04/12/2020

0.1 Collapse Loads for Portal Frames

Portal frames are rigid structures designed to offer rigidity and stability in their plane.

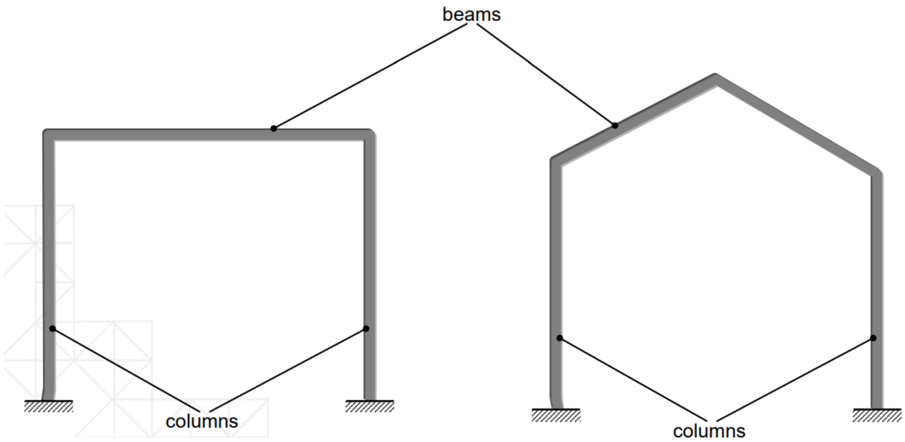
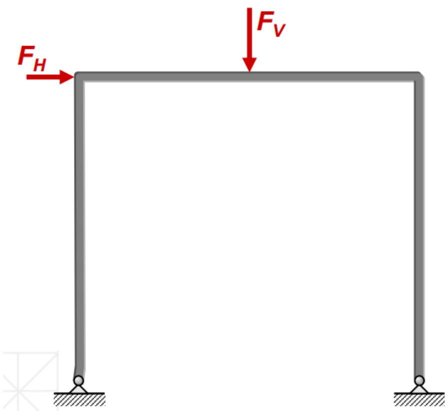


Figure 1: Left - rectangular portal frame, Right - pitched-roof portal frame

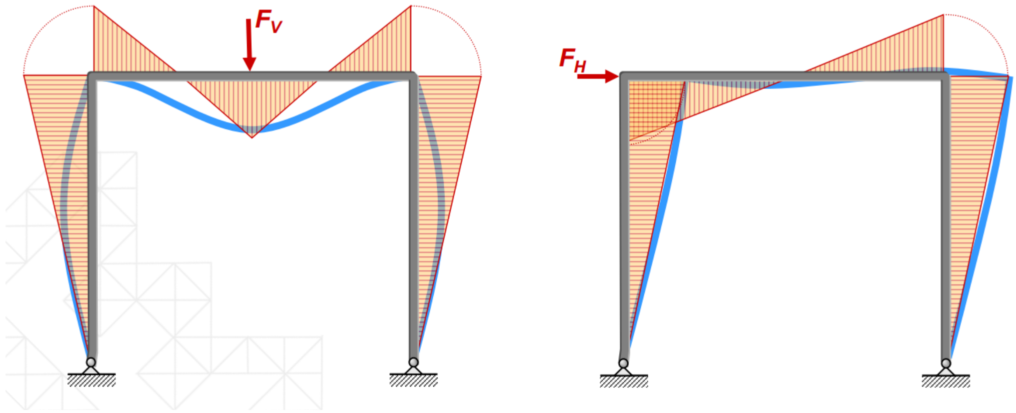
0.2 Portal Frames with Hinged Bases

Consider a flat-roofed portal frame with pinned feet, carrying a vertical concentrated load F_V at the centre of the beam and an horizontal concentrated load F_H at the top of one column.



0.2.1 Deformations and Bending Moments

In this case, the deformations and bending moment distribution produced by the horizontal and vertical loads considered independently are respectively:



Vertical Loads

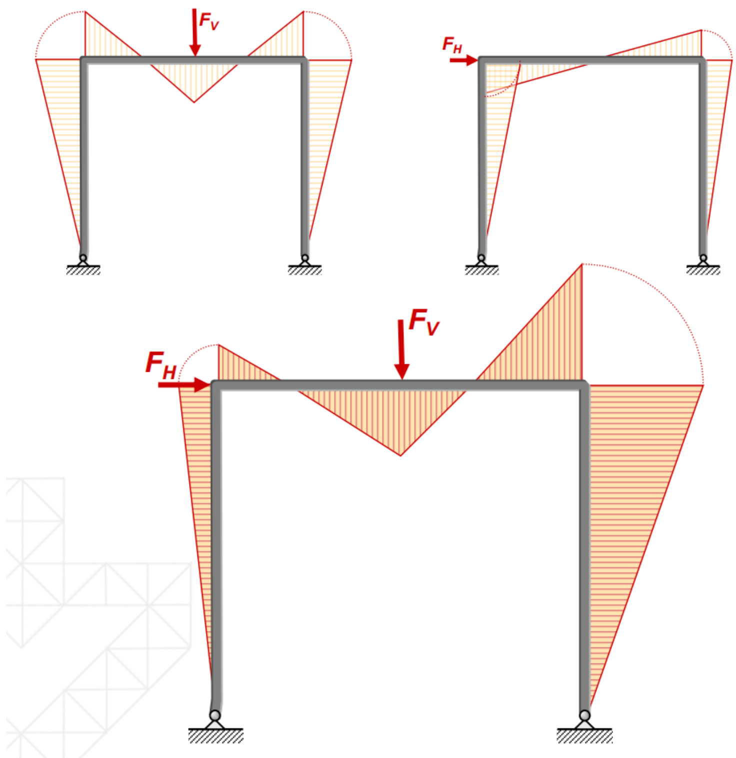
- The mid span off the horizontal beam deforms downwards
- The corners are rigid joints
- The corner points are rotated (from the vertical column's POV)
  - Right Corner - CCW
  - Left Corner - CW
- The bending moment is highest at the mid-span
- The magnitude of the bending moments at the corner (left & right) are the same
  - The arc indicates that they are the same

Horizontal Loads

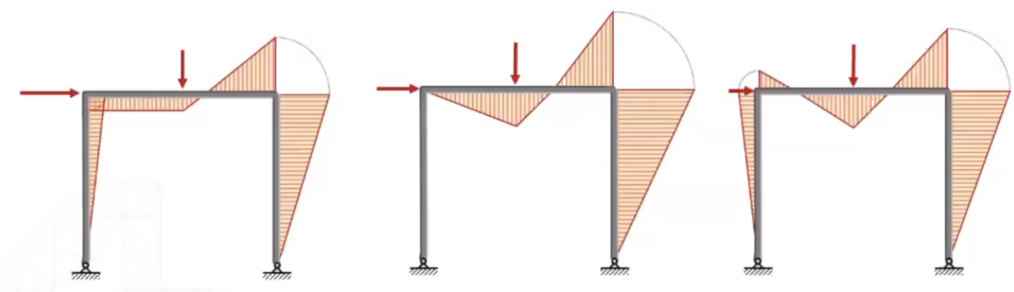
- The frame is displaced to the side
- The corners are rigid joints
- Due to the rigidity of the beam, the top corners are rotated
- The magnitude of the bending moments at the corner (right) are the same

0.2.2 Bending Moment Distribution

By adding the contributions together, we obtain the bending moment distribution below:



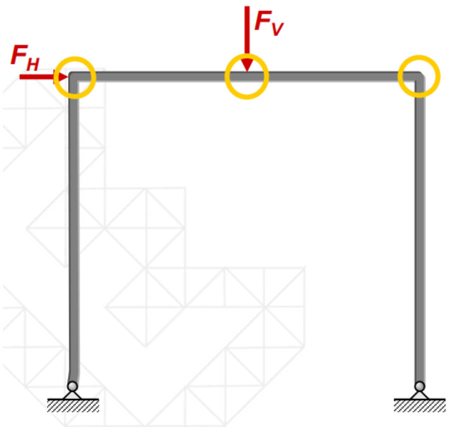
Of course, the exact distribution will depend on the relative magnitude of the two acting forces, and may vary significantly:



However, for any distribution, the points of maximum bending moment are located at the joints or/and at the points of application of the load.

0.2.3 Plastic Hinge Sections

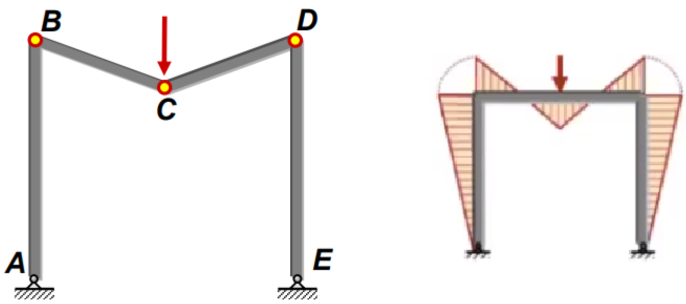
In a framework with rigid joints, the points of local maximum bending moment will occur at the joints as well as under any applied loads.



