MATH 609

Programming Assignment #4

Fall, 2015

Hasan Tahir Abbas

1 Specifications

This programming assignment is implemented through two pieces of algorithms *spline3_coeff* and *spline3_eval* as presented in *Numerical Mathematics and Computing* by Kincaid and Cheney. The first code, *spline3_coeff* accepts a data file and finds the unknown parameters z_i based on the conditions provided at the ends. The second code, *spline3_eval* computes the spline along with the first and second derivatives of the spline function.

Table 1: Car Traveling on a Straight road

Time	Distance
0	0
3	225
5	385
8	623
13	933

1.1 Traveling Car Example with free ends

This example is solved without any modifications to the core algorithms. With the data given in Table 1, the position of the car is shown in the Fig. 1. The speed at $t = 10$ sec is 62.44 ft/s.

1.2 Traveling Car Example with fixed ends

Since the end conditions are specified in the case of fixed ends, z_0 and z_n are calculated through 1 and 2. With the data given in 1, the position of the car is shown in the Fig. 1. The speed at $t = 10$ sec is 64.17 ft/s.

$$z_0 = 6 \times \frac{\alpha_0 + z_1 \times h_0/6 + y_0/h_0 - y_1/h_0}{4h_0} \tag{1}$$

$$z_n = \left((\alpha_n - z_{n-1}) \times h_{n-1}/6 - \frac{y_n - y_{n-1}}{h_{n-1}} \right) \times 3/h_{n-1} \tag{2}$$

1.3 Approximation of US Population

This example is solved without any modifications to the core algorithms. The required numbers are tabulated in Table 2 and the spline curve is shown in Fig. 3.

1.4 Parametric Splines

In the first example, we have two trigonometric functions, $x(t) = cos(t)$ and $y(t) = sin(t)$ with $0 <= t <= \pi$. Two plots of the parametric spline are shown for different number of knots are shown in Fig. 4. The errors are tabulated in Table 3.

1.5 Parametric Splines Curve Approximation

In this example, a curve is first hand-drawn and the co-ordinates of a number of nodes are noted. The list of the co-ordinates is then passed to the code to generate Fig. 5.

