An Integral Equation Scheme for Plasma based Thin Sheets

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***Abstract*—We discuss the dispersion relation of a thin plasma sheet embedded in a stack of semiconductor materials. The conditions under which propagation of wave is ensured are investigated by a comparison between III-V compounds and perovskite structured oxide materials. Steps to miniature antennas and microwave devices are also proposed in light of the dispersion relation.**

I. Introduction

Advances in semiconductor engineering technology has changed our perception of materials existing only in a volumetric and bulk sense. Deposition processes to engineer extremely thin layers beyond the nanoscale has led to realization of truly two-dimensional materials. Graphene may be the first that springs to mind but owing to extreme difficulties in its engineering, it has created a divided opinion regards to its usage in the scientific community, although in theory it has remarkable electrical and mechanical properties. Looking beyond, there are other 2D materials that inherently exist in commonplace transistor devices due to formation of an extremely thin layer of free electrons in the stack of multilayer semiconductor substrate. The electrical properties have to be considered in a different way than traditional bulk materials where permittivity and permeability are customarily considered in a bulk sense. Most commercial electromagnetic simulation tools model thin structures with equivalent surface conductivity where the thickness is incorporated in computing the corresponding parameters.

II. Theory

A. Derivation of Dispersion Relation

Consider two dielectric half-spaces of permittivity ε_1 and ε_2 respectively, separated by an infinitesimally thin sheet of charge at $z = 0$ that represents an idealized 2DEG in which the electrons are allowed to flow only in one direction. The propagation constants for the materials can be found starting with field expressions for transverse magnetic (TM) plane wave as shown in **??**. In region 1 ($z > 0$):

$$\mathbf{E}_1 = \left(\hat{\mathbf{x}}E_{x1} + \hat{\mathbf{y}}E_{z1}\right) e^{-j(k_x x + k_{z1} z)}, \tag{1a}$$

$$\mathbf{H}_1 = \hat{\mathbf{y}}H_{y1} e^{-j(k_x x + k_{z1} z)} \tag{1b}$$

Similarly, in region 2 ($z < 0$), the fields are expressed as:

$$\mathbf{E}_2 = \left(\hat{\mathbf{x}}E_{x2} + \hat{\mathbf{y}}E_{z2}\right) e^{-j(k_x x - k_{z2} z)}, \tag{2a}$$

$$\mathbf{H}_2 = \hat{\mathbf{y}}H_{y2} e^{-j(k_x x - k_{z1} z)}. \tag{2b}$$

where k_x and k_{zi} for $i = 1, 2$ are the x and z directed propagation constants respectively, related by the dispsersion equation:

$$\begin{aligned} k_{zi} &= \sqrt{k_i^2 - k_x^2} \\ &= \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_i(\omega) - k_x^2} \end{aligned} \tag{3}$$

$$\frac{\varepsilon_1(\omega)}{k_{z1}} + \frac{\varepsilon_2(\omega)}{k_{z2}} = -\frac{\sigma_s}{\omega} \tag{4}$$

The dispersive dielectric constants $\varepsilon_i(\omega)$ are based on a Lorentz osciallator model:

$$\varepsilon(\omega) = \varepsilon^{\infty} + \prod_i \frac{\omega_{li}^2 - \omega^2 - j\gamma_i\omega}{\omega_{ti}^2 - \omega^2 - j\gamma_i\omega} \tag{5}$$

where the i is the number of resonances, ε^{∞} is the high frequency limit value of dielectric constant, ω_{ti} and ω_{li} the i-th low and high phonon frequencies and, γ_i the damping constant of the material. Applying boundary conditions by maintaining the continuity of tangential field components at the interface $z = 0$ yield:

$$E_{x1} = E_{x2} = E_x, \tag{6a}$$

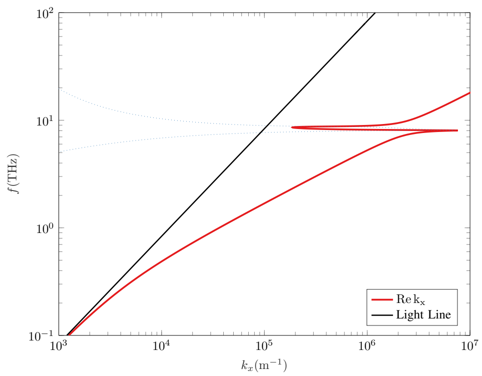
$$H_{y1} - H_{y2} = J_s \tag{6b}$$

where J_s is the surface current due to the sheet of charge. For a TM mode, H is related to E by:

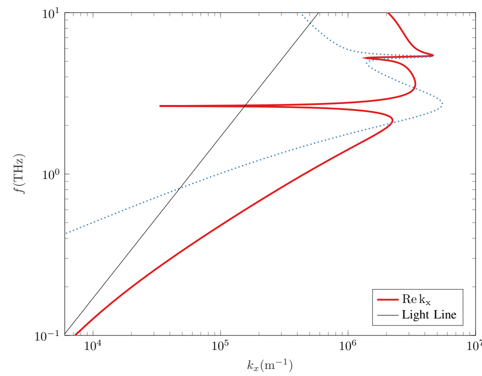
$$\mathbf{H} = \frac{1}{\eta_{TM}} \hat{\mathbf{n}} \times \mathbf{E} \tag{7}$$

where η_{TM} is the TM mode wave impedance equal to $k_z/\omega\varepsilon$. The surface current J_s in (6b) is related to the electric field and surface conductivity σ_s by Ohm's law:

$$J_s = \sigma_s E_x \tag{8}$$



(a) Case A



(b) Case B

Fig. 1. Dispersion relation of 2DEG embedded in region 2 of the heterostructure. Solid line: real part, dashed line: imaginary part

In the microwave and terahertz regions, the surface conductivity can be approximated by a Drude-type formula:

$$\sigma_s(\omega) = \frac{N_s e^2 \tau}{m^*} \frac{1}{1 + j\omega\tau} \tag{9}$$

The parameters e and m^* are the effective charge and effective mass of an electron respectively, N_s is surface-charge density, and the scattering time τ is the reciprocal of damping constant Γ. From Eqs. (6)-(8), we obtain the TM mode dispersion relation:

$$\frac{\varepsilon_1(\omega)}{k_{z1}} + \frac{\varepsilon_2(\omega)}{k_{z2}} = -\frac{\sigma_s}{\omega} \tag{10}$$

In the terahertz region, the surface conductivity (9) can case be approximated as [1]:

$$\sigma_s(\omega) \approx \frac{N_s e^2}{jm^*\omega}. \tag{11}$$

For a homogeneous environment where $\varepsilon_1 = \varepsilon_2$, Eq. (4), can therefore be written in a simplified form as:

$$\frac{2\varepsilon_1(\omega)}{k_{z1}} \approx -\frac{N_s e^2}{jm^*\omega^2} \tag{12}$$

Solving for the propagation constant k_x, using (3) by squaring (12) yields:

$$k_x^2 \approx \left(\frac{\omega}{c}\right)^2 \varepsilon_1(\omega) \left[1 + \left(\frac{2\omega^2}{\omega_p}\right)^2 \varepsilon_1(\omega)\right] \tag{13}$$

where ω_p is the 2DEG plasma frequency given expressed as:

$$\omega_p = \sqrt{\frac{4\pi e^2 N_s}{m^* c}}. \tag{14}$$

In a semiconductor heterostructure environment, the solution of (10) is cumbersome requiring careful complex root search analysis tools. However, when the propagation constant k_x is much larger than the material wave-vectors k_i, which is the case for a non-radiating wave, the solution can be accurately approximated as [2]:

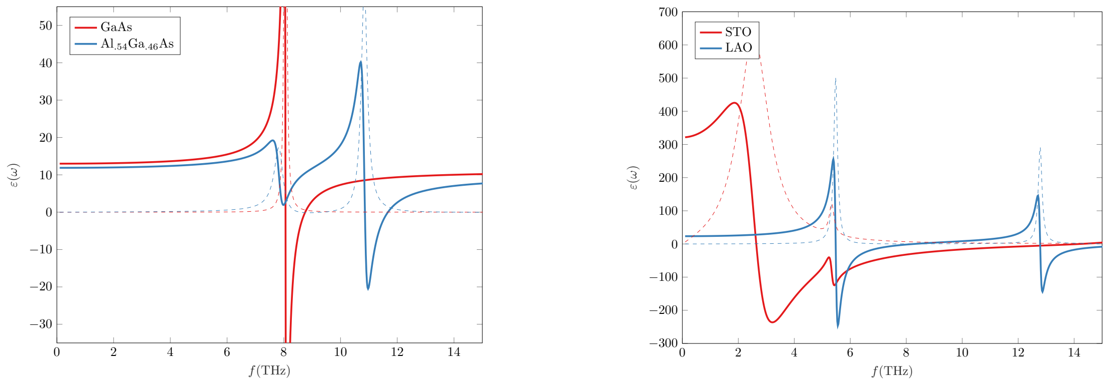
$$k_x \approx -\varepsilon_1(\omega) + \varepsilon_2(\omega) \frac{j\omega}{\sigma_s(\omega)} \tag{15}$$

