Code : 105202

B.Tech 2nd Semester Exam., 2019

MATHEMATICS—II

(Probability and Statistics)

(New Course)

Time : 3 hours   *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

(v) The relevant data are given at the end of this question paper.

1. Answer any *seven* of the following questions : 2×7=14

(a) If A and B are independent, then show that $\bar{A}$ and B are also independent.

(b) Let A and B are two possible outcomes of an experiment and suppose $P(A) = 0 \cdot 4$, $P(A \cup B) = 0 \cdot 7$ and $P(B) = k$.

(i) For what choice of 'k' A and B are mutually exclusive?

(ii) For what choice of 'k' A and B are independent?

(c) If n people are seated at a round table, what is the chance that two named individuals will be next to each other?

(d) A five figure number is formed by digits 0, 1, 2, 3, 4 (without repetition). Find the probability that the number formed is divisible by 4.

(e) A point is chosen at random out of four points in 3-dimensional space (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1). Let E_i = i-th coordinate is 1. Check if the events E_1, E_2 and E_3 are independent.

(f) The members of a consulting firm rent cars from three rental agencies; 60% from agency A, 30% from agency B and remaining from agency C. If 9% of the cars from agency A need a tune-up, 20% from B, and 6% of the cars from C need a tune-up, what is the probability that a rental car needs a tune-up, when it came from agency B?

(g) If X and Y are two independent random variables with $E(X) = \alpha$, $E(X^2) = \beta$ and $E(Y^k) = a_k$, $k = 1, 2, 3, 4$.

Find $E(XY + Y^2)^2$.

(h) A discrete random variable X has probability mass function

$$f(x) = k\left(\frac{1}{2}\right)^x, \; x = 1, 2, 3, 4, \cdots, \cdots, \cdots, \cdots$$

Find the value of k.

(i) The incidence of occupational disease in an industry is such that workers have a 25% chance of suffering from it. What is the probability that out of 13 workers chosen at random, 5 or more will suffer from the disease?

(j) A random variable X follows the binomial distribution with $B\left(40, \frac{1}{4}\right)$.

Use Chebyshev's inequality to find bound for $P(|X-10|>10)$.

2. (a) The following marks have been obtained by a class of students in Statistics (out of 100) :

Paper—I :	80	45	55	56	58	60	65	68	70	75	85
Paper—II :	82	56	50	48	60	62	64	65	70	74	90

Compute the coefficient of correlation for the above data.

(b) Fit a second degree parabola to the following data, x is the independent variable :

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

7+7=14

3. (a) A language class has only three students A, B, C and they independently attend the classes. The probability of attendance of A, B and C on any given day is $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ respectively. Find the probability that total number of attendance in two consecutive days is exactly 3.

(b) A bag contains n white and 2 black balls. Balls are drawn at random one by one without replacement until a black ball is drawn. If K white balls are drawn before first black ball, a man is to receive ₹ K^2. Find his expected gain.

7+7=14

4. (a) The lines of regression for a bivariate population are $Y = X$ and $4X - Y = 3$, and that the second moment of X about the origin is 2. Find (i) the correlation coefficient (r) and (ii) the standard deviation of Y.

(b) Ball bearings of a given brand weighs 15 gram with a standard deviation of 0·5 gram. What is the probability that two lots of 1000 ball bearings each will differ in weight by more than 50 gram? 7+7=14

5. For any three events A, B and C, show that—

(i) $P\left\{\frac{(A \cup B)}{C}\right\} = P\left(\frac{A}{C}\right) + P\left(\frac{B}{C}\right) - P\left\{\frac{(A \cap B)}{C}\right\}$;

(ii) $P\left\{\frac{(A \cap \overline{B})}{C}\right\} = P\left(\frac{A}{C}\right) - P\left\{\frac{(A \cap B)}{C}\right\}$. 7+7=14

6. A continuous random variable has probability function as

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Check whether this is a probability density function.

(b) Find the mean of X.

(c) Find the variance of X.

(d) Find a and b such that

$$P(X \le a) = P(X > a)$$

and $P(X > b) = 0 \cdot 05$.

(e) Find $P(0 \cdot 2 < x < 0 \cdot 5)$.

(f) Find $P(x < 0 \cdot 3)$.

(g) Find $P\left(\frac{x > 0 \cdot 75}{x > 0 \cdot 50}\right)$.

2×7=14

7. Let X and Y be continuous random variables having joint density function

$$f(x, y) = \begin{cases} C(x^2 + y^2) & 0 < x < 1,\ 0 < y < 1, \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Find—

(a) C;

(b) $P\left(X < \frac{1}{2}, Y < \frac{1}{2}\right)$;

(c) $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$;

(d) $P(|X| > 1)$. 3½×4=14

8. A random variable follows Gamma (α, β) distribution with probability density function

$$f_X(x) = \begin{cases} \frac{1}{\Gamma\alpha\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}, & x \ge 0 \\ 0 \quad , & \text{elsewhere} \end{cases}$$

If X and Y are independent gamma (α_1, β) and gamma (α_2, β), then find the probability density function $X + Y$. 14

9. *(a)* It has been found from experience that the mean breaking strength of a particular brand of thread is 275·6 gram with a standard deviation of 39·7 gram. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253·2 gram. Can one conclude at a significance level *(i)* 0·05 and *(ii)* 0·01 that the thread has become inferior?

(b) On an elementary school examination in spelling, the mean grade of 32 boys was 72 with a standard deviation of 8, while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis at a *(i)* 0·05 and *(ii)* 0·01 level of significance that the girls are better in spelling than the boys. 7+7=14

[Given that

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{x^2}{2}}\, dx,$$

then $f(1 \cdot 645) = 0 \cdot 45$, $f(1 \cdot 96) = 0 \cdot 475$,
$f(2 \cdot 0) = 0 \cdot 4772$, $f(2 \cdot 23) = 0 \cdot 4871$,
$f(2 \cdot 33) = 0 \cdot 4900$, $f(2 \cdot 50) = 0 \cdot 4938$,
$f(2 \cdot 58) = 0 \cdot 4950$].

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