Surface wave supporting structures in the terahertz and optical frequency domains

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Outline

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- Overview
- Background
- Theory and Methods
  - Subwavelength phenomena - Dispersion relations
  - Existence of plasmonic behavior - Sommerfeld Integral analysis
  - Surface Integral equation scheme
- Applications
  - Super-resolution Imaging scheme
- Conclusions

Overview

- Plasmonics: subwavelength localization of electromagnetic (EM) fields
- Bridging the THz gap

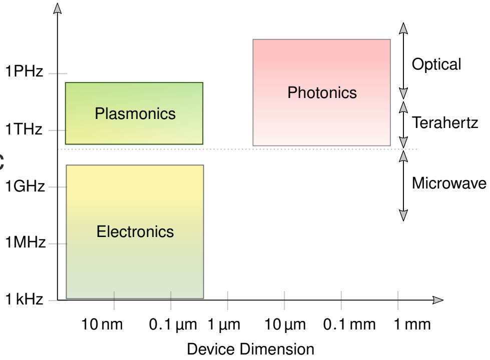


Figure: Communication Technologies at various frequencies

Overview

Terahertz 2DES

- Graphene
  - Grown separately, transfered to substrate
  - Currently not integrable to current electronics technology
  - Superior electronic properties
- Semiconductor heterostructures
  - Conventional epitaxial semiconductor device fabrication techniques
  - On-chip integration with silicon electronics

Background

Plasmonics Overview

- Interfacial wave phenomena
  - Metal-dielectric interface
  - Semiconductor heterostructure
- Surface plasmon polaritons (SPPs)
- Plasma frequency
  - Metals - Optical frequency
  - Semiconductors - Terahertz

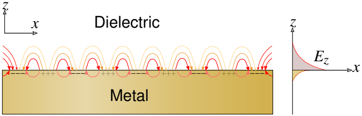


Figure: SPPs at optical frequencies

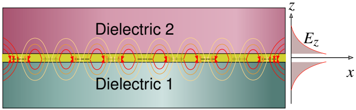


Figure: SPPs in the THz regime

Background

Surface Plasmon Polaritons

- Slow surface waves
- Reduced Wavelength
- Focusing beyond the diffraction limit

- Optical SPP

$$\mathrm{Re}\left[\varepsilon_{\text{metal}}(\omega)\right] < 0$$

- THz SPP

$$\mathrm{Im}\left[\sigma_s(\omega)\right] < 0$$

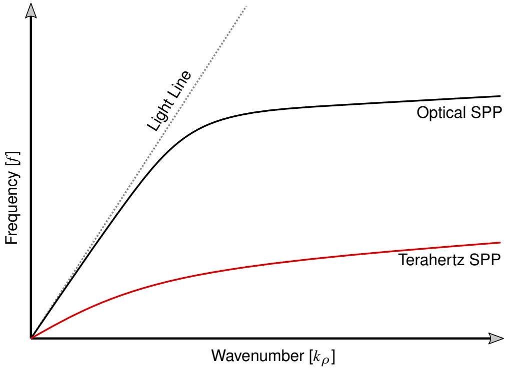


Figure: Dispersion Curve comparison

Background

Optical Nanoantennas

- Convert Localized near-field to efficient far-field radiation
- Low Q-factor
- Extremely small size
- High Purcell Factor

$$P = \frac{Q}{V}$$

- Directive radiation

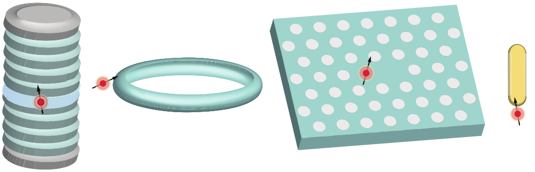
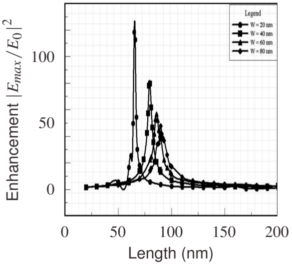


