

# MECH0005 Topic Notes

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# Chapter 1

## Fluids

### 1.1 Fluid Properties

Fluids have many properties including:

- Surface Tension -  $\sigma$ .
- Viscosity -  $\mu$ .
- Compressibility.
- Density  $\rho$ .
- Temperature.
- Pressure -  $P$ .

What is a fluid?

A fluid is a substance that deforms continuously when acted on by a shearing stress of any magnitude.

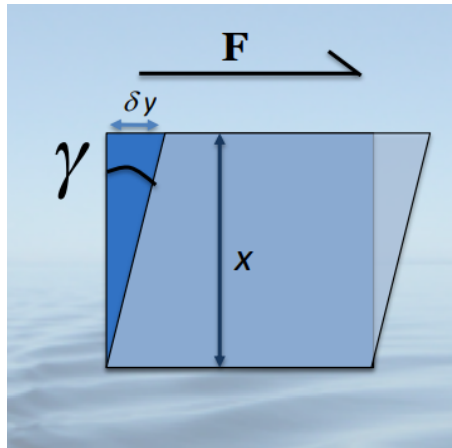
This includes gases and liquids but not silly putty, gels or glass. Some soft materials such as toothpaste will only start to flow once a critical shear stress has been reached. This is the study of rheology and not studied in this module.

In classical fluid mechanics, we treat fluids as a continuum because there is an extremely large number of particles. Thus, we can conclude that fluid parameter such as pressure and density vary continuously throughout the fluid.

### 1.1.1 Shear strain for solid bodies

What is shear strain?

Shear strain is the change in angle as an element experience a force tangential to its surface.



For a fluid, we are interested in the rate of shear. A specific tangential force will cause a fixed amount of shear strain per unit time. If the speed of the top layer is  $u_y$ , the shear rate is  $\delta u_y / \delta x$ .

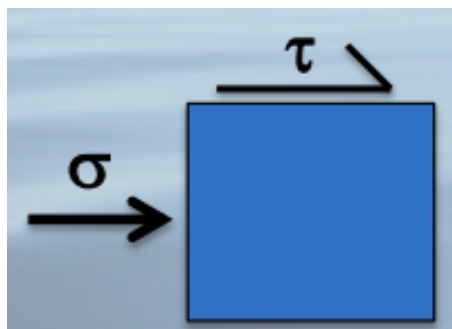
$$\tan(\gamma) = \frac{\delta y}{x} \approx \gamma \quad (1.1)$$

### 1.1.2 Shear stress on a solid body

For a solid, the shear force is the force applied tangentially to a surface.

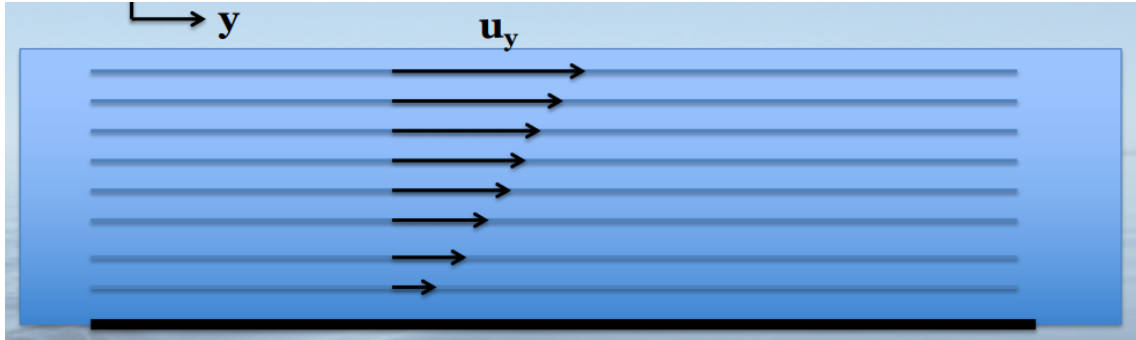
*Shear stress* is the tangential force per unit area  $\tau$ .

*Normal stress* is the perpendicular force per unit area  $\sigma$ .



### 1.1.3 Simple laminar flow case

Consider flat layers of fluid sliding over each other. The sideways velocity of the fluid changes as you move away from the stationary boundary. The velocity of the  $y$  direction,  $u_y$  is changing with position,  $x$ .



