An Integral Equation Scheme for Plasma based Thin Sheets

Hasan T. Abbas
Department of Electrical and
Computer Engineering
Texas A&M University
College Station, TX 77843-3128
Email: hasantahir@tamu.edu

Robert D. Nevels
Department of Electrical and
Computer Engineering
Texas A&M University
College Station, TX 77843-3128
Email: nevels@ece.tamu.edu

***Abstract*—We discuss the dispersion relation of a thin plasma sheet embedded in a stack semiconductor materials. The conditions under which propagation of wave is ensured are investigated. Miniaturization of antennas and microwave devices is presented in light of the dispersion relation.**

I. Introduction

With the advancements in semiconductor manufacturing techniques, our perception of materials existing only in a volumetric sensse has changed. Of late, two-dimensional materials have been discovered. Graphene may the first that springs to mind but owing to difficulties in its engineering, it has created a divided opinion regards to its usage in the scientific community, although it has remarkable electrical and mechanical properties. Looking beyond, there are other 2D materials that inherently exist in commonplace transistor devices due to formation of an extremely thin layer of free electrons in the stack of multilayer semiconductor substrate. The electrical properties have to be considered in a different way than traditional bulk materials where permittivity and permeability are customarily considered in a bulk sense. Most commercial electromagnetic simulation tools model thin structures with equivalent surface conductivity where the thickness is incorporated in computing the corresponding parameters.

II. Theory

A. Derivation of Dispersion Relation

Consider two dielectric half-planes of permittivity ε_1 and ε_2 respectively, separated by an infinitesimally thin sheet of charge at $z = 0$. The propagation constants for the materials can be found starting with field expressions for transverse magnetic (TM) plane wave as shown in **??**. In region 1 ($z > 0$):

$$\mathbf{E}_1 = \left(\hat{\mathbf{x}} E_{x1} + \hat{\mathbf{y}} E_{z1}\right) e^{-j(k_x x + k_{z1} z)}, \tag{1a}$$
$$\mathbf{H}_1 = \hat{\mathbf{y}} H_{y1} e^{-j(k_x x + k_{z1} z)} \tag{1b}$$

Similarly, in region 2 ($z < 0$), the fields are expressed as:

$$\mathbf{E}_2 = \left(\hat{\mathbf{x}} E_{x2} + \hat{\mathbf{y}} E_{z2}\right) e^{-j(k_x x - k_{z2} z)}, \tag{2a}$$
$$\mathbf{H}_2 = \hat{\mathbf{y}} H_{y2} e^{-j(k_x x - k_{z1} z)}. \tag{2b}$$

where k_x and k_{zi} for $i = 1, 2$ are the x and z directed propagation constants respectively, related by the dispsersion equation:

$$\begin{aligned} k_{zi} &= \sqrt{k_i^2 - k_x^2} \\ &= \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon_i(\omega) - k_x^2} \end{aligned} \tag{3}$$

$$\frac{\varepsilon_1(\omega)}{k_{z1}} + \frac{\varepsilon_2(\omega)}{k_{z2}} = -\frac{\sigma_s}{\omega} \tag{4}$$

The dispersive dielectric constants $\varepsilon_i(\omega)$ are based on a Lorentz osciallator model:

$$\varepsilon(\omega) = \varepsilon^{\infty} + \prod_i \frac{\omega_{li}^2 - \omega^2 - j\gamma_i\omega}{\omega_{ti}^2 - \omega^2 - j\gamma_i\omega} \tag{5}$$

where the i is the number of resonances, ε^{∞} and ε^0 are the high and zero frequency limit values of dielectric constant, ω_{ti} is the i-th resonance frequency and, Γ_i the damping constant of the material. Applying boundary conditions by maintaining the continuity of tangential field components at the interface $z = 0$ yield:

$$E_{x1} = E_{x2} = E_x, \tag{6a}$$
$$H_{y1} - H_{y2} = J_s \tag{6b}$$

where J_s is the surface current due to the sheet of charge. For a TM mode, H is related to E by:

