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***Abstract*—An integral equation formulation for a thin dielectric sheet is presented using the surface equivalence theorem. The advantageous properties of plasma waves, chief among them surface wave propagation are briefly discussed. Numerical results are presented to illustrate the scattering properties of the sheet with different material properties.**

I. Introduction

The emergence of high-precision nanoscale fabrication techniques has led to an increased interest in two-dimensional (2D) materials and electronic systems of late, especially in the terahertz frequency regime. One particular intriguing example is the two-dimensional electron gas (2DEG) existing in the multilayer stack semiconductor structures like high-electron mobility transistors (HEMTs), with remarkable electrical properties such as high conductivity with high values of free-carrier densities. The 2DEG is extremely thin as compared to other layer thicknesses in the stack and therefore, its scattering properties can be found by modeling it as a two-dimensional plasma. An interaction between an external electromagnetic radiation and plasma results in 2D plasmons (surface waves). In this paper, we formulate the scattering response of an infinitesimally thin flat layer of plasma surrounded by free-space using the surface equivalence theorem.

II. Theory

A. Surface Plasmons

The electrical properties of any material can be characterized by a frequency-dependent permittivity:

$$\varepsilon(\omega) = \varepsilon_r - j\frac{\sigma(\omega)}{\omega\varepsilon_0} \tag{1}$$

where ε_r is the permittivity of the material at dc frequency and ω is the conductivity given by a Drude-type model [1]:

$$\sigma(\omega) = \frac{Ne^2\tau}{m^*}\frac{1}{1+j\omega\tau} \tag{2}$$

The parameters e and m^* are the charge and effective mass of electron respectively, N is free-charge density, and τ is the scattering time of free charges in the 2DEG and can determined with the physical quantity, mobility μ_e can be computed from:

$$\tau = \frac{m^*\mu_e}{e}. \tag{3}$$

The dispersion relation of the 2D plasma waves can be written in terms plasma frequency ω_p and the wave-vector k:

$$\omega_p = \sqrt{\frac{2\pi e^2 N}{m^*}k}. \tag{4}$$

B. Surface Integral Equation

Consider a flat plasma sheet of length L and thickness t excited by a TM polarized plane wave as illustrated in Fig. 1. The plasma is assumed nonmagnetic and the dielectric constant is determined from (1)-(4). By applying the surface equivalence theorem [2, p. 328-333], the plasma sheet is replaced by an equivalent set of surface currents. For the case of 2DEG plasma, the thickness is treated in its limiting case of $t \to 0$. The resulting electric field integral equation (EFIE) in a homogeneous free-space is written as:

$$E_i = \frac{\omega\mu}{4}\int_0^L J_z(x')\left[H_0^{(2)}(k_1|x-x'|) + H_0^{(2)}(k_2|x-x'|)\right]\mathrm{d}x' \tag{5}$$

where μ is the free-space permeability, J_z is the yet-unknown surface electric current, $H_0^{(2)}(\cdot)$ is the zero-order Hankel function of the second kind and k_i with $i = 1, 2$ being the corresponding wave-vectors of the free-space and plasma respectively.

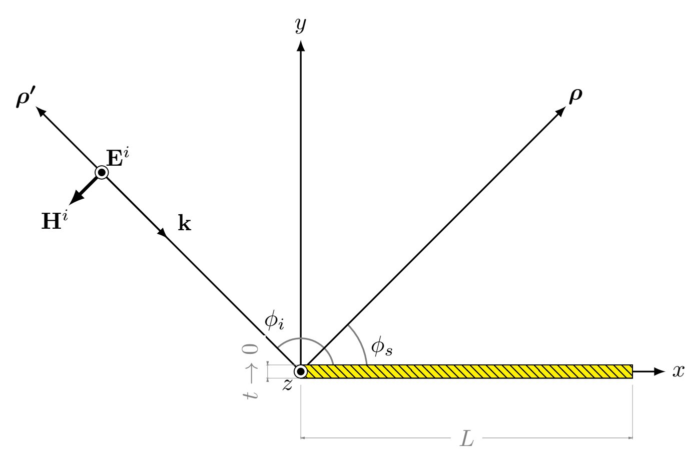

Fig. 1. Thin Plasma sheet with $\mathrm{TM_z}$ polarization

III. Numerical Results

A method of moments (MoM) solution to compute the current J_z in (5) is implemented using pulse basis functions with point matching method. The resulting integrals contain a logarithmic singularity due to self terms ($x = x'$), however, it can be easily removed through a simple step invovling integration by parts followed by using recurrence property of Hankel functions [3], [4, p. 361], resulting in a well-behaved integral:

$$Z_{self} = \frac{\omega\mu}{4}\Delta\left[H_0^{(2)}(k_1\Delta) + H_0^{(2)}(k_2\Delta)\right] + \frac{\omega\mu}{4}\int_0^{\Delta} x'\left[k_1 H_1^{(2)}(k_1 x') + k_2 H_1^{(2)}(k_2 x')\right]\mathrm{d}x' \quad (6)$$

where Δ is the width of the pulse basis functions.

