20-8-19

at $x = x_2$ $\left\{ \begin{array}{l} a_0 = y_0 \\ a_1 = f[x_0, x_1] \end{array} \right\}$

$$\phi(x_2) = y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

$$y_2 = y_0 + f(x_0, x_1](x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

$$\frac{y_2 - y_0}{x_2 - x_0} = f[x_0, x_1] + a_2(x_2 - x_1)$$

$$F(x_0, x_2] = f[x_0, x_1] + a_2(x_2 - x_1)$$

$$\frac{f[x_2, x_0] - f[x_0, x_1]}{x_2 - x_1}$$ → depend on f

$$= f[x_2, x_0, x_1]$$

$$= f[x_0, x_1, x_2]$$

$$a = f[x_0, x_1, x_2]$$

$$a_n = f[x_0, x_1, \ldots, x_n]$$

Newton's Devided difference formula —

$$\boxed{\begin{aligned} \phi(x) = y_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) \\ \cdots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \cdots (x - x_{n-1}) \end{aligned}}$$

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Pblm:

Find $F(0.9)$ by using Newton's devided difference formula: Given

x	0	1	2	4
$F(x)$	5	14	41	98

Soln:

x	$F(x)$	1st diff	2nd diff	3rd diff
0	5	$\frac{14-5}{1}=9$		
1	14	$\frac{41-14}{2-1}=27$	$\frac{27-9}{2-0}=9$	
2	41	$\frac{98-41}{4-2}=28.5$	$\frac{28.5-27}{4-1}=0.5$	$\frac{0.5-9}{4-0}=-2.125$
4	98			

$$f(0.5) = f(0) + 9(0.9-0) + 9(0.9-0)(0.9-1) + (-2.125)(0.9-0)(0.9-1)(0.9-2)$$

$$= 5 + 9\times 0.9 + 9\times 0.9 \times -0.1 + (-2.125) \times 0.9 \times -0.1 \times (-1.1)$$

$$= 5 + 8.1 \cancel{-} .81 + 0.210375$$

$$= 13.1 - 1.02$$

$$= 12.08 \text{ Ans.}$$

Pblm using NDDIF find $F(0.05)$ from the following Table:

Ans → 1.05072

x	0.0	0.2	0.4	0.6	0.8
$F(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

x	$F(x)$	1st d	2nd d	3rd d	4th diff
0.0	1.00000	$\frac{1.22140-1.0000}{0.2} = 1.107$	$\frac{1.3521-1.107}{0.4} = 0.61275$	0.22625	0.061975
0.2	1.22140	$\frac{1.49182-1.22140}{0.2} = \cancel{0.13501}$ 1.3521	$\frac{1.6515-1.3521}{0.4} = 0.7485$	0.27583	
0.4	1.49182	$\frac{1.82212-1.49182}{0.2} = 1.6515$	$\frac{2.0171-1.6515}{0.4} = 0.914$		
0.6	1.82212	$\frac{2.22554-1.82212}{0.2} = 2.0171$			
0.8	2.22554				

$$F(0.05) = 1.00000 + 1.1007(0.05-0.0) + 0.61275(0.05-0.0)(0.05-0.2)$$

$$+ 0.22625(0.05-0)(0.05-0.2)(0.05-0.4)$$

$$+ 0.061975(0.05-0)(0.05-0.2)(0.05-0.4)(0.05-0.6)$$

$$= 1.00000 + 1.1007 \times 0.5 + 0.61275 \times 0.5 \times (-0.15)$$

$$+ 0.22625 \times 0.05 \times (-0.15) \times (-0.35)$$

$$+ 0.061975 \times 0.05 \times (-0.15) \times (-0.35) \times (-0.55)$$

Pblm

Using NDDIF find F(1) and F(9) from the Following Table:

Soln:

x	0	2	3	4	7	8
$F(x)$	4	26	58	112	466	668

Soln:

x		1st	2nd	3rd	4th	5th
0	4					
		$\frac{22}{2} = 11$				
			$\frac{32-11}{3} = 7$			
2	26			$\frac{11-7}{4} = 1$		
		$\frac{32}{1} = 32$				
3	58		$\frac{54-32}{2} = 11$		$\frac{1-1}{7-0} = 0$	
		54				$\frac{0-0}{8} = 0$
4	112			$\frac{16-11}{5} = 1$		
		118	$\frac{118-54}{4} = 16$			
7	466				$\frac{1-1}{8-0} = 0$	
		202	$\frac{202-118}{4} = 21$	$\frac{21-16}{5} = 1$		
8	668					

922

$$F(9) = 668 + 202(9-8) + 21(9-8)(9-7) + 1(9-8)(9-7)(9-4)$$

$$= 668 + 202 + 21 \times 2 + 1 \times (2) \times 5$$

$$= 668 + 202 + 42 + 10$$

$$= 922 \text{ A}$$

$$F(1) = 4 + 11(1-0) + 7(1-0)(1-2) + 1(1-0)(1-2)(1-3)$$

$$= 4 + 11 - 7 + 2$$

$$= 10$$ Ans

26/8

$$I_{TC} = \frac{h}{2}\left[y_0 + (2y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

$$I_{SC} = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2(y_2 + y_2 + \cdots + y_{n-2}) + y_n\right]$$

Q compute the value of π from the formula —

$\frac{\pi}{4} = \int_0^1 \frac{1}{1+x^2}\,dx$ using composite trapizoidal rule with take 10 interval and compare it with the exact value.

Soln

$n = 10$

$h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1$

$\frac{1}{1+0.01} =$

$F(x) = \frac{1}{1+x^2}$

i	x_i	$f(x_i), y_{i0}$	$F(x_i) / y_i / i=1$
0	0	1	
1	0.1		0.990
2	0.2		0.961
3	0.3		0.917
4	0.4		0.862
5	0.5		0.8000
6	0.6		0.735
7	0.7		0.671
8	0.8		0.600
9	0.9		0.552
10	1.0	0.5	
		1.5	7.098

$= (1 + 0.5 + 2 \times 7.098) = 15.696$

$\frac{h}{2}[15.696] = \frac{0.1}{2}[15.696] = \frac{1.570}{2} = .785$

$\pi = 4 \times (0.785)$

$= 3.140$

$[\pi = 3.1416]$

Error $= 3.1416 - 3.1400$

$= 0.0016$

$3.1416 = \pi$

Q calculate by simpson's 1/3 rule 0 to 1 $= \int_0^1 \frac{x}{1+x} dx$ correct upto 3 s.f by taking 6th intervels, what is the value of h.