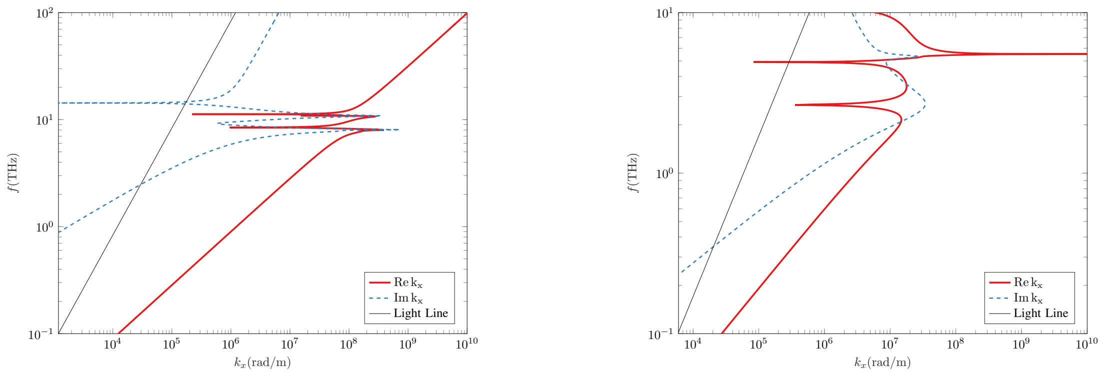
$$\mathbf{H} = \frac{1}{\eta_{TM}} \hat{\mathbf{n}} \times \mathbf{E} \tag{7}$$

where η_{TM} is the TM mode wave impedance equal to $k_z/\omega\varepsilon$. The surface current J_s in (5b) is related to the electric field and surface conductivity σ_s by Ohm's law:

$$J_s = \sigma_s E_x \tag{8}$$

In the microwave and terahertz regions, the surface conductivity can be approximated by a Drude-type formula:

$$\sigma_s(\omega) = \frac{N_s e^2 \tau}{m^*} \frac{1}{1 + j\omega\tau} \tag{9}$$



(a) Case A   (b) Case B

Fig. 1. Dispersion relation of 2DEG embedded in a heterostructure. Solid line: real part, dashed line: imaginary part

The parameters e and m^* are the effective charge and effective mass of an electron respectively, N_s is surface-charge density, and the scattering time τ is the reciprocal of damping constant Γ. From Eqs. (5)-(7), we obtain the TM mode dispersion relation:

In the terahertz region, the surface conductivity (8) can case be approximated as [1]:

$$\sigma_s(\omega) \approx \frac{N_s e^2}{jm^* \omega}. \tag{10}$$

For a homogeneous environment where $\varepsilon_1 = \varepsilon_2$, Eq. (9), can therefore be written in a simplified form as:

$$\frac{2\varepsilon_1(\omega)}{k_{z1}} \approx -\frac{N_s e^2}{jm^* \omega^2} \tag{11}$$

Solving for the propagation constant k_x, using (3) by squaring (11) yields:

$$k_x^2 \approx \left(\frac{\omega}{c}\right)^2 \varepsilon_1(\omega) \left[1 + \left(\frac{\omega^2 m^*}{\pi N_s e^2}\right) \varepsilon_1(\omega)\right] \tag{12}$$

In a semiconductor heterostructure environment, the solution of (9) is cumbersome requiring careful complex root search analysis tools. However, when the propagation constant k_x is much larger than the material wave-vectors k_i, which is the case for a non-radiating wave, the solution can be accurately approximated as [2]:

$$k_x \approx -\varepsilon_1(\omega) + \varepsilon_2(\omega) \frac{j\omega}{\sigma_s(\omega)} \tag{13}$$

III. Results and Discussion

In this section, we compare two different 2DEGs formed at heterostructure interfaces. The first is Gallium Arsenide/Aluminum Gallium Arsenide (GaAs/AlGaAs) based which is commonly used in contemporary microwave frequency devices. The second one is formed at the interface of Strontium Titanate/ Lanthanium Aluminum Oxide abbreviated as STO/LAO, that has exhibited exceptional electrical properties mainly due to metal-like free electron concentration at the interface. From here onwards, we identify the structures discussed above by cases A and B respectively. The dispersion relations for both cases are shown in Fig. 1. The lower part of the curve lies in the non-radiative plasmon region followed by an anomalous dispersion region and above that a Brewster mode radiative region. The plasmon region lies to the right of the light line and therefore, it is a slow wave region. Of particular significance is the much shorter wavelength of plasmons than light avoiding radiation from a flat planar surface. In other words, this can be thought of as a wave-guiding structure with highly miniaturized dimensions.