A. Current Distribution

Fig. 2 shows the absolute value of the tangential surface electric current on a TM-wave excited plate of length 3λ at edge-on ($\phi_i = \pi$). A Gallium Arsenide (GaAs) based 2DEG is considered where material data was taken from measurements in [1] and the results are compared with a PEC plate of same length and a lossy sheet of dielectric constant $4 - .4j$. Fig. 3 shows the current distribution under normal incidence ($\phi_i = \pi/2$).

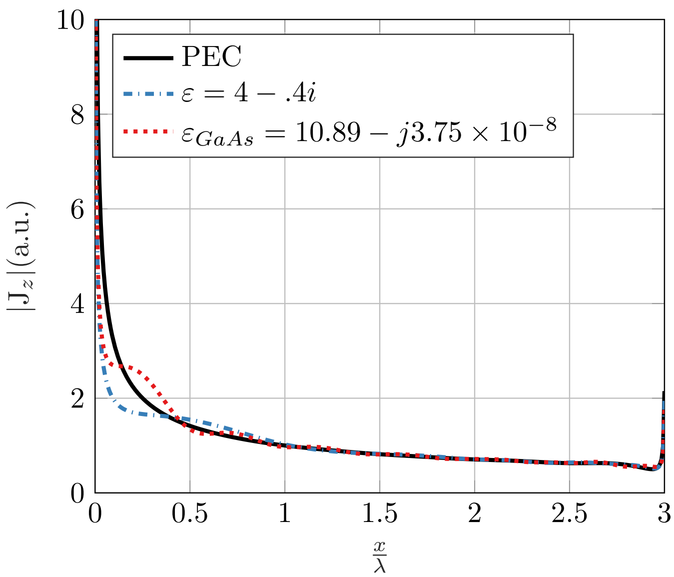

Fig. 2. Current Distributions on a 3λ plate at edge-on incidence

B. Far-field

The scattered electric field in the far-zone can be expressed using the large argument approximation of Hankel functions as:

$$\lim_{k_1|\mathbf{r}-\mathbf{r}'|\to\infty} E_z(\mathbf{r}) = -\frac{\omega\mu e^{jk_1 r - \pi/4}}{\sqrt{8\pi k_1 r}}\int_0^L J_z(\mathbf{r}')e^{jk_1 x'\cos(\phi_i)}\mathrm{d}x' \quad (7)$$

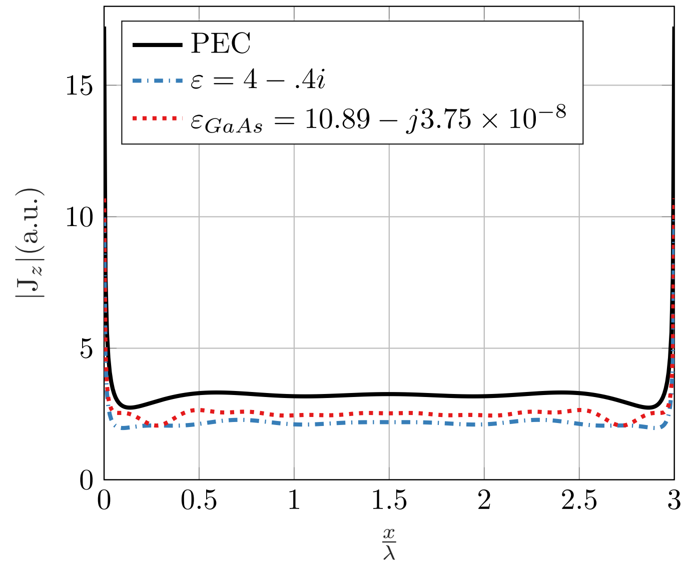

Fig. 3. Current Distributions on a 3λ plate at normal incidence

where ϕ_i is the angle of incidence. The results are shown in Fig. 4.

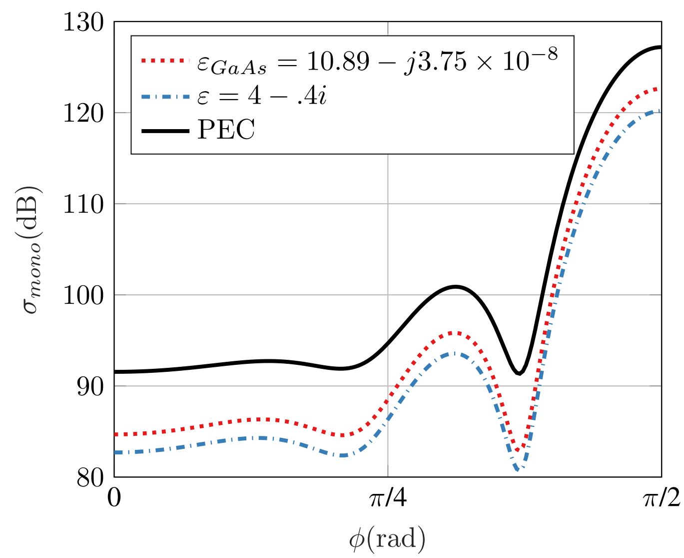

Fig. 4. Bacscetterd fields from a sheet of length 1.25λ

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