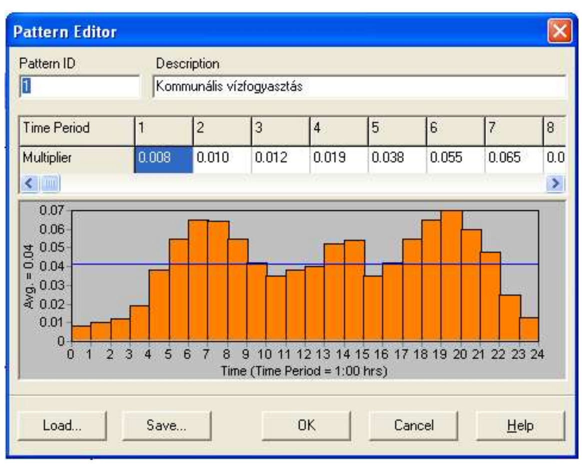
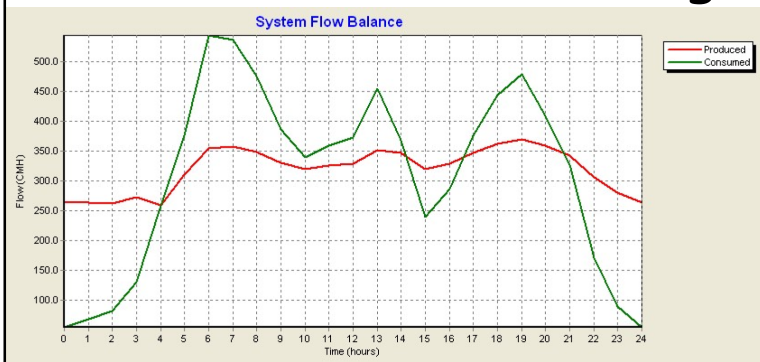
Water demands

- specific water demand: q [l/cap/d]
- 1970's USA 250 lt/cap/day now 150lt/cap/day
- average daily demand:
  $Q_d = \Sigma n_i q_i + Q_{industry} + Q_{agriculture} + Q_{loss}$
  [m^3/d]
- maximum daily demand:
  $Q_{dmax} = \beta \cdot Q_d$ [m^3/d]
- hourly demand:
  Q_h [m^3/h]

Daily consumption variation

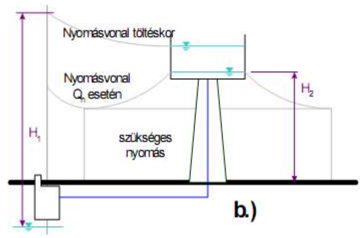


The role of storage



Continuity equation:

$Q_c + Q_p + Q_s = 0$



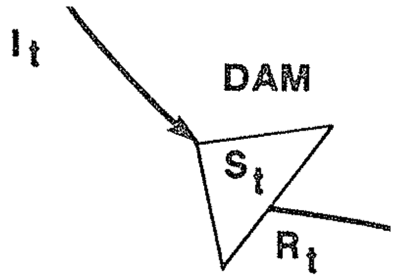
RESERVOIR PROBLEMS

Assume a river with known Inflow pattern.

If the inflow can not satisfy the demand at all times a dam can be built to control the releases.

Excess water is stored when **inflow > demand**

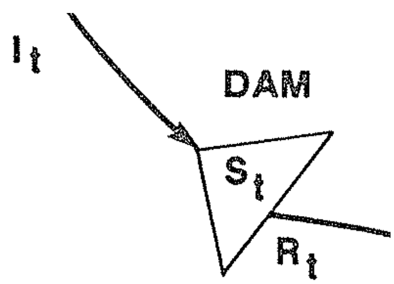
and released when **demand > inflow**



S_t Reservoir volume at the beginning of time period t
CAP Capacity of the reservoir
I_t Inflow to reservoir during time period t (deterministic)
R_t Release from reservoir during time period t

Minimize Z =

Subject to



Reservoir Continuity

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,...,T$$

Storage limitations

$$SMIN \le S_{t+1} \le CAP \quad t=1,2,...,T$$

Release requirements

$$RMIN \le R_t \le RMAX \quad t=1,2,...,T$$

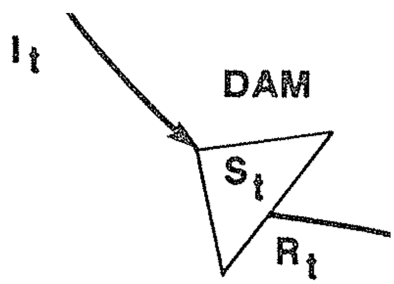
All variables non negative

Subject to

$S_2 - S_1 + R_1 = 8$
$S_3 - S_2 + R_2 = 20$
$S_4 - S_3 + R_3 = 10$
$S_5 - S_4 + R_4 = 21$
……
$S_{18} - S_{17} + R_{17} = 6$
$S_{19} - S_{18} + R_{17} = 22$
$S_{20} - S_{19} + R_{19} = 6$
$S_{21} - S_{20} + R_{20} = 15$

