Student Information

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Answer 1

a) The condition of independence is f(x,y)=f(x)f(y) where f(x,y) is a joint density function and f(x) f(y) are marginal pdfs for X and Y.

From Part b we know the marginal pdfs of X and Y : $\frac{1}{\pi} \neq 4\frac{\sqrt{(1-y^2)(1-x^2)}}{\pi^2}$ Hence they are dependent.

b) f(x) = $\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{\pi} dy = \frac{2\sqrt{1-x^2}}{\pi}$

f(y) = $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi} dx = \frac{2\sqrt{1-y^2}}{\pi}$

c) E[x]= $\int_{-1}^{1} xf(x)dx = \int_{-1}^{1} x\frac{2\sqrt{1-x^2}}{\pi} = 0$ E[x] = 0

d) Var[x] = E(x-μ)^2 = 0.25

Answer 2

a) Since t_A and t_A are independent and uniformly distributed on [0, 100] seconds :

f(t_A)= $\begin{cases} \frac{1}{100}, & 0 \leq t_A \leq 100 \\ 0, & otherwise \end{cases}$

f(t_B)= $\begin{cases} \frac{1}{100}, & 0 \leq t_B \leq 100 \\ 0, & otherwise \end{cases}$

Joint density function:

$f_{t_A t_B}(t_A,t_B)= \begin{cases} \frac{1}{10000}, & 0 \leq t_B \leq 100, 0 \leq t_A \leq 100 \\ 0, & otherwise \end{cases}$

Joint cdf function :

$F_{t_A t_B}(t_A,t_B)=\int_0^{t_B} \int_0^{t_A} \frac{1}{10000}\, dt_A\, dt_B$

b) P(t_B-$t_A \leq 20$) $\int_{90}^{100} \int_0^{10} \frac{1}{10000}\, dt_A\, dt_B = \frac{1}{100}$=0.01

c) Until t_B equals 80,t_a need to be [0,t_B+20] interval.Then t_A can also take every value. P(t_B-t_A $\leq$ 20) + P($t_B \geq$ 80) $=\int_0^{80}\int_0^{t_B+20}\frac{1}{10000}\,dt_A\,dt_B+\int_0^{80}\int_0^{100}\frac{1}{10000}\,dt_A\,dt_B=\frac{68}{100}$=0.68

d) We determine t_A 's value according to t_B 's value. When t_B = [0,30] , t_A = [0,t_B+30]; $When$ t_B = [30,70] , t_A = [t_B-30,t_B+30] ; When t_B = [70,100] , t_A = [t_B-30,100]
$\int_0^{30}\int_0^{t_B+30}\frac{1}{10000}\,dt_A\,dt_B+\int_{30}^{70}\int_{t_B-30}^{t_B+30}\frac{1}{10000}\,dt_A\,dt_B+\int_{70}^{100}\int_{t_B-30}^{100}\frac{1}{10000}\,dt_A\,dt_B=0.51$

Answer 3

a) f_{xi}= λ_i e $^{-\lambda ix}$
$F_x i$(x)=P($X_i \leq$ x) = $\int_0^x \lambda_i e^{-\lambda ix}\,dx$=1-e$^{-\lambda ix}$ for x $\geq$ 0
T=F_{min}(x)=1-(1-$F_{\lambda 1}$(x))-(1-$F_{\lambda 2}$(x))...-(1-$F_{\lambda n}$(x))=1-(1-$(1-e^{-\lambda 1x})-(1-$(1-e$^{-\lambda 2x}$)...-(1-$(1-e^{-\lambda nx}$)
T=1-$e^{-(\lambda_1+\lambda_2+..+\lambda_n)x}$

b) λ_n=n/10;
T=1-$e^{-(\lambda_1+\lambda_2+..+\lambda_n)x}$ from part A. When we substiute λs with their values we get $1-e^{-5.5x}$ as a cdf.To get pdf we need to take the derivative of cdf: =5.5$e^{-5.5x}$
E[x]=$\int_0^\infty xf(x)\,dx=\int_0^\infty 5.5xe^{-5.5x}\,dx$=0.18
Expected time before one of the computers fails is 0.18 years.

Answer 4

a)

Let X_i, i=1,2,3,..,100 represent the outcome of the vote of the i-th person polled, where X_i=1 if they enrolled in an undergraduate program and X_i=0 otherwise. Let S_n be the sum of all X_i. Then the probability that at least 70 of participants are undergraduate students is:
$P(S_n \geq 70) = 1 - P(S_n \leq 70)$
we can approximate Z=$\frac{S_n-n\mu}{\sqrt{\sigma}}$ as the Standard Normal,
P($S_n \leq$ 70) =P(Z $\leq \frac{70-n\mu}{\sqrt{\sigma}}$) = P(Z $\leq \frac{70-74}{4.386}$)
P($S_n \geq$ 70)= 1$-\Phi$(-1.026)=1-0.1515=0.8485

b)

With the same logic from Part A ,this time S_n represents doctoral degree students,
P($S_n \leq$ 5)=P(Z $\leq \frac{5.5-10}{3}$)= Φ(-1.5)=0.0668