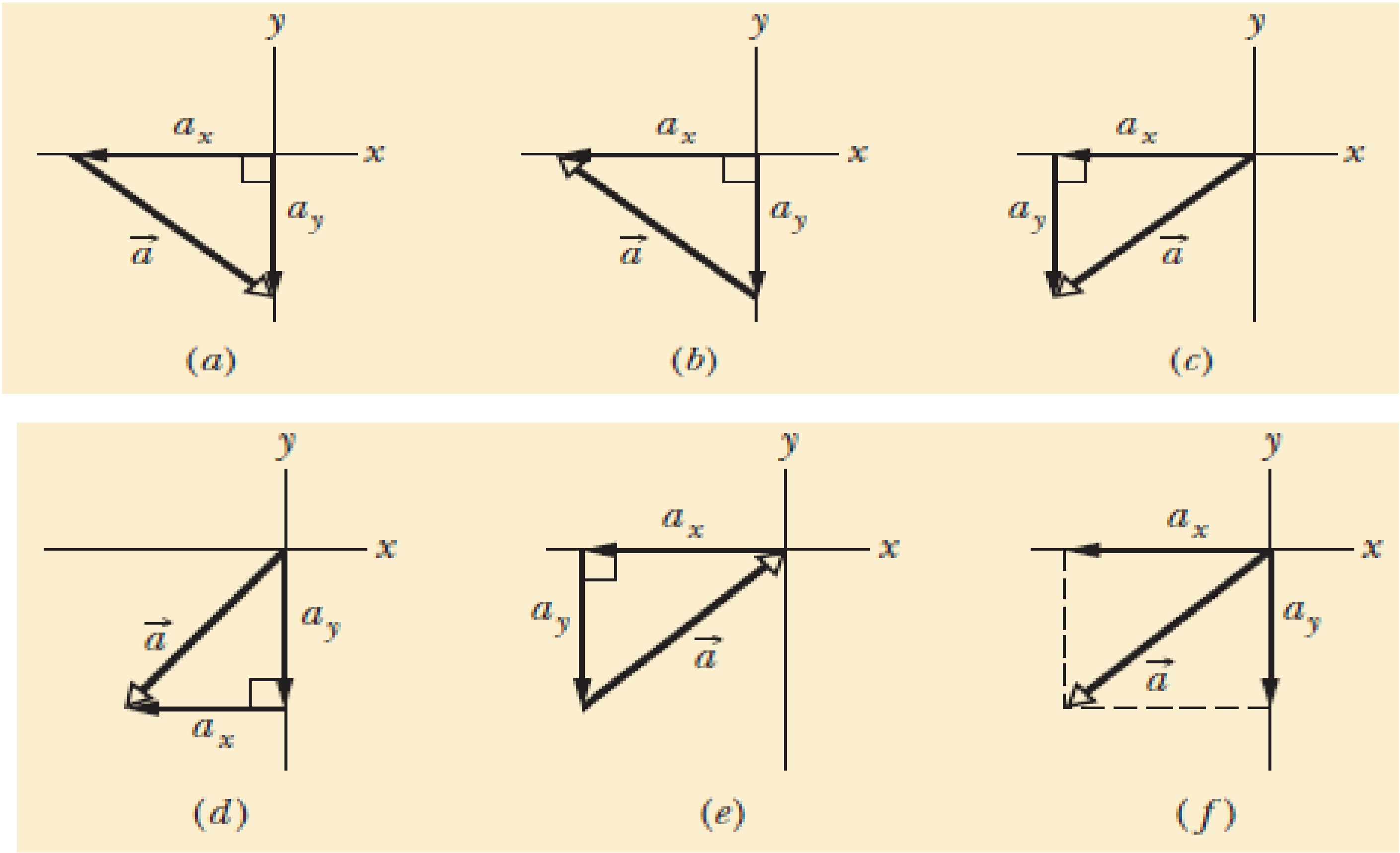
Vector Components

In the figure, which of the indicated methods for combining the x and y components of vector $\vec{a}$ are proper to determine that vector?



Sample Problem 3.02 Finding components, airplane flight

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?

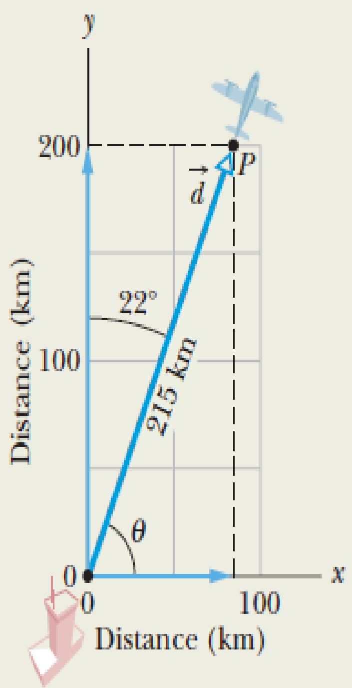


Figure 3-10 A plane takes off from an airport at the origin and is later sighted at *P*.

KEY IDEA

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

Calculations: We draw an *xy* coordinate system with the positive direction of *x* due east and that of *y* due north (Fig. 3-10). For convenience, the origin is placed at the airport. (We don't have to do this. We could shift and misalign the coordinate system but, given a choice, why make the problem more difficult?) The airplane's displacement $\vec{d}$ points from the origin to where the airplane is sighted.