Soln

$n = 6$

$h = \frac{1-0}{6} = 1/6$

i	x_i	$y_i / i = 0,6$	$y_i / i = 1,3,5$	$y_i / i = 2,4$
0	0	0	0.143	0.149
1	1/6		0.250	0.149
2	2/6	0.333		2.90 0.250
3	3/6		0.333	
4	4/6			0.400
5	5/6		0.454	
6	1	0.5		
8		0.5	0.930	0.65

$\frac{1}{6}[0.5 + 4 \times 0.930 + 2 \times 0.65]$

$\cancel{\frac{1}{6}[0.5 + 0.92 + 1.3]}$

$\cancel{\frac{1}{6} \times 9.65 = 1.587}$

$\frac{5.52}{6} = \frac{0.92}{2}$ [$\frac{h}{3}$ in formula]

$= 0.307$ correct upto 3 s.f

H.W

Q1. find the value of $\log_2^{1/3}$ from $\int_0^1 \frac{x^2}{1+x^3}dx$ using simpson's 1/3 rule where $h = 0.25$

2. Evaluate $\int_0^1 \frac{dx}{1+x}$ by simpsons composite rule taking eleven ordinates and hence find the value of $\log_e^2$ correct upto 5 s.f.

n=10

3 calculate the approximate value of $\int_0^{\pi/2} \sin x\,dx$ by composite trapizoidal rall and composite simpson's 1/3 rule having eleven ordinates also compare it with actual value of the ~~intervap~~ integral

Exp $\int_0^1 \frac{\sin x}{x} \rightarrow 0^+ \quad 1$

① $f(x) = \int_0^1 \frac{x^2}{1+x^3} dx$

$z = 1+x^3$

$dz = 3x^2 dx \quad = \frac{1}{3}\int_1^2 \frac{dz}{z} = \left[\frac{1}{3}\log z\right]_1^2$

$\frac{1}{3}\log 2^{1/3}$

②

$x_0 = 0,\ x_1 = \frac{\pi}{20},\ x_2 = \frac{\pi}{10},\ x_3 = \frac{3\pi}{20},\ x_4 = \frac{\pi}{4},\ x_5 = \frac{\pi}{4}$

$x_6 = \frac{3\pi}{10};\ x_7 = \frac{7\pi}{20},\ x_8 = \frac{2\pi}{5},\ x_9 = \frac{9\pi}{20},\ x_{10} = \frac{\pi}{2}$

Next

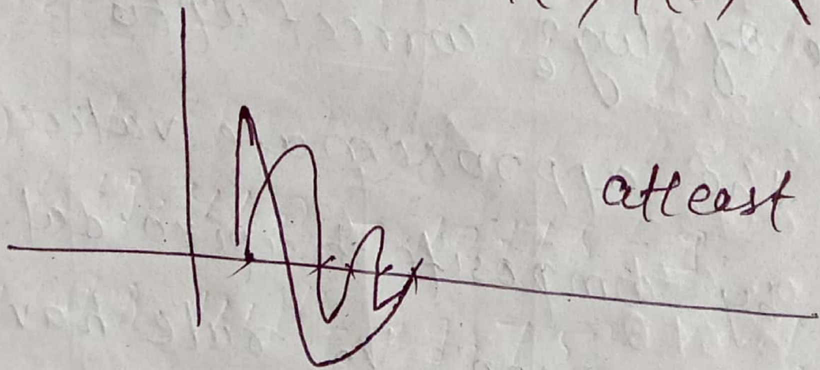
* Transidantal eqn

⇒ eqn consist of every Fn. like, log, exp etc.

$F(x) = 0$

$\left.\begin{array}{l} F(a) = +ve \\ F(b) = -ve \end{array}\right\}$ then atleast one root exist

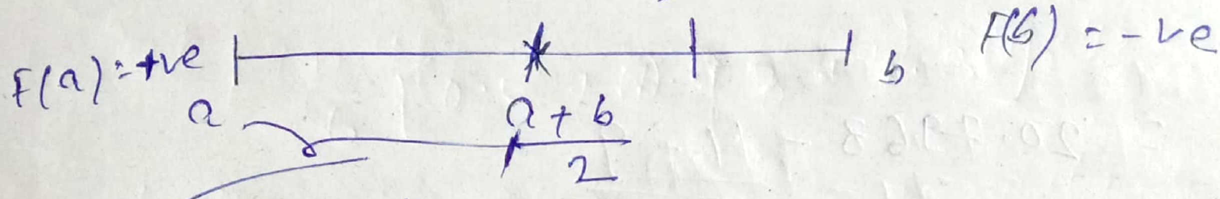
$F(a)\,F(b) < 0$



atleast 1, or 3, or 5, or 7 — —

2/9/19

* Bisection Method:-



$F(a)\,F(b) < 0$

$F\left(\frac{a+b}{2}\right) > 0$

eliminate this

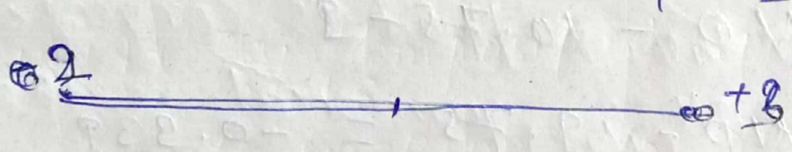
$$F\left(\frac{b + \frac{a+b}{2}}{2}\right) < 0$$

Pblm

Find a real root of the eqn $x^3 - 4x - 9 = 0$ correct upto 2 S.F. taking the interval of the root as [2, 3]

$F(a) = 8 - 8 - 9 = -9 < 0$

$= 27 - 12 - 9 = 27 - 21 = 6 > 0$



$x_1 = \frac{2+3}{2} = 2.5$

$F(2.5) = (2.5)^3 - 4 \times 2.5 - 9$

$15.625 = -3.375 < 0$

$$x_2 = \frac{2.5+3}{2} = 2.75$$

$$= 20.7968 - \underline{\underline{11 - 9}}$$

$$= +0.7968 > 0$$

$$x_3 = \cancel{\frac{2.75+3}{2}} = \cancel{2.875}\ \frac{2.5+2.75}{2} = 2.625$$

$$F(2.625) = 18.0878 - 10.5 - 9$$

$$18.0875 - 19.5$$

$$= -1.41$$

$$x_4 = \frac{2.625+2.75}{2} = 2.6875$$

$F(2.6875)$

$$19.410 - 10.75 - 9$$

$$= 19.410 - 19.75 = -0.339$$

$$x_5 = 2.6875 +$$

$$F(2.7188) = 20.09 - 19.8752$$

$$= 0.22$$

No of iteration	a_n $F(a_n)<0$	b_n $F(b_n)>0$	$x_{n+1} = \frac{a_n+b_n}{2}$	$F(x_{n+1})$
0	2	3	2.5	-3.37
1	2.5	3	2.75	0.75
2	2.5	2.75	2.625	-1.41
3	2.625	2.75	2.61875	-0.339
4	2.6875	2.75	2.7188	0.22
5	2.	2.7188	2.7032	-0.059

