An Integral Equation Scheme for Plasma based Thin Sheets

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***Abstract*—We discuss the dispersion relation of a thin plasma sheet embedded in a stack semiconductor materials. The miniaturization of antennas and microwave devices is presentes in light of the dispersion relation. Numerical results are presented to illustrate the scattering properties of the plasma sheet.**

I. Introduction

With the advancements in semiconductor manufacturing techniques, our perception of materials existing only in a volumetric sensse has changed. Of late, two-dimensional materials have been discovered. Graphene may the first that springs to mind but owing to difficulties in its engineering, it has created a divided opinion regards to its usage in the scientific community, although it has remarkable electrical and mechanical properties. Looking beyond, there are other 2D materials that inherently exist in commonplace transistor devices due to formation of an extremely thin layer of free electrons in the stack of multilayer semiconductor substrate. The electrical properties have to be considered in a different way than traditional bulk materials where permittivity and permeability are customarily considered in a bulk sense. Most commercial electromagnetic simulation tools model thin structures with equivalent surface conductivity where the thickness is incorporated in computing the corresponding parameters.

II. Theory

A. Derivation of Dispersion Relation

Consider two dielectric half-planes of permittivity ε_1 and ε_2 respectively, separated by an infinitesimally thin sheet of charge at $z = 0$. The propagation constants for the materials can be found starting with field expressions for transverse magnetic (TM) plane wave as shown in **??**. In region 1 ($z > 0$):

$$\mathbf{E}_1 = \left(\hat{\mathbf{x}} E_{x1} + \hat{\mathbf{y}} E_{z1}\right) e^{-j(k_x x + k_{z1} z)}, \tag{1a}$$

$$\mathbf{H}_1 = \hat{\mathbf{y}} H_{y1} e^{-j(k_x x + k_{z1} z)} \tag{1b}$$

Similarly, in region 2 ($z < 0$), the fields are expressed as:

$$\mathbf{E}_2 = \left(\hat{\mathbf{x}} E_{x2} + \hat{\mathbf{y}} E_{z2}\right) e^{-j(k_x x - k_{z2} z)}, \tag{2a}$$

$$\mathbf{H}_2 = \hat{\mathbf{y}} H_{y2} e^{-j(k_x x - k_{z1} z)}. \tag{2b}$$

where k_x and k_{zi} for $i = 1, 2$ are the x and z directed propagation constants respectively, related by the dispsersion equation:

$$\begin{aligned} k_{zi} &= \sqrt{k_i^2 - k_x^2} \\ &= \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_i(\omega) - k_x^2} \end{aligned} \tag{3}$$

The dispersive dielectric constants $\varepsilon_i(\omega)$ are based on a Lorentz osciallator model:

$$\varepsilon(\omega) = \varepsilon^{\infty} + \frac{\omega_p^2(\varepsilon^0 - \varepsilon^{\infty})}{\omega_p^2 - \omega^2 + j\Gamma\omega} \tag{4}$$

Here ε^{∞} and ε^0 are the high and zero frequency limit values of dielectric constant, ω_p is the plasma resonance frequency and Γ is the damping constant of the material. Applying boundary conditions by maintaining the continuity of tangential field components at the interface $z = 0$ yield:

$$E_{x1} = E_{x2} = E_x, \tag{5a}$$

$$H_{y1} - H_{y2} = J_s \tag{5b}$$

where J_s is the surface current due to the sheet of charge. For a TM mode, H is related to E by:

$$\mathbf{H} = \frac{1}{\eta_{TM}} \hat{\mathbf{n}} \times \mathbf{E} \tag{6}$$

where η_{TM} is the TM mode wave impedance equal to $k_z/\omega\varepsilon$. The surface current J_s in (**??**) is related to the electric field and surface conductivity σ_s by Ohm's law:

$$J_s = \sigma_s E_x \tag{7}$$

In the microwave and terahertz regions, the surface conductivity can be approximated by a Drude-type formula:

$$\sigma_s(\omega) = \frac{N_s e^2 \tau}{m^*} \frac{1}{1 + j\omega\tau} \tag{8}$$

The parameters e and m^* are the effective charge and effective mass of an electron respectively, N_s is surface-charge density, and the scattering time τ is the reciprocal of damping constant

Γ. The plasma frequency for the bulk material is similarly expressed as:

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m^*}}. \tag{9}$$

It should be noted that N is the free-electron concentration which is different from N_s. The former quantity involves volume and is only valid for bulk materials. From Eqs. (**??**)-(**??**), we obtain the TM mode dispersion relation:

$$\frac{\varepsilon_1(\omega)}{k_{z1}} + \frac{\varepsilon_2(\omega)}{k_{z2}} = -\frac{\sigma_s}{\omega} \tag{10}$$

B. Homogeneous Background

To explore the solutions of Eq. (**??**), we consider first a simple case where the plasma sheet is surrounded by a single medium, i.e., $\varepsilon_1 = \varepsilon_2$ assumed to be a dispersive semiconductor material. The surface conductivity in (**??**) can in this case be approximated as [**?**]:

$$\sigma_s(\omega) \approx \frac{N_s e^2}{jm^*\omega}. \tag{11}$$

The dispersion relation in (**??**) can now be written as:

$$\frac{2\varepsilon_1(\omega)}{k_{z1}} \approx -\frac{N_s e^2}{jm^*\omega^2} \tag{12}$$

By squaring (**??**) and using (**??**), we obtain:

$$k_x^2 \approx \left(\frac{\omega}{c}\right)^2 \varepsilon_1(\omega)\left[1 + \left(\frac{\omega^2 m^*}{\pi N_s e^2}\right)\varepsilon_1(\omega)\right] \tag{13}$$

C. Heterogeneous Background

Next we assume that region 1 is composed of a non-dispersive material (constant dielectric) and region 2 is a dispersive semiconductor material. The solution of (**??**) is cumbersome involving careful complex analysis. However, when $k_x >> k_1$, a simplified solution can be written as [**?**]:

$$k_x \approx -\varepsilon_2 + 1\frac{j\omega}{\sigma_s(\omega)} \tag{14}$$

III. Results

In this section, we first show the dispersion relation of a plasma sheet with Gallium Arsenide (GaAs) as the background material. The data required to model the frequency-dependent dielectric and conductivity are listed in Table **??** [**?**]. The complex permittivity of GaAs is plotted in Fig. **??**. The result is shown in Fig. **??**.

Next, the dispersion relation for a heterostructure based environment is plotted where the region 1 is assumed free-space and region 2 is GaAs.

TABLE I
GaAs Material Properties

ε^0	12.9
ε^∞	11.0
ω_t	5.52×10^{13}rad/s
ω_l	5.06×10^{13}rad/s
Γ	4.52×10^{11}rad/s
N_s	5×10^{12}cm^{-2}
m^*	$.063 m_e$
τ	1.3889×10^{12}s

IV. Conclusion

References

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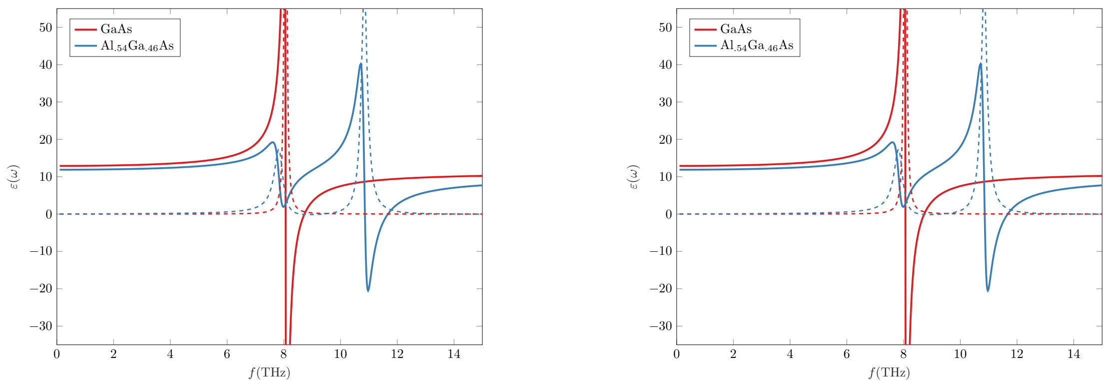
[2] M. Jablan, H. Buljan, and M. Soljačić, "Plasmonics in graphene at infrared frequencies," *Physical review B*, vol. 80, no. 24, p. 245435, 2009.