As a consequence, if plastic collapse is reached, the plastic hinges develop in some of these sections

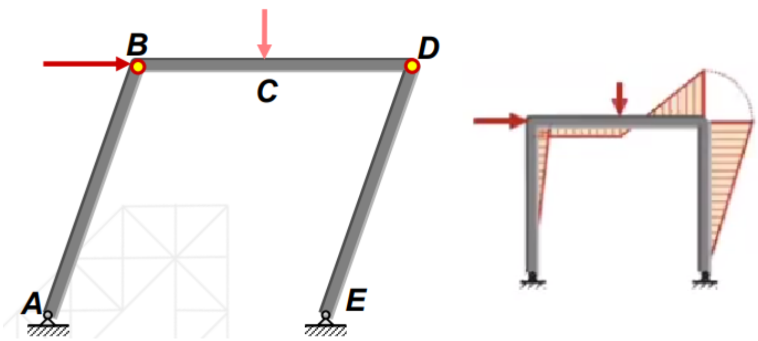
0.2.4 Possible Plastic Collapse Mechanisms

Beam Collapse:



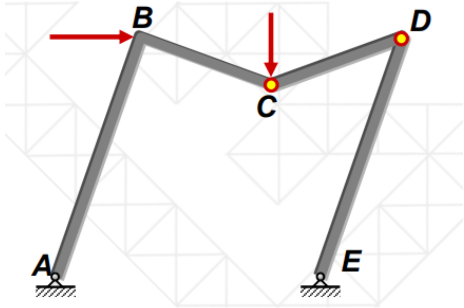
Similar mechanism to fixed-ended beams – hinges form at sections B, C and D. (vertical load much larger than horizontal)

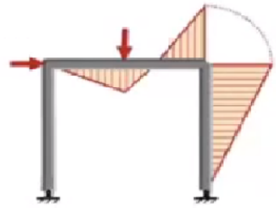
Sway Collapse:



Occurs by overall sway of the frame – hinges form at sections B and D. (horizontal load much larger than vertical)

Combined Beam and Sway Collapse:





Combined beam and sway collapse – hinges form at sections C and D and section B keeps elastic. (horizontal and vertical loads comparable)

0.3 Principle of Virtual Works

0.3.1 Energy

Energy of a system is the ability of the system to do work. Energy may exist in many forms, and can be transformed from one for to another. All forms of energy can be put into two main categories:

1. Kinetic Energy - Motion (of waves, electrons, atoms, molecules, substances and objects)

2. Potential Energy - Stored energy and energy of position (gravitational)

0.3.2 Principle of Virtual Works

If the possible collapse mechanisms are easy to recognise (as in this case), the **principle of virtual works** can be very useful to analyse the collapse mechanisms.

Consider a structure subjected to a set of external mechanical actions. Assume for the structure a set of virtual (imaginary) displacements. The principle of virtual works states that:

The virtual work done by the real external actions as effect of the virtual displacements is equal to the virtual work done by the real internal reactions as effect of the virtual deformations.

0.3.3 Applications to Portal Frames

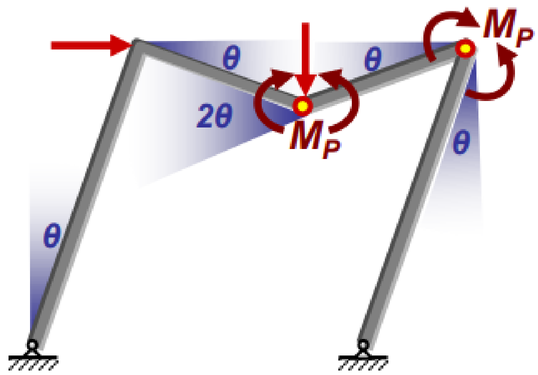
In the case of portal frames:

- The set of external mechanical actions corresponds to the real forces applied to the structure;

- The set of internal reactions are the internal forces and moments reacting to the forces.

- Shear force
- Bending moment

- It is convenient to chose the possible collapse mechanisms as set virtual displacements (only one of them will occur as effect of the applied load).
- In this case the deformations are concentrated at plastic hinges, and are the rotations produced by the internal plastic moments.

