An Integral Equation Scheme for Plasma based Thin Sheets

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Outline

- Motivation and Objective
- Background
- Theory and Methods
  - Subwavelength phenomena - Dispersion relations
  - Existence of plasmonic behavior - Sommerfeld Integral analysis
  - Surface Integral equation scheme
- Conclusions

Motivation and Objective

- Plasmonics: subwavelength localization of electromagnetic (EM) fields
- Bridging the THz gap

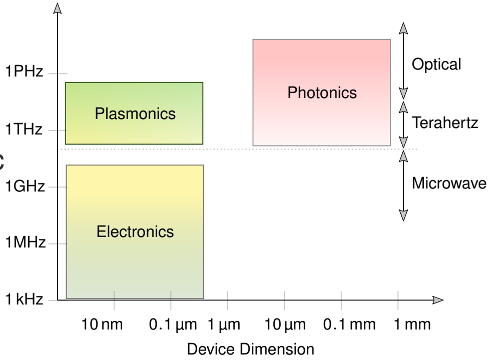


Figure: Communication Technologies at various frequencies

Background

Two-dimensional Electron Gas (2DEG)

- Semiconductor Heterostructure in high electron mobility transistor (HEMT)
- High concentration of free electrons ($\sim 1 \times 10^{11} - 1 \times 10^{14}\,\text{cm}^{-2}$)
- Very high Mobility ($\sim 1 \times 10^{3} - 1 \times 10^{6}\,\text{cm}^{2}/\text{V}/\text{s}$)
- Formation of Quantum Well
  - Two-dimensional confinement of electrons

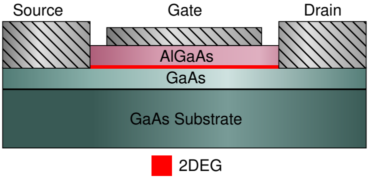


Figure: Typical GaAs/AlGaAs HEMT

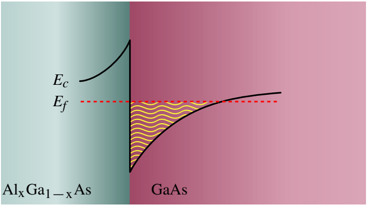
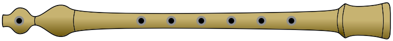


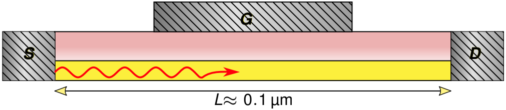
Figure: Band diagram of a GaAs/AlGaAs heterostructure

Background

2DEG (contd.)

- Plasma waves in 2DEG
- Dyakonov-Shur instability
  - Voltage bias at source and drain terminals
  - Plasma resonance
  - THz emission
- Electronic Flute
  - Tunable resonance with gate voltage



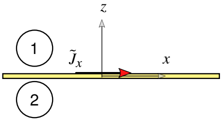


$$\lambda = \frac{c}{f}$$

$$\implies 300\mu\text{m}$$

Theory and Methods

Field Computation - Thin sheet



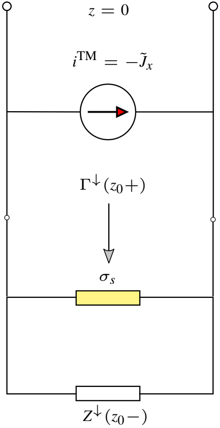
- Thin conductive sheet in free-space

$$Z^{\downarrow}(z_0^+) = \frac{Z_0}{1 + \sigma_s Z_0}$$

$$\Gamma^{\downarrow,\mathrm{TE}} = \frac{k_{z1} - \omega\mu_1\sigma_s}{k_{z1} + \omega\mu_1\sigma_s}$$

$$\Gamma^{\downarrow,\mathrm{TM}} = \frac{\omega\varepsilon_1 - \sigma_s k_{z1}}{\omega\varepsilon_1 + \sigma_s k_{z1}}$$

$$G_{zx}^{\mathrm{A}} = \frac{j\mu}{2}\cos\phi\,\mathcal{S}_1\left\{\frac{\Gamma^{\downarrow,\mathrm{TM}} - \Gamma^{\downarrow,\mathrm{TE}}}{k_\rho}\right\}.$$



