

# Student Information

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## Answer 1

$$f(x) = \sum_{i=2}^{\infty} a_n * x^n = \sum_{i=2}^{\infty} (3 * a_{n-1} + 4 * a_{n-2}) x^n = f(x) = 3x * \sum_{i=2}^{\infty} a_{n-1} * x^{n-1} + 4x^2 * \sum_{i=2}^{\infty} a_{n-2} * x^{n-2} = 3x * (x + f(x)) + 4x^2 * (1 + x + f(x)) = f(x) * (4x^2 + 3x) + 7x^2 + 4x^3 = f(x) \rightarrow f(x) * (1 - 4x^2 - 3x) = 4x^3 + 7x^2 \rightarrow f(x) = \frac{4x^3 + 7x^2}{1 - 4x^2 - 3x}$$

$$f(x) = -x^2 * \left( \frac{a}{4x - 1} + \frac{b}{x + 1} \right) \rightarrow (a = 6.4, b = -0.6)$$

For  $\frac{1}{1+x}$ :

$$1, -1, 1 \dots (-1)^n : \left( \frac{1}{1+x} \right)$$

$$0, 0, 0.6, -0.6 \dots 0.6 * (-1)^n : \left( \frac{0.6x^2}{1+x} \right)$$

For  $\frac{1}{1-4x}$ :

$$1, 4, 16 \dots 4^n : \left( \frac{1}{1-4x} \right)$$

$$0, 0, 6.4 \dots 6.4 * 4^{n+2} : \left( \frac{6.4x^2}{1-4x} \right)$$

Summing up:

$$0, 0, 7, 25 \dots 6.4 * 4^{n+2} + 0.6 * (-1)^n$$

$$a_n = 6.4 * 4^{n+2} + 0.6 * (-1)^n$$

## Answer 2

a)

Consider as sum:  $\langle 2 + 0, 2 + 3, 2 + 9, 2 + 27, 2 + 81, 2 + 243 \dots \rangle$

$$\text{Multiply by 2: } \langle 2, 2, 2, 2, 2, 2 \dots \rangle : \left( \frac{2}{1-x} \right)$$

(i) Substitute  $x$  with  $3x$  (ii) Shift 1 unit to right (iii) Multiply by 3:  $\langle 1, 3, 9, 27, 81, 243 \dots \rangle : \left( \frac{3x}{1-3x} \right)$

$$\text{Sum them up: } \frac{2}{1-x} + \frac{3x}{1-3x}$$

b)

$$G(x) = \frac{a}{2x-1} + \frac{b}{x-1} \longrightarrow (a = -5, b = -2)$$

$$\left(\frac{2}{1-x}\right) : < 2, 2, 2, 2, 2, 2... >$$

$$\left(\frac{5}{1-2x}\right) : < 5, 10, 20, 40, 80, 160... >$$

$$G(x) : < 5 + 2, 10 + 2, 20 + 2, 40 + 2, 80 + 2, 160 + 2... >$$

$$G(x) : < 7, 12, 22, 42, 82, 162... >$$

## Answer 3

a)

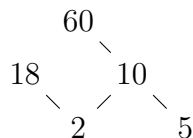
An equivalence relation must be transitive, meaning that if  $a$  is related to  $b$  and  $b$  is related to  $c$ , then  $a$  must also be related to  $c$ . However, the given relation is not transitive because there are pairs  $(a, b)$  and  $(b, c)$  that satisfy the relation, but  $(a, c)$  does not. For example,  $(5, 12)$  and  $(12, 16)$  both satisfy the relation, but  $(5, 16)$  does not. Therefore, this relation cannot be an equivalence relation.

b)

The given relation satisfies the reflexive property because it holds for all pairs  $(a, b)$ . It also satisfies the symmetric property because if  $(a, b)$  is related to  $(c, d)$ , then  $(c, d)$  is related to  $(a, b)$ . Finally, it satisfies the transitive property because if  $(a, b)$  is related to  $(c, d)$  and  $(c, d)$  is related to  $(e, f)$ , then  $(a, b)$  is related to  $(e, f)$ . Therefore, this relation is an equivalence relation. The equivalence class consists of all pairs  $(a, b)$  such that  $2a + b = 0$ . This is a line with slope  $-2$  that passes through the origin.

## Answer 4

a)



**b)**

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

**c)**

Symmetric closure:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

List of all pairs:

$$(10, 2), (10, 5), (18, 2), (60, 2), (60, 5), (60, 10)$$

**d)**

To establish a complete ordering of all pairs of numbers  $(a, b)$ , it is necessary that either  $a$  divides  $b$  or  $b$  divides  $a$ . Some pairs, like  $(2, 5)$  and  $(5, 18)$ , do not follow this rule. To order these pairs, we can delete two numbers from the sequence, such as 18 and 5, and add a new number, such as 30.