ELEC 341 – 101 (2025 W1)

Final Exam
100 Marks

Required Files

Available on Canvas

- **e341-x2.pdf** *Exam description (this document)*
- **x2DSPlot.p** Data-Sheet curve generator
- **x2Submit.p** *Grading script (**LATEST** version)*
- **e341-APE.pdf** *Instructions for submitting graded work (for reference)*

Regulations

Open Book/Computer

- **ANY** reference material **OK**
- **ANY** Matlab scripts **OK**
- ***NO*** *communication*
- ***NO*** *file sharing*

Fig 1a: Plant

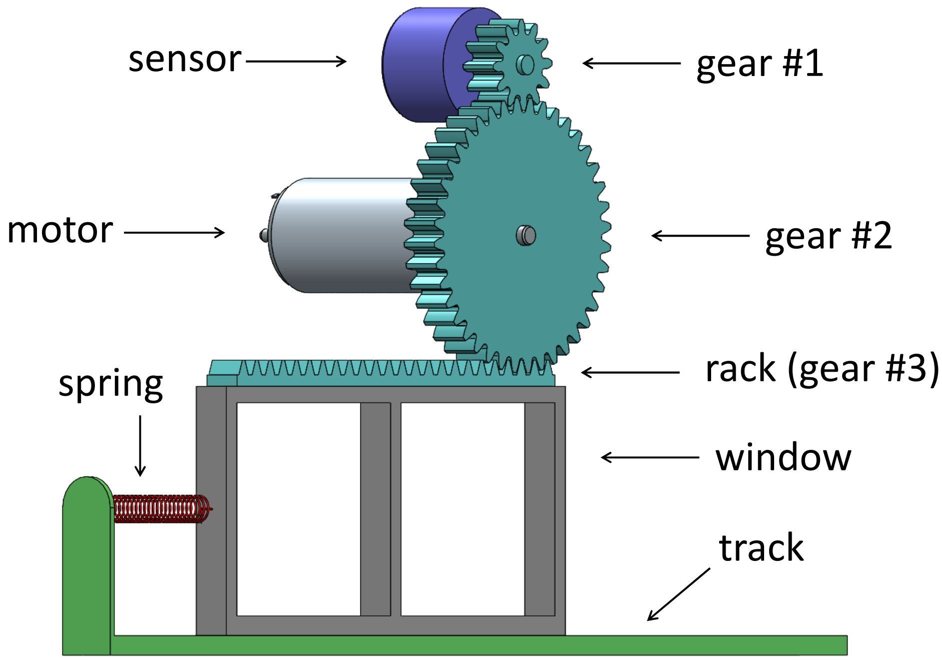


Fig 1b: Plant Parameters

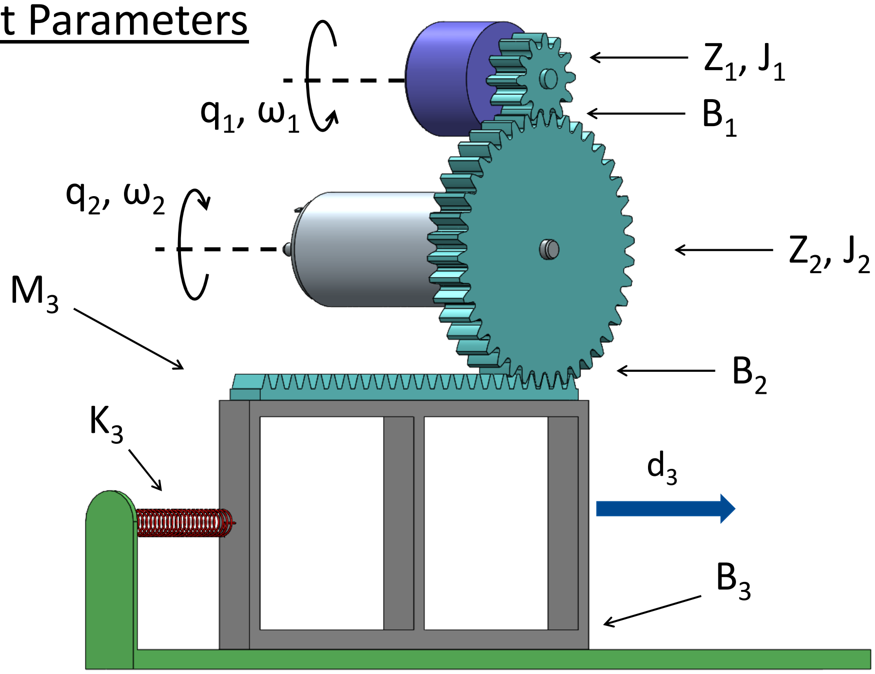
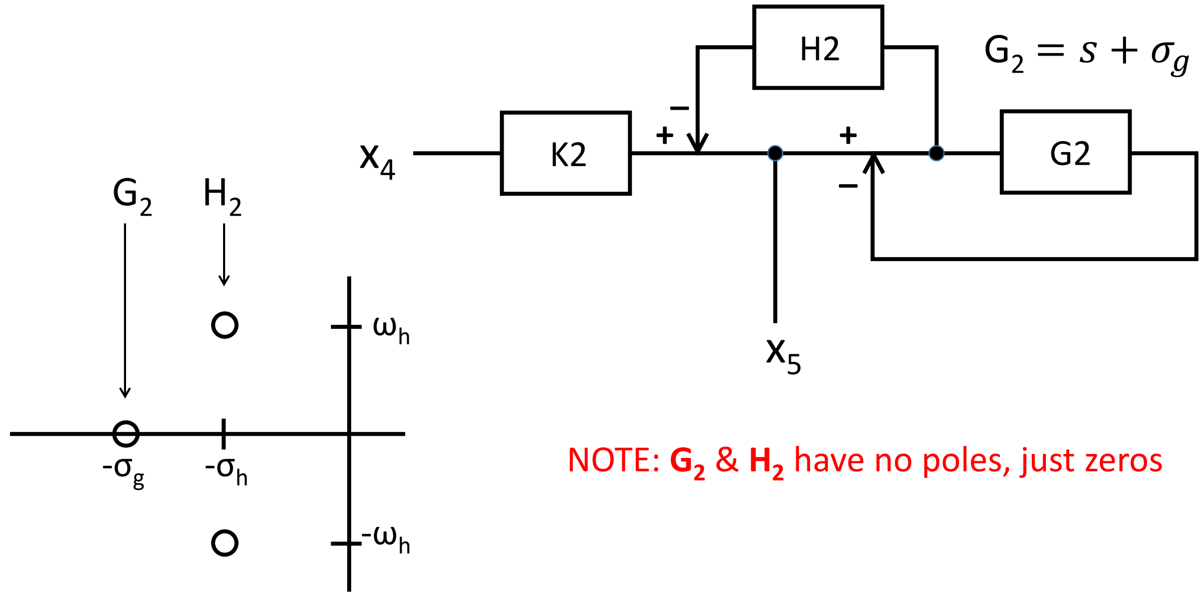


Fig 2: Sensor

$$H_2 = (s + \sigma_h + j\omega_h)(s + \sigma_h - j\omega_h)$$

$$G_2 = s + \sigma_g$$



NOTE: **G_2** & **H_2** have no poles, just zeros

Fig 3: Plant (Pos-Control)

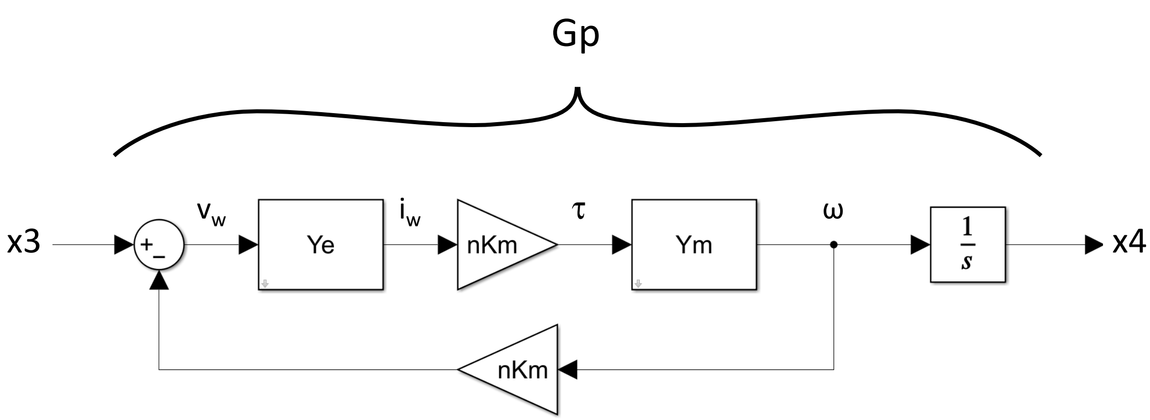


Fig 4: Control System (Pos-Control)

