SOLUTION OF LINEAR SYSTEMS

Minimize F = DUMMY

Subject to

$$3X + 5Y + 4Z = 25$$
$$2X + 4Y + 6Z = 28$$
$$2X + 3Y + 3Z = 17$$

X , Y, Z non negative

In standart LP, all variables are non negative.

However, if X, Y, or Z can take a negative value

We have to replace these
with difference of two positive variables

Handling Unrestricted Variables

Assume X1 is unrestricted in the problem.

Let

$$X1 = X1P - X1N$$

where X1P and X1N are nonnegative.

The value of X1 is positive or negative depending on wheter

$$X1P > X1N \qquad \text{or} \qquad X1P < X1N$$

$$X = XP - XN$$
$$Y = YP - YN$$
$$Z = ZP - ZN$$

Minimize F = DUMMY

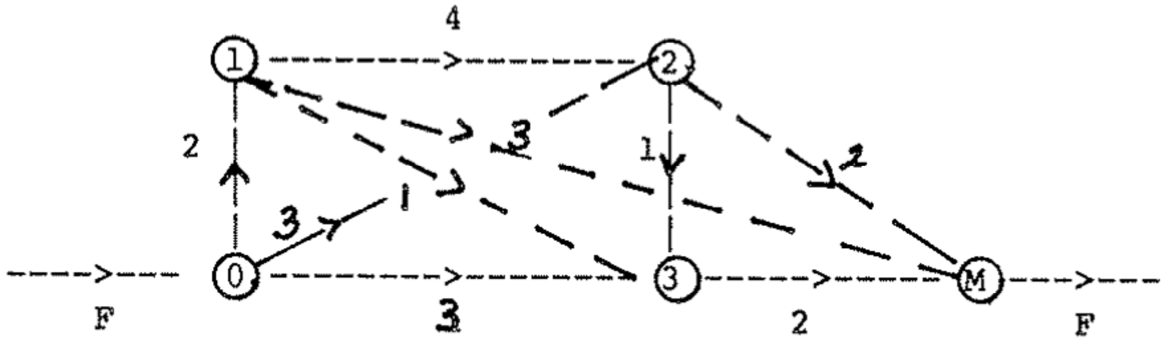
Subject to

$$3(XP - XN) + 5(YP - YN) + 4(ZP - ZN) = 25$$
$$2(XP - XN) + 4(YP - YN) + 6(ZP - ZN) = 28$$
$$2(XP - XN) + 3(YP - YN) + 3(ZP - ZN) = 17$$

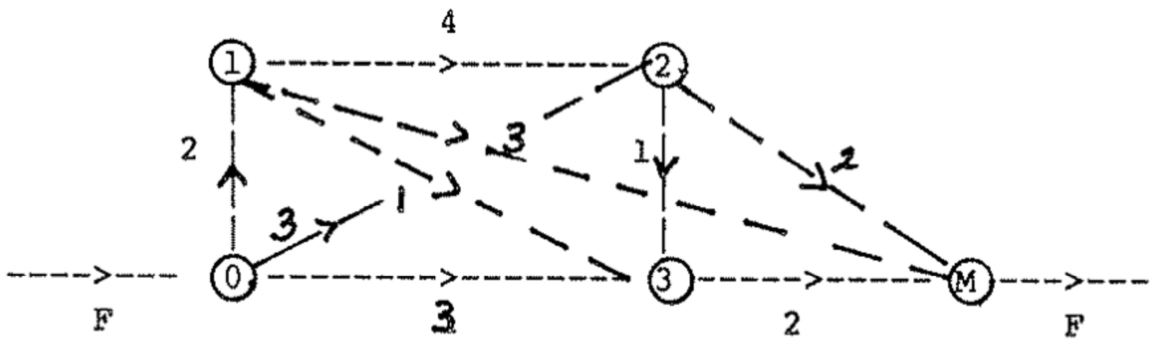
XP, XN YP, YN ZP, ZN non-negative

TRANSPORTATION NETWORKS

The goal is sending a homogeneous commodity from a particular point of the network to a designated destination.