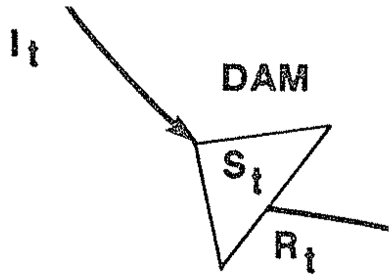
$S_{t+1} \leq CAP$
$S_{t+1} \geq SMIN$
$R_t \leq RMAX$
$R_t > RMIN$

	Flow in Season	
Year	**1**	**2**
1	8	20
2	10	21
3	9	20
4	2	10
5	6	14
6	8	17
7	12	17
8	9	15
9	6	22
10	6	15



What will be the minimum capacity of the reservoir to satisfy the demand (RMIN) at all times?

CAPACITY DETERMINATION !!!!



Minimize Z = CAP

Subject to

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,...,T$$

$$SMIN \leq S_{t+1} \leq CAP \quad t=1,2,...,T$$

$$RMIN \leq R_t \leq RMAX \quad t=1,2,...,T$$

Subject to

$S_2 - S_1 + R_1 = 8$
$S_3 - S_2 + R_2 = 20$
$S_4 - S_3 + R_3 = 10$
$S_5 - S_4 + R_4 = 21$
......
$S_{18} - S_{17} + R_{17} = 6$
$S_{19} - S_{18} + R_{17} = 22$
$S_{20} - S_{19} + R_{19} = 6$
$S_{21} - S_{20} + R_{20} = 15$

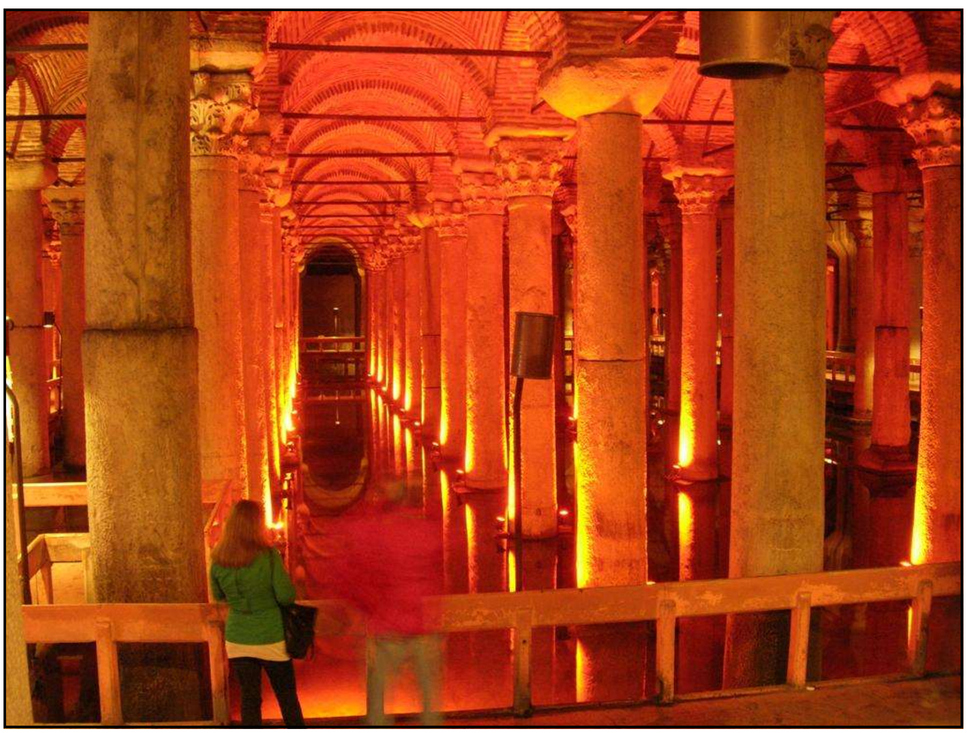
$S_{t+1} \leq CAP$
$S_{t+1} \geq SMIN$
$R_t \leq RMAX$
$R_t > RMIN$

	Flow in Season	
Year	**1**	**2**
1	8	20
2	10	21
3	9	20
4	2	10
5	6	14
6	8	17
7	12	17
8	9	15
9	6	22
10	6	15

Objective function

Minimize CAP


Buda Gellert dağında 80,000 m3 (40 K'lık 2 oda) 70'lerde yapılmış.