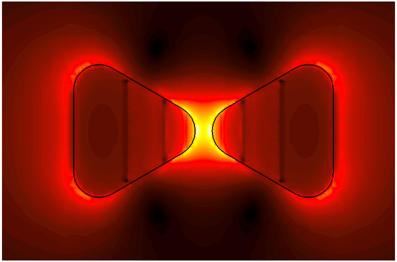
Figure: Optical resonant cavities for electric field enhancement

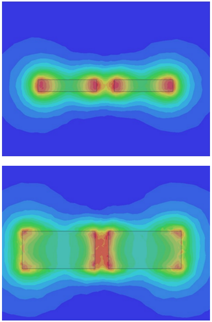
Background

Optical Nanoantennas (contd.)

- Scaled-down microwave antenna designs
- Shift of resonance with width unlike in microwave regime







Background

Two-dimensional Electron Gas (2DEG)

- Semiconductor Heterostructure in high electron mobility transistor (HEMT)
- High concentration of free electrons ($\sim 1 \times 10^{11} - 1 \times 10^{14}\,\mathrm{cm}^{-2}$)
- Very high Mobility ($\sim 1 \times 10^{3} - 1 \times 10^{6}\,\mathrm{cm}^2/\mathrm{V}/\mathrm{s}$)
- Formation of Quantum Well
    - Two-dimensional confinement of electrons

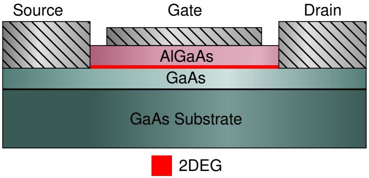


Figure: Typical GaAs/AlGaAs HEMT

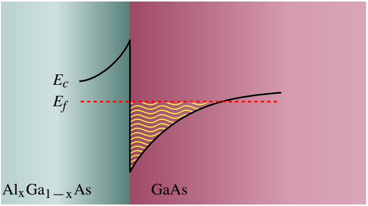
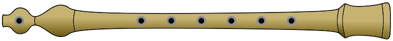


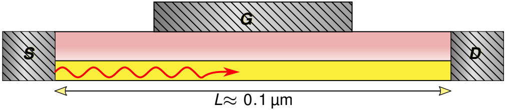
Figure: Band diagram of a GaAs/AlGaAs heterostructure

Background

2DEG (contd.)

- Plasma waves in 2DEG
- Dyakonov-Shur instability
  - Voltage bias at source and drain terminals
  - Plasma resonance
  - THz emission
- Electronic Flute
  - Tunable resonance with gate voltage





$$\lambda = \frac{c}{f}$$

$$\implies 300\mu\text{m}$$

Theory

SPP Dispersion Relation

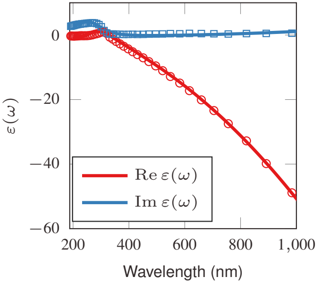
- SPP pole

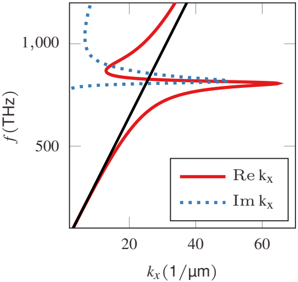
$$k_{sp} = \frac{\omega}{c}\sqrt{\frac{\varepsilon_1 \varepsilon_2(\omega)}{\varepsilon_1 + \varepsilon_2(\omega)}}$$

- Accurate material description

$$\varepsilon_2(\omega) = \varepsilon_\infty - \frac{\omega_d^2}{\omega^2 - j\gamma\omega} + \sum_{i=1}^{N} G_i(\omega)$$

$$G_i(\omega) = C_i \left[\frac{e^{j\phi_i}}{\omega_i + \omega - j\Gamma_i} + \frac{e^{-j\phi_i}}{\omega_i - \omega + j\Gamma_i} \right]$$





