homework5

December 13, 2023

1 Homework 5 (100 points)

This homework will focus on Neural Networks and visualization.

a) Write a function that takes a keras network and outputs an image (png format) of the network. (10points)

You can assume the model is sequential and only uses dense layers. The input and output neurons must be blue circles. The hidden neurons must be green circles. The edges must be directed red arrows.

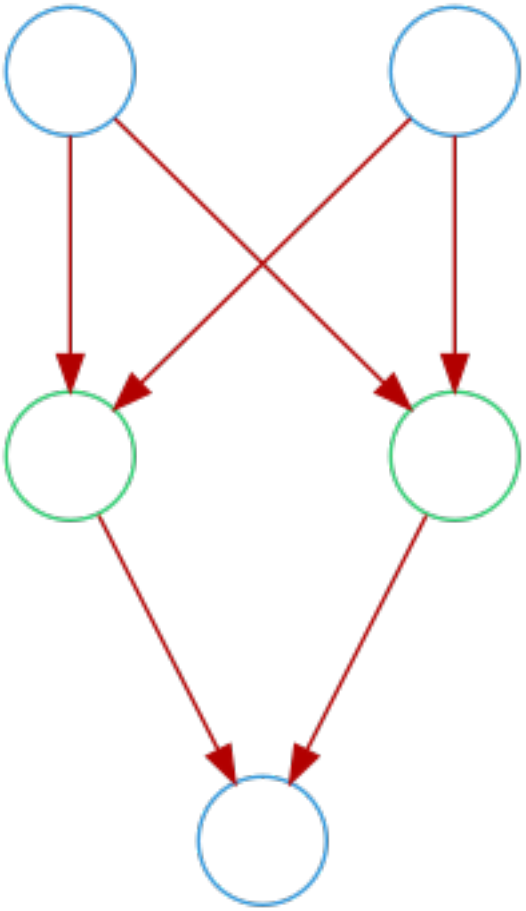
For example, the output image for

```
model = keras.models.Sequential()
model.add(layers.Dense(2, input_dim=2))
model.add(layers.Dense(1))
model.compile(loss="binary_crossentropy")
```

should look exactly like this:

[1]:
```
from IPython.display import Image
Image(filename="example.png")
```

[1]:



Hint: use the networkx library (specifically the to_agraph method)

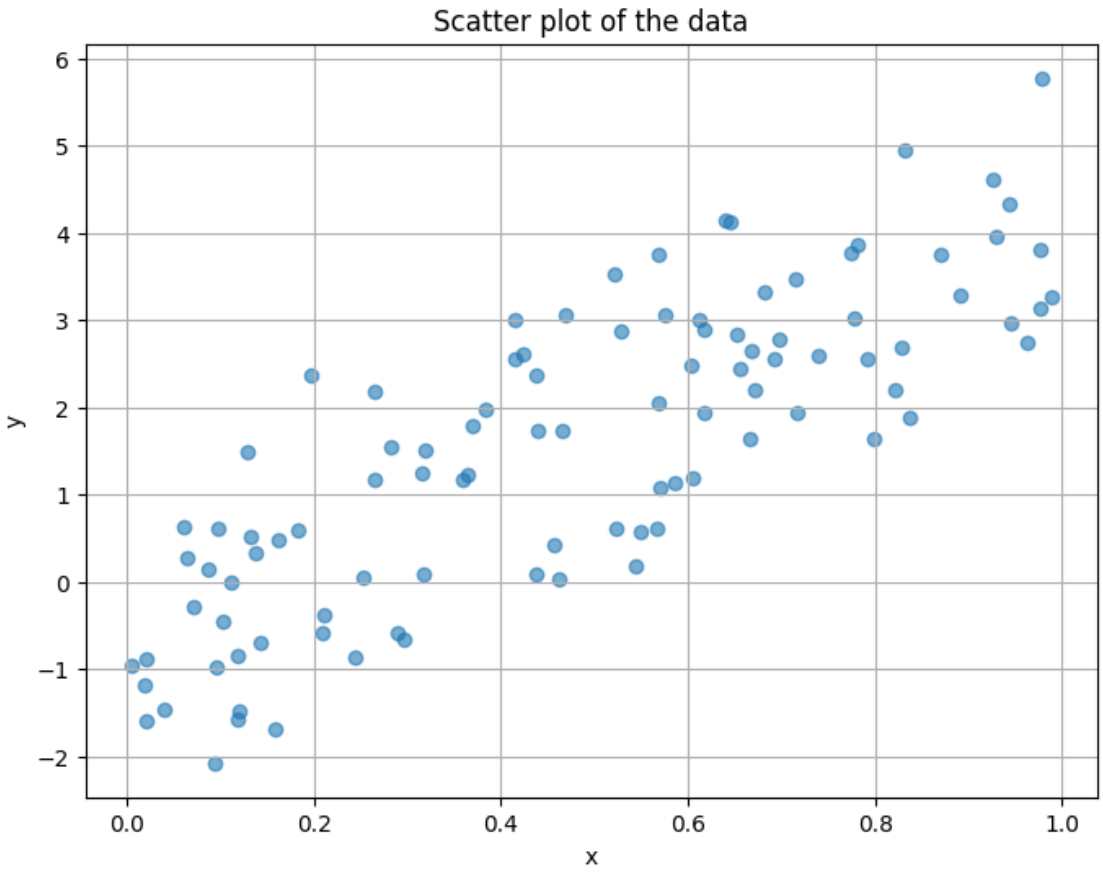
[15]:
```
import numpy as np
import networkx as nx
from PIL import Image
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation, PillowWriter
from networkx.drawing.nx_agraph import to_agraph

from tensorflow.keras import layers, models
from tensorflow.keras.utils import plot_model
from networkx.drawing.nx_agraph import to_agraph

from IPython.display import Image as img_display
```

[22]:
```
import matplotlib.pyplot as plt
from keras.models import Sequential
from keras.layers import Dense
```

```
def plot_model_architecture(model, to_file='model_architecture.png'):
    # Determine the number of layers
    n_layers = len(model.layers) + 1  # Including the input layer
    max_neurons = max([layer.output_shape[1] for layer in model.layers] +␣
 ↪[model.layers[0].input_shape[1]])

    fig, ax = plt.subplots()

    # Plotting the neurons
    for i, layer in enumerate([model.layers[0]] + model.layers):
        n_neurons = layer.input_shape[1] if i == 0 else layer.output_shape[1]
        y_offset = (max_neurons - n_neurons) / 2
        for j in range(n_neurons):
            color = 'blue' if i == 0 or i == (n_layers - 1) else 'green'
            circle = plt.Circle((i * 2, j + y_offset), 0.5, color=color,␣
 ↪zorder=3)
            ax.add_artist(circle)

    # Plotting the connections
    for i in range(n_layers - 1):
        n_neurons = model.layers[i].output_shape[1]
        n_neurons_next = model.layers[i].input_shape[1] if i == 0 else model.
 ↪layers[i-1].output_shape[1]
        y_offset = (max_neurons - n_neurons) / 2
        y_offset_next = (max_neurons - n_neurons_next) / 2
        for j in range(n_neurons):
            for k in range(n_neurons_next):
                x = np.array([i * 2 + 0.5, (i + 1) * 2 - 0.5])
                y = np.array([j + y_offset, k + y_offset_next])
                line = plt.Line2D(x, y, lw=0.5, color='red', zorder=2)
                ax.add_line(line)

    # Setting the plot limits
    ax.set_xlim(-1, n_layers * 2)
    ax.set_ylim(-1, max_neurons)
    ax.axis('off')

    # Saving the figure
    plt.savefig(to_file)
    plt.close()

model = Sequential()
model.add(Dense(2, input_dim=2))
model.add(Dense(1))
model.compile(loss="binary_crossentropy")

plot_model_architecture(model)
```

b) Generate 100 datapoints of the form y = 5x - 1 + e where e ~ N(0, 1) and plot the data in a scatter plot. Create a Neural Network with no hidden layers (just input to ouput each with just one neuron), using the `mean_squared_error` loss and no activation function. Create an image of this model using a) then train this model on the dataset produced such that it learns a good fit to the points. Plot that fitted line. (10points)

[6]:
```
# Generate the Data
np.random.seed(0)  # For reproducibility
x_data = np.random.rand(100, 1)  # 100 data points in the shape (100, 1)
e_data = np.random.randn(100, 1)  # Noise ~ N(0, 1)
y_data = 5 * x_data - 1 + e_data  # y = 5x - 1 + e

# Create the Scatter Plot
plt.figure(figsize=(8, 6))
plt.scatter(x_data, y_data, alpha=0.6)
plt.title('Scatter plot of the data')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True)
plt.show()

# Create the Neural Network
model_b = models.Sequential()
model_b.add(layers.Dense(1, input_dim=1, use_bias=True))


plot_keras_model(model_b, "model_b.png")

# Train the Model
model_b.compile(optimizer='sgd', loss='mean_squared_error')
model_b.fit(x_data, y_data, epochs=100)

# Plot the Fitted Line
x_values = np.linspace(0, 1, 100).reshape(-1, 1)  # Reshape for Keras model␣
 ↪input
y_pred = model_b.predict(x_values)

plt.figure(figsize=(8, 6))
plt.scatter(x_data, y_data, alpha=0.6, label='Data')
plt.plot(x_values, y_pred, color='red', label='Fitted Line')
plt.title('Data Points and Fitted Line')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.grid(True)
plt.show()
```



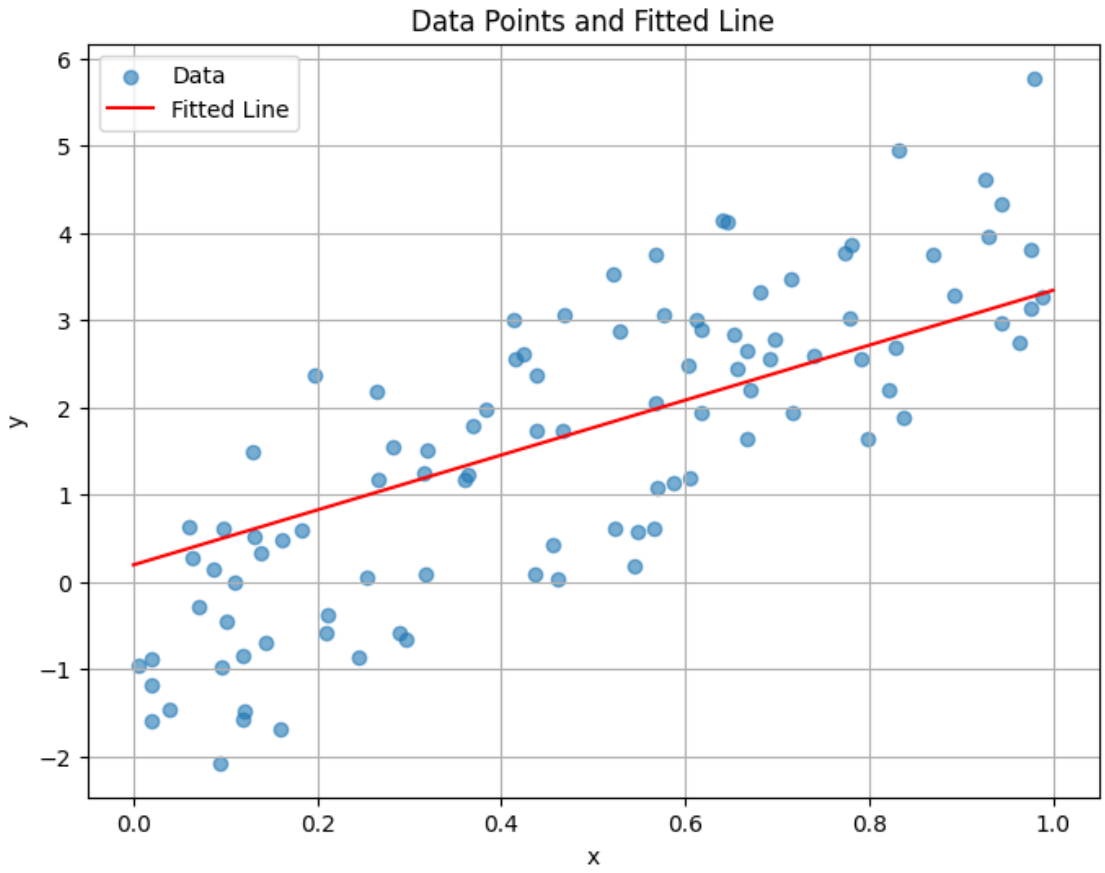

```
Epoch 1/100
4/4 [==============================] - 0s 2ms/step - loss: 2.7334
Epoch 2/100
4/4 [==============================] - 0s 1ms/step - loss: 2.5529
Epoch 3/100
4/4 [==============================] - 0s 1ms/step - loss: 2.3974
Epoch 4/100
4/4 [==============================] - 0s 979us/step - loss: 2.2831
Epoch 5/100
4/4 [==============================] - 0s 1ms/step - loss: 2.2361
Epoch 6/100
4/4 [==============================] - 0s 1ms/step - loss: 2.1401
Epoch 7/100
4/4 [==============================] - 0s 1ms/step - loss: 2.0687
Epoch 8/100
4/4 [==============================] - 0s 775us/step - loss: 1.9584
Epoch 9/100
4/4 [==============================] - 0s 724us/step - loss: 1.9085
Epoch 10/100
4/4 [==============================] - 0s 1ms/step - loss: 1.8635
```

```
Epoch 11/100
4/4 [==============================] - 0s 735us/step - loss: 1.8329
Epoch 12/100
4/4 [==============================] - 0s 713us/step - loss: 1.8143
Epoch 13/100
4/4 [==============================] - 0s 672us/step - loss: 1.7801
Epoch 14/100
4/4 [==============================] - 0s 780us/step - loss: 1.7619
Epoch 15/100
4/4 [==============================] - 0s 912us/step - loss: 1.7419
Epoch 16/100
4/4 [==============================] - 0s 979us/step - loss: 1.7222
Epoch 17/100
4/4 [==============================] - 0s 1ms/step - loss: 1.7046
Epoch 18/100
4/4 [==============================] - 0s 1ms/step - loss: 1.6904
Epoch 19/100
4/4 [==============================] - 0s 880us/step - loss: 1.6750
Epoch 20/100
4/4 [==============================] - 0s 913us/step - loss: 1.6681
Epoch 21/100
4/4 [==============================] - 0s 2ms/step - loss: 1.6633
Epoch 22/100
4/4 [==============================] - 0s 4ms/step - loss: 1.6534
Epoch 23/100
4/4 [==============================] - 0s 1ms/step - loss: 1.6428
Epoch 24/100
4/4 [==============================] - 0s 1ms/step - loss: 1.6362
Epoch 25/100
4/4 [==============================] - 0s 846us/step - loss: 1.6252
Epoch 26/100
4/4 [==============================] - 0s 652us/step - loss: 1.6131
Epoch 27/100
4/4 [==============================] - 0s 611us/step - loss: 1.6077
Epoch 28/100
4/4 [==============================] - 0s 1ms/step - loss: 1.6046
Epoch 29/100
4/4 [==============================] - 0s 644us/step - loss: 1.5986
Epoch 30/100
4/4 [==============================] - 0s 808us/step - loss: 1.5870
Epoch 31/100
4/4 [==============================] - 0s 621us/step - loss: 1.5809
Epoch 32/100
4/4 [==============================] - 0s 1ms/step - loss: 1.5749
Epoch 33/100
4/4 [==============================] - 0s 2ms/step - loss: 1.5701
Epoch 34/100
4/4 [==============================] - 0s 707us/step - loss: 1.5621
```

```
Epoch 35/100
4/4 [==============================] - 0s 670us/step - loss: 1.5561
Epoch 36/100
4/4 [==============================] - 0s 765us/step - loss: 1.5479
Epoch 37/100
4/4 [==============================] - 0s 622us/step - loss: 1.5430
Epoch 38/100
4/4 [==============================] - 0s 831us/step - loss: 1.5372
Epoch 39/100
4/4 [==============================] - 0s 613us/step - loss: 1.5308
Epoch 40/100
4/4 [==============================] - 0s 1ms/step - loss: 1.5247
Epoch 41/100
4/4 [==============================] - 0s 1ms/step - loss: 1.5203
Epoch 42/100
4/4 [==============================] - 0s 743us/step - loss: 1.5138
Epoch 43/100
4/4 [==============================] - 0s 1ms/step - loss: 1.5067
Epoch 44/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4987
Epoch 45/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4947
Epoch 46/100
4/4 [==============================] - 0s 2ms/step - loss: 1.4894
Epoch 47/100
4/4 [==============================] - 0s 731us/step - loss: 1.4858
Epoch 48/100
4/4 [==============================] - 0s 973us/step - loss: 1.4806
Epoch 49/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4763
Epoch 50/100
4/4 [==============================] - 0s 679us/step - loss: 1.4722
Epoch 51/100
4/4 [==============================] - 0s 543us/step - loss: 1.4648
Epoch 52/100
4/4 [==============================] - 0s 900us/step - loss: 1.4594
Epoch 53/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4540
Epoch 54/100
4/4 [==============================] - 0s 702us/step - loss: 1.4494
Epoch 55/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4454
Epoch 56/100
4/4 [==============================] - 0s 915us/step - loss: 1.4395
Epoch 57/100
4/4 [==============================] - 0s 832us/step - loss: 1.4354
Epoch 58/100
4/4 [==============================] - 0s 648us/step - loss: 1.4311
```

```
Epoch 59/100
4/4 [==============================] - 0s 1ms/step - loss: 1.4274
Epoch 60/100
4/4 [==============================] - 0s 793us/step - loss: 1.4228
Epoch 61/100
4/4 [==============================] - 0s 666us/step - loss: 1.4188
Epoch 62/100
4/4 [==============================] - 0s 595us/step - loss: 1.4151
Epoch 63/100
4/4 [==============================] - 0s 2ms/step - loss: 1.4103
Epoch 64/100
4/4 [==============================] - 0s 689us/step - loss: 1.4060
Epoch 65/100
4/4 [==============================] - 0s 882us/step - loss: 1.3999
Epoch 66/100
4/4 [==============================] - 0s 666us/step - loss: 1.3952
Epoch 67/100
4/4 [==============================] - 0s 1ms/step - loss: 1.3900
Epoch 68/100
4/4 [==============================] - 0s 841us/step - loss: 1.3890
Epoch 69/100
4/4 [==============================] - 0s 624us/step - loss: 1.3830
Epoch 70/100
4/4 [==============================] - 0s 581us/step - loss: 1.3785
Epoch 71/100
4/4 [==============================] - 0s 3ms/step - loss: 1.3746
Epoch 72/100
4/4 [==============================] - 0s 684us/step - loss: 1.3692
Epoch 73/100
4/4 [==============================] - 0s 677us/step - loss: 1.3656
Epoch 74/100
4/4 [==============================] - 0s 1ms/step - loss: 1.3608
Epoch 75/100
4/4 [==============================] - 0s 8ms/step - loss: 1.3567
Epoch 76/100
4/4 [==============================] - 0s 2ms/step - loss: 1.3534
Epoch 77/100
4/4 [==============================] - 0s 2ms/step - loss: 1.3506
Epoch 78/100
4/4 [==============================] - 0s 1ms/step - loss: 1.3462
Epoch 79/100
4/4 [==============================] - 0s 921us/step - loss: 1.3442
Epoch 80/100
4/4 [==============================] - 0s 636us/step - loss: 1.3390
Epoch 81/100
4/4 [==============================] - 0s 577us/step - loss: 1.3364
Epoch 82/100
4/4 [==============================] - 0s 2ms/step - loss: 1.3341
```

```
Epoch 83/100
4/4 [==============================] - 0s 762us/step - loss: 1.3317
Epoch 84/100
4/4 [==============================] - 0s 771us/step - loss: 1.3282
Epoch 85/100
4/4 [==============================] - 0s 678us/step - loss: 1.3231
Epoch 86/100
4/4 [==============================] - 0s 983us/step - loss: 1.3204
Epoch 87/100
4/4 [==============================] - 0s 731us/step - loss: 1.3156
Epoch 88/100
4/4 [==============================] - 0s 582us/step - loss: 1.3117
Epoch 89/100
4/4 [==============================] - 0s 679us/step - loss: 1.3085
Epoch 90/100
4/4 [==============================] - 0s 668us/step - loss: 1.3046
Epoch 91/100
4/4 [==============================] - 0s 967us/step - loss: 1.3016
Epoch 92/100
4/4 [==============================] - 0s 675us/step - loss: 1.2987
Epoch 93/100
4/4 [==============================] - 0s 937us/step - loss: 1.2948
Epoch 94/100
4/4 [==============================] - 0s 615us/step - loss: 1.2924
Epoch 95/100
4/4 [==============================] - 0s 999us/step - loss: 1.2908
Epoch 96/100
4/4 [==============================] - 0s 710us/step - loss: 1.2882
Epoch 97/100
4/4 [==============================] - 0s 593us/step - loss: 1.2843
Epoch 98/100
4/4 [==============================] - 0s 807us/step - loss: 1.2818
Epoch 99/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2790
Epoch 100/100
4/4 [==============================] - 0s 596us/step - loss: 1.2767
4/4 [==============================] - 0s 689us/step
```



