UCL Mechanical Engineering 2020/2021

ENGF0004 Coursework 2

NCWT3

April 11, 2021

1 Question 1

1.1 a

For the line integral to be independent from the path of integration, the following conditions must be fulfilled:

$$I = \int_A^B \left(\frac{\partial u}{\partial x}\,\mathrm{d}x + \frac{\partial u}{\partial y}\,\mathrm{d}y \right) \tag{1.1}$$

$$P\left(x,\, y\right) = \frac{\partial u}{\partial x} \text{ and } Q\left(x,\, y\right) = \frac{\partial u}{\partial y} \tag{1.2}$$

$$\frac{\partial P\left(x,\, y\right)}{\partial y} = \frac{\partial Q\left(x,\, y\right)}{\partial x} \tag{1.3}$$

Considering the integral:

$$I = \int_A^B \left[e^{-\alpha x y} \left(\frac{\alpha - 2}{x} \right) \mathrm{d}x - \frac{1}{\alpha y} \left(e^{-\alpha x y} - 1 \right) \mathrm{d}y \right] \tag{1.4}$$

$$P\left(x,\, y\right) = e^{-\alpha x y} \left(\frac{\alpha - 2}{x} \right) \text{ and } Q\left(x,\, y\right) = -\frac{1}{\alpha y} \left(e^{-\alpha x y} - 1 \right) \tag{1.5}$$

$$\frac{\partial P\left(x,\, y\right)}{\partial y} = -\alpha x \left(\frac{\alpha - 2}{x} \right) e^{-\alpha x y} = \left(2\alpha - \alpha^2 \right) e^{-\alpha x y} \tag{1.6}$$

$$\frac{\partial Q\left(x,\, y\right)}{\partial x} = -\frac{1}{\alpha y} \left(-\alpha y \right) e^{-\alpha x y} = e^{-\alpha x y} \tag{1.7}$$

$$\therefore 2\alpha e^{-\alpha x y} - \alpha^2 e^{-\alpha x y} = e^{-\alpha x y} \tag{1.8}$$

$$e^{-\alpha x y} \left(\alpha^2 - 2\alpha + 1 \right) = 0 \tag{1.9}$$

$$e^{-\alpha x y} = 0 \rightarrow \text{no solutions} \tag{1.10}$$

$$\left(\alpha - 1 \right)^2 = 0 \tag{1.11}$$

$$\alpha = 1 \tag{1.12}$$

1.2 b

Calculating the line integral of 1.13 from $O\,(0,\,0)$ to $A(1,\,e-1)$ along $y = e^x - 1$:

$$I = \int_O^A \left(ye^{-2x}\right)(\mathrm{d}x + \mathrm{d}y) \tag{1.13}$$

$$y = e^x - 1 \tag{1.14}$$

$$\mathrm{d}y = e^x\,\mathrm{d}x \tag{1.15}$$

$$I = \int_0^1 \left((e^x - 1)\left(e^{-2x}\right) + (e^x - 1)\left(e^{-2x}\right)(e^x)\right)\mathrm{d}x \tag{1.16}$$

$$= \int_0^1 \left(e^{-x} - e^{-x} - e^{-2x} + 1\right)\mathrm{d}x \tag{1.17}$$

$$= \int_0^1 \left(1 - e^{-2x}\right)\mathrm{d}x \tag{1.18}$$

$$= \left[x + \frac{e^{-2x}}{2}\right]_0^1 \tag{1.19}$$

$$= 1 + \frac{e^{-2}}{2} - 0 - \frac{1}{2} \tag{1.20}$$

$$I = \frac{1}{2}\left(e^{-2} + 1\right) \tag{1.21}$$

1.3 c

1.3.1 i

$$\underline{F}\,(x,\,y,\,z) = \begin{pmatrix} \frac{y}{x^2} \\ \frac{x}{y^2} \end{pmatrix} \tag{1.22}$$

$$\nabla \cdot \underline{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \cdot \begin{pmatrix} \frac{y}{x^2} \\ \frac{x}{y^2} \end{pmatrix} \tag{1.23}$$

$$= \frac{\partial}{\partial x}\left(\frac{y}{x^2}\right) + \frac{\partial}{\partial y}\left(\frac{x}{y^2}\right) \tag{1.24}$$

$$= -\frac{2y}{x^3} - \frac{2x}{y^3} \tag{1.25}$$

$$= -2\left(\frac{y}{x^3} + \frac{x}{y^3}\right) \tag{1.26}$$

1.3.2 ii

$$I = \int_1^2 \int_1^2 \left(-2\left(\frac{y}{x^3} + \frac{x}{y^3}\right)\right) \mathrm{d}x\,\mathrm{d}y \tag{1.27}$$

$$= \int_1^2 \left[-2\left(\frac{y}{-2x^2} + \frac{x^2}{2y^3}\right)\right]_1^2 \mathrm{d}y \tag{1.28}$$

$$= \int_1^2 \left[-2\left(-\frac{y}{8} + \frac{2}{y^3} + \frac{y}{2} - \frac{1}{2y^3}\right)\right] \mathrm{d}y \tag{1.29}$$

$$= \int_1^2 \left(-\frac{3y}{4} - \frac{3}{y^3}\right) \mathrm{d}y \tag{1.30}$$

$$= \left[-\frac{3y^2}{8} + \frac{3}{2y^2}\right]_1^2 \tag{1.31}$$

$$= -\frac{3}{2} + \frac{3}{8} + \frac{3}{8} - \frac{3}{2} \tag{1.32}$$

$$I = -\frac{9}{4} \tag{1.33}$$

1.4 d

1.4.1 i

$$I = \int (\sin x \cos y\,\mathrm{d}y + \cos x \sin y\,\mathrm{d}y) \tag{1.34}$$

