

0.1 Fluids formulas

Density

$$v = \frac{1}{\rho} \quad (1)$$

Specific weight

$$\gamma = \rho g \quad (2)$$

Specific gravity

$$SG = \frac{\rho}{\rho_{H_2O@4^\circ C}} \quad (3)$$

Ideal gas law

$$P = \rho RT \quad (4)$$

$$Pv = RT \quad (5)$$

$$PV = nR_u T \quad (6)$$

Kinematic viscosity

$$v = \frac{\mu}{\rho} \quad (7)$$

Surface tension

$$\sigma = \frac{F}{L} \quad (8)$$

Capillary action

$$h = \frac{2\sigma \cos \theta}{\gamma R} \quad (9)$$

Centre of mass

$$\bar{x} = \frac{\int_m(x)dm}{m} \quad (10)$$

$$= \frac{\int_m(x)dm}{\int_m dm} \quad (11)$$

Stable equilibrium: the centre of gravity is directly below the centre of buoyancy.

Reynolds Number

$$Re = \frac{\rho Lu}{\mu} \quad (12)$$

$Re < 2000 \rightarrow$ Laminar

$Re > 2000 \rightarrow$ Turbulent

Material derivative

$$\frac{D}{Dt}() = \frac{\partial}{\partial t}() + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \quad (13)$$

$$\underline{a} = \frac{D}{Dt}(\underline{v}) = \frac{\partial \underline{v}}{\partial t} + u\frac{\partial \underline{v}}{\partial x} + v\frac{\partial \underline{v}}{\partial y} + w\frac{\partial \underline{v}}{\partial z} \quad (14)$$

Streamline coordinates

$$\underline{a} = \frac{D\underline{v}}{Dt} = a_s\hat{s} + a_n\hat{n} \quad (15)$$

$$= v\frac{\partial \underline{v}}{\partial s}\hat{s} + \frac{v^2}{R}\hat{n} \quad (16)$$

$$a_s = \frac{-v \sin \theta - \frac{\partial P}{\partial s}}{\rho} = v\frac{\partial \underline{v}}{\partial s} // a_n = \frac{-v \cos \theta - \frac{\partial P}{\partial n}}{\rho} = \frac{v^2}{R} \quad (17)$$

RTT

$$\left(\frac{DB}{Dt}\right)_{sys} = \frac{\partial}{\partial t} \int_{cv} (\rho b) d\forall + \int_{cs} (\rho b(\underline{v} \cdot \underline{\hat{n}})) dA \quad (18)$$

$$\text{When } B=M \rightarrow 0 = \frac{\partial}{\partial t} \int_{cv} (\rho) d\forall + \int_{cs} (\rho(\underline{v} \cdot \underline{\hat{n}})) dA \quad (19)$$

$$\text{When } \underline{B} = m\underline{v} \rightarrow \sum F_{sys} = \frac{\partial}{\partial t} \int_{cv} (\rho \underline{v}) d\forall + \int_{cs} (\rho \underline{v}(\underline{v} \cdot \underline{\hat{n}})) dA \quad (20)$$

$$(21)$$

Propeller stages

1. Bernoulli equation between 1 and 2.

$$P_1 = P_2 + 0.5\rho(v_2^2 - v_1^2)$$

2. Bernoulli equation between 3 and 4.

$$P_4 = P_3 + 0.5\rho(v_3^2 - v_4^2)$$

3. $P_1 = P_4$.

$$\therefore \Delta P = P_3 - P_2 = 0.5\rho(v_4^2 - v_1^2)$$

4. $F = \Delta P \times A$.

$$\therefore F_{thrust} = \Delta P \times A = 0.5\rho A(v_4^2 - v_1^2)$$

5. Apply momentum equation.

$$\begin{aligned}\sum F_x &= \int_{inlet} \rho v_1(-v_1)dA + \int_{outlet} \rho v_4(v_4)dA \\ &= -\dot{m}_1 v_1 + \dot{m}_2 v_4 \\ &= \dot{m}(v_4 - v_1) \text{ (assume steady state, no viscous force, neglect g)}\end{aligned}$$