MATH 609

Homework #5

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1 Specifications

1.1 Tridiagonal System of Equations

In the first case, the numerical solution is compared with the exact solution when the k parameter is approximated by a linear function. We see that the performance of Conjugate-Gradient (CG) method is much faster than the Steepest-Descent (SD) method. In the second part, numerical solution is computed when the k parameter is a piece-wise function [1].

Method	$n = 19$	$n = 39$	$n = 79$
CG	20	42	89
SD	2430	9761	38467

Table 1: Iterations for Convergence required in Example 1, part a

$n \downarrow$	$K = 2$	$K = 100$	$K = 1000$
19	21	30	37
39	46	84	107
79	96	21	311

Table 2: Iterations for Convergence required in Example 1, part b for CG method

$n \downarrow$	$K = 2$	$K = 100$	$K = 1000$
19	2715	43676	ς
39	10553	ς	ς
79	40509	ς	ς

Table 3: Iterations for Convergence required in Example 1, part b for SD method

1.2 Approximate Solution of 2D Elliptic Equation

The numerical solution of a two-dimensional elliptic equation is computed by applying the given boundary conditions.

Method	$n = 8$	$n = 16$	$n = 32$
CG	6	27	67
SD	254	1291	5639

Table 4: Iterations for Convergence required in Example 2

[1] ς: Convergence not achieved within 100000 iterations

1.3 Numerical Solution of Trough Potential

The numerical solution of an electric potential in a trough is computed. The top boundary has a voltage boundary condition of 100 volts and the rest of the boundaries are perfect electric conductors (PEC) having zero potential. The region is assumed square.