### 1.1.4 Viscosity

$$\tau = \mu \frac{du_y}{dx} \quad (1.2)$$

Where:

- $\tau$  = Shear stress = force/area.
- $\mu$  = Dynamic viscosity.
- $du_y/dx$  = Shear rate.
- Units: Velocity change per unit perpendicular distance.

Viscosity is the constant of proportionality, telling us how much shear stress is required to produce a given shear rate. Shear stress can be different at different places in the same fluid. Equation (2) is Newton's equation for viscosity and he assumed that the viscosity was a constant. However, this is not always true and the fluids for which that does not apply are called *non-Newtonian fluids*. An example of a shear *thickening* fluid is corn-starch, i.e. viscosity increases with shear rate. An example of shear *thinning* fluid is blood or ketchup, i.e. viscosity decreases with shear rate. We cannot use laminar flow analysis to study blood vessels because they constrict and dilate as fluids pass through - the rigid boundary condition does not apply (or you would not be able to feel your pulse.) We will only be studying Newtonian fluids and rigid boundaries.

The standard symbol for the viscosity as defined above is  $\mu$  and this is known as the *dynamic viscosity* with units  $\text{kg m}^{-1} \text{s}^{-1}$  or Pa.s. This will be our definition

of viscosity. For some applications it is easier to do calculations in terms of the viscosity per unit density:

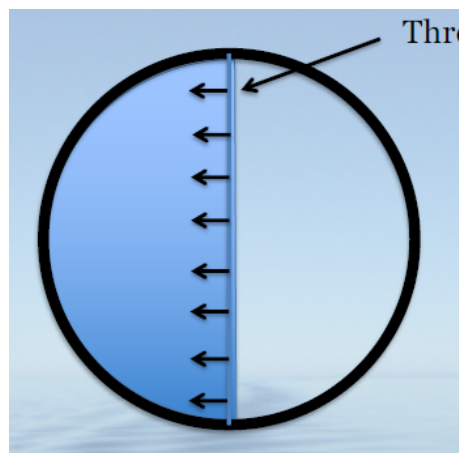
$$v = \frac{\mu}{\rho} \quad (1.3)$$

### 1.1.5 Surface tension

What is surface tension?

Surface tension is the effective tangential force per unit length ( $\text{N m}^{-1}$ ) exerted on an imaginary line through a surface, due to intermolecular interaction.

Surface tension is always considered perpendicular to the line it's pulling on. In this case, we want the force exerted on the thread. If the thread has length  $L$ , the total force on one side of the soap film is  $\sigma L$ , so the total force from both sides is  $2\sigma L$ .

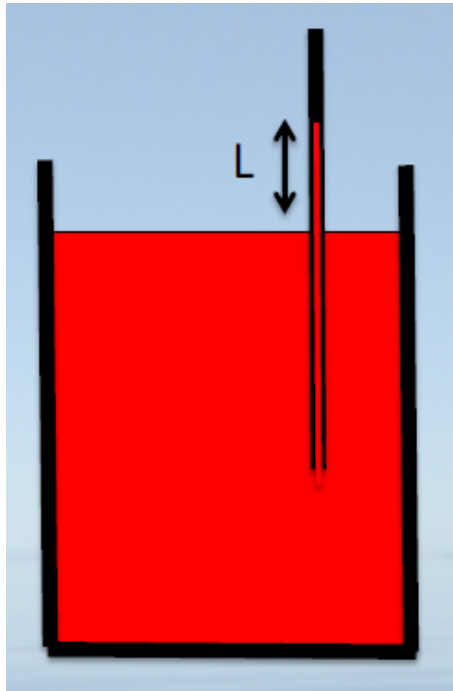


### 1.1.6 Capillary action

This is caused by two processes:

- *Adhesion* between the water molecules and the tube wall.
- *Cohesion* between water molecules.

However, the surface tension (cohesion) must still be strong enough to drag the fluid up the pipe.