Newton Raphson's Method :-

⇒ Let x_0 be an initial app. of the desire root α of the eqn $f(x)=0$ and $x_1 = x_0 + h$ is the correct root say ie $f(x_1) = 0$

$$F(x_0 + h) = 0$$

$$f(x_0) + hF'(x_0) + \frac{h^2}{2!}F''(\xi) = 0 \quad , \quad x_0 < \xi < x_1$$

neglect higher order term of h / h^2 onwards

$$F(x_0) + hF'(x) = 0$$

$$h = -\frac{F(x_0)}{F'(x_0)}$$

$$x_1 = x_0 + h$$

$$= x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$\cancel{\left(x_2 = x_1 + h \Rightarrow x_1 - \frac{F(x_1)}{F'(x_1)}\right)}$$

repeating the above process and replacing x_0 by x_1 we obtain the 2nd app of the root α as

$$x_2 = x_1 + h$$

$$= x_1 - \frac{F(x_1)}{F'(x_1)}$$

Prociding in this way we get the successive app we get $x_1, x_2 \dots x_{n+1}$

$$x_{n+1} = x_n - \frac{f(x_n)}{F'(x_n)}$$

$$F'(x_n) \neq 0$$

$$n = 0, 1, 2 \dots$$

This result is known as Newton Raphson method.

advantage and disadv. of NRM.

The rate of convergens of this method is quadratic so the method is converges more rapidly another numerical method

(i) In this method the initial app. must be chosen very close to the root other wise the method will be failed (ii) Since the method depends on the derivative $F'(x)$ it may not be suitable for a F^n $F(x)$ whose derivetive is difficult to compute.

iii) The method faild if $F'(x) = 0$ or very small in the neighbour root of the root

* Newton Raphson method Algo:-

step 1. start the program

" 2. define the F^n $f(x), F'(x)$

" 3. enter the initial app. of the root, x_0

" 4. calculate $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

5. If $|x_{n+1} - x_n| < \epsilon$, ϵ being prescribe accuracy ($.001$, $.0001$ app. apb)
then go to step 7.

6. set $x_n = x_{n+1}$ and go to step 4.

7. print the value of x_n

8. stop

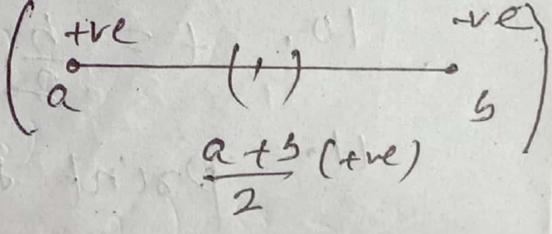
* Bisection method Algo:-

step 1. start the prgm

2. define the function $F(x)$

3. select the interval a, b in which the root lies where $F(a) F(b) < 0$

4. calculate $x = \frac{a+b}{2}$



5. If ($F(b) F(x) < 0$)
set $a = x$
Otherwise set $b = x$

6. If $|a-b| < \epsilon$, ~~the root is a and b~~
ϵ being the presize accuracy
then goto step 7
else go to step 4

7. print the value of x

8. stop

* Trapizoidal Algo.

step 1. start the prgm

2. Define the F^n $F(x)$

3. read $a, b, n, s=0, t$

4. set $x_0 = a$, and $x_n = b$ ~~x_0, x_n~~

5. take $h = \frac{b-a}{n}$

6. for (i=0; i<=n; i++) / For $i = 0(1)n$

7. $x_i = x_0 + ih$, $y_i = F(x_i)$

8. for $i = 1(1)n-1$

9. $s = s + F(x_i)$

10. $t = \frac{h}{2}(y_0 + y_n + 2s)$ ~~step 4~~

11. print the value

* Simpson's 1/3 Algo :-

Step 1.

Q. Find a root of the eqn $x\sin x + \cos x = 0$ using Newton Rapshon method correct upto 5 places of the decimal, root lies b/w 2 and 3

Soln:

$$F(x) = x\sin x + \cos x = 0$$

$x_0 = 2$

Ans: 2.79838

$F(x_0) = 2\sin 2 + \cos 2$

$F'(x) = x\cos x + \cancel{\sin x} - \cancel{\sin x}$
$= x\cos x$

$$x_1 = 2 - \frac{F(2)}{F'(2)} = 2 - \frac{2\sin x + \cos x}{2\cos x}$$

$= 2 - \left(\tan x + \frac{1}{x}\right)$

$= 2 - (2.18503 + 0.5)$

$=$

$x_1 = 3.68$ out from the criteria not exceptable

$x_0 = 2.5$

$$x_1 = 2.5 - \frac{F(2.5)}{F'(2.5)} = 2.5 - \left(\tan(2.5) + \frac{1}{2.5}\right)$$

$2.5 - (-0.74)$

$= 2.5 + 0.34 = 2.15298$

$$x_2 = 2.15298 - \left(\tan 2.15298 + \frac{1}{2.15298}\right)$$

$$= 2.15298 - (-2 + 0.464472)$$

$$= 2.15298 - (-1.53552)$$

9/9/19

Q. Find the 4 place decimal the smallest root of the eqn:

$e^{-x} = \sin x$ by N.R Method.

$F(x) = e^{-x} - \sin x = 0$ / $\sin x - e^{-x}$

$F'(x) = -e^{-x} - \cos x$ / $e^{-x} + \cos x$

$F(0) = e^0 - \sin 0$

$= 0 - 0 = -1$

$F'(1) = 0.3678$

$= 0.908$

$F(1) = \frac{1}{e} - \sin 1$

$= \frac{1}{e} - 0.84$

$= 0.8415 - 0.3678$

$= 0.48$

root lies b/w [0, 1]

$$\frac{F(x)}{F'(x)} = \frac{\sin x - e^{-x}}{\cos x + e^{-x}}$$

$F(0.5) = \sin(0.5) - e^{-0.5} = -0.127$

$F'(0.5) = 1.484$

$$\frac{F(0.5)}{F'(0.5)} = \frac{-0.127}{1.484} = -0.085$$

n	x_n	$\frac{F(x)}{F'(x)}$	$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$
0	0	$-1/2$	$x_1 = 0 + \frac{1}{2} = 0.5$
1	0.5	$\frac{F(0.5)}{F'(0.5)}$	$x_2 = 0.58564$
2	0.58564	-0.00288	$x_3 = 0.58852$
3	0.58852	-0.0000127	$x_4 = 0.5885327$

compare ~~if both~~ are equal

$F(0.58564) = -4.01658 \times 10^{-3}$ $\quad -0.00401$

$F'(\ '' \ ) = 1.39010$

$$\frac{F(0.58564)}{F'(\ '' \ )} = -2.88940 \times 10^{-3}$$

$= 0.00288$

~~0.58564 + 0.00288~~

~~= 0.58275~~

$= 0.58852$

$F(.58852) = -0.00001768$

$F'(.58852) = 1.38691$

$= -1.2747 \times 10^{-5}$

$$\frac{F(.58852)}{F'(.58852)} = -0.0000127$$

~~= 0.58853~~

0.0000127
+0.58852

x_4 = 0.5885327

= 0.58853 Ans.