To find the components of $\vec{d}$, we use Eq. 3-5 with $\theta = 68°\ (= 90° - 22°)$:

$$d_x = d\cos\theta = (215\text{ km})(\cos 68°) = 81\text{ km} \qquad \text{(Answer)}$$

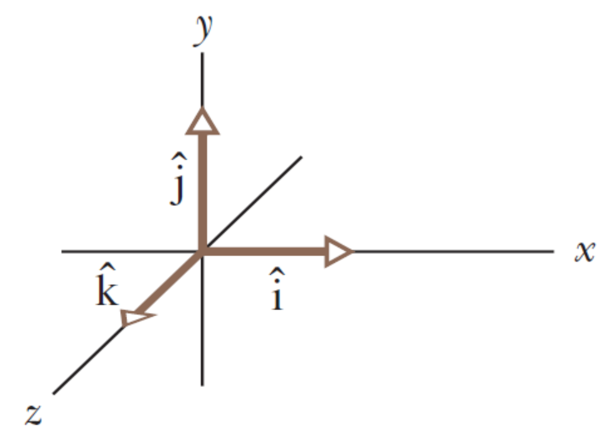
$$d_y = d\sin\theta = (215\text{ km})(\sin 68°) = 199\text{ km} \approx 2.0\times10^2\text{ km}. \qquad \text{(Answer)}$$

Thus, the airplane is 81 km east and 2.0×10^2 km north of the airport.

Unit Vectors

- A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction
- Its sole purpose is to point.
- The unit vectors in the positive directions of the x, y, and z axes are labeled $\hat{\imath}, \hat{\jmath}$ and $\widehat{k}$
- The arrangement of axes in Figure shown is said to be a **right-handed coordinate system.**

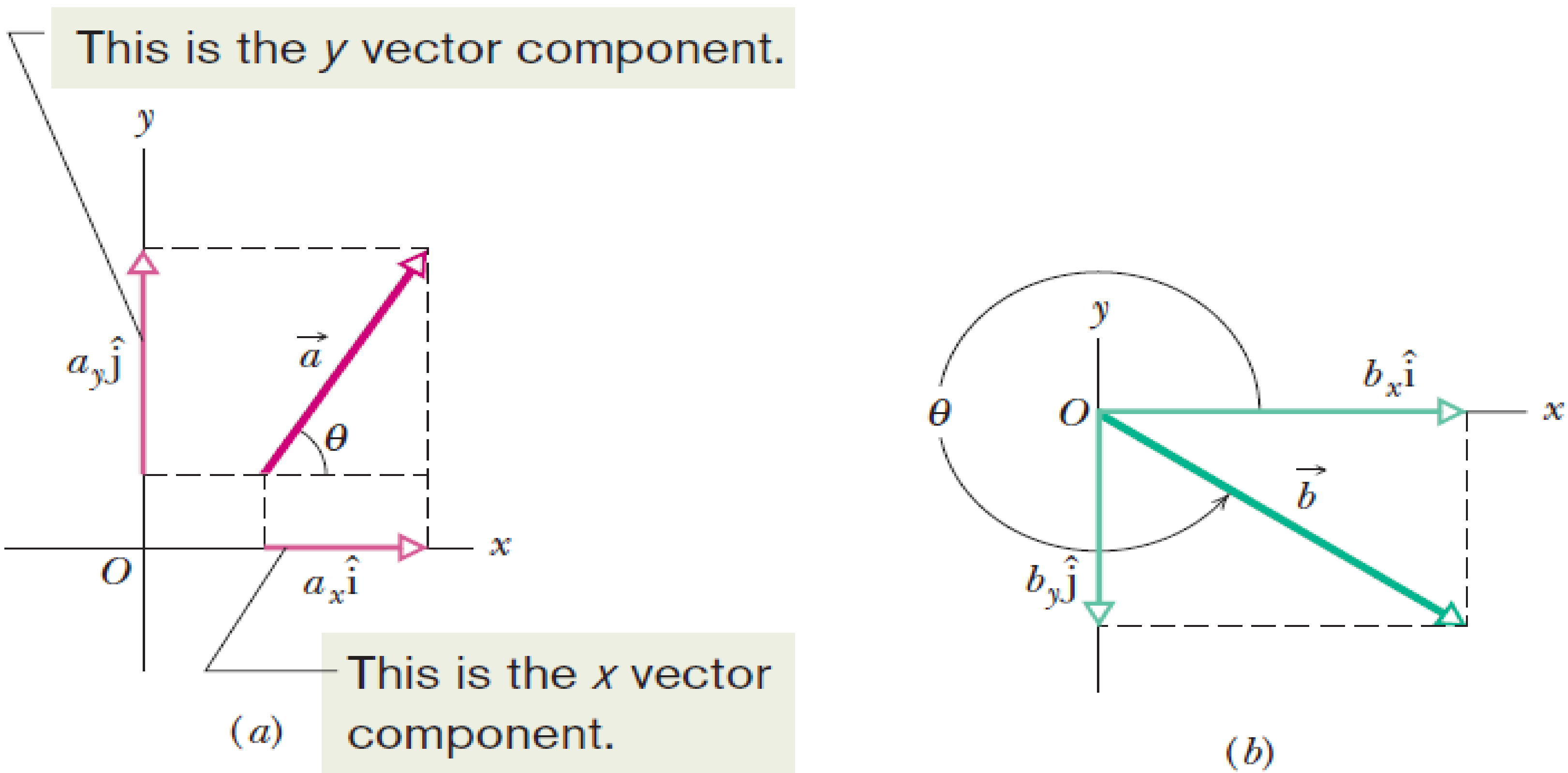
The unit vectors point along axes.