Theory and Methods

2DEG Circuit model

- Drude-Lorentz Surface Conductivity

$$\sigma_s = \frac{N_s e^2}{m^*} \frac{\tau}{1 + j\tau\omega}$$

N_s - Surface charge density

τ - Scattering time

m^* - Effective electron mass

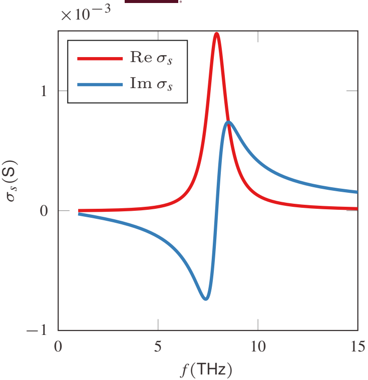
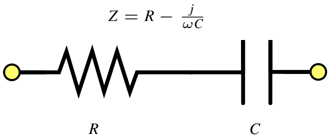


Figure: Room temperature GaN/AlGaN 2DEG surface conductivity

- Equivalent Circuit

$$\sigma_s = \frac{1}{Z} = \frac{1}{R + 1/j\omega C}$$



Theory and Methods

Dispersion Relation for a 2D Sheet

- Conductive Sheet in freespace
- TM mode surface wave

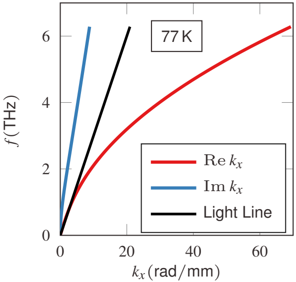
$$k_{\mathrm{P}}^{\mathrm{TM}} = \frac{\omega}{c}\sqrt{1 - \left(\frac{2}{\eta_0 \sigma_s}\right)^2}$$

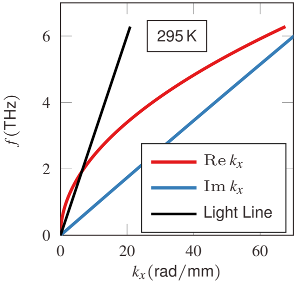
- Below plasma frequency

$$\operatorname{Im}\sigma_s < 0$$

- At low temperature

$$\operatorname{Im}|\sigma_s| \gg \operatorname{Re}|\sigma_s|$$





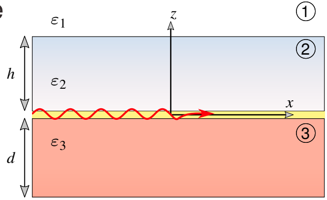
Theory and Methods

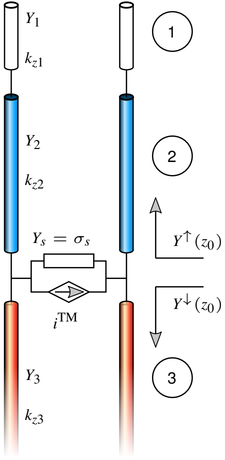
Dispersion Relation - Multilayer structures

- Equivalent transmission line (TL) network
- Dispersion relation
  - Transverse resonance condition

