OT7DEE Probability and statistics. HW-3

Q. No. 01.

(a) Solution.

We know CDF by defination.

$$F_X(t) = P(x < t) = P(\max(u, u_2) < t).$$

Since the x is $\max(u_1, u_2)$ so event occur iff $u_1 < t$ and $u_2 < t$ because $\max(u_1, u_2) < t$.

$$\Rightarrow \quad F_x(t) = P(u_1 < t, u_2 < t)$$

$\because$ $u_1$ and $u_2$ are Independent

$$\Rightarrow \quad F_x(t) = P(u_1 < t) \cdot P(u_2 < t)$$

$$= t \cdot t = t^2$$

$$\Rightarrow \quad F_x(t) = \begin{cases} 0 & , t < 0 \\ t^2 & , 0 \leq t \leq 1 \\ 1 & , t > 1. \end{cases}$$

(b) Solution.

Since we know CDF so by takinf deaivative we can find density

$$f_x(t) = \frac{d}{dx} F_x(t) = \frac{d}{dx}(t^2) = 2t$$

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$\Rightarrow \quad f_x(t) = \begin{cases} 2t, & 0 \le t \le 1 \\ 0, & \text{otherwise.} \end{cases}$

(3) Mean.

$$E(x=t) = \int_0^1 t \cdot 2t \, dt = 2\int_0^1 t^2 = 2\left[\frac{t^3}{3}\right]_0^1$$

$= 2 \cdot \frac{1}{3} = \frac{2}{3}$ so the mean is $2/3$.

(4) variance of $x$.

$$\text{var}(X) = \underline{E}(X^2) - E(X)^2$$

$$\underline{E}(X^2) = \int_0^1 \cancel{t}^2 \cdot 2t\,dt = 2\int_0^1 \cancel{t}^3 \, dx = 2\left[\frac{\cancel{t}^4}{4}\right]_0^1$$

$$= 2 \cdot 1/4 = 1/2.$$

$$\text{var}(X) = \frac{1}{2} - \left(\frac{2}{3}\right)^2$$

$$= \frac{1}{2} - \frac{4}{9} = \frac{9}{18} - \frac{9}{18} = 1/18$$

So the variance of $x$ is

$$\text{Var}(X) = 1/18. \quad \underline{\text{Ans}}$$

Question 04. OT7DBE

Part d.

① $F_X(t) = P(X \le t) = P(\min(U_1, U_2) \le t)$.

Since ~~bot~~ the $X$ is $\min(U_1, U_2)$ so event if $U_1 \le t$ or $U_2 \le t$ because both are independent

$$F_X(t) = 1 - P(U_1 > t \text{ and } U_2 > t)$$

$$F_X(t) = 1 - P(U_1 > t)P(U_2 > t) = 1 - (1-t)^2$$

So the CDF of $X = \min(U_1, U_2)$ is

$$F_X(t) = 1 - (1-t)^2 \quad \text{for } 0 \le x \le 1$$

②

$$X = 1 - \sqrt{U_1}$$

$$F_X(y) = P(1 - \sqrt{U_1} \le y)$$

$$1 - \sqrt{U_1} \le y \Rightarrow \sqrt{U_1} \ge 1 - y \Rightarrow U_1 \ge (1-y)^2$$

$$F_X(y) = P(U_1 \le (1-y)^2) = (1-y)^2$$

$$F_X(y) = 1 - (1-y)^2$$

So both are same distribution.