CENG 3549 – Functional Programming

Modules & Lists and Strings & Recursive Functions

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Outline

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- **note:** separate namespaces for functions and types

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Example

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module Stack where
type Stack a = [a]
empty = []
push  = (:)
pop s = (head s, tail s)
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push :: a -> Stack a -> Stack a
push = (:)
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- note the partial application of `(:)`
- this is equivalent to `push x s = x : s`

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- `import Stack`
- `import Stack (push, pop)`
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- what are the results of `unwords . words` and `words . unwords`?

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- `concat xss = [x | xs <- xss, x <- xs]`

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- `primes = [n | n <- [1..], factors n == [1,n]]`

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  5. generalize and simplify (e.g., `product :: Num a => [a] -> a` and `product = foldr (*) 1`)

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- generalize and simplify:

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drop n xs | n <= 0  =  xs
drop _ []           =  []
drop n (_:xs)       =  drop (n - 1) xs
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  ```
- define simple cases:
  ```
  init (x:xs) | null xs = []
  ```
- define other cases:
  ```
              | otherwise = x : init xs
  ```
- generalize and simplify:
  ```
  init :: [a] -> [a]
  init [_]    = []
  init (x:xs) = x : init xs
  ```

Outline

Printing a Calendar

- given a month and a year, print the corresponding calendar
- separate construction phase (computation of days, leap year, ... in file `Calendar.hs`) from printing
- we concentrate on printing, assuming machinery for construction

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Example (October 2022)

```
Su Mo Tu We Th Fr Sa
                   1
 2  3  4  5  6  7  8
 9 10 11 12 13 14 15
16 17 18 19 20 21 22
23 24 25 26 27 28 29
30 31
```

Helper Functions: Combining two Lists via a Function

- `myZipWith :: (a -> b -> c) -> [a] -> [b] -> [c]`
- `myZipWith f` $[x_1,\ldots,x_m]$ $[y_1,\ldots,y_n] = [x_1 \grave{} f \grave{} y_1,\ldots,x_{\min\{m,n\}} \grave{} f \grave{} y_{\min\{m,n\}}]$
- specialization `myZip :: [a] -> [b] -> [(a, b)]`

  `myZip = myZipWith (,)`

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Example

- `myZip [1,2,3] ['a','b'] = [(1,'a'),(2,'b')]`
- `myZipWith (*) [1,2] [3,4,5] = [1*3,2*4] = [3,8]`
- `myZipWith drop [1,0] ["a","b"] = [drop 1 "a",drop 0 "b"] = ["","b"]`

Helper Functions: Combining two Lists via a Function

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Helper Functions (cont'd): Right Folding without a base

- `myFoldr1` – special version of `myFoldr`, without base value (does not work on empty list)

```
myFoldr1 :: (a -> a -> a) -> [a] -> a
myFoldr1 f l =
    case l of
        [x]  -> x
        x:xs -> f x (myFoldr1 f xs)
```

Helper Functions (cont'd): Right Justifying a String & Grouping a List

- `myRjustify` – right-justify given text inside box of given width

```
myRjustify :: Int -> String -> String
myRjustify n l =
    if length l < n then replicate (n-(length l)) ' ' ++ l
    else error ("list of length " ++ show (length l) ++ "does not fit in a box of width " ++ show n)
```

Helper Functions (cont'd): Right Justifying a String & Grouping a List

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```

- `myGroupsOfSize` – split list into sublists of given length

```
myGroupsOfSize :: Int -> [a] -> [[a]]
myGroupsOfSize n l =
    let (xs, ys) = mySplitAt n l
    in if null xs then [] else xs : myGroupsOfSize n ys
```

Helper Functions (cont'd): Right Justifying a String & Grouping a List

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myGroupsOfSize n l =
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    in if null xs then [] else xs : myGroupsOfSize n ys
```

- `mySplitAt :: Int -> [a] -> ([a], [a])` – split list at given position

The Picture Analogon

pictures:

- atomic part: pixel
- height and width
- white pixel

strings:

- atomic part: character
- number of rows and columns
- blank character

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Auxiliary Types

```
type Height  = Int
type Width   = Int
type Picture = (Height, Width, [[Char]])
```

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type Width   = Int
type Picture = (Height, Width, [[Char]])
```

- consider `(h, w, rs)`

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- consider `(h, w, rs)`
- `rs :: [[Char]]` – “list of rows”

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type Picture = (Height, Width, [[Char]])
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- consider (h, w, rs)
- rs :: [[Char]] – “list of rows”
- invariant 1: length of rs is height h

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Auxiliary Types

```
type Height  = Int
type Width   = Int
type Picture = (Height, Width, [[Char]])
```

- consider (h, w, rs)
- rs :: [[Char]] – “list of rows”
- invariant 1: length of rs is height h
- invariant 2: all rows (that is, lists in rs) have length w

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pictures:

- atomic part: pixel
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Auxiliary Types

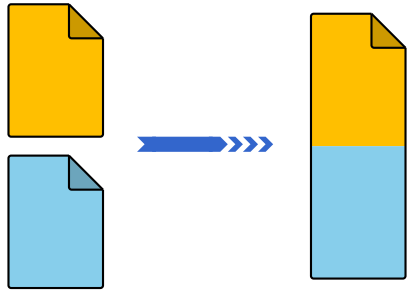
```
type Height  = Int
type Width   = Int
type Picture = (Height, Width, [[Char]])
```

- consider `(h, w, rs)`
- `rs :: [[Char]]` – "list of rows"
- invariant 1: length of `rs` is height `h`
- invariant 2: all rows (that is, lists in `rs`) have length `w`

Showing Pictures

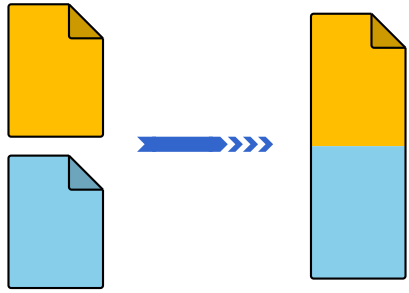
transform a `Picture` into a `String` and print it out on the standard IO considering escape characters

```
printPic (h, w, css) = putStr (unlines css)
```



above

```
above :: Picture -> Picture -> Picture
above p q =
    case p of
        (h1,w1,l1) -> case q of
                        (h2,w2,l2) -> if w1 == w2 then (h1+h2, w1, l1 ++ l2) else error "different widths"
```



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above p q =
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```

Stacking Several Pictures Above Each Other

```
stack :: [Picture] -> Picture
stack = myFoldr1 above
```

beside

```
beside :: Picture -> Picture -> Picture
beside p q =
    case p of
        (h1,w1,l1) -> case q of
                        (h2,w2,l2) -> if h1 == h2 then (h1, w1+w2, myZipWith (++) l1 l2)
                                      else error "different heights"
```

beside

```
beside :: Picture -> Picture -> Picture
beside p q =
    case p of
        (h1,w1,l1) -> case q of
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                                        else error "different heights"
```

Spreading Several Pictures Beside Each Other

```
spread :: [Picture] -> Picture
spread = myFoldr1 beside
```

beside

```
beside :: Picture -> Picture -> Picture
beside p q =
    case p of
        (h1,w1,l1) -> case q of
                        (h2,w2,l2) -> if h1 == h2 then (h1, w1+w2, myZipWith (++) l1 l2)
                                      else error "different heights"
```

Spreading Several Pictures Beside Each Other

```
spread :: [Picture] -> Picture
spread = myFoldr1 beside
```

Tiling Several Pictures

```
tile :: [[Picture]] -> Picture
tile l = stack (myMap spread l)
```

Creating Pictures

- single‘pixels’

```
pixel :: Char -> Picture
pixel c = (1, 1, [[c]])
```

Creating Pictures

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```
pixel :: Char -> Picture
pixel c = (1, 1, [[c]])
```

- rows

```
row :: String -> Picture
row r = (1, length r, [r])
```

Creating Pictures

- single‘pixels’

```
pixel :: Char -> Picture
pixel c = (1, 1, [[c]])
```

- rows

```
row :: String -> Picture
row r = (1, length r, [r])
```

- blank

```
blank :: Int -> Int -> Picture
blank h w = (h, w, replicate h (replicate w ' '))
```

Creating Pictures

- single‘pixels’

```
pixel :: Char -> Picture
pixel c = (1, 1, [[c]])
```

- rows

```
row :: String -> Picture
row r = (1, length r, [r])
```

- blank

```
blank :: Int -> Int -> Picture
blank h w = (h, w, replicate h (replicate w ' '))
```

Remark

`replicate :: Int -> a -> [a]` – replicates single element given number of times

Constructing a Month

- assume function `monthInfo :: Int -> Int -> (Int, Int)`, returning the first weekday of the month together with the number of days for the month
- where days are 0 (Sunday), 1 (Monday), . . .
- e.g., `monthInfo 10 2022` = `(6, 31)`, meaning that the first weekday of October 2022 is a Saturday and the month has 31 days

```
daysOfMonth :: Month -> Year -> [Picture]
daysOfMonth m y =
    let (d, t) = monthInfo m y
        pic n  = if 1 <= n && n <= t then show n else ""
    in myMap row (myMap (myRjustify 3) (myMap pic [1-d..42-d]))

month :: Month -> Year -> Picture
month m y = tile (myGroupsOfSize 7 (daysOfMonth m y))
```

Printing a Month

- print result of `month m y`

```
printMonth :: Month -> Year -> IO()
printMonth m y =
    let weekdays = row " Su Mo Tu We Th Fr Sa"
    in printPic (above weekdays (month m y))
```

Thanks! & Questions?