$$Y^{\uparrow}(z_0) + Y^{\downarrow}(z_0) + Y_{\sigma} = 0.$$

- Different configurations
  - Gated
  - Ungated
  - Backgated





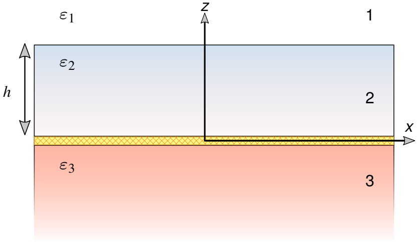
Theory and Methods

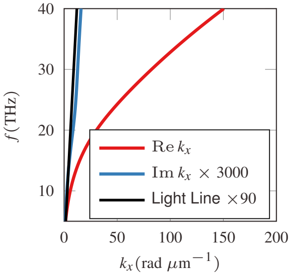
Dispersion Relation - Ungated 2DEG

$$Y^{\uparrow} = Y_2 \frac{1 - \Gamma^{\uparrow}}{1 + \Gamma^{\uparrow}}$$

$$\Gamma^{\uparrow} = \frac{Y_2 - Y_1}{Y_2 + Y_1} \mathrm{e}^{-2jk_{z2}\,h}$$

$$Y^{\downarrow} = Y_3$$





GaN/AlGaN heterostructure

N_s - $5 \times 10^{13}\,\mathrm{cm}^{-2}$

τ - $1.4 \times 10^{-10}\,\mathrm{s}$

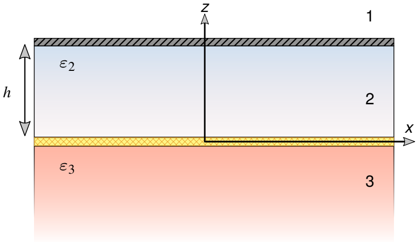
m^*- $0.2 m_e$

h - 20 nm

Theory and Methods

Dispersion Relation - Gated 2DEG

$$Y^{\uparrow} = -Y_2 \coth(k_{z2}h)$$
$$Y^{\downarrow} = Y_3$$



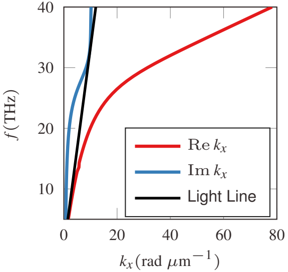
GaN/AlGaN heterostructure

N_s - $5 \times 10^{13}\,\mathrm{cm}^{-2}$

τ - $1.4 \times 10^{-10}\,\mathrm{s}$

m^*- $0.2m_e$

h - 20 nm



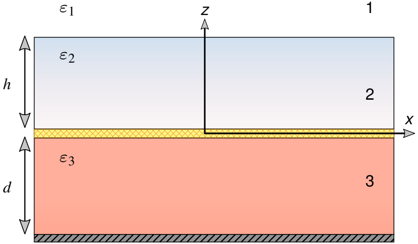
Theory and Methods

Dispersion Relation - Backgated 2DEG

$$Y^{\uparrow} = Y_2 \frac{1 - \Gamma^{\uparrow}}{1 + \Gamma^{\uparrow}}$$

$$\Gamma^{\uparrow} = \frac{Y_2 - Y_1}{Y_2 + Y_1} \mathrm{e}^{-2jk_{z2}\, h}$$

$$Y^{\downarrow} = -Y_3 \coth(k_{z3} d)$$



GaN/AlGaN heterostructure

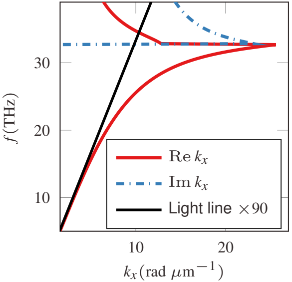
N_s - $5 \times 10^{13}\,\mathrm{cm}^{-2}$

τ - $1.4 \times 10^{-10}\,\mathrm{s}$

m^* - $0.2m_e$

h - 20 nm

d - 50 nm



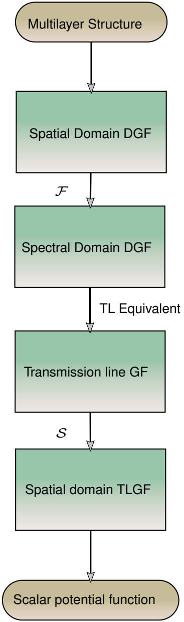
Theory and Methods

Field Computation of layered media

- Methods using discretization computationally expensive
  - Finite element method (FEM)
  - Finite Difference Time Domain (FDTD)
- Integral equation (IE) approach most suitable
  - Formulation of dyadic Green function (DGF)

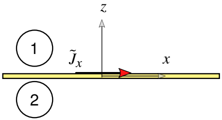
$$\mathbf{E} = \int_{r'} \underline{\underline{\mathbf{G}}}^{\mathrm{EJ}}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\,\mathrm{d}r',$$

$$\mathbf{H} = \int_{r'} \underline{\underline{\mathbf{G}}}^{\mathrm{HJ}}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\,\mathrm{d}r'.$$



