Student Information

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Answer 1

a)

The joint density functions for $f(t_B)$ and $f(t_A)$ respectively are:

$$f(t_B) = \frac{1}{100}$$

$$f(t_A) = \frac{1}{100}$$

Using the $f(t_B, t_A) = f(t_B) * f(t_A)$ **formula for joint densities** and substituting the above-found values:

$$f(t_B, t_A) = \frac{1}{100} * \frac{1}{100} = \frac{1}{10000}$$

Using the $F(t_B, t_A) = \int_0^{t_B} \int_0^{t_A} f(u, v) dudv$ formula for joint densities and substituting the above-found value:

$$F(t_B, t_A) = \int_0^{t_B} \int_0^{t_A} \frac{1}{10000} dt_A dt_B = \frac{t_B * t_A}{10000}$$

b)

We can select $t_B = 60 - 40 = 20$ and $t_B = 30 - 0 = 30$ thanks to the **Uniform property** of uniform distribution. (the probability is only determined by the length of the interval). Using the formula from part (a) and substituting the given values:

$$F(t_B, t_A) = \frac{20 * 30}{10000} = \frac{6}{100}$$

c)

Firstly, we find the cases where $t_B \leq 90$:

$$F(t_B, t_A) = \int_0^{t_B} \int_0^{t_A} \frac{1}{10000} dt_A dt_B$$

$$= \int_0^{90} \int_0^{t_B+10} \frac{1}{10000} dt_A dt_B$$

$$= \int_0^{90} \frac{t_B + 10}{10000} dt_B$$

$$= \frac{t_B^2 + 20 * t_B}{20000}\Big|_0^{90}$$

$$= \frac{90^2 + 20 * 90}{20000} = 0.495$$

Secondly, we find the cases where $t_B > 90$:
Since it is guaranteed that the given condition will always be satisifed ($t_B + 10$ always exceeds 100 which is over the given interval for t_A), we can take:

$$\frac{10}{100} * 1 = 0.1$$

And finally, we sum our findings making the result complete:

$$0.495 + 0.1 = 0.595$$

d)

Firstly, we find the cases where $t_B < 20$ & $t_B > 80$:

$$F(t_B, t_A) = \int_0^{t_B} \int_0^{t_A} \frac{1}{10000} dt_A dt_B$$

$$= \int_0^{20} \int_0^{t_B+20} \frac{1}{10000} dt_A dt_B$$

$$= \int_0^{20} \frac{t_B + 20}{10000} dt_B$$

$$= \frac{t_B^2 + 40 * t_B}{20000}\Big|_0^{20}$$

$$= \frac{20^2 + 40 * 20}{20000} = 0.06$$

This value will also hold true for $t_B > 80$ by **symmetry**. Therefore we find $0.06 * 2 = 0.12$.
Secondly, we find the cases where $20 \leq t_B \leq 80$:

$$F(t_B, t_A) = \int_0^{t_B} \int_0^{t_A} \frac{1}{10000} dt_A dt_B$$

$$= \int_{80}^{20} \int_{t_B-20}^{t_B+20} \frac{1}{10000} dt_A dt_B$$

$$= \int_{80}^{20} \frac{40}{10000} dt_B$$

$$= \frac{t_B}{250}\Big|_{20}^{80}$$

$$= \frac{60}{250} = 0.24$$

And finally, we sum our findings making the result complete:

$$0.12 + 0.24 = 0.36$$

Answer 2

a)

Given $n = 150$ and $p = 0.6$, we use the **Normal approximation of Binomial distribution**:

$$\mu = n * p = 150 * 0.6 = 90$$

$$\sigma = \sqrt{n * p * (1 - p)} = \sqrt{150 * 0.6 * (1 - 0.6)} = 6$$

Then,

$$P\{X \geq 97.5\}$$
$$\approx P\{X > 97\}$$
$$= P\{\frac{X - \mu}{\sigma} > \frac{97 - 90}{6}\}$$
$$= 1 - P\{\frac{X - \mu}{\sigma} \leq 1.16\}$$
$$= 1 - \phi(1.16) = 0.123$$

b)

Given $n = 150$ and $p = 0.1$, we use the **Normal approximation of Binomial distribution**:

$$\mu = n * p = 150 * 0.1 = 15$$

$$\sigma = \sqrt{n * p * (1 - p)} = \sqrt{150 * 0.1 * (1 - 0.1)} = 3.674$$

Then,

$$P\{X \leq 150 * 0.15\}$$
$$= P\{\frac{X - \mu}{\sigma} \leq \frac{150 * 0.15 - 15}{3.674}\}$$
$$= P\{\frac{X - \mu}{\sigma} \leq 2.04\}$$
$$= \phi(2.04) = 0.9793$$

