(CT)
CSE-08

Markov Analysis

*** Example-13.1**

Let state 1 be the condition of owning a TCC car and state 2 that of owning a non-TCC car.

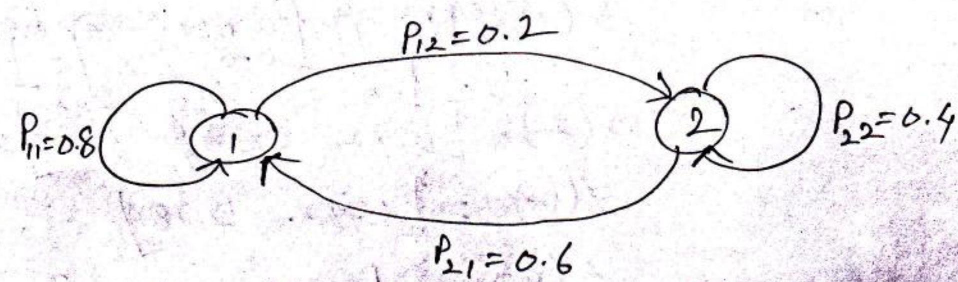
The transition probabilities are

$$P_{11} = 0.8, \quad P_{12} = 0.2, \quad P_{21} = 0.6, \quad P_{22} = 0.4$$

The transition prob. matrix is

Current state i	Next state j: 1 (P_{i1})	Next state j: 2 (P_{i2})
1 (TCC car)	0.8	0.2
2 (non-TCC car)	0.6	0.4

The transition diagram is



Case-1: Present car is a TCC car, i.e.,

$$s_{11}(0)=1 \quad \text{and} \quad s_{12}(0)=0$$

so, $s(0)=[1 \quad 0]$

Now,

$$s(1)=s(0)P=[1 \quad 0]\begin{bmatrix}0.8 & 0.2\\0.6 & 0.4\end{bmatrix}=[0.8 \quad 0.2]$$

$$s(2)=s(1)P=[0.76 \quad 0.24]$$

$$s(3)=s(2)P=[0.752 \quad 0.248]$$

$$s(4)=s(3)P=[0.7504 \quad 0.2496]$$

$$\cdots \cdots \cdots$$

$$s(\infty)=[0.75 \quad 0.25]$$

i.e., $s_{11}(\infty)=0.75, \quad s_{12}(\infty)=0.25$

Case-2: Present car is a non-TCC car, i.e.,

$$s_{21}(0)=0 \quad \text{and} \quad s_{22}(0)=1$$

so, $s(0)=[0 \quad 1]$

The first four state row vectors are then

$$s(1)=[0 \quad 1]\begin{bmatrix}0.8 & 0.2\\0.6 & 0.4\end{bmatrix}=[0.6 \quad 0.4]$$

$$s(2)=[0.72 \quad 0.28]$$

$$s(3)=[0.744 \quad 0.256]$$

$$s(4)=[0.7488 \quad 0.2512]$$

$$\cdots \cdots$$

$$s(\alpha) = [0.75 \quad 0.25]$$

i.e. $s_{21}(\alpha) = 0.75$ and $s_{22}(\alpha) = 0.25$

(**) Calculation of row vectors in a different way:

since $s(2) = s(1)P$ and $s(1) = s(0)P$

$$\therefore s(2) = s(1)P = s(0)P.P = s(0)P^2$$

and in general

$$s(n) = s(0)\, P^n$$

(*) steady state Analysis

suppose $P_{12} = a$ and $P_{21} = b$

$$\therefore P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

Let $s = s(n) = s(\alpha)$

$\therefore s = sP$ for n large

If we denote s_1 and s_2 as the limiting state prob., then

$$[s_1 \quad s_2] = [s_1 \quad s_2] \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

$$\therefore s_1 = (1-a)s_1 + b s_2$$

$$s_2 = a s_1 + (1-b) s_2$$

$$\Rightarrow \begin{array}{l} as_1 - bs_2 = 0 \\ -as_1 + bs_2 = 0 \end{array} \Big| \text{ one is redundant}$$