Theory and Methods

Field Computation - Thin sheet



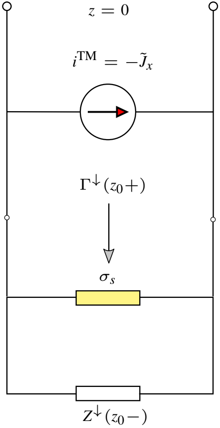
- Thin conductive sheet in free-space

$$Z^{\downarrow}(z_0^{+}) = \frac{Z_0}{1 + \sigma_s Z_0}$$

$$\Gamma^{\downarrow,\mathrm{TE}} = \frac{k_{z1} - \omega\mu_1\sigma_s}{k_{z1} + \omega\mu_1\sigma_s}$$

$$\Gamma^{\downarrow,\mathrm{TM}} = \frac{\omega\varepsilon_1 - \sigma_s k_{z1}}{\omega\varepsilon_1 + \sigma_s k_{z1}}$$

$$G_{zx}^{\mathrm{A}} = \frac{j\mu}{2}\cos\phi\,\mathcal{S}_1\left\{\frac{\Gamma^{\downarrow,\mathrm{TM}} - \Gamma^{\downarrow,\mathrm{TE}}}{k_\rho}\right\}.$$



Theory and Methods

Computed Fields - Thin sheet

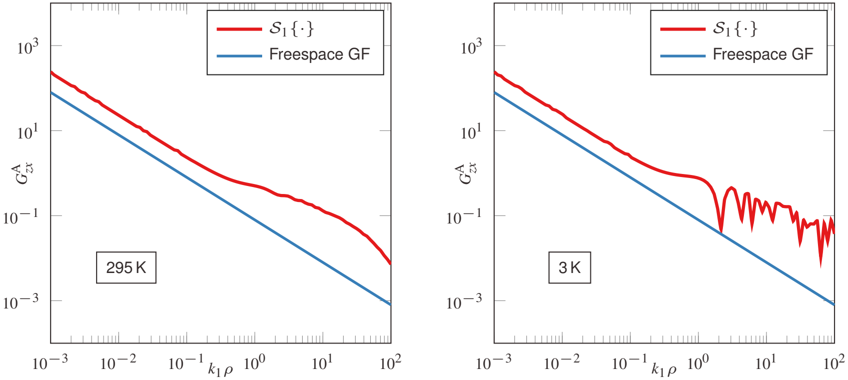


Figure: G^{A}_{zx} computed for a GaN/AlGaN based 2DEG sheet suspended in freespace at 5.6 THz. The surface conductivity of the sheet is (a) $\sigma_s = 7.6 \times 10^{-5} - j2.98 \times 10^{-3}$S at room temperature (300 K), and (b) $\sigma_s = 7.6 \times 10^{-8} - j2.98 \times 10^{-3}$S at 3 K

Theory and Methods

Thin Sheet Simulation

- Volume Integral formulation

$$\mathbf{A} = \frac{\mu}{4\pi} \int_V \mathbf{J}_v(\mathbf{r}') \frac{e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \,\mathrm{d}v'$$

$$\mathbf{E}_1^{scat} = -\frac{j\omega}{k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\mathbf{A}$$

$$\mathbf{J}_v = \frac{-jk_1}{Z_0}(\varepsilon_2 - 1)\mathbf{E}_2$$

- Surface current J_s approximated from J_v

- Impedance (Leontovich) Boundary Condition

$$\mathbf{E}_{tan} = \eta Z_0 \hat{\mathbf{n}} \times \mathbf{H}$$

$$E^i = \eta Z_0 J_s(x') + \frac{\omega\mu}{4} \int_l J_s(x') H_0^{(2)}(k_2|x-x'|) \,\mathrm{d}x'$$

Theory and Methods

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Proposed Surface Integral Equation (SIE) scheme

