Sommerfeld Integral

Horizontally Oriented Magnetic Dipole above Silver Half-plane

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Theory

Thin Sheet Simulation

- Volume Integral formulation

$$\mathbf{A} = \frac{\mu}{4\pi} \int_V \mathbf{J}_v(\mathbf{r}') \frac{e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \, \mathrm{d}v'$$

$$\mathbf{E}_1^{scat} = -\frac{j\omega}{k_1^2} \left(k_1^2 + \nabla\nabla\cdot\right) \mathbf{A}$$

$$\mathbf{J}_v = \frac{-jk_1}{Z_0} (\varepsilon_2 - 1) \mathbf{E}_2$$

- Surface current J_s approximated from J_v

- Impedance (Leontovich) Boundary Conditions

$$\mathbf{E}_{tan} = \eta Z_0 \hat{\mathbf{n}} \times \mathbf{H}$$

$$E^i = \eta Z_0 J_s(x') + \frac{\omega\mu}{4} \int_l J_s(x') H_0^{(2)}(k_2|x - x'|) \, \mathrm{d}x'$$

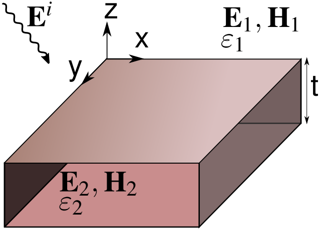

Figure: Dielectric Slab geometry

Proposed Scheme

Surface Integral Equation

- Surface Equivalence Theorem

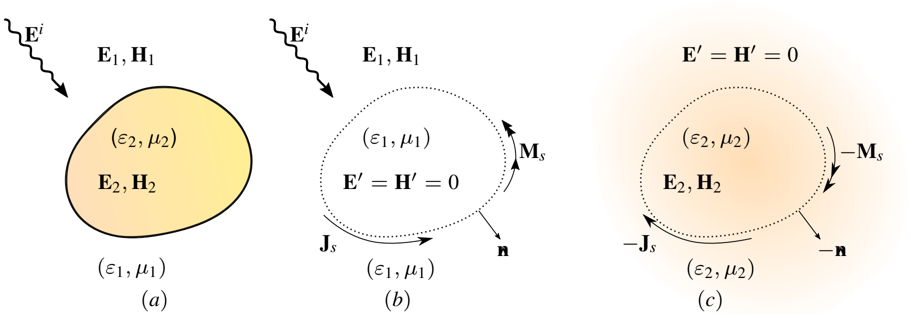

Figure: (a). Actual and its equivalent models for the (b) external and, (c) Internal region

Proposed Scheme

Surface Integral Equation

- Exterior Region

$$
\begin{aligned}
\mathbf{E}_1 &= \mathbf{E}_i + \mathbf{E}_1^{scat} \\
&= -\frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_C \mathbf{J}_s(\mathbf{p}')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' \\
&\quad -\frac{1}{4\varepsilon j}\nabla\times\int_l \mathbf{M}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' + \mathbf{E}_i
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}_1 &= \mathbf{H}_i + \mathbf{H}_1^{scat} \\
&= \frac{1}{4j}\nabla\times\int_l \mathbf{J}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' \\
&\quad -\frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_l \mathbf{M}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' + \mathbf{H}_i
\end{aligned}
$$

Proposed Scheme

Surface Integral Equation

- Interior Region

$$\mathbf{E}_1 = \mathbf{E}_i + \mathbf{E}_1^{scat}$$

$$= -\frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_C \left(-\mathbf{J}_s(\mathbf{p}')\right) H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l'$$

$$-\frac{1}{4j}\nabla\times\int_l \left(-\mathbf{M}_s(\rho')\right) H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l'$$

$$\mathbf{H}_1 = \mathbf{H}_i + \mathbf{H}_1^{scat}$$

$$= \frac{1}{4j}\nabla\times\int_l \left(-\mathbf{J}_s(\rho')\right) H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l'$$

$$-\frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_l \left(-\mathbf{M}_s(\rho')\right) H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l'$$

Proposed Scheme

Thin Flat Sheet TM_z

$$\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{0}$$

$$E_i = \frac{\omega}{4} \int\limits_L J_z(x') \left[H_0^{(2)}(k_1|x - x'|) + H_0^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

$$\hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}$$

$$H_i^{tan} = \frac{-j\omega}{4} \int\limits_L M_x(x') \left[\varepsilon_1 H_0^{(2)}(k_1|x - x'|) + \varepsilon_1 H_2^{(2)}(k_1|x - x'|) \right.$$

$$\left. + \varepsilon_2 H_0^{(2)}(k_2|x - x'|) + \varepsilon_2 H_2^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

Proposed Scheme

Method of moments

- Integral equations to sytem of linear equations

$$\begin{bmatrix} Z_{mn} & 0 \\ 0 & Y_{mn} \end{bmatrix} \begin{bmatrix} J_n \\ M_n \end{bmatrix} = \begin{bmatrix} E_m^i \\ H_m^i \end{bmatrix}$$

- Pulse basis functions and Point matching used
- Far-field

$$RCS(\phi) \simeq \int_0^L \left[J_z(x')\eta_1 + M_x(x') \sin(\phi_i) \right] e^{jk_1 x' \cos(\phi_i)} \mathrm{d}x'$$

Two-dimensional Electon Gas (2DEG)

Existence of Surface Waves

$$\varepsilon_1(\omega) \cdot \varepsilon_2(\omega) < 0$$

- Criterion met at terahertz frequency range
- Opposite signs of dielectric constant at Semiconductor interface
- GaAs/AlGaAs semiconductor heterostructures
- Strontium Titanate/Lanthanum Aluminate (STO/LAO) oxide