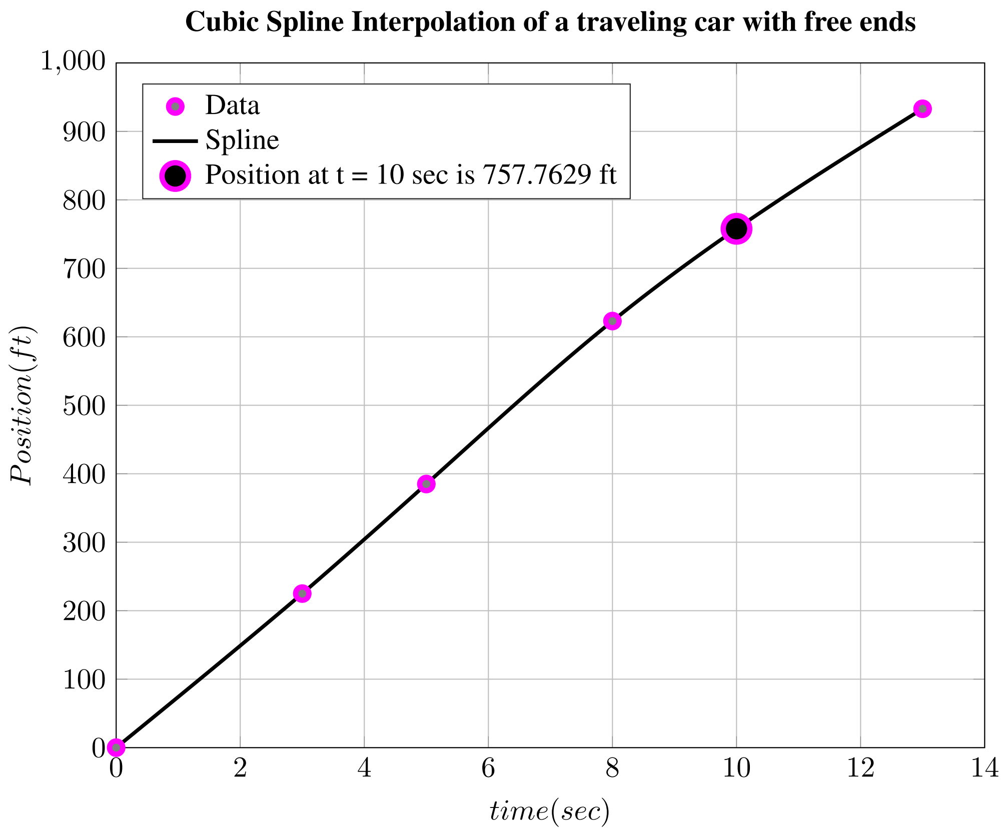


Figure 1: Cubic Spline Interpolation of a traveling car with free ends

Table 2: US Population

Year	Population
1930	123203
1940	131669
1950	150697
1960	179323
1965	191847
1970	203212
1975	214773
1980	226505
1985	238123
1990	249643

Table 3: Error $\max_{t\in[0,\pi]}\left(\left|x^{(l)}(t)\right| - \left|S_x^{(l)}(t)\right| + \left|y^{(l)}(t)\right| - \left|S_y^{(l)}(t)\right|\right)$