The scatter plot and the decreasing loss over 100 epochs indicate that the network learned to approximate the true relationship between x and y. The fitted line on the second plot demonstrates that the model's predictions closely follow the expected trend, implying a successful training outcome where the model has captured the underlying linear relationship of the data.

c) Create a 3D animation (.gif) of the (weight, bias, loss) point over the training period. (15 points)

[8]:
```
from tensorflow.keras.callbacks import Callback
# Step 1: Create a custom callback
class RecordWeightBiasCallback(Callback):
    def on_train_begin(self, logs=None):
        self.weights = []
        self.biases = []
        self.losses = []

    def on_epoch_end(self, epoch, logs=None):
        # Get the weight and bias from the single Dense layer
        weight, bias = self.model.layers[0].get_weights()
        self.weights.append(weight[0][0])
```

```
        self.biases.append(bias[0])
        self.losses.append(logs['loss'])

# Instantiate the callback
record = RecordWeightBiasCallback()

# Step 2: Train the model with the callback
model_b.fit(x_data, y_data, epochs=100, callbacks=[record])

# Step 3: Create the 3D animation
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Prepare the data for animation
data = np.array(list(zip(record.weights, record.biases, record.losses)))

# Initialize the plot with the first frame's data
line, = ax.plot(data[:1, 0], data[:1, 1], data[:1, 2], marker='o')

# Function to update the plot with each frame
def update(num, data, line):
    line.set_data(data[:num, 0:2].T)
    line.set_3d_properties(data[:num, 2])
    return line,

# Create the animation
ani = FuncAnimation(fig, update, frames=len(record.weights), fargs=(data,␣
 ↪line), interval=100, blit=True)

# Set labels and title
ax.set_xlabel('Weight')
ax.set_ylabel('Bias')
ax.set_zlabel('Loss')
ax.set_title('3D Animation of Weight, Bias, and Loss during Training')

# Step 4: Save the animation as a .gif file
ani.save('training_animation.gif', writer=PillowWriter(fps=10))

# Display the animation
plt.show()
```

```
Epoch 1/100
4/4 [==============================] - 0s 2ms/step - loss: 1.2749
Epoch 2/100
4/4 [==============================] - 0s 867us/step - loss: 1.2741
Epoch 3/100
4/4 [==============================] - 0s 5ms/step - loss: 1.2697
```

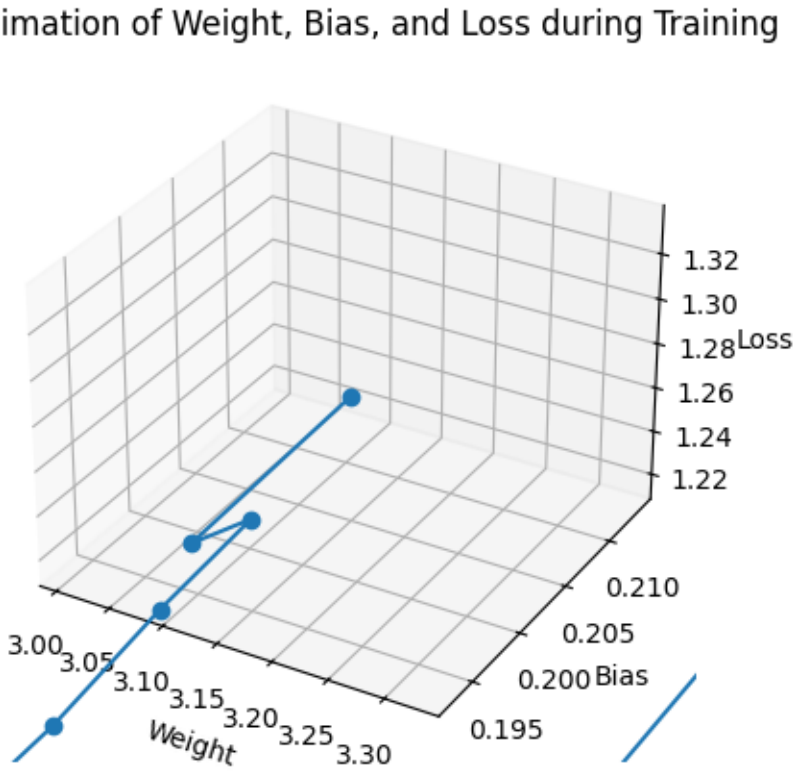
```
Epoch 4/100
4/4 [==============================] - 0s 881us/step - loss: 1.2688
Epoch 5/100
4/4 [==============================] - 0s 3ms/step - loss: 1.2633
Epoch 6/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2605
Epoch 7/100
4/4 [==============================] - 0s 631us/step - loss: 1.2565
Epoch 8/100
4/4 [==============================] - 0s 988us/step - loss: 1.2545
Epoch 9/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2529
Epoch 10/100
4/4 [==============================] - 0s 689us/step - loss: 1.2505
Epoch 11/100
4/4 [==============================] - 0s 755us/step - loss: 1.2502
Epoch 12/100
4/4 [==============================] - 0s 2ms/step - loss: 1.2449
Epoch 13/100
4/4 [==============================] - 0s 773us/step - loss: 1.2401
Epoch 14/100
4/4 [==============================] - 0s 656us/step - loss: 1.2374
Epoch 15/100
4/4 [==============================] - 0s 639us/step - loss: 1.2390
Epoch 16/100
4/4 [==============================] - 0s 879us/step - loss: 1.2317
Epoch 17/100
4/4 [==============================] - 0s 783us/step - loss: 1.2267
Epoch 18/100
4/4 [==============================] - 0s 641us/step - loss: 1.2254
Epoch 19/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2247
Epoch 20/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2223
Epoch 21/100
4/4 [==============================] - 0s 655us/step - loss: 1.2203
Epoch 22/100
4/4 [==============================] - 0s 715us/step - loss: 1.2185
Epoch 23/100
4/4 [==============================] - 0s 606us/step - loss: 1.2155
Epoch 24/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2158
Epoch 25/100
4/4 [==============================] - 0s 686us/step - loss: 1.2117
Epoch 26/100
4/4 [==============================] - 0s 763us/step - loss: 1.2100
Epoch 27/100
4/4 [==============================] - 0s 806us/step - loss: 1.2069
```

```
Epoch 28/100
4/4 [==============================] - 0s 1ms/step - loss: 1.2049
Epoch 29/100
4/4 [==============================] - 0s 767us/step - loss: 1.2026
Epoch 30/100
4/4 [==============================] - 0s 654us/step - loss: 1.1997
Epoch 31/100
4/4 [==============================] - 0s 670us/step - loss: 1.1979
Epoch 32/100
4/4 [==============================] - 0s 1ms/step - loss: 1.1954
Epoch 33/100
4/4 [==============================] - 0s 866us/step - loss: 1.1937
Epoch 34/100
4/4 [==============================] - 0s 2ms/step - loss: 1.1926
Epoch 35/100
4/4 [==============================] - 0s 865us/step - loss: 1.1890
Epoch 36/100
4/4 [==============================] - 0s 986us/step - loss: 1.1858
Epoch 37/100
4/4 [==============================] - 0s 661us/step - loss: 1.1840
Epoch 38/100
4/4 [==============================] - 0s 643us/step - loss: 1.1816
Epoch 39/100
4/4 [==============================] - 0s 1ms/step - loss: 1.1817
Epoch 40/100
4/4 [==============================] - 0s 671us/step - loss: 1.1796
Epoch 41/100
4/4 [==============================] - 0s 614us/step - loss: 1.1756
Epoch 42/100
4/4 [==============================] - 0s 671us/step - loss: 1.1742
Epoch 43/100
4/4 [==============================] - 0s 755us/step - loss: 1.1713
Epoch 44/100
4/4 [==============================] - 0s 825us/step - loss: 1.1696
Epoch 45/100
4/4 [==============================] - 0s 639us/step - loss: 1.1687
Epoch 46/100
4/4 [==============================] - 0s 634us/step - loss: 1.1653
Epoch 47/100
4/4 [==============================] - 0s 648us/step - loss: 1.1626
Epoch 48/100
4/4 [==============================] - 0s 1ms/step - loss: 1.1624
Epoch 49/100
4/4 [==============================] - 0s 679us/step - loss: 1.1594
Epoch 50/100
4/4 [==============================] - 0s 610us/step - loss: 1.1578
Epoch 51/100
4/4 [==============================] - 0s 758us/step - loss: 1.1557
```

```
Epoch 52/100
4/4 [==============================] - 0s 691us/step - loss: 1.1542
Epoch 53/100
4/4 [==============================] - 0s 908us/step - loss: 1.1522
Epoch 54/100
4/4 [==============================] - 0s 692us/step - loss: 1.1503
Epoch 55/100
4/4 [==============================] - 0s 575us/step - loss: 1.1486
Epoch 56/100
4/4 [==============================] - 0s 697us/step - loss: 1.1480
Epoch 57/100
4/4 [==============================] - 0s 619us/step - loss: 1.1477
Epoch 58/100
4/4 [==============================] - 0s 748us/step - loss: 1.1442
Epoch 59/100
4/4 [==============================] - 0s 658us/step - loss: 1.1411
Epoch 60/100
4/4 [==============================] - 0s 665us/step - loss: 1.1402
Epoch 61/100
4/4 [==============================] - 0s 791us/step - loss: 1.1400
Epoch 62/100
4/4 [==============================] - 0s 615us/step - loss: 1.1383
Epoch 63/100
4/4 [==============================] - 0s 814us/step - loss: 1.1374
Epoch 64/100
4/4 [==============================] - 0s 955us/step - loss: 1.1354
Epoch 65/100
4/4 [==============================] - 0s 2ms/step - loss: 1.1350
Epoch 66/100
4/4 [==============================] - 0s 2ms/step - loss: 1.1327
Epoch 67/100
4/4 [==============================] - 0s 887us/step - loss: 1.1315
Epoch 68/100
4/4 [==============================] - 0s 720us/step - loss: 1.1304
Epoch 69/100
4/4 [==============================] - 0s 708us/step - loss: 1.1301
Epoch 70/100
4/4 [==============================] - 0s 610us/step - loss: 1.1279
Epoch 71/100
4/4 [==============================] - 0s 624us/step - loss: 1.1256
Epoch 72/100
4/4 [==============================] - 0s 846us/step - loss: 1.1230
Epoch 73/100
4/4 [==============================] - 0s 845us/step - loss: 1.1214
Epoch 74/100
4/4 [==============================] - 0s 653us/step - loss: 1.1207
Epoch 75/100
4/4 [==============================] - 0s 705us/step - loss: 1.1199
```

```
Epoch 76/100
4/4 [==============================] - 0s 927us/step - loss: 1.1199
Epoch 77/100
4/4 [==============================] - 0s 888us/step - loss: 1.1179
Epoch 78/100
4/4 [==============================] - 0s 617us/step - loss: 1.1161
Epoch 79/100
4/4 [==============================] - 0s 745us/step - loss: 1.1169
Epoch 80/100
4/4 [==============================] - 0s 665us/step - loss: 1.1142
Epoch 81/100
4/4 [==============================] - 0s 757us/step - loss: 1.1131
Epoch 82/100
4/4 [==============================] - 0s 955us/step - loss: 1.1119
Epoch 83/100
4/4 [==============================] - 0s 660us/step - loss: 1.1111
Epoch 84/100
4/4 [==============================] - 0s 606us/step - loss: 1.1081
Epoch 85/100
4/4 [==============================] - 0s 1ms/step - loss: 1.1068
Epoch 86/100
4/4 [==============================] - 0s 602us/step - loss: 1.1058
Epoch 87/100
4/4 [==============================] - 0s 580us/step - loss: 1.1035
Epoch 88/100
4/4 [==============================] - 0s 674us/step - loss: 1.1024
Epoch 89/100
4/4 [==============================] - 0s 723us/step - loss: 1.1002
Epoch 90/100
4/4 [==============================] - 0s 800us/step - loss: 1.0992
Epoch 91/100
4/4 [==============================] - 0s 971us/step - loss: 1.0989
Epoch 92/100
4/4 [==============================] - 0s 676us/step - loss: 1.0988
Epoch 93/100
4/4 [==============================] - 0s 667us/step - loss: 1.0961
Epoch 94/100
4/4 [==============================] - 0s 749us/step - loss: 1.0949
Epoch 95/100
4/4 [==============================] - 0s 688us/step - loss: 1.0949
Epoch 96/100
4/4 [==============================] - 0s 644us/step - loss: 1.0927
Epoch 97/100
4/4 [==============================] - 0s 1ms/step - loss: 1.0921
Epoch 98/100
4/4 [==============================] - 0s 2ms/step - loss: 1.0910
Epoch 99/100
4/4 [==============================] - 0s 3ms/step - loss: 1.0900
```

```
Epoch 100/100
4/4 [==============================] - 0s 698us/step - loss: 1.0899
```

