Student Information

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Answer 1

a) 1. The given function is not surjective:
Consider y_0 in f's codomain such that $y_0 = -4$ ($\{-4\} \in \mathbb{R}$). Plugging in the value for f, we get:

$$-4 = x^2$$
$$\sqrt{-2} = x$$
$$\{\emptyset\} = x$$

bringing us to the conclusion that f is not surjective.

2. The given function is not injective:
Consider y_0 in f's codomain such that $y_0 = -4$ ($\{4\} \in \mathbb{R}$). Plugging in the value for f, we get:

$$4 = x^2$$
$$2 = |x|$$
$$\{-2, 2\} = x$$
$$\{-2, 2\} \ni \mathbb{R}$$

bringing us to the conclusion that f is not surjective.

b) 1. The given function is not surjective:
Consider y_0 in f's codomain such that $y_0 = -4$ ($\{-4\} \in \mathbb{R}$). Plugging in the value for f, we get:

$$-4 = x^2$$
$$\sqrt{-2} = x$$
$$\{\emptyset\} = x$$

bringing us to the conclusion that f is not surjective.

2. The given function is injective:
Consider $f(x) = 4$ and $f(y) = 4$ $(x, y \in \mathbb{R})$

$$4 = x^2$$

$$2 = |x|$$

Since only nonnegative values exist for x:

$$2 = x$$

The same holds for y, so x=y
bringing us to the conclusion that f is injective.

c) 1. The given function is surjective:

$$y = x^2$$
$$y = \sqrt{x}$$

where $\sqrt{x}$ can only be nonnegative for an arbitrary x.
which matches the description for y given that $y \in \mathbb{R}^+$ bringing us to the conclusion that f is surjective.

2. The given function is not injective:

Consider y_0 in f's codomain such that $y_0 = -4$ ($\{4\} \in \mathbb{R}$). Plugging in the value for f, we get:

$$4 = x^2$$
$$2 = |x|$$
$$\{-2, 2\} = x$$
$$\{-2, 2\} \ni \mathbb{R}$$

bringing us to the conclusion that f is not surjective.

d) 1. The given function is surjective:

$$y = x^2$$
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where $\sqrt{x}$ can only be nonnegative for an arbitrary x.
which matches the description for y given that $y \in \mathbb{R}^+$ bringing us to the conclusion that f is surjective.

2. The given function is injective:
Consider $f(x) = 4$ and $f(y) = 4$ $(x, y \in \mathbb{R})$

$$4 = x^2$$
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Since only nonnegative values exist for x:

$$2 = x$$

The same holds for y, so x=y
bringing us to the conclusion that f is injective.

Answer 2

a) Let $f : \mathbb{Z} \longrightarrow \mathbb{R}$, $x_0 \in \mathbb{Z}$ and $\epsilon > 0$
For an arbitrary $x \in \mathbb{Z}$, suppose that $|x - x_0| < \delta$ where $\delta = 1/4$
Since the only integer within $\delta = 1/4$ distance of x_0 is itself, we must have $x = x_0$

$$f(x) = f(x_0)$$
$$|f(x) - f(x_0)| = 0$$
$$0 < \epsilon$$

Hence,

$$|x - x_0| < \delta \implies |f(x) - f(x_0)| < \epsilon$$

Bringing us to the conclusion that it is continious.

b)

Answer 3

a) by Mathematical Induction,
Basis:(i) $n = 2 \implies X_2 = A_1 \times A_2$ $(A_1, A_2$ being countable)
Induction Hyphoteiss: (ii) Assume X_k is countable
Induction Step: (iii) $X_{k+1} = X_k \times A_{k+1}$ $(X_k, A_{k+1}$ being countable)
Therefore, finite Cartesian product of countable sets, i.e. $X_n = A_1 \times A_2 \times ... \times A_n$ for all $n \leq 2$, is countable.

b)