Theory and Methods

Thin Sheet Simulation

- Volume Integral formulation

$$\mathbf{A} = \frac{\mu}{4\pi} \int_V \mathbf{J}_v(\mathbf{r}') \frac{\mathrm{e}^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \, \mathrm{d}v'$$

$$\mathbf{E}_1^{scat} = -\frac{j\omega}{k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\mathbf{A}$$

$$\mathbf{J}_v = \frac{-jk_1}{Z_0}(\varepsilon_2 - 1)\mathbf{E}_2$$

- Surface current J_s approximated from J_v

- Impedance (Leontovich) Boundary Condition

$$\mathbf{E}_{tan} = \eta Z_0 \hat{\mathbf{n}} \times \mathbf{H}$$

$$E^i = \eta Z_0 J_s(x') + \frac{\omega\mu}{4} \int_l J_s(x') H_0^{(2)}(k_2|x - x'|) \, \mathrm{d}x'$$

Theory and Methods

Proposed Surface Integral Equation (SIE) scheme

- Surface Equivalence Theorem

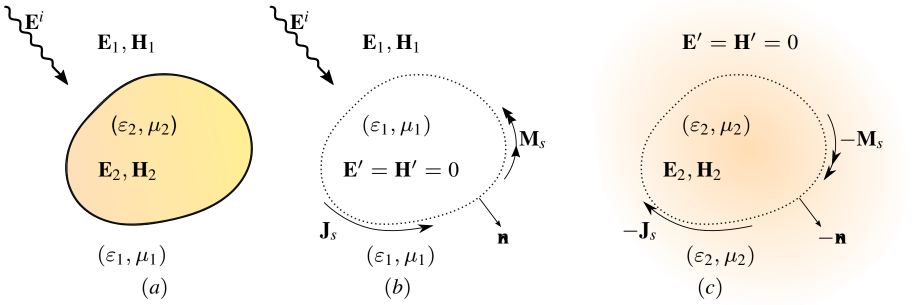


Figure: (a). Actual and its equivalent models for the (b) external and, (c) Internal region

Proposed Scheme

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Surface Integral Equation

- Exterior Region

$$
\begin{aligned}
\mathbf{E}_1 &= \mathbf{E}_i + \mathbf{E}_1^{scat} \\
&= -\frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_C \mathbf{J}_s(\mathbf{p}')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' \\
&- \frac{1}{4\varepsilon j}\nabla\times\int_l \mathbf{M}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' + \mathbf{E}_i
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}_1 &= \mathbf{H}_i + \mathbf{H}_1^{scat} \\
&= \frac{1}{4j}\nabla\times\int_l \mathbf{J}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' \\
&- \frac{\omega}{4k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_l \mathbf{M}_s(\rho')H_0^{(2)}(k_1|\rho-\rho'|)\,\mathrm{d}l' + \mathbf{H}_i
\end{aligned}
$$

Proposed Scheme

Surface Integral Equation

- Interior Region

$$
\begin{aligned}
\mathbf{E}_2 &= \mathbf{E}_2^{scat} \\
&= -\frac{\omega}{4k_2^2}\left(k_2^2 + \nabla\nabla\cdot\right)\int\limits_C \left(-\mathbf{J}_s(\mathbf{p}')\right) H_0^{(2)}(k_2|\rho - \rho'|)\,\mathrm{d}l' \\
&- \frac{1}{4j}\nabla \times \int\limits_l \left(-\mathbf{M}_s(\rho')\right) H_0^{(2)}(k_2|\rho - \rho'|)\,\mathrm{d}l'
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}_2 &= \mathbf{H}_1^{scat} \\
&= \frac{1}{4j}\nabla \times \int\limits_l \left(-\mathbf{J}_s(\rho')\right) H_0^{(2)}(k_2|\rho - \rho'|)\,\mathrm{d}l' \\
&- \frac{\omega}{4k_2^2}\left(k_2^2 + \nabla\nabla\cdot\right)\int\limits_l \left(-\mathbf{M}_s(\rho')\right) H_0^{(2)}(k_2|\rho - \rho'|)\,\mathrm{d}l'
\end{aligned}
$$

Theory and Methods

TM_z SIE for Thin Flat Sheet

$$\hat{\mathbf{n}} \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{0}$$

$$E_i = \frac{\omega}{4} \int_L J_z(x') \left[H_0^{(2)}(k_1|x - x'|) + H_0^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

$$\hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}$$

$$H_i^{tan} = \frac{-j\omega}{2} \int_L M_x(x') \left[\varepsilon_1 H_0^{(2)}(k_1|x - x'|) + \varepsilon_1 H_2^{(2)}(k_1|x - x'|) \right.$$
$$\left. + \varepsilon_2 H_0^{(2)}(k_2|x - x'|) + \varepsilon_2 H_2^{(2)}(k_2|x - x'|) \right] \mathrm{d}x'$$

