff between complexity and accuracy in 3D modelling.

(3 marks)

Model Answer:

Detailed model: higher computational cost. Less detail: might lose realism.

2. If you are designing a computer animation package (like Blender) how would you decide to represent orientations internally and why?

(4 marks)

What representation would you choose for user-defined orientations and why?

(2 marks)

Model Answer:

The internal representation would be based on quaternions (1 mark) because they can easily be interpolated and composed while not suffering from the gimbal lock problem. (3 marks, one each) For user input I would pick Euler/Fixed angles (1 mark) because they are intuitively understood by humans (1 mark).

3. Describe the **Angle+Axis** representation for rotations. State the theorem on which it is based. How do we interpolate orientation using this representation?

(5 marks)

Model Answer:

The Angle+axis representation uses a direction in space that is denoted by a 3d vector $(x, y, z)^T$ (1 mark) and an angle θ by which the object is rotated around that axis (1 mark). Euler's theorem states that one can relate any 2 orientations by a single orientation about some axis (1 mark). To interpolate between two orientations each of which is represented by an axis and angle, we interpolate separately between the two axes and the two angles (2 marks).

4. When rotating an object using the fixed angles representation, when does **gimbal lock** occur?

(4 marks)

Model Answer:

Gimbal lock occurs when two rotation axes coincide resulting in the loss of one degree of freedom. (In other words, changing the angles corresponding to each of those two axes gives *the same rotation.)*

5. How many parameters are needed to describe the configuration of a pair of scissors in 3D space? Explain your answer.

(4 marks)

Model Answer:

Seven parameters. (1 mark) *Three of these are used to describe the position of the object in space (x-y-z)* (1 mark) *three are used to describe orientation (yaw-pitch-roll).* (1 mark) *and one parameter is used to describe the angle of the scissor blades.*

6. Which animation technique would you use to model each of the following:

a) Debris flying out of an explosion

b) A field of flowers

c) A jellyfish swimming

d) A snooker table

e) An alien space ship

f) A lizard slowly changing colour

g) A ”monster” character walking

(7 marks)

Model Answer:

a) Particle systems

b) L-trees

c) Free Form Deformation or Vertex displacement

d) Simulation

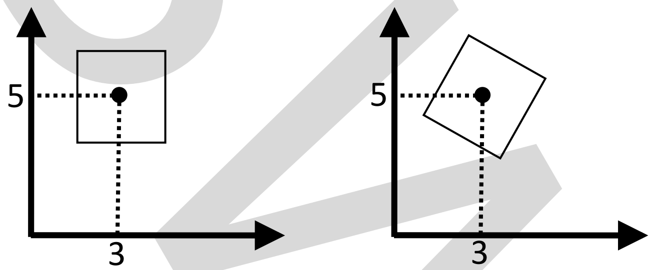
e) Keyframing position & orientation

f) Keyframing colour/texture

g) Skeleton deformation

7. The figure on the left shows the initial configuration of a 2D square centred on point $(3, 5)$. Assume that $T(x, y)$ is the matrix that translates by x and y units in along the x and y axes respectively and that $R(\theta)$ is the matrix that rotates counter-clockwise by θ. Both types of matrix are assumed to operate on homogeneous coordinates. Write down the transformation that achieves the figure on the right, i.e. the square remaining centred on $(3, 5)$ while rotating by 30 degrees clockwise.

(4 marks)



Model Answer:

$T(3, 5)R(-30)T(-3, -5)$

8. Explain the steps involved in object deformation based on **FFD (free-form deformation)**.

(5 marks)

Model Answer:

- *Object embedded in 3-d lattice/grid. (1 mark)*
- *Local co-ordinates of object vertices determined. (1 mark)*
- *Lattice distorted. (1 mark)*
- *Object vertex co-ordinates interpolated (usually Bezier or B-spline interpolation) (1 mark)*
- *Vertices mapped back to global space. (1 mark)*

9. Name the FIVE different spaces that form part of the **display pipeline**.

(5 marks)

Model Answer:

Object space, World space, Eye space, Image space,Screen space (1 mark each)

10. Give definitions of **forward and inverse kinematics**. Which one is harder to achieve? Give an example of an animation task for which EACH technique is more suitable.

(7 marks)

Model Answer:

In forward kinematics we specify the configuration of joints over a time period and the animation system will calculate the resulting animation of the object (2 marks). In inverse kinematics we specify a joint tree and give the system the start and end positions. From this the animation system shall infer the configuration of the joints over time (and create the animation) (2 marks). Inverse kinematics is much harder to achieve (1 mark). FK is suitable for modelling simple curved motion (e.g. arms swinging during walk) (1 mark). IK is suitable for placing the end effector in a specific location (e.g. a character grabs an object, a robotic arm lifts a load, the character's feet must touch the ground while walking etc) (1 mark).