Components in Unit Vector Form

Unit vectors are very useful for expressing other vectors

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}}$$
$$\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}}.$$



Adding Vectors

We can add vectors

- Geometrically on a sketch, or
- Directly on a vector-capable calculator.
- A third way is to combine their components axis by axis.

Consider the statement,

$$\vec{r} = \vec{a} + \vec{b},$$

which says that the vector $\vec{r}$ is the same as the vector $(\vec{a} + \vec{b})$. Thus, each component of $\vec{r}$ must be the same as the corresponding component of $(\vec{a} + \vec{b})$:

$$r_x = a_x + b_x$$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z.$$

Adding Vectors(Steps)

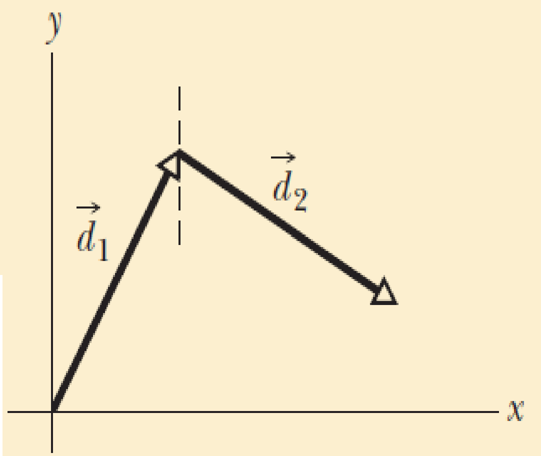
to add vectors $\vec{a}$ and $\vec{b}$, we must (1) resolve the vectors into their scalar components; (2) combine these scalar components, axis by axis, to get the components of the sum $\vec{r}$; and (3) combine the components of $\vec{r}$ to get $\vec{r}$ itself. We have a choice in step 3. We can express $\vec{r}$ in unit-vector notation or in magnitude-angle notation.

This procedure for adding vectors by components also applies to vector subtractions. Recall that a subtraction such as $\vec{d} = \vec{a} - \vec{b}$ can be rewritten as an addition $\vec{d} = \vec{a} + (-\vec{b})$. To subtract, we add $\vec{a}$ and $-\vec{b}$ by components, to get

$$d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$$

where $$\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}.$$

(a) In the figure here, what are the signs of the x components of $\vec{d}_1$ and $\vec{d}_2$? (b) What are the signs of the y components of $\vec{d}_1$ and $\vec{d}_2$? (c) What are the signs of the x and y components of $\vec{d}_1 + \vec{d}_2$?



Sample Problem 3.04 Adding vectors, unit-vector components

Figure 3-17*a* shows the following three vectors:

$$\vec{a} = (4.2\ \text{m})\hat{i} - (1.5\ \text{m})\hat{j},$$
$$\vec{b} = (-1.6\ \text{m})\hat{i} + (2.9\ \text{m})\hat{j},$$
and
$$\vec{c} = (-3.7\ \text{m})\hat{j}.$$

What is their vector sum $\vec{r}$ which is also shown?