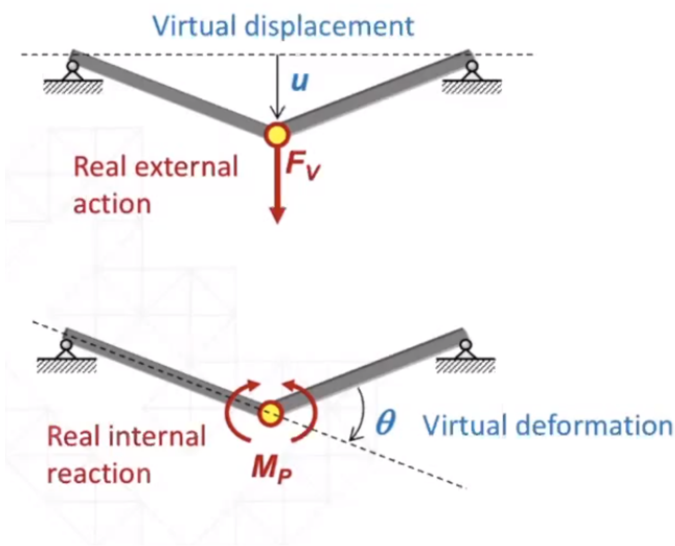


Then, the virtual work principle can be written as:

$$\sum_{i=1}^{n} F_i \cdot u_i = \sum_{j=1}^{m} M_{Pj} \cdot \theta_j \tag{1}$$

Where:

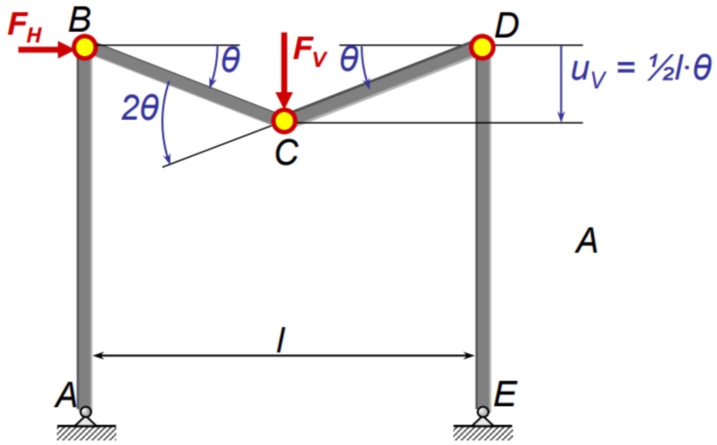
- F_i are the applied forces
- M_{Pj} are the plastic moments at the hinges
- u_i is the relative linear displacement
- θ_i is the relative angular displacement



We do this to find out the level of F_V such that the frame is collapse:

$$F_V = f(M_P) \tag{2}$$

Beam Collapse:



- F_H is ignored in this example as it is not causing any sort of displacement
- F_i is the F_V in this example, causing a vertical displacement at the middle
- The linear displacement u_i is considered for point C
- The angular displacement θ is assumed to be very small $\therefore |CD| = \frac{l}{2}$

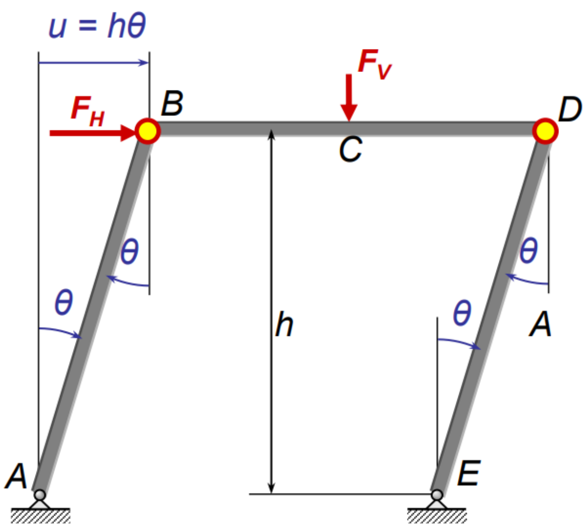
$$F_V \cdot \frac{l}{2}\theta = M_P \cdot \theta + M_P \cdot 2\theta + M_P \cdot \theta \tag{3}$$

The RHS is considered for points B,C,D respectively.

$$F_V \cdot \frac{l}{2}\theta = 4M_P \cdot \theta \tag{4}$$

$$F_V = 8\frac{M_P}{l} \tag{5}$$

Sway Collapse:



- F_V is ignored in this example as it is not causing any sort of displacement

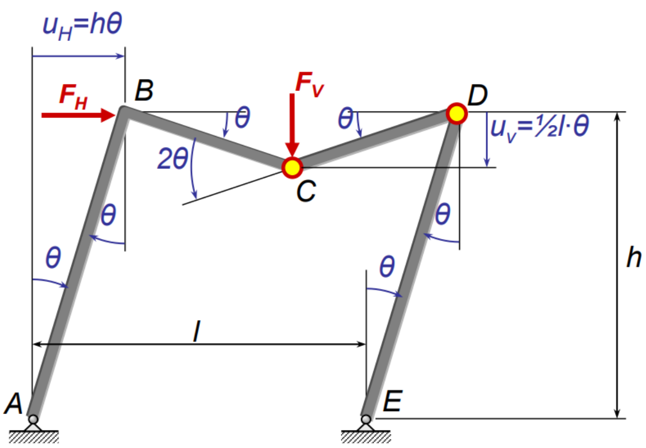
- $F_i = F_H$, causing a horizontal displacement
- $u_i = h\theta$
- $\theta \approxeq 0 \therefore |CD| = \frac{l}{2}$

$$F_H \cdot h\theta = M_P \cdot \theta + M_P \cdot \theta \tag{6}$$

The RHS is considered for points B and D.

$$F_H \cdot h\theta = 2M_P \cdot \theta \tag{7}$$
$$F_H = 2\frac{M_P}{h} \tag{8}$$

Combined Collapse:



- Only points C and D are under collapse
- $F_i = F_H$ and F_V
- $u_h = h\theta$
- $u_v = \frac{l}{2}\theta$
- $\theta \approxeq 0 \therefore |CD| = \frac{l}{2}$

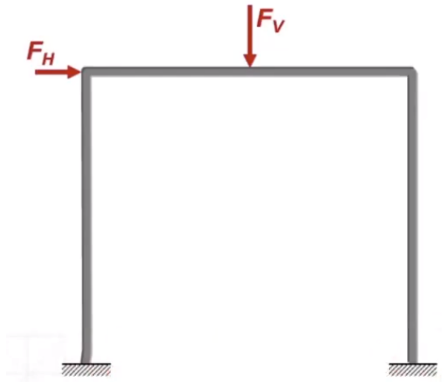
$$F_H h + F_V \frac{l}{2} = 4M_P \tag{9}$$

0.3.4 Collapse Mechanism

The collapse mechanism of the portal frame, under the effect of the external forces considered is **the one that requires lower critical forces** (the one that is reached first).

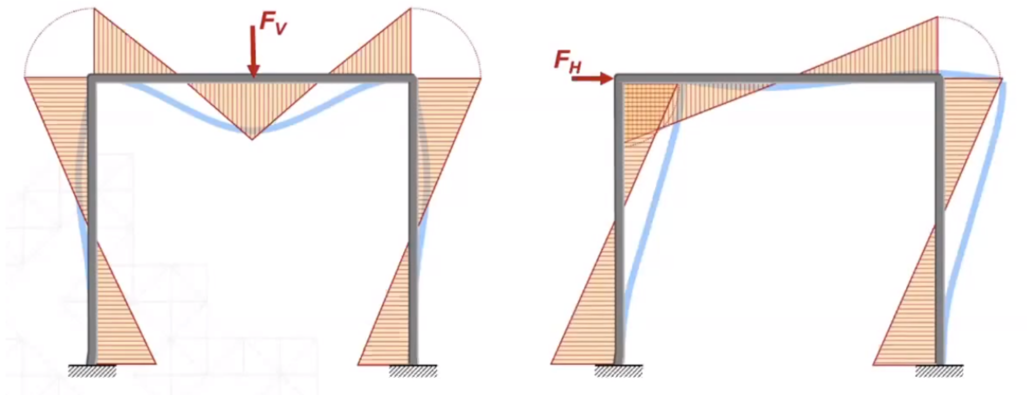
0.4 Portal Frames with Fixed Bases

Consider a flat-roofed portal frame with pinned feet, carrying a vertical concentrated load F_V at the centre of the beam and an horizontal concentrated load F_H at the top of one column.



