Fast**FAST- National University of Computer & Emerging Sciences, Karachi.  
Department of Computer Science,  
Mid Term IExaminations, Fall 2018.  
2nd October, 2018, 1:00 pm – 2:00 pm**

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| **Course Code:**CS 211 | **Course Name:**Discrete Structures |
| **Instructors:** Dr. Jalal ud Din, Dr. Nouman M Durrani and Shoaib Raza | |
| **Student Roll No:** | **Section:** |

**Instructions:**

* Return the question paper.
* Read each question completely before answering it. There are**4questions on2 pages.**
* In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
* All the answers must be solved according to the sequence given in the question paper,otherwise marks will be deducted.
* This paper is subjective.
* The sign of difference is denoted by both symbols "\" and "-" in the set theory.

**Time Allowed**: 60 minutes. **MaximumPoints**: 24 points

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**Propositional Logic and Equivalences** [3x2=6 points]

Q No. 1 (i)Express these system specifications using the propositions p “The message is scanned for viruses”

and q “The message was sent from an unknown system” together with logical connectives (includingnegation).

1. “The message is scanned for viruses whenever the message was sent from an unknown system.”

SOLUTION: q implies p

1. “The message was sent from an unknown system but it was not scanned for viruses.”

SOLUTION:q and not p

(ii) Let p, q and r be statements. Determine, using a truth table or otherwise, that if S1 is the statement (p V q) ∧ (q V r) ∧ (r V p) and if S2 is the statement (p ∧ q) V (q ∧ r) V (r ∧ p), then S1 and S2 are logically equivalent.

SOLUTION: You can either use algebra of proposition or truth table to determine the solution of the question. I initially started with algebra of proposition, but after 3-5 mins of workout, I realized that the solution will be very time consuming, not something which you can do during an exam. A more efficient approach would be to quickly use the truth table.

Using a C++ program I was able to get the following solution:

p=0 q=0 r=0

(p|q)&(q|r)&(r|p)=0

(p&q)|(q&r)|(r&p)=0

p=0 q=0 r=1

(p|q)&(q|r)&(r|p)=0

(p&q)|(q&r)|(r&p)=0

p=0 q=1 r=0

(p|q)&(q|r)&(r|p)=0

(p&q)|(q&r)|(r&p)=0

p=0 q=1 r=1

(p|q)&(q|r)&(r|p)=1

(p&q)|(q&r)|(r&p)=1

p=1 q=0 r=0

(p|q)&(q|r)&(r|p)=0

(p&q)|(q&r)|(r&p)=0

p=1 q=0 r=1

(p|q)&(q|r)&(r|p)=1

(p&q)|(q&r)|(r&p)=1

p=1 q=1 r=0

(p|q)&(q|r)&(r|p)=1

(p&q)|(q&r)|(r&p)=1

p=1 q=1 r=1

(p|q)&(q|r)&(r|p)=1

(p&q)|(q&r)|(r&p)=1

(iii) Prove the following logical equivalence using the laws of logic(Algebra of Proposition):

¬(P ↔ Q) ≡ P ↔ ¬Q

SOLUTION:  
¬(P ↔ Q) = ¬( (not(P) and not(Q) or (P and Q) ) = (P or Q) and (not(P) or not(Q) ) = P xor Q

By using the idea of Karaugh map, De morgan’s law and the well known form of XOR addition

P ↔ ¬Q = (not(P) and Q) or (P and not(Q) = P xor Q

**Predicate, Quantifiers and Rules of Inference** [3x2=6 points]

Q. No. 2 (i) Let P(x, y) means “x+y >10”, where x and y are integers. Determine the truth value of the statement.

a)∀x ∃y P(x, y).  
 SOLUTION:

True, provide a short justification. Just writing true will result in zero marks.

1. ¬∀x∃y ¬P(x, y).

SOLUTION:

Negation of universal statement means existential statement. Hence the above statement is true. Provide an example to illustrate your answer.

(ii) Translate the following statements into English, where F(p) is "Printer p is out of service", B(p) is "Printer p is busy", L(j) is "Print job j is lost", Q(j) is "Print job j is queued" and U(s, r) is “Student s uses Printer r”.

1. p (B(p) F(p))

SOLUTION:

There exist a printer such that the printer is busy and out of service.

1. j (Q(j) L(j)) p F(p)

SOLUTION:

If there exist a print job which is queued and the job is lost, then there exist a printer which is out of service.

1. What relevant conclusion or conclusions can be drawn from the following premises? Also, explain the rules of inference used to obtain each conclusion from the premises.

It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it is sunny.

If we do not go swimming, then we will take a canoe trip.If we take a canoe trip, then we will be home by sunset.

Where: s: “it is sunny this afternoon" c: “it is colder than yesterday"

w: “we will go swimming" t: “we will take a canoe trip.

h: “we will be home by the sunset."

SOLUTION:

Not (s) and c (simplification: not(s) )

W 🡪 s (modulus Tollen: not (w)

Not (s) 🡪t (Modus Ponus, t)

t 🡪 h (Modus Ponus, h)

**Set Theory** [4x2=8points]

Q. No. 3 (i) A computer company receives 350 applications. Suppose that 220 majored in computer science, 147 majored in business, and 51 majored in both. How many of these applicants majored neither incomputer science nor in business?

SOLUTION:

|CS U B|= |CS| +|B| - |CS intersect B| =220+147-51=316

As the universal set has 350 element, the answer is 350-316=34

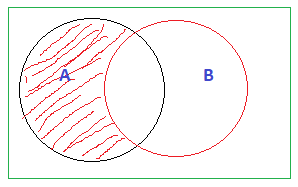
1. If A ⊆ B, and B ⊄ A, express A− B using set builder notation.

SOLUTION:

A-B = {x: x belongs to A and x does not belongs to B}

1. Using logical equivalences, prove or disprove the following set operations:

A − (A∩B) = (A − B)  
SOLUTION:

By definition A-B means remove from A those elements which are also present in B.  


1. Use a Venn diagram to determine which relationship,⊆ or = is true for the following pair of sets.

A ∪B and A ∪ (B − A).

SOLUTION:

The solution is intuitive. Both the relationship apply due to equality, as improper subset includes the possibility of equality.

**Relations:** [2x2=4 points]

1. No. 4 (i) Using the relation R = {(x,y)∈ Z : 12x + 3y = 12}, determine whether or not each is true.
2. 2R3

SOLUTION:

Substitute these values in x and y and compute the value and verify. In this case,  
12\*2 + 3\*3 = 24+9=33, but the relation is true when 12x+3y=12. Hence it is false.

1. -3R5

SOLUTION:

12\*-3+3\*5=-36+15=-21 Hence it is false

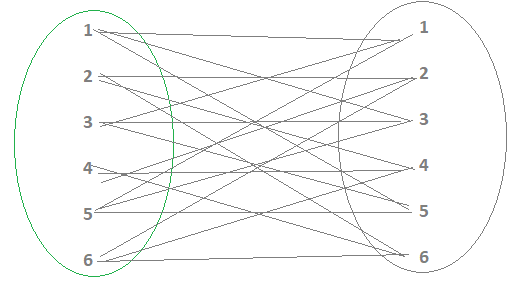
1. Let A = {1, 2, 3, 4, 5, 6}. Define a relation R on Set A as follows:

∀ (x, y) ∈ A, x R y↔ 2 divides (x – y).

Draw the arrow diagram of the above relation.

SOLUTION:

It seems that there has been a slight printing mistake in the question. I will consider both approaches, provided you give justification in your answer.



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***BEST OF LUCK!***