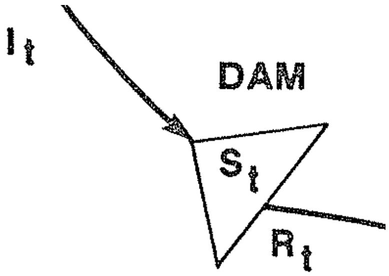


3,000 m^3 Tuna üzerinde adada 80'lerde

200 m^3 Buda Dağında 1920 de inşa (RC)

Szeged 1904'de ilk RC yapı 500-800 m^3
2005'de restorasyon

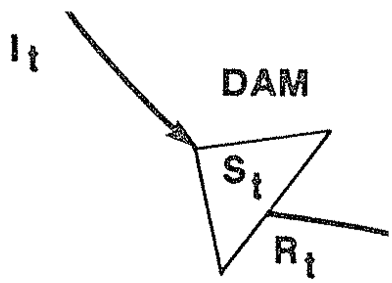
Ufak bir tarım kasabasıHodmezövasarhely
800 m^3 RC 1970 ' sonlarında



Assume CAP is known

What will be the maximum quantity of constant water which can be supplied from the reservoir at all times?

YIELD Determination !!!



Maximize Z = RMIN

Subject to

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,\ldots,T$$

$$SMIN \le S_{t+1} \le CAP \quad t=1,2,\ldots,T$$

$$RMIN \le R_t \le RMAX \quad t=1,2,\ldots,T$$

Subject to

$S_2 - S_1 + R_1 = 8$
$S_3 - S_2 + R_2 = 20$
$S_4 - S_3 + R_3 = 10$
$S_5 - S_4 + R_4 = 21$
......
$S_{18} - S_{17} + R_{17} = 6$
$S_{19} - S_{18} + R_{17} = 22$
$S_{20} - S_{19} + R_{19} = 6$
$S_{21} - S_{20} + R_{20} = 15$

$S_{t+1} \leq CAP$
$S_{t+1} \geq SMIN$
$R_t \leq RMAX$
$R_t > RMIN$

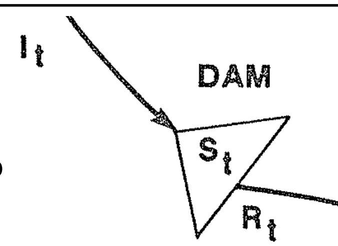
	Flow in Season	
Year	1	2
1	8	20
2	10	21
3	9	20
4	2	10
5	6	14
6	8	17
7	12	17
8	9	15
9	6	22
10	6	15

Objective function

Maximize RMIN

SAMPLE PROBLEM I

The table below shows the forecasted (estimated) monthly flows in ($10^6 m^3$) to a systems shown on the side.



Months (t)	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Now	Dec
Discharge	12	14	11	10	8	7	7	5	10	12	14	15

Write mathematical models that will answer the following questions and solve via LINDO.

a) What will be the capactiy of the reservoir if the minimum required release is $9*10^6$ m^3?

b) What is the maximum yield (RMIN) if the existing reservoir's capacity is $10*10^6$ m^3?

Minimize the maximum release deviations from a target release

Let RTARGET be the target release

Minimize Z = DUMMY

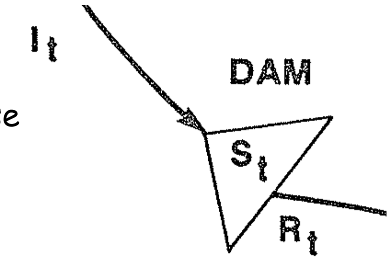
Subject to

$$RTARGET - R_t < Dummy$$

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,...,T$$

$$SMIN \leq S_{t+1} \leq CAP \quad t=1,2,...,T$$

$$RMIN \leq R_t \leq RMAX \quad t=1,2,...,T$$



Minimize the sum of release deviations from a target release

Let RTARGET be the target release

Minimize $Z = \Sigma\ (POS_t + NEG_t)$

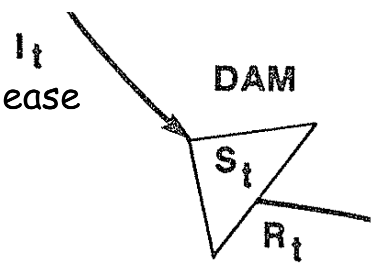
Subject to

$$R_t - RTARGET < POS_t - NEG_t$$

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,...,T$$

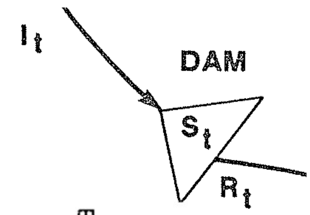
$$SMIN \leq S_{t+1} \leq CAP \quad t=1,2,...,T$$