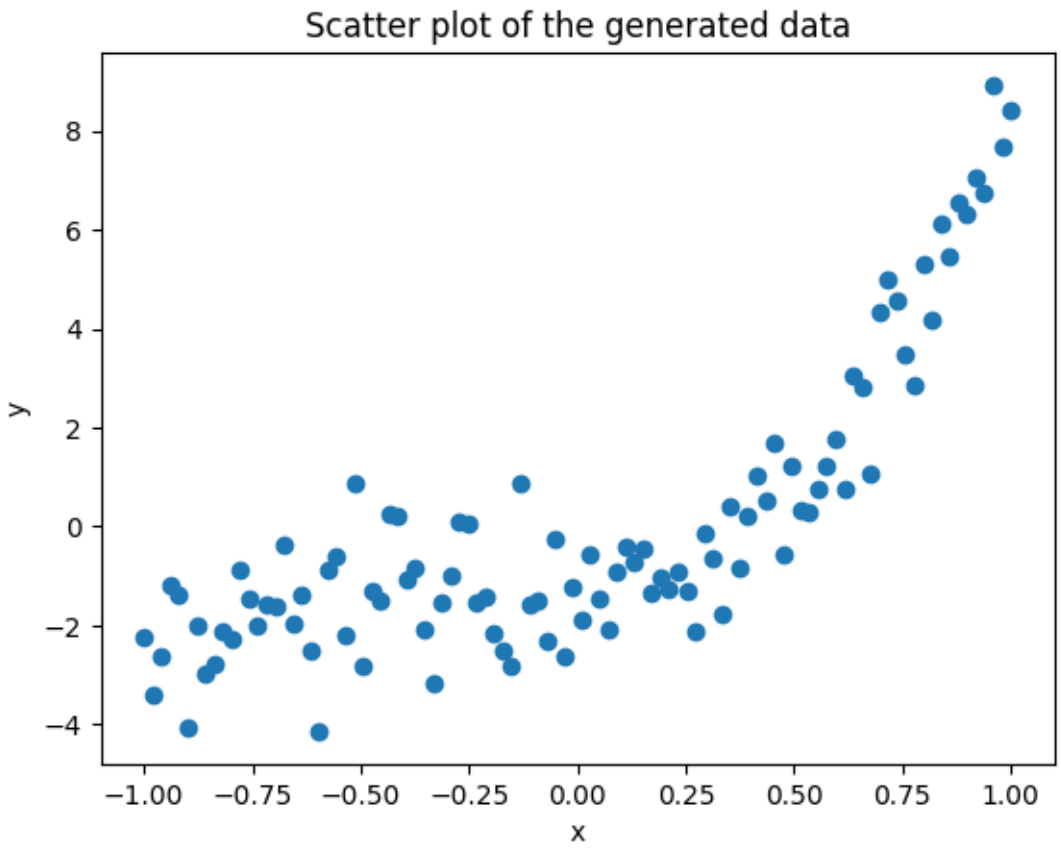


The trajectory indicates that as training progresses, the weight increases and the loss decreases, which suggests the model is learning and its performance is improving. The path of the connected data points represents the optimization path, with decreasing loss values implying the model's predictions are converging towards the training data's actual values. This frame suggests successful learning

d) Generate data of the form y = 5x^3 + 3x^2 + x - 1 + e where e ~ N(0, 1) and plot the data in a scatter plot. Create and train a neural network on this dataset and plot the resulting curve through the scatter plot. Explain your choice of model architecture (number of layers, and neurons) as well as your choice of activation function. (5points)