The problem is to determine the max amount of flow F which can be transported from source to sink, if the flow from node I to J is limited to FIJ.



Let non-negative **XIJ** be the flow in arc IJ

Continuity of flow at source requires

$$\Sigma \ (X0J - XJ0) = F$$

Continuity of flow at sink

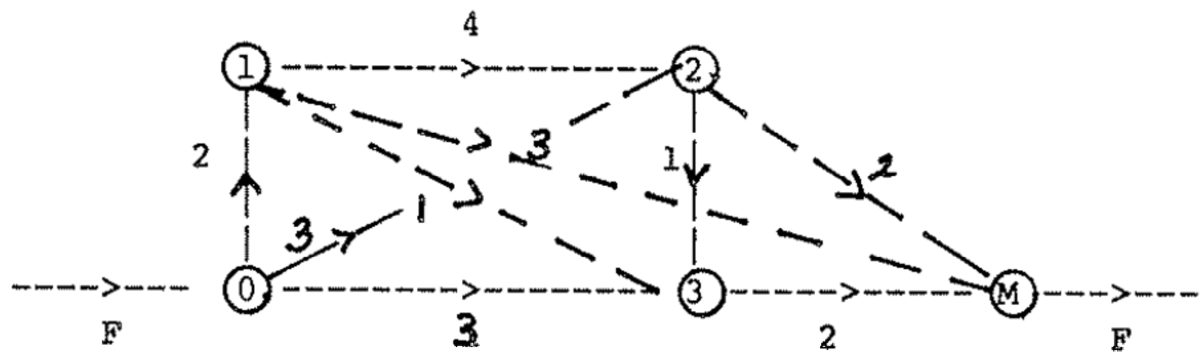
$$F = \Sigma \ (XJM - XMJ)$$

Node continuities

$$\Sigma XIJ - \Sigma XJI = 0 \quad \text{for all I}$$

Flow at arcs are limited

$$XIJ \leq FIJ$$



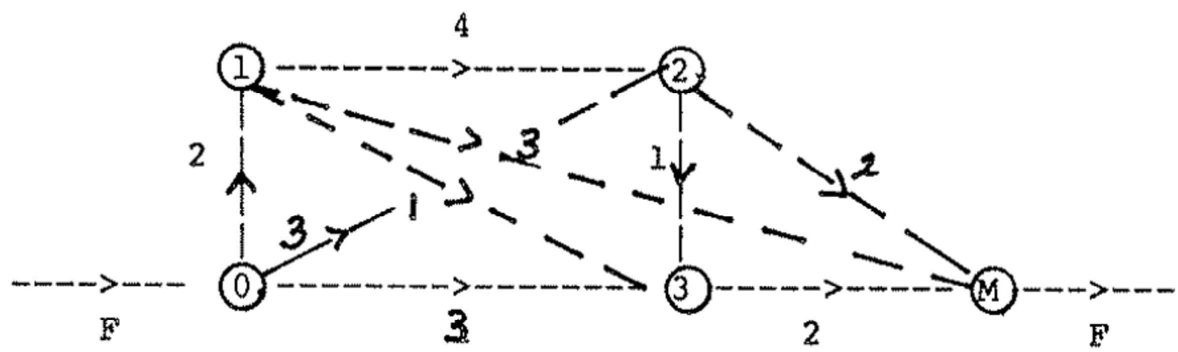
Maximize F

Subject to $\sum (X0J - XJ0) = F$

$F = \sum (XJM - XMJ)$

$\sum XIJ - \sum XJI = 0$ for all I

$0 \le XIJ \le FIJ$ for all IJ



Maximize F

Subject to $F = X01 + X02 + X03$

$X1M + X2M + X3M = F$

$X01 - X12 - X13 - X1M = 0$

$X02 + X12 - X23 - X2M = 0$

$X03 + X13 + X23 - X3M = 0$

$X01 \le 2 \quad X02 \le 3 \quad X03 \le 3$

$X12 \le 4 \quad X13 \le 1 \quad X1M \le 3$

$X23 \le 1 \quad X2M \le 2 \quad X3M \le 2$

All variables nonnegative

Handling Unrestricted Variables

Assume X1 is unrestricted in the problem.

