UCL Mechanical Engineering 2020/2021
MECH0013 Coursework 1

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Contents

1 Question 1

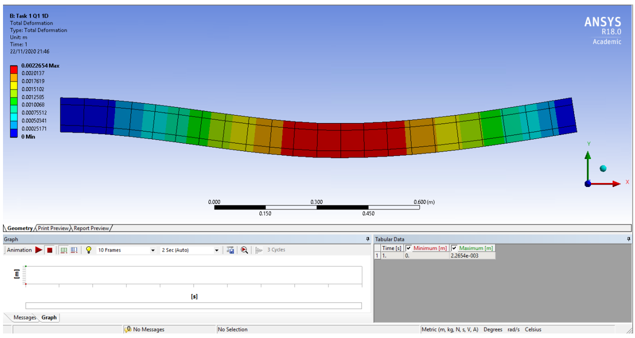


Figure 1: Total deformation in beam

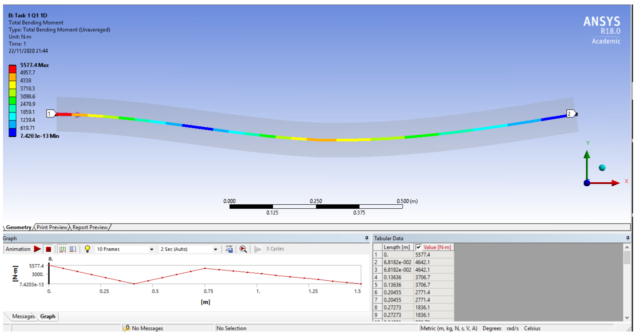


Figure 2: Bending moment in beam

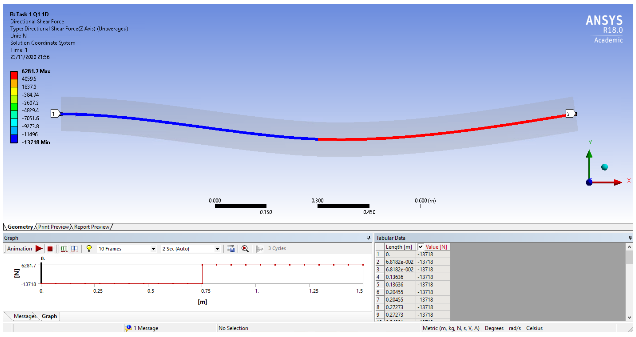


Figure 3: Directional shear in beam

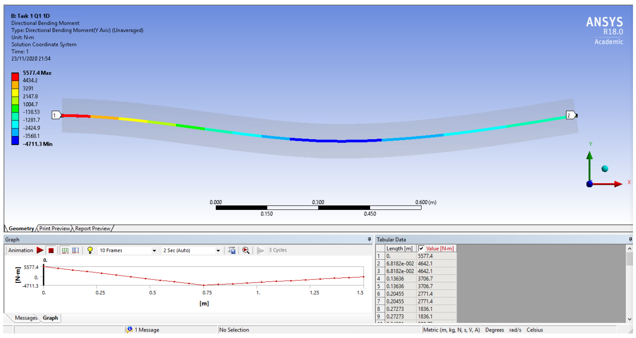


Figure 4: Directional bending in beam

FIX LINK to see numerical data of the directional deformation and the bending moment of the beam.

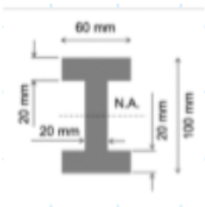
2 Question 2

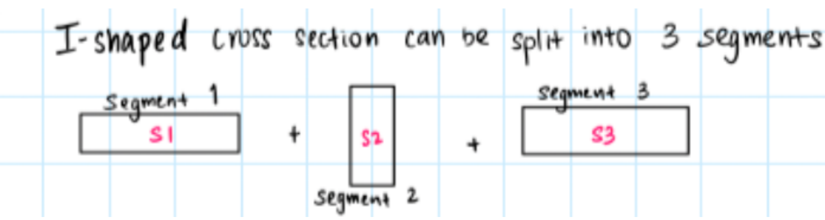
$$\sum F_y = 0 \rightarrow R_A + R_B = 20000 \tag{2.1}$$

$$\sum M_B = 0 \rightarrow M_B + 20000(0.75) - R_A(1.5) = 0 \tag{2.2}$$

$$M_A + 15000 - 1.5R_A = 0 \tag{2.3}$$

Determine second moment of area (I):





Segment 1

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \text{m}^4 \tag{2.4}$$

Segment 2

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0)^2 = 3.6 \times 10^{-7}\ \text{m}^4 \tag{2.5}$$

Segment 3

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \text{m}^4 \tag{2.6}$$

$$I_{\text{total}} = 4.28 \times 10^{-6}\ \text{m}^4 \tag{2.7}$$

Macaulay's Method

$$M = M_A + F(x - 0.75) - R_A(x) \tag{2.8}$$

$$\theta = -\frac{1}{EI}\int M\,\mathrm{d}x = -\frac{1}{EI}\left[M_A x + \frac{F(x-0.75)^2}{2} + \frac{R_A(x)^2}{2}\right] + \theta_0 \tag{2.9}$$

$$y = \int \theta\,\mathrm{d}x = -\frac{1}{EI}\left[\frac{M_A x^2}{2} + \frac{F(x-0.75)^3}{6} + \frac{R_A(x)^3}{6}\right] + \theta_0 x + y_0 \tag{2.10}$$

Boundary conditions. At $y = 0$, $x = 0$:

$$y(0) = 0 = \theta_0 \cdot (0) + y_0 \rightarrow y_0 = 0 \tag{2.11}$$

At $\theta = 0$, $x = 0$:

$$\theta(0) = 0 = \theta_0 \rightarrow \theta_0 = 0 \tag{2.12}$$

At $y = 0$, $x = 1.5$:

$$y(1.5) = 0 = -\frac{1}{EI}\left[\frac{M_A(1.5)^2}{2} + \frac{F(1.5-0.75)^3}{6} + \frac{R_A(1.5)^3}{6}\right] + 0 \cdot 1.5 + 0 \tag{2.13}$$

$$0 = \frac{9}{8}M_A + 1406.25 - \frac{9}{16}R_A \tag{2.14}$$

Multiply equation (2.3) by $\frac{9}{8}$:

$$\frac{9}{8}M_A + 16875 - \frac{27}{16}R_A = 0 \tag{2.15}$$

Equations (2.15) - (2.14):

$$15468.75 = \frac{9}{8}R_A \rightarrow R_A = 13\,750\ \text{N} \tag{2.16}$$

$$\therefore M_A = 1.5(13750) - 15000 \rightarrow M_A = 5625\ \text{N} \tag{2.17}$$

$$\therefore R_B = 20000 - 13750 \rightarrow R_B = 6250\ \text{N} \tag{2.18}$$

We know y_{max} occurs at $\theta = 0$

$$M_A x + \frac{F(x=0.75)^2}{2} - \frac{R_A x^2}{2} = 3125x^2 - 9325x + 5625 = 0 \tag{2.19}$$

$$x \neq 2.171\ \text{m} \rightarrow x = 0.829\ \text{m}\ (3dp) \tag{2.20}$$

$$y_{max} = -\frac{1}{EI}\left[\frac{M_A(0.829)^2}{2} + \frac{F(0.829-0.75)^3}{6} + \frac{R_A(0.829)^3}{6}\right] = -2.099 \times 10^{-3}\ \text{m}\ (3dp) \tag{2.21}$$