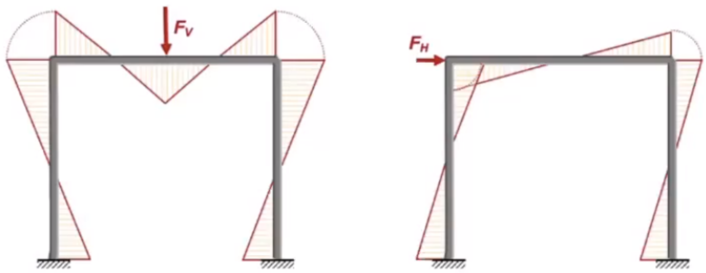
0.4.1 Deformations and Bending Moments

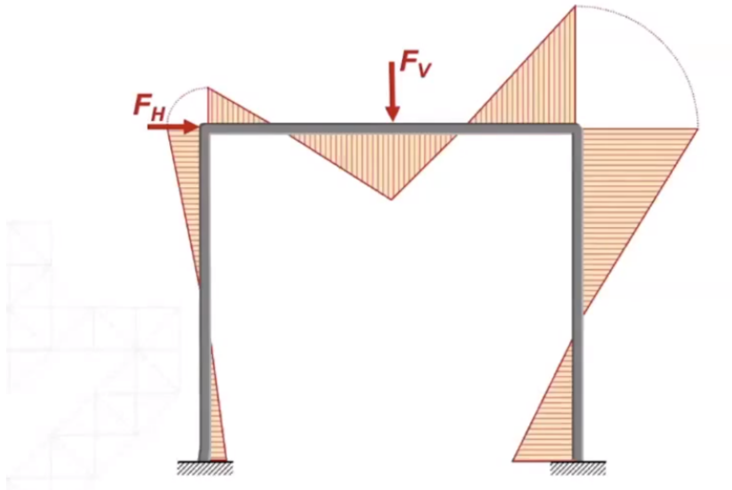
In this case, the deformations and bending moment distribution produced by the horizontal and vertical loads considered independently are respectively:



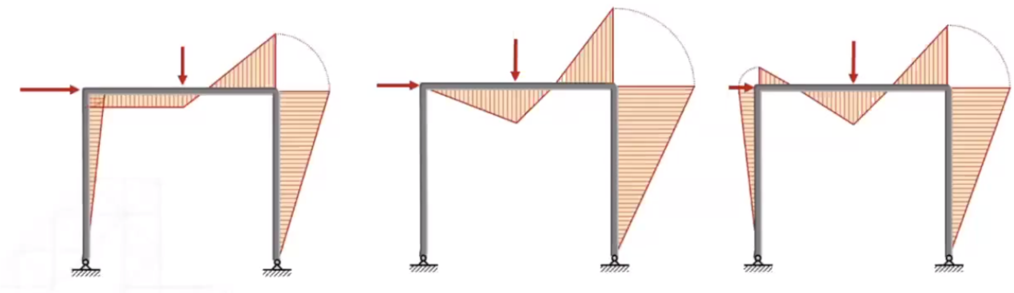
0.4.2 Bending Moment Distribution

By adding the contributions together, we obtain the bending moment distribution below:





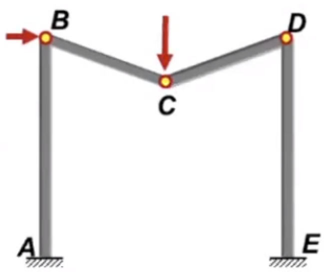
Again, the exact distribution will depend on the relative magnitude of the two acting forces, and may vary significantly (with one column left unstressed in one of the possible configurations):

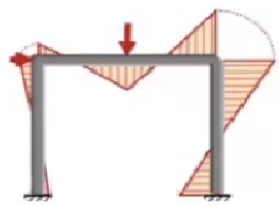


It is confirmed again that the sections of maximum bending moment are located at the joints or/and at the points of application of the load. Therefore, the plastic hinges develop in some of these sections.