$$RMIN \leq R_t \leq RMAX \quad t=1,2,...,T$$



Linear Decision Rule in Reservoir Design and Management

Minimize Z = CAP



Subject to

$R_t = S_t - b_i$

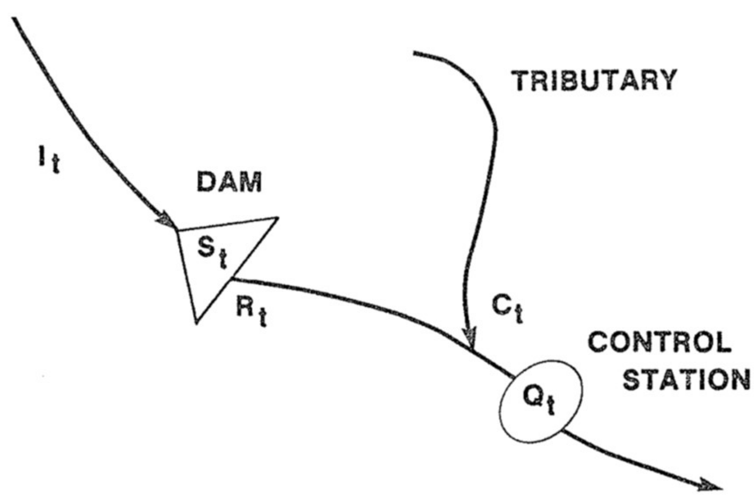
$S_{t+1} - S_t + R_t = I_t \quad t=1,2,...,T$

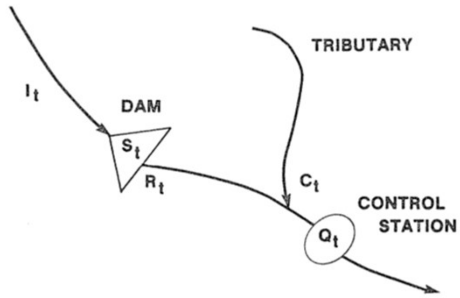
$SMIN \leq S_{t+1} \leq CAP \quad t=1,2,...,T$

$RMIN \leq R_t \leq RMAX \quad t=1,2,...,T$

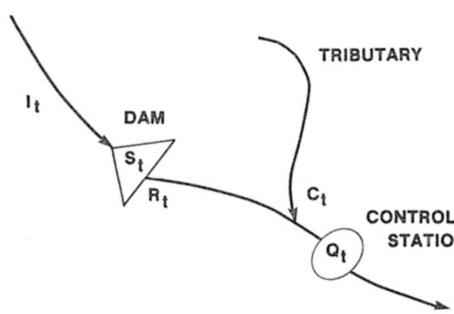
Problem minimizes the capacity as well as determines b.

The problem is finding smallest reservoir that will deliver Flows in the specified range over the entire record under the added constraint of LDR.





S_t	Reservoir volume at the beginning of time period t
CAP	Capacity of the reservoir
I_t	Inflow to reservoir during time period t (deterministic)
C_t	Tributary flow during time period t (known)
R_t	Release from reservoir during time period t
Q_t	Control station flow during time period t

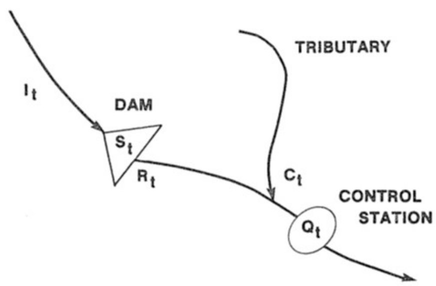


Reservoir Continuity

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,\ldots,T$$

Storage limitation

$$SMIN \le S_{t+1} \le CAP \quad t=1,2,\ldots,T$$

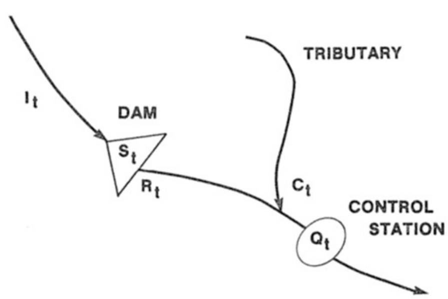


Release requirements

$$\text{RMIN} \le R_t \le \text{RMAX} \quad t=1,2,\ldots,T$$

Release difference requirement for scour, etc

$$|R_t - R_{t-1}| \le \text{RDIF} \quad t=1,2,\ldots,T$$

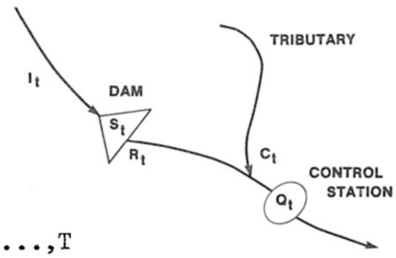


Control station flow equation

$$Q_t - aR_t - bR_{t-1} = C_t \quad t=1,2,\ldots,T$$

Downstream requirement

$$Q_t \ge \text{QMIN} \quad t=1,2,\ldots,T$$



$S_{t+1} - S_t + R_t = I_t \quad t=1,2,\ldots,T$

$Q_t - aR_t - bR_{t-1} = C_t \quad t=1,2,\ldots,T$

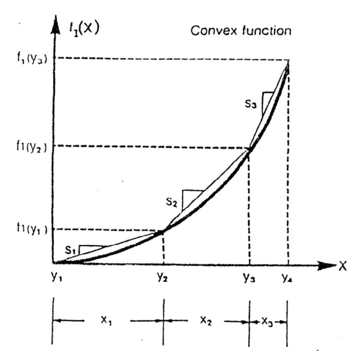
$|R_t - R_{t-1}| \leq RDIF \quad t=1,2,\ldots,T$