[9]:
```
np.random.seed(0)
x_data = np.linspace(-1, 1, 100)  # Generate 100 data points in the range [-1,␣
↪1]
e_data = np.random.randn(100)  # Noise ~ N(0, 1)
y_data = 5 * x_data**3 + 3 * x_data**2 + x_data - 1 + e_data

plt.scatter(x_data, y_data)
plt.title('Scatter plot of the generated data')
plt.xlabel('x')
```

```
plt.ylabel('y')
plt.show()

model_d = models.Sequential([
    layers.Dense(64, input_dim=1, activation='relu'),  # Hidden layer with ReLU␣
 ↪activation
    layers.Dense(64, activation='relu'),  # Additional hidden layer for more␣
 ↪complexity
    layers.Dense(1)  # Output layer for regression
])

model_d.compile(optimizer='adam', loss='mean_squared_error')
model_d.fit(x_data, y_data, epochs=1000)

# Step 4: Plot the resulting curve
x_values = np.linspace(-1, 1, 200)
y_pred = model_d.predict(x_values)

plt.scatter(x_data, y_data, label='Data')
plt.plot(x_values, y_pred, color='red', label='Model Prediction')
plt.title('Data and Model Prediction')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.show()
```




```
Epoch 1/1000
4/4 [==============================] - 0s 1ms/step - loss: 8.4277
Epoch 2/1000
4/4 [==============================] - 0s 735us/step - loss: 8.0989
Epoch 3/1000
4/4 [==============================] - 0s 812us/step - loss: 7.8199
Epoch 4/1000
4/4 [==============================] - 0s 900us/step - loss: 7.5661
Epoch 5/1000
4/4 [==============================] - 0s 875us/step - loss: 7.3079
Epoch 6/1000
4/4 [==============================] - 0s 699us/step - loss: 7.0763
Epoch 7/1000
4/4 [==============================] - 0s 766us/step - loss: 6.8444
Epoch 8/1000
4/4 [==============================] - 0s 700us/step - loss: 6.5964
Epoch 9/1000
4/4 [==============================] - 0s 697us/step - loss: 6.3443
Epoch 10/1000
4/4 [==============================] - 0s 1ms/step - loss: 6.1002
```

```
Epoch 11/1000
4/4 [==============================] - 0s 719us/step - loss: 5.8678
Epoch 12/1000
4/4 [==============================] - 0s 769us/step - loss: 5.5995
Epoch 13/1000
4/4 [==============================] - 0s 669us/step - loss: 5.3785
Epoch 14/1000
4/4 [==============================] - 0s 1ms/step - loss: 5.1858
Epoch 15/1000
4/4 [==============================] - 0s 899us/step - loss: 5.0000
Epoch 16/1000
4/4 [==============================] - 0s 715us/step - loss: 4.8467
Epoch 17/1000
4/4 [==============================] - 0s 683us/step - loss: 4.6873
Epoch 18/1000
4/4 [==============================] - 0s 1ms/step - loss: 4.4879
Epoch 19/1000
4/4 [==============================] - 0s 748us/step - loss: 4.3379
Epoch 20/1000
4/4 [==============================] - 0s 729us/step - loss: 4.1471
Epoch 21/1000
4/4 [==============================] - 0s 4ms/step - loss: 3.9664
Epoch 22/1000
4/4 [==============================] - 0s 2ms/step - loss: 3.8096
Epoch 23/1000
4/4 [==============================] - 0s 817us/step - loss: 3.6606
Epoch 24/1000
4/4 [==============================] - 0s 673us/step - loss: 3.4958
Epoch 25/1000
4/4 [==============================] - 0s 1ms/step - loss: 3.3609
Epoch 26/1000
4/4 [==============================] - 0s 692us/step - loss: 3.2141
Epoch 27/1000
4/4 [==============================] - 0s 628us/step - loss: 3.0849
Epoch 28/1000
4/4 [==============================] - 0s 610us/step - loss: 2.9593
Epoch 29/1000
4/4 [==============================] - 0s 1ms/step - loss: 2.8500
Epoch 30/1000
4/4 [==============================] - 0s 2ms/step - loss: 2.7289
Epoch 31/1000
4/4 [==============================] - 0s 2ms/step - loss: 2.6159
Epoch 32/1000
4/4 [==============================] - 0s 3ms/step - loss: 2.4978
Epoch 33/1000
4/4 [==============================] - 0s 3ms/step - loss: 2.3955
Epoch 34/1000
4/4 [==============================] - 0s 2ms/step - loss: 2.2986
```

```
Epoch 35/1000
4/4 [==============================] - 0s 1ms/step - loss: 2.1982
Epoch 36/1000
4/4 [==============================] - 0s 2ms/step - loss: 2.1070
Epoch 37/1000
4/4 [==============================] - 0s 3ms/step - loss: 2.0183
Epoch 38/1000
4/4 [==============================] - 0s 2ms/step - loss: 1.9155
Epoch 39/1000
4/4 [==============================] - 0s 2ms/step - loss: 1.8386
Epoch 40/1000
4/4 [==============================] - 0s 2ms/step - loss: 1.7503
Epoch 41/1000
4/4 [==============================] - 0s 868us/step - loss: 1.6919
Epoch 42/1000
4/4 [==============================] - 0s 691us/step - loss: 1.6388
Epoch 43/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.5890
Epoch 44/1000
4/4 [==============================] - 0s 871us/step - loss: 1.5409
Epoch 45/1000
4/4 [==============================] - 0s 626us/step - loss: 1.4973
Epoch 46/1000
4/4 [==============================] - 0s 629us/step - loss: 1.4792
Epoch 47/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.4485
Epoch 48/1000
4/4 [==============================] - 0s 782us/step - loss: 1.3998
Epoch 49/1000
4/4 [==============================] - 0s 858us/step - loss: 1.3588
Epoch 50/1000
4/4 [==============================] - 0s 4ms/step - loss: 1.3216
Epoch 51/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.2890
Epoch 52/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.2656
Epoch 53/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.2388
Epoch 54/1000
4/4 [==============================] - 0s 798us/step - loss: 1.2220
Epoch 55/1000
4/4 [==============================] - 0s 709us/step - loss: 1.2081
Epoch 56/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.1933
Epoch 57/1000
4/4 [==============================] - 0s 910us/step - loss: 1.1708
Epoch 58/1000
4/4 [==============================] - 0s 651us/step - loss: 1.1450
```

```
Epoch 59/1000
4/4 [==============================] - 0s 657us/step - loss: 1.1347
Epoch 60/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.1315
Epoch 61/1000
4/4 [==============================] - 0s 844us/step - loss: 1.1245
Epoch 62/1000
4/4 [==============================] - 0s 766us/step - loss: 1.1001
Epoch 63/1000
4/4 [==============================] - 0s 699us/step - loss: 1.0882
Epoch 64/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.0784
Epoch 65/1000
4/4 [==============================] - 0s 656us/step - loss: 1.0643
Epoch 66/1000
4/4 [==============================] - 0s 640us/step - loss: 1.0583
Epoch 67/1000
4/4 [==============================] - 0s 713us/step - loss: 1.0499
Epoch 68/1000
4/4 [==============================] - 0s 851us/step - loss: 1.0480
Epoch 69/1000
4/4 [==============================] - 0s 827us/step - loss: 1.0396
Epoch 70/1000
4/4 [==============================] - 0s 744us/step - loss: 1.0371
Epoch 71/1000
4/4 [==============================] - 0s 653us/step - loss: 1.0238
Epoch 72/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.0165
Epoch 73/1000
4/4 [==============================] - 0s 889us/step - loss: 1.0215
Epoch 74/1000
4/4 [==============================] - 0s 637us/step - loss: 1.0239
Epoch 75/1000
4/4 [==============================] - 0s 665us/step - loss: 1.0436
Epoch 76/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.0366
Epoch 77/1000
4/4 [==============================] - 0s 1ms/step - loss: 1.0269
Epoch 78/1000
4/4 [==============================] - 0s 3ms/step - loss: 1.0186
Epoch 79/1000
4/4 [==============================] - 0s 763us/step - loss: 1.0006
Epoch 80/1000
4/4 [==============================] - 0s 687us/step - loss: 0.9883
Epoch 81/1000
4/4 [==============================] - 0s 764us/step - loss: 0.9805
Epoch 82/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9755
```

```
Epoch 83/1000
4/4 [==============================] - 0s 742us/step - loss: 0.9813
Epoch 84/1000
4/4 [==============================] - 0s 747us/step - loss: 0.9743
Epoch 85/1000
4/4 [==============================] - 0s 716us/step - loss: 0.9677
Epoch 86/1000
4/4 [==============================] - 0s 636us/step - loss: 0.9759
Epoch 87/1000
4/4 [==============================] - 0s 890us/step - loss: 0.9822
Epoch 88/1000
4/4 [==============================] - 0s 731us/step - loss: 0.9828
Epoch 89/1000
4/4 [==============================] - 0s 675us/step - loss: 0.9874
Epoch 90/1000
4/4 [==============================] - 0s 731us/step - loss: 0.9852
Epoch 91/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9684
Epoch 92/1000
4/4 [==============================] - 0s 724us/step - loss: 0.9632
Epoch 93/1000
4/4 [==============================] - 0s 924us/step - loss: 0.9570
Epoch 94/1000
4/4 [==============================] - 0s 628us/step - loss: 0.9666
Epoch 95/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9635
Epoch 96/1000
4/4 [==============================] - 0s 832us/step - loss: 0.9451
Epoch 97/1000
4/4 [==============================] - 0s 677us/step - loss: 0.9543
Epoch 98/1000
4/4 [==============================] - 0s 667us/step - loss: 0.9700
Epoch 99/1000
4/4 [==============================] - 0s 822us/step - loss: 0.9531
Epoch 100/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9443
Epoch 101/1000
4/4 [==============================] - 0s 803us/step - loss: 0.9494
Epoch 102/1000
4/4 [==============================] - 0s 644us/step - loss: 0.9478
Epoch 103/1000
4/4 [==============================] - 0s 688us/step - loss: 0.9462
Epoch 104/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9383
Epoch 105/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9400
Epoch 106/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.9520
```

```
Epoch 107/1000
4/4 [==============================] - 0s 889us/step - loss: 0.9549
Epoch 108/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9555
Epoch 109/1000
4/4 [==============================] - 0s 829us/step - loss: 0.9470
Epoch 110/1000
4/4 [==============================] - 0s 734us/step - loss: 0.9456
Epoch 111/1000
4/4 [==============================] - 0s 703us/step - loss: 0.9340
Epoch 112/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9384
Epoch 113/1000
4/4 [==============================] - 0s 693us/step - loss: 0.9370
Epoch 114/1000
4/4 [==============================] - 0s 694us/step - loss: 0.9354
Epoch 115/1000
4/4 [==============================] - 0s 710us/step - loss: 0.9297
Epoch 116/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9239
Epoch 117/1000
4/4 [==============================] - 0s 727us/step - loss: 0.9229
Epoch 118/1000
4/4 [==============================] - 0s 746us/step - loss: 0.9265
Epoch 119/1000
4/4 [==============================] - 0s 689us/step - loss: 0.9231
Epoch 120/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9212
Epoch 121/1000
4/4 [==============================] - 0s 953us/step - loss: 0.9222
Epoch 122/1000
4/4 [==============================] - 0s 670us/step - loss: 0.9192
Epoch 123/1000
4/4 [==============================] - 0s 607us/step - loss: 0.9139
Epoch 124/1000
4/4 [==============================] - 0s 850us/step - loss: 0.9281
Epoch 125/1000
4/4 [==============================] - 0s 714us/step - loss: 0.9213
Epoch 126/1000
4/4 [==============================] - 0s 723us/step - loss: 0.9149
Epoch 127/1000
4/4 [==============================] - 0s 787us/step - loss: 0.9207
Epoch 128/1000
4/4 [==============================] - 0s 650us/step - loss: 0.9292
Epoch 129/1000
4/4 [==============================] - 0s 941us/step - loss: 0.9398
Epoch 130/1000
4/4 [==============================] - 0s 974us/step - loss: 0.9374
```

```
Epoch 131/1000
4/4 [==============================] - 0s 660us/step - loss: 0.9464
Epoch 132/1000
4/4 [==============================] - 0s 685us/step - loss: 0.9538
Epoch 133/1000
4/4 [==============================] - 0s 801us/step - loss: 0.9392
Epoch 134/1000
4/4 [==============================] - 0s 691us/step - loss: 0.9230
Epoch 135/1000
4/4 [==============================] - 0s 754us/step - loss: 0.9246
Epoch 136/1000
4/4 [==============================] - 0s 742us/step - loss: 0.9308
Epoch 137/1000
4/4 [==============================] - 0s 681us/step - loss: 0.9267
Epoch 138/1000
4/4 [==============================] - 0s 888us/step - loss: 0.9036
Epoch 139/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9115
Epoch 140/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9141
Epoch 141/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9070
Epoch 142/1000
4/4 [==============================] - 0s 728us/step - loss: 0.9018
Epoch 143/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8992
Epoch 144/1000
4/4 [==============================] - 0s 625us/step - loss: 0.8970
Epoch 145/1000
4/4 [==============================] - 0s 661us/step - loss: 0.8982
Epoch 146/1000
4/4 [==============================] - 0s 864us/step - loss: 0.8986
Epoch 147/1000
4/4 [==============================] - 0s 690us/step - loss: 0.9062
Epoch 148/1000
4/4 [==============================] - 0s 710us/step - loss: 0.9011
Epoch 149/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8926
Epoch 150/1000
4/4 [==============================] - 0s 737us/step - loss: 0.8980
Epoch 151/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8986
Epoch 152/1000
4/4 [==============================] - 0s 662us/step - loss: 0.8911
Epoch 153/1000
4/4 [==============================] - 0s 720us/step - loss: 0.8898
Epoch 154/1000
4/4 [==============================] - 0s 981us/step - loss: 0.8973
```

```
Epoch 155/1000
4/4 [==============================] - 0s 874us/step - loss: 0.8969
Epoch 156/1000
4/4 [==============================] - 0s 680us/step - loss: 0.8920
Epoch 157/1000
4/4 [==============================] - 0s 673us/step - loss: 0.8923
Epoch 158/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8953
Epoch 159/1000
4/4 [==============================] - 0s 671us/step - loss: 0.8955
Epoch 160/1000
4/4 [==============================] - 0s 861us/step - loss: 0.8953
Epoch 161/1000
4/4 [==============================] - 0s 661us/step - loss: 0.8950
Epoch 162/1000
4/4 [==============================] - 0s 836us/step - loss: 0.8914
Epoch 163/1000
4/4 [==============================] - 0s 809us/step - loss: 0.8863
Epoch 164/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8833
Epoch 165/1000
4/4 [==============================] - 0s 694us/step - loss: 0.8826
Epoch 166/1000
4/4 [==============================] - 0s 679us/step - loss: 0.8859
Epoch 167/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8820
Epoch 168/1000
4/4 [==============================] - 0s 755us/step - loss: 0.8863
Epoch 169/1000
4/4 [==============================] - 0s 868us/step - loss: 0.8853
Epoch 170/1000
4/4 [==============================] - 0s 930us/step - loss: 0.8826
Epoch 171/1000
4/4 [==============================] - 0s 720us/step - loss: 0.8841
Epoch 172/1000
4/4 [==============================] - 0s 745us/step - loss: 0.8820
Epoch 173/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8828
Epoch 174/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8822
Epoch 175/1000
4/4 [==============================] - 0s 808us/step - loss: 0.8797
Epoch 176/1000
4/4 [==============================] - 0s 663us/step - loss: 0.8810
Epoch 177/1000
4/4 [==============================] - 0s 962us/step - loss: 0.8801
Epoch 178/1000
4/4 [==============================] - 0s 757us/step - loss: 0.8936
```

```
Epoch 179/1000
4/4 [==============================] - 0s 663us/step - loss: 0.9062
Epoch 180/1000
4/4 [==============================] - 0s 655us/step - loss: 0.9016
Epoch 181/1000
4/4 [==============================] - 0s 642us/step - loss: 0.8871
Epoch 182/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8951
Epoch 183/1000
4/4 [==============================] - 0s 770us/step - loss: 0.8880
Epoch 184/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8817
Epoch 185/1000
4/4 [==============================] - 0s 847us/step - loss: 0.8821
Epoch 186/1000
4/4 [==============================] - 0s 762us/step - loss: 0.8742
Epoch 187/1000
4/4 [==============================] - 0s 738us/step - loss: 0.8771
Epoch 188/1000
4/4 [==============================] - 0s 688us/step - loss: 0.8904
Epoch 189/1000
4/4 [==============================] - 0s 705us/step - loss: 0.8833
Epoch 190/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8855
Epoch 191/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8938
Epoch 192/1000
4/4 [==============================] - 0s 643us/step - loss: 0.8874
Epoch 193/1000
4/4 [==============================] - 0s 669us/step - loss: 0.8819
Epoch 194/1000
4/4 [==============================] - 0s 933us/step - loss: 0.8718
Epoch 195/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8778
Epoch 196/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8786
Epoch 197/1000
4/4 [==============================] - 0s 770us/step - loss: 0.8801
Epoch 198/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8745
Epoch 199/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8735
Epoch 200/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8793
Epoch 201/1000
4/4 [==============================] - 0s 694us/step - loss: 0.8828
Epoch 202/1000
4/4 [==============================] - 0s 918us/step - loss: 0.8805
```

```
Epoch 203/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8751
Epoch 204/1000
4/4 [==============================] - 0s 922us/step - loss: 0.8699
Epoch 205/1000
4/4 [==============================] - 0s 633us/step - loss: 0.8723
Epoch 206/1000
4/4 [==============================] - 0s 724us/step - loss: 0.8733
Epoch 207/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8726
Epoch 208/1000
4/4 [==============================] - 0s 773us/step - loss: 0.8691
Epoch 209/1000
4/4 [==============================] - 0s 878us/step - loss: 0.8746
Epoch 210/1000
4/4 [==============================] - 0s 669us/step - loss: 0.8780
Epoch 211/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8773
Epoch 212/1000
4/4 [==============================] - 0s 872us/step - loss: 0.8849
Epoch 213/1000
4/4 [==============================] - 0s 777us/step - loss: 0.8716
Epoch 214/1000
4/4 [==============================] - 0s 679us/step - loss: 0.8715
Epoch 215/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8823
Epoch 216/1000
4/4 [==============================] - 0s 775us/step - loss: 0.8807
Epoch 217/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8696
Epoch 218/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8854
Epoch 219/1000
4/4 [==============================] - 0s 874us/step - loss: 0.8841
Epoch 220/1000
4/4 [==============================] - 0s 666us/step - loss: 0.8705
Epoch 221/1000
4/4 [==============================] - 0s 658us/step - loss: 0.8665
Epoch 222/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8723
Epoch 223/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8664
Epoch 224/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8716
Epoch 225/1000
4/4 [==============================] - 0s 923us/step - loss: 0.8716
Epoch 226/1000
4/4 [==============================] - 0s 682us/step - loss: 0.8816
```

```
Epoch 227/1000
4/4 [==============================] - 0s 806us/step - loss: 0.8796
Epoch 228/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8689
Epoch 229/1000
4/4 [==============================] - 0s 721us/step - loss: 0.8671
Epoch 230/1000
4/4 [==============================] - 0s 663us/step - loss: 0.8710
Epoch 231/1000
4/4 [==============================] - 0s 758us/step - loss: 0.8762
Epoch 232/1000
4/4 [==============================] - 0s 732us/step - loss: 0.8644
Epoch 233/1000
4/4 [==============================] - 0s 671us/step - loss: 0.8705
Epoch 234/1000
4/4 [==============================] - 0s 707us/step - loss: 0.8752
Epoch 235/1000
4/4 [==============================] - 0s 632us/step - loss: 0.8677
Epoch 236/1000
4/4 [==============================] - 0s 643us/step - loss: 0.8596
Epoch 237/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8729
Epoch 238/1000
4/4 [==============================] - 0s 750us/step - loss: 0.8749
Epoch 239/1000
4/4 [==============================] - 0s 664us/step - loss: 0.8682
Epoch 240/1000
4/4 [==============================] - 0s 948us/step - loss: 0.8676
Epoch 241/1000
4/4 [==============================] - 0s 647us/step - loss: 0.8652
Epoch 242/1000
4/4 [==============================] - 0s 696us/step - loss: 0.8642
Epoch 243/1000
4/4 [==============================] - 0s 640us/step - loss: 0.8596
Epoch 244/1000
4/4 [==============================] - 0s 589us/step - loss: 0.8805
Epoch 245/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8967
Epoch 246/1000
4/4 [==============================] - 0s 666us/step - loss: 0.8881
Epoch 247/1000
4/4 [==============================] - 0s 672us/step - loss: 0.8755
Epoch 248/1000
4/4 [==============================] - 0s 724us/step - loss: 0.8658
Epoch 249/1000
4/4 [==============================] - 0s 639us/step - loss: 0.8647
Epoch 250/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8597
```

```
Epoch 251/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8650
Epoch 252/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8866
Epoch 253/1000
4/4 [==============================] - 0s 672us/step - loss: 0.8848
Epoch 254/1000
4/4 [==============================] - 0s 786us/step - loss: 0.8671
Epoch 255/1000
4/4 [==============================] - 0s 613us/step - loss: 0.8616
Epoch 256/1000
4/4 [==============================] - 0s 929us/step - loss: 0.8763
Epoch 257/1000
4/4 [==============================] - 0s 719us/step - loss: 0.9103
Epoch 258/1000
4/4 [==============================] - 0s 677us/step - loss: 0.9004
Epoch 259/1000
4/4 [==============================] - 0s 661us/step - loss: 0.8678
Epoch 260/1000
4/4 [==============================] - 0s 617us/step - loss: 0.8629
Epoch 261/1000
4/4 [==============================] - 0s 961us/step - loss: 0.8637
Epoch 262/1000
4/4 [==============================] - 0s 765us/step - loss: 0.8627
Epoch 263/1000
4/4 [==============================] - 0s 738us/step - loss: 0.8764
Epoch 264/1000
4/4 [==============================] - 0s 701us/step - loss: 0.8829
Epoch 265/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8960
Epoch 266/1000
4/4 [==============================] - 0s 644us/step - loss: 0.8758
Epoch 267/1000
4/4 [==============================] - 0s 657us/step - loss: 0.8691
Epoch 268/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8818
Epoch 269/1000
4/4 [==============================] - 0s 840us/step - loss: 0.8748
Epoch 270/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8693
Epoch 271/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8665
Epoch 272/1000
4/4 [==============================] - 0s 745us/step - loss: 0.8597
Epoch 273/1000
4/4 [==============================] - 0s 969us/step - loss: 0.8612
Epoch 274/1000
4/4 [==============================] - 0s 796us/step - loss: 0.8639
```

```
Epoch 275/1000
4/4 [==============================] - 0s 722us/step - loss: 0.8718
Epoch 276/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8781
Epoch 277/1000
4/4 [==============================] - 0s 990us/step - loss: 0.8770
Epoch 278/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8677
Epoch 279/1000
4/4 [==============================] - 0s 917us/step - loss: 0.8661
Epoch 280/1000
4/4 [==============================] - 0s 659us/step - loss: 0.8639
Epoch 281/1000
4/4 [==============================] - 0s 855us/step - loss: 0.8750
Epoch 282/1000
4/4 [==============================] - 0s 844us/step - loss: 0.9213
Epoch 283/1000
4/4 [==============================] - 0s 731us/step - loss: 0.9244
Epoch 284/1000
4/4 [==============================] - 0s 684us/step - loss: 0.8944
Epoch 285/1000
4/4 [==============================] - 0s 702us/step - loss: 0.8755
Epoch 286/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8759
Epoch 287/1000
4/4 [==============================] - 0s 718us/step - loss: 0.8606
Epoch 288/1000
