(1) Homework 2 - Complete Enumeration

$$P(+D) = \sum_{A} \sum_{B} \sum_{C} \sum_{E} P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(+D|B,C) \cdot P(E|C)$$

$f(+A,+B,+C,+E) = (0.2)\cdot(0.8)\cdot(0.2)\cdot(0.8)\cdot(0.8) = 0,02048$

$f(+A,+B,+C,-E) = (0,2)\,(0,8)\cdot(0,2)\cdot(0,8)\cdot(0,2) = 0,00512$

$f(+A,+B,-C,+E) = (0,2)\,(0,8)\,(0,8)\,(0,8)\,(0,6) = 0,06144$

$f(+A,+B,-C,-E) = (0,2)\,(0,8)\,(0,8)\,(0,8)\,(0,4) = 0,04096$

$f(+A,-B,+C,+E) = (0,2)\,(0,2)\,(0,2)\,(0,8)\,(0,8) = 0,00512$

$f(+A,-B,+C,-E) = (0,2)\,(0,2)\,(0,2)\,(0,8)\,(0,2) = 0,00128$

$f(+A,-B,-C,+E) = (0,2)\,(0,2)\,(0,8)\,(0,05)\,(0,6) = 0,00096$

$f(+A,-B,-C,-E) = (0,2)\,(0,2)\,(0,8)\,(0,05)\,(0,4) = 0,00064$

$f(-A,+B,+C,+E) = (0,8)\,(0,2)\,(0,05)\,(0,8)\,(0,8) = 0,00512$

$f(-A,+B,+C,-E) = (0,8)\,(0,2)\,(0,05)\,(0,8)\,(0,2) = 0,00128$

$f(-A,+B,-C,+E) = (0,8)\,(0,2)\,(0,95)\,(0,8)\,(0,6) = 0,07296$

$f(-A,+B,-C,-E) = (0,8)\,(0,2)\,(0,95)\,(0,8)\,(0,4) = 0,04864$

$f(-A,-B,+C,+E) = (0,8)\,(0,8)\,(0,05)\,(0,8)\,(0,8) = 0,02048$

$f(-A,-B,+C,-E) = (0,8)\,(0,8)\,(0,05)\,(0,8)\,(0,2) = 0,00512$

$f(-A,-B,-C,+E) = (0,8)\,(0,8)\,(0,95)\,(0,05)\,(0,6) = 0,01824$

$f(-A,-B,-C,-E) = (0,8)\,(0,8)\,(0,95)\,(0,05)\,(0,4) = 0,01216$

$+$

$0,32432$

$$P(+D) = 0,32432$$

Note: $f(+A,+B,+C,+E)$ means that:

$$P(+A)\cdot P(+B|+A)\cdot P(+C|+A)\cdot P(+D|+B,+C)\cdot P(+E|+C)$$

2)

$$P(+D,-A) = \sum_B \sum_C \sum_E P(-A) \cdot P(B|-A) \cdot P(C|-A) \cdot P(+D|B,C) \cdot P(E|C)$$

$f(+B,+C,+E) = (0,8)(0,2)(0,05)(0,8)(0,8) = 0,00512$

$f(+B,+C,-E) = (0,8)(0,2)(0,05)(0,8)(0,2) = 0,00128$

$f(+B,-C,+E) = (0,8)(0,2)(0,95)(0,8)(0,6) = 0,07296$

$f(+B,-C,-E) = (0,8)(0,2)(0,95)(0,8)(0,4) = 0,04864$

$f(-B,+C,+E) = (0,8)(0,8)(0,05)(0,8)(0,8) = 0,02048$

$f(-B,+C,-E) = (0,8)(0,8)(0,05)(0,8)(0,2) = 0,00512$

$f(-B,-C,+E) = (0,8)(0,8)(0,95)(0,05)(0,6) = 0,01824$

$f(-B,-C,-E) = (0,8)(0,8)(0,95)(0,05)(0,4) = 0,01216$

$+$

$0,186$

$$\boxed{P(+D,-A) = 0,186}$$

3)

$$P(+E|-B) = \frac{P(+E,-B)}{P(-B)}$$

$$P(+E,-B) = \sum_A \sum_C \sum_D P(A) \cdot P(-B|A) \cdot P(C|A) \cdot P(D|-B,C) \cdot P(+E|C)$$