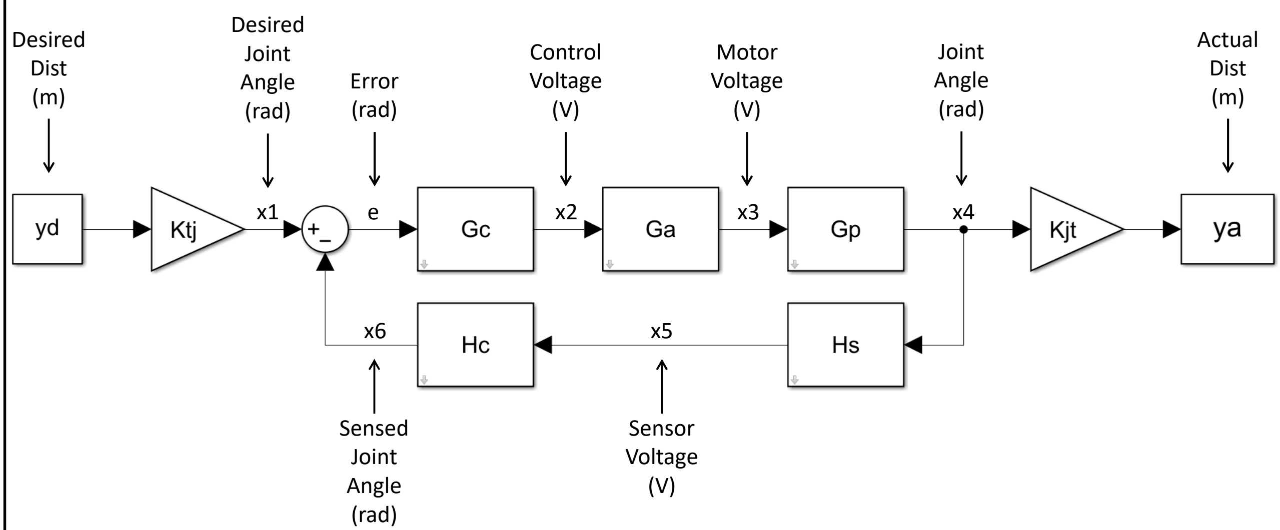


Fig 5: Worksheet

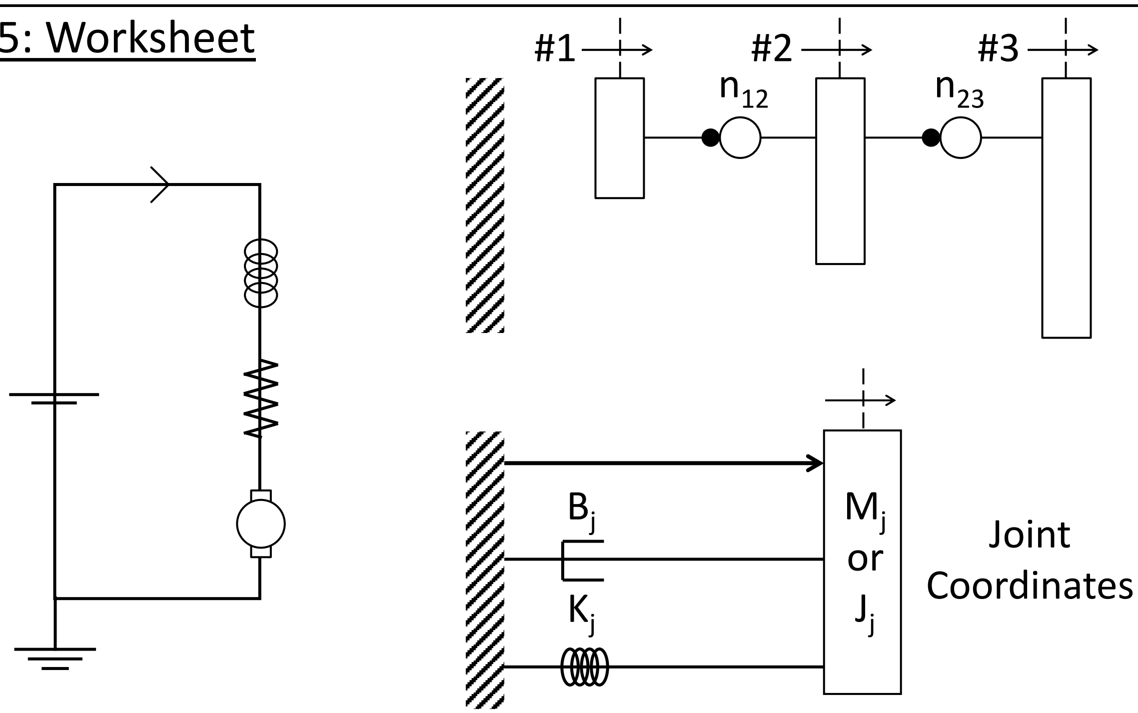


Table 1: Plant

Parameter	Value	Physical Units
Z_1	#A	teeth
Z_2	#B x 3	teeth
Mod	2	mm
J_1	#C x 3×10^{-6}	Nms^2/rad
J_2	#D x 3×10^{-5}	Nms^2/rad
M_3	#E x 5×10^{-2}	Kg
B_1	#F x 3×10^{-6}	Nms/rad
B_2	#G x 3×10^{-4}	Nms/rad
B_3	#H x 3×10^{-1}	Ns/m
K_3	#A	N/m

Table 2: Sensor

Parameter	Value	Physical Units
K_2	#A x 10^3	V/rad
σ_g	#B x 10^2	rad/s
σ_h	#C x 10^2	rad/s
ω_h	#D x 10^2	rad/s

Table 3: Motor

Parameter	Value	Physical Units
R_w	#A / 5	Ω
L_w	#B	mH
K_m	#C x 10^{-3}	Nm/A
J_m	#E x 2×10^{-6}	Nms^2/rad
B_m	#D x 2×10^{-5}	Nms/rad

Table 4: Control System

Parameter	Value	Physical Units
CF	#B x 200	Hz
DC	#C x 2	%

Table 5: RCGs

Parameter	Value	Physical Units
Res	0.1	rad/s
TargPM	40	deg
OS_u	≤ 50 (WRT ref OS_u)	%
T_s	≤ 80 (WRT ref T_s)	%
T_r	< final T_s	sec
E_{ss}	0	%

SYSTEM IDENTIFICATION

$$\mathrm{PD} = Z \times Mod$$

The Plant is shown in **Fig 1**.

- An automated sliding window is opened and closed by a motor & spring.
- A motor is attached to Gear #2 which rotates to angle $\mathbf{q_2}$.
- The sensor is attached to Gear #1 which rotates to angle $\mathbf{q_1}$.