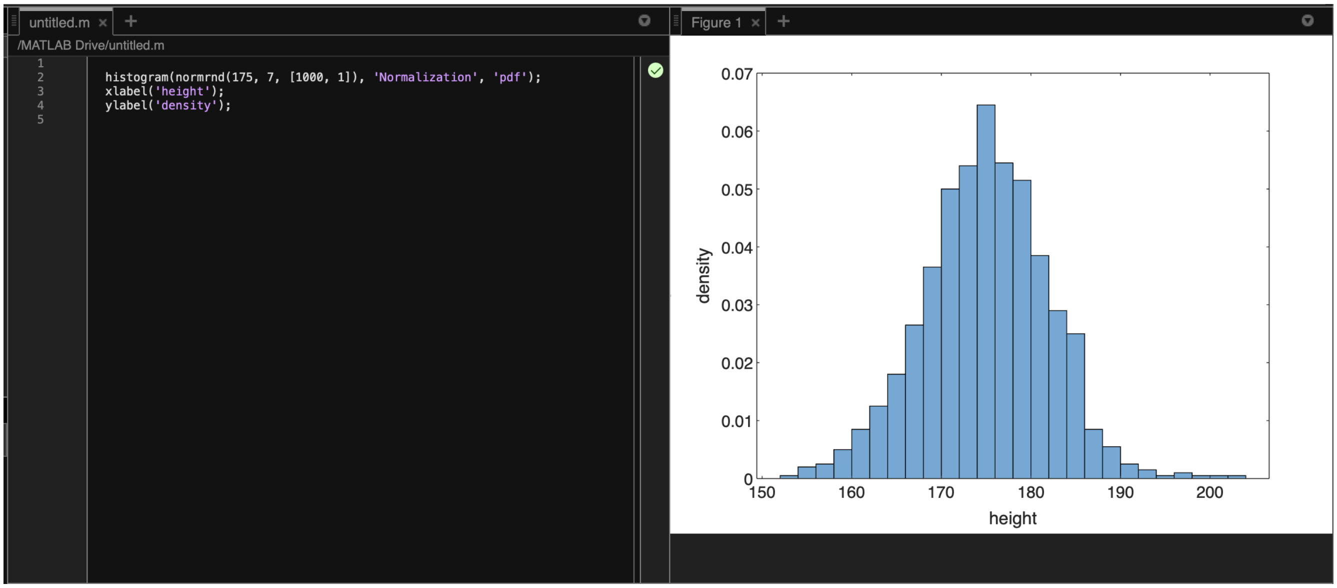
Answer 3

Given $\mu = 175$ and $\sigma = 7$, to satisfy the conditions:

$$P\{\frac{170 - 175}{7} < Z < \frac{180 - 175}{7}\}$$
$$= P{-0.714} < Z < 0.714$$
$$= \phi(-0.714) - \phi(0.714) = 0.7611 - 0.2389 = 0.522$$

Answer 4

a)

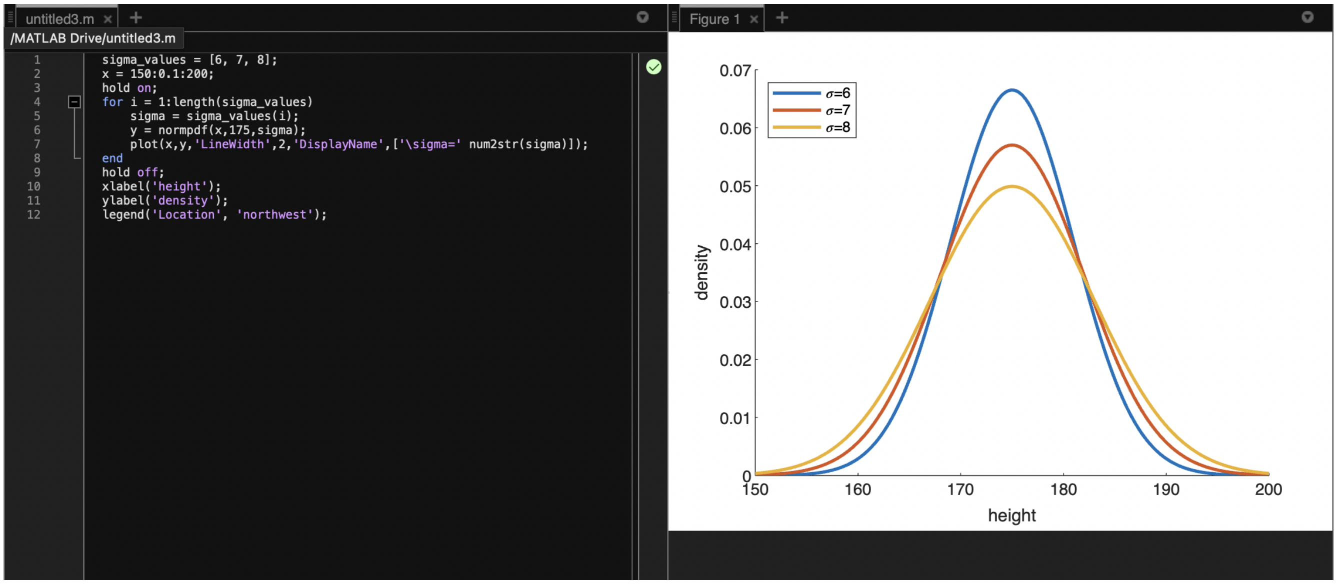


Histogram shows a normal distribution showing a mean of 175 cm and an standard deviation of 7 cm. Bell-shaped, mean peak, standard deviation-spread distribution. (normalized pdf)

code:

```
histogram(normrnd(175, 7, [1000, 1]), 'Normalization', 'pdf');
xlabel('height');
ylabel('density');
```

b)

