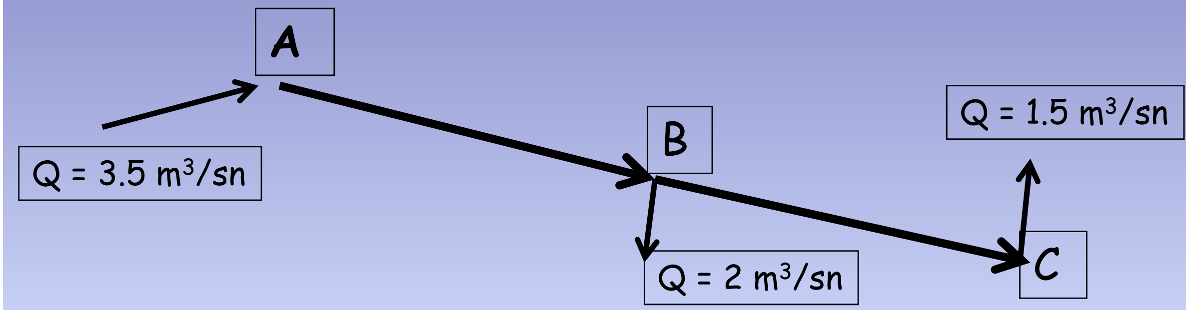
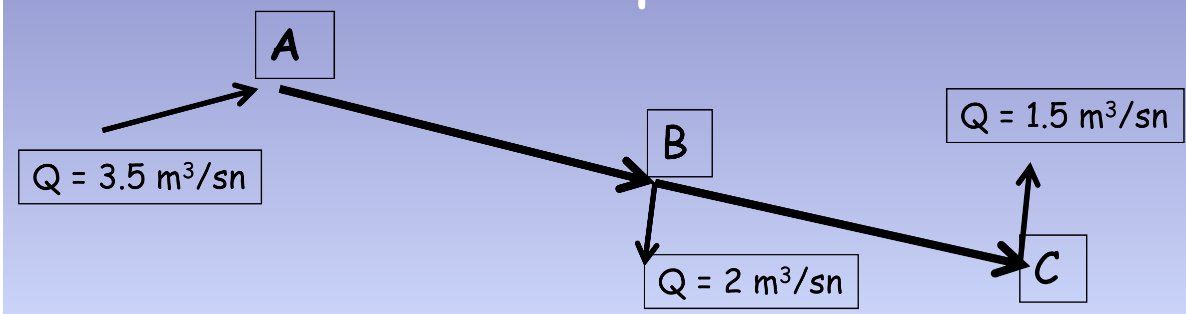
OPTIMIZATION OF PIPE SYSTEMS



- The problem is the determination of most economical pipe diameters
- If the reservoir pool level is known, the energy loss should not exceed the head difference between nodes.
- Upstream elevation is also variable for pumped cases.

OPTIMIZATION OF PIPE SYSTEMS
Known pool level



Variables : X_j, length of different diameter pipes

Constraints : Pipe lengths for each link
Head loss in each link

Objective : Cost Minimization

OPTIMIZATION OF PIPE SYSTEMS BY LINEAR PROGRAMMING

Charles A. Calhoun

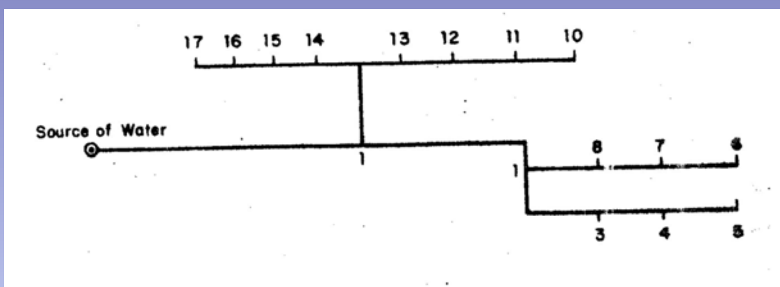


Figure 2. Plan view - branched lines pipe system.