$SMIN \leq S_{t+1} \leq CAP \quad t=1,2,\ldots,T$

$RMIN \leq R_t \leq RMAX \quad t=1,2,\ldots,T$

$Q_t \geq QMIN \quad t=1,2,\ldots,T$

Piecewise Linearization of objective function



$X_j \leq Y_j+1 - Y_j$ for all j

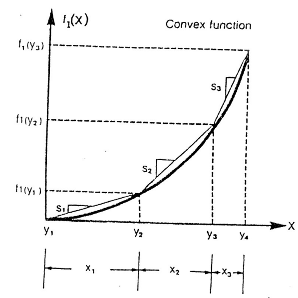
$X = X_1 + X_2 + \ldots + X_n + Y_1$

The objective now can be written as

Min $f(X) = (S_1 * X_1) + (S_2 * X_2) + \ldots + (S_n * X_n)$

- Note that since S_1 is the smallest coeficient in the objective function to be minimized, first X_1 will have a positive value.
- At the optimal solution, all other variables can NOT assume any value but zero unless X_1 is equal to its upper bound.
- Next X_2 will take positive value since it has the next lowest coefficient in the objective function.

Piecewise Linearization of objective function



Example : Consider the objective function

Minimize $F(X) = 3X^2 + 5X$

Subject to $0 \le X \le 8$

Let's asume equal spacing

Xi	F(Xi)	Si
0	0	
2	22	11
4	68	23
6	138	35
8	232	47

One can rewrite the objective function

Min $F(X) = 11\,X1 + 23\,X2 + 35\,X3 + 47\,X4$

Subject to $X = X1 + X2 + X3 + X4$

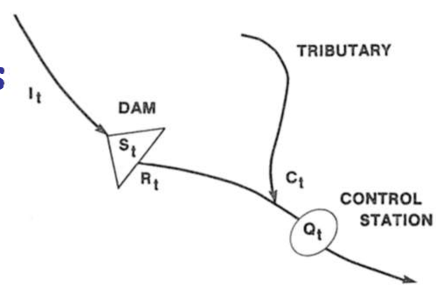
$0 \le X1 \le 2$

$0 \le X2 \le 2$

$0 \le X3 \le 2$

$0 \le X4 \le 2$

Real – time Operations



Small changes in the operation of an existing system may result significant increases in benefits and/or decreases in damages.

Thus, real - time operation of reservoir systems should be studied in detail.

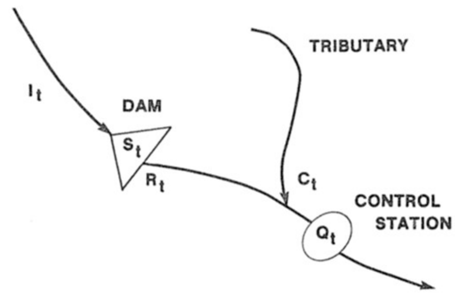
The systems is checked to see the current levels

Current reservoir levels, releases, etc

Forecasts are made for

inflows, tributary flows, preciptations, evaporations, seepage

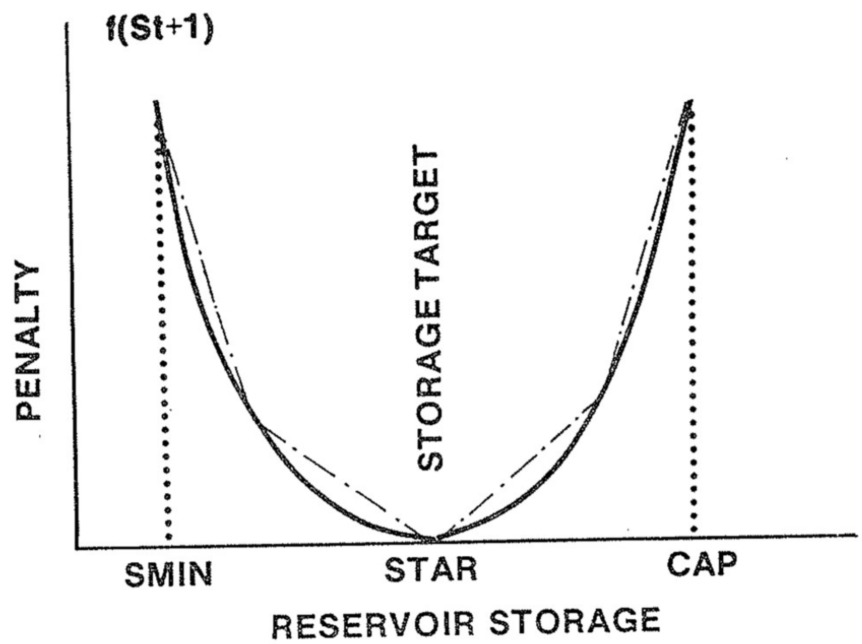
Decision is made in short time periods (daily even shorter)