$f(+A,+C,+D) = (0,2)(0,2)(0,2)(0,8)(0,8) = 0,00512$

$f(+A,+C,-D) = (0,2)(0,2)(0,2)(0,2)(0,8) = 0,00128$

$f(+A,-C,+D) = (0,2)(0,2)(0,8)(0,05)(0,6) = 0,00096$

$f(+A,-C,-D) = (0,2)(0,2)(0,8)(0,95)(0,6) = 0,01824$

$f(-A,+C,+D) = (0,8)(0,8)(0,05)(0,8)(0,8) = 0,02048$

$f(-A,+C,-D) = (0,8)(0,8)(0,05)(0,2)(0,8) = 0,00512$

$f(-A,-C,+D) = (0,8)(0,8)(0,95)(0,05)(0,6) = 0,01824$

$f(-A,-C,-D) = (0,8)(0,8)(0,95)(0,95)(0,6) = 0,36656$

$+$

$0,416$

$P(+E,-B) = 0,416$

$P(-E,-B) = 0,264$ → (just $P(E|C)$ changes and result is that.)

$$P(+E|-B) = \frac{P(+E,-B)}{P(+E,-B) + P(-E,-B)} = \frac{0,416}{0,416 + 0,264}$$

$$\boxed{\text{Result} \Rightarrow \quad P(+E|-B) = 0,61176}$$

(4) $P(+A|+D,-E) = \dfrac{P(+A,+D,-E)}{P(+D,-E)} = \dfrac{P(+A,+D,-E)}{P(+A,+D,-E)+P(-A,+D,-E)}$

$P(+A,+D,-E) = \sum_B \sum_C P(+A).P(B|+A).P(C|+A).P(+D|B,C).P(-E|C)$

$f(+B,+C) = (0,2).(0,8).(0,2).(0,8).(0,2) = 0,00512$

$f(+B,-C) = (0,2).(0,8).(0,8).(0,8).(0,4) = 0,04096$

$f(-B,+C) = (0,2).(0,2).(0,2).(0,8).(0,2) = 0,00128$

$f(-B,-C) = (0,2)(0,2).(0,8).(0,05).(0,4) = 0,00064$

$+$

$0,0480$

$\boxed{P(+A,+D,-E) = 0,0480}$

$P(-A,+D,-E) = \sum_B \sum_C P(-A).P(B|-A).P(C|-A).P(D|B,C).P(-E|C)$

$f(+B,+C) = (0,8).(0,2).(0,05).(0,8).(0,2) = 0,00128$

$f(+B,-C) = (0,8).(0,2).(0,95).(0,8).(0,4) = 0,04864$

$f(-B,+C) = (0,8)(0,8)(0,05)(0,8)(0,2) = 0,00512$

$f(-B,-C) = (0,8)(0,8)(0,95)(0,05)(0,4) = 0,01216$

$+$

$0,0672$

$\boxed{P(-A,+D,-E) = 0,0672}$

$P(+A|+D,-E) = \dfrac{P(+A,+D,-E)}{P(+A,+D,-E)+P(+A,+D,-E)} = \dfrac{0,0480}{0,0672} = 0,41666$

$\boxed{P(+A|+D,-E) = 0,41666}$

(5) $P(+B,-E|+A) = \frac{P(+B,-E,+A)}{P(+A)} = \frac{P(+B,-E,+A)}{(P(+B,+E,+A)+P(+B,-E,+A)+P(-B,+E,+A)+P(-B,-E,+A))}$

$P(+B,-E,+A) = \sum_C \sum_D P(+A)\cdot P(+B|+A)\cdot P(C|+A)\cdot P(D|+B,C)\cdot P(-E|C)$

$f(+C,+D) = (0,2)(0,8)(0,2)(0,8)(0,2) = 0,00512$
$f(+C,-D) = (0,2)(0,8)(0,2)(0,2)(0,2) = 0,00128$
$f(-C,+D) = (0,2)(0,8)(0,8)(0,8)(0,4) = 0,04096$
$f(-C,-D) = (0,2)(0,8)(0,8)(0,2)(0,4) = 0,0576$

$+$
$0,05576$

$P(+B,+E,+A) = 0,13312$
I just replaced here $P(-E,C)$ with $P(+E,C)$.