- Surface Equivalence Theorem

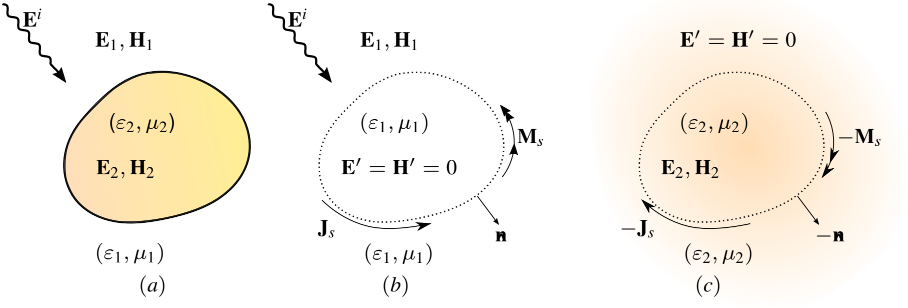

Figure: (a). Actual and its equivalent models for the (b) external and, (c) Internal region

Theory and Methods

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TM_z SIE for Thin Flat Sheet

$$\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{0}$$

$$E_i = \frac{\omega}{4} \int_L J_z(x') \left[H_0^{(2)}(k_1|x - x'|) + H_0^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

$$\hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}$$

$$H_i^{tan} = \frac{-j\omega}{2} \int_L M_x(x') \left[\varepsilon_1 H_0^{(2)}(k_1|x - x'|) + \varepsilon_1 H_2^{(2)}(k_1|x - x'|) \right.$$
$$\left. + \varepsilon_2 H_0^{(2)}(k_2|x - x'|) + \varepsilon_2 H_2^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

Theory and Methods

Method of moments

- Integral equations to system of linear equations

$$\begin{bmatrix} Z_{mn} & 0 \\ 0 & Y_{mn} \end{bmatrix} \begin{bmatrix} J_n \\ M_n \end{bmatrix} = \begin{bmatrix} E_m^i \\ H_m^i \end{bmatrix}$$

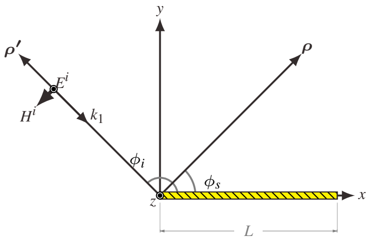
- Pulse basis functions and Point matching used
- Far-field

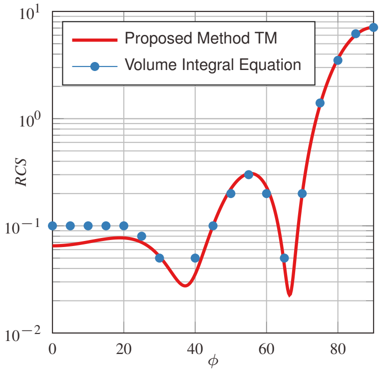
$$RCS(\phi) \simeq \int_0^L \left[J_z(x')\eta_1 + M_x(x')\sin(\phi_i) \right] e^{jk_1 x' \cos(\phi_i)} \mathrm{d}x'$$

Results

Thin Sheet Simulation (TM_z)

- TM_z polarization
- Dielectric Rod of length 2.5 λ
- $\varepsilon = 4$, $\mu = 1$



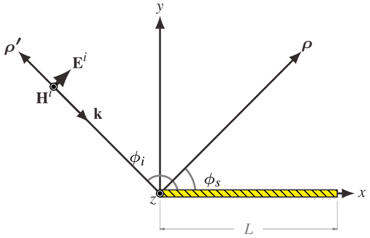


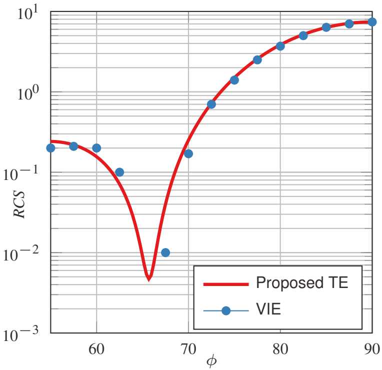
- Thickness of $.05\lambda$ assumed in Volume Integral equation model

