STAT/CS 5525 Homework 1

Kazi Hasan Ibn Arif — Student ID: 906614469

September 2023

1 Question 1

We will show that the solution to the optimization problem with the L1 loss function, i.e., $L(Y, f(X)) = E|Y - f(X)|$, is the **median** of the conditional distribution of Y given X which is $P(Y|X)$.

The optimization problem can be formulated as finding f^* that minimizes the expected L1 loss:

$$f^* = \arg\min_f E|Y - f(X)|$$

Now, we will prove that the solution f^* to this optimization problem is the median of the conditional distribution $P(Y|X)$.

To prove this, we will use the fact that the median of a random variable Y is the value m that minimizes the following expected value:

$$E|Y - m|$$

Now, let's apply this property to our optimization problem:

$$f^* = \arg\min_f E|Y - f(X)|$$

This is equivalent to finding f^* such that:

$$f^* = \arg\min_f E|Y - f(X)| = \arg\min_f E|Y - f(X)|_{Y \sim P(Y|X)}$$

Here, we are conditioning on X, so we are dealing with the conditional distribution $P(Y|X)$. Now, let's rewrite this in terms of finding the median of $P(Y|X)$:

$$f^* = \arg\min_f E|Y - f(X)|_{Y \sim P(Y|X)} = \arg\min_f E|Y - m|_{Y \sim P(Y|X)}$$

This is the form of the problem where we are looking for the value m that minimizes the expected L1 loss for the conditional distribution $P(Y|X)$, which

is the definition of the median.

Therefore, we have shown that the solution f^* to the optimization problem with the L1 loss function **is indeed the median of the conditional distribution** $P(Y|X)$.