

1 Student Information

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Answer 1

a)

Getting at least 1 white ball probability equals to subtracting getting 3 black balls from 1 so: $1 - \frac{8}{10} \cdot \frac{11}{15} \cdot \frac{9}{12} = \frac{42}{75} = 0.56$

b)

Getting white ball from each boxes: $WWW = \frac{2}{10} \cdot \frac{4}{15} \cdot \frac{3}{12} = \frac{1}{75} = 0.013$

c)

Box 1: $\frac{2}{10} \cdot \frac{1}{9} = \frac{1}{45}$

Box 2: $\frac{4}{15} \cdot \frac{3}{14} = \frac{2}{35}$

Box 3: $\frac{3}{12} \cdot \frac{2}{11} = \frac{1}{22}$

I would choose box 2 since it has greater probability to get 2 white balls.

d)

Since we need to get 2 white balls, we just focus on drawing white balls. Firstly, box 2 has greatest probability with $\frac{4}{15}$, after drawing there remains $\frac{3}{14}$ for box 2, however; box 3 has $\frac{3}{12}$ to get white ball. So the order is box 2 - box 3

e)

	0	1	2	3
P(X=x)	0.44	0.4167	0.13	0.013
	WWW	WBB,BWB,BBW	WBB,WBW,BWW	WWW

$$\text{Expected Value} = 0 * 0.44 + 1 * 0.4167 + 2 * 0.13 + 3 * 0.013 = 0.7167$$

f)

$$\text{Box 1: } \frac{1}{3} \cdot \frac{2}{10} = \frac{1}{15}$$

$$\text{Box 2: } \frac{1}{3} \cdot \frac{4}{15} = \frac{4}{45}$$

$$\text{Box 3: } \frac{1}{3} \cdot \frac{3}{12} = \frac{1}{12}$$

$$\frac{BOX1}{BOX1+BOX2+BOX3} = 0.279$$

Answer 2

F=Frodo is corrupted S=Sam is corrupted R=Ring is destroyed

a)

$$P(R | \bar{S}) = 0.9$$

$$P(R | S) = 0.5$$

$$P(S) = 0.1$$

If we add the probabilities of ring being destroyed where Sam is corrupted and not corrupted, we can find the total probability of ring being destroyed:

$$P(R) = P(\bar{S}) * P(R | \bar{S}) + P(S) * P(R | S) = 0.86$$

From Bayes Theorem;

$$P(S | R) = \frac{P(R|S)*P(S)}{P(R)} = \frac{0.5*0.1}{0.86} = 0.058$$

b)

$$P(F) = 0.25$$

$$P(D | F) = 0.2$$

$$P(D | \bar{F} \cap \bar{S}) = 0.9$$

$$P(D | F \cap S) = 0.05$$

We need to find the probability that the ring destroyed both Sam and Frodo are corrupted :

$$P(F \cap S | D)$$

From Bayes Theorem:

$$P(F \cap S | D) = \frac{P(D|F \cap S) * P(F \cap S)}{P(D)}$$

$$P(F \cap S) = P(F) * P(S) \text{ since these are independent events. } P(F \cap S) = 0.25 * 0.1 = 0.025$$

$$P(D) = P(D | F) * P(F) + P(D | S) * P(S) - P(D | F \cap S) * P(F \cap S) + P(D | \bar{F} \cap \bar{S}) * P(\bar{F} \cap \bar{S})$$

$$P(D) = 0.2 * 0.25 + 0.5 * 0.1 - 0.05 * 0.025 + 0.9 * 0.75 * 0.9 = 0.70625$$

By using Inclusion-Exclusion Principle $P(D | F \cap S) * P(F \cap S)$ part is subtracted to avoid calculating intersection part twice.

When we use these values in Bayes Theorem we can find $P(F \cap S | D)$.

$$P(F \cap S | D) = \frac{0.05 * 0.025}{0.70625} = 0.018$$

Answer 3

a)

We can see that there are only 2 cases that conform 4 snowy days :
 $P(A=2, I=2)=0.2$ or $P(A=3, I=1)=0.12$ from the law of probability;
 $P(A=2, I=2) + P(A=3, I=1) = 0.32$

b)

	$P(A=1)$	$P(A=2)$	$P(A=3)$	$P(A=a)$
$P(I=1)$	0.18	0.3	0.12	0.6
$P(I=2)$	0.12	0.2	0.08	4
$P(I=i)$	0.3	0.5	0.2	1

Events A and B are independent if occurrence of B does not affect the probability of A. From the table we can infer that $P(A|B)=P(A)$ and $P(A \cap B)=P(A)P(B)$ for all values of A and B. Hence we can say that snowy days in Ankara and Istanbul are independent.