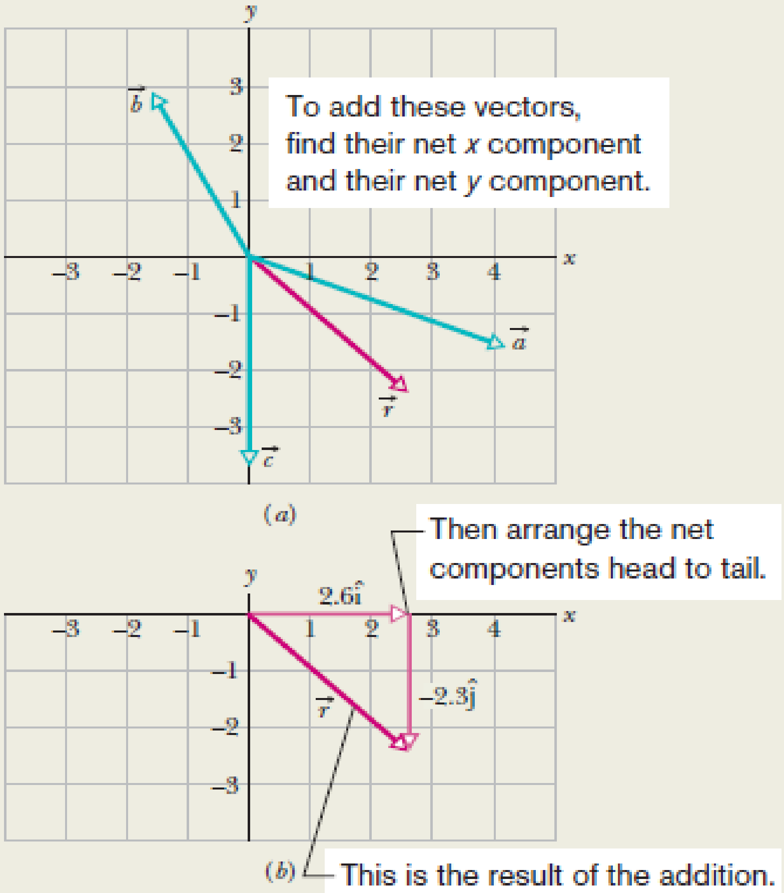


Figure 3-17 Vector $\vec{r}$ is the vector sum of the other three vectors.

KEY IDEA

We can add the three vectors by components, axis by axis, and then combine the components to write the vector sum $\vec{r}$.

Calculations: For the x axis, we add the x components of $\vec{a}$, $\vec{b}$, and $\vec{c}$, to get the x component of the vector sum $\vec{r}$:

$$r_x = a_x + b_x + c_x$$
$$= 4.2\ \text{m} - 1.6\ \text{m} + 0 = 2.6\ \text{m}.$$

Similarly, for the y axis,

$$r_y = a_y + b_y + c_y$$
$$= -1.5\ \text{m} + 2.9\ \text{m} - 3.7\ \text{m} = -2.3\ \text{m}.$$

We then combine these components of $\vec{r}$ to write the vector in unit-vector notation:

$$\vec{r} = (2.6\ \text{m})\hat{i} - (2.3\ \text{m})\hat{j}, \qquad \text{(Answer)}$$

where $(2.6\ \text{m})\hat{i}$ is the vector component of $\vec{r}$ along the x axis and $-(2.3\ \text{m})\hat{j}$ is that along the y axis. Figure 3-17*b* shows one way to arrange these vector components to form $\vec{r}$. (Can you sketch the other way?)

We can also answer the question by giving the magnitude and an angle for $\vec{r}$. From Eq. 3-6, the magnitude is

$$r = \sqrt{(2.6\ \text{m})^2 + (-2.3\ \text{m})^2} \approx 3.5\ \text{m} \qquad \text{(Answer)}$$

and the angle (measured from the $+x$ direction) is

$$\theta = \tan^{-1}\left(\frac{-2.3\ \text{m}}{2.6\ \text{m}}\right) = -41°, \qquad \text{(Answer)}$$

where the minus sign means clockwise.

Practice Problems

Q1. What are (a) the *x* component and (b) the *y* component of a vector in the *xy* plane if its direction is 250° counterclockwise from the positive direction of the *x* axis and its magnitude is 7.3 m?

Q2. A displacement vector in the *xy* plane is 15 m long and directed at angle $\theta = 30^\circ$ in Fig. 1.1.
Determine (a) the *x* component (b) the *y* component of the vector.

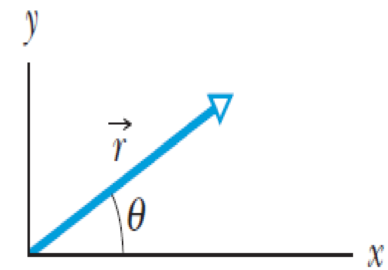


Fig.1.1

Q3 The x component of vector $\vec{A}$ is -25.0 m and the y component is $+40.0$ m. (a) What is the magnitude of $\vec{A}$? (b) What is the angle between the direction of $\vec{A}$ and the positive direction of x?

Q4. A ship sets out to sail to a point 120 km due north. An unexpected storm blows the ship to a point 100 km due east of its starting point. (a) How far and (b) in what direction must it now sail to reach its original destination?

Practice Problems

Q 5. Two vectors are given by

$$\vec{a} = (4.0\text{ m})\hat{\text{i}} - (3.0\text{ m})\hat{\text{j}} + (1.0\text{ m})\hat{\text{k}}$$

and $$\vec{b} = (-1.0\text{ m})\hat{\text{i}} + (1.0\text{ m})\hat{\text{j}} + (4.0\text{ m})\hat{\text{k}}.$$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

Q 6. Find the (a) x, (b) y, and (c) z components of the sum $\vec{r}$ of the displacements $\vec{c}$ and $\vec{d}$ whose components in meters are $c_x = 7.4, c_y = -3.8, c_z = -6.1; d_x = 4.4, d_y = -2.0, d_z = 3.3$.

Product of a Vector

Multiplying Vectors

- **Multiplying a Vector by a Scalar**

If we multiply a vector $\vec{a}$ by a scalar s, we get a new vector. Its magnitude is the product of the magnitude of $\vec{a}$ and the absolute value of s. Its direction is the direction of $\vec{a}$ if s is positive but the opposite direction if s is negative. To divide $\vec{a}$ by s, we multiply $\vec{a}$ by $1/s$.