The liquid will rise by itself up the tube and this gives us a crude method for calculating surface tension. The weight of the water must be balanced by the surface tension forces at the water surface.

$$F = 2\pi\sigma = L\pi r^2 \rho g \quad (1.4)$$

$$\sigma = \frac{1}{2} \rho r g L \quad (1.5)$$

### 1.1.7 Pressure

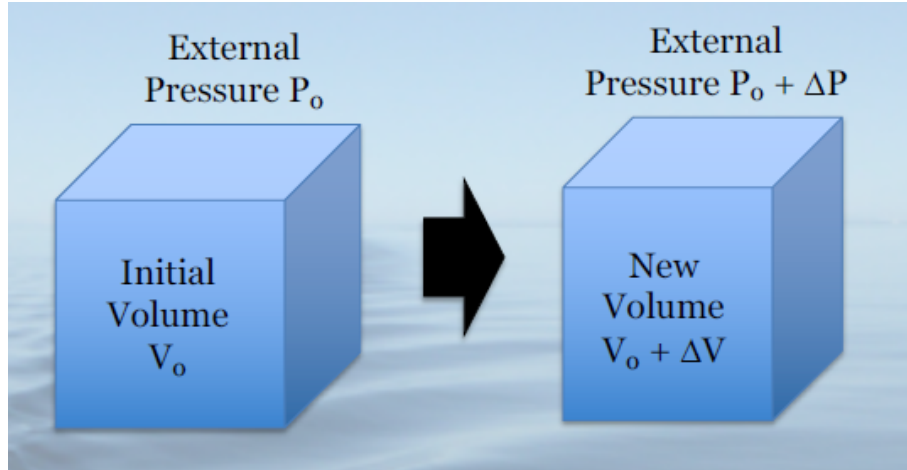
What is Pascal's Law?

The pressure at a point in a fluid at rest or in motion is independent of direction as long as there are no shearing forces present.

The volume of a fluid under pressure will depend on the magnitude of the pressure. However, for many liquids (e.g. water), the change in volume is so small that it can be considered *negligible*. Even in everyday situations, water obviously is not really incompressible because sound waves can travel through it. For a truly incompressible fluid, the speed of sound would be infinite. Seawater density varies with temperature, pressure and salinity but below the top one kilometre, where



salinity and temperature are nearly uniform, there is almost no variation in density with depth. The relationship between the pressure, volume and temperature of a fluid is described by an *equation of state*. Any equation that relates pressure and volume tells us something about the compressibility of a fluid.



We are interested in finding  $dV/dP$  which is (almost) always negative. A fluid is considered *compressible* when its volume varies significantly with temperature and pressure in the parameter space of interest. If there is no significant variation, the fluid can be considered *incompressible*. We will only be covering compressible fluids.

### 1.1.8 Example Question: Cartesian diver

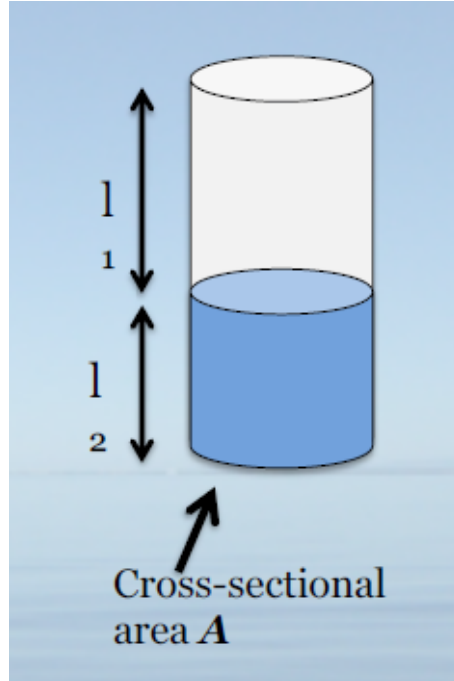
Consider a closed vessel that is completely full of water but with flexible sides (plastic bottle). Inside is a cylinder that is closed at the top but open at the bottom. It is partly full of air and has some extra mass added at the base. The plastic parts of the diver have a volume of  $1.067 \times 10^{-6} \text{ m}^3$  and a density of  $2400 \text{ kg m}^{-3}$ . At atmospheric pressure, the length of the air column is 1.5 cm and the cross-sectional area is  $1 \times 10^{-4} \text{ m}^2$ . The temperature in the room is  $20^\circ\text{C}$ .

Question: How much pressure must be applied to the bottle to make the diver sink?

We must ask an important question: is the total density of the diver and the air and the water greater than or less than the density of the water around it? We know that  $\rho_w = 1000 \text{ kg/m}^3$  and if the water is compressible, the total density of the whole diver is given by:

$$\rho_D = \frac{\rho_A l_1 A + \rho_w l_2 A + M_D}{(l_1 + l_2)A}$$

Where  $M_D$  is the mass of the plastic parts of the diver.



