

Q 1) Calculate probability from the given dataset for the below cases

Data _set: Cars.csv

Calculate the probability of MPG of Cars for the below cases.

MPG <- Cars\$MPG

- a. $P(\text{MPG} > 38)$
- b. $P(\text{MPG} < 40)$
- c. $P(20 < \text{MPG} < 50)$

Ans: Mean = 34.422 Standard deviation = 9.1314

- a) $P(\text{MPG} > 38) = 34.83\%$
- b) $P(\text{MPG} < 40) = 72.935\%$
- c) $P(20 < \text{MPG} < 50) = 89.83\%$

Q 2) Check whether the data follows normal distribution

- a) Check whether the MPG of Cars follows Normal Distribution
Dataset: Cars.csv

Ans: Not normally distributed

- b) Check Whether the Adipose Tissue (AT) and Waist Circumference (Waist) from wc-at data set follows Normal Distribution
Dataset: wc-at.csv

Ans: Waist Circumference not normally distributed, Adipose Tissue skewed to the right.

Q 3) Calculate the Z scores of 90% confidence interval, 94% confidence interval, 60% confidence interval

Q 4) Calculate the t scores of 95% confidence interval, 96% confidence interval, 99% confidence interval for sample size of 25

Q 5) A Government company claims that an average light bulb lasts 270 days. A researcher randomly selects 18 bulbs for testing. The sampled bulbs last an average of 260 days, with a standard deviation of 90 days. If the CEO's claim were true, what is the probability that 18 randomly selected bulbs would have an average life of no more than 260 days

Hint:

rcode \rightarrow pt(tscore,df)

df \rightarrow degrees of freedom

Ans: t-statistics = -0.471 , degrees of freedom = 17

The probability that $t < -0.471$ with 17 degrees of freedom assuming the population mean is true, the t-value is less than the t-value obtained With 17 degrees of freedom and a t score of - 0.471, the probability of the bulbs lasting less than 260 days on average of 0.3218 assuming the mean life of the bulbs is 300 days

Q 6) The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

- A. 0.3875
- ☒ B. 0.2676
- C. 0.5
- D. 0.6987

Q 7) The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

A. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False (Probability of employees older than 44 = 15.87% while probability of employees between 38 and 44 = 34.13%)

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True (Probability of employees under age of 30 = 9.18%, so 9.18% of 400 employee = 36)

Q 8) If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are iid normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans:

$2X_1 \sim N(2\mu, 4\sigma^2)$ and

$X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$

Q 9) Let $X \sim N(100, 202)$. Find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

☒ D. 48.5, 151.5

E. 90.1, 109.9

Q 10) Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 32)$ and $\text{Profit}_2 \sim N(7, 42)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Range is Rs (99.00810347, 980.9918965) in Millions

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in Million Rupees) is 170.0

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Loss = profit < 0 = $P(X < 0)$

Division 1 has a larger probability of making a loss.