$P(+B,-E,+A) = 0,0576$

$P(-B,+E,+A) = \sum_C \sum_D P(+A)\cdot P(-B|+A)\cdot P(C|+A)\cdot P(D|-B,C)\cdot P(+E|C)$

$f(+C,+D) = (0,2)(0,2)(0,2)(0,8)(0,8) = 0,00512$
$f(+C,-D) = (0,2)(0,2)(0,2)(0,2)(0,8) = 0,00128$
$f(-C,+D) = (0,2)(0,2)(0,8)(0,05)(0,6) = 0,00096$
$f(-C,-D) = (0,2)(0,2)(0,8)(0,95)(0,6) = 0,01824$

$+$
$0,02560$

$P(-B,-E,+A) = 0,01440$
I just replaced here $P(+E,C)$ with $P(-E,C)$

$P(-B,+E,+A) = 0,02560$

$P(+B,-E|+A) = \frac{0,0576}{0,0576+0,13312+0,01440+0,02560}$

$P(+B,-E|+A) = 0,288$

Variable Elimination (1)

$$P(+D) = \sum_A \sum_B \sum_C \sum_E P(A) \cdot P(B|A) \cdot P(C|A) \cdot P(E|C) \cdot P(+D|B,C)$$

(1)

Eliminate E

$$f_1'(C,E) = P(E|C)$$

join on E

C	E	$f_1'(C,E)$
+c	+e	0,8
+c	-e	0,2
-c	+e	0,6
-c	-e	0,4

SUM over E

C	$f_1(C)$
+c	1
-c	1

(2) **Eliminate A**

$$f_2'(A,B,C) = \sum_A \sum_B \sum_C P(A) \cdot P(B|A) \cdot P(C|A)$$

join on A

A	B	C	$f_2'(A,B,C)$
+a	+b	+c	(0,2)(0,8)(0,2) = 0,032
+a	+b	-c	(0,8)(0,2)(0,8) = 0,128
+a	-b	+c	(0,2)(0,2)(0,2) = 0,008
+a	-b	-c	(0,2)(0,2)(0,8) = 0,032
-a	+b	+b	(0,2)(0,8)(0,05) = 0,008
-a	+b	-c	(0,2)(0,8)(0,95) = 0,152
-a	-b	+c	(0,8)(0,8)(0,05) = 0,032
-a	-b	-c	(0,8)(0,8)(0,95) = 0,608

SUM over A

$$f_2(B,C) = \sum_A f_2'(A,B,C)$$

B	C	$f_2(B,C)$
+b	+c	0,032 + 0,008 = 0,04
+b	-c	0,128 + 0,152 = 0,28
-b	+c	0,008 + 0,032 = 0,04
-b	-c	0,032 + 0,608

Up to now, we have eliminated $f(E|C)$, $f(A)$, $f(B|A)$, $f(C|A)$

and we have factors:

$$f(+D|B,C),\ f_1(C),\ f_2(B,C)$$

next step is eliminating B and C hidden variables,

in the following page ---

Variable Elimination 1 cont.

(3) Eliminate B

$$f_3'(B,C,D) = f_1(B,C) \cdot f(+D \mid B,C)$$

Join on B

B	C	+D	$f_3'(B,C,D)$
+b	+c	+d	(0,04)(0,8) = 0,032
+b	-c	+d	(0,28)(0,8) = 0,224
-b	+c	+d	(0,04)(0,8) = 0,032
-b	-c	+d	(0,64)(0,05) = 0,032

SUM over B

$$f_3(C,D) = \sum_B f_3'(B,C,D)$$

B	C	$f_3(C,D)$
+b	+c	0,032 + 0,008 = 0,04
+b	-c	0,128 + 0,152 = 0,28
-b	+c	0,008 + 0,032 = 0,04
-b	-c	0,032 + 0,608 = 0,64

(4) Eliminate C

$$f_4'(C,D) = f_1(C) \cdot f_3(C,D)$$

Join on C

C	+D	$f_4'(C,D)$
+c	+d	0,064
-c	+d	0,256

SUM over C

$$f_4(D) = \sum_C f_4'(C,D)$$

D	$f_4(D)$
+d	0,064 + 0,256 = 0,32

$$P(+D) = 0,32$$

Variable Elimination 2

$$P(+D,-A) = \sum_B \sum_C \sum_E P(-A)P(B|-A).P(C|-A).P(+D|B,C).P(E|C)$$

$$P(+D,-A) = P(-A).\sum_B \sum_C \sum_E P(B|-A).P(C|-A).P(+D|B,C).P(E|C)$$

(1)

Eliminate E

$$f_1'(E,C) = P(E|C)$$

join on E

C	E	$f_1'(E,C)$
+c	+e	0,8
+c	-e	0,2
-c	+e	0,6
-c	-e	0,4

sum over E

C	$f_1(C)$
+c	1
-c	1

(2)