Method	$n = 8$	$n = 16$	$n = 32$
CG	13	48	165
SD	236	1128	4689

Table 5: Iterations for Convergence required in Example 2

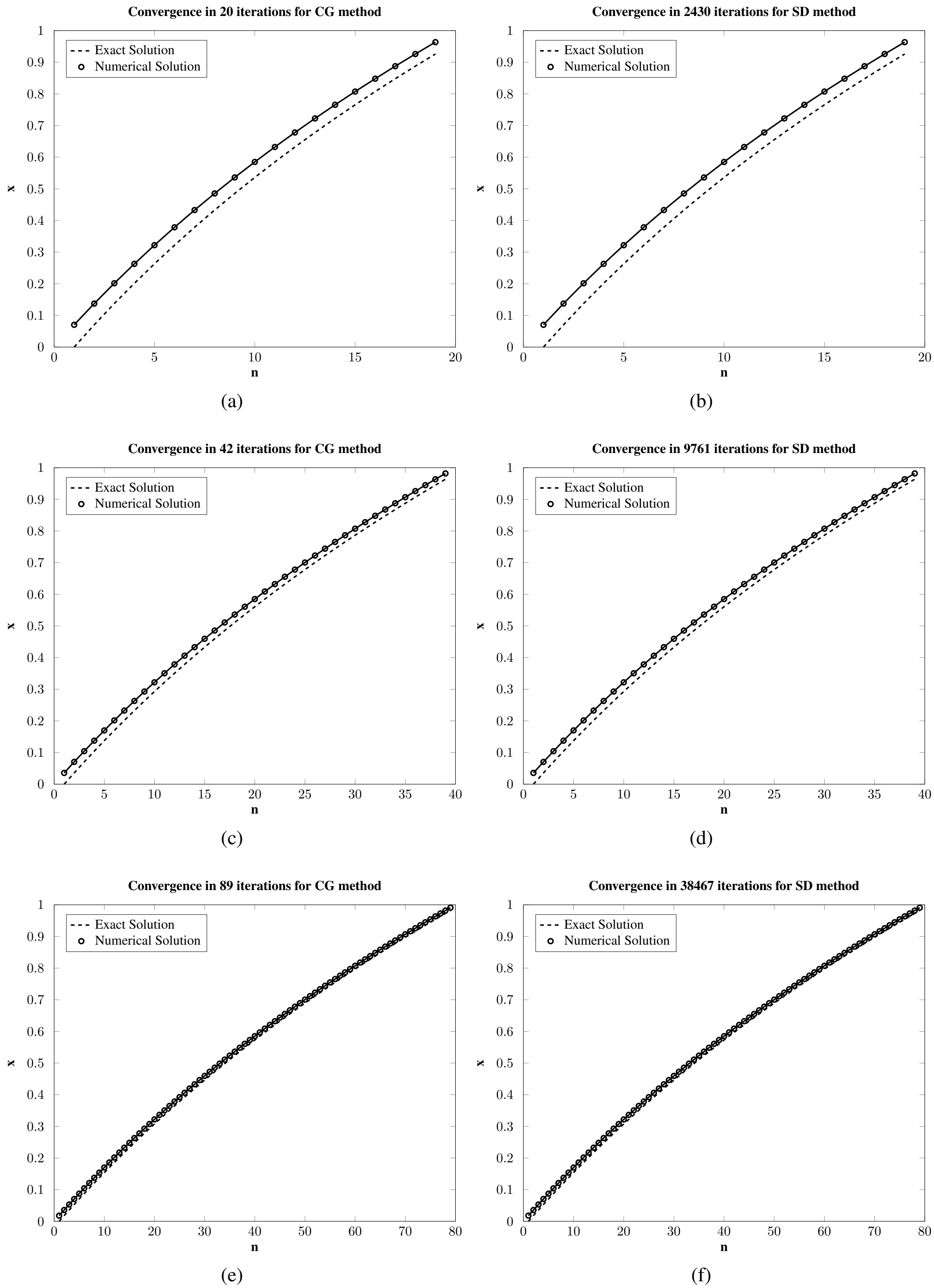

Figure 1: Solution of Tridiagonal System with linear function $k(t)$ (a)-(b) $n = 19$ (c)-(d) $n = 39$ (e)-(f) $n = 79$ with CG and SD method as shown

