

Student Information

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Answer 1

Confidence Interval for population is:

CI= $\bar{x} \pm Z_{\alpha/2} * \frac{\sigma}{\sqrt{n}}$ when the population standard deviation is known.

$$\bar{x} = \frac{20.1+12.8+18.9+16.4+20.3+10.1+15.4+12.4+24.7+18.5}{10} = 16.96$$

a) The critical value Z for % 90 CI from z table=1.6449

$$\text{Hence: CI} = 16.96 \pm 1.6449 * \frac{3}{\sqrt{10}} = 16.96 \pm 1.5605$$

$$\text{CI} = (15.40, 18.52)$$

The critical value Z for % 99 CI from z table=2.5758

$$\text{Hence: CI} = 16.96 \pm 2.5758 * \frac{3}{\sqrt{10}} = 16.96 \pm 2.4436$$

$$\text{CI} = (14.52, 19.40)$$

b) $n = \left(\frac{Z * \sigma}{\Delta}\right)^2$ is the formula to calculate required sample size to attain a specific margin.

The critical value Z for % 98 CI from z table=2.3263

$$\text{When we substitute the values on the formula: } n = \left(\frac{2.3263 * 3}{1.55}\right)^2 = 20.27 \approx 21$$

Answer 2

a) If standard deviation is also known, mean rating and the sample size would be enough as statistics because standard deviation provides us information about the spread of the data. However, mean rating and sample size are not enough in this situation.

b) We can use hypothesis testing.

$$H_0: \mu = 7.5$$

$$H_a: \mu < 7.5$$

$\alpha = 0.05$ (level of significance)

Since population standard deviation is not known and $n \geq 30$; we need to use z-test.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{7.4 - 7.5}{\frac{0.8}{\sqrt{256}}} = -2 \quad \text{The critical value Z for } \alpha = 0.05 \text{ from z table} = -1.645 \text{ since it is one-tailed (left)}$$

region. We will reject H_0 if $Z < -1.645$, and not reject otherwise. When we compare these two z values, the computed one is less than the critical value, it means we need to reject H_0 . With 95% confidence $(1 - \alpha)$, Restaurant A will not be in list of candidate restaurants.

c) When the standard deviation equals to 1, the computed z value is equal to -1.6. Unlike part b, it is greater than the critical value so with 95 % confidence Restaurant A will be in list of candidate

restaurants.

$$z = \frac{7.4 - 7.5}{\frac{1}{\sqrt{256}}} = -1.6, -1.6 > -1.645$$

d) Since $7.6 > 7.5$ we do not need to do any other statistical test, it will be in our list definitely.

Answer 3

Computer A: $\bar{x}_1 = 211$ $s_1 = 5.2$ $n_1 = 20$

Computer B: $\bar{x}_1 = 133$ $s_1 = 22.8$ $n_1 = 32$

$H_0 : \mu_1 - \mu_2 = 90$

$H_1 : \mu_1 - \mu_2 < 90$

a) Pooled variance: $s_p^2 = \frac{(n_1 * s_1^2 + n_2 * s_2^2)}{n_1 + n_2 - 2} = \frac{(20-1)*5.2^2 + ((32-1)*22.8)^2}{20+32-2} = 332.576$

Test Statistic: $T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_p^2(1/n_1 + 1/n_2)}} = (211 - 133 - 90) / (\sqrt{332.576 * (1/20 + 1/32)}) = -2.3085$

Degree of freedom = $n_1 + n_2 - 2 = 50$

P-value = $T.DIST(-2.3085, 50, 1) = 0.0126$

Since P-value $> \alpha$, H_0 can not be rejected. So it can not be inferred that Computer B provides a 90-minute or better improvement at 1% level of significance.

b) Test Statistic: $T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{(s_1^2/n_1 + s_2^2/n_2)}} = (211 - 133 - 90) / (\sqrt{5.2^2/20 + 22.8^2/32}) = -2.8606$

Degree of freedom = $\frac{(s_1^2/n_1 + s_2^2/n_2)^2}{((s_1^2/n_1)/(n_1-1)) + ((s_2^2/n_2)/(n_2-1))} = 35.97 \approx 35$

P-value = $T.DIST(-2.8606, 35, 1) = 0.0035$

Since P-Value $< \alpha$, We can reject the H_0 so it can be inferred that Computer B provides a 90-minute or better improvement at 1% level of significance.