4/4 [==============================] - 0s 638us/step - loss: 0.8682
Epoch 289/1000
4/4 [==============================] - 0s 980us/step - loss: 0.8702
Epoch 290/1000
4/4 [==============================] - 0s 996us/step - loss: 0.8664
Epoch 291/1000
4/4 [==============================] - 0s 771us/step - loss: 0.8596
Epoch 292/1000
4/4 [==============================] - 0s 717us/step - loss: 0.8573
Epoch 293/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8608
Epoch 294/1000
4/4 [==============================] - 0s 845us/step - loss: 0.8630
Epoch 295/1000
4/4 [==============================] - 0s 708us/step - loss: 0.8759
Epoch 296/1000
4/4 [==============================] - 0s 623us/step - loss: 0.8683
Epoch 297/1000
4/4 [==============================] - 0s 977us/step - loss: 0.8744
Epoch 298/1000
4/4 [==============================] - 0s 726us/step - loss: 0.8774
```

```
Epoch 299/1000
4/4 [==============================] - 0s 739us/step - loss: 0.8679
Epoch 300/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8712
Epoch 301/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8797
Epoch 302/1000
4/4 [==============================] - 0s 832us/step - loss: 0.8748
Epoch 303/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8638
Epoch 304/1000
4/4 [==============================] - 0s 834us/step - loss: 0.8623
Epoch 305/1000
4/4 [==============================] - 0s 762us/step - loss: 0.8596
Epoch 306/1000
4/4 [==============================] - 0s 608us/step - loss: 0.8567
Epoch 307/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8574
Epoch 308/1000
4/4 [==============================] - 0s 878us/step - loss: 0.8627
Epoch 309/1000
4/4 [==============================] - 0s 701us/step - loss: 0.8568
Epoch 310/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8602
Epoch 311/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8732
Epoch 312/1000
4/4 [==============================] - 0s 778us/step - loss: 0.8607
Epoch 313/1000
4/4 [==============================] - 0s 669us/step - loss: 0.8621
Epoch 314/1000
4/4 [==============================] - 0s 696us/step - loss: 0.8813
Epoch 315/1000
4/4 [==============================] - 0s 899us/step - loss: 0.8831
Epoch 316/1000
4/4 [==============================] - 0s 986us/step - loss: 0.8651
Epoch 317/1000
4/4 [==============================] - 0s 729us/step - loss: 0.8780
Epoch 318/1000
4/4 [==============================] - 0s 909us/step - loss: 0.8903
Epoch 319/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8754
Epoch 320/1000
4/4 [==============================] - 0s 839us/step - loss: 0.8586
Epoch 321/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8561
Epoch 322/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8581
```

```
Epoch 323/1000
4/4 [==============================] - 0s 685us/step - loss: 0.8565
Epoch 324/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8535
Epoch 325/1000
4/4 [==============================] - 0s 878us/step - loss: 0.8570
Epoch 326/1000
4/4 [==============================] - 0s 858us/step - loss: 0.8647
Epoch 327/1000
4/4 [==============================] - 0s 769us/step - loss: 0.8646
Epoch 328/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8627
Epoch 329/1000
4/4 [==============================] - 0s 613us/step - loss: 0.8609
Epoch 330/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8584
Epoch 331/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8646
Epoch 332/1000
4/4 [==============================] - 0s 759us/step - loss: 0.8541
Epoch 333/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8641
Epoch 334/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8596
Epoch 335/1000
4/4 [==============================] - 0s 669us/step - loss: 0.8608
Epoch 336/1000
4/4 [==============================] - 0s 653us/step - loss: 0.8604
Epoch 337/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8558
Epoch 338/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8616
Epoch 339/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8631
Epoch 340/1000
4/4 [==============================] - 0s 673us/step - loss: 0.8592
Epoch 341/1000
4/4 [==============================] - 0s 740us/step - loss: 0.8588
Epoch 342/1000
4/4 [==============================] - 0s 973us/step - loss: 0.8575
Epoch 343/1000
4/4 [==============================] - 0s 757us/step - loss: 0.8538
Epoch 344/1000
4/4 [==============================] - 0s 709us/step - loss: 0.8531
Epoch 345/1000
4/4 [==============================] - 0s 655us/step - loss: 0.8536
Epoch 346/1000
4/4 [==============================] - 0s 748us/step - loss: 0.8581
```

```
Epoch 347/1000
4/4 [==============================] - 0s 883us/step - loss: 0.8630
Epoch 348/1000
4/4 [==============================] - 0s 634us/step - loss: 0.8614
Epoch 349/1000
4/4 [==============================] - 0s 805us/step - loss: 0.8558
Epoch 350/1000
4/4 [==============================] - 0s 703us/step - loss: 0.8528
Epoch 351/1000
4/4 [==============================] - 0s 678us/step - loss: 0.8552
Epoch 352/1000
4/4 [==============================] - 0s 683us/step - loss: 0.8512
Epoch 353/1000
4/4 [==============================] - 0s 700us/step - loss: 0.8600
Epoch 354/1000
4/4 [==============================] - 0s 694us/step - loss: 0.8709
Epoch 355/1000
4/4 [==============================] - 0s 847us/step - loss: 0.8690
Epoch 356/1000
4/4 [==============================] - 0s 815us/step - loss: 0.8669
Epoch 357/1000
4/4 [==============================] - 0s 737us/step - loss: 0.8553
Epoch 358/1000
4/4 [==============================] - 0s 716us/step - loss: 0.8580
Epoch 359/1000
4/4 [==============================] - 0s 648us/step - loss: 0.8589
Epoch 360/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8510
Epoch 361/1000
4/4 [==============================] - 0s 774us/step - loss: 0.8700
Epoch 362/1000
4/4 [==============================] - 0s 723us/step - loss: 0.8671
Epoch 363/1000
4/4 [==============================] - 0s 721us/step - loss: 0.8630
Epoch 364/1000
4/4 [==============================] - 0s 626us/step - loss: 0.8522
Epoch 365/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9011
Epoch 366/1000
4/4 [==============================] - 0s 907us/step - loss: 0.9114
Epoch 367/1000
4/4 [==============================] - 0s 845us/step - loss: 0.8704
Epoch 368/1000
4/4 [==============================] - 0s 754us/step - loss: 0.8717
Epoch 369/1000
4/4 [==============================] - 0s 775us/step - loss: 0.8914
Epoch 370/1000
4/4 [==============================] - 0s 660us/step - loss: 0.8671
```

```
Epoch 371/1000
4/4 [==============================] - 0s 677us/step - loss: 0.8488
Epoch 372/1000
4/4 [==============================] - 0s 798us/step - loss: 0.8580
Epoch 373/1000
4/4 [==============================] - 0s 730us/step - loss: 0.8634
Epoch 374/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8708
Epoch 375/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8600
Epoch 376/1000
4/4 [==============================] - 0s 912us/step - loss: 0.8519
Epoch 377/1000
4/4 [==============================] - 0s 780us/step - loss: 0.8572
Epoch 378/1000
4/4 [==============================] - 0s 976us/step - loss: 0.8655
Epoch 379/1000
4/4 [==============================] - 0s 715us/step - loss: 0.8617
Epoch 380/1000
4/4 [==============================] - 0s 688us/step - loss: 0.8495
Epoch 381/1000
4/4 [==============================] - 0s 673us/step - loss: 0.8525
Epoch 382/1000
4/4 [==============================] - 0s 652us/step - loss: 0.8783
Epoch 383/1000
4/4 [==============================] - 0s 998us/step - loss: 0.8625
Epoch 384/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8509
Epoch 385/1000
4/4 [==============================] - 0s 874us/step - loss: 0.8640
Epoch 386/1000
4/4 [==============================] - 0s 696us/step - loss: 0.8682
Epoch 387/1000
4/4 [==============================] - 0s 882us/step - loss: 0.8592
Epoch 388/1000
4/4 [==============================] - 0s 680us/step - loss: 0.8556
Epoch 389/1000
4/4 [==============================] - 0s 883us/step - loss: 0.8476
Epoch 390/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8491
Epoch 391/1000
4/4 [==============================] - 0s 933us/step - loss: 0.8514
Epoch 392/1000
4/4 [==============================] - 0s 888us/step - loss: 0.8515
Epoch 393/1000
4/4 [==============================] - 0s 681us/step - loss: 0.8586
Epoch 394/1000
4/4 [==============================] - 0s 958us/step - loss: 0.8588
```

```
Epoch 395/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8448
Epoch 396/1000
4/4 [==============================] - 0s 703us/step - loss: 0.8676
Epoch 397/1000
4/4 [==============================] - 0s 636us/step - loss: 0.9278
Epoch 398/1000
4/4 [==============================] - 0s 583us/step - loss: 0.9023
Epoch 399/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8649
Epoch 400/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8576
Epoch 401/1000
4/4 [==============================] - 0s 764us/step - loss: 0.8718
Epoch 402/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8814
Epoch 403/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8829
Epoch 404/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8650
Epoch 405/1000
4/4 [==============================] - 0s 856us/step - loss: 0.8421
Epoch 406/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.9055
Epoch 407/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9154
Epoch 408/1000
4/4 [==============================] - 0s 867us/step - loss: 0.8832
Epoch 409/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8567
Epoch 410/1000
4/4 [==============================] - 0s 685us/step - loss: 0.8729
Epoch 411/1000
4/4 [==============================] - 0s 665us/step - loss: 0.8700
Epoch 412/1000
4/4 [==============================] - 0s 996us/step - loss: 0.8484
Epoch 413/1000
4/4 [==============================] - 0s 830us/step - loss: 0.8680
Epoch 414/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8799
Epoch 415/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8551
Epoch 416/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8509
Epoch 417/1000
4/4 [==============================] - 0s 832us/step - loss: 0.8602
Epoch 418/1000
4/4 [==============================] - 0s 965us/step - loss: 0.8582
```

```
Epoch 419/1000
4/4 [==============================] - 0s 990us/step - loss: 0.8560
Epoch 420/1000
4/4 [==============================] - 0s 643us/step - loss: 0.8534
Epoch 421/1000
4/4 [==============================] - 0s 763us/step - loss: 0.8504
Epoch 422/1000
4/4 [==============================] - 0s 660us/step - loss: 0.8529
Epoch 423/1000
4/4 [==============================] - 0s 747us/step - loss: 0.8441
Epoch 424/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8572
Epoch 425/1000
4/4 [==============================] - 0s 678us/step - loss: 0.8744
Epoch 426/1000
4/4 [==============================] - 0s 861us/step - loss: 0.8662
Epoch 427/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8503
Epoch 428/1000
4/4 [==============================] - 0s 704us/step - loss: 0.8526
Epoch 429/1000
4/4 [==============================] - 0s 737us/step - loss: 0.8492
Epoch 430/1000
4/4 [==============================] - 0s 657us/step - loss: 0.8478
Epoch 431/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8520
Epoch 432/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8557
Epoch 433/1000
4/4 [==============================] - 0s 732us/step - loss: 0.8470
Epoch 434/1000
4/4 [==============================] - 0s 783us/step - loss: 0.8494
Epoch 435/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8552
Epoch 436/1000
4/4 [==============================] - 0s 755us/step - loss: 0.8580
Epoch 437/1000
4/4 [==============================] - 0s 799us/step - loss: 0.8481
Epoch 438/1000
4/4 [==============================] - 0s 710us/step - loss: 0.8583
Epoch 439/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8346
Epoch 440/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8758
Epoch 441/1000
4/4 [==============================] - 0s 680us/step - loss: 0.8475
Epoch 442/1000
4/4 [==============================] - 0s 886us/step - loss: 0.8614
```

```
Epoch 443/1000
4/4 [==============================] - 0s 918us/step - loss: 0.9235
Epoch 444/1000
4/4 [==============================] - 0s 957us/step - loss: 0.8908
Epoch 445/1000
4/4 [==============================] - 0s 684us/step - loss: 0.8484
Epoch 446/1000
4/4 [==============================] - 0s 884us/step - loss: 0.8950
Epoch 447/1000
4/4 [==============================] - 0s 908us/step - loss: 0.8942
Epoch 448/1000
4/4 [==============================] - 0s 873us/step - loss: 0.8497
Epoch 449/1000
4/4 [==============================] - 0s 662us/step - loss: 0.8472
Epoch 450/1000
4/4 [==============================] - 0s 929us/step - loss: 0.8469
Epoch 451/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8458
Epoch 452/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8492
Epoch 453/1000
4/4 [==============================] - 0s 646us/step - loss: 0.8478
Epoch 454/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8448
Epoch 455/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8457
Epoch 456/1000
4/4 [==============================] - 0s 668us/step - loss: 0.8466
Epoch 457/1000
4/4 [==============================] - 0s 784us/step - loss: 0.8446
Epoch 458/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8447
Epoch 459/1000
4/4 [==============================] - 0s 967us/step - loss: 0.8464
Epoch 460/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8591
Epoch 461/1000
4/4 [==============================] - 0s 723us/step - loss: 0.8501
Epoch 462/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8498
Epoch 463/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8507
Epoch 464/1000
4/4 [==============================] - 0s 738us/step - loss: 0.8561
Epoch 465/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8651
Epoch 466/1000
4/4 [==============================] - 0s 730us/step - loss: 0.8584
```

```
Epoch 467/1000
4/4 [==============================] - 0s 706us/step - loss: 0.8440
Epoch 468/1000
4/4 [==============================] - 0s 609us/step - loss: 0.8517
Epoch 469/1000
4/4 [==============================] - 0s 608us/step - loss: 0.8920
Epoch 470/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8984
Epoch 471/1000
4/4 [==============================] - 0s 908us/step - loss: 0.8509
Epoch 472/1000
4/4 [==============================] - 0s 836us/step - loss: 0.8515
Epoch 473/1000
4/4 [==============================] - 0s 783us/step - loss: 0.8864
Epoch 474/1000
4/4 [==============================] - 0s 821us/step - loss: 0.8825
Epoch 475/1000
4/4 [==============================] - 0s 776us/step - loss: 0.8508
Epoch 476/1000
4/4 [==============================] - 0s 713us/step - loss: 0.8592
Epoch 477/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8946
Epoch 478/1000
4/4 [==============================] - 0s 616us/step - loss: 0.8668
Epoch 479/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8576
Epoch 480/1000
4/4 [==============================] - 0s 680us/step - loss: 0.8451
Epoch 481/1000
4/4 [==============================] - 0s 690us/step - loss: 0.8468
Epoch 482/1000
4/4 [==============================] - 0s 908us/step - loss: 0.8470
Epoch 483/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8469
Epoch 484/1000
4/4 [==============================] - 0s 712us/step - loss: 0.8470
Epoch 485/1000
4/4 [==============================] - 0s 633us/step - loss: 0.8458
Epoch 486/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8471
Epoch 487/1000
4/4 [==============================] - 0s 650us/step - loss: 0.8445
Epoch 488/1000
4/4 [==============================] - 0s 658us/step - loss: 0.8489
Epoch 489/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8468
Epoch 490/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8450
```

```
Epoch 491/1000
4/4 [==============================] - 0s 809us/step - loss: 0.8547
Epoch 492/1000
4/4 [==============================] - 0s 732us/step - loss: 0.8531
Epoch 493/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8492
Epoch 494/1000
4/4 [==============================] - 0s 667us/step - loss: 0.8461
Epoch 495/1000
4/4 [==============================] - 0s 785us/step - loss: 0.8479
Epoch 496/1000
4/4 [==============================] - 0s 592us/step - loss: 0.8520
Epoch 497/1000
4/4 [==============================] - 0s 829us/step - loss: 0.8468
Epoch 498/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8399
Epoch 499/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8472
Epoch 500/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8792
Epoch 501/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8751
Epoch 502/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8577
Epoch 503/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8501
Epoch 504/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8471
Epoch 505/1000
4/4 [==============================] - 0s 812us/step - loss: 0.8455
Epoch 506/1000
4/4 [==============================] - 0s 751us/step - loss: 0.8553
Epoch 507/1000
4/4 [==============================] - 0s 851us/step - loss: 0.8586
Epoch 508/1000
4/4 [==============================] - 0s 983us/step - loss: 0.8519
Epoch 509/1000
4/4 [==============================] - 0s 644us/step - loss: 0.8424
Epoch 510/1000
4/4 [==============================] - 0s 755us/step - loss: 0.8406
Epoch 511/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8423
Epoch 512/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8352
Epoch 513/1000
4/4 [==============================] - 0s 928us/step - loss: 0.8536
Epoch 514/1000
4/4 [==============================] - 0s 594us/step - loss: 0.8680
```

```
Epoch 515/1000
4/4 [==============================] - 0s 992us/step - loss: 0.8523
Epoch 516/1000
4/4 [==============================] - 0s 877us/step - loss: 0.8455
Epoch 517/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8420
Epoch 518/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8381
Epoch 519/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8416
Epoch 520/1000
4/4 [==============================] - 0s 711us/step - loss: 0.8421
Epoch 521/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8415
Epoch 522/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8422
Epoch 523/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8421
Epoch 524/1000
4/4 [==============================] - 0s 678us/step - loss: 0.8406
Epoch 525/1000
4/4 [==============================] - 0s 662us/step - loss: 0.8418
Epoch 526/1000
4/4 [==============================] - 0s 702us/step - loss: 0.8413
Epoch 527/1000
4/4 [==============================] - 0s 786us/step - loss: 0.8428
Epoch 528/1000
4/4 [==============================] - 0s 862us/step - loss: 0.8421
Epoch 529/1000
4/4 [==============================] - 0s 626us/step - loss: 0.8513
Epoch 530/1000
4/4 [==============================] - 0s 593us/step - loss: 0.8636
Epoch 531/1000
4/4 [==============================] - 0s 663us/step - loss: 0.8470
Epoch 532/1000
4/4 [==============================] - 0s 984us/step - loss: 0.8395
Epoch 533/1000
4/4 [==============================] - 0s 999us/step - loss: 0.8368
Epoch 534/1000
4/4 [==============================] - 0s 759us/step - loss: 0.8461
Epoch 535/1000
4/4 [==============================] - 0s 987us/step - loss: 0.8333
Epoch 536/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8611
Epoch 537/1000
4/4 [==============================] - 0s 646us/step - loss: 0.8589
Epoch 538/1000
4/4 [==============================] - 0s 725us/step - loss: 0.8458
```

```
Epoch 539/1000
4/4 [==============================] - 0s 668us/step - loss: 0.8376
Epoch 540/1000
4/4 [==============================] - 0s 970us/step - loss: 0.8424
Epoch 541/1000
4/4 [==============================] - 0s 936us/step - loss: 0.8439
Epoch 542/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8440
Epoch 543/1000
4/4 [==============================] - 0s 766us/step - loss: 0.8400
Epoch 544/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8364
Epoch 545/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8396
Epoch 546/1000
4/4 [==============================] - 0s 736us/step - loss: 0.8433
Epoch 547/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8562
Epoch 548/1000
4/4 [==============================] - 0s 773us/step - loss: 0.8502
Epoch 549/1000
4/4 [==============================] - 0s 10ms/step - loss: 0.8418
Epoch 550/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8322
Epoch 551/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8537
Epoch 552/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8548
Epoch 553/1000
4/4 [==============================] - 0s 860us/step - loss: 0.8519
Epoch 554/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8349
Epoch 555/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8454
Epoch 556/1000
4/4 [==============================] - 0s 964us/step - loss: 0.8461
Epoch 557/1000
4/4 [==============================] - 0s 992us/step - loss: 0.8408
Epoch 558/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8407
Epoch 559/1000
4/4 [==============================] - 0s 685us/step - loss: 0.8449
Epoch 560/1000
4/4 [==============================] - 0s 693us/step - loss: 0.8406
Epoch 561/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8470
Epoch 562/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8423
```

```
Epoch 563/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8420
Epoch 564/1000
4/4 [==============================] - 0s 796us/step - loss: 0.8455
Epoch 565/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8521
Epoch 566/1000
4/4 [==============================] - 0s 706us/step - loss: 0.8565
Epoch 567/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8627
Epoch 568/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8602
Epoch 569/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8427
Epoch 570/1000
4/4 [==============================] - 0s 809us/step - loss: 0.8707
Epoch 571/1000
4/4 [==============================] - 0s 848us/step - loss: 0.8729
Epoch 572/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8454
Epoch 573/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8454
Epoch 574/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8375
Epoch 575/1000
4/4 [==============================] - 0s 9ms/step - loss: 0.8456
Epoch 576/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8498
Epoch 577/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8460
Epoch 578/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8434
Epoch 579/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8412
Epoch 580/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8438
Epoch 581/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8436
Epoch 582/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8438
Epoch 583/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8457
Epoch 584/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8390
Epoch 585/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8405
Epoch 586/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8495
```

```
Epoch 587/1000
4/4 [==============================] - 0s 911us/step - loss: 0.8418
Epoch 588/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8442
Epoch 589/1000
4/4 [==============================] - 0s 915us/step - loss: 0.8387
Epoch 590/1000
4/4 [==============================] - 0s 856us/step - loss: 0.8290
Epoch 591/1000
4/4 [==============================] - 0s 724us/step - loss: 0.8545
Epoch 592/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8782
Epoch 593/1000
4/4 [==============================] - 0s 5ms/step - loss: 0.8686
Epoch 594/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8523
Epoch 595/1000