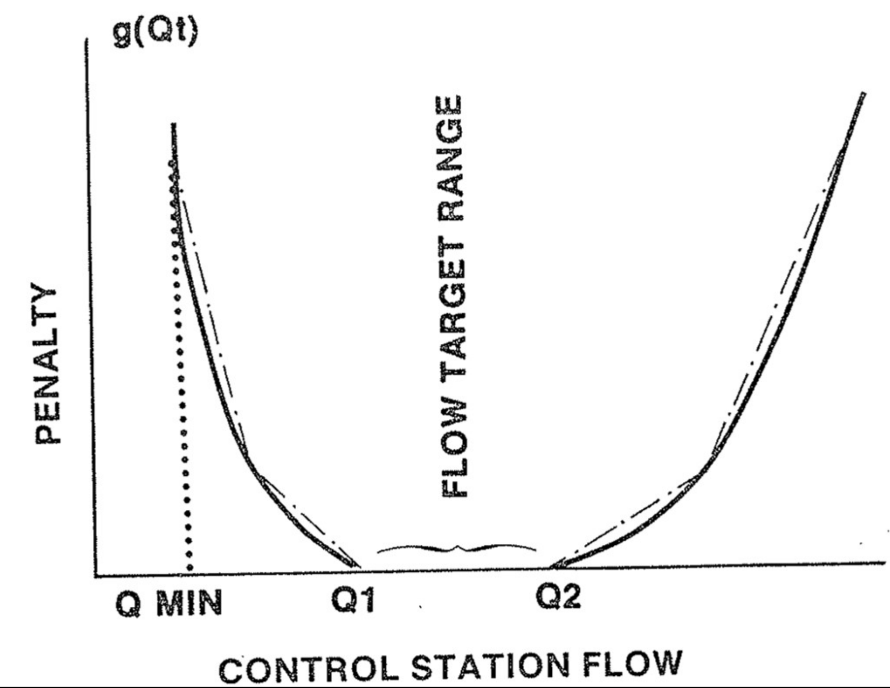


If target levels can be set for

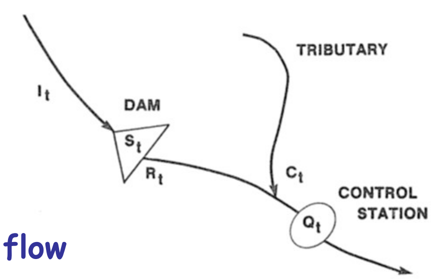
- **Storage levels**
- **Releases**
- **Downstream control station flow**

and deviations from these target levels are penalized





If target levels can be set for

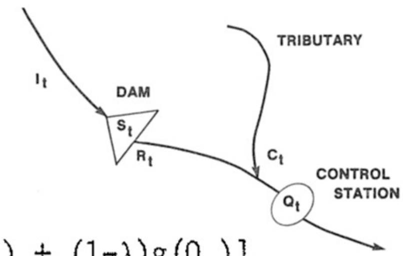


- **Storage levels**
- **Releases**
- **Downstream control station flow**

and deviations from these target levels are penalized

Then the best operation may be defined as that minimizes the sum of penalties incurred over the horizon.

$$\text{Minimize } Z = \sum_{t=1}^{T} [\lambda\ f(S_{t+1}) + (1-\lambda) g(Q_t)]$$



$$\text{Minimize } Z = \sum_{t=1}^{T} [\lambda f(S_{t+1}) + (1-\lambda) g(Q_t)]$$

$$S_{t+1} - S_t + R_t = I_t \quad t=1,2,\ldots,T$$

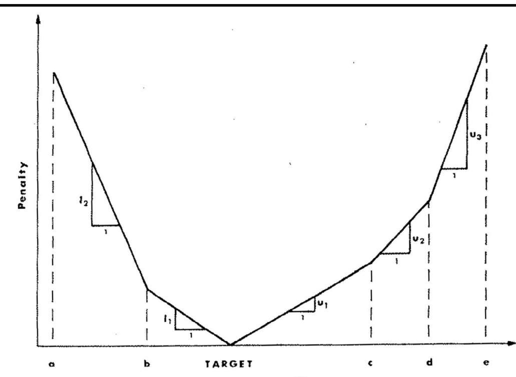
$$Q_t - aR_t - bR_{t-1} = C_t \quad t=1,2,\ldots,T$$

$$|R_t - R_{t-1}| \leq \text{RDIF} \quad t=1,2,\ldots,T$$

$$\text{SMIN} \leq S_{t+1} \leq \text{CAP} \quad t=1,2,\ldots,T$$

$$\text{RMIN} \leq R_t \leq \text{RMAX} \quad t=1,2,\ldots,T$$

$$Q_t \geq \text{QMIN} \quad t=1,2,\ldots,T$$



Point	Storage	Flow
a	beaches unusable	fish just surviving
b	boat ramps unusable	no rafting possible
TAR	storage target	flow target
c	picnic tables underwater	minor flooding
d	spillway elevation	major flooding
e	top of dam	disaster

