Probability and Statistics.

HW4.

Part (a):

$X_1, X_2 ..$ are Independent Random variables with exponentiel distribution also given

$$\mathbb{P}(X_i \geq 1) = e^{-3}$$

As we know

$$\mathbb{P}(x_i > x) = e^{-\lambda x}$$ for exponential distribution with parameter λ

here $x = 1$

$$P(x_i \geq 1) = e^{-\lambda \cdot 1}$$

$$e^{-3} = e^{-\lambda} \Rightarrow \boxed{\lambda = 3}$$

Hence

$$\mathbb{E}(x_i) = \frac{1}{\lambda} = 1/3$$

Now using law of large Numbers

$$\frac{x_1 + \dots x_n}{n} \rightarrow [E(x_i)] = \frac{1}{\lambda} = 1/3.$$

Ans

Part (B).

here we will use centeral limit theorem

$$\frac{1}{\sqrt{n}}\left(\sum_{i=1}^{n} x_i - nu\right] \xrightarrow{\gamma} N(0, \sigma^2)$$

x_i and i.i.d with the

mean u variance σ^2

For our Problem.

$$\text{Mean} = E(x_i) = 1/3 = u$$

$$\text{Variance} = \sigma^2 = \frac{1}{\lambda^2} = 1/3^2 = 1/9$$

So by CLT

$$\frac{1}{\sqrt{n}}\left(x_1 + \dots x_n - \frac{n}{3}\right) \xrightarrow{d} N\left(0, \frac{1}{9}\right)$$

$$= \frac{1}{3} N(0,1)$$

but we have to find limit

$$\mathbb{P}\left(\frac{1}{\sqrt{n}}\left((x_1 + \dots + x_n) - \frac{n}{3}\right) \leq 1\right)$$

$$= \mathbb{P}\left(\frac{1}{3} z \leq 1\right)$$

$$= \mathbb{P}(z \leq 3)$$

$$= \Phi(3) = 0.9987.$$ Ans