# Sommerfeld Integral

## Horizontally Oriented Magnetic Dipole above Silver Half-plane

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# Two-dimensional Electon Gas (2DEG)

## Introduction

- Semiconductor Heterostructure Interface
- High electron Mobility
- High concentration of electric charge
- **Surface waves**
  - Surface Plasmon-Polaritons
- Formation of Quantum Well
  - Two-dimensional confinement of electrons

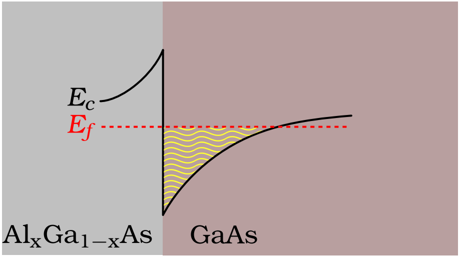



Figure: Band diagram of a GaAs/AlGaAs heterostructure

$E_c$ - Conduction band edge

$E_f$ - Fermi level

# Two-dimensional Electon Gas (2DEG)

## Material Description

- Drude-Lorentz conductivity model

$$\sigma_s(\omega) = \frac{Ne^2\tau}{m^*}\frac{\omega}{\omega + j\tau\left(\omega^2 - \omega_0^2\right)}$$

$$\omega_0 = \chi\sqrt{\frac{Ne^2}{m^*\varepsilon_\infty}}$$

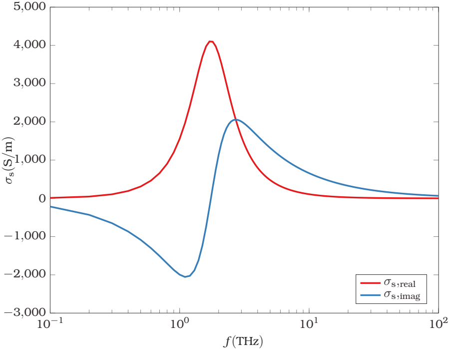
$N$ - charge density ($cm^{-3}$)

$e$ - electron charge

$\tau$ - scattering time

$m^*$- effective mass of electron

$\omega_0$- effective plasma frequency




$\varepsilon_\infty$- high frequency limit of dielectric constan

$\chi$ - geometrical factor (1/3)

# Two-dimensional Electon Gas (2DEG)

## Material Description (contd.)

- Drude-Lorentz conductivity model

$$\varepsilon(\omega) = \varepsilon^{\infty} + \prod_i \frac{\omega_{li}^2 - \omega^2 - j\gamma_{li}\omega}{\omega_{ti}^2 - \omega^2 - j\gamma_{ti}\omega}$$

$\omega_{ti}$- TO phonon frequencies

$\omega_{li}$- LO phonon frequencies

$\gamma$ - Damping constants

# Two-dimensional Electon Gas (2DEG)

## Dispersion Relation

- TE mode

$$k_{z1} + k_{z2} = \omega\sigma_s(\omega)$$

No real solutions for an isotropic environment

- TM mode

$$\boxed{\frac{\varepsilon_1(\omega)}{k_{z1}} + \frac{\varepsilon_2(\omega)}{k_{z2}} = -\frac{\sigma_s(\omega)}{\omega}}$$

Real solution(s). Surface waves exist.

$$\begin{aligned} k_{zi} &= \sqrt{k_i^2 - k_x^2} \\ &= \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_i(\omega) - k_x^2} \end{aligned}$$

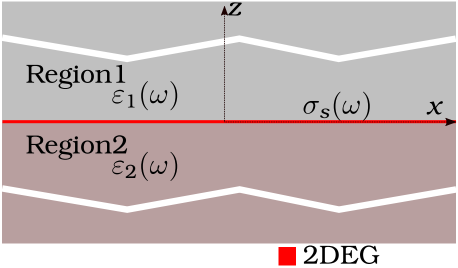



Figure: 2DEG at a semiconductor heterojunction

# Two-dimensional Electon Gas (2DEG)

## Existence of Surface Waves

$$\varepsilon_1(\omega) \cdot \varepsilon_2(\omega) < 0$$

- Criterion met at terahertz frequency range
- Opposite signs of dielectric constant at Semiconductor interface
- GaAs/AlGaAs semiconductor heterostructures
- Strontium Titanate/Lanthanum Aluminate (STO/LAO) oxide interfaces

# Optical Nanoantennas

## Introduction

- Near-field Scanning Electron Microscopy (NSOM)

    Subwavelength confinement

- Directivity enhancement of Quantum emitters
- **Surface waves**

    Surface Plasmon-Polaritons

- Radiation Mechanism

    Wavenumber Mismatch

$E_c$ - Conduction band edge

$E_f$ - Fermi level

# Optical Nanoantennas

## Dispersion Relation

- Metal-dielectric Interface

$$k_{sp} = k_1 \sqrt{\frac{\varepsilon_1 \varepsilon_2(\omega)}{\varepsilon_1 + \varepsilon_2(\omega)}}$$

$$\varepsilon_2(\omega) = \varepsilon_\infty - \frac{\omega_d^2}{\omega^2 + j\gamma\omega} + \mathrm{C.P.terms}$$