Let

$$X1 = X1P - X1N$$

where X1P and X1N are nonnegative.

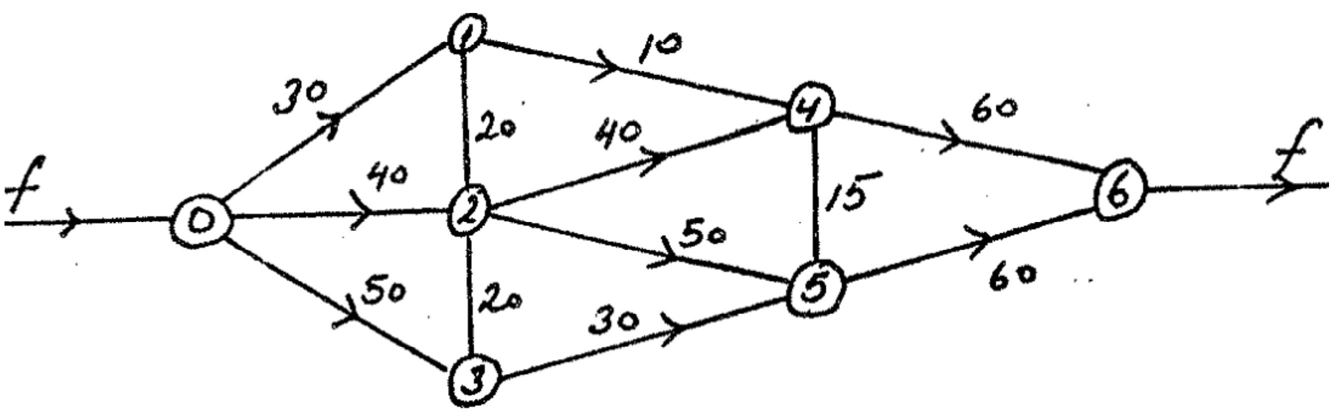
The value of X1 is positive or negative depending on wheter

$X1P \geq X1N$ or $X1P \leq X1N$

Street networks

The numbers on the arcs represent the traffic flow capacities.
The arrows indicate the trafic direction.

The problem is to place one-way signs so as to
maximize the traffic flow.

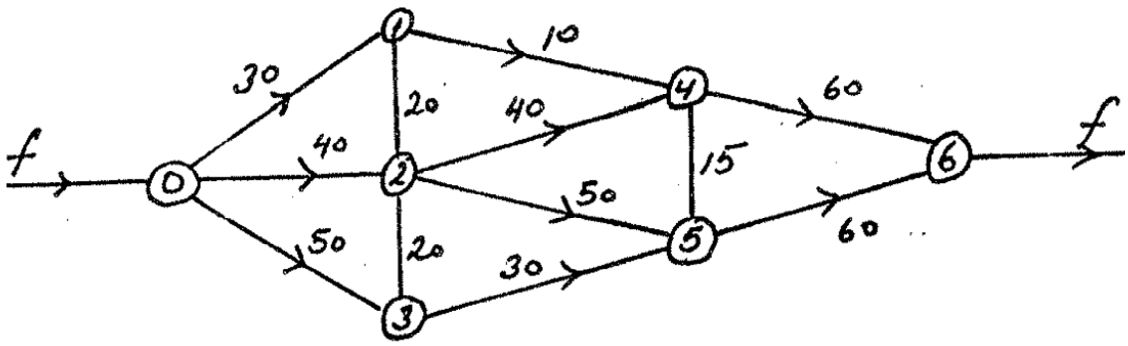


The trick is to replace each undirected arc by a pair of oppositely directed arcs with same limitations.

