CS301 Assignment-5

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Problem - NP

1. So, original version of the problem is finding the maximum profit that can be obtained from selected projects where total effort is less than the given limit (c). Now, we are asked to write decision version of the problem. Then we need to convert the problem into version whose answer (or output) is "Yes" or "No" and add new variable 'u' as lower boundary. New question with same inputs is does there exist a subset of projects with total effort ≤ c and have at least u profit.

Input: Set T of projects, positive integer e_t (effort of it), positive integer p_t (profit of it) for each project t in T, positive integer c maximum effort can be spent and non-negative integer u (u is lower bound of profit gained).

$$Output: \begin{cases} Yes, & \text{if there is a subset S} \subseteq \text{T such that } \Sigma_{k\in S} e_k \leq c, \Sigma_{k\in S} p_k \geq u \\ No, & \text{Otherwise (such subset does not exist)} \end{cases}$$

2. To prove that the decision version is in NP (membership) we can have a look at solvability and verifiability. For solvability as brute force we can generate all project subsets of T and one by one check if it satisfies the conditions which takes exponential time because of generating every subset.

So for verifiability, lets say we have certificate solution S' = (t1, t2, t3 .... tk) of size k. Then to verify if it is a solution, all operation we need to is traverse the projects in the S' and while traversing we add effort spent for that project to total effort and add profit to the total profit. After that, if total effort is less than or equal to c (maximum effort), and if total profit is greater or equal to u (lower bound) will be checked.

Hence, all these verifying process can be conducted in polynomial time since the most expensive task here is traversing the certificate and doing constant operation for each iteration. Then problem's solvability is exponential, whereas verifying process is polynomial that shows that problem is IN NP.