16/9/19

* NR for finding pth root of a +ve real no r.

$\Rightarrow$ Let $x = \sqrt[p]{R}$ or $x^p = R$

$$f(x) = x^p - R = 0$$

using NR method

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

we have

$$x_{n+1} = x_n - \frac{x_n^p - R}{p x_n^{p-1}}$$

Special case:

$p=2$, $x_{n+1} = x_n - \frac{x_n^2 - R}{2x_n}$

$$= \frac{2x_n^2 - x_n^2 + R}{2x_n}$$

$$= \frac{1}{2}\left[x_n + \frac{R}{x_n}\right]$$

{ Ex: $x = \sqrt{13}$

$x_{n+1} = x_n + \frac{13}{x_n}$ }

for $p=3$

$$x_{n+1} = \frac{2x_n^3 + R}{3x_n^2} \quad n = 0, 1, 2 \ldots$$

H.W
Q. Find cube root of 17 ($\sqrt[3]{17}$) correct upto 5 s.F by NR.

Method of Regula Falsi :–

The method is the oldest method of computing of real root of an eqn $F(x)=0$ to find the real root we 1st chose a sufficiently small interval $[x_0, x_1]$ in which the root lies, then we take chord which joins the curve b/w these two points.

Eqn of the chord is —

$$y - F(x_1) = \frac{F(x_0) - F(x_1)}{x_0 - x_1}(x - x_1)$$

$$\left.\begin{array}{l}(x_0, F(x_0)) \\ (x_1, F(x_1))\end{array}\right\} \text{points}$$

The chord ~~process~~ crosses the X-axis at $y=0$

$$-F(x_1)(x_0 - x_1) = [F(x_0) - F(x_1)](x - x_1)$$

$$x - x_1 = \frac{-F(x_1)(x_0 - x_1)}{F(x_0) - F(x_1)}$$

$$x = x_1 - F(x_1)\left[\frac{x_0 - x_1}{F(x_0) - F(x_1)}\right]$$

here this x is said to be the 2nd app. of the real root i.e,

$$x_2 = x_1 - F(x_1)\left[\frac{x_0 - x_1}{F(x_0) - F(x_1)}\right]$$

we can repeat the above procedure with $[x_1, x_2]$ or $[x_0, x_2]$ as the next interval where $F(x_1)F(x_2) < 0$ or $F(x_0)F(x_2) < 0$

$$x_3 = x_2 - f(x_2)\left[\frac{x_1 - x_2}{F(x_1) - F(x_2)}\right]$$

and ultimetly we get a recussion

$$\left\{ x_{n+1} = x_n - \frac{F(x_n)\left[x_n - x_{n-1}\right]}{F(x_n) - F(x_{n-1})} \right\}$$

Pblm Find out root of the eqn $x^3 - 5x - 7 = 0$ b/w $[2, 3]$ by regula falsi correct upto 4 decimal

soln

$F(x) = x^3 - 5x - 7 = 0$

$f(2) = 8 - 10 - 7 = -9$

$F(3) = 27 - 15 - 7 = 27 - 22 = 5$

n	x_{n-1} / $F(x_{n-1}) = -ve$	x_n / $F(x_n) = +ve$	$f(x_{n-1})$	$F(x_n)$	x_{n+1}	$F(x_{n+1})$
0	2	3	-9	5	2.642857	-1.754720
1	2.642857	3	-1.754720	5	2.7357	-0.20553
2	2.7357	3	-0.20553	5	2.7.	
3						
4						
5						

$$x_{n+1} = x_n - \frac{F(x_n)[x_{n-1} - x_n]}{F(x_{n-1}) - F(x_n)}$$

$$x_2 = x_1 - \frac{F(x_1)[x_0 - x_1]}{F(x_0) - F(x_1)}$$

$$= 3 - \frac{5[2-3)}{-9-5}$$

$$= 3 + \frac{5}{-14}$$

$$= 3 - \frac{5}{14}$$

$$= 3 - 0.357142$$

$$= 2.642857$$

$$F(x_1) = 13.480499 - 13.214285 - 7$$

$$= 13.480499 - 20.214285$$

$$= -1.754780$$

$x_1 = 2.642857$, $x_2 = 3$, $F(x_1) = -1.754780$

$F(x_2) = 5$

$$x_3 = x_2 - \frac{F(x_2)[x_1 - x_2]}{F(x_1) - F(x_2)}$$

$$x_3 = 3 - \frac{5[2.642857 - 3]}{-1.754720 - 5}$$

$$= 3 - \frac{5[-0.357143]}{-6.75472}$$

$$= 3 - \frac{(-1.785715)}{-6.75472}$$

$$= 3 - 0.26436551$$

$$= \underline{2.7357}345$$

$$F(2.7357) = 20.47412 - 13.6785 - 7$$

$$= 20.67$$

$$= 20.47412 - 20.6785$$

$$= -0.20553$$

17/9 Algo. **Regula Falsi Method**

Step 1: Start

2: Define the F^n^ F(x)

3: Select the inter [a, b] in which the root lies, where F(a) F(b) < 0

4: calculate $x = \frac{b-a}{F(b)-F(a)} F(b)$

5: If F(x) F(b) < 0, set a = x otherwise b = x

6: If |a − b| < ε,

ε being the prescribe accuracy then goto step 7, else go to step 4

7: print the value of x.

8: stop.