Results

Thin Sheet Simulation (TE_z)

- TE_z polarization
- Dielectric Rod of length 2.5 λ
- $\varepsilon = 4$, $\mu = 1$



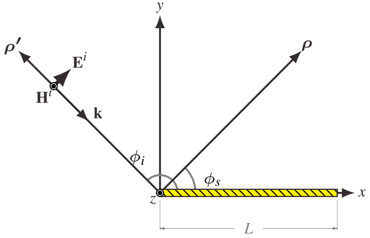


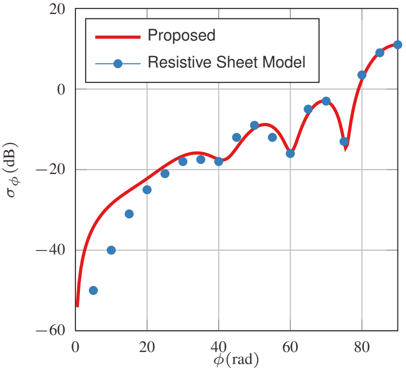
- Thickness of $.05\lambda$ assumed in Volume Integral equation model

Results

Thin Sheet Simulation (TE_z)

- TE_z polarization
- Dielectric Rod of length 2 λ
- $\varepsilon = 4$, $\mu = 1$





- Thickness of $.628/k_1$ assumed in resistive model

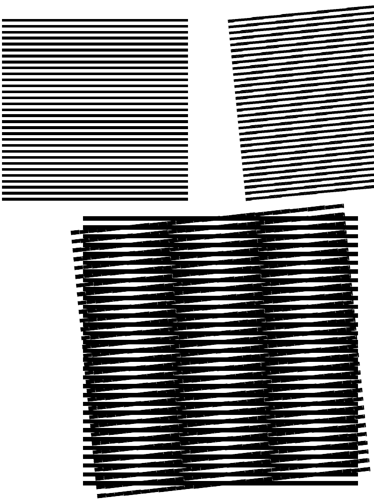
Nanoscale Imaging

Basic Concept

- Conventional microscopy
  - Uniform illumination

$$\text{Resolution} \approx \frac{\lambda}{2}$$

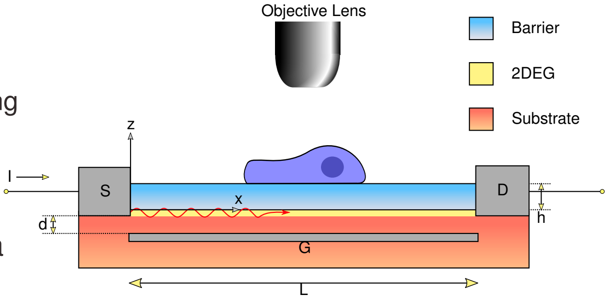
- Structured Illumination
  - Periodic sine pattern
- Moiré Fringes
  - Frequency modulation of two fine patterns results in a coarse pattern



Nanoscale Imaging

Imaging Setup

- Backgated high electron mobility transistor (HEMT)
  - Plasmonic standing wave
  - Subwavelength profile
- Tunable response via gate voltage control
- Low light intensity
- Performance limited by presence of barrier layer



Nanoscale Imaging

Working Principle

- Illumination signal

$$I(\mathbf{r}) = 1 + \cos(\mathbf{k}_\rho \cdot \mathbf{r} + \phi)$$

- Observed Image under ideal circumstances (Spatial domain)

$$M(\mathbf{r}) = [F(\mathbf{r}) \cdot I(\mathbf{r})]$$

- Fourier transformed Image

$$\begin{aligned}\tilde{M}(\mathbf{k}) &= \tilde{F}(\mathbf{k}) \otimes \tilde{I}(\mathbf{k}) \\ &= \frac{1}{2}\left[2\tilde{F}(\mathbf{k}) + \tilde{F}(\mathbf{k} - \mathbf{k}_\rho)\mathrm{e}^{-j\phi} + \tilde{F}(\mathbf{k} + \mathbf{k}_\rho)\mathrm{e}^{j\phi}\right]\end{aligned}$$