- The window is attached to a rack (Gear #3) and moves to distance $\mathbf{d_3}$.

The Plant parameters are shown in **Table 1**.

- Each gear has the number of teeth ($\mathbf{Z_1 \rightarrow Z_2}$). The number of teeth on the rack is not relevant.
- Each gear has the tooth module **Mod**.
- The diameter of any gear (**PD**) is the product of its number of teeth (**Z**) and tooth module (**Mod**).
- Gears #1 & #2 have inertia $\mathbf{J_1}$ and $\mathbf{J_2}$, respectively.
- The rack and window have a combined mass $\mathbf{M_3}$.
- The friction between gears #1 & #2 is represented by damping $\mathbf{B_1}$, applied to Gear #1.
- The friction between gears #2 & #3 is represented by damping $\mathbf{B_2}$, applied to Gear #2.
- The friction between the window and track is represented by damping $\mathbf{B_3}$, applied to Gear #3.
- The spring joining the window to the track (reference) has stiffness $\mathbf{K_3}$.

The Sensor shown in **Fig 2** has parameters shown in **Table 2**.

- The sensor is attached to Gear #1 so input ($\mathbf{x_4}$) is gear angle $\mathbf{q_1}$, (see **Fig 1**).
- The output ($\mathbf{x_5}$) is sensor voltage.
- The inertia and friction of the sensor are negligible.