Matrix

* Gauss Elemination:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \qquad a_{11} \neq 0$$

$$-\frac{a_{21}}{a_{11}} \quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$-\frac{a_{31}}{a_{11}} \quad a_{31}x_1 + a_{22}x_2 + a_{33}x_3 = b_3$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 = b_2^{(1)}$$

$$\frac{a_{32}}{a_{22}} \quad - \quad a_{32}^{(1)}x_3 \quad a_{33}^{(1)}x_3 = b_3^{(1)}$$

$$\left.\begin{aligned} a_{21}^{(1)} &= a_{21} - \frac{a_{21}}{a_{11}} \times a_{11} = 0 \\ a_{31}^{(1)} &= a_{31} = a_{31} - \frac{a_{31}}{a_{11}} \times a_{11} = 0 \end{aligned}\right\}$$

✓ $a_{22}^{(1)} = a_{22} - \frac{a_{21}}{a_{11}} \times a_{12}$, $a_{23}^{(1)}$ ✓ $= a_{23} - \frac{a_{21}}{a_{11}} \times a_{12}$

✓ $b_2 = b_2 - \frac{a_{21}}{a_{11}} \times b_1$

✓ $a_{32}^{(1)} = a_{32} - \frac{a_{31}}{a_{11}} \times a_{12}$

✓ $a_{33}^{(1)} = a_{33}^{(1)} - \frac{a_{31}}{a_{11}} \times a_{13}$

✓ $b_3^{(1)} = b_3 - \frac{a_{31}}{a_{11}} \times b_1$

$$a_{33}^{(2)} x_3 = b_3^{(2)}$$

$$a_{33}^{(2)} = a_{33}^{(1)} - \frac{a_{32}^{(1)}}{a_{22}^{(1)}} \times a_{23}^{(1)}$$

$$b_3^{(2)} = b_3^{(1)} - \frac{a_{32}}{a_{22}^{(1)}} \times b_2$$

⇒ we consider a system of 3 eqn with three unknown as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

Let $a_{11} \neq 0$

then the element of the 2nd & 3rd line is to be added to the corresponding element of the 1st line with multi. of

$$-\frac{a_{21}}{a_{11}}, \quad -\frac{a_{31}}{a_{11}} \text{ respectivily}$$

this will give the following 3 eqns.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{32}x_3 + a_{33}x_3 = b_3$$

$$a_{21} = a_{21} - \frac{a_{21}}{a_{11}} \times a_{11}$$

$$a_{31} = a_{31} - \frac{a_{31}}{a_{11}} \times a_{11}$$

(✓ previous 6 are written)

If $a_{21} \neq 0$, the we add the element of 2nd row with the element of 3rd row multi. by $\frac{-a_{32}}{a_{22}}$ and we get the following three eqn.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{22}^{(1)}x_2 + a_{23}^{(1)}x_3 = b_2$$
$$a_{33}^{(2)}x_3 = b_3$$

$$\left\{ \begin{array}{l} a_{33}^{(2)} = a_{33}^{(1)} - \frac{a_{32}^{(1)}}{a_{22}^{(1)}} \times a_{23}^{(1)} \\ b_3^{(2)} = b_3^{(1)} - \frac{a_{32}}{a_{22}} \times b_2^{(1)} \end{array} \right\}$$

So, by back calculation we can find the values of x_1, x_2, x_3

Q. Solve the eqn using gauss elemination method.

$$\cancel{6x_1 + 3x_2 + 2x_3 = 6}$$
$$x_1 + x_2 + 4x_3 = 12$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$8x_1 - 3x_2 + 12x_3 = 20$$

rearranging the eqns we write

$$8x_1 - 3x_2 + 12x_3 = 20$$
$$-\tfrac{4}{8} \quad 4x_1 + 11x_2 - x_3 = 33$$
$$\tfrac{1}{8} \quad x_1 + x_2 + 4x_3 = 12$$

$$a_{22} = 11 - \frac{4}{8} \times (-3)$$

$$= 11 + \frac{3}{2}$$

$$= \frac{25}{2} = 12.5$$

$$a_{23} = -1 + \left(-\frac{4}{8}\right) \times 2$$

$$= -2$$

$$b_2 = 33 - \frac{4}{8} \times 20 = 23$$

$$a_{32} = 1 + \left(-\frac{1}{8}\right) \times (-3) = 1 + \frac{3}{8}$$

$$= \cancel{6/8} \cancel{= 3.075}$$

$$1.375$$

$$a_{33} = 4 - \left(-\frac{1}{8}\right) \times \cancel{4}\ 2$$

$$= \cancel{\left(4 + \frac{1}{2}\right) = 4.5}\ \ 3.75$$

$$b_{33} = 12 - \left(-\frac{1}{8}\right) \times 20$$

$$= 12 + \frac{20}{8} = \cancel{13.50}\ \ 9.5$$

$$= 9.5$$

$$\left.\begin{aligned} 8x_1 - 3x_2 + 2x_3 &= 20 \\ 12.5x_2 - 2x_3 &= 23 \\ 1.375x_2 + 3.75x_3 &= \underline{9.5} \end{aligned}\right\}$$

$$a_{33} = 3.75 - \frac{1.375}{12.5} \times (-2)$$

$$= 3.75 + 0.22$$

$$= \cancel{4.19}\ 3.97$$

$$b_3 = 9.5 -$$

$$= 6.97$$

$$x_3 = \frac{b_3}{a_{33}}$$

$$= \frac{6.97}{3.97} = 1.756$$

(a)

$$12.5x_2 - 2 \times 1.756 = 23$$

$$12.5x_2 = 23 + 3.511$$

$$x_2 = \frac{30.022}{12.5} = \cancel{4.241}$$

$$2.120$$

$$8x_1 - 3 \times 2.40 + 2 \times 1.756 = 20$$

$$8x_1 = 20 + 7.2 - 3.512$$

$$= 27.2 - 3.8512$$

$$x_1 = \frac{23.688}{8} = 2.960 \quad 2.1856$$

Table

multiplier	x_1	x_2	x_3	b	check
	8	-3	2	20	27
$\left(-\frac{4}{8}\right)$	4	11	-1	33	47
$\left(-\frac{1}{8}\right)$	1	1	4	12	18
		12.5	-2	23	33.5
		1.375	3.75	9.5	14.625
			3.97	6.97	

Q

$$\left.\begin{aligned} 3x + 4y + 5z &= 40 \\ 2x - 3y + 4z &= 13 \\ x + y + z &= 9 \end{aligned}\right\} \quad \left.\begin{aligned} x &= 1 \\ y &= 3 \\ z &= 5 \end{aligned}\right\} \text{Ans.}$$

Q.

$$\left.\begin{aligned} 0.34x_1 - 0.58x_2 + 0.94x_3 &= 2 \\ 0.27x_1 + 0.42x_2 + 0.13x_3 &= 1.5 \\ 0.2x_1 - 0.51x_2 + 0.54x_3 &= 0.8 \end{aligned}\right\} \quad \begin{aligned} x_1 &= 0.89 \\ x_2 &= 2.1 \\ x_3 &= 3.1 \end{aligned}$$