- The basic problem is the determination of optimal pipe diameters.
- For the gravity case,
  - → The head at the source is known
- For the pumped case
  - →The head at the source is not known

OPTIMIZATION OF PIPE SYSTEMS GRAVITY CASE

$$\text{MIN} \quad Z = c_1X_1 + c_2X_2 + c_3X_3 + \dots + c_nX_n$$

SUBJECT TO

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1j}X_j (\le, =, \ge) b_1$$
$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2j}X_j (\le, =, \ge) b_2$$
$$\dots$$
$$\dots$$
$$a_{i1}X_1 + a_{i2}X_2 + \dots + a_{ij}X_j (\le, =, \ge) b_j$$
$$X_1 \ge 0, \; X_2 \ge 0, \; \dots, \; X_j \ge 0.$$

- x_j, the unknown lengths of various sizes of pipe
- c_j , costs per foot of pipe installed
- aij, constants evaluating the friction slopes and/or lengths
- bj, constants evaluating the head losses and/or lengths

OPTIMIZATION OF PIPE SYSTEMS : GRAVITY CASE

Scobey's Friction Formula is used.

$$H_{loss} = \frac{V^2}{c^2 d^{1.25}}$$

- H , head loss in feet per 1000 feet of pipe
- V, velocity in feet per second
- c, Scobey's coefficient of friction for concrete pipe
  - c = 0.345 for pipe sizes 4 to 21 inches
  - c = 0.370 for pipe sizes larger than 21 inches
- d, inside diameter of pipe in inches

Other common pipe friction formulas are Darcy-Weisbach, Mannings, and Hazen-Williams .

Example

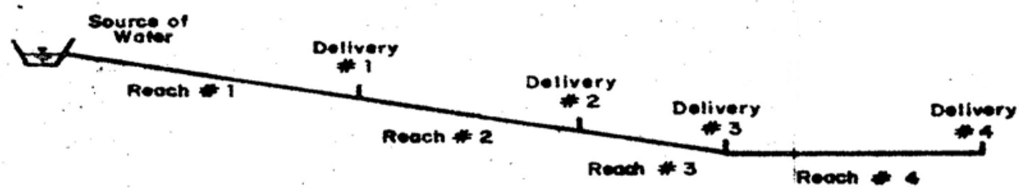


Figure 1. Profile view - single line pipe system.

TABLE 1. DATA FOR EXAMPLE 1

Reach	Length	Discharge	Delivery	Elevation Required	Delivery Discharge
1	2000'	16 cfs	1	90	4 cfs
2	1500'	12 cfs	2	80	6 cfs
3	1000'	6 cfs	3	70	4 cfs
4	2200'	2 cfs	4	70	2 cfs

TABLE 2. SLOPE COMPUTATIONS FOR EXAMPLE 1

Delivery	Available Head (100-Del. Elev. Req'd)	Distance (from Source)	Slope
1	10'	2000'	.005
2	20'	3500'	.00571
3	30'	4500'	.00667
4	30'	6700'	.00448

EXAMPLE FOR THE GRAVITY CASE

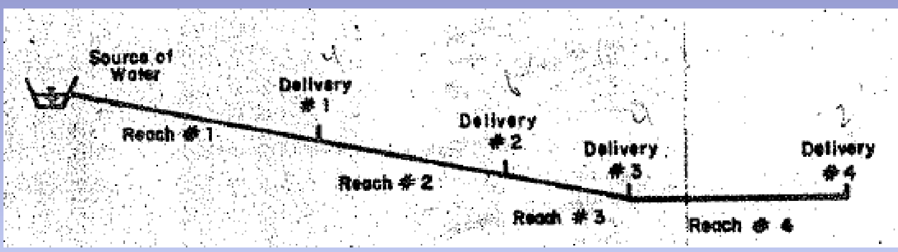
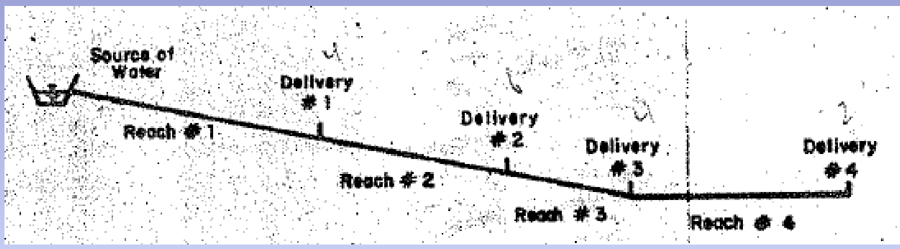


TABLE 3. POSSIBLE PIPE CHOICES FOR EXAMPLE 1

Reach	Control Slope (ft/ft)	Unknown Length	Pipe Diameter	Friction Slope	Cost ($/ft)
1	.00448	X_1	24"	.00357	12.30
		X_2	21"	.00827	10.35
2		X_3	24"	.00201	12.30
		X_4	21"	.00465	10.35
3		X_5	18"	.00261	8.48
		X_6	15"	.00680	6.68
4		X_7	12"	.00244	5.00
		X_8	10"	.00635	3.92