l	Error
0	0.0061
1	0.0791
2	0.6362

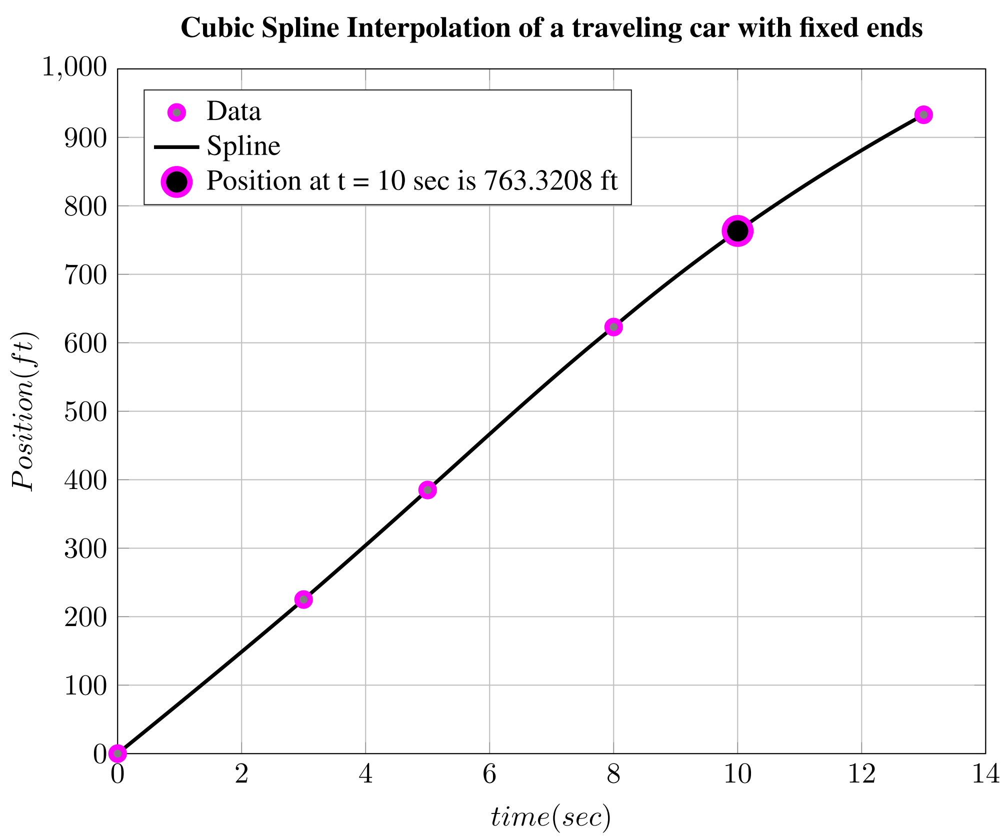


Figure 2: Cubic Spline Interpolation of a traveling car with fixed ends