END OF SECTION A

Total: 50 marks

Section B - Answer any TWO of the following three questions

11. This question is about **physics simulation**.

a) Why is physics simulation used in computer animation? (2 marks)

Model Answer:

Physics simulation helps us to quickly create animations of lifeless objects that move and deform under the laws of physics (E.g. a billiard table or a football that is kicked). Instead of key-framing the motion or deformation we can just let the laws of physics determine how the object reacts to forces like gravity, friction, viscosity as well as how it interacts with other objects through collisions.

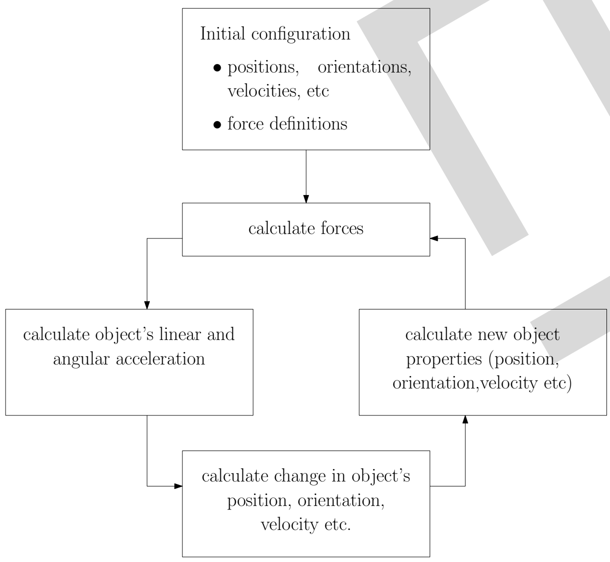
b) Briefly explain what is meant by broad phase and narrow phase algorithms in the context of **collision detection**. (4 marks)

Model Answer:

Testing for possible collisions of all object pairs is computationally expensive. Broad phase collision detection involves discarding those object pairs that definitely do not collide. (2 marks) In a subsequent phase which is known as narrow phase collision, the algorithm examines in more detail the remaining object pairs. (2 marks)

c) Describe the **simulation loop** employed in physics-based animation of rigid bodies. (7 marks)

Model Answer:



(1 mark for each block + 2 for perfect explanation)

d) We wish to simulate an object of mass $m = 2$ that moves on the x-y plane under the influence of a constant gravitational field $F = (0,\ -2)$. Its velocity initially (at frame 0) is $v = (2,\ 6)$ and its position is at $x = (0,\ 0)$. Use Euler's algorithm to calculate the position of the object in frames 1 to 6.

(12 marks)

Model Answer:

Frame=0: $x = (0,\ 0)$, $v = (2,\ 6)$.

Frame=1:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 6) * 1 = (2,\ 6)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (0,\ 0) + (2,\ 6) = (2,\ 6)$

$v := v + \Delta v = (2,\ 6) + (0,\ -1) = (2,\ 5)$

Frame=2:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 5) * 1 = (2,\ 5)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (2,\ 6) + (2,\ 5) = (4,\ 11)$

$v := v + \Delta v = (2,\ 5) + (0,\ -1) = (2,\ 4)$

Frame=3:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 4) * 1 = (2,\ 4)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (4,\ 11) + (2,\ 4) = (6,\ 15)$

$v := v + \Delta v = (2,\ 4) + (0,\ -1) = (2,\ 3)$

Frame=4:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 3) * 1 = (2,\ 3)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (6,\ 15) + (2,\ 3) = (8,\ 18)$

$v := v + \Delta v = (2,\ 3) + (0,\ -1) = (2,\ 2)$

Frame=5:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 2) * 1 = (2,\ 2)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (8,\ 18) + (2,\ 2) = (10,\ 20)$

$v := v + \Delta v = (2,\ 2) + (0,\ -1) = (2,\ 1)$

Frame=6:

$a := \frac{F}{m} = \frac{1}{2} = (0,\ -1)$

$\Delta x := v * \Delta t = (2,\ 1) * 1 = (2,\ 1)$

$\Delta v := a * \Delta t = (0,\ -1) * 1 = (0,\ -1)$

$x := x + \Delta x = (10,\ 20) + (2,\ 1) = (12,\ 21)$

$v := v + \Delta v = (2,\ 1) + (0,\ -1) = (2,\ 0)$

12. This question is about **homogeneous transformations**.

An animator wishes to move a three-dimensional object in a scene, that has its centre at $[1\ 3\ 2]^T$. The object needs to be rotated by 90 degrees about the x-axis followed by -90 degrees about the y-axis. It should then be scaled by a factor of 3 along the z axis and finally it needs to be translated by $[-1\ \ -1\ 5]^T$. The scaling and rotation are to be performed using the object centre as pivot point.