0.4.3 Possible Plastic Collapse Mechanisms

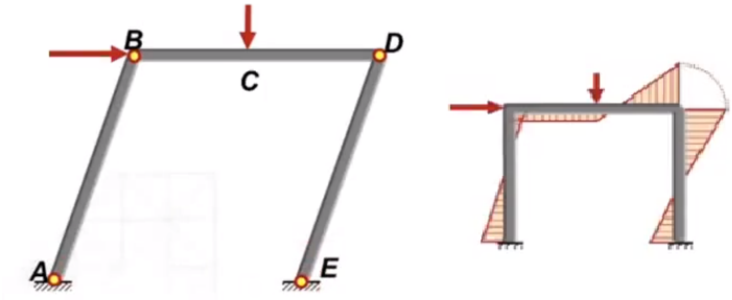
Beam Collapse:





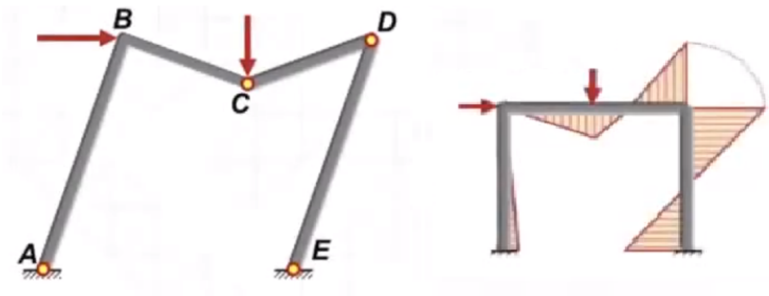
Similar mechanism to fixed-ended beams – hinges form at sections B, C and D. (occurs when vertical load is dominant)

Sway Collapse:



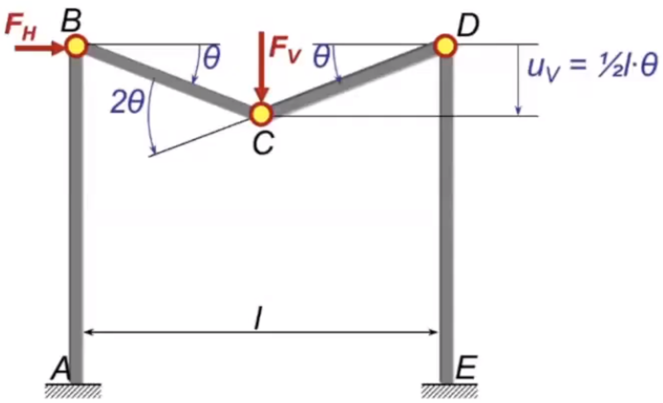
Occurs by overall sway of the frame – hinges form at sections A, B, D and E. (due to a dominant horizontal load)

Combined Beam and Sway Collapse:



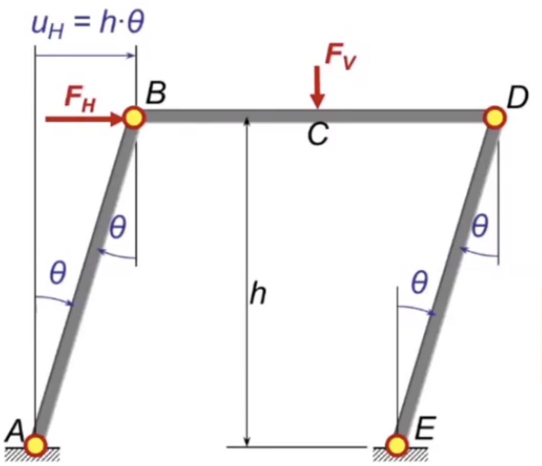
Combined beam and sway collapse – hinges form at sections A, C, D and E and section B keeps elastic. (horizontal and vertical loads comparable)

Beam Collapse:



$$F_V = 8\frac{M_P}{l} \tag{10}$$

Sway Collapse:

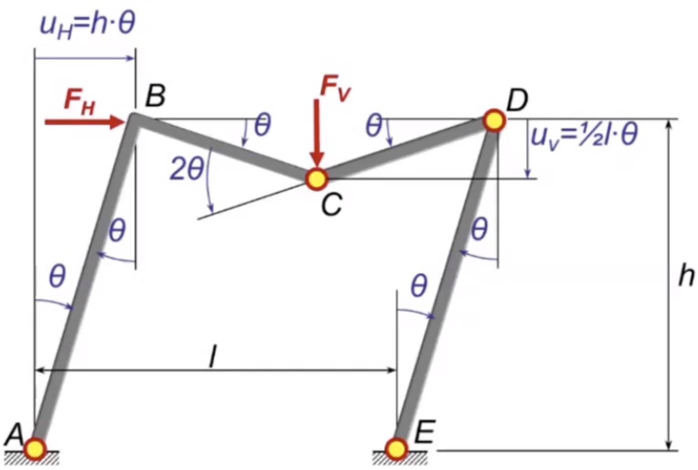


$$F_H \cdot h\theta = M_P\theta + M_P\theta + M_P\theta + M_P\theta \tag{11}$$

$$F_H \cdot h\theta 4M_P\theta \tag{12}$$

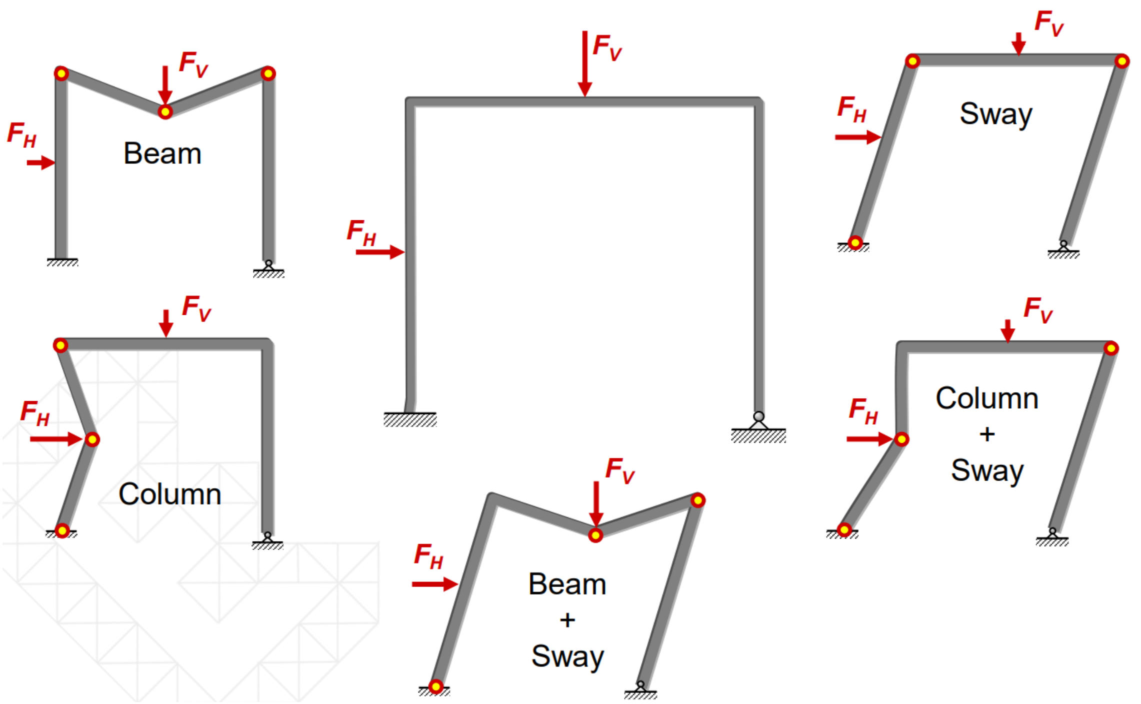
$$F_H = 4\frac{M_P}{h} \tag{13}$$

Combined Collapse:

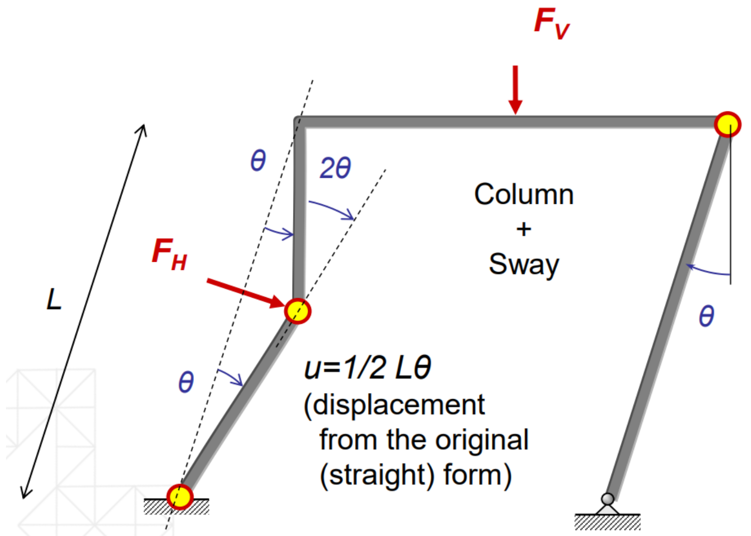


$$F_H h + F_V \frac{l}{2} = 6M_P \tag{14}$$

Example



These are all possible modes of collapse. Find out which mode is the easiest to occur and under what level of load.



Other modes are the same as previous examples.