4/4 [==============================] - 0s 835us/step - loss: 0.8455
Epoch 596/1000
4/4 [==============================] - 0s 669us/step - loss: 0.8509
Epoch 597/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8331
Epoch 598/1000
4/4 [==============================] - 0s 671us/step - loss: 0.8455
Epoch 599/1000
4/4 [==============================] - 0s 806us/step - loss: 0.8405
Epoch 600/1000
4/4 [==============================] - 0s 925us/step - loss: 0.8364
Epoch 601/1000
4/4 [==============================] - 0s 866us/step - loss: 0.8385
Epoch 602/1000
4/4 [==============================] - 0s 638us/step - loss: 0.8426
Epoch 603/1000
4/4 [==============================] - 0s 682us/step - loss: 0.8353
Epoch 604/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8409
Epoch 605/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8374
Epoch 606/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8351
Epoch 607/1000
4/4 [==============================] - 0s 977us/step - loss: 0.8382
Epoch 608/1000
4/4 [==============================] - 0s 894us/step - loss: 0.8451
Epoch 609/1000
4/4 [==============================] - 0s 631us/step - loss: 0.8676
Epoch 610/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8896
```

```
Epoch 611/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8691
Epoch 612/1000
4/4 [==============================] - 0s 770us/step - loss: 0.8428
Epoch 613/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8419
Epoch 614/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8457
Epoch 615/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8501
Epoch 616/1000
4/4 [==============================] - 0s 902us/step - loss: 0.8514
Epoch 617/1000
4/4 [==============================] - 0s 779us/step - loss: 0.8392
Epoch 618/1000
4/4 [==============================] - 0s 907us/step - loss: 0.8348
Epoch 619/1000
4/4 [==============================] - 0s 756us/step - loss: 0.8417
Epoch 620/1000
4/4 [==============================] - 0s 700us/step - loss: 0.8386
Epoch 621/1000
4/4 [==============================] - 0s 653us/step - loss: 0.8335
Epoch 622/1000
4/4 [==============================] - 0s 820us/step - loss: 0.8408
Epoch 623/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8447
Epoch 624/1000
4/4 [==============================] - 0s 730us/step - loss: 0.8422
Epoch 625/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8522
Epoch 626/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8454
Epoch 627/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8509
Epoch 628/1000
4/4 [==============================] - 0s 8ms/step - loss: 0.8505
Epoch 629/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8381
Epoch 630/1000
4/4 [==============================] - 0s 948us/step - loss: 0.8392
Epoch 631/1000
4/4 [==============================] - 0s 766us/step - loss: 0.8471
Epoch 632/1000
4/4 [==============================] - 0s 729us/step - loss: 0.8414
Epoch 633/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8323
Epoch 634/1000
4/4 [==============================] - 0s 698us/step - loss: 0.8365
```

```
Epoch 635/1000
4/4 [==============================] - 0s 720us/step - loss: 0.8418
Epoch 636/1000
4/4 [==============================] - 0s 606us/step - loss: 0.8358
Epoch 637/1000
4/4 [==============================] - 0s 906us/step - loss: 0.8370
Epoch 638/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8397
Epoch 639/1000
4/4 [==============================] - 0s 624us/step - loss: 0.8347
Epoch 640/1000
4/4 [==============================] - 0s 731us/step - loss: 0.8330
Epoch 641/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8351
Epoch 642/1000
4/4 [==============================] - 0s 704us/step - loss: 0.8429
Epoch 643/1000
4/4 [==============================] - 0s 658us/step - loss: 0.8328
Epoch 644/1000
4/4 [==============================] - 0s 668us/step - loss: 0.8428
Epoch 645/1000
4/4 [==============================] - 0s 860us/step - loss: 0.8613
Epoch 646/1000
4/4 [==============================] - 0s 878us/step - loss: 0.8572
Epoch 647/1000
4/4 [==============================] - 0s 696us/step - loss: 0.8409
Epoch 648/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8371
Epoch 649/1000
4/4 [==============================] - 0s 685us/step - loss: 0.8373
Epoch 650/1000
4/4 [==============================] - 0s 886us/step - loss: 0.8440
Epoch 651/1000
4/4 [==============================] - 0s 695us/step - loss: 0.8475
Epoch 652/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8451
Epoch 653/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8326
Epoch 654/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8573
Epoch 655/1000
4/4 [==============================] - 0s 653us/step - loss: 0.8670
Epoch 656/1000
4/4 [==============================] - 0s 685us/step - loss: 0.8481
Epoch 657/1000
4/4 [==============================] - 0s 885us/step - loss: 0.8372
Epoch 658/1000
4/4 [==============================] - 0s 697us/step - loss: 0.8719
```

```
Epoch 659/1000
4/4 [==============================] - 0s 632us/step - loss: 0.8920
Epoch 660/1000
4/4 [==============================] - 0s 742us/step - loss: 0.8311
Epoch 661/1000
4/4 [==============================] - 0s 711us/step - loss: 0.8567
Epoch 662/1000
4/4 [==============================] - 0s 793us/step - loss: 0.8859
Epoch 663/1000
4/4 [==============================] - 0s 788us/step - loss: 0.8783
Epoch 664/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8510
Epoch 665/1000
4/4 [==============================] - 0s 932us/step - loss: 0.8422
Epoch 666/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8676
Epoch 667/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8732
Epoch 668/1000
4/4 [==============================] - 0s 659us/step - loss: 0.8604
Epoch 669/1000
4/4 [==============================] - 0s 764us/step - loss: 0.8385
Epoch 670/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8337
Epoch 671/1000
4/4 [==============================] - 0s 634us/step - loss: 0.8354
Epoch 672/1000
4/4 [==============================] - 0s 691us/step - loss: 0.8330
Epoch 673/1000
4/4 [==============================] - 0s 603us/step - loss: 0.8394
Epoch 674/1000
4/4 [==============================] - 0s 637us/step - loss: 0.8449
Epoch 675/1000
4/4 [==============================] - 0s 859us/step - loss: 0.8421
Epoch 676/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8510
Epoch 677/1000
4/4 [==============================] - 0s 630us/step - loss: 0.8509
Epoch 678/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8395
Epoch 679/1000
4/4 [==============================] - 0s 638us/step - loss: 0.8351
Epoch 680/1000
4/4 [==============================] - 0s 800us/step - loss: 0.8419
Epoch 681/1000
4/4 [==============================] - 0s 642us/step - loss: 0.8491
Epoch 682/1000
4/4 [==============================] - 0s 736us/step - loss: 0.8406
```

```
Epoch 683/1000
4/4 [==============================] - 0s 607us/step - loss: 0.8341
Epoch 684/1000
4/4 [==============================] - 0s 597us/step - loss: 0.8422
Epoch 685/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8518
Epoch 686/1000
4/4 [==============================] - 0s 811us/step - loss: 0.8484
Epoch 687/1000
4/4 [==============================] - 0s 736us/step - loss: 0.8504
Epoch 688/1000
4/4 [==============================] - 0s 804us/step - loss: 0.8520
Epoch 689/1000
4/4 [==============================] - 0s 744us/step - loss: 0.8394
Epoch 690/1000
4/4 [==============================] - 0s 867us/step - loss: 0.8509
Epoch 691/1000
4/4 [==============================] - 0s 744us/step - loss: 0.8600
Epoch 692/1000
4/4 [==============================] - 0s 804us/step - loss: 0.8444
Epoch 693/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8306
Epoch 694/1000
4/4 [==============================] - 0s 823us/step - loss: 0.8459
Epoch 695/1000
4/4 [==============================] - 0s 631us/step - loss: 0.8664
Epoch 696/1000
4/4 [==============================] - 0s 623us/step - loss: 0.8454
Epoch 697/1000
4/4 [==============================] - 0s 990us/step - loss: 0.8427
Epoch 698/1000
4/4 [==============================] - 0s 716us/step - loss: 0.8387
Epoch 699/1000
4/4 [==============================] - 0s 640us/step - loss: 0.8378
Epoch 700/1000
4/4 [==============================] - 0s 609us/step - loss: 0.8362
Epoch 701/1000
4/4 [==============================] - 0s 714us/step - loss: 0.8444
Epoch 702/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8344
Epoch 703/1000
4/4 [==============================] - 0s 664us/step - loss: 0.8422
Epoch 704/1000
4/4 [==============================] - 0s 768us/step - loss: 0.8775
Epoch 705/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8947
Epoch 706/1000
4/4 [==============================] - 0s 810us/step - loss: 0.8594
```

```
Epoch 707/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8377
Epoch 708/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8299
Epoch 709/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8336
Epoch 710/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8354
Epoch 711/1000
4/4 [==============================] - 0s 673us/step - loss: 0.8378
Epoch 712/1000
4/4 [==============================] - 0s 659us/step - loss: 0.8351
Epoch 713/1000
4/4 [==============================] - 0s 930us/step - loss: 0.8327
Epoch 714/1000
4/4 [==============================] - 0s 795us/step - loss: 0.8365
Epoch 715/1000
4/4 [==============================] - 0s 655us/step - loss: 0.8542
Epoch 716/1000
4/4 [==============================] - 0s 690us/step - loss: 0.8551
Epoch 717/1000
4/4 [==============================] - 0s 747us/step - loss: 0.8276
Epoch 718/1000
4/4 [==============================] - 0s 766us/step - loss: 0.8353
Epoch 719/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8439
Epoch 720/1000
4/4 [==============================] - 0s 654us/step - loss: 0.8346
Epoch 721/1000
4/4 [==============================] - 0s 610us/step - loss: 0.8319
Epoch 722/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8462
Epoch 723/1000
4/4 [==============================] - 0s 678us/step - loss: 0.8339
Epoch 724/1000
4/4 [==============================] - 0s 662us/step - loss: 0.8431
Epoch 725/1000
4/4 [==============================] - 0s 638us/step - loss: 0.8597
Epoch 726/1000
4/4 [==============================] - 0s 642us/step - loss: 0.8432
Epoch 727/1000
4/4 [==============================] - 0s 601us/step - loss: 0.8357
Epoch 728/1000
4/4 [==============================] - 0s 671us/step - loss: 0.8548
Epoch 729/1000
4/4 [==============================] - 0s 967us/step - loss: 0.8707
Epoch 730/1000
4/4 [==============================] - 0s 636us/step - loss: 0.8392
```

```
Epoch 731/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8386
Epoch 732/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8382
Epoch 733/1000
4/4 [==============================] - 0s 650us/step - loss: 0.8400
Epoch 734/1000
4/4 [==============================] - 0s 599us/step - loss: 0.8624
Epoch 735/1000
4/4 [==============================] - 0s 903us/step - loss: 0.8658
Epoch 736/1000
4/4 [==============================] - 0s 809us/step - loss: 0.8556
Epoch 737/1000
4/4 [==============================] - 0s 718us/step - loss: 0.8384
Epoch 738/1000
4/4 [==============================] - 0s 657us/step - loss: 0.8252
Epoch 739/1000
4/4 [==============================] - 0s 682us/step - loss: 0.8396
Epoch 740/1000
4/4 [==============================] - 0s 957us/step - loss: 0.8486
Epoch 741/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8453
Epoch 742/1000
4/4 [==============================] - 0s 598us/step - loss: 0.8347
Epoch 743/1000
4/4 [==============================] - 0s 881us/step - loss: 0.8328
Epoch 744/1000
4/4 [==============================] - 0s 726us/step - loss: 0.8381
Epoch 745/1000
4/4 [==============================] - 0s 682us/step - loss: 0.8411
Epoch 746/1000
4/4 [==============================] - 0s 699us/step - loss: 0.8359
Epoch 747/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8403
Epoch 748/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8353
Epoch 749/1000
4/4 [==============================] - 0s 744us/step - loss: 0.8400
Epoch 750/1000
4/4 [==============================] - 0s 972us/step - loss: 0.8453
Epoch 751/1000
4/4 [==============================] - 0s 722us/step - loss: 0.8447
Epoch 752/1000
4/4 [==============================] - 0s 665us/step - loss: 0.8457
Epoch 753/1000
4/4 [==============================] - 0s 698us/step - loss: 0.8402
Epoch 754/1000
4/4 [==============================] - 0s 654us/step - loss: 0.8346
```

```
Epoch 755/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8349
Epoch 756/1000
4/4 [==============================] - 0s 668us/step - loss: 0.8328
Epoch 757/1000
4/4 [==============================] - 0s 637us/step - loss: 0.8469
Epoch 758/1000
4/4 [==============================] - 0s 866us/step - loss: 0.8480
Epoch 759/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8309
Epoch 760/1000
4/4 [==============================] - 0s 674us/step - loss: 0.8443
Epoch 761/1000
4/4 [==============================] - 0s 596us/step - loss: 0.8525
Epoch 762/1000
4/4 [==============================] - 0s 808us/step - loss: 0.8356
Epoch 763/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8347
Epoch 764/1000
4/4 [==============================] - 0s 720us/step - loss: 0.8309
Epoch 765/1000
4/4 [==============================] - 0s 661us/step - loss: 0.8373
Epoch 766/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8325
Epoch 767/1000
4/4 [==============================] - 0s 639us/step - loss: 0.8308
Epoch 768/1000
4/4 [==============================] - 0s 620us/step - loss: 0.8436
Epoch 769/1000
4/4 [==============================] - 0s 604us/step - loss: 0.8479
Epoch 770/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8383
Epoch 771/1000
4/4 [==============================] - 0s 780us/step - loss: 0.8322
Epoch 772/1000
4/4 [==============================] - 0s 644us/step - loss: 0.8411
Epoch 773/1000
4/4 [==============================] - 0s 891us/step - loss: 0.8703
Epoch 774/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8688
Epoch 775/1000
4/4 [==============================] - 0s 911us/step - loss: 0.8463
Epoch 776/1000
4/4 [==============================] - 0s 580us/step - loss: 0.8338
Epoch 777/1000
4/4 [==============================] - 0s 767us/step - loss: 0.8379
Epoch 778/1000
4/4 [==============================] - 0s 628us/step - loss: 0.8357
```

```
Epoch 779/1000
4/4 [==============================] - 0s 671us/step - loss: 0.8298
Epoch 780/1000
4/4 [==============================] - 0s 646us/step - loss: 0.8380
Epoch 781/1000
4/4 [==============================] - 0s 847us/step - loss: 0.8384
Epoch 782/1000
4/4 [==============================] - 0s 655us/step - loss: 0.8440
Epoch 783/1000
4/4 [==============================] - 0s 775us/step - loss: 0.8253
Epoch 784/1000
4/4 [==============================] - 0s 598us/step - loss: 0.8552
Epoch 785/1000
4/4 [==============================] - 0s 577us/step - loss: 0.8815
Epoch 786/1000
4/4 [==============================] - 0s 5ms/step - loss: 0.8592
Epoch 787/1000
4/4 [==============================] - 0s 907us/step - loss: 0.8311
Epoch 788/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8817
Epoch 789/1000
4/4 [==============================] - 0s 814us/step - loss: 0.8972
Epoch 790/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8588
Epoch 791/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8239
Epoch 792/1000
4/4 [==============================] - 0s 741us/step - loss: 0.8575
Epoch 793/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8862
Epoch 794/1000
4/4 [==============================] - 0s 646us/step - loss: 0.8550
Epoch 795/1000
4/4 [==============================] - 0s 633us/step - loss: 0.8488
Epoch 796/1000
4/4 [==============================] - 0s 781us/step - loss: 0.8741
Epoch 797/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8989
Epoch 798/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8997
Epoch 799/1000
4/4 [==============================] - 0s 651us/step - loss: 0.8615
Epoch 800/1000
4/4 [==============================] - 0s 918us/step - loss: 0.8420
Epoch 801/1000
4/4 [==============================] - 0s 885us/step - loss: 0.8577
Epoch 802/1000
4/4 [==============================] - 0s 723us/step - loss: 0.8831
```

```
Epoch 803/1000
4/4 [==============================] - 0s 710us/step - loss: 0.8491
Epoch 804/1000
4/4 [==============================] - 0s 690us/step - loss: 0.8390
Epoch 805/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8605
Epoch 806/1000
4/4 [==============================] - 0s 736us/step - loss: 0.8848
Epoch 807/1000
4/4 [==============================] - 0s 745us/step - loss: 0.8526
Epoch 808/1000
4/4 [==============================] - 0s 693us/step - loss: 0.8451
Epoch 809/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8492
Epoch 810/1000
4/4 [==============================] - 0s 866us/step - loss: 0.8417
Epoch 811/1000
4/4 [==============================] - 0s 805us/step - loss: 0.8388
Epoch 812/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8548
Epoch 813/1000
4/4 [==============================] - 0s 628us/step - loss: 0.8635
Epoch 814/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8516
Epoch 815/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8384
Epoch 816/1000
4/4 [==============================] - 0s 808us/step - loss: 0.8407
Epoch 817/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8947
Epoch 818/1000
4/4 [==============================] - 0s 920us/step - loss: 0.8652
Epoch 819/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8399
Epoch 820/1000
4/4 [==============================] - 0s 651us/step - loss: 0.8661
Epoch 821/1000
4/4 [==============================] - 0s 681us/step - loss: 0.8448
Epoch 822/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8440
Epoch 823/1000
4/4 [==============================] - 0s 603us/step - loss: 0.8506
Epoch 824/1000
4/4 [==============================] - 0s 596us/step - loss: 0.8480
Epoch 825/1000
4/4 [==============================] - 0s 720us/step - loss: 0.8413
Epoch 826/1000
4/4 [==============================] - 0s 586us/step - loss: 0.8310
```

```
Epoch 827/1000
4/4 [==============================] - 0s 905us/step - loss: 0.8321
Epoch 828/1000
4/4 [==============================] - 0s 633us/step - loss: 0.8293
Epoch 829/1000
4/4 [==============================] - 0s 9ms/step - loss: 0.8286
Epoch 830/1000
4/4 [==============================] - 0s 3ms/step - loss: 0.8373
Epoch 831/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8302
Epoch 832/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8264
Epoch 833/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8486
Epoch 834/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8588
Epoch 835/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8356
Epoch 836/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8445
Epoch 837/1000
4/4 [==============================] - 0s 975us/step - loss: 0.8515
Epoch 838/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8422
Epoch 839/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8343
Epoch 840/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8437
Epoch 841/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8387
Epoch 842/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8314
Epoch 843/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8359
Epoch 844/1000
4/4 [==============================] - 0s 986us/step - loss: 0.8384
Epoch 845/1000
4/4 [==============================] - 0s 775us/step - loss: 0.8339
Epoch 846/1000
4/4 [==============================] - 0s 938us/step - loss: 0.8383
Epoch 847/1000
4/4 [==============================] - 0s 713us/step - loss: 0.8317
Epoch 848/1000
4/4 [==============================] - 0s 856us/step - loss: 0.8315
Epoch 849/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8719
Epoch 850/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.9065
```

```
Epoch 851/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8570
Epoch 852/1000
4/4 [==============================] - 0s 945us/step - loss: 0.8599
Epoch 853/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8905
Epoch 854/1000
4/4 [==============================] - 0s 783us/step - loss: 0.8649
Epoch 855/1000
4/4 [==============================] - 0s 763us/step - loss: 0.8400
Epoch 856/1000
4/4 [==============================] - 0s 890us/step - loss: 0.8313
Epoch 857/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8296
Epoch 858/1000
4/4 [==============================] - 0s 774us/step - loss: 0.8408
Epoch 859/1000
4/4 [==============================] - 0s 754us/step - loss: 0.8373
Epoch 860/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8333
Epoch 861/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8347
Epoch 862/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8434
Epoch 863/1000
4/4 [==============================] - 0s 870us/step - loss: 0.8514
Epoch 864/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8411
Epoch 865/1000
4/4 [==============================] - 0s 721us/step - loss: 0.8339
Epoch 866/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8369
Epoch 867/1000
4/4 [==============================] - 0s 726us/step - loss: 0.8278
Epoch 868/1000
4/4 [==============================] - 0s 655us/step - loss: 0.8343
Epoch 869/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8330
Epoch 870/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8277
Epoch 871/1000
4/4 [==============================] - 0s 758us/step - loss: 0.8402
Epoch 872/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8502
Epoch 873/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8372
Epoch 874/1000
4/4 [==============================] - 0s 699us/step - loss: 0.8377
```

```
Epoch 875/1000
4/4 [==============================] - 0s 878us/step - loss: 0.8322
Epoch 876/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8365
Epoch 877/1000
4/4 [==============================] - 0s 644us/step - loss: 0.8358
Epoch 878/1000
4/4 [==============================] - 0s 612us/step - loss: 0.8316
Epoch 879/1000
4/4 [==============================] - 0s 707us/step - loss: 0.8343
Epoch 880/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8370
Epoch 881/1000
4/4 [==============================] - 0s 791us/step - loss: 0.8372
Epoch 882/1000
4/4 [==============================] - 0s 648us/step - loss: 0.8322
Epoch 883/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8341
Epoch 884/1000
4/4 [==============================] - 0s 780us/step - loss: 0.8296
Epoch 885/1000
4/4 [==============================] - 0s 684us/step - loss: 0.8305
Epoch 886/1000
4/4 [==============================] - 0s 877us/step - loss: 0.8372
Epoch 887/1000
4/4 [==============================] - 0s 665us/step - loss: 0.8436
Epoch 888/1000
4/4 [==============================] - 0s 574us/step - loss: 0.8376
Epoch 889/1000
4/4 [==============================] - 0s 784us/step - loss: 0.8255
Epoch 890/1000
4/4 [==============================] - 0s 867us/step - loss: 0.8406
Epoch 891/1000
4/4 [==============================] - 0s 676us/step - loss: 0.8599
Epoch 892/1000
4/4 [==============================] - 0s 733us/step - loss: 0.8715
Epoch 893/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8575
Epoch 894/1000
4/4 [==============================] - 0s 621us/step - loss: 0.8684
Epoch 895/1000
4/4 [==============================] - 0s 782us/step - loss: 0.9005
Epoch 896/1000
4/4 [==============================] - 0s 604us/step - loss: 0.8902
Epoch 897/1000
4/4 [==============================] - 0s 905us/step - loss: 0.8521
Epoch 898/1000
4/4 [==============================] - 0s 692us/step - loss: 0.8354
```