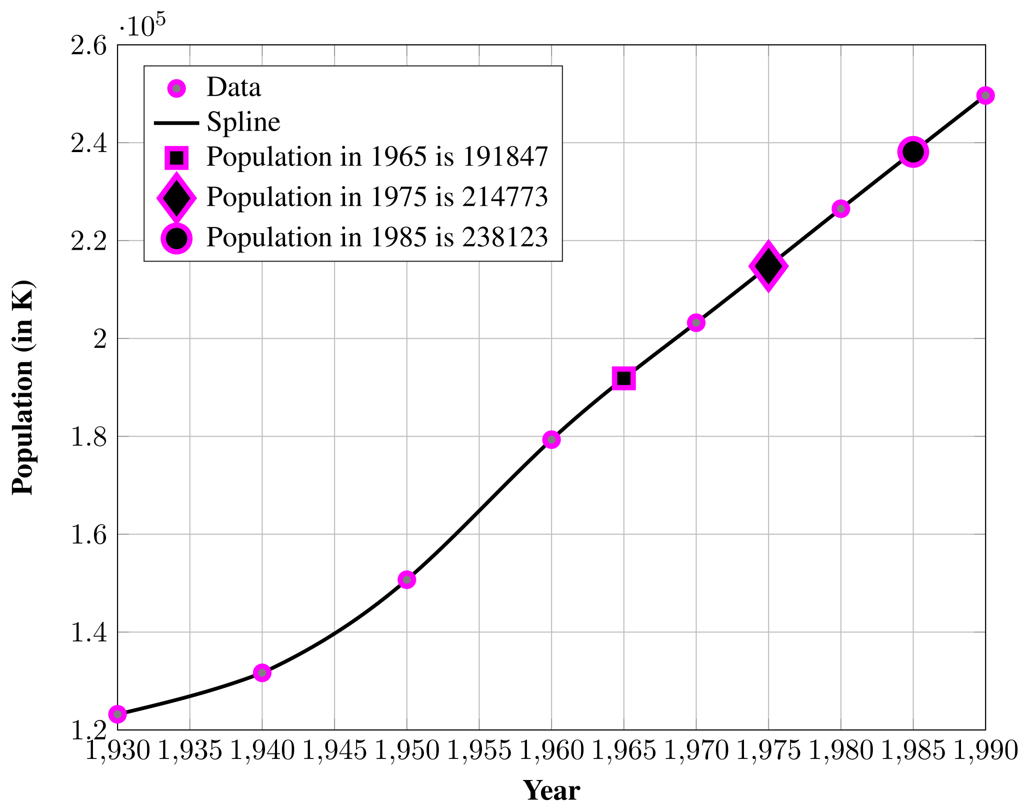
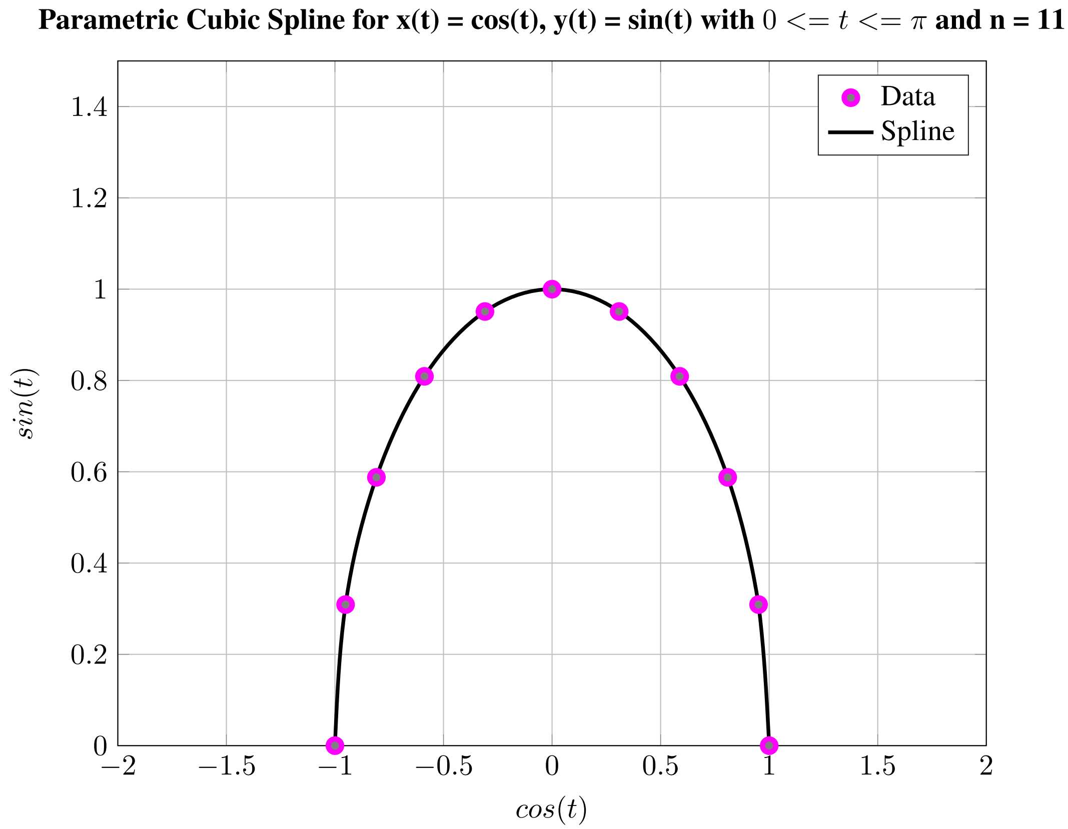
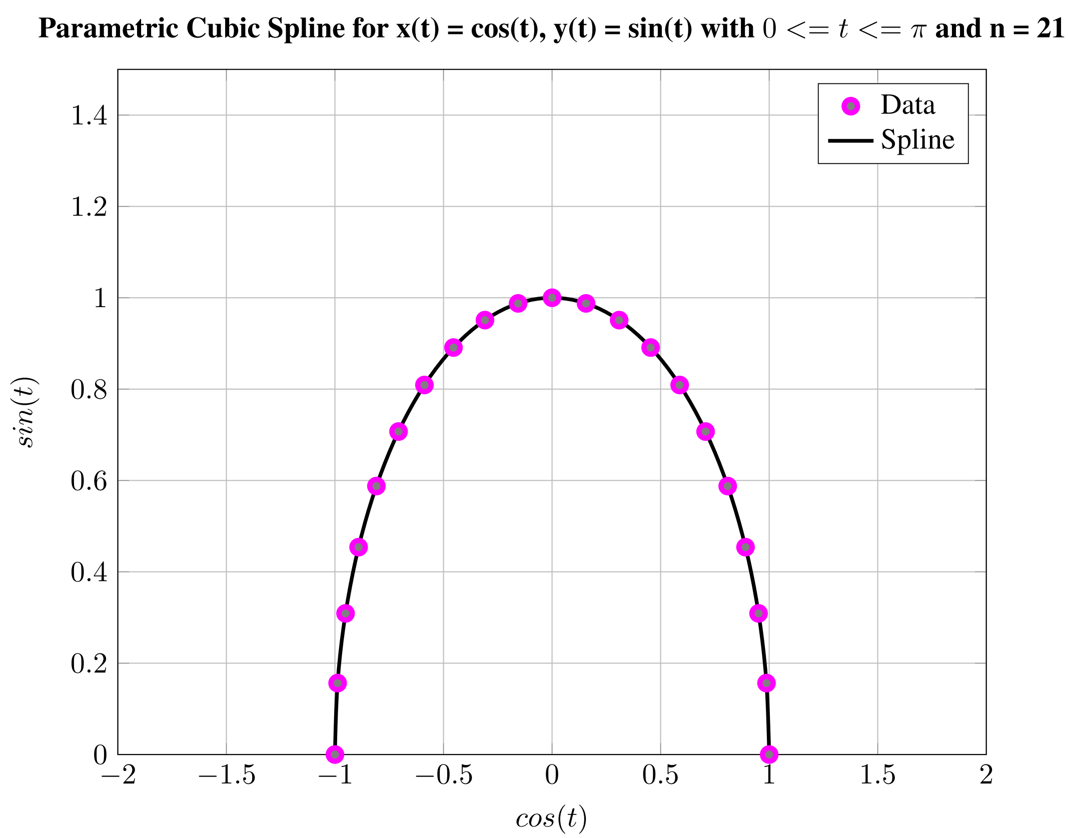