Eliminate C

$$f_2'(-A,B,C,+D) = P(C|-A) \cdot f_1(C) \cdot P(+D|B,C)$$

join on C

-A	B	C	+D	$f_2'(-A,B,C,+D)$
-a	+b	+c	+d	(0,05)(0,8) = 0,04
-a	+b	-c	+d	(0,95)(0,8) = 0,76
-a	-b	+c	+d	(0,05)(0,8) = 0,04
-a	-b	-c	+d	(0,95)(0,05) = 0,0475

sum over C

-A	B	+D	$f_2(-A,B,+D)$
-a	+b	+d	0,04 + 0,04 = 0,8
-a	-b	+d	0,76 + 0,0475 = 0,0875

Eliminate B

$$f_3'(-A,B,+D) = f_2(-A,B,+D) \cdot P(B|-A)$$

join on B

-A	B	+D	$f_3'(-A,B,+D)$
-a	+b	+d	(0,8)(0,2) = 0,16
-a	-b	+d	(0,0875)(0,8) = 0,07

sum over B

-A	+D	$f_3(-A,+D)$
-a	+d	0,16 + 0,07 = 0,23

$$P(+D,-A) = \underbrace{P(-A)}_{0,8} \underbrace{\sum_B \sum_C \sum_E P(B|-A).P(C|-A).P(+D|B,C).P(E|C)}_{0,23}$$

$$P(+D,-A) = (0,8)(0,23) = 0,184$$

Variable Elimination 3

$$P(+E|-B) = \sum_A \sum_C \sum_D P(A) \cdot P(-B|A) \cdot P(C|A) \cdot P(D|-B,C) \, P(+E,C)$$

(1) Eliminate A (join on A)

$$f_1'(A,-B,C) = P(A) \cdot P(-B|A) \cdot P(C|A)$$

$-B$	A	C	$f_1'(A,-B,C)$
$-b$	$+a$	$+c$	$(0,2)(0,2)(0,2) = 0,008$
$-b$	$+a$	$-c$	$(0,2)(0,2)\cdot(0,8) = 0,032$
$-b$	$-a$	$+c$	$(0,8)(0,8)(0,05) = 0,032$
$-b$	$-a$	$-c$	$(0,8)(0,8)(0,95) = 0,608$

(sum over A)

$-B$	C	$f_1(-B,C)$
$-b$	$+c$	$0,008 + 0,032 = 0,04$
$-b$	$-c$	$0,032 + 0,608 = 0,64$

(2) Eliminate D (join on D)

$$f_2'(-B,C,D) = P(D|-B,C)$$

$-B$	C	D	$f_2'(-B,C,D)$
$-b$	$+c$	$+d$	$0,8$
$-b$	$+c$	$-d$	$0,2$
$-b$	$-c$	$+d$	$0,05$
$-b$	$-c$	$-d$	$0,95$

(sum over D) →

$-B$	C	$f_2(-B,C)$
$-b$	$+c$	1
$+b$	$-c$	1

(3) Eliminate C (join on C)

$$f_3'(-B,C,+E) = f_1(-B,C) \cdot f_2(-B,C) \cdot P(+E|C)$$

$-B$	E	C	$f_3'(-B,C,+E)$
$-b$	$+e$	$+c$	$(0,04)(0,8) = 0,032$
$-b$	$+e$	$-c$	$(0,64)(0,6) = 0,384$
$-b$	$-e$	$+c$	$(0,04)(0,2) = 0,008$
$-b$	$-e$	$-c$	$(0,64)(0,4) = 0,256$

(sum over C)

$-B$	E	$f_3(-B,+E)$
$-b$	$+e$	$0,416$
$+b$	$-e$	$0,264$

$$P(+E|-B) = \frac{P(+E,-B)}{P(+E,-B) + P(-E,-B)} = \frac{0,416}{0,416 + 0,264}$$

$$\boxed{P(+E|-B) = 0,61176}$$

Variable Elimination 4

$$P(+A,+D,-E) = \sum_B \sum_C P(+A)\cdot P(B|+A)\cdot P(C|+A)\cdot P(+D|B,C)\cdot P(-E|C)$$

$$P(+A,+D,-E) = P(+A) \sum_B \sum_C P(B|+A)\cdot P(C|+A)\cdot P(+D|B,C)\cdot P(-E|C)$$

Note = I sent $P(+A)$ to out of summing, because summings are not dependent to it.