In the terahertz frequency range, case A enables reduction of the resonant wavelength by at least three orders compared to the wavelength of light in region 2 whereas, case B provides a two order reduction. However, the dispersion relation does not indicate the existence of a propagating surface wave for which the following condition is necessary [3]:

$$\varepsilon_1(\omega) \cdot \varepsilon_2(\omega) < 0 \tag{14}$$

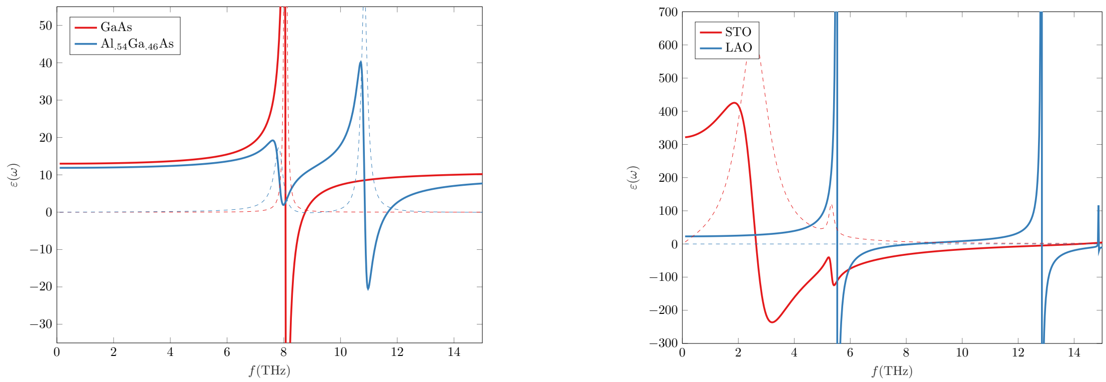
To investigate this, the dielectric functions of the materials used are shown in Fig. 2 computed from (5a). The material parameters are taken from [4], [5]. As an example, GaAs parameters are listed in Table. I.

In the frequency range of $1 - 10$THz, the necessary condition (14) holds true only for case B. This suggests that a 2DEG based on materials used in case B is an excellent choice for guiding devices.

TABLE I
GaAs Material Properties

ε^0	12.9
ε^∞	11.0
ω_t	5.52×10^{13}rad/s
ω_l	5.06×10^{13}rad/s
Γ	4.52×10^{11}rad/s
m^*	$.063m_e$
τ	1.3889×10^{12}s

The complex permittivity of GaAs is plotted in Fig. **??** where the real part changes sign at the plasma frequency. The



(a) Case A (b) Case B

Fig. 2. Dielectric Functions of the materials in bulk form. Solid line: real part, dashed line: imaginary part

result is shown in Fig. **??**. The lower part of the curve lies in the non-radiative plasmon region followed by an anomalous dispersion region and above that a Brewster mode radiative region. The plasmon region lies to the right of the light line and therefore, it is a slow wave region. Of particular significance is the much shorter wavelength of plasmons than light preventing from radiating from a flat planar surface. In other words, this can be thought of as a wave-guiding structure with highly miniaturized dimensions.

IV. Conclusion

References

[1] F. Stern, "Polarizability of a two-dimensional electron gas," *Physical Review Letters*, vol. 18, no. 14, p. 546, 1967.

[2] M. Jablan, H. Buljan, and M. Soljačić, "Plasmonics in graphene at infrared frequencies," *Physical review B*, vol. 80, no. 24, p. 245435, 2009.

[3] R. D. Nevels and H. T. Abbas, "Optical nanoantennas," *Handbook of Antenna Technologies*, pp. 527–566, 2016.

[4] E. D. Palik, "Gallium arsenide (GaAs)," in *Handbook of Optical Constants of Solids*, E. D. Palik, Ed. Burlington: Academic Press, 1997, pp. 429 – 443.

[5] Z. Zhang, B. Choi, M. Flik, and A. C. Anderson, "Infrared refractive indices of $\mathrm{LaAlO_3}$, $\mathrm{LaGaO_3}$, and $\mathrm{NdGaO_3}$," *JOSA B*, vol. 11, no. 11, pp. 2252–2257, 1994.