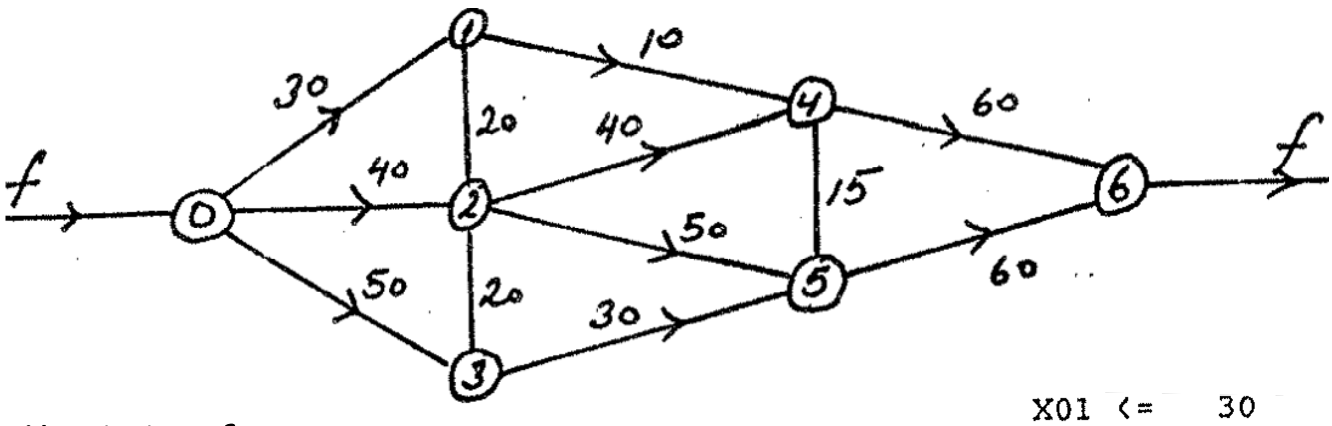
Similar to replacing unrestricted decision variable with Difference of two non-negative variables.