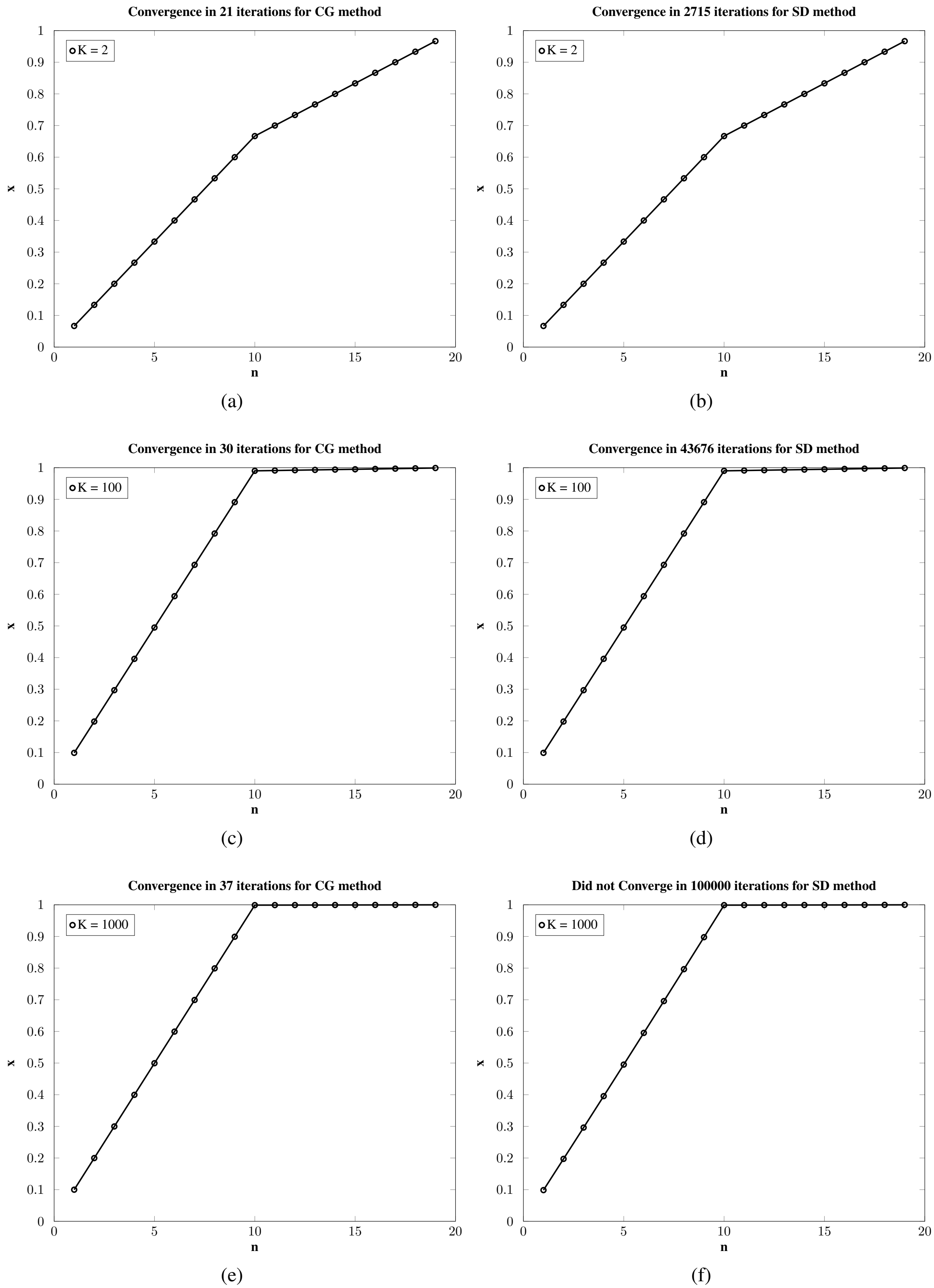

Figure 2: Solution of Tridiagonal System with piecewise function $k(t)$ and $n = 19$ (a)-(b) $K = 2$ (c)-(d) $K = 100$ (e)-(f) $K = 1000$ with CG and SD methods as shown