Answer 4

$$(n!)^2 \geq 5^n \geq 2^n \geq n^{n^{51}+n^{49}} \geq n^{50} \geq \sqrt{n}logn \geq (logn)^2$$

a) To show that $5^n = \Theta((n!)^2)$, we use Mathematical Induction:
Basis: (i) $n = 5 \implies 3125 < 14400$
Induction Hyphotesis: (ii) $5^n \leq (n!)^2$
Induction Step: (ii) $5^{n+1} \leq ((n+1)!)^2$

$$5^{n+1} \leq ((n+1)!)^2$$
$$5 \times 5^n \leq (n+1)^2 \times (n!)^2$$

So, if we can prove that $5 \leq (n+1)^2$ $\forall n \geq 5$, our proof will hold TRUE.
Therefore, Consider:

$$5 \leq (n+1)^2$$
$$\lim_{n\to\infty} 5 \leq \lim_{n\to\infty} (n+1)^2$$
$$5 \leq \infty$$

Hence, our proof is done.

b) To show that $2^n = \Theta(5^n)$,

$$2^n \leq 5^n$$
$$log(2^n) \leq log(5^n)$$
$$nlog(2) \leq nlog(5)$$
$$log(2) \leq log(5)$$
$$2 \leq 5$$

c) To show that $n^{51} + n^{49} = \Theta(2^n)$,

$$n^{51} + n^{49} \leq 2^n$$
$$log(n^{51} + n^{49}) \leq log(2^n)$$
$$\frac{log(n^{51} + n^{49})}{n} \leq log(2)$$
$$\lim_{n\to\infty} \frac{log(n^{51} + n^{49})}{n} \leq \lim_{n\to\infty} log(2)$$
$$\lim_{n\to\infty} \frac{log(n^{51} + n^{49})}{n} \leq log(2)$$

Applying the L'Hopital's Rule:

$$\lim_{n\to\infty} \frac{51n^2 + 49}{n(n^2+1)} \leq log(2)$$
$$0 \leq log(2)$$

d) To show that $n^{50} = \Theta(n^{51} + n^{49})$,

$$n^{50} \leq n^{51} + n^{49}$$
$$\frac{n^{50}}{n^{50}} \leq \frac{n^{51} + n^{49}}{n^{50}}$$
$$1 \leq n + \frac{1}{n}$$
$$\lim_{n\to\infty} 1 \leq \lim_{n\to\infty} n + \frac{1}{n}$$
$$1 \leq \infty$$

e) To show that $\sqrt{n}logn = \Theta(n^{50})$,

$$\sqrt{n}logn \leq n^{50}$$
$$logn \leq n^{49\frac{1}{2}}$$
$$\lim_{n\to\infty} logn \leq \lim_{n\to\infty} n^{49\frac{1}{2}}$$
$$\lim_{n\to\infty} \frac{1}{n} \leq \lim_{n\to\infty} \frac{99}{2} n^{48\frac{1}{2}}$$
$$0 \leq \infty$$

f) To show that $(logn)^2 = \Theta(\sqrt{n}logn)$,

$$(logn)^2 \leq \sqrt{n}logn$$
$$(logn)(logn) \leq \sqrt{n}(logn)$$
$$logn \leq \sqrt{n}$$
$$\frac{logn}{\sqrt{n}} \leq \frac{\sqrt{n}}{\sqrt{n}}$$
$$\lim_{n\to\infty} \frac{logn}{\sqrt{n}} \leq \lim_{n\to\infty} 1$$

Applying the L'Hopital's Rule:

$$\lim_{n\to\infty} \frac{2 + lnn}{2\sqrt{n}} \leq 1$$

Applying the L'Hopital's Rule:

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \leq 1$$
$$0 \leq 1$$

Answer 5

a) by euclid's algorithm:

$$\begin{aligned}
&gcd(94, 134)\\
&= gcd(94, 40)\\
&= gcd(40, 14)\\
&= gcd(14, 12)\\
&= gcd(12, 2)\\
&= gcd(2, 0)\\
&= 2
\end{aligned}$$

b)