Theory and Methods

Method of moments

- Integral equations to system of linear equations

$$\begin{bmatrix} Z_{mn} & 0 \\ 0 & Y_{mn} \end{bmatrix} \begin{bmatrix} J_n \\ M_n \end{bmatrix} = \begin{bmatrix} E_m^i \\ H_m^i \end{bmatrix}$$

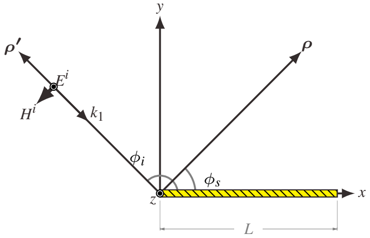
- Pulse basis functions and Point matching used
- Far-field

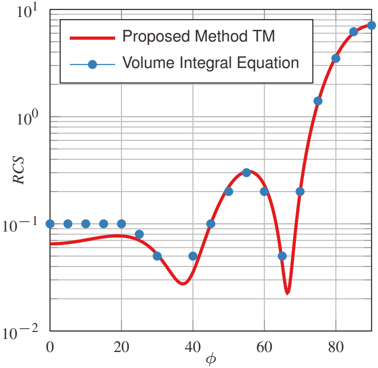
$$RCS(\phi) \simeq \int_0^L \left[J_z(x')\eta_1 + M_x(x') \sin(\phi_i) \right] e^{jk_1 x' \cos(\phi_i)} \mathrm{d}x'$$

Results

Thin Sheet Simulation (TM_z)

- TM_z polarization
- Dielectric Rod of length 2.5 λ
- $\varepsilon = 4$, $\mu = 1$



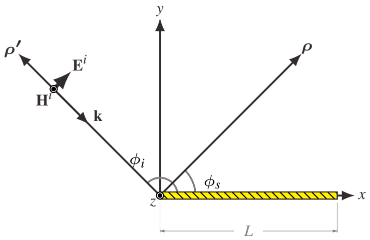


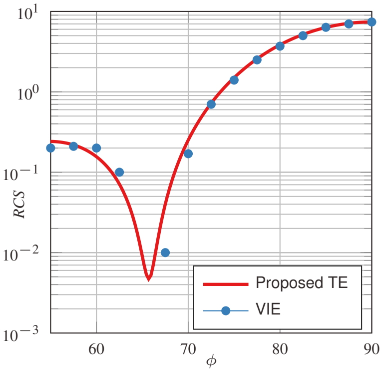
- Thickness of $.05\lambda$ assumed in Volume Integral equation model

Results

Thin Sheet Simulation (TE_z)

- TE_z polarization
- Dielectric Rod of length 2.5 λ
- $\varepsilon = 4$, $\mu = 1$



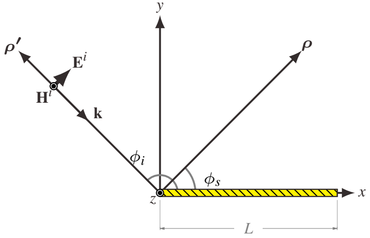


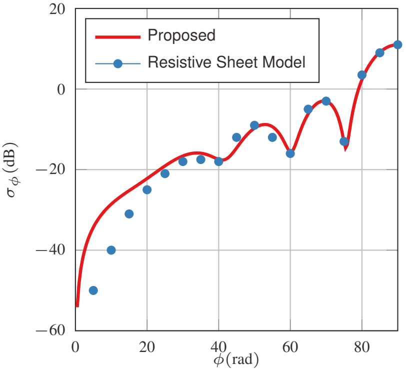
- Thickness of $.05\lambda$ assumed in Volume Integral equation model

Results

Thin Sheet Simulation (TE_z)

- TE_z polarization
- Dielectric Rod of length 2 λ
- $\varepsilon = 4$, $\mu = 1$





- Thickness of $.628/k_1$ assumed in resistive model

Summary

Two-dimensional plasmonic devices

- Subwavelength wave phenomena at optical and terahertz frequencies
- Realization of terahertz sources and sensors
- 2D nature of waves permits subwavelength confinement
- Plasmonic activity
- Nanoscale imaging using terahertz plasma waves

Thank you!

Questions?