We can see that the diver will sink if  $\rho_D > \rho_w$  and it rises if  $\rho_D < \rho_w$ .

Method: set the densities to be equal and deduce an expression for the pressure in this situation.

$$\rho_w = \rho_D = \frac{\rho_A l_1 A + \rho_w l_2 A + \rho_P V_P}{(l_1 + l_2)A + V_P}$$

To calculate the density of air, we need the equation of state. Assume that  $R_g = 287 \text{ J kg}^{-1} \text{ K}^{-1}$

$$P = \rho_A R_g T$$

The necessary pressure is determined by the necessary density of air. So, we start by rearranging the equation to specify the density of air.

$$\rho_A = \frac{1}{l_1 A} (\rho_w ((l_1 + l_2)A + V_P) - \rho_w l_2 A - \rho_P V_P)$$

$$\rho_A = \frac{1}{l_1 A} (\rho_w (l_1 A + V_P)) - \rho_P V_P$$

We can now use the equation of state to find an expression for the pressure.

$$P = \frac{R_g T}{l_1 A} (\rho_w (l_1 A + V_P)) - \rho_P V_P$$

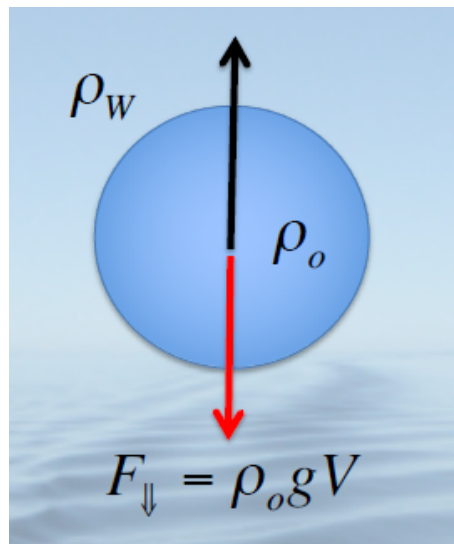
From here we can substitute our values to arrive at an answer of  $P = 3.48 \times 10^5 \text{ N m}^{-2}$  or approximately 3.5 atmospheres.

In many situations, we assume that liquids are incompressible. However, this is an *assumption* that must be justified.

## 1.2 Buoyancy and stability

The resultant force acts at the centre of gravity of the fluid, the centroid of the volume. This is called the centre of buoyancy. Whatever is filling the space also has a force acting on it: it's weight. This acts at its centre of mass. The resultant force is given by:

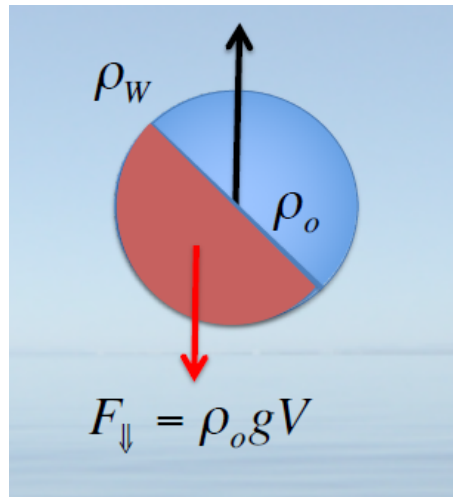
$$F_{\uparrow} - F_{\downarrow} = (\rho_w - \rho_o)gV \quad (1.6)$$



An object is neutrally buoyant if the weight and the buoyant forces are equal, so the system is in equilibrium.

### 1.2.1 Instability

Consider a submerged ping pong ball which is half filled with plasticine and the other half just filled with air.



Assume that the ball overall is neutrally buoyant. The forces are equal and opposite but there is a *couple*. The ball will rotate in response to the couple until the forces are aligned. At this point, the object is stable. The buoyancy and the weight do not necessarily have the same line of action. We must also consider whether the object will maintain its orientation.

- *Stable* equilibrium: when displaced slightly, there is a net force acting to push the object back to its initial orientation.
- *Unstable* equilibrium: when displaced slightly, there is a net force acting to push the object further from its initial orientation.
- *Neutral* equilibrium: when displaced slightly, there is no net force.

If the centre of buoyancy is above the centre of gravity, the object is in a stable equilibrium.

