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Answer 1

First, consider:

$$(6^{2n} - 1|5) \wedge (6^{2n} - 1|7)$$
$$\equiv 36^n - 1|35$$

By using Mathematical Induction:

(i) for $n = 1$:

$$36^n - 1 = 36 - 1 = 35$$

Since $35|35$ is true, the claim holds true for $n = 1$.

(ii) For n, assume $((36^n - 1)|35)$ is true.

(iii) For $n + 1$:

$$(36^{n+1} - 1)|35$$
$$\equiv (36^n \times 36 - 1)|35$$
$$\equiv ((36^n \times 35) + (36^n - 1))|35$$

Dividing the statement to 2 parts, we get:

1. $(36^n \times 35)|35$ is true since 35 is a factor of the expression and $35|35$ is true.

2. $(36^n - 1)|35$ is true by part (ii).

Hence, the statement $((36^n \times 35) + (36^n - 1))|35$ is true as it is sum of 1 and 2 above, making the proof complete.

Answer 2

First, let:

$$P(n) = (H_n = 8 * H_{n-1} + 8 * H_{n-2} + 9 * H_{n-3} \leq 9^n)$$

By using strong induction:

(i) For $n = \{3, 4, 5\}$:

$$1.\ H_3 = 8 * H_2 + 8 * H_1 + 9 * H_0 = 105 \leq 9^3$$
$$2.\ H_4 = 8 * H_3 + 8 * H_2 + 9 * H_1 = 941 \leq 9^4$$
$$3.\ H_5 = 8 * H_4 + 8 * H_3 + 9 * H_2 = 583583 \leq 9^5$$

From the calculations above, $P(3)$, $P(4)$ and $P(5)$ holds true.

(ii) For n, assume $(P(3) \wedge P(4) \wedge ... \wedge P(n))$ is true.

(iii) For $n + 1$:

$$H_{n+1} = 8 * H_n + 8 * H_{n-1} + 9 * H_{n-2} \leq 9^{n+1}$$

$$1.\ H_n \leq 9^n \equiv 8 * H_n \leq 8 * 9^n \text{ is true by } (ii)$$
$$2.\ H_{n-1} \leq 9^{n-1} \equiv 8 * H_{n-1} \leq 8 * 9^{n-1} \text{ is true by } (ii)$$
$$3.\ H_{n-2} \leq 9^{n-2} \equiv 9 * H_{n-2} \leq 9^{n-1} \text{ is true by } (ii)$$

Sum of 1, 2, and 3 from above gives us:

$$H_{n+1} = 8 * H_n + 8 * H_{n-1} + 9 * H_{n-2} \leq 9^{n-1} + 8 * 9^{n-1} + 8 * 9^n$$
$$\equiv H_{n+1} = 8 * H_n + 8 * H_{n-1} + 9 * H_{n-2} \leq 9^{n+1}$$

Hence, the statement $P(n + 1)$ holds true, making the proof complete.

Answer 3

First, consider:

$$X \text{ as a variable that can be either 0 or 1}$$

$$Y \text{ as a variable that can be either 0000 or 1111}$$

Then, our string with length of 8 would consist of 4 Xs and 1 Y in order to contain 4 consecutive 0s or 4 consecutive 1s.

To find the possible number of orders, we use the Permutation Formula since the order matters for Y:

$$P(n,r) = \frac{n!}{(n-r)!}$$
$$P(5,1) = \frac{5!}{(5-1)!}$$
$$\equiv P(5,1) = 5$$

And since both X and Y have 2 possible values, we multiply 5 by 2^5:

$$5 * 2^5 = 160$$

SInce the cases of 11110000 and 00001111 are duplicate, we subtract the result by 2:

$$160 - 2 = 158$$

Answer 4

First, to find the possible order of planets such that there is at least 6 nonhabitable planets between 2 habitable ones, we consider the planets lined up in their orbits on a line:

Let N be a nonhabitable planet and P be a planet that can be both habitable and nonhabitable for illustration purposes:

$$P, P, P, P, P, P, P, P, P, P$$

Since there should be 6 nonhabitable planets between 2 habitable ones, it is guaranteed that the middle 4 planets should all be nonhabitable ones:

$$P, P, P, N, N, N, N, P, P, P$$

So if we divide the problem into 3 pieces, the outer 3 planets should contain at least 1 habitable planet each. Then we get:

1. If the habitable planet is the first among left 3, then there is 3 possibilites for habitable planet's posititon for the right side in order to have at least 6 nonhabitable planets in between.
2. If the habitable planet is the second among left 3, then there is 2 possibilites for habitable planet's posititon for the right side in order to have at least 6 nonhabitable planets in between.
3. If the habitable planet is the third among left 3, then there is 1 possibility for habitable planet's posititon for the right side in order to have at least 6 nonhabitable planets in between.

So, there is 6 total possible orders (3+2+1) for planets in order to satisfy the given condition.

Then, we continue with the Permutation Formula for finding the possible selection and order of stars (1), habitable planets (2) and nonhabitable planets (3):

$$P(n,r) = \frac{n!}{(n-r)!}$$

$$1.\ P(10,1) = \frac{10!}{(10-1)!}$$

$$2.\ P(20,2) = \frac{20!}{(20-2)!}$$

$$3.\ P(80,8) = \frac{80!}{(80-8)!}$$

To find the final result, we multiply the results together:

$$\frac{80!}{(80-8)!} * \frac{20!}{(20-2)!} * \frac{10!}{(10-1)!} * 6$$

Answer 5

a) First, let the number of ways that robot can move to n cells away from its initial location as a number sequence for every n possible. So, we can denote the n-th element by a_n. Since the robot can only jump 1, 2, and 3 cells away:

Assuming the last jump was 1 cell away, we have a_{n-1} number of ways for a_n:

$$n = 1 + (n-1)$$

Assuming the last jump was 2 cell away, we have a_{n-2} number of ways for a_n:

$$n = 2 + (n-2)$$

Assuming the last jump was 3 cell away, we have a_{n-3} number of ways for a_n:

$$n = 3 + (n-3)$$

After covering all possibilities for a_n, we have:

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}$$

b) Since the robot should jump at least 1 cells away, and there is only 1 way to do so, $a_1 = 1$ is the first term of the above mentioned **(part a)** sequence. And for the recurrence relation to hold true, a_4 should be the smallest term.

$$a_{n-3} \geq a_1$$

$$n \geq 4$$

Then, for the initial case, we have the initial conditions:

$$a_1 = 1,\ a_2 = 2,\ a_3 = 4,\ n \geq 4$$

1. $a_1 : (1) \implies n = 1$

2. $a_2 : (1,1),(2) \implies n = 2$

3. $a_3 : (1,1,1),(1,2),(2,1),(3) \implies n = 4$

c) To find a_9 from the above mentioned sequence which responds to the robot jumping $n = 9$ cells away we start with the initial case:

$$a_4 = a_3 + a_2 + a_1 = 4 + 2 + 1 = 7$$

and continue by increasing n by 1 every time:

$$a_5 = a_4 + a_3 + a_2 = 7 + 4 + 2 = 13$$

$$a_6 = a_5 + a_4 + a_3 = 13 + 7 + 4 = 24$$

$$a_7 = a_6 + a_5 + a_4 = 24 + 13 + 7 = 44$$

$$a_8 = a_7 + a_6 + a_5 = 44 + 24 + 13 = 81$$

$$a_9 = a_8 + a_7 + a_6 = 81 + 44 + 24 = 149$$

$$a_9 = 149$$

The robot can move to 9 cells away from its initial location in 149 different ways.