Figure 3: Cubic Spline Interpolation of US Population



(a)



(b)

Figure 4: Parametric Spline for trigonometric functions, $x(t) = cos(t)$ and $y(t) = sin(t)$ with

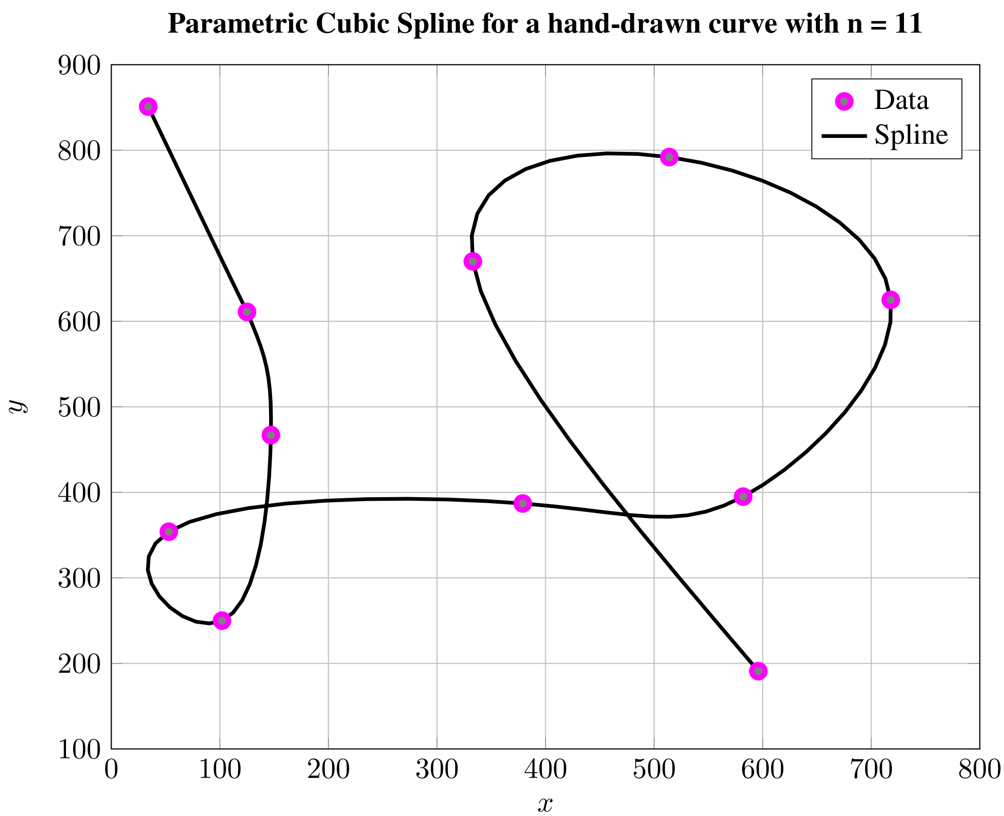


Figure 5: Parametric Cubic Spline for a hand-drawn curve with n = 11