### 1.2.2 Centre of mass

The centre of mass is calculated in an analogous way to the centroid of the volume. If we split any object in half with a flat plane that passes through its centre of mass, there will be equal mass in both halves.

$$x_{CoM} = \frac{1}{m} \int_m x dm$$

$$y_{CoM} = \frac{1}{m} \int_m y dm$$

$$z_{CoM} = \frac{1}{m} \int_m z dm$$

Where  $m$  is the total mass of the object.

In a semi-submerged body, the upward thrust is equal to the weight of the *displaced* fluid. For a body to be in vertical equilibrium, the immersed body must generate a buoyancy force equal and opposite to the weight of the object.

## 1.3 Dimensionless Numbers

### 1.3.1 Notation

We denote a dimension by using a capital letter in square brackets. Here are some common dimensions.

- $[M]$  - mass (unit: kilogram).
- $[T]$  - time (unit: second).
- $[L]$  - length (unit: metre).
- $[\Theta]$  - temperature (unit: kelvin).

Thus, we can derive that the dimensions of acceleration (which has units  $\text{ms}^{-2}$ )  $[L][T]^{-2}$ . Some dimensions of common measurements are shown below:

- Force -  $[M][L][T]^{-2}$ .
- Energy -  $[M][L]^2[T]^{-2}$ .

We use dimensional analysis to check derivations. The dimensions of both sides of any equation must match. Physical constants also often have units associated with them - these must also be considered. Some variables are dimensionless such as the Reynolds number.

$$Re = \frac{\rho l u}{\mu}$$
$$[Re] = \frac{[M][L]^{-3} \cdot [L] \cdot [L][T]^{-1}}{[L][M][T]^{-2} \cdot [T][L]^{-2}} = \frac{[M][L]^{-1}[T]^{-1}}{[M][L]^{-1}[T]^{-1}}$$

Hence, we can see that the Reynolds number is a dimensionless quantity as all the dimensions cancel.

### 1.3.2 Example

We can use dimensional analysis to derive basic forms of equations. We want to work out the pressure drop as oil flows through a pipe. Let us consider the parameters this may depend on.

- Viscosity -  $[M][L]^{-1}[T]^{-1}$ .
- Pipe length -  $[L]$ .
- Pipe diameter -  $[L]$ .
- Velocity -  $[L][T]^{-1}$ .
- Pressure -  $[M][L]^{-1}[T]^{-2}$ .

Next we can assume that the pressure is a function of the other four. Some combination of the others must have the same dimension as the quantity we want.

$$[M][L]^{-1}[T]^{-2} = ([M][L]^{-1}[T]^{-1})^\alpha \cdot ([L])^\beta \cdot ([L])^\gamma \cdot ([L][T]^{-1})^\delta$$

$$[L] : -1 = -\alpha + \beta + \gamma + \delta$$

$$[M] : 1 = \alpha$$

$$[T] : -2 = -\alpha - \delta$$

$$\alpha = 1, \delta = 1, \beta + \gamma = -1$$

So it must be true that:

$$\Delta P = \mu \cdot v \cdot I^\beta \cdot D^\gamma$$

Where  $\beta + \gamma = -1$  The actual answer for laminar flow is:

$$\Delta P = \frac{2\mu Lv}{D^2}$$

This sort of analysis is useful for checking on the functional form of relationships, but it won't give you the exact relationship, or the value of any dimensionless constants involved.

### 1.3.3 Similarity

- Geometrical similarity: fixed ratio of lengths.
- Kinematic similarity: fixed ratio of velocities.
- Dynamic similarity - fixed ratio of forces.

Note on inertia: Inertia is not a force. However, for considering its importance to dynamic similarity, we can use the force needed to slow down a moving object. So we quantify inertia for these purposes as  $ma$ , from  $F = ma$ . Since the forces on flow change fluid motion, we use this often.

### Dynamic similarity: viscosity

Compare the inertia "force" and the viscous force for a fluid:

$$\frac{[Inertia\ force]}{[Viscous\ force]} = \frac{\rho L^2 u^2}{\mu u L} = \frac{\rho L u}{\mu}$$

The Reynolds number is something very specific - it allows us to calculate the ratio of inertial and viscous forces in order to check for dynamical similarity.

- Honey:  $Re \approx 1.3 \times 10^{-4}$
- Tea:  $Re \approx 1100$

Therefore, they are not dynamically similar with respect to viscosity.

