Stat/CS 5525 Homework 2

Kazi Hasan Ibn Arif / hasanarif@vt.edu / SID: 906614469

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Question 1
We want to show that the Ridge Regression estimates given by

$$\hat{\beta}_{ridge} = (X^T X + \lambda I)^{-1} X^T y$$

and

$$\hat{\beta}_{ridge} = X^T (X X^T + \lambda I)^{-1} y$$

are equivalent.
Solution:

Let $Z = X\hat{\beta}_{ridge}$

Rewrite the Original Equation Using Z

$$Z = X(X^T X + \lambda I)^{-1} X^T y$$

Multiply Both Sides by $(XX^T + \lambda I)$

$$(XX^T + \lambda I)Z = (XX^T + \lambda I)X(X^T X + \lambda I)^{-1} X^T y$$

Simplifying,
The equation becomes –

$$(XX^T + \lambda I)Z = XX^T X(X^T X + \lambda I)^{-1} X^T y + \lambda X(X^T X + \lambda I)^{-1} X^T y$$

Now, Factoring Out Common Terms,

$$(XX^T + \lambda I)Z = (X^T X + \lambda I)X(X^T X + \lambda I)^{-1} X^T y$$

Here, $(X^T X + \lambda I)$ and $(X^T X + \lambda I)^{-1}$ cancel out, leaving:

$$(XX^T + \lambda I)Z = XX^T y$$

Therefore, the solve for Z

$$Z = X^T (XX^T + \lambda I)^{-1} y$$

Since $Z = X\hat{\beta}_{\{ridge}$, this shows that

$$\hat{\beta}_{ridge} = X^T (XX^T + \lambda I)^{-1} y$$

is equivalent to the original formula.