Definition of objective function

$$S_t + B1_t + B2_t - A1_t - A2_t - A3_t = \text{TAR}$$

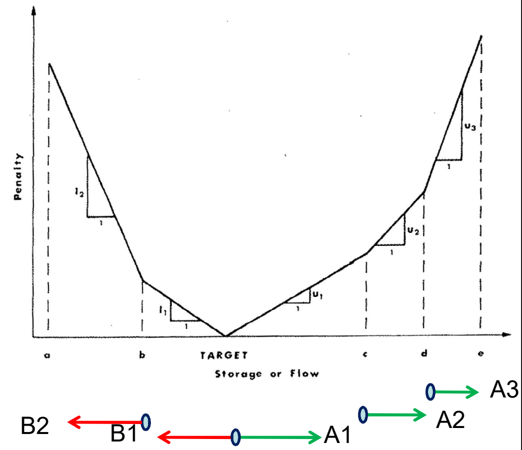
$$0 \le B1_t \le (\text{TAR} - b)$$

$$0 \le B2_t \le (b - a)$$

$$0 \le A1_t \le (c - \text{TAR})$$

$$0 \le A2_t \le (d - c)$$

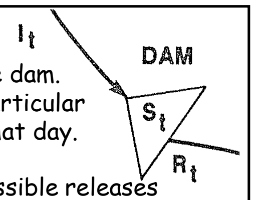
$$0 \le A3_t \le (e - d)$$



Minimize

$$Z1 = \sum_{t=2}^{T+1} (U_1A1_t + U_2A2_t + U_3A3_t + L_1B1_t + L_2B2_t)$$

HOMEWORK



Imagine a single reservoir with a flood zone directly below the dam. Assume that the decision about how much to release on any particular day is based only on the forecasted inflow to the reservoir that day.

There exists some guidelines on maximum and minimum permissible releases and storages. Any day's release can not exceed the previous day's release by more than 50% and can not be less than 62.5% of the previous release.

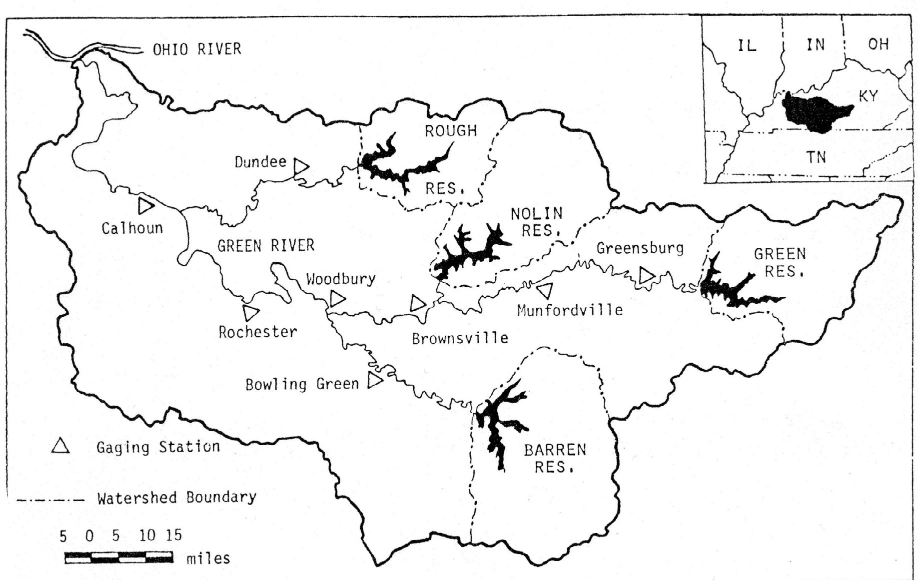
Also the rule curve elevation for the reservoir and the target for the flood zone are given. It has been estimated that each unit of deviation of actual storage from the rule curve storage volume causes a penalty of 20 units and each unit deviation of the release from the target flow causes a penalty of 30 units.