### 1.3.4 Dimensionless groups

We have identified some dimensionless groups such as Reynolds number and Froude number. There are many more such as:

- Bond number: ratio of gravitational to surface tension forces.
- Capillary number: ratio of surface tension to viscous forces.
- Euler number: ratio of pressure force to inertial force.
- Grashof number: ratio of buoyancy to viscous forces.
- Cauchy number: ratio of inertial to elastic forces.
- Weber number: ratio of inertial to surface tension forces.

## 1.4 Buckingham Pi

insert theory here

# Chapter 2

## Fluids Worksheets

### 2.1 Introduction to Fluid Dynamics

#### 2.1.1 Question 1

**a**

Let us start with the equation for viscosity:

$$\tau = \mu \frac{\partial u_y}{\partial x} \quad (2.1)$$

Rearranging the following equation for  $\partial u_y / \partial x$  gives us:

$$\frac{\partial u_y}{\partial x} = \frac{\tau}{\mu}$$

Inputting our variables:

$$\frac{\partial u_y}{\partial x} = \frac{0.4}{5} = 0.08 \text{ s}^{-1}$$

**b**

We need to find the shear stress exerted by the block on the oil, found using the following equation:

$$\tau = \frac{F_t}{A} \quad (2.2)$$



The force that the block exerts tangentially on the oil covered inclined plane surface is given by:

$$F_t = 10 \cdot g \cdot \sin(20) \text{ N}$$

The area of contact between the oil and the block is

$$A = 0.1 \text{ m}^2$$

Inputting into equation (2):

$$\tau = 100 \cdot g \cdot \sin(20)$$

We are told the velocity distribution through the oil is linear and hence can make the following simplification:

$$\frac{\partial u_x}{\partial y} = \frac{u_{block}}{D}$$

$$\tau = \mu \frac{u_{block}}{D}$$

$$u_{block} = \frac{\tau D}{\mu}$$

$$u_{block} = \frac{100 \cdot 9.81 \cdot \sin(20) \cdot 1 \times 10^{-4}}{5}$$

$$u_{block} = 0.0067 \text{ m s}^{-1}$$

### 2.1.2 Question 2

# Chapter 3

## Thermodynamics

### 3.1 Second Law of Thermodynamics

First, let us remind ourselves of the first law of thermodynamics and some of its limitations.

Energy can neither be created or destroyed during a process; it can only change forms.

- A certain energy balance will hold when a system undergoes change or a thermodynamic process.
  - But does not give information on whether the change of state or the process is at all feasible or not.
- It cannot indicate whether a metallic bar of uniform temperature can spontaneously become warmer at one end and cooler at the other.
  - However, if that process did occur, all that law can state is that the energy gained at one end would exactly equal the energy lost at the other.

The second law of thermodynamics provides the criterion as to the *probability* of various processes. Spontaneous processes in nature occur only in one direction. Heat flows from a body at high temperature to a body at low temperature; water always flows downwards etc. The spontaneity of the process is due to finite driving potential, sometimes called 'force' or 'cause', e.g. a temperature or concentration

gradient or an electric potential. What happens as a result of this finite driving potential is called the 'flux' or the 'current' or the 'effect' (heat transfer, mass transfer, flow of electric current). This directional law puts a limitation on energy transformation other than that imposed by the first law.

There is also a qualitative difference between heat and work. Energy supplied as work can be completely converted to heat, e.g. paddle wheel work on a liquid in an adiabatic vessel. However, the complete conversion of heat into work is not possible, thus making heat and work not completely interchangeable forms of energy. Also, we considered the problem of a simple steam power plant and by using the Steady Flow Energy Equation (i.e. first law) and the properties of steam were able to calculate the work done and heat transfers for individual components. However, we are not yet able to understand ways of improving steam engine efficiency.

The second law is based on experimental observation and was the result of the question, 'how efficient can one make a steam engine?' From now we will start by considering engines and define them with the precision that thermodynamics requires. The sort of steam engines we shall discuss are heat devices in boxes with no fluid entering or leaving but with just heat and work crossing the boundaries.

### 3.1.1 Heat engine

Definition:

A heat engine (or Cyclic Heat Power Plant - CHPP) is a continuously operating thermodynamic system at the boundary of which there are heat and work transfers.

Notes:

- 'Continuously operating' means that the state of the system exhibits only periodic (cyclic) changes.
- The heat engine is a thermodynamic system and so no matter crosses the boundary e.g. simple steam power plant and closed-cycle gas turbine plant.