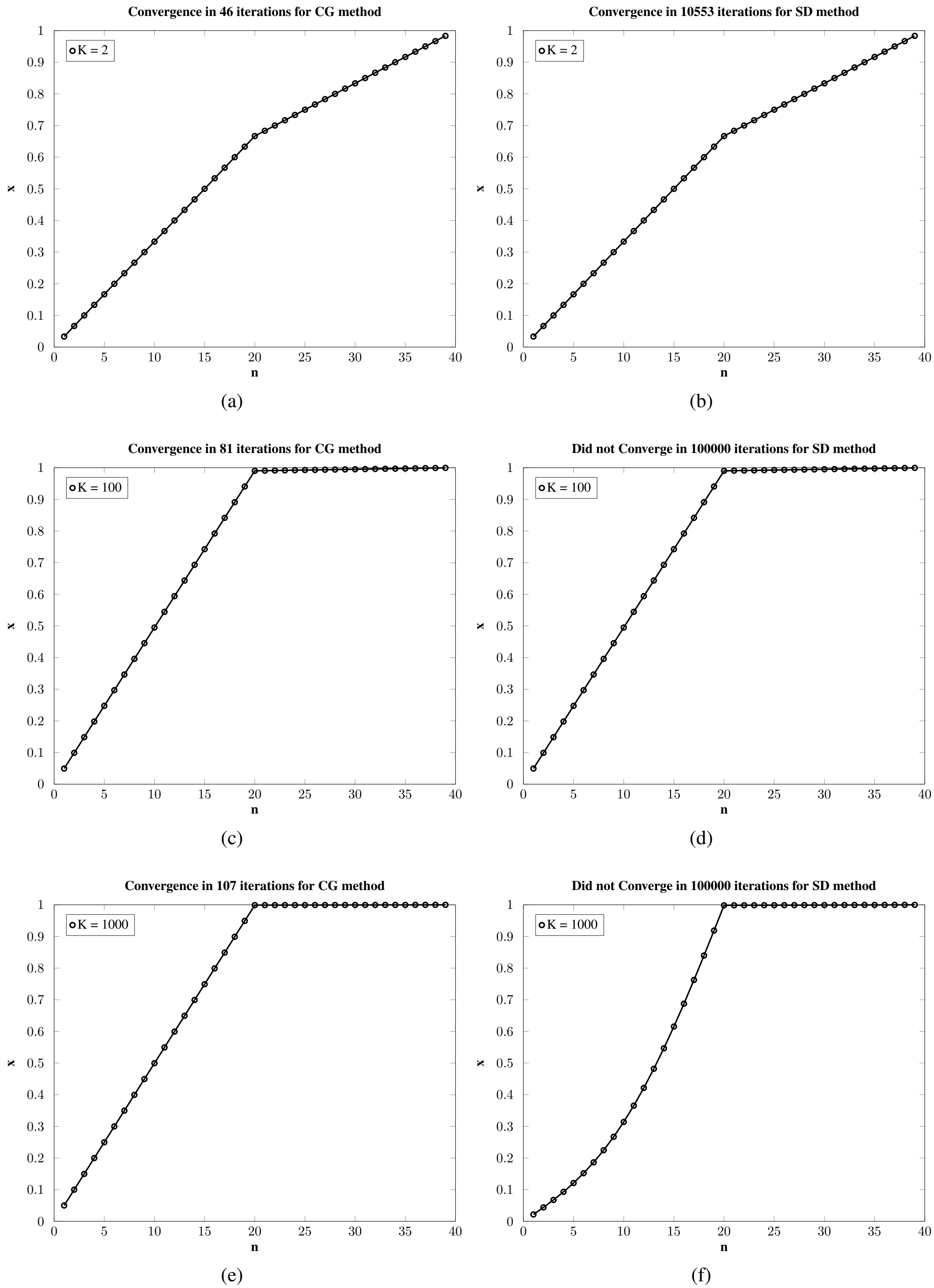

Figure 3: Solution of Tridiagonal System with piecewise function $k(t)$ and $n = 39$ (a)-(b) $K = 2$ (c)-(d) $K = 100$ (e)-(f) $K = 1000$ with CG and SD methods as shown

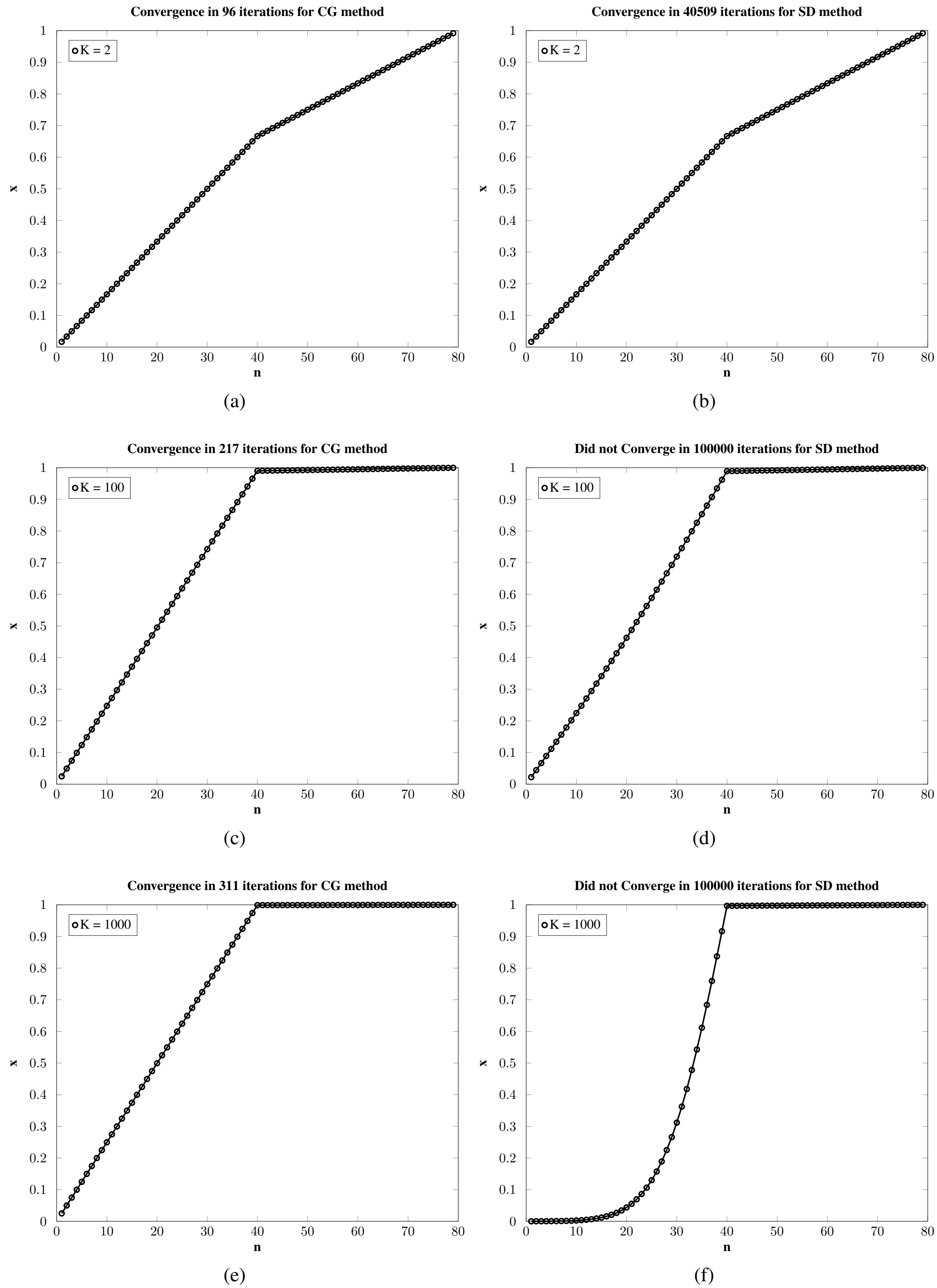

Figure 4: Solution of Tridiagonal System with piecewise function $k(t)$ and $n = 79$ (a)-(b) $K = 2$ (c)-(d) $K = 100$ (e)-(f) $K = 1000$ with CG and SD methods as shown

