CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

(b) Find the solution with $a_0 = 1$.

Problem 1

$a_n = 3a_{n-1} + 2^n.$

$a_n = -2^{n+1}$, $a_0 = 1$

a)

Step 1:

$a_{(n-1)} \stackrel{?}{=} -2^{n-1+1} = -2^n$

Step 2:

$a_n = a_n$

$-2^{n+1} \stackrel{?}{=} 3 \cdot a_{n-1} + 2^n$

$-2^{n+1} \stackrel{?}{=} 3 \cdot (-2^n) + 2^n$

$-2^{n+1} \stackrel{?}{=} -2^n(3-1)$

$-2^{n+1} \stackrel{?}{=} -2^n \cdot 2$

$-2^{n+1} = -2^{n+1}$ ✓

Yes. It is a solution.

b)

Step 1: Homogeneous part.

$a_n(h) = 3a_{n-1}$

$a_n = r$

$a_{n-1} = 1$

$r = 3$

$a_n^{(h)} = C.3^n$

Step 2: Particular part.

$a_n^{(p)} = A.2^n$

$a_{n-1} = A.2^{n-1}$

$A.2^n = 3.(A.2^{n-1}) + 2^n$

$A.2^n = \frac{3A.2^n}{2} + 2^n$

$A = \frac{3A}{2} + 1$

$A - \frac{3A}{2} = 1$

$A = -2$

$a_n^{(p)} = -2.2^n$

Step 3:

$a_n = C.3^n - 2.2^n$

$a_0 = 1 = C.3^0 - 2.2^0$

$1 = C - 2$

$C = 3$

Result:

$a_n = 3.3^n - 2.2^n$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.

Problem 2

$f(n) = 4f(n-1) - 4f(n-2) + n^2$, $f(0) = 2$, $f(1) = 5$

$f(n) = f(n)^{(h)} + f(n)^{(p)}$

Step 1: Homogeneous part:

$f(n)^{(h)} = 4f(n-1) - 4f(n-2)$

$f(n) = r^2$
$f(n-1) = r$ $\rightarrow$ $r^2 - 4r + 4 = 0 \Rightarrow r_1 = r_2 = 2$
$f(n-2) = 1$

$f(n)^{(h)} = c_1 \cdot 2^n + c_2 \cdot 2^n \cdot n$

Step 2: Particular Part

$f(n) = An^2 + Bn + C$

$f(n-1) = A(n-1)^2 + B(n-1) + C = An^2 - 2An + 1 + Bn - B + C$

$f(n-2) = A(n-2)^2 + B(n-2) + C = An^2 - 4An + 4 + Bn - 2B + C$

$An^2 + Bn + C = 4(An^2 - 2An + 1 + Bn - B + C) - 4(An^2 - 4An + 4 + Bn - 2B + C) + n^2$

$An^2 + Bn + C = \cancel{4An^2} - 8An + 4 + 4Bn - 4B + \cancel{4C} - \cancel{4An^2} + 16An - 16 - 4Bn + 8B - \cancel{4C} + n^2$

$An^2 + Bn + C = 8An + 4B - 12 + n^2$

$An^2 + Bn + C - 8An - 4B + 12 = n^2$

$An^2 + (B - 8A)n + (C - 4B + 12) = n^2$

$A = 1$

$B - 8A = 0$

$B = 8$

$C - 4B + 12 = 0$

$C = 20$

Step 3

$f(n) = c_1 \cdot 2^n + c_2 \cdot 2^n \cdot n + n^2 + 8n + 20$

$f(0) = 2 = c_1 \cdot 2^0 + c_2 \cdot 2^0 \cdot 0 + 0^2 + 8 \cdot 0 + 20$

$2 = c_1 + 20$

$c_1 = -18$

$f(1) = 5 = -18 \cdot 2^1 + c_2 \cdot 2^1 \cdot 1 + 1^2 + 8 \cdot 1 + 20$

$5 = 2c_2 + 11$

$c_2 = -3$

Result:

$f(n) = -18 \cdot 2^n - 3 \cdot 2^n \cdot n + n^2 + 8n + 20$

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

$\boxed{\text{Problem 3}}$

$a_n = 2a_{n-1} - 2a_{n-2}$ → It is homogeneous.

Step 1: $a_n = r^2$
$a_{n-1} = r$
$a_{n-2} = 1$

$\Rightarrow r^2 = 2r - 2$
$r^2 - 2r + 2 = 0$

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} \Rightarrow \begin{array}{l} r_1 = 1 + i \\ r_2 = 1 - i \end{array}$$

$\rightarrow$ Characteristic Roots

→ Part b is not available.