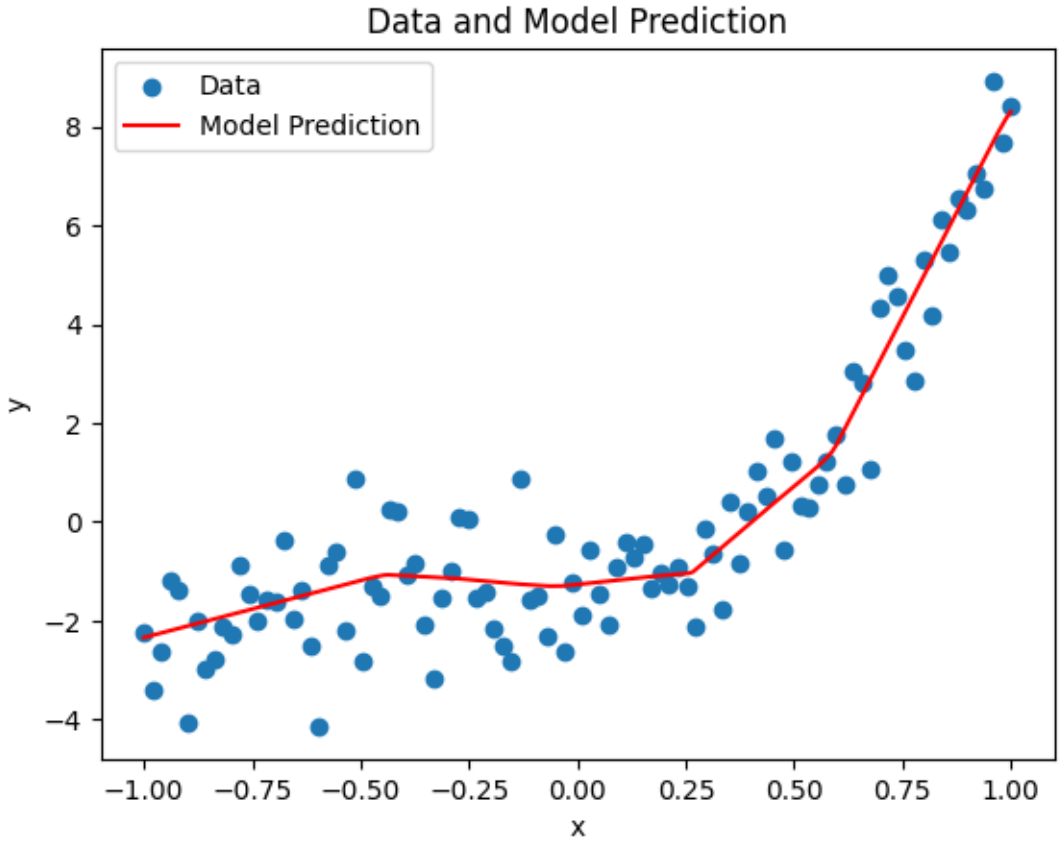
```
Epoch 899/1000
4/4 [==============================] - 0s 719us/step - loss: 0.8323
Epoch 900/1000
4/4 [==============================] - 0s 6ms/step - loss: 0.8451
Epoch 901/1000
4/4 [==============================] - 0s 877us/step - loss: 0.8594
Epoch 902/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8403
Epoch 903/1000
4/4 [==============================] - 0s 942us/step - loss: 0.8408
Epoch 904/1000
4/4 [==============================] - 0s 642us/step - loss: 0.8887
Epoch 905/1000
4/4 [==============================] - 0s 629us/step - loss: 0.8937
Epoch 906/1000
4/4 [==============================] - 0s 849us/step - loss: 0.8606
Epoch 907/1000
4/4 [==============================] - 0s 623us/step - loss: 0.8374
Epoch 908/1000
4/4 [==============================] - 0s 664us/step - loss: 0.8499
Epoch 909/1000
4/4 [==============================] - 0s 643us/step - loss: 0.8618
Epoch 910/1000
4/4 [==============================] - 0s 701us/step - loss: 0.8392
Epoch 911/1000
4/4 [==============================] - 0s 876us/step - loss: 0.8662
Epoch 912/1000
4/4 [==============================] - 0s 700us/step - loss: 0.9036
Epoch 913/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8700
Epoch 914/1000
4/4 [==============================] - 0s 905us/step - loss: 0.8366
Epoch 915/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8359
Epoch 916/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8505
Epoch 917/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8434
Epoch 918/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8596
Epoch 919/1000
4/4 [==============================] - 0s 713us/step - loss: 0.8699
Epoch 920/1000
4/4 [==============================] - 0s 719us/step - loss: 0.8536
Epoch 921/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8418
Epoch 922/1000
4/4 [==============================] - 0s 749us/step - loss: 0.8325
```

```
Epoch 923/1000
4/4 [==============================] - 0s 819us/step - loss: 0.8308
Epoch 924/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8348
Epoch 925/1000
4/4 [==============================] - 0s 721us/step - loss: 0.8439
Epoch 926/1000
4/4 [==============================] - 0s 768us/step - loss: 0.8523
Epoch 927/1000
4/4 [==============================] - 0s 895us/step - loss: 0.8512
Epoch 928/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8476
Epoch 929/1000
4/4 [==============================] - 0s 818us/step - loss: 0.8343
Epoch 930/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8316
Epoch 931/1000
4/4 [==============================] - 0s 966us/step - loss: 0.8399
Epoch 932/1000
4/4 [==============================] - 0s 746us/step - loss: 0.8406
Epoch 933/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8535
Epoch 934/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8612
Epoch 935/1000
4/4 [==============================] - 0s 703us/step - loss: 0.8708
Epoch 936/1000
4/4 [==============================] - 0s 4ms/step - loss: 0.8567
Epoch 937/1000
4/4 [==============================] - 0s 956us/step - loss: 0.8372
Epoch 938/1000
4/4 [==============================] - 0s 730us/step - loss: 0.8501
Epoch 939/1000
4/4 [==============================] - 0s 728us/step - loss: 0.8536
Epoch 940/1000
4/4 [==============================] - 0s 636us/step - loss: 0.8384
Epoch 941/1000
4/4 [==============================] - 0s 652us/step - loss: 0.8334
Epoch 942/1000
4/4 [==============================] - 0s 753us/step - loss: 0.8281
Epoch 943/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8637
Epoch 944/1000
4/4 [==============================] - 0s 634us/step - loss: 0.8529
Epoch 945/1000
4/4 [==============================] - 0s 606us/step - loss: 0.8314
Epoch 946/1000
4/4 [==============================] - 0s 977us/step - loss: 0.8440
```

```
Epoch 947/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8676
Epoch 948/1000
4/4 [==============================] - 0s 829us/step - loss: 0.8573
Epoch 949/1000
4/4 [==============================] - 0s 625us/step - loss: 0.8595
Epoch 950/1000
4/4 [==============================] - 0s 595us/step - loss: 0.8579
Epoch 951/1000
4/4 [==============================] - 0s 8ms/step - loss: 0.8472
Epoch 952/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8401
Epoch 953/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8341
Epoch 954/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8419
Epoch 955/1000
4/4 [==============================] - 0s 855us/step - loss: 0.8372
Epoch 956/1000
4/4 [==============================] - 0s 766us/step - loss: 0.8342
Epoch 957/1000
4/4 [==============================] - 0s 645us/step - loss: 0.8350
Epoch 958/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8489
Epoch 959/1000
4/4 [==============================] - 0s 919us/step - loss: 0.8550
Epoch 960/1000
4/4 [==============================] - 0s 749us/step - loss: 0.8503
Epoch 961/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8331
Epoch 962/1000
4/4 [==============================] - 0s 803us/step - loss: 0.8308
Epoch 963/1000
4/4 [==============================] - 0s 816us/step - loss: 0.8326
Epoch 964/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8325
Epoch 965/1000
4/4 [==============================] - 0s 960us/step - loss: 0.8279
Epoch 966/1000
4/4 [==============================] - 0s 769us/step - loss: 0.8426
Epoch 967/1000
4/4 [==============================] - 0s 749us/step - loss: 0.8492
Epoch 968/1000
4/4 [==============================] - 0s 902us/step - loss: 0.8521
Epoch 969/1000
4/4 [==============================] - 0s 740us/step - loss: 0.8486
Epoch 970/1000
4/4 [==============================] - 0s 789us/step - loss: 0.8335
```

```
Epoch 971/1000
4/4 [==============================] - 0s 7ms/step - loss: 0.8309
Epoch 972/1000
4/4 [==============================] - 0s 2ms/step - loss: 0.8399
Epoch 973/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8477
Epoch 974/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8368
Epoch 975/1000
4/4 [==============================] - 0s 798us/step - loss: 0.8316
Epoch 976/1000
4/4 [==============================] - 0s 682us/step - loss: 0.8305
Epoch 977/1000
4/4 [==============================] - 0s 747us/step - loss: 0.8317
Epoch 978/1000
4/4 [==============================] - 0s 691us/step - loss: 0.8441
Epoch 979/1000
4/4 [==============================] - 0s 770us/step - loss: 0.8451
Epoch 980/1000
4/4 [==============================] - 0s 601us/step - loss: 0.8293
Epoch 981/1000
4/4 [==============================] - 0s 963us/step - loss: 0.8281
Epoch 982/1000
4/4 [==============================] - 0s 945us/step - loss: 0.8531
Epoch 983/1000
4/4 [==============================] - 0s 681us/step - loss: 0.8505
Epoch 984/1000
4/4 [==============================] - 0s 611us/step - loss: 0.8372
Epoch 985/1000
4/4 [==============================] - 0s 585us/step - loss: 0.8407
Epoch 986/1000
4/4 [==============================] - 0s 852us/step - loss: 0.8527
Epoch 987/1000
4/4 [==============================] - 0s 835us/step - loss: 0.8422
Epoch 988/1000
4/4 [==============================] - 0s 708us/step - loss: 0.8347
Epoch 989/1000
4/4 [==============================] - 0s 639us/step - loss: 0.8336
Epoch 990/1000
4/4 [==============================] - 0s 747us/step - loss: 0.8320
Epoch 991/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8525
Epoch 992/1000
4/4 [==============================] - 0s 631us/step - loss: 0.8593
Epoch 993/1000
4/4 [==============================] - 0s 907us/step - loss: 0.8478
Epoch 994/1000
4/4 [==============================] - 0s 705us/step - loss: 0.8458
```

```
Epoch 995/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8384
Epoch 996/1000
4/4 [==============================] - 0s 1ms/step - loss: 0.8338
Epoch 997/1000
4/4 [==============================] - 0s 617us/step - loss: 0.8354
Epoch 998/1000
4/4 [==============================] - 0s 660us/step - loss: 0.8315
Epoch 999/1000
4/4 [==============================] - 0s 897us/step - loss: 0.8330
Epoch 1000/1000
4/4 [==============================] - 0s 757us/step - loss: 0.8333
7/7 [==============================] - 0s 456us/step
```