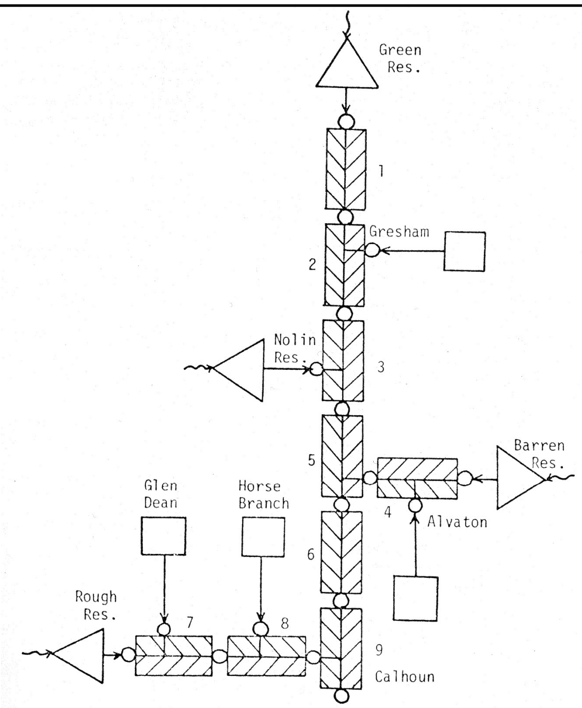
S1 : Storage at the beginning of today : 55,100,000 m^3
S2 : Anticipated storage volume at the beginning of tomorrow (m^3)
R0 : Yesterday's actual release = 850,000 m^3
R1 : Today's anticipated release (m^3)
I1 : Today's forecasted inflow volume = 800,000 m^3
RCS : Rule curve storage for today = 55,000,000 m^3
TFL : Target flow volume for today = 400,000 m^3
CAP : Reservoir storage volume capacity = 60,000,000 m^3
RMIN : Today's minimum allowable release = 200,000 m^3
RMAX : Today's maximum allowable release = 1,500,000 m^3

FORMULATE THE PROBLEM and SOLVE with the help of LINDO software.

Green River Basin
Optimization – Simulation Model

Yazıcıgil, H., G.H. Toebes and M.H. Houck, "Green River Basin Optimization – Simulation Model" Purdue University, Water Resources Research Center, West Lafayette, Indiana, November, 1980.





Hydroelectric Power Production

A cubic meter of water (weighing 10^3 kg), falling a distance of 1 m acquires 9.81 * 10^3 joules (newton-meters) of kinetic energy.

The energy generated in 1 second equals the **Watts** (joules per second) of power produced.

Hence, an average flow of q_t m^3/sec falling a height of H_t meters in period t yields **9.81 q_t H_t** kilowatts of power.

Multiplying by the number of hours in period t yields the kwh of energy produced from an average rate of q_t.

Hydroelectric Power Production

The total kwh of energy production of hydroelectrical energy during any period at any site

$$\mathbf{KWH_t = 2730 \quad q_t \; H_t} \; \varepsilon$$

- KWH_t : Energy produced in period t (kwh)
- q_t : the total flow through turbines during t (m^3/sec)
- H_t : the average productive storage head (m)
- ε : : plant efficiency
- Non linearity may be handled by

$$\mathbf{q_t \; H_t = (q_t^0 \; H_t) + (q_t \; H_t^0) - (q_t^0 \; H_t^0)}$$

q_t^0 : Average flow H_t^0: Average head

Hydroelectric Power Production

The amount of electrical energy produced is limited by the installed kw of plant capacity **P** a well as on the plant factor.

The plant factor is a measure of hydroelectric power plant use and is usually dictated by the characteristics of the power sytem supply and demand.

It is defined as the average load on the plant for the period divided by the installed plant capacity.

It accounts for the variability in the flow rate during each period t, and this variability mut be specified by those responsible for energy production and distribution.

HYDROELECTRIC POWER SYSTEMS PLANNING

An agency controls the operation of a system consisting of two water reservoirs with one hydroelectric power generation plant attached to each as shown in Fig. 2.1. The planning horizon for the system is broken into two periods. When the reservoir is at full capacity, additional inflowing water is spilled over a spillway. In addition, water can also be released through a spillway as desired for flood protection purposes. Spilled water does not produce any electricity.

Assume that on an average 1 kilo-acre-foot (KAF) of water is converted to 400 megawatt hours (MWh) of electricity by power plant A and 200 MWh by power plant B. The capacities of power plants A and B are 60,000 and 35,000 MWh per period. During each period, up to 50,000 MWh of electricity can be sold at \$20.00/MWh, and excess power above 50,000 MWh can only be sold for \$14.00/MWh. The following table gives additional data on the reservoir operation and inflow in kilo-acre-feet:

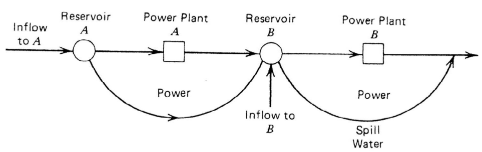


Figure 2.1

	Reservoir A	Reservoir B
Capacity	2000	1500
Predicted inflow		
Period 1	200	40
Period 2	130	15
Minimum allowable level	1200	800
Level at the beginning of period 1	1900	850

Develop a linear programming model for determining the optimal operating policy that will maximize the total revenue from electricity sales.