

Haseeb Raza

OT70FF

Prob and stat.

HW6: Solution.

1: Hypotheses.

- H_0 : This Prob of 1 is $p_0 = 0.5$ (for coin)
- H_1 : This Prob of 1 is $p_1 = 1/6$ (for dice)

2: Test Statistic.

$$\hat{p} = \frac{\sum_{i=1}^n X_i}{n}$$

X_i is (0 or 1), n no of observation.

3: Decision Rule.

- Accept H_0 if \hat{p} is close to 0.5.

- Accept H_1 if \hat{p} is close to $1/6$

Mathematically,

- If $|\hat{p} - 0.5| < |\tilde{p} - 1/6|$ we accept H_0

- If $|\hat{p} - 0.5| > |\tilde{p} - 1/6|$ we accept H_1

Part b. Error Probabilities.

- Type I Error: Rejecting H_0 when it is true.

- Type II Error: Failing to reject H_0 when H_1 is true.

The Prob of Type I error is α , as set $\alpha = 0.05$ (5% significance level)

To calculate the Errors:

- Under H_0 , \hat{p} follows a binomial distribution with $p = 0.5$.
- Under H_1 , \hat{p} follows a binomial distribution with $p = 1/6$

Type I Error.

$$\hat{p} \sim \text{binomial}(n, p = 0.5)$$

This happens when \hat{p} is closer to $1/6$.

Type II Error.

$$\hat{p} \sim \text{binomial}(n, p = 1/6)$$

This happens when \hat{p} is closer to 0.5.

Part c: Simulation.

- uses the binomial for TYPE 1 and TYPE 2 Errors for n .
- Decision boundary between H_0 and H_1 is midpoint between 0.5 and $1/6$ which is 0.3333.

To calculate the Error statistic:

- Under H_0 , \hat{p} follows

$$\text{distribution with } \sum_{i=1}^n X_i = \hat{p}$$

- Under H_1 , \hat{p} follows

distribution for n , $(1, 0, 2)$ is X_i

Type I Error.

Decision Rule:

- Accept H_0 if \hat{p} is close to 0.5

- Accept H_1 if \hat{p} is close to $1/6$

Mathematically,

- If $|p - 0.5| \geq |2.0 - p|$ we accept H_0

- If $|p - 0.5| < |2.0 - p|$ we accept H_1