[3] E. D. Palik, "Gallium arsenide (GaAs)," in *Handbook of Optical Constants of Solids*, E. D. Palik, Ed. Burlington: Academic Press, 1997, pp. 429 – 443.



(a) Case I (b) Case II

Fig. 1. Simulation results for the network.

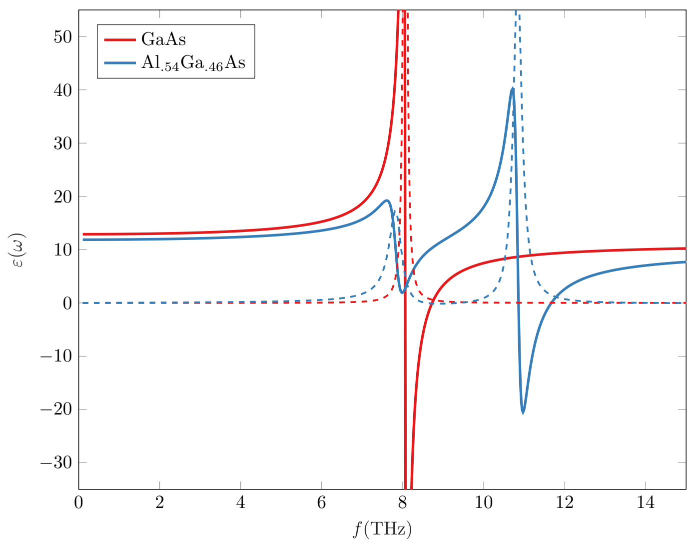


Fig. 2. Complex Dielectric constant of GaAs

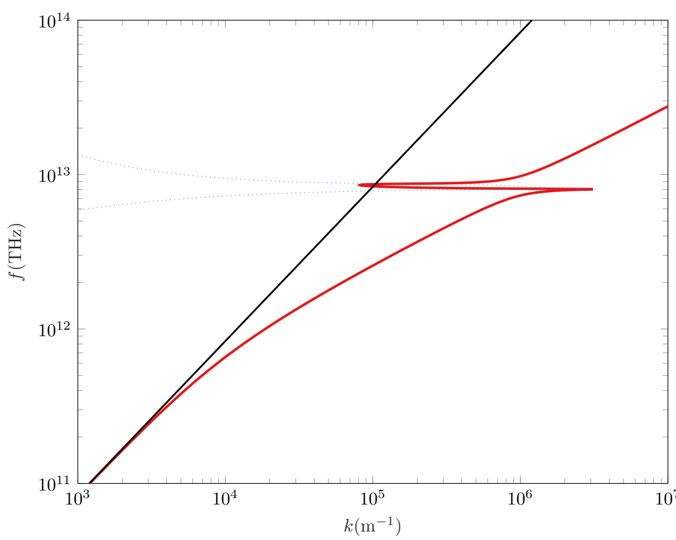


Fig. 3. Dispersion relation of 2DEG embedded in GaAs. Solid: real part, dashed:

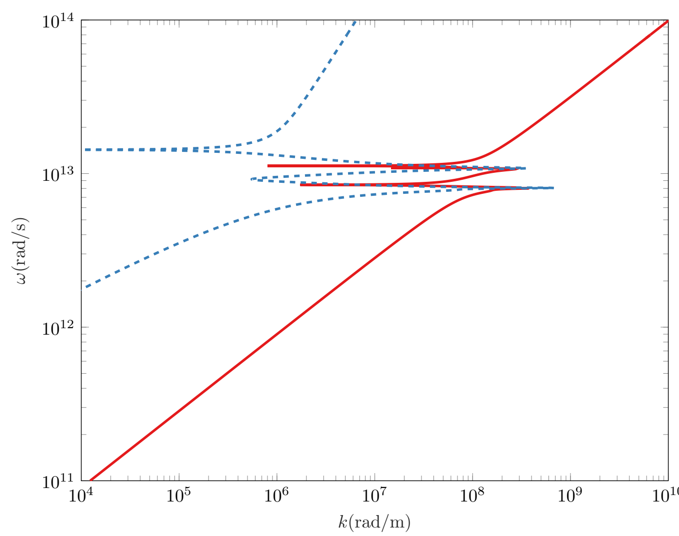


Fig. 4. Dispersion relation of 2DEG embedded in GaAs