As an aid the 3 rotational transformation matrices are listed below together with common sine and cosine values so you can perform the calculations without a calculator:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos 0^\circ = 1,\ \sin 0^\circ = 0,\ \cos 90^\circ = 0,\ \sin 90^\circ = 1,\ \cos(-90^\circ) = 0,\ \sin(-90^\circ) = -1$$

a) Write down (in homogeneous coordinate form) the transformations necessary to achieve the motion described above, stating clearly the order in which the transformations should be applied.

(10 marks)

Model Answer:

1. *Translation of obj centre to origin* $T_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. *An x-rotation* $R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. *A y-rotation* $R_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. *A z-scaling* $S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. *Translation of origin to obj centre* $T_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6. *Translation* $T_3 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b) Explain how the transformations from (a) can be combined into one single transformation and perform the necessary calculations to arrive at this composite transformation.

(8 marks)

Model Answer:

They should be combined into a single composite transformation M defined by $M = T_3 T_2 S R_2 R_1 T_1$

$$M = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 4 \\ 3 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Before this motion is applied a vertex on the object has global coordinates $[1\ 5\ 3]^T$. Use the transform computed in (b) to calculate the new global coordinates of the vertex. By considering how one would transform a million vertices, illustrate the computational efficiency of homogeneous coordinates.

(4 marks)

Model Answer:

$$M \begin{pmatrix} 1 \\ 5 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 7 \\ 1 \end{pmatrix}$$

Transforming a million vertices costs 1 million matrix-vector multiplications plus the almost-negligible cost of computing M. Without homogeneous coordinates we would need 6 matrix-vector multiplications per vertex, i.e. 6 times as much computational cost.

d) Without performing the calculation, describe how you would efficiently undo the motion.

(1 mark)

Model Answer:

We can obtain the opposite transformation by calculating the matrix inverse of M and multiplying all vertices with that matrix.

e) Explain why storing a 3d orientation in matrix form is wasteful in terms of memory.

(2 marks)

Model Answer:

We need 3DOF to describe an orientation while a matrix uses 9 DOF (non-homogeneous) or 16DOF (homogeneous). Either of those cases suffices as an explanation.

13. Two widely used methods for generating and animating natural objects and phenomena are **L-systems** and **particle systems**.

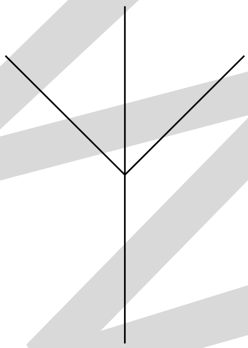
a) Explain what L-systems are and how they are used to generate models of natural objects

(8 marks)

Model Answer:

L-systems are compact, iterative systems designed to generate fractal (self-similar) graphics (2 marks). They use a string representation with small vocabulary where string symbols are mapped to graphics commands (2 marks). An L-system consists of an axiom (starting configuration) (1 mark) and one or more production rules that are applied in an iterative manner (1 mark). The string characters correspond to turtle graphics commands that, when executed, produce a graphical rendering of the L-system (1 mark). L-systems can be used to model natural objects (e.g. trees) that exhibit a high degree of self-similarity (1 mark).

b) The following figure represents the rendered output of the second iteration of an L-system.



Write down the axiom, production rule and constants for this tree, assuming that each branch makes an angle of 45 degrees with the main tree trunk.

(4 marks)

Model Answer:

```
Axiom F
F→F[+F][-F]F angle = 45°
```

Write down the output of the second iteration both in string representation and in graphical form.

(5 marks)

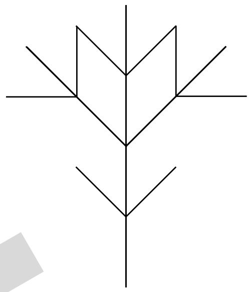
Model Answer:

String:

```
F[+F][-F]F[+F[+F][-F]F][-F[+F][-F]F]F[+F][-F]F
```

(2.5 marks)

Graphical:



(2.5 marks)

c) Using your answer to (b) as illustration, explain the advantage of using L-systems for modelling natural objects, compared to ordinary modelling techniques.

(4 marks)

Model Answer:

L-systems are a more compact representation. A short production rule can give rise to very complicated shapes by applying it recursively. For example, just 4 geometric primitives in the rule in (b) give rise to 16 primitives in the second iteration, 64 in the 3d iteration and so on, without adding to the complexity of the representation.

d) Name FOUR examples of phenomena that can be modelled by particle systems.

(4 marks)

Model Answer:

Particles systems can be used to model amorphous, dynamic and fluid objects/phenomena like clouds, smoke, water, explosions, fire, etc (1 for each example).

END OF EXAMINATION PAPER

Total: 125 marks