Nanoscale Imaging

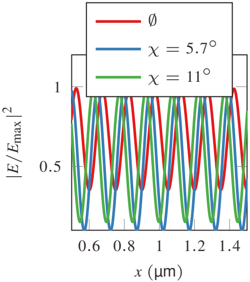
Image Reconstruction

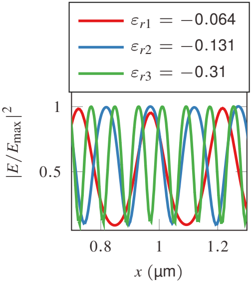
- Three shifted versions of the sample in the observed image
- Phased shifts achieved through external TM-polarized plane wave

$$\mathbf{E}_{ext} = \hat{\mathbf{x}}\, a + \hat{\mathbf{z}}\, b$$

$$|E|^2 = (a + \cos k_\rho x)^2 + (b + \sin k_\rho x)^2$$
$$= a^2 + b^2 + 1 + 2\chi \cos(k_\rho x + \psi)$$

- Change in 2DEG plasma frequency by gate voltage

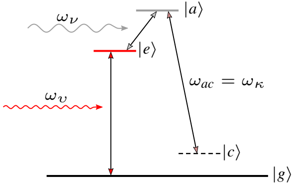


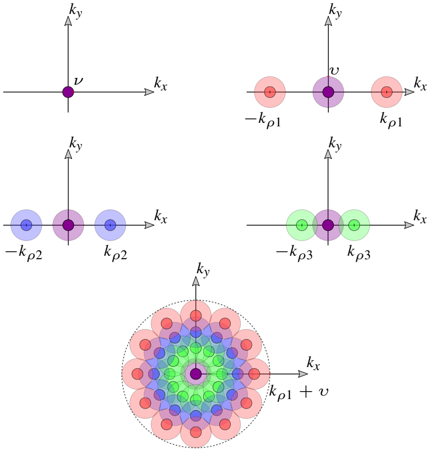


Nanoscale Imaging

Frequency domain illustration of the scheme

- Slow imaging process
  - Many plasmonic resonances required
- Process expedited by an additional illumination





Nanoscale Imaging

Simulation Results

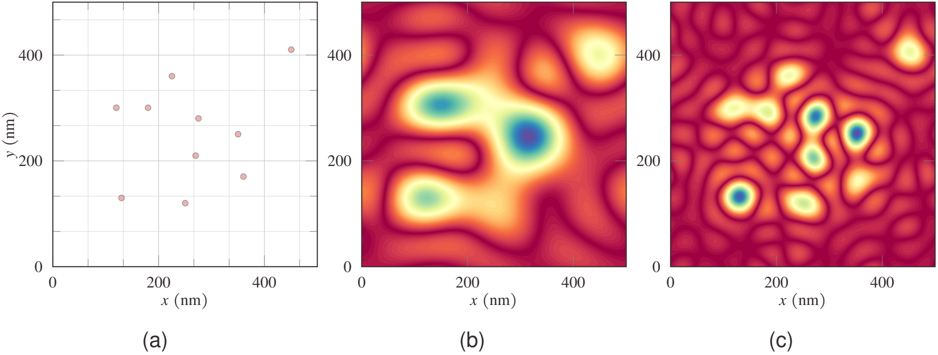


Figure: (a) Sample distribution. Simulation of the reconstructed sample image at: (b) $\mathrm{Re}\, k_\rho = 39.5$ (c) $\mathrm{Re}\, k_\rho = 80$

Summary

Two-dimensional plasmonic devices

- Subwavelength wave phenomena at optical and terahertz frequencies
- Realization of terahertz sources and sensors
- 2D nature of waves permits subwavelength confinement
- Plasmonic activity
- Nanoscale imaging using terahertz plasma waves

Acknowledgements

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- The Fulbright Program

Thank you!

Questions?