X12 ===➔ X12 – X21 X23 ===➔ X23 – X32

X45 ===➔ X45 – X54



Once the solution is obtained place one way signs accordingly



Maximize f

Subject to f – (X01 + X02 + X03) = 0

X01 + X21 – X12 – X14 = 0

X02 + X12 + X32 – X21 – X23 – X24 – X25 = 0

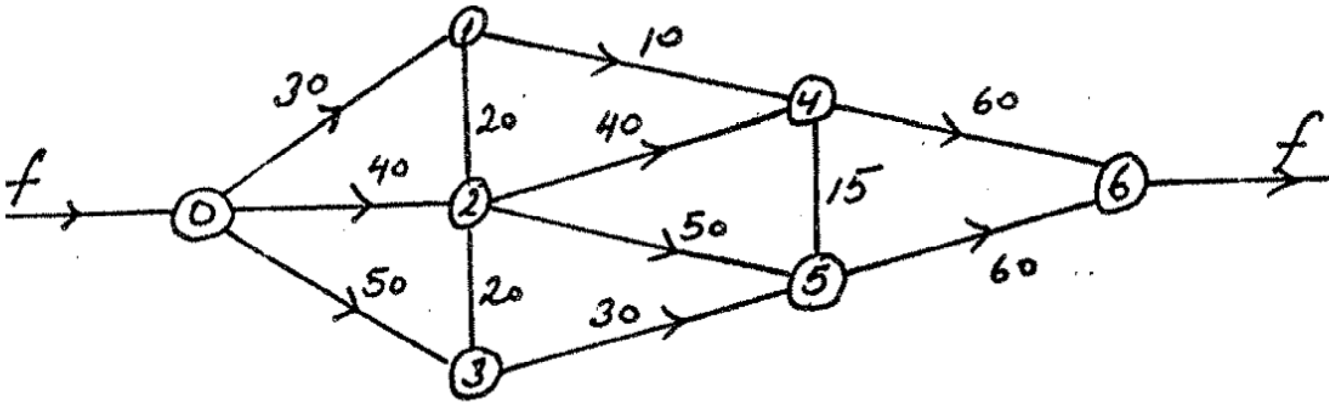
X03 + X23 – X32 – X35 = 0

X14 + X24 + X54 – X45 – X46 = 0

X25 + X35 + X45 – X54 – X56 = 0

X46 + X56 –f = 0

X01	<=	30
X02	<=	40
X03	<=	50
X14	<=	10
X24	<=	40
X25	<=	50
X35	<=	30
X46	<=	60
X56	<=	60
X12	<=	20
X21	<=	20
X23	<=	20
X32	<=	20
X45	<=	15
X54	<=	15



VARIABLE	VALUE	
F	120.000000	
X01	30.000000	
X02	40.000000	
X03	50.000000	
X14	10.000000	
X12	20.000000	⇐=== X12 > X21 One way from 1 to 2
X21	0.000000	
X24	40.000000	
X25	40.000000	
X23	0.000000	
X32	20.000000	⇐=== X32 > X23 One way from 3 to 2
X35	30.000000	
X46	60.000000	
X45	0.000000	
X54	10.000000	⇐=== X54 > X45 One way from 5 to 4
X56	60.000000	

ASSIGNMENT MODELS

Optimal assignment of resources (man, machine, vehicle) to tasks,

Let a machine shop has N machines **M1, M2,…,MN.**

A group of N different jobs **J1, J2,…., JN** are to be assigned to these machines.

For each job (i) assigned to machine (j), the machining cost is Cij

The problem is to assign the jobs to the machines which will minimize the total cost of machining.

$$\text{MIN} \quad Z = \sum \sum C_{ij} * X_{ij}$$

Subject to $\sum X_{ij} \le 1 \qquad i = 1,2,\ldots,I$

$\sum X_{ij} \ge 1 \qquad j = 1,2,\ldots,J$

$X_{ij} \ge 0$ for all links

Homework 1 :

In a company the time required for a given personnel to do a certain job is given below.

Find the minimum time required for jobs to be completed if each personel has to be assigened to a job.

Person	Job A	Job B	Job C
1	3	2	5
2	4	1	6
3	3	4	7

RESOURCE ALLOCATION MODELS

Consider the allocation of a resource available at I different sources

$S_1,\ S_2,\ \ldots\ldots,\ S_i$

in the amounts $A_1,\ A_2,\ \ldots\ldots A_i$, respectively.

Let the allocations be needed over J destinations,

$D_1,\ D_2,\ldots\ldots D_j$

where it is needed

in the amounts $B_1,\ B_2,\ldots\ B_j$ respectively.

Note that the feasibility requires

Requirement should not exceed the availability

$$\sum A_i \geq \sum B_j$$

If C_{ij} is the constant transportation cost from source I to J

Let X_{ij} be the allocation from I to J

Then the allocation that minimizes the transportation cost is the solution of

$$\text{MIN} \quad Z = \sum \sum C_{ij} * X_{ij}$$

Subject to

$$\sum X_{ij} \leq A_{ij} \text{ for all } i$$

$$\sum X_{ij} \geq B_{ij} \text{ for all } j$$

$$X_{ij} \geq 0$$

Example

Consider an irrigation district serviced by several groundwater or surface water sources. Assume a known supply of water AI is available at each source of supply I and a known demand BJ at each irrigation area J. The problem is to find the quantity of water XIJ to pump or transport from source site I to use in site J so that the total cost is minimized.

Assuming that CIJ is the cost of moving a unit water from origin I to destination J, solve the above transportation problem using the following data.