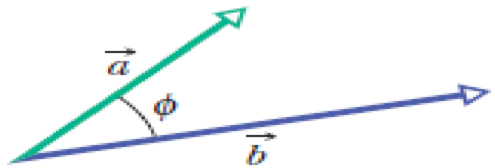
- **Multiplying a Vector by a Vector**

There are two ways to multiply a vector by a vector: one way produces a scalar (called the *scalar product*), and the other produces a new vector (called the *vector product*).

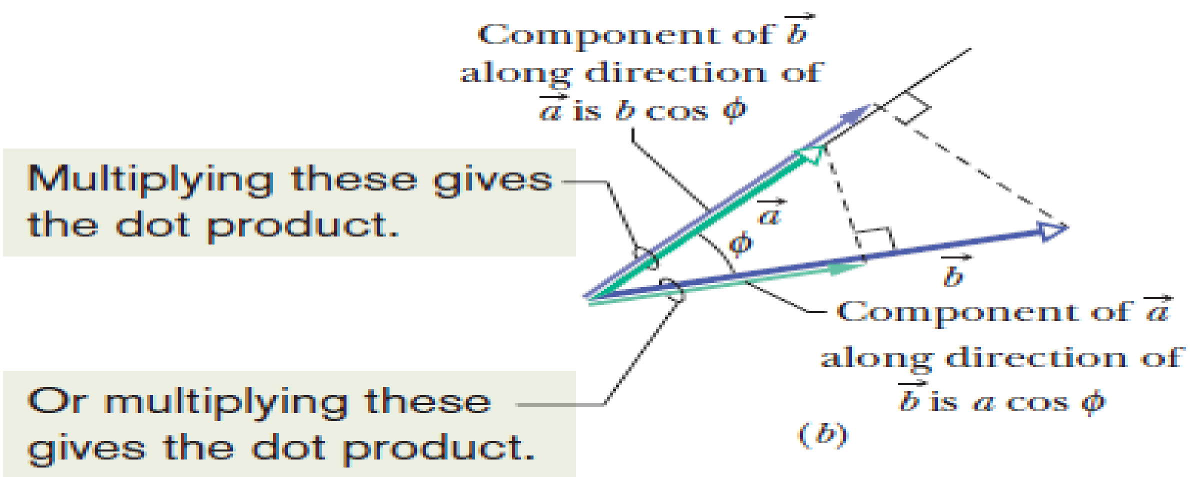
Fundamental concept of DOT PRODUCT

DOT PRODUCT

A dot product can be regarded as the product of two quantities: (1) the magnitude of one of the vectors and (2) the scalar component of the second vector along the direction of the first vector. For example, in Fig. below , $\vec{a}$ has a scalar component $a \cos \phi$ along the direction of $\vec{b}$; note that a perpendicular dropped from the head of $\vec{a}$ onto $\vec{b}$ determines that component. Similarly, $\vec{b}$ has a scalar component $b \cos \phi$ along the direction of $\vec{a}$.



(a)



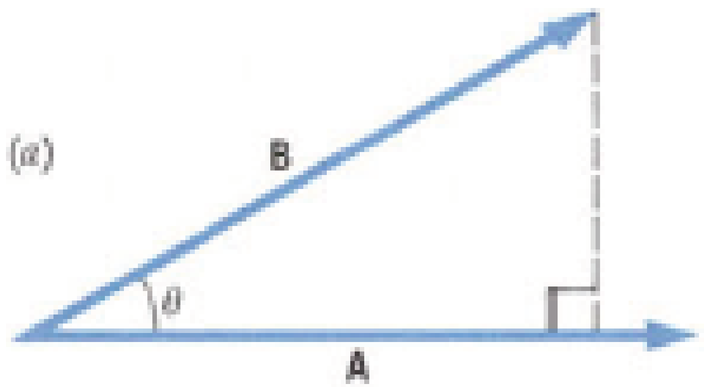
(b)

Vector Multiplication: Scalar Product or "Dot Product"

Very often in physics we have two vectors with an angle θ between them, and we wish to find the product of their components that lie in the direction of one or the other vector.

Consider Fig. 2-12. If we, for instance, select the **A** direction, then the component of vector **B** in that direction is given by dropping a perpendicular (Fig. 2-12a) and noting from the resulting right triangle that the component of **B** in the **A** direction is $B\cos\theta$ and the product of this component and vector **A** is

$$AB\cos\theta$$



Vector Multiplication: Scalar Product or "Dot Product"

If, instead, we had selected the **B** direction we could equally have dropped a perpendicular from vector A to the line of vector **B** (Fig. 2-12b) and obtained the identical result. Because there is no specified direction for the resulting product, we define such a product as a scalar. We use the shorthand notation of a dot $(\cdot)$ to represent this type of product, which is referred to as the *dot product*

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta \tag{2.1}$$

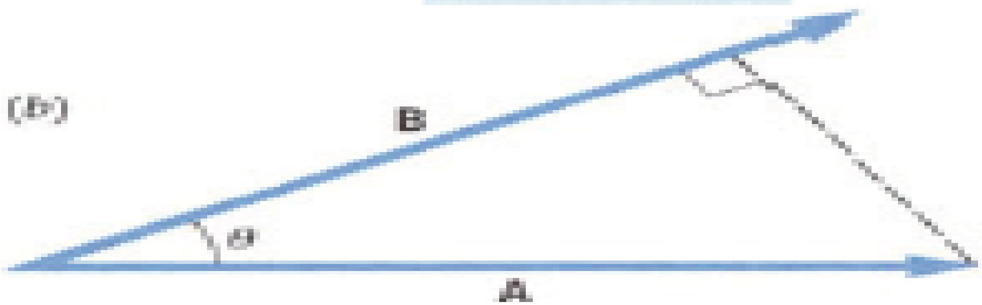


FIGURE 2-12 Geometric representation of two ways of forming a dot product of vectors **A** and **B**.