LU factorization

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{3} \end{bmatrix}$$

LUX = b

$$U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{3} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Q. solve using LU factorization :-

$$3x + 4y + 2z = 15$$
$$5x + 2y + z = 18$$
$$2x + 3y + 2z = 10$$

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$L_{11} = 3$, $L_{21} = 5$, $L_{31} = 2$, $L_{11} u_{13} = 2$

$u_{13} = 2/3$

$L_{11} \times u_{12} = 4$, $u_{12} = 4/3$

$L_{21} u_{12} + L_{22} = 2 \Rightarrow 5 \times \frac{4}{3} \times L_{22} = 2$; $L_{22} = 2 - \frac{20}{3}$

$= -14/3$

$L_{31} \cdot u_{12} + L_{32} = 2$

$2 \times \frac{4}{3} + L_{32} = 3$

$L_{32} = 3 - \frac{8}{3} = 1/3$

$\cancel{L_{31} u_{12} + L_{32} u_{23} + L_{3} = 2}$

$\cancel{2 \times \frac{1}{14} + \frac{1}{3} \times}$

$L_{21} u_{13} + L_{22} u_{23} \Rightarrow 5 \times \frac{2}{3} - \frac{14}{3} u_{23} = 1$

$-\frac{14}{3} u_{23} = 1 - \frac{10}{3} = -\frac{7}{3}$

$u_{23} = 1/2$

$L_{31} u_{12} + L_{32} u_{23} + L_{33} = 2$

$2 \times \frac{4}{3} + \frac{1}{3} \times \frac{1}{2} + L_{33} = 2$

$\frac{8}{3} + \frac{1}{6} - 2 = -L_{33}$

$\frac{16 + 1 - 12}{6} = -5/6$

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$$L = \begin{bmatrix} 3 & 0 & 0 \\ 5 & -14/3 & 0 \\ 2 & 1/3 & 1/2 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 4/3 & 2/3 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$LUX = b$

$LY = b$, where $UX = Y$

$$\begin{bmatrix} 3 & 0 & 0 \\ 5 & -14/3 & 0 \\ 2 & 1/3 & 1/2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 18 \\ 10 \end{bmatrix}$$

$3y_1 = 15 \Rightarrow y_1 = 5$

$5y_1 - \frac{14}{3}y_2 = 18 \Rightarrow 25 - 18 = \frac{14}{3}y_2$

$7 = \frac{14}{3}y_2$

$y_2 = 3/2$

$2y_1 + \frac{1}{3}y_2 + \frac{1}{2}y_3 = 10$

$\cancel{10} + \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{2} + \frac{1}{2}y_3 = \cancel{10}$

$\frac{1}{\cancel{2}}y_3 = \cancel{-\frac{1}{2}} \Rightarrow y_3 = -1$

$$\begin{bmatrix} 1 & 4/3 & 2/3 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3/2 \\ -1 \end{bmatrix}$$

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$$x + \frac{4}{3}y + \frac{2}{3}z = 5$$

$$x + \frac{4}{3} \times 2 - \frac{2}{3} = 5$$

$$x + \frac{8-2}{3} = 5$$

$$z = -1 \qquad x = 5 - \frac{8}{3} + \frac{2}{3}$$

$$x = 3$$

$$y + \frac{1}{2}z = \frac{3}{2}$$

$$y = 2$$

$$x = 3,\ y = 2,\ z = -1$$

Q. solve the following system of eqn

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$4x_1 + 11x_2 - x_3 = 33$$

$$6x_1 + 3x_2 + 12x_3 = 36$$

$$\cancel{\begin{bmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 6 & 3 & 12 \end{bmatrix}}$$

$$\cancel{L = \begin{bmatrix} 8 & 0 & 0 \\ 4 & 11 & 0 \\ 6 & 3 & 12 \end{bmatrix}} \qquad \cancel{U = \begin{bmatrix} 8 & -3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}$$

$LY = b$

$$\begin{bmatrix} 8 & 0 & 0 \\ 4 & 11 & 0 \\ 6 & 3 & 12 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 33 \\ 36 \end{bmatrix}$$

$8y_1 = 20 \Rightarrow y_1 = 20/8 = 5/2$

$4y_1 + 11y_2 = 33$

$4 \times \frac{5}{2} + 11y_2 = 33$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 6 & 3 & 12 \end{bmatrix}$$

$L_{11} = 8, \; L_{21} = 4, \; L_{31} = 6$

$L_{11}u_{12} = -3 = 8u_{12} = -3/8$

$L_{21}u_{12} + L_{22} = 11$

$4 \times \frac{-3}{\cancel{8}\,2} + L_{22} = 11$

$L_{22} = 11 + \frac{3}{2} = 25/2$

$L_{31} + L_{33}$ L_{33}

$\cancel{L_{31}U_{12} + L_{32} + L_{33} = 3}$

$\cancel{6 \times -\frac{3}{8} +}$ $L_{32} = \frac{567}{60}$

$L_{21}U_{13} + L_{22}U_{23} =$

$$\left.\begin{aligned} x_1 &= 3 \\ x_2 &= 2 \\ x_3 &= 1 \end{aligned}\right\} \text{Ans.}$$

$$\left\{\begin{aligned} y_1 &= 5/2 \\ y_2 &= 46/25 \\ y_3 &= 1 \end{aligned}\right.$$

$$A = \begin{bmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 6 & 3 & 12 \end{bmatrix}$$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$L_{11} = 8,\ L_{21} = 4,\ L_{31} = 6$

$L_{11} u_{12} = -3,\quad u_{12} = -3/8$

$L_{21} u_{12} + L_{22} = 11 \Rightarrow \cancel{L_{21}u_{12}}\ \cancel{4} \times -\frac{3}{\cancel{8}\,2} + L_{22} = 11$

$L_{22} = 11 + \frac{3}{2} = 25/2$

$L_{31} u_{12} + L_{32} + L_{33} = 3$

$L_{11} u_{13} = 2 \Rightarrow u_{13} = 2/8 = 1/4$

$L_{21} u_{13} + L_{22} u_{23} = -1$

$\cancel{4}^{2} \times \frac{1}{\cancel{4}7} + \cancel{L_{22}}\ \frac{25}{2} \times u_{23} = -1 \quad = \frac{25}{2} u_{23} = -1 - \frac{1}{7}$

$= \frac{25}{2} u_{23} = \frac{-8}{7}$

$u_{23} = -16/175$

$L_{31} u_{13} + L_{32} u_{23} + L_{33} = 12$

24/9/19

Gauss Seidel

খুব important

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}\right\}$$ diagonally dominated.