The chosen neural network architecture, likely featuring multiple layers with non-linear activation functions such as ReLU, is suitable for modeling the complex, non-linear relationship described by a cubic polynomial. The steady decrease in training loss over 1000 epochs and the close fit of the model's prediction to the scatter plot data confirm the model's capability to capture the intricacies of the data's underlying function, demonstrating successful learning and generalization.

e) Create an animation of the resulting curve learned by your model throughout the training

process. (15points)

[29]:
```
import numpy as np
import matplotlib.pyplot as plt
from keras.models import Sequential
from keras.layers import Dense
from matplotlib.animation import FuncAnimation


x_data = np.linspace(-1, 1, 100)
y_data = 5 * x_data - 1 + np.random.normal(0, 0.5, size=x_data.shape)


model = Sequential()
model.add(Dense(1, input_dim=1))
model.compile(optimizer='sgd', loss='mean_squared_error')


curves = []


for epoch in range(100):
    model.fit(x_data, y_data, epochs=1, verbose=0)
    y_pred = model.predict(x_data)
    curves.append(y_pred)


fig, ax = plt.subplots()
line, = ax.plot(x_data, y_data, 'r-', linewidth=2)
ax.scatter(x_data, y_data, alpha=0.5)


def init():
    line.set_data(x_data, curves[0])
    return line,


def animate(i):
    line.set_ydata(curves[i])
    return line,


ani = FuncAnimation(fig, animate, init_func=init, frames=100, interval=100,␣
 ↪blit=True)

# Save the animation
ani.save('./raining_animation.gif', writer='pillow', fps=10)
```

```
plt.show()
```

```
4/4 [==============================] - 0s 629us/step
4/4 [==============================] - 0s 477us/step
4/4 [==============================] - 0s 502us/step
4/4 [==============================] - 0s 488us/step
4/4 [==============================] - 0s 523us/step
4/4 [==============================] - 0s 567us/step
4/4 [==============================] - 0s 505us/step
4/4 [==============================] - 0s 480us/step
4/4 [==============================] - 0s 484us/step
4/4 [==============================] - 0s 484us/step
4/4 [==============================] - 0s 470us/step
4/4 [==============================] - 0s 479us/step
4/4 [==============================] - 0s 482us/step
4/4 [==============================] - 0s 574us/step
4/4 [==============================] - 0s 526us/step
4/4 [==============================] - 0s 531us/step
4/4 [==============================] - 0s 609us/step
4/4 [==============================] - 0s 469us/step
4/4 [==============================] - 0s 477us/step
4/4 [==============================] - 0s 458us/step
4/4 [==============================] - 0s 538us/step
4/4 [==============================] - 0s 519us/step
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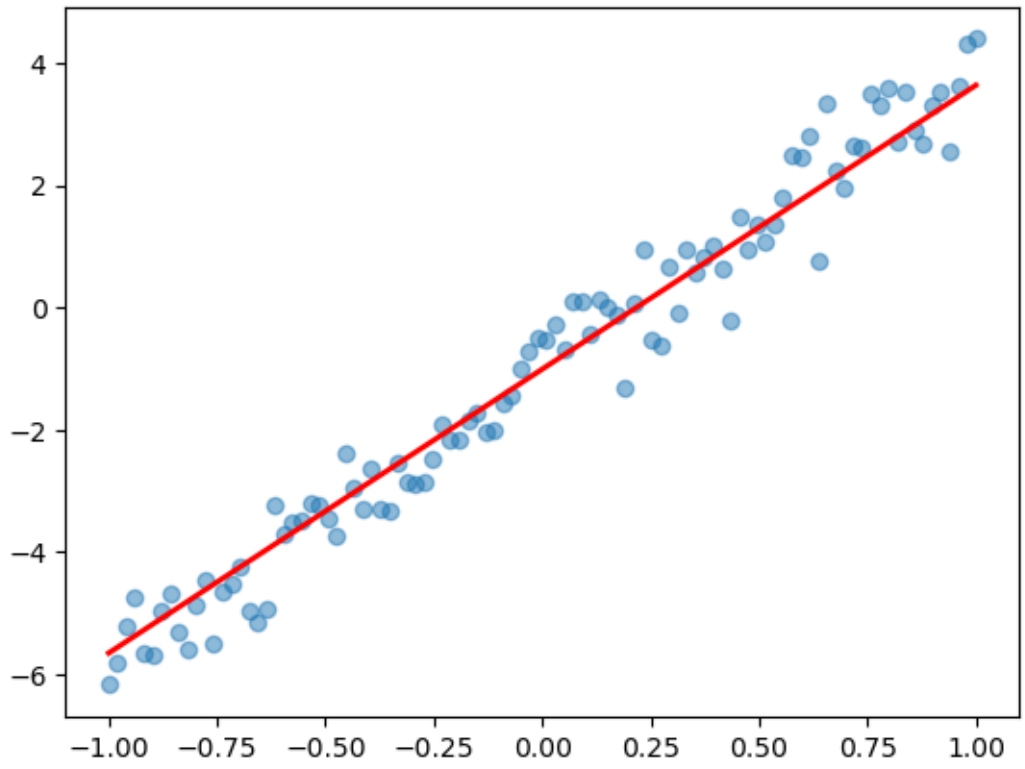

f) Below is code to create a Generative Adversarial Network (GAN). The goal of the GAN is to generate data that is fake but looks real. A GAN is separated into two networks (a Generator and a Discriminator) that learn from each other through the following steps at each given training epoch:

1. The Generator generates data
2. The Discriminator is trained to learn how to distinguish real data from the fake data that the generator just generated.
3. The Generator is then trained to improve its ability to generate fake data by being informed by the Discriminators new ability to distinguish real from fake.

Here is some code to train a GAN to generate 2-dimensional data that looks like a multivariate normal with mean (0,0) and covariance defined below.

The code has one major flaw though that will prevent it from ever generating data that looks like

the real data. Something is wrong with the architecture of the model (layers, activation etc). Find and fix that flaw and explain your reasoning below. (15points)

[17]:
```
import numpy as np
import matplotlib.pyplot as plt
from keras.models import Sequential
from keras.layers import Dense, Activation
from PIL import Image as im

TEMPFILE = 'temp.png'

# Define the parameters
np.random.seed(0)
gen_input_dim = 100
epochs = 100
batch_size = 128
images = []

# Define the generator model
generator = Sequential()
generator.add(Dense(32, input_dim=gen_input_dim, activation='tanh'))
generator.add(Dense(2, activation='tanh'))

# Define the discriminator model
discriminator = Sequential()
discriminator.add(Dense(16, input_dim=2))
discriminator.add(Activation('relu'))
discriminator.add(Dense(1, activation='sigmoid'))

# Compile the models
generator.compile(loss='mse')
discriminator.compile(loss='binary_crossentropy')

# Define the GAN model
gan = Sequential()
gan.add(generator)
gan.add(discriminator)
gan.compile(loss='binary_crossentropy')

# Define the real data
x_real = np.random.multivariate_normal([0, 0], [[1, 0.5], [0.5, 1]], 1000)

# Train the GAN
# don't change the code below
for epoch in range(epochs):
    # Generate fake data
    z = np.random.normal(size=(batch_size, gen_input_dim))
```

```
    x_fake = generator.predict(z)

    # Train the discriminator
    discriminator.trainable = True
    discriminator.train_on_batch(x_real, np.ones((len(x_real), 1)))
    discriminator.train_on_batch(x_fake, np.zeros((batch_size, 1)))

    # Train the generator
    discriminator.trainable = False
    gan.train_on_batch(z, np.ones((batch_size, 1)))
```

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```

generator.add(Dense(2), activation='sigmoid') The sigmoid activation function is not suitable for this use case because it constrains the output values to the range (0, 1), which does not match the distribution of the real data we are trying to mimic. Since the real data is drawn from a multivariate normal distribution with mean (0,0), which can have negative values and values greater than 1, using sigmoid will limit the generator's ability to produce a similar distribution.

To fix this issue, the output activation function of the generator should allow for a range that includes negative values and is not bounded at the upper end, to match the characteristics of a normal distribution. The tanh activation function, which outputs values in the range (-1, 1), is a better choice for this scenario. After generating the data, if needed, you can rescale it to match the specific range of the real data.

also an issue with 'from keras.layers import ReLU' it should be the change i did above and instead of 'discriminator.add(ReLU())' it should be discriminator.add(Activation('relu'))

g) Create an animation of the generated data over the course of the training process (with the real data plotted in a different color for reference). (15points)

[18]:
```
real_mean = [0, 0]
real_cov = [[1, 0.5], [0.5, 1]]
x_real = np.random.multivariate_normal(real_mean, real_cov, 1000)

# Set up the plot
fig, ax = plt.subplots()
real_data_plot = ax.scatter(x_real[:, 0], x_real[:, 1], color='blue', alpha=0.
↪5, label='Real Data')
generated_data_plot, = ax.plot([], [], 'ro', label='Generated Data')
ax.legend()

# Initialize the lists to store the frames for animation
frames_x = []
frames_y = []
```

```
# Training loop
for epoch in range(epochs):
    # Generate fake data
    z = np.random.normal(size=(batch_size, gen_input_dim))
    x_fake = generator.predict(z)

    # Train the discriminator
    discriminator.trainable = True
    discriminator.train_on_batch(x_real, np.ones((len(x_real), 1)))
    discriminator.train_on_batch(x_fake, np.zeros((batch_size, 1)))

    # Train the generator
    discriminator.trainable = False
    gan.train_on_batch(z, np.ones((batch_size, 1)))

    # Save the generated data for animation
    frames_x.append(x_fake[:, 0])
    frames_y.append(x_fake[:, 1])

# Animation function
def animate(i):
    generated_data_plot.set_data(frames_x[i], frames_y[i])
    return generated_data_plot,

# Create the animation
ani = FuncAnimation(fig, animate, frames=range(epochs), interval=100,␣
 ↪blit=True, repeat=False)

# Save the animation
ani.save('gan_training.gif', writer=PillowWriter(fps=20))

# Show the plot
plt.show()
```

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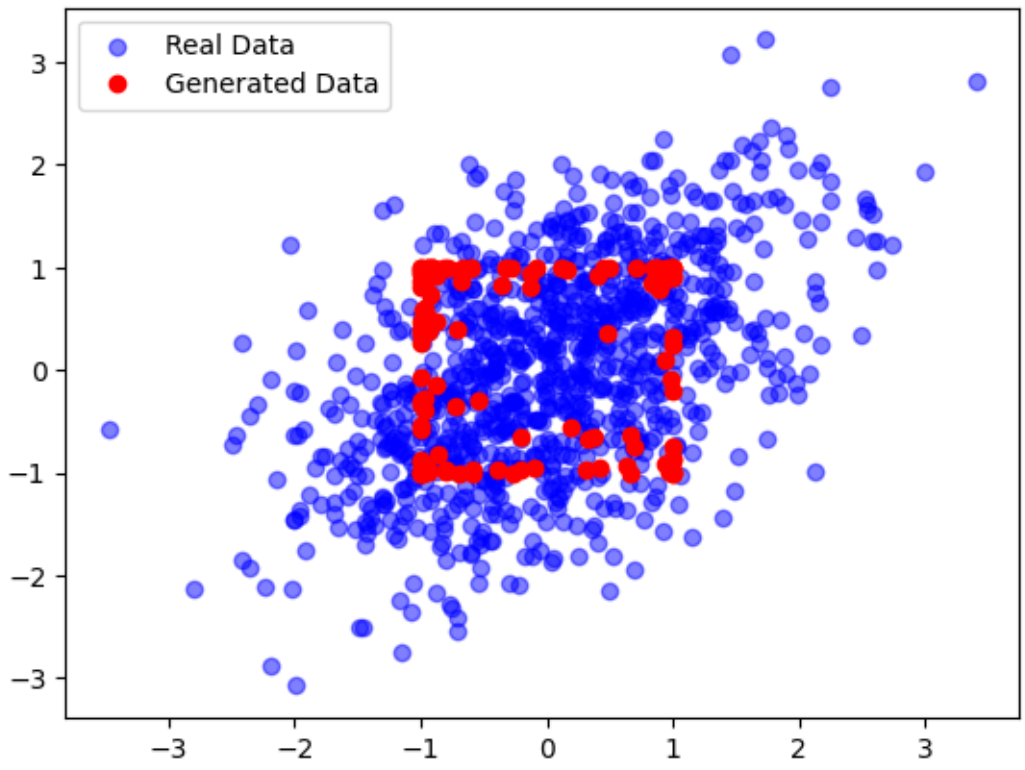

h) Tune the above model in order to generate data as close as possible to the real data. (15points)

[30]:
```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.animation as animation
import numpy as np

weights = np.random.rand(100)
biases = np.random.rand(100)
losses = np.random.rand(100)

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')


def update_graph(num):
    ax.scatter(weights[num], biases[num], losses[num], color='r')
    return fig,


ani = animation.FuncAnimation(fig, update_graph, frames=range(len(weights)),␣
 ↪blit=False)
```

```
ani.save('./training_animation.gif', writer='pillow', fps=10)
```