Student Information

Full Name : Unal Hasanli
Id Number : 2589877

Answer 1

a) 14
b) 14
c) 21
d) G does not have a complete graph with at least four veritces as a subgraph.
e) G is not bipartite because it contains an odd cycle (a, b, c, d, e, a).
f) $2^7 = 128$
g) The simple longest path in G is a $->$ b $->$ c $->$ d $->$ e $->$ a $->$ c, and its length is 6.
h) G has 1 connected component. All of the nodes in the graph are connected to one another, so there is only one connected component.
i) No. If a graph is connected and all vertices have even degree, it has an Euler circuit. Whereas, G has 4 vertices with odd degrees {a, b, c, e} which makes it impossible to have an Euler circuit.
j) No. If a graph is connected and has exactly 0 or 2 vertices with odd degree, it has an Euler path. Whereas, G has 4 vertices with odd degrees {a, b, c, e} which makes it impossible to have an Euler path.
k) a $->$ b $->$ c $->$ d $->$ e $->$ a.
l) a $->$ b $->$ c $->$ d $->$ e.

Answer 2

First, consider $G = (V, E)$ and $H = (W, F)$

The function f with $f(a) = a'$, $f(b) = b'$, $f(c) = c'$, $f(d) = d'$, and $f(e) = e'$, is a one-to-one correspondence between V and W. To see that this correspondence preserves adjacency, note that adjacent vertices in G are (a, b), (b, c), (c, d), (d, e), and (e, a), and each of the pairs $(f(a) = a', f(b) = b')$, $(f(b) = b', f(c) = c')$, $(f(c) = c', f(d) = d')$, $(f(d) = d', f(e) = e')$, and $(f(e) = e', f(a) = a')$ consists of two adjacent vertices in H.

Answer 3

1. Initialize 3 arrays:
unvisited: $\{s, u, v, w, x, y, z, t\}$
distances: $\{0, \infty, \infty, \infty, \infty, \infty, \infty, \infty\}$
previous: $\{s, null, null, null, null, null, null, null\}$

2.
current node: s (first node)
unvisited: $\{u, v, w, x, y, z, t\}$
distances: $\{0, 4, 5, 3, \infty, \infty, \infty, \infty\}(0 + 4 < \infty)(0 + 5 < \infty)(0 + 3 < \infty)$
previous: $\{s, s, s, s, null, null, null, null\}$

3.
current node: w (shortest distance, unvisited)
unvisited: $\{u, v, x, y, z, t\}$
distances: $\{0, 4, 5, 3, 11, \infty, 15, \infty\}(3 + 3 > 5)(3 + 8 < \infty)(3 + 12 < \infty)$
previous: $\{s, s, s, s, w, null, w, null\}$

4.
current node: u (shortest distance, unvisited)
unvisited: $\{v, x, y, z, t\}$
distances: $\{0, 4, 5, 3, 11, 15, 15, \infty\}(4 + 8 > 5)(4 + 11 < \infty)$
previous: $\{s, s, s, s, w, u, w, null\}$

5.
current node: v (unvisited)
unvisited: $\{x, y, z, t\}$
distances: $\{0, 4, 5, 3, 7, 11, 15, \infty\}(5 + 8 > 4)(5 + 3 > 3)(5 + 6 < 15)(5 + 2 < \infty)$
previous: $\{s, s, s, s, v, v, w, null\}$

6.
current node: x (shortest disance, unvisited)
unvisited: $\{y, z, t\}$
distances: $\{0, 4, 5, 3, 7, 8, 13, \infty\}(7 + 1 > 11)(7 + 8 > 3)(7 + 6 < 15)$
previous: $\{s, s, s, s, v, x, x, null\}$

7.
current node: y (shortest disance, unvisited)
unvisited: $\{z, t\}$
distances: $\{0, 4, 5, 3, 11, 8, 12, 17\}(8 + 11 > 4)(8 + 6 > 5)(8 + 4 < 13)(8 + 9 < \infty)$
previous: $\{s, s, s, s, v, x, y, y\}$

8.
current node: z (shortest disance, unvisited)
unvisited: $\{t\}$
distances: $\{0, 4, 5, 3, 11, 8, 12, 15\}(12 + 12 > 3)(12 + 6 > 7)(12 + 3 < 17)$
previous: $\{s, s, s, s, v, x, y, z\}$

9.
current node: t

Tracing back using the "previous" array, we can find that the shortest path from s to t is $\{s, v, x, y, z, t\}$ with a total distance of 15.

Answer 4

a)

1. [b,c]

2. [c,f], [d,k], [h,i]

3. [c,d], [a,b], [f,j]

4. [e,f], [g,j], [f,i]

b)

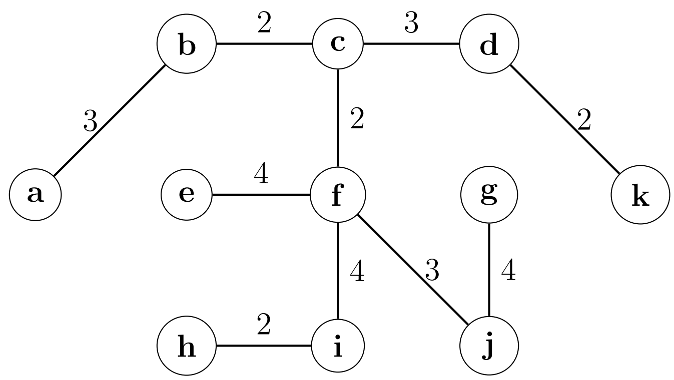

Figure 1: Graph G in Q4.

c) No. [f,g] could have been chosen instead of [g,j]