The Plant detail shown in **Fig 3** has motor parameters shown in **Table 3**.

- The windings have resistance $\mathbf{R_w}$ and Inductance $\mathbf{L_w}$.
- The motor constant is $\mathbf{K_m}$.
- The motor has total inertia $\mathbf{J_m}$.
- The motor bearings have friction $\mathbf{B_m}$.

The Control System shown in **Fig 4** has parameters shown in **Table 4**.

- The input & output is the desired & actual **distance** of the **Window** ($\mathbf{d_3}$ in **Fig 1**).
- $\mathbf{G_p}$ is the plant (motor and mechanism) shown in **Fig 1** & **Fig 3**.
- The controller operates at a control frequency **CF**.
- The duty-cycle is **DC**.

The Worksheet in **Fig 5** is generic. **Add or remove elements as needed.**

- The Worksheet may be used to model the plant.
- It includes 3 reference frames (#1 - #3) joined by transmissions with ratios $\mathbf{n_{12}}$ & $\mathbf{n_{23}}$.
- In joint coordinates, it has equivalent mass $\mathbf{M_j}$ or inertia $\mathbf{J_j}$.
- In joint coordinates, it has equivalent damping $\mathbf{B_j}$.
- In joint coordinates, it has equivalent stiffness $\mathbf{K_j}$.

The **unit** step response of voltage amplifier $G_a = x_3/x_2$ in **Fig 4** is shown by **x2DSPlot.p**.
If G_a is **under-damped**, use **PEAK TIME** to find a 2[nd] order approximation.
If G_a is **over-damped**, find a **1[st]** or **2[nd]** order approximation, whichever is preferred.

1. 10 mark(s) Amplifier

- Q1.Ga (V/V) LTI Object

Find sensor gain: $H_s = x_5/x_4$ (see **Figs 2&4**)
Separate H_s into Gain K_s and Dynamics D_s.

2. 10 mark(s) Sensor

- Q2.Ks (V/rad) Scalar
- Q2.Ds (pure) LTI Object

Identify Joint, Task, and all other coordinate systems in **Fig 5**.
Transform all impedances to **Joint** coordinates.
Find the total equivalent inertia J_j, damping B_j and stiffness K_j.

3. 5 mark(s) Equivalent Impedance

- Q3.Jj (Nms^2/rad) Scalar
- Q3.Bj (Nms/rad) Scalar
- Q3.Kj (Nm/rad) Scalar

Find ELEC admittance: $Y_e = i_w/v_w$ (see **Fig 3**)
Find MECH admittance: $Y_m = \omega/\tau$ (see **Fig 3**)
Find plant gain: $G_p = x_4/x_3$ (see **Fig 3**)

4. 5 mark(s) Plant Model

- Q4.Ye (A/V) LTI Object
- Q4.Ym (rad/Nms) LTI Object
- Q4.Gp (rad/V) LTI Object

Find the state-space matrices (**A**,**B**,**C**,**D**) for the plant G_p.
Use the state $\bar{x}$, input $\bar{u}$, and output $\bar{y}$ vectors shown (see **Fig 5b**).
The output f_{Kw} is the force (N) of the spring (K_3) on the window (Task coordinates).
The output f_{Bw} is the force (N) of the friction (B_3) on the window (Task coordinates).

5. 10 mark(s) State Space

- Q5.A (mixed) 3x3 Matrix
- Q5.B (mixed) 3x1 Matrix
- Q5.C (mixed) 2x3 Matrix
- Q5.D (mixed) 2x1 Matrix

$$\bar{x} = \begin{bmatrix} i_w \\ \omega_j \\ \tau_{Kj} \end{bmatrix} \quad \bar{u} = [x_3] \quad \bar{y} = \begin{bmatrix} f_{Kw} \\ f_{Bw} \end{bmatrix}$$

Find system gain: $GH_s = x_5/x_2$ (see **Fig 4**)

Find the frequency of the most dominant pole w_d of GH_s.

Find the maximum filter delay N_f for non-dominant controller FB dynamics.

For the purpose of finding N_f, **ROUND-UP** w_d to the next highest **1/10 (rad/s)**.

Find the corresponding **IIR** filter time constant **tau** and weighting factor **beta**.