EXAMPLE FOR THE GRAVITY CASE



MIN $Z = 12.3\ X_1 + 10.35\ X_2 + 12.3\ X_3 + 10.35\ X_4 + 8.48\ X_5 + 6.68\ X_6 + 5.\ X_7 + 3.92\ X_8$

SUBJECT TO

$X_1 + X_2 = 2000$

$X_3 + X_4 = 1500$

$X_5 + X_6 = 1000$

$X_7 + X_8 = 2200$

$100. - .00357\ X_1 - .00827\ X_2 \geq 90$

$100. - .00357\ X_1 - .00827\ X_2 - .00201\ X_3 - .00465\ X_4 \geq 80$

$100. - .00357\ X_1 - .00827\ X_2 - .00201\ X_3 - .00465\ X_4 - .00261\ X_5 - .0068\ X_6 \geq 70.$

$100. - .00357\ X_1 - .00827\ X_2 - .00201\ X_3 - .00465\ X_4 - .00261\ X_5 - .0068\ X_6 - .00244\ X_7 - .00635\ X_8 \geq 70.$

EXAMPLE FOR THE GRAVITY CASE

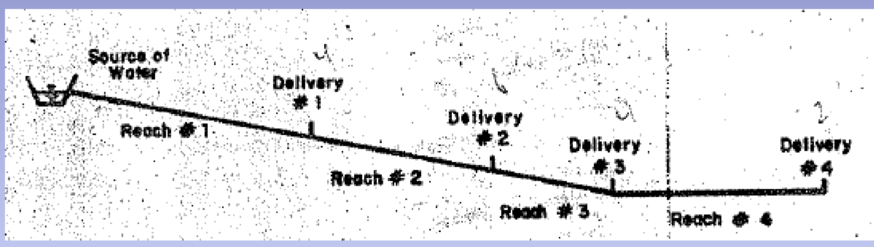


TABLE 4. SOLUTION TO EXAMPLE 1

Reach	Variable	Pipe Diameter	Solution Length
1	X_1	24"	1391'
	X_2	21"	609'
2	X_3	24"	0'
	X_4	21"	1500'
3	X_5	18"	0'
	X_6	15"	1000'
4	X_7	12"	1981'
	X_8	10"	219'

OPTIMIZATION OF PIPE SYSTEMS : PUMPED CASE

$$\text{MIN} \quad Z = c_1X_1 + c_2X_2 + c_3X_3 + \cdots + c_nX_n$$

SUBJECT TO

$$
\begin{aligned}
&a_{11}X_1 + a_{12}X_2 + \cdots + a_{1j}X_j \,(\le,=,\ge)\, b_1 \\
&a_{21}X_1 + a_{22}X_2 + \cdots + a_{2j}X_j \,(\le,=,\ge)\, b_2 \\
&\qquad\qquad \vdots \\
&a_{i1}X_1 + a_{i2}X_2 + \cdots + a_{ij}X_j \,(\le,=,\ge)\, b_j \\
&X_1 \ge 0,\ X_2 \ge 0,\ \ldots,\ X_j \ge 0.
\end{aligned}
$$

- x_1, the unknown pump head
- $x_2,\ldots,x_n$, the unknown lengths of various sizes of pipe
- c_1 , cost per incremental foot of head
- $c_2,\ldots,c_n$,costs per foot of pipe installed
- a_{i1}, constants evaluating the unknown pump head 0-1 variable.
- a_{ij}, constants evaluating the friction slopes and/or lengths
- b_j, constants evaluating the head losses and/or lengths

OPTIMIZATION OF PIPE SYSTEMS : PUMPED CASE

Example : A pipe system which has the following given data is to be sized and optimized.

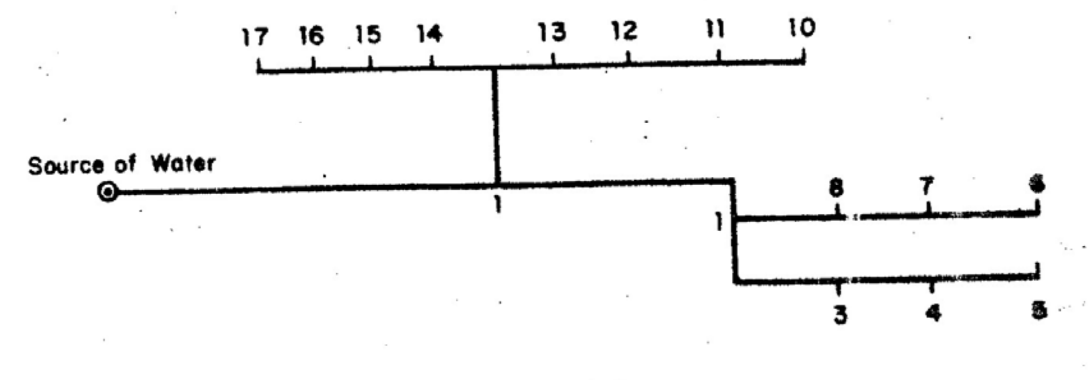


Figure 2. Plan view - branched lines pipe system.