III. Results and Discussion

In this section, we compare two different 2DEGs formed at the heterojunction of, first in a III-V compound and second in a perovskite structured oxide interface. We consider particular the first to be a Gallium Arsenide/Aluminum Gallium Arsenide (GaAs/AlGaAs) case which has been at the forefront in manufacturing microwave frequency devices. A surface charge density N_s of $10^{11}\mathrm{cm}^{-2}$ is assumed. The second example involves a Strontium Titanate/ Lanthanium Aluminum Oxide based 2DEG abbreviated as STO/LAO, that has attracted a lot of interest in the scientific community ever since its discovery in the last decade [**?**], owing to its remarkable electrical properties mainly due to very high electron mobility and metal-like free electron concentration which is taken as $10^{14}\mathrm{cm}^{-2}$ [**?**]. From here onwards, we identify the structures discussed above by cases A and B respectively. The dispersion relations for both cases are shown in Fig. 1. The lower part of the curve lies in the non-radiative plasmon region followed by an anomalous dispersion region and above that a Brewster mode radiative region. The plasmon region lies to the right of the light line and therefore, it is a slow wave region. Of particular significance is the much shorter wavelength of plasmons than light that prevents them to radiate from a flat planar surface. In other words, the plasma waves are tightly confined to the 2DEG where they propagate along the interface. This mechanism has led to waveguiding structures that have miniaturized dimensions [**?**]. The propagation constant in case B is at least an order greater than for case B as shown in Fig. 1. However, the dispersion relation does not indicate the existence of a propagating surface wave for which the following condition is necessary [3]:

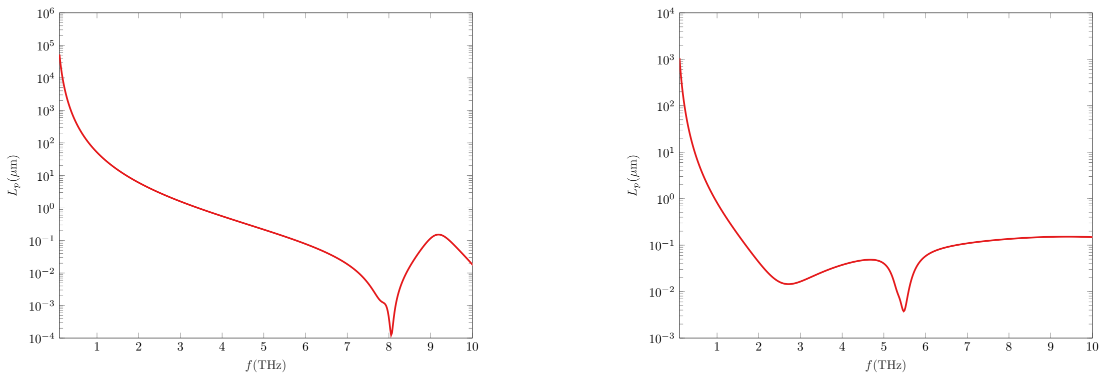
$$\varepsilon_1(\omega) \cdot \varepsilon_2(\omega) < 0 \tag{16}$$

To investigate this, the dielectric functions of the materials used are shown in Fig. 2 computed from (6a). The material parameters are taken from [4], [5]. As an example, GaAs parameters are listed in Table. I. In the frequency range of $3-5$THz, the necessary condition (16) holds true only for case B. This suggests that a 2DEG based on materials used



(a) Case A (b) Case B

Fig. 2. Dielectric Functions of the materials in bulk form. Solid line: real part, dashed line: imaginary part



(a) Case A (b) Case B

Fig. 3. Propagation Lengths of the 2DEG plasma waves

in case B is an excellent choice for guiding devices. The complex permittivity of GaAs is plotted in Fig. **??** where the real part changes sign at the plasma frequency. The result is shown in Fig. **??**. The lower part of the curve lies in the non-radiative plasmon region followed by an anomalous dispersion region and above that a Brewster mode radiative region. The plasmon region lies to the right of the light line and therefore, it is a slow wave region. Of particular significance is the much shorter wavelength of plasmons than light preventing from radiating from a planar surface. A limiting value of the wavelength confinement can be estimated from:

TABLE I
GaAs Material Properties

ε^0	12.9
ε^∞	11.0
ω_t	5.52×10^{13}rad/s
ω_l	5.06×10^{13}rad/s
Γ	4.52×10^{11}rad/s
m^*	$.063m_e$
τ	1.3889×10^{-12}s

$$\frac{\lambda_t}{\lambda_{sw}} \approx \sqrt{\varepsilon_1(\omega_t) + \varepsilon_2(\omega_t)} \tag{17}$$

where $\lambda_t = 2\pi c/\omega_t$ is the start wavelength of the dispersiive region that corresponds to the first phonon resonance in the dielectric functions shown in Fig. 2 dispersive region. We have calculated the ratios of 1.32 and 27.3 for cases A and B respectively.

The decay of the plasma wave along the interface is determined by the propagation length defined as:

$$L(\omega) = \frac{1}{\mathrm{Im}(k_x(\omega))} \tag{18}$$

which is plotted in Fig. 3. although the plasma waves travel farther in GaAs based 2DEG, the distance in an STO based 2DEG is still very large considering the dimensions of devices at terahertz frequency range. This behavior is similar to surface plasmons found at metal-dielectric interfaces at optical frequencies [**?**]. The lower value can be alluded to the large dielectric constant of STO. The behavior of

IV. Conclusion

In this paper, we have have presented basic theory of plasma wave propagation in the 2DEG. A comparison between III-

V class of materials and perovskite based oxides has been done by an example. The results show that the latter is a better option primarily due to its exceptional two-dimensional properties.

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