6. 10 mark(s) IIR Filter

- Q6.GHs (V/V) LTI Object
- Q6.wd (rad/s) Scalar
- Q6.Nf (pure) Scalar
- Q6.tau (s) Scalar
- Q6.beta (pure) Scalar

COW: *If you round w_d down rather than up, the pole becomes dominant.*

Round **tau** to 2 significant digits.

Find the associated number of **FIR** filter coefficients **num**.

7. 3 mark(s) FIR Filter

- Q7.num (pure) Scalar

Find the controller feedback delay multiple **N**, assuming an IIR filter.

Find controller feedback gain: $H_c = x_6/x_5$ (see **Fig 4**)

8. 2 mark(s) Feedback Path

- Q8.N (pure) Scalar
- Q8.Hc (rad/V) LTI Object

Begin with a unity gain, 0-order controller. In other words, G_c**=1**.

Find forward gain: $G = x_4/x_2$ (see **Fig 4**)

Find feedback gain: $H = x_6/x_4$ (see **Fig 4**)

Find loop gain: $GH = x_6/x_2$ (see **Fig 4**)

Find static gains: K_{tj} & K_{jt} (see **Fig 4**)

9. 5 mark(s) System Model

- Q9.G (rad/V) LTI Object
- Q9.H (pure) LTI Object
- Q9.GH (rad/V) LTI Object
- Q9.Ktj (rad/m) Scalar
- Q9.Kjt (m/rad) Scalar

CONTROL

Develop a **PI** controller. Refer to **Table 5** for RCGs.

Find the Partial Dynamics $\mathbf{D_p}$.

Find the initial gain for marginal stability $\mathbf{K_0}$ using the partial dynamics $\mathbf{D_p}$.

Find the cross-over frequency $\mathbf{w_{xo}}$.

10. 5 mark(s) Partial Dynamics

- Q10.Dp (pure) LTI Object
- Q10.K0 (V/rad) Scalar
- Q10.wxo (rad/s) Scalar

Find the zero **Z** that maximizes phase margin **PM** using search resolution **Res** shown in **Table 5**. Only search values rounded to the search resolution.

Find the full dynamics **D**.

11. 10 mark(s) Controller Zero(s)

- Q11.Z (rad/s) Scalar
- Q11.PM (deg) Scalar
- Q11.D (pure) LTI Object

Find the master gain **K** that delivers the **TargPM** shown in **Table 5**.

12. 2 mark(s) Master Gain

- Q12.K (V/rad) Scalar

Find the **Reference** gains $\mathbf{K_p}$, $\mathbf{K_i}$, corresponding to unity master gain **K=1**.

13. 3 mark(s) Reference Gains

- Q13.Kp (pure) Scalar
- Q13.Ki (sec^{-1}) Scalar

Find the **Reference** performance metrics, rise time $\mathbf{T_r}$, peak time $\mathbf{T_p}$, settle time $\mathbf{T_s}$, input overshoot $\mathbf{OS_u}$, output overshoot $\mathbf{OS_y}$, and steady-state error $\mathbf{E_{ss}}$.

14. 3 mark(s) Reference Performance Metrics

- Q14.Tr (sec) Scalar
- Q14.Tp (sec) Scalar
- Q14.Ts (sec) Scalar
- Q14.OSu (%) Scalar
- Q14.OSy (%) Scalar
- Q14.Ess (%) Scalar

Use heuristic tuning to satisfy the RCGs shown in **Table 5**.

All RCGs are relative to the **Reference** performance metrics.

Find the **Tuned** gains $\mathbf{K_p}$, $\mathbf{K_i}$, corresponding to unity master gain **K=1**.

15. 12 mark(s) Tuned Gains

- Q15.Kp (pure) Scalar
- Q15.Ki (sec^{-1}) Scalar

Find the **Tuned** performance metrics, rise time $\mathbf{T_r}$, peak time $\mathbf{T_p}$, settle time $\mathbf{T_s}$, input overshoot $\mathbf{OS_u}$, output overshoot $\mathbf{OS_y}$, and steady-state error $\mathbf{E_{ss}}$.

16. 5 mark(s) Tuned Performance Metrics

- Q16.Tr (sec) Scalar
- Q16.Tp (sec) Scalar
- Q16.Ts (sec) Scalar
- Q16.OSu (%) Scalar
- Q16.OSy (%) Scalar
- Q16.Ess (%) Scalar

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