Vector Multiplication: Scalar Product or ”Dot Product”

Let us apply our definition of the dot product to the unit vectors **i**, **j**, and **k**.

$$\mathbf{i} \cdot \mathbf{j} = (1)(1)\cos 90^\circ = 0$$

$$\mathbf{i} \cdot \mathbf{k} = (1)(1)\cos 90^\circ = 0$$

$$\mathbf{j} \cdot \mathbf{k} = (1)(1)\cos 90^\circ = 0$$

$$\mathbf{i} \cdot \mathbf{i} = (1)(1)\cos 0^\circ = 1$$

$$\mathbf{j} \cdot \mathbf{j} = (1)(1)\cos 0^\circ = 1$$

$$\mathbf{k} \cdot \mathbf{k} = (1)(1)\cos 0^\circ = 1$$

We see that when a unit vector is dotted with a different unit vector the result is zero, whereas when a unit vector is dotted with itself the result is unity.

Vector Multiplication: Scalar Product or "Dot Product"

$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta_{AB}}$$

$$A = |\mathbf{A}| = \sqrt[+]{\mathbf{A}\cdot\mathbf{A}}$$

$$\theta_{AB} = \cos^{-1}\left[\frac{\mathbf{A}\cdot\mathbf{B}}{\sqrt[+]{\mathbf{A}\cdot\mathbf{A}}\;\sqrt[+]{\mathbf{B}\cdot\mathbf{B}}}\right]$$

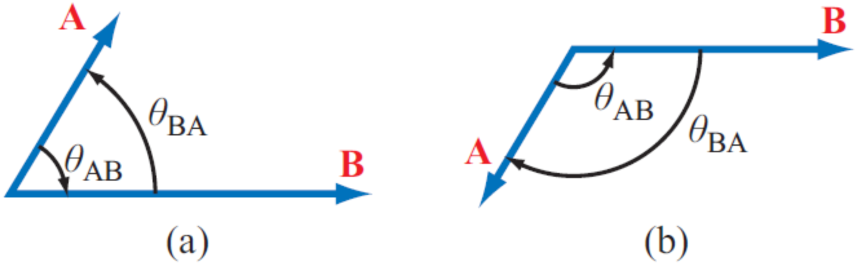


Figure 3-5: The angle θ_{AB} is the angle between $\mathbf{A}$ and $\mathbf{B}$, measured from $\mathbf{A}$ to $\mathbf{B}$ between vector tails. The dot product is positive if $0 \le \theta_{AB} < 90°$, as in (a), and it is negative if $90° < \theta_{AB} \le 180°$, as in (b).

$$\boxed{\begin{aligned}\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} &= 1,\\ \hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} &= 0.\end{aligned}}$$

If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$, then

$$\mathbf{A}\cdot\mathbf{B} = (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z)\cdot(\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z).$$

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A} \quad \text{(commutative property)},$$

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C} \quad \text{(distributive property)}$$

Hence:

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z.$$

Example to solve dot product

- $\langle 2, 4 \rangle \cdot \langle 3, -1 \rangle = 2(3) + 4(-1) = 2$

- $\langle -1, 7, 4 \rangle \cdot \langle 6, 2, -\tfrac{1}{2} \rangle = (-1)(6) + 7(2) + 4(-\tfrac{1}{2})$
  $= 6$

- $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \cdot (2\mathbf{j} - \mathbf{k}) = 1(0) + 2(2) + (-3)(-1)$
  $= 7$

Example 2

Solve:

- If the vectors **a** and **b** have lengths 4 and 6, and the angle between them is $\pi/3$, find $\mathbf{a} \cdot \mathbf{b}$
- **Ans : 12**

Example 3

Solve:

- Find the angle between the vectors

$$\mathbf{a} = \langle 2, 2, -1 \rangle \text{ and } \mathbf{b} = \langle 5, -3, 2 \rangle$$

Example 3: Solution

$$|\mathbf{a}| = \sqrt{2^2 + 2^2 + (-1)^2} = 3$$

and

$$|\mathbf{b}| = \sqrt{5^2 + (-3)^2 + 2^2} = \sqrt{38}$$

Also, $\mathbf{a} \cdot \mathbf{b} = 2(5) + 2(-3) + (-1)(2) = 2$

Example 3 Solution:Cont'd

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{2}{3\sqrt{38}}$$

So, the angle between **a** and **b** is:

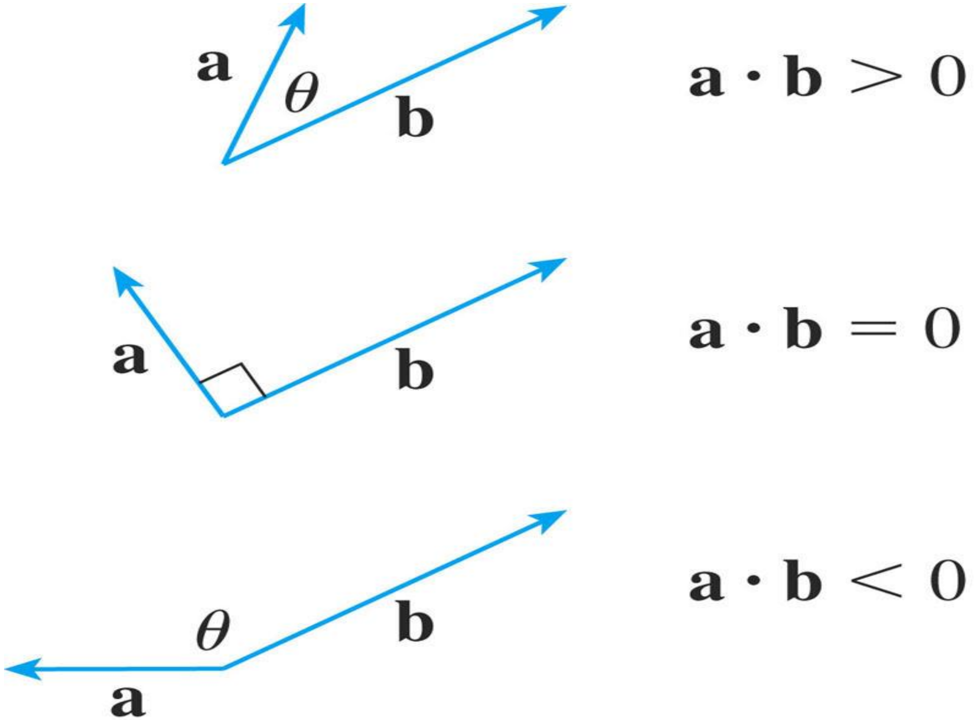
$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{38}}\right) \approx 1.46 \quad (\text{or } 84^\circ)$$

Orthogonal Vectors

- Two nonzero vectors **a** and **b** are called perpendicular or orthogonal if the angle between them is $\theta = \pi/2$.
- $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\pi/2) = 0$
- Conversely, if $\mathbf{a} \cdot \mathbf{b} = 0$, then $\cos \theta = 0$; so, $\theta = \pi/2$.
- Two vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$
- The zero vector **0** is considered to be perpendicular to all vectors.

THE DOT PRODUCT

- The dot product **a** · **b** is:
  - Positive, if **a** and **b** point in the same general direction
  - Zero, if they are perpendicular
  - Negative, if they point in generally opposite directions



Properties of Dot Product

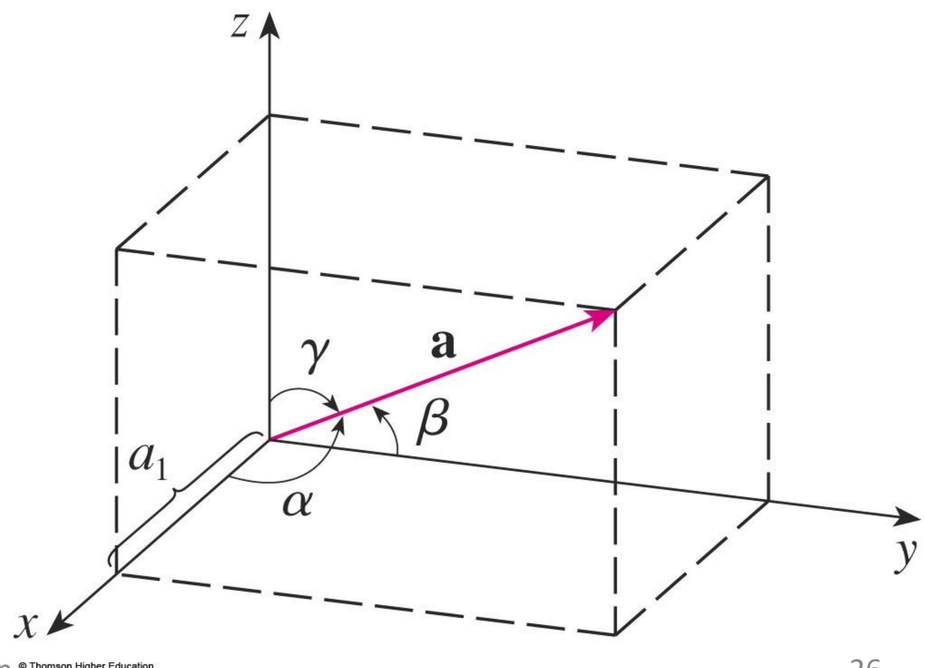
The dot product obeys many of the laws that hold for ordinary products of real numbers. These are stated as follows

If a, b, and c are vectors in V_3 and c is a scalar, then

1. $\mathbf{a}\cdot\mathbf{a}=|\mathbf{a}|^2$
2. $\mathbf{a}\cdot\mathbf{b}=\mathbf{b}\cdot\mathbf{a}$
3. $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})=\mathbf{a}\cdot\mathbf{b}+\mathbf{a}\cdot\mathbf{c}$
4. $(c\mathbf{a})\cdot\mathbf{b}=c(\mathbf{a}\cdot\mathbf{b})=\mathbf{a}\cdot(c\mathbf{b})$
5. $0\cdot\mathbf{a}=0$

Direction Angles

- The direction angles of a nonzero vector **a** are the angles α, β, and γ (in the interval $[0, \pi]$) that **a** makes with the positive x-, y-, and z-axes.