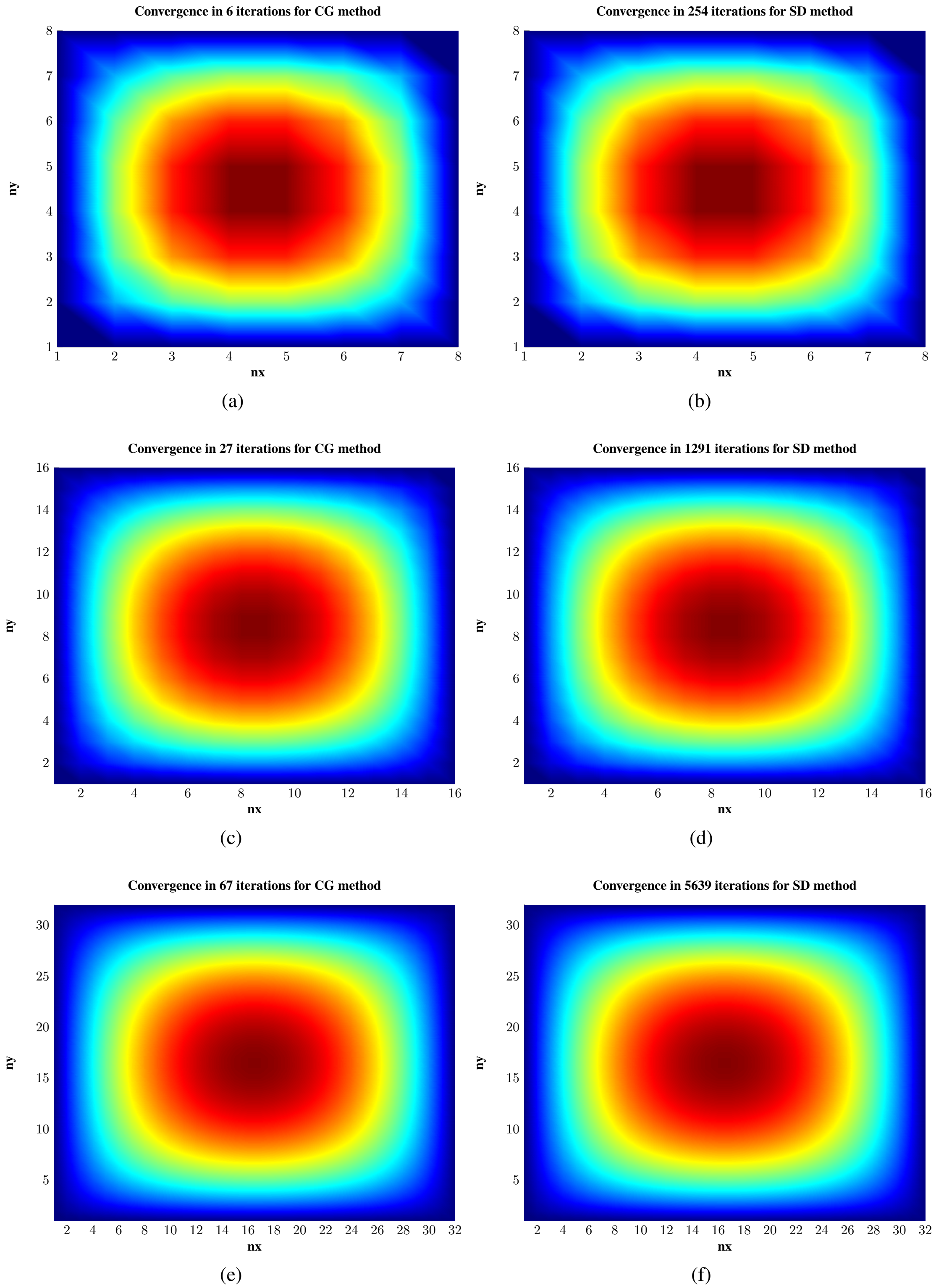

Figure 5: Approximate Solution of 2D Elliptic Equation $-\Delta u + u = 1$ with $\Omega = (0,1) \times (0,1)$ and $u(\Omega) = 0$ (a)-(b) $n = 8$ (c)-(d) $n = 16$ (e)-(f) $n = 32$ with CG and SD methods as shown