$$\rightarrow |a_{11}| \geq |a_{12}| + |a_{13}|$$

$$|a_{22}| \geq |a_{21}| + |a_{23}|$$

$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

$$x_1^{(k+1)} = \frac{1}{a_{11}}\left[b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}\right]$$

$$x_2^{(k+1)} = \frac{1}{a_{22}}\left[b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}\right]$$

$$x_3^{(k+1)} = \frac{1}{a_{33}}\left[b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}\right]$$

pblm: using Gauss seidel method find the solⁿ of the following system of linear eqⁿ correct upto 2 places of decimal. Eqns are -

$$3x + y + 5z = 13$$
$$5x - 2y + z = 14$$
$$x + 6y + 2z = -1$$

$$|5| \geq |-2| + |1|$$

re-arrange

$$\left\{\begin{aligned} 5x - 2y + z &= 14 \\ x + 6y + 2z &= -1 \\ 3x + y + 5z &= 13 \end{aligned}\right.$$ diagonally dominated

$$x^{k+1} = \frac{1}{5}\left(4 + 2y^{(k)} - z^{(k)}\right)$$

$$y^{k+1} = \frac{1}{6}\left(-1 - x^{(k+1)} + 2z^{(k)}\right)$$

$$z^{k+1} = \frac{1}{5}\left(13 - 3x^{k+1} - y^{k+1}\right)$$

$$x^{(n)} = \frac{1}{5}\left(4 + 2y^{(n-1)} + z^{(n-1)}\right)$$

$$y^{(n)} = \frac{1}{6}\left(-1 - x^{(n)} - 2z^{(n-1)}\right)$$

$$z^{n} = \frac{1}{5}\left(13 - 3x^{(n)} - y^{(n)}\right)$$

$x^{(0)} = 0$
$y^{(0)} = 0$
$z^{(0)} = 0$ } initial values always

$$y^{(1)} = \frac{1}{6}(-1 - 0.8) = -0.3$$

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	0	0	0
1	0.8	-0.3	2.18
2	0.244	0.519	2.349

} every value check in 2 interval

$x = 0.55$
$y = 0.47$
$z = 2.18$
Ans

$$z^{(1)} = \frac{1}{5}\left(13 - 3\times(0.8) - (-0.3)\right)$$

$$x^{(2)} = \frac{1}{5}\left(4 + 2(-0.3) - 2.18\right) = 0.244$$

$$y^{(2)} = \frac{1}{6}\left(-1 - 0.244 + 2(2.18)\right) = 0.519$$

$$z^{(2)} = \frac{1}{5}\left(13 - 3\times 0.244 + 0.519\right) = 2.349$$

Q. Using G S find the sol^n of the system of Linear eq^n correct upto 2 places of decimal

$$9x_1 + 2x_2 + 3x_3 = -7$$
$$x_1 - 6x_2 + 2x_3 = -2$$
$$x_1 + x_2 + 3x_3 = 5$$

Above eq^n are diagonally dominated.

$$x_1^{(k+1)} = \frac{1}{9}\left(-7 - 2x_2^{(k)} - 3x_3^{(k)}\right)$$

$$x_2 = \frac{1}{-6}\left(-2 - x_1^{(k+1)} - 2x_3^{k}\right) = \frac{1}{6}\left(2 + x_1^{(k+1)} + 2x_3^{(k)}\right)$$

$$x_3 = \frac{1}{3}\left(5 - x_1^{(k+1)} - x_2^{(k+1)}\right)$$

K	x_1^K	x_2^K	x_3^K
1	-0.778	0.2036	1.7909
2	-1.479	0.6985	1.908
3	-1.568	0.708	1.953
4	-1.586	0.72	1.956
5	-1.589	0.7205	1.956

So Ans $\Rightarrow x_1 = -1.59,\ x_2 = 0.72$

$x_3 = 1.96$

$x^{(1)} = \frac{1}{9}(-7) = -0.778$

$y^{(1)} = \frac{1}{6}(2 + 0.77) = \cancel{0.461}\ 0.2036$

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$$x_3^{(1)} = \frac{1}{3}(5 + 0.77 - 0.208)$$

$$= \cancel{1.7714}\ 1.798$$

$$\cancel{x_1^{(2)} = \frac{1}{9}(-7 - 2\times(-0.77) - 3(1.7714))}$$

$$\cancel{= -1.}$$

$$\cancel{x_1^{(2)} = \frac{1}{9}(-7 - 2\times(0.4628) - 3(1.7714))}$$

$$\cancel{= -1.471}$$

$$\cancel{x_2^{(2)} = +\frac{1}{6}(+2 + (-1.471) + 2\times(1.7714))}$$

$$\cancel{= 0.678}$$

$$\cancel{x_3^{(2)} = \frac{1}{3}(5 - (-1.471) - 0.678)}$$

$$\cancel{= 1.931}$$

5th iteration

$x_1 = -1.59$

$x_2 = 0.72$

$x_3 = 1.96$

Ans

$$x_1^{(2)} = \frac{1}{9}(-7 - 2\times 0.2036 - 3\times 1.79) = -1.419$$

$$x_2^{(2)} = \frac{1}{6}(2 + (-1.419) + 2\times 1.79) = 0.6935$$

$$x_3^{(2)} = \frac{1}{3}(5 + 1.419 - 0.6935) = 1.908$$

$$x_1^{(3)} = \frac{1}{9}(-7 - 2\times(0.6935) - 3(1.908)) = -1.568$$

$$x_2^{(3)} = \frac{1}{6}(2 + (-1.568) + 2(1.908)) = 0.708$$

$$x_3^{(3)} = \frac{1}{3}(5 + 1.568 - 0.708) = 1.953$$

$x_1^{(4)} = \frac{1}{9}(-7-2(0.708)-3(1.953)) = -1.586$

$x_2^{(4)} = \frac{1}{6}(2-1.586+2(1.953)) = 0.72$

$x_3^{(4)} = \frac{1}{3}(5+1.586-0.72) = 1.956$

$x_1^{(5)} = \frac{1}{9}(-7-2(0.72)-3(1.956)) = -1.589$

$x_2^{(5)} = \frac{1}{6}(2-1.589+2(1.956)) = 0.7205$

$x_3^{(5)} = \frac{1}{3}(5+1.589-0.72) = 1.956$

Ans correct upto 2 D.P.