Direction Cosine

- The direction cosine of a vector are the cosines of the angles between the vector and the three co ordinate axis.
- The cosines of these direction angles—cos α, cos β, and cos γ

As $$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

b replaced by **i**, we obtain:

$$\cos\alpha = \frac{\mathbf{a}\cdot\mathbf{i}}{|\mathbf{a}||\mathbf{i}|} = \frac{a_1}{|\mathbf{a}|}$$

Similarly we also will have ,

$$\cos\beta = \frac{a_2}{|\mathbf{a}|} \qquad \cos\gamma = \frac{a_3}{|\mathbf{a}|}$$

Fundamental concept of CROSS PRODUCT

CROSS PRODUCT

The **vector product** of $\vec{a}$ and $\vec{b}$, written $\vec{a} \times \vec{b}$, produces a third vector $\vec{c}$ whose magnitude is

$$c = ab \sin \phi,$$

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ and $\vec{b}$. shows how to determine the direction of $\vec{c} = \vec{a} \times \vec{b}$ with what is known as a **right-hand rule.**

CROSS PRODUCT

In unit-vector notation, we write

$$\vec{a} \times \vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}}),$$

$$a_x\hat{\mathrm{i}} \times b_x\hat{\mathrm{i}} = a_x b_x(\hat{\mathrm{i}} \times \hat{\mathrm{i}}) = 0,$$

$$a_x\hat{\mathrm{i}} \times b_y\hat{\mathrm{j}} = a_x b_y(\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = a_x b_y\hat{\mathrm{k}}.$$

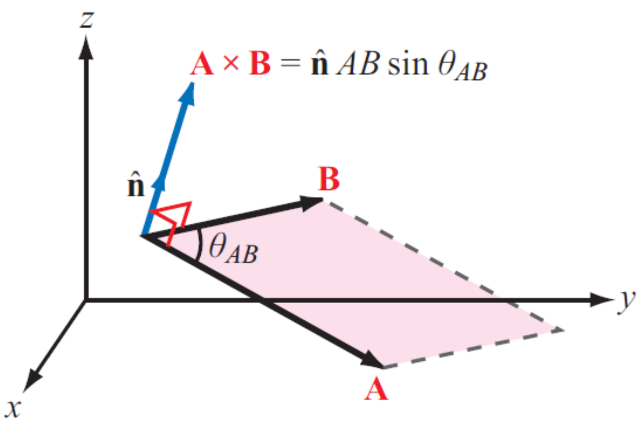
Continuing to expand you can show that

$$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}} + (a_x b_y - b_x a_y)\hat{\mathrm{k}}.$$

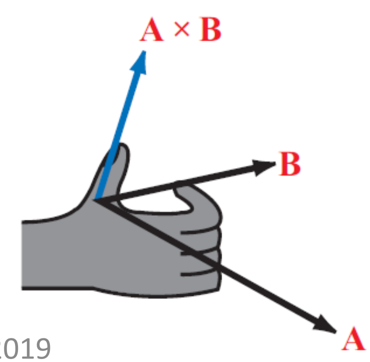
A determinant or a vector-capable calculator can also be used to calculate vector product.

Vector Multiplication: Vector Product or "Cross Product"

$$\boxed{\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}}\, AB \sin\theta_{AB}}$$



(a) Cross product



(b) Right-hand rule

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \qquad \text{(anticommutative)}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} \qquad \text{(distributive)}$$

$$\mathbf{A} \times \mathbf{A} = 0$$

$$\boxed{\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}, \qquad \hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}, \qquad \hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}. \qquad (3.25)}$$

Note the cyclic order $(xyzxyz\ldots)$. Also,

$$\boxed{\hat{\mathbf{x}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0. \qquad (3.26)}$$

If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$,

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

DOT & CROSS PRODUCTS

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if the magnitude of the vector product $\vec{C} \times \vec{D}$ is (a) zero and (b) 12 units?

Vectors $\vec{C}$ and $\vec{D}$ have magnitudes of 3 units and 4 units, respectively. What is the angle between the directions of $\vec{C}$ and $\vec{D}$ if $\vec{C} \cdot \vec{D}$ equals (a) zero, (b) 12 units, and (c) -12 units?

Practice Problems

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

Q2. Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a} \cdot (\vec{b} \times \vec{c})$, (b) $\vec{a} \cdot (\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

Q3. In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 2$,

$\vec{v} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k}$ and $\vec{F} = 4.0\hat{i} - 20\hat{j} + 12\hat{k}$.

What then is $\vec{B}$ in unit-vector notation if $B_x = B_y$?

Table 3-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\quad |\mathbf{A}| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\quad \overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$