C11 = 5 ; C12 = 3 ; C13 = 6 ; A1 = 10 ; B1 = 25
C21 = 4 ; C22 = 5 ; C23 = 6 ; A2 = 23 ; B2 = 25
C31 = 2 ; C32 = 1 ; C33 = 7 ; A3 = 37 ; B3 = 20

C11 = 5 ; C12 = 3 ; C13 = 6 ; A1 = 10 ; B1 = 25
C21 = 4 ; C22 = 5 ; C23 = 6 ; A2 = 23 ; B2 = 25
C31 = 2 ; C32 = 1 ; C33 = 7 ; A3 = 37 ; B3 = 20

Feasbility check :

A1 + A2 + A3 = 70

B1 + B2 + B3 = 70

OK.

C11 = 5 ; C12 = 3 ; C13 = 6 ; A1 = 10 ; B1 = 25
C21 = 4 ; C22 = 5 ; C23 = 6 ; A2 = 23 ; B2 = 25
C31 = 2 ; C32 = 1 ; C33 = 7 ; A3 = 37 ; B3 = 20

$$\text{Minimize} \quad (5X11 + 3X12 + 6X13) + (4X21 + 5X22 + 6X23) + (2X31 + X32 + 7X33)$$

Subject to

$$X11 + X12 + X13 \le 10$$
$$X21 + X22 + X23 \le 23$$
$$X31 + X32 + X33 \le 37$$

$$X11 + X21 + X31 \ge 25$$
$$X12 + X22 + X32 \ge 25$$
$$X13 + X23 + X33 \ge 20$$

$$Xij \ge 0 \quad \text{for all } ij$$

HOMEWORK 2 :

Workers must be transported from two locations to 4 sites

The number of workers available at each site are :

S1 = 30 and S2 = 50

Requirements at each sites are determined as :

D1 = 20 D2 = 20 D3 = 30 D4 = 10

Unit cost of transportations are estimated as :

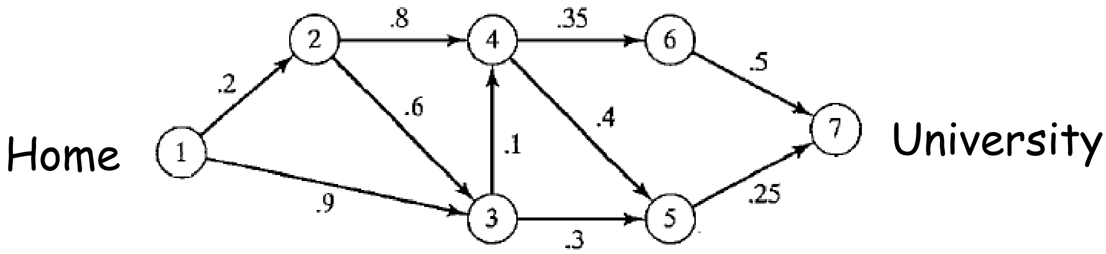
C11 = 40 C12 = 20 C13 = 20 C14 = 10

C21 = 20 C22 = 50 C23 = 50 C24 = 60

Write the mathematical model that minimizes the total cost of transportation.

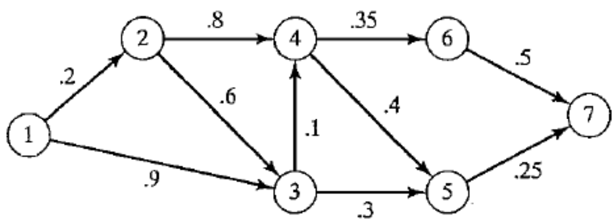
SHORTEST ROUTE PROBLEMS

Determine the shortest time it will take the driver to reach the destination if the numbers on the arcs indicate the driving times between nodes.



One way to solve this problem is to reinterpret the situation as you would like to send one unit of flow from node 1 to node 7 at "minimum cost".

SHORTEST ROUTE PROBLEMS



If T_{ij} is the travel time between i &j,

Let X_{ij} [0-1] variables for every link

X_{ij} = 1 if the link is used X_{ij} = 0 if the link is not used

Then LP can be formulated as to

$$MIN\ Z = \Sigma\ \Sigma\ T_{ij} * X_{ij}$$

Subject to Continuities at nodes

& nonnegativities