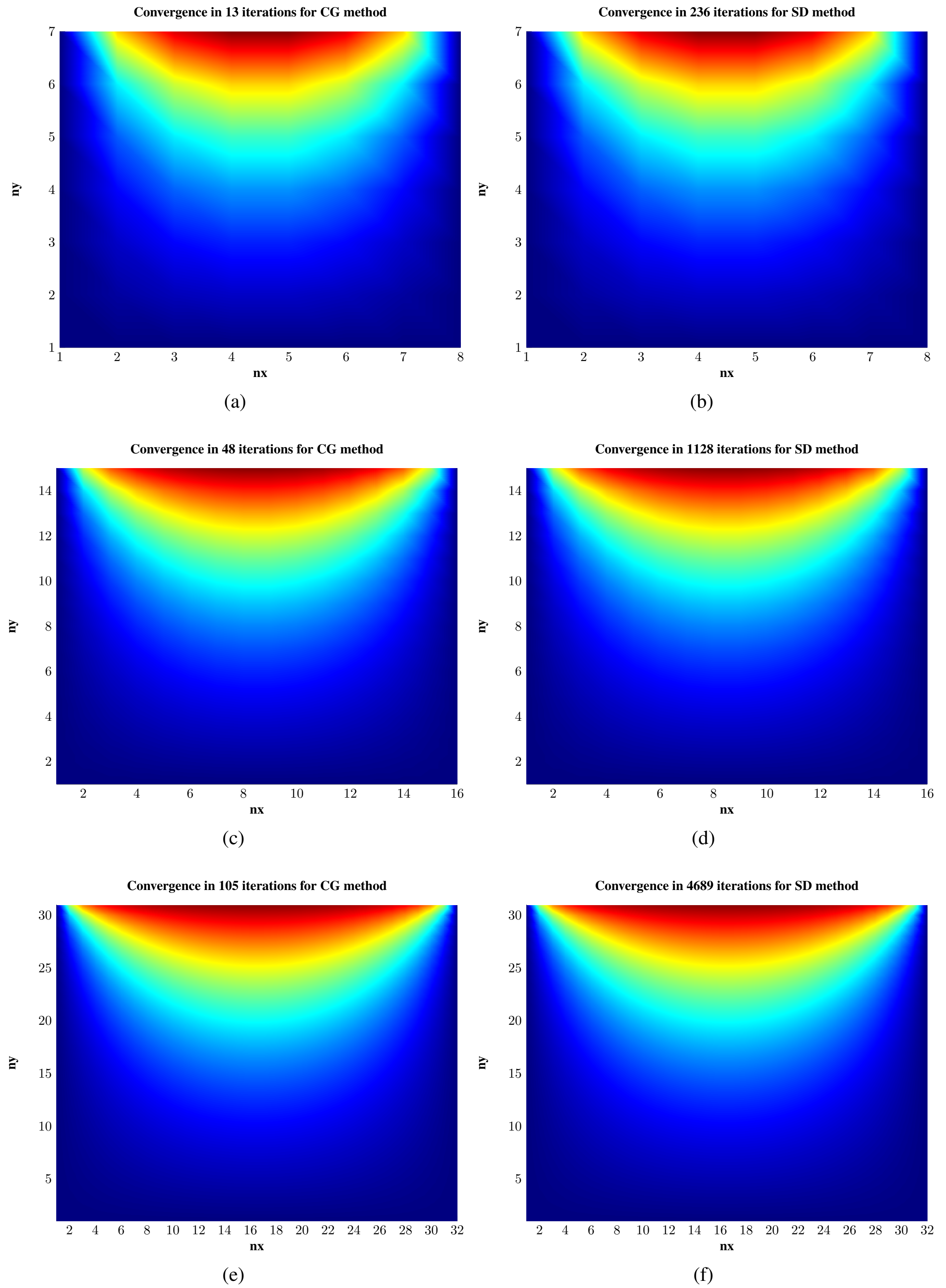

Figure 6: Approximate Electrical Potential Solution of 2D Poisson's Equation $-\Delta\Phi + \Phi = V$ with three PEC boundaries $\Phi(\Omega) = 0$ and 100 volts plate at the top and $\Phi(\chi) = 100$ (a)-(b) $n = 32$ with CG and SD methods as shown