Student Information

Full Name : Unal Hasanli
Id Number : 2589877

Answer 1

$f(x) = \sum_{i=2}^{\infty} a_n * x^n = \sum_{i=2}^{\infty}(3 * a_{n-1} + 4 * a_{n-2})x^n = f(x) = 3x * \sum_{i=2}^{\infty} a_{n-1} * x^{n-1} + 4x^2 * \sum_{i=2}^{\infty} a_{n-2} * x^{n-2} = 3x * (x + f(x)) + 4x^2 * (1 + x + f(x)) = f(x) * (4x^2 + 3x) + 7x^2 + 4x^3 = f(x) \rightarrow f(x) * (1 - 4x^2 - 3x) = 4x^3 + 7x^2 \rightarrow f(x) = \frac{4x^3+7x^2}{1-4x^2-3x}$

$$f(x) = -x^2 * (\frac{a}{4x-1} + \frac{b}{x+1}) \longrightarrow (a = 6.4, b = -0.6)$$

For $\frac{1}{1+x}$:

$$1, -1, 1...(-1)^n : (\frac{1}{1+x})$$

$$0, 0, 0.6, -0.6...0.6 * (-1)^n : (\frac{0.6x^2}{1+x})$$

For $\frac{1}{1-4x}$:

$$1, 4, 16...4^n : (\frac{1}{1-4x})$$

$$0, 0, 6.4...6.4 * 4^{n+2} : (\frac{6.4x^2}{1-4x})$$

Summing up:

$$0, 0, 7, 25...6.4 * 4^{n+2} + 0.6 * (-1)^n$$

$$a_n = 6.4 * 4^{n+2} + 0.6 * (-1)^n$$

Answer 2

a)

$$\text{Consider as sum: } < 2+0, 2+3, 2+9, 2+27, 2+81, 2+243... >$$

$$\text{Multiply by 2: } < 2, 2, 2, 2, 2, 2... >: (\frac{2}{1-x})$$

(i) Substitute x with $3x$ (ii) Shift 1 unit to right (iii) Multiply by 3: $< 1, 3, 9, 27, 81, 243... >: (\frac{3x}{1-3x})$

$$\text{Sum them up:} \frac{2}{1-x} + \frac{3x}{1-3x}$$

b)

$$G(x) = \frac{a}{2x-1} + \frac{b}{x-1} \longrightarrow (a = -5, b = -2)$$

$$(\frac{2}{1-x}) :< 2, 2, 2, 2, 2, 2... >$$

$$(\frac{5}{1-2x}) :< 5, 10, 20, 40, 80, 160... >$$

$$G(x) :< 5+2, 10+2, 20+2, 40+2, 80+2, 160+2... >$$

$$G(x) :< 7, 12, 22, 42, 82, 162... >$$

Answer 3

a)

An equivalence relation must be transitive, meaning that if a is related to b and b is related to c, then a must also be related to c. However, the given relation is not transitive because there are pairs (a, b) and (b, c) that satisfy the relation, but (a, c) does not. For example, $(5, 12)$ and $(12, 16)$ both satisfy the relation, but $(12, 16)$ does not. Therefore, this relation cannot be an equivalence relation.

b)

The given relation satisfies the reflexive property because it holds for all pairs (a, b). It also satisfies the symmetric property because if (a, b) is related to (c, d), then (c, d) is related to (a, b). Finally, it satisfies the transitive property because if (a, b) is related to (c, d) and (c, d) is related to (e, f), then (a, b) is related to (e, f). Therefore, this relation is an equivalence relation. The equivalence class consists of all pairs (a, b) such that $2a + b = 0$. This is a line with slope -2 that passes through the origin.

Answer 4

a)

```
    60
      \
18      10
  \   /    \
    2        5
```

b)

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

Symmetric closure:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

List of all pairs:

$$(10, 2), (10, 5), (18, 2), (60, 2), (60, 5), (60, 10)$$

d)

To establish a complete ordering of all pairs of numbers (a, b), it is necessary that either a divides b or b divides a. Some pairs, like $(2, 5)$ and $(5, 18)$, do not follow this rule. To order these pairs, we can delete two numbers from the sequence, such as 18 and 5, and add a new number, such as 30.