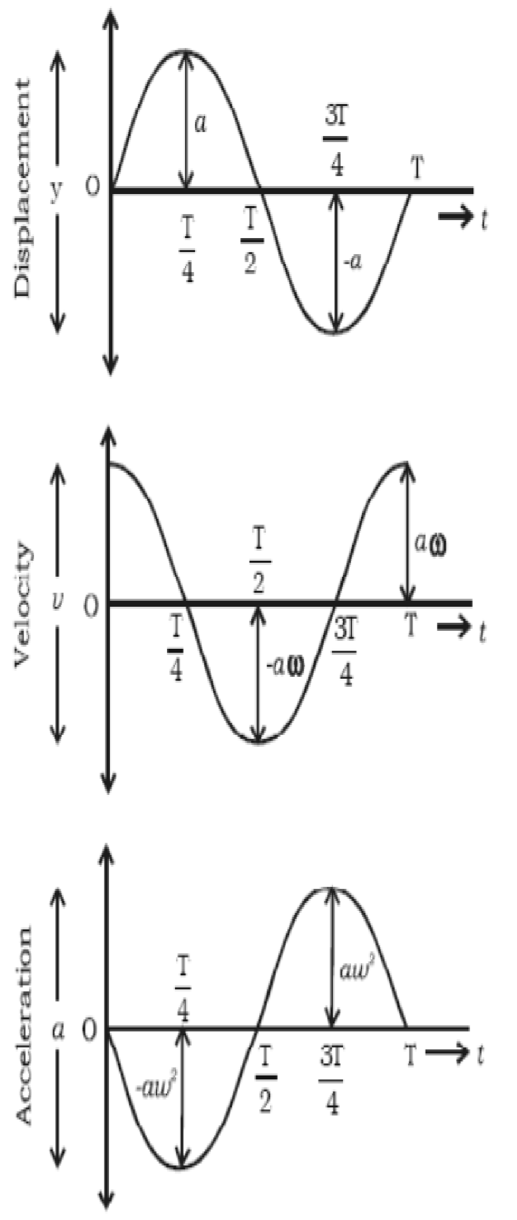
Chapter: Oscillations & Wave Motion (EE117)

Equations:

When we take displacement along y-axis



$y = A \sin(kx - \omega t)$,
(displacement at time t)

$v_y = -A\,\omega \cos(kx - \omega t)$,
(Transverse Velocity at time t)

Also, $v_y = \omega\sqrt{A^2 - y^2}$ (when time is not given)

$a_y = -A\,\omega^2 \sin(kx - \omega t)$,
(Transverse acceleration at time t)

Max Transverse Speed & Acceleration:

The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions

$$v_{y,\,\max} = \omega A$$

$$a_{y,\,\max} = \omega^2 A$$

The transverse speed reaches its maximum value ($A\,\omega$) when $y = 0$, whereas the magnitude of the transverse acceleration reaches its maximum value ($A\,\omega^2$) when $y = \pm A$.
(here in figure amplitude is taken as "a")

Energy of a particle doing oscillations:

$KE = \frac{1}{2} m A^2 \omega^2 \sin(kx - \omega t)$,

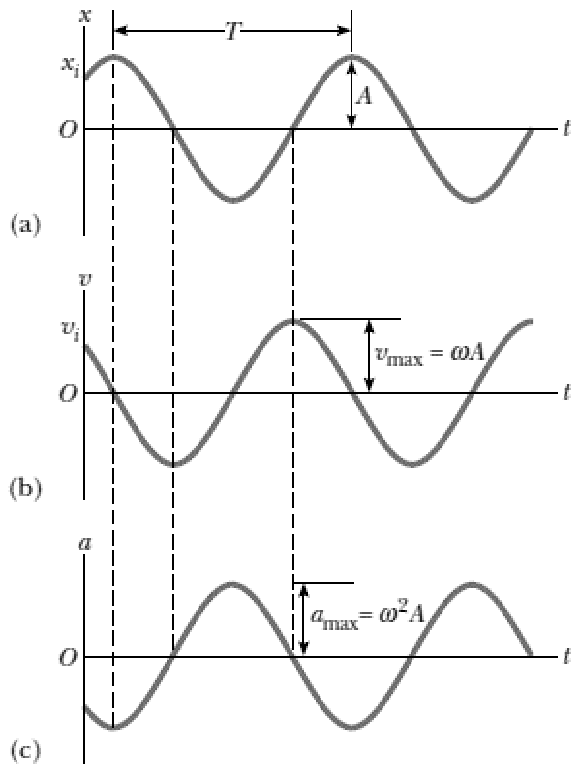
(KE at time t)

$PE = \frac{1}{2} m A^2 \omega^2 \cos(kx - \omega t)$

(PE at time t)

Designed by Sonia Nasir
Lecturer NU-FAST

When the particle doing simple harmonic motion along x-axis (from Extreme position)



$x = A \cos(\omega t + \emptyset)$

(in slides amplitude is represented by x_m)

(displacement at time t , $\emptyset$ *is the initial phase*)

$v_x = -A\,\omega \sin(\omega t + \emptyset)$,
(velocity at time t)

$a_x = -A\,\omega^2 \cos(\omega t + \emptyset)$,
(acceleration at time t)

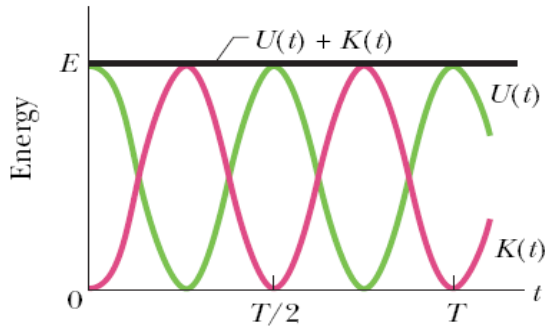
Maximum acceleration and velocity of the

Particle

we see that, because the sine and cosine functions oscillate between ±1, the extreme values of the velocity *v* are & $\pm A\,\omega$. Likewise, the extreme values of the acceleration *a* are $\pm A\,\omega^2$the *maximum* values of the magnitudes of the velocity and acceleration are,

$$v_{max,x} = A\,\omega$$
$$a_{max,x} = A\,\omega^2$$

Energy of a particle doing oscillations:

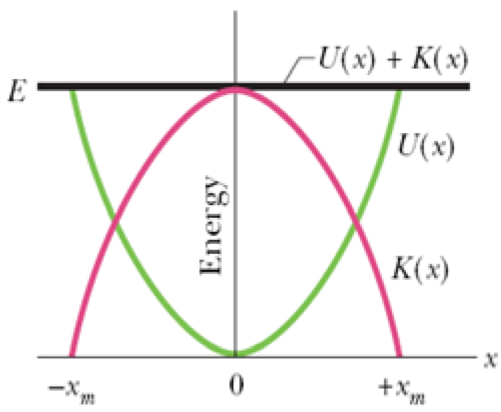


$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

Because $\omega^2 = k/m$,

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

mechanical energy of the simple harmonic oscillator



$$E = K + U = \tfrac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

$$E = \tfrac{1}{2}kA^2$$

$$E = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}m\omega^2 A^2 = \tfrac{1}{2}m\frac{k}{m}A^2 = \tfrac{1}{2}kA^2 \qquad (\text{at } x = 0)$$

Designed by Sonia Nasir
Lecturer NU-FAST