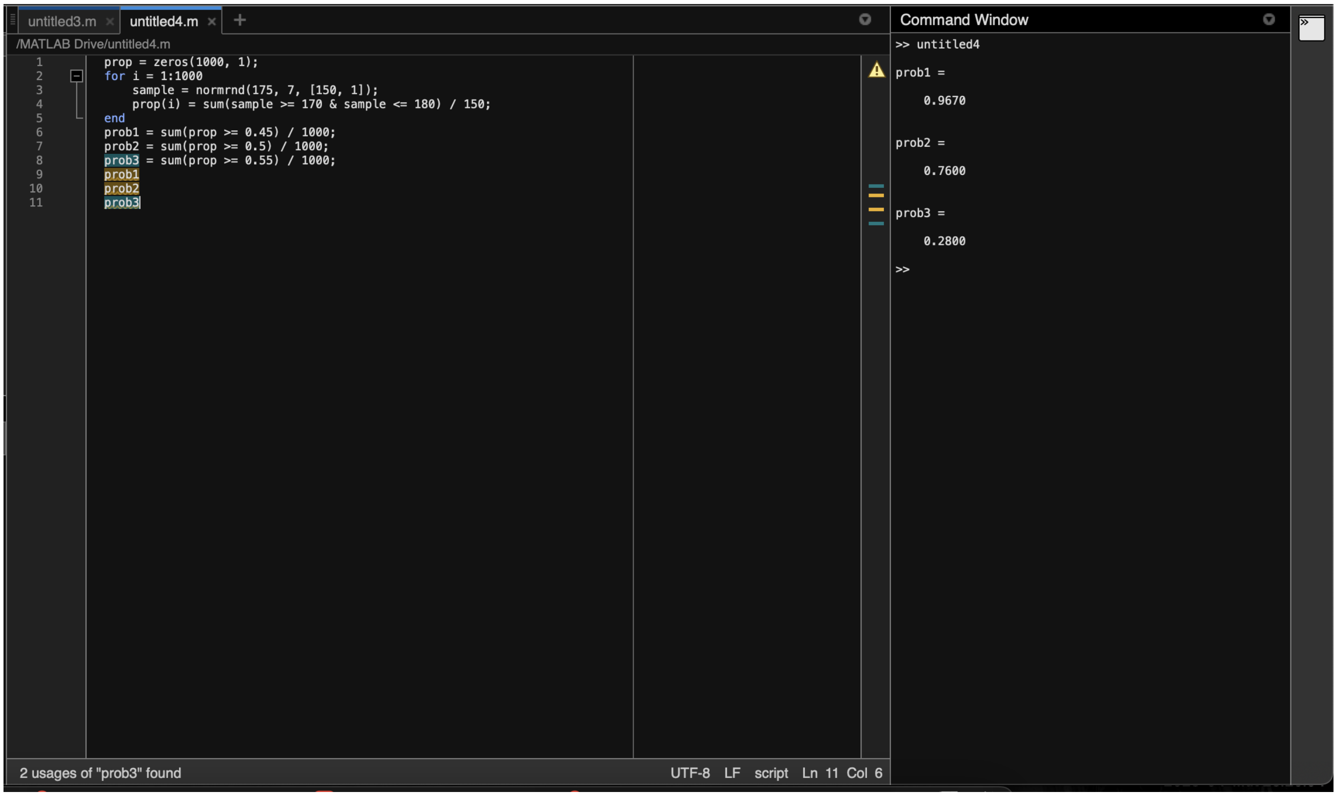


A PDF of normal distributions with varying standard deviations—6, 7, and 8—are displayed in the visualization, all of which are centered on a mean height of 175 cm.

code:

```
sigma_values = [6, 7, 8];
x = 150:0.1:200;
hold on;
for i = 1:length(sigma_values)
    sigma = sigma_values(i);
    y = normpdf(x,175,sigma);
    plot(x,y,'LineWidth',2,'DisplayName',['\sigma=' num2str(sigma)]);
end
hold off;
xlabel('height');
ylabel('density');
legend('Location', 'northwest');
```

c)



Despite the fact that probabilites rise 5% themselves at each step, their results fall in a more swift manner.

code:

```
prop = zeros(1000, 1);
for i = 1:1000
    sample = normrnd(175, 7, [150, 1]);
    prop(i) = sum(sample >= 170 & sample <= 180) / 150;
end
prob1 = sum(prop >= 0.45) / 1000;
prob2 = sum(prop >= 0.5) / 1000;
prob3 = sum(prop >= 0.55) / 1000;
prob1
prob2
prob3
```