Consider the constraint, $s_1 + s_2 = 1$, we have,

$$as_1 - bs_2 = 0$$
$$s_1 + s_2 = 1$$

$$\therefore s_1 = \frac{b}{a+b} \text{ and } s_2 = \frac{a}{a+b}$$

$$\therefore S = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix} = \begin{bmatrix} \frac{P_{21}}{P_{12}+P_{21}} & \frac{P_{12}}{P_{12}+P_{21}} \end{bmatrix}$$

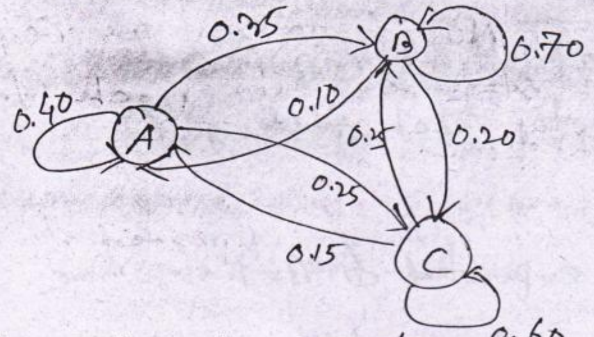
The limiting multistep transition prob. matrix

$$P^{\alpha} = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \\ \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix} = \begin{bmatrix} \frac{P_{21}}{P_{12}+P_{21}} & \frac{P_{12}}{P_{12}+P_{21}} \\ \frac{P_{21}}{P_{12}+P_{21}} & \frac{P_{12}}{P_{12}+P_{21}} \end{bmatrix}$$

* Example-13.2:

This week's Leader		Next week's Leader station		
		A	B	C
station	A	0.40	0.35	0.25
	B	0.10	0.70	0.20
	C	0.15	0.25	0.60

The transition diagram is



Using the principles of the equilibrium of probabilistic flows, i.e.,

At steady state

Input = output ——— (1)

$\sum$ probability = 1 ——— (2)

For state A: $(0.35+0.25)S_A = 0.10S_B + 0.15S_C$

For state B: $(0.10+0.20)S_B = 0.35S_A + 0.25S_C$

For state C: $(0.15+0.25)S_C = 0.25S_A + 0.20S_B$

Here S_A, S_B, S_C be the limiting state prob. be remaining in state A, B, C respectively.

$S_A + S_B + S_C = 1$

Solving ⇒

$S_A = 0.168$	$S_A = 16.8\%$
$S_B = 0.485$	$S_B = 48.5\%$
$S_C = 0.347$	$S_C = 34.7\%$

Img 007

$\Rightarrow$,

$$0.30\mu_{BA} - 0.20\mu_{CA} = 1$$

$$-0.25\mu_{BA} + 0.75\mu_{CA} = 1$$

Multiplying the 1st eq^n by 3.75 and then adding,

$$0.875\mu_{BA} = 4.75$$

$$\therefore \mu_{BA} = 5.43$$

(b)

Expected recurrence time

$$\mu_{ii} = \frac{1}{s_i}$$

Here $s_A = 0.168$, $s_B = 0.485$, $s_C = 0.347$

$$\therefore \mu_{AA} = \frac{1}{s_A} = 5.95 \text{ weeks}$$

$$\mu_{BB} = \frac{1}{s_B} = 2.06 \text{ weeks}$$

$$\mu_{CC} = \frac{1}{s_C} = 2.88 \text{ weeks}$$

—— x ——

* Important Definitions:

1. Recurrent state:

2. Transient or Non-recurrent state:

3. Absorbing state:

4. Irreducible Markov chain:

5. Ergodic Process:

* Exercise – 2
P. No – 324
Ref. – Trivedi

Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1 and 2. Observing its state at 2 P.M. each day, we beleive that the system approx. behaves like a homogeneous Markov chain with the transition probability Matrix:

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}.$$

* Prove that the chain is irreducible and determine the steady-state probabilities.