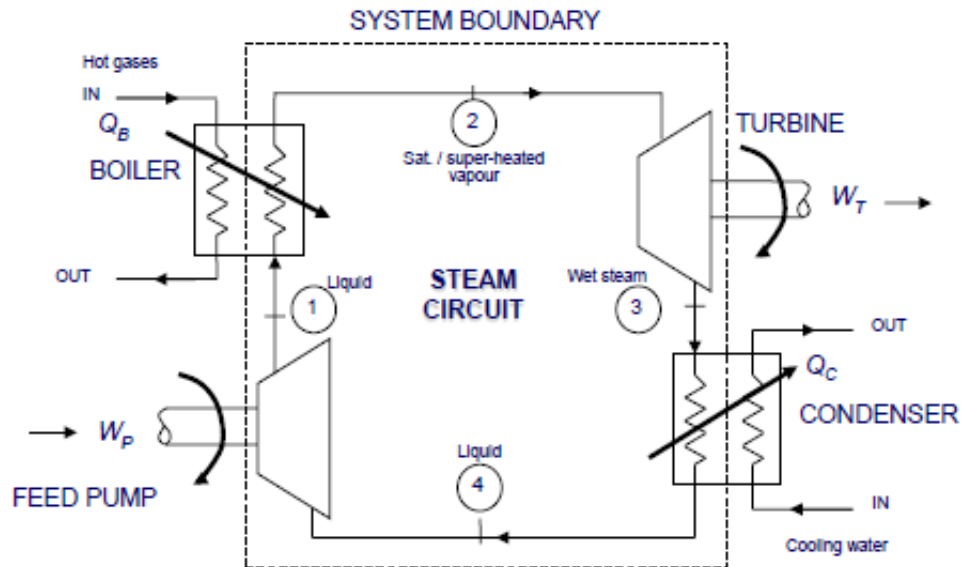


Figure 3.1: A simple steam power plant

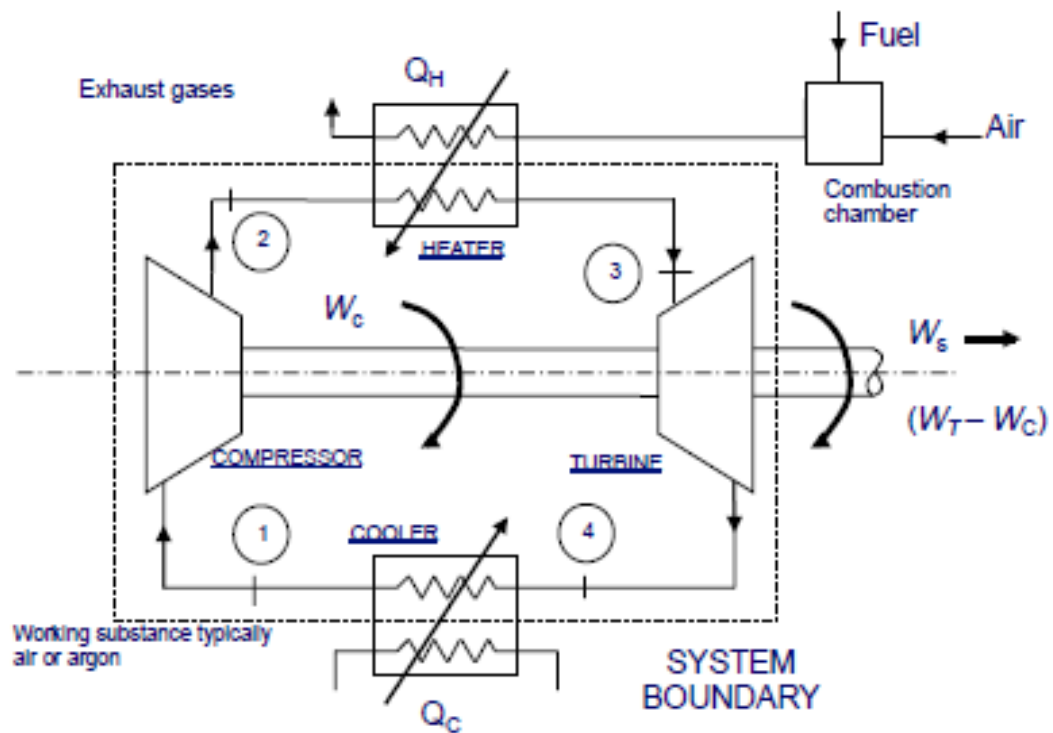


Figure 3.2: A closed cycle gas power plant

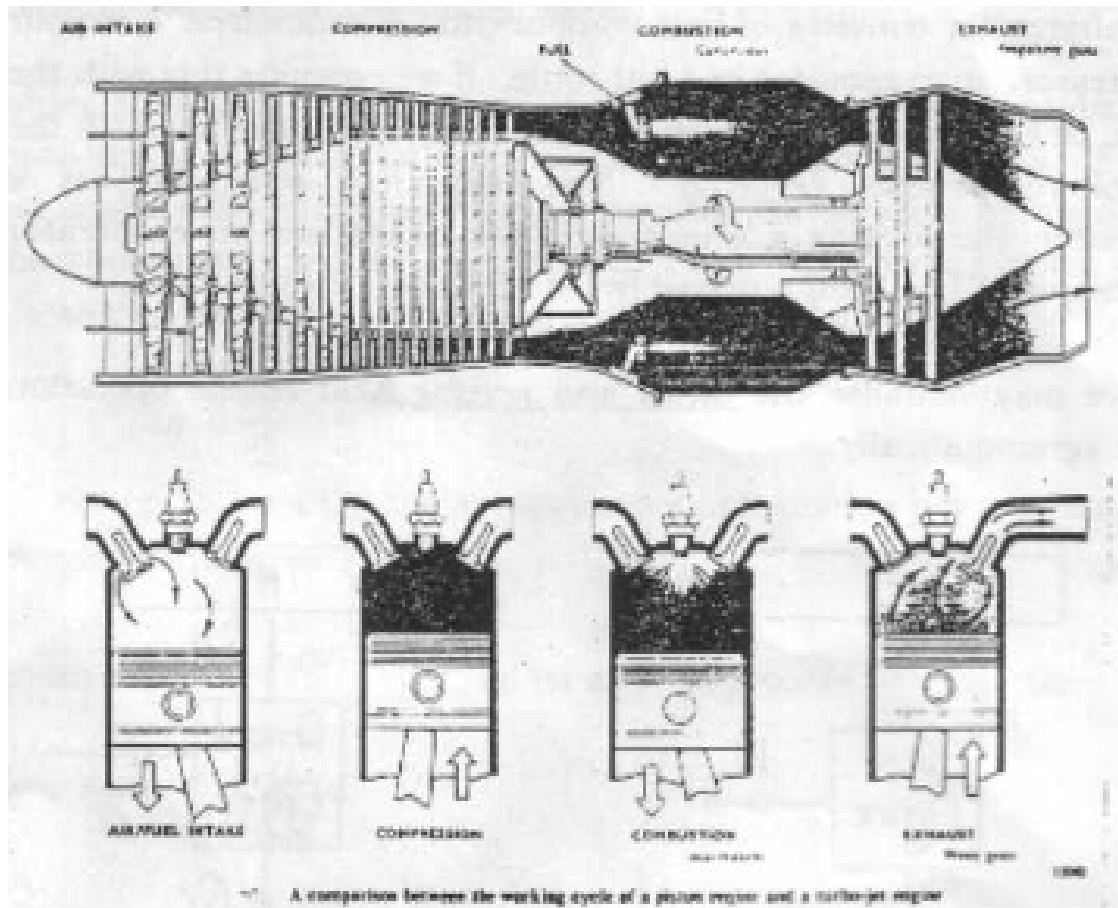


Figure 3.3: A diesel engine

Diesel engines (and internal combustion engines generally) are not heat engines (CHPP) because matter crosses its boundaries. A jet engine is also not a CHPP because matter, air, fuel and exhaust cross the boundary of the system.

### 3.1.2 Reversed heat engine

The definition of a heat engine says nothing about the direction of heat and work transfers. Thus, this makes a domestic refrigerator a heat engine. The domestic refrigerator consists of:

- Condenser.
- Vapour compressor.
- Evaporator.
- Throttle.

Compare this to other heat engines e.g. steam/gas turbine power plants, where the component roles are reversed.

- Boiler > Condenser.
- Turbine > Compressor.
- Condenser > Evaporator.
- Pump > Throttle.

## 3.2 Reversibility

## 3.3 Entropy