$x_1 = -1.59,\ x_2 = 0.72,\ x_3 = 1.96\}$

Differential Equation — Date → 1.10.19

Euler's Differential Equation

⇒ Let us consider the 1st order linear differential eqn $\frac{dy}{dx}(x,y)$ with some boundary condn $y(x_0)=?$, $x_0=?$ then the euler's method is–

$$y_n = y_{n-1} + hf(x_{n-1}, y_{n-1}) \quad , x_0 \Rightarrow y_0 \Rightarrow x_1 = x_0 + h$$

Q. Given $dy/dx + \frac{y}{x} = \frac{1}{x^2}$ with $y(1)=1$ Evaluate $y(1.2)$ by Eulers method.

Soln: Initial boundary condn

$$\left.\begin{array}{l} x_0 = 1 \\ y_0 = 1 \end{array}\right\}$$

take $h = 0.1$ $\quad \left\{ 1 \longmapsto 1.1 \right.$

$x_1 = x_0 + h = 1 + 0.1 = 1.1$

$y_1 = y_0 + hF(x_0, y_0)$

$y_1 = 1 + 0.1(0)$

$y_1 = 1$

$$\left\{ \frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x} \right.$$

$$= \frac{1-xy}{x^2} = f(x,y)$$

$$F(1,1) = \frac{1-1\times1}{1} = 0$$

$x_2 = 1.1 + 0.1 = 1.2$

$y_2 = y_1 + hF(x_1, y_1)$

$= 1 + 0.1 \times \left(\frac{1 - x_1 y_1}{x_1^2}\right)$

$= 1 + (0.1)\left(\frac{1-(1.1)\times1}{(1.1)^2}\right) \Rightarrow \cancel{1} + 0.1 \times \cancel{0} = 0.0991736$

1.008264

Q. Solve by using Euler's method the following differential eqn for $x=1$ taking $h=0.2$

$dy/dx = xy$, $y=1, x=0$ correct upto 5 d.p.

$x_0 = 0$

$y_0 = 1$

$x_1 = 0 + 0.2 = 0.2$

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.2 \times 0 = 1$

$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$

$y_2 = y_1 + hf(x_1, y_1)$

$= y_1 + 0.2 \times (0.2)$

$= y_1 + 0.04 \Rightarrow 1.04$

$x_3 = x_2 + h = 0.4 + 0.2 = 0.6$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.04 + 0.2 \times (0.4 \times 1.04)$

$= 1.04 + 0.2 \times$

$= \cancel{1.05664}\ 1.12320$

$x_4 = x_3 + h = 0.6 + .2 = 0.8$

$y_4 = 1.12320 + 0.2(0.6 \times 1.12320)$

$= 1.257984$

Q. $x_5 = 0.8 + 0.2 = 1$

$y_5 = 1.257984 + 0.2(0.8 \times 1.257984)$

$= 1.45926$ Ans.

Q. Given $dy/dx = \frac{y-x}{y+x}$ with initial condn $y=1$ at $x=0$ find y for $x=0.1$ by Euler's method correct upto 4 D.places. taking step lenght $h = 0.02$

solⁿ

$x_0 = 0$

$y_0 = 1$

$f(x,y) = \frac{y-x}{y+x}$

$x_1 = 0 + 0.02 = 0.02$

$y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.02\left(\frac{1-0}{1+0}\right)$

$= 1.02$

$x_2 = 0.02 + 0.02 = 0.04$

$\frac{(1.02 - 0.02)}{1.02 + 0.02}$

$y_2 = 1.02 + 0.02\left(\frac{1}{1.04}\right)$

$= 1.02 +$

$= 1.0392$

$x_3 = 0.04 + 0.02 = 0.06$

$\frac{1.0392 - 0.04}{1.0392 + 0.04}$

$y_3 = 1.0392 + 0.02$

$= 1.05774 8$

$= \frac{0.9992}{1.0792}$

$x_4 = 0.06 + 0.02 = 0.08$

$y_4 = 1.057748 + 0.02\left(\frac{1.057748 - 0.06}{1.057748 + 0.06}\right) = 0.92587$

$= 1.057748 + 0.02 \times \left(\frac{0.997748}{1.117748} \right)$

$= 1.07560$

(9) $x_5 = 0.08 + 0.02 = 0.1$

$y_5 = 1.07560 + 0.02 \left(\frac{1.07560 - 0.08}{1.07560 + 0.08} \right)$

$\left(\frac{0.9956}{1.1556} \right)$

$= 1.07594$

$= 1.02928$ Ans.

Algo: (Euler's DE)

1. Start the progm.
2. Define $F(x, y)$
3. Input x_0, y_0, h, x_n
4. calculate the number of interval $(n) = (x_n - x_0)/h$
5. Set $i = 0$
6. For $i = 0$ to $n-1$
7. compare $x_{i+1} = x_i + h$
8. compute $y_{i+1} = y_i + h(F x_i, y_i)$
9. print x_n, y_n
10. stop.

RK-4 method   Range-kutta

$\frac{dy}{dx} = F(x, y)$ (y_0, x_0)

x_1, y_1 $x_1 = x_0 + h$
$y_1 = y_0 + K$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hF(x_0, y_0)$

$K_2 = hF(x_0 + h/2, y_0 + K_1/2)$

$K_3 = hF(x_0 + h/2, y_0 + K_2/2)$

$K_4 = hF(x_0 + h, y_0 + K_3)$

Q. use RK method to calculate $y(1.1)$ given that $dy/dx = y^2 + xy$, $x_0 = 1$, $y_0 = 1$ with $h = 0.1$

Soln

$x_0 = 1$, $y_0 = 1$

$x_1 = 1 + 0.1 = 1.1$

$y_1 = y_0 + K$ $K_1 = hF(x_0, y_0)$

$= 0.1[1^2 + 1 \times 1]$

$= 0.1 \times 1 = 0.2$

$K_2 = hF(x_0 + h/2, y_0 + K_1/2)$

$= \cancel{\quad} x = (1 + 0.05) = 1.05$

$y_2 \ (1 + 0.1) = 1.1$

$K_2 = (0.1)[(1.1)^2 + (1.1) \times (1.05)] = 0.2365$

$K_3 = hf(x_0 + h/2, y_0 + \frac{K_2}{2})$

$x = 1.1, \quad y = 1 + 0.011825$

$= 1.011825$

$K_3 = 0.1[(1.011825)^2 + 1.1 \times 1.011825]$

$= \cancel{0.2130}\ 0.24246$

$K_4 = hf(x_0 + h/2, y_0 + K_3)$

$x = 1.1, \quad y = 1 + 0.24246$

$= 1.24246$

$= 0.1[(1.24246)^2 + 1.1 \times 1.24246)$

$= 0.2910$

$y_1 = y_0 + K$

$= \cancel{1 + [0.2 + .2365 + 0.24246 + 0.2910]}$

$K = [0.2 + 2 \times 0.2365 + 2 \times 0.24246 + 0.2910]$

$K = 1.44892$

$y_1 = \frac{1 + 1.44892}{6} = \frac{2.44892}{6}$

$=$