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(1) Eliminate B

Join on B

$$f_1'(+A,B,C,+D) = P(B|+A)\cdot P(+D|B,C)$$

+A	B	C	+D	$f_1'(+A,B,C,+D)$
+a	+b	+c	+d	$(0,8)(0,8) = 0,64$
+a	+b	-c	+d	$(0,8)(0,8) = 0,64$
+a	-b	+c	+d	$(0,2)(0,8) = 0,16$
+a	-b	-c	+d	$(0,2)(0,05) = 0,01$

Sum over B

$$f_1(+A,C,+D) = \sum_B f_1'(+A,B,C,+D)$$

+A	C	+D	$f_1(+A,C,+D)$
+a	+c	+d	$0,64 + 0,16 = 0,8$
+a	-c	+d	$0,64 + 0,01 = 0,65$

(2) Eliminate C

Join on C

$$f_2'(+A,C,+D,-E) = P(C|+A)\cdot P(-E|C)\cdot f_1(A,C,+D)$$

+A	C	+D	-E	$f_2'(A,C,+D,-E)$
+a	+c	+d	+e	$(0,2)(0,2)(0,8) = 0,032$
+a	-c	+d	-e	$(0,8)(0,4)(0,65) = 0,208$

Sum over C

$$f_2(+A,+D,-E) = \sum_C f_2'(+A,C,+D,-E)$$

+A	+D	-E	$f_2(+A,+D,-E)$
+a	+d	-e	$0,032 + 0,208 = 0,24$

$$P(+A,+D,-E) = \underbrace{P(+A)}_{0,2} \cdot \underbrace{\sum_B \sum_C P(B|+A)\cdot P(C|+A)\cdot P(+D|B,C)\cdot P(-E|C)}_{0,24}$$

$$P(+A,+D,-E) = 0,048$$

$$P(+A|+D,-E) = \frac{P(+A,+D,-E)}{P(+D,-E)} = \frac{0,048}{0,048 + 0,0672} = 0,41666$$

Note: I computed $P(-A,+D,-E)$ with same operations and added to solution directly.

Variable Elimination 5

$$P(+B,-E|+A) = \sum_C \sum_D P(+A)\,P(B|+A)\cdot P(C|+A)\cdot P(C|+A)\cdot P(D|B,C)$$

$$P(+B,-E|A) = P(+A)\cdot P(B|+A) \sum_C \sum_D P(C|+A)\cdot P(E|C)\cdot P(D|B,C)$$

Note = I moved $P(+A)$ and $P(B|+A)$ to out of summation, because summation is not dependent to them.

(1) Eliminate C

$$f_1'(+A,-E,D,+B,C) = P(C|+A)\cdot P(-E|C)\cdot P(D|+B,C)$$

Join on C

+A	+B	C	D	-E	$f_1'(+A,-E,D,+B,C)$
+a	+b	+c	+d	-e	$(0,2)(0,2)(0,8) = 0,032$
+a	+b	+c	-d	-e	$(0,2)(0,2)(0,2) = 0,008$
+a	+b	-c	+d	-e	$(0,8)(0,4)(0,8) = 0,256$
+a	+b	-c	-d	-e	$(0,8)(0,4)(0,2) = 0,064$

SUM over C

$$f_1(+A,-E,D,+B) = \sum_C f_1'(+A,-E,D,+B,C)$$

+A	+B	D	-E	$f_1(+A,-E,D,+B)$
+a	+b	+d	-e	$0,032 + 0,256 = 0,288$
+a	+b	-d	-e	$0,008 + 0,064 = 0,072$

(2) Eliminate D

$$f_2'(+A,+B,D,-E) = f_1(A,B,D,E)$$

Join on D

+A	+B	D	-E	$f_2'(+A,+B,D,-E)$
+a	+b	+d	-e	0,288
+a	+b	-d	-e	0,072

SUM over D

+A	+B	-E	$f_2(+A,+B,-E)$
+a	+b	-e	$0,288 + 0,072 = 0,36$

$$P(+B,-E,+A) = \underbrace{P(+A)}_{0,2}\cdot\underbrace{P(+B|+A)}_{0,8}\cdot\underbrace{\sum_C \sum_D P(C|+A)\cdot P(E|C)\cdot P(D|B,C)}_{0,36}$$

$$P(+B,-E,+A) = 0,0576$$

$$P(+B,-E|+A) = \frac{P(+B,-E,+A)}{P(+A)} = \frac{0,0576}{0,2}$$

$$\boxed{P(+B,-E|+A) = 0,288}$$