EXAMPLE FOR THE PUMPED CASE

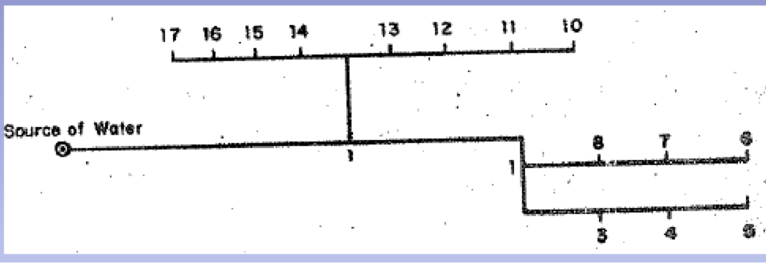


TABLE 5. DATA

Section	Q. in cubic feet per second	Length, in feet	Diameters, in inches	Cost, in dollars per foot installed	Head loss, in feet per foot
1	25	4,500	36, 33, 30	19.48, 17.32, 15.23	.00104, .00164, .0027
2	9	4,500	27, 24, 21	13.22, 11.28, 9.42	.00061, .00113, .00262
3	4.5	3,660	24, 21, 18	11.28, 9.42, 7.65	.00028, .00065, .00147
4	3	1,200	21, 18, 15	9.42, 7.65, 5.98	.00029, .00065, .0017
5	1.5	1,200	18, 15, 12	7.65, 5.98, 4.43	.00016, .00043, .00137
6	1.5	1,200	15, 12, 10	5.98, 4.43, 3.45	.00043, .00137, .00357
7	3	1,200	18, 15, 12	7.65, 5.98, 4.43	.00065, .0017, .00549
8	4.5	1,160	21, 18, 15	9.42, 7.65, 5.98	.00065, .00147, .00383
9	16	1,200	24, 21, 18	11.28, 9.42, 7.65	.00357, .00827, .01858
10	2	360	12, 10, 8	4.43, 3.45, 2.63	.00244, .00635, .0205
11	4	540	15, 12, 10	5.98, 4.43, 3.45	.00302, .00976, .02541
12	6	360	18, 15, 12	7.65, 5.98, 4.43	.00261, .0068, .02195
13	8	450	21, 18, 15	9.42, 7.65, 5.98	.00207, .00464, .0121
14	8	450	21, 18, 15	9.42, 7.65, 5.98	.00207, .00464, .0121
15	6	360	18, 15, 12	7.65, 5.98, 4.43	.00261, .0068, .02195
16	4	540	15, 12, 10	5.98, 4.43, 3.45	.00302, .00976, .02541
17	2	360	15, 12, 10	5.98, 4.43, 3.45	.00076, .00244, [illegible]

EXAMPLE FOR THE PUMPED CASE

MIN $Z = 2,400\, X_1 + 19.48\, X_2 + 17.32\, X_4 + 13.22\, X_5 + \ldots + 3.45\, X_{52}$

SUBJECT TO

$$X_1 + X_2 + X_3 = 4500.$$
$$X_4 + X_5 + X_6 = 4500.$$
$$+ \ldots$$
$$\ldots + X_{49} + X_{50} + X_{51} = 360.$$

$$X_1 - .00104\, X_2 - .00164\, X_3 - .0027\, X_4 - .00061\, X_5 - .00113\, X_6$$
$$- .00262\, X_7 - .00028\, X_8 - .00065\, X_9 - .00147\, X_{10} - .00029\, X_{11}$$
$$- .00065\, X_{12} - .0017\, X_{13} - .00016\, X_{14} - .00043\, X_{15}$$
$$- .00137\, X_{16} \geq 151.$$

EXAMPLE FOR THE PUMPED CASE

TABLE 7. PUMPED CASE SOLUTION

Pump Head = 172.53 ft

Section No.	Diameters in inches	Length in feet	Cost in dollars
1	33	4,500	77,940
2	24	4,500	50,760
3	18	3,660	27,999
4	15	1,200	7,176
5	12	1,200	5,316
6	12	774	3,429
	10	426	1,470
7	15	1,200	7,176
8	15	1,160	6,937
9	24	473	5,335
	21	727	6,848
10	10	360	1,242
11	12	540	2,392
12	15	360	2,153
13	15	450	2,691
14	18	450	3,442
15	15	360	2,153
16	15	170	1,017
	12	370	1,639
17	10	360	1,242

CONCLUSION

The optimal design of pipe networks can be achived through the use of linear programming. This paper illustrated the use of linear programming to optimize several types of pipe networks. The several types are broken into two general cases which are the gravity case and the pumped case.