$$y = 0 \qquad \mathrm{d}y = 0 \tag{1.35}$$

$$I_{AB} = \int_{x=0}^{\pi} (\sin x)\,\mathrm{d}x = [-\cos x]_0^{\pi} = 2 \tag{1.36}$$

$$x = \pi \qquad \mathrm{d}x = 0 \tag{1.37}$$

$$I_{BC} = \int_{y=0}^{\pi} (-\sin y)\,\mathrm{d}y = [\cos y]_0^{\pi} = -2 \tag{1.38}$$

$$\therefore I = I_{AB} + I_{BC} = 2 - 2 = 0 \tag{1.39}$$

1.4.2 ii

$$I = \int (\sin x \cos y\,\mathrm{d}y + \cos x \sin y\,\mathrm{d}y) \tag{1.40}$$

$$y = x \qquad \mathrm{d}y = \mathrm{d}x \tag{1.41}$$

$$I_{AC} = \int_0^{\pi} (\sin x \cos x + \sin x \cos x)\,\mathrm{d}x \tag{1.42}$$

$$= \int_0^{\pi} \big(\sin(2x)\big)\,\mathrm{d}x \tag{1.43}$$

$$I_{AC} = \left[-\frac{1}{2}\cos(2x)\right]_0^{\pi} = \frac{1}{2} - \frac{1}{2} = 0 \tag{1.44}$$

$$\tag{1.45}$$

1.5 e

1.5.1 i

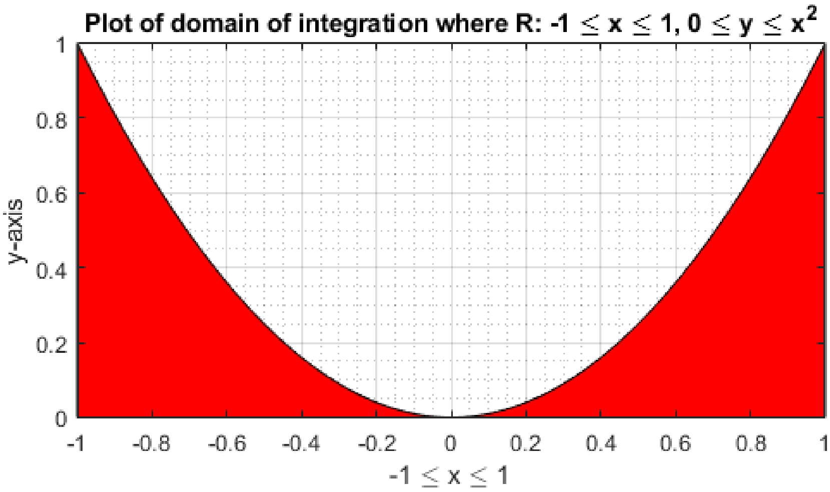

Figure 1: Domain of integration where $R: -1 \leq x \leq 1$, $0 \leq y \leq x^2$.

```
clf
x=-1:0.001:1;
y=x.^2;
A=area(x,y);
set(A(1),'FaceColor','red');
axis('image');
xlabel('-1 \leq x \leq 1')
ylabel('y-axis')
title('Plot of domain of integration where R: -1 \leq x \leq 1, 0 \leq y \leq x^2')
grid on
grid minor
```

1.5.2 ii

1.6 f

1.6.1 ii

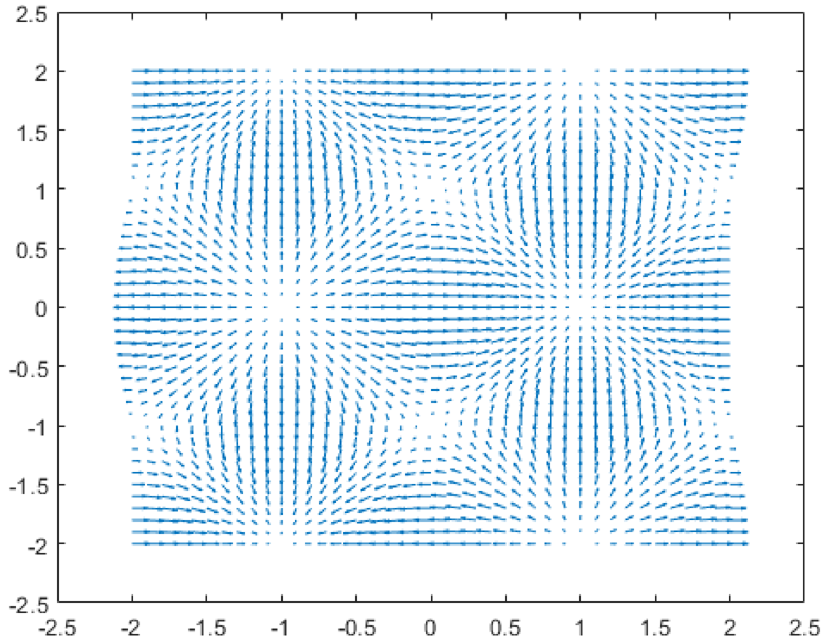

Figure 2:

```
clc
clear
close all

syms x y
z = x*y*exp(-sqrt(x^2 + y^2));
f = (sin((pi/2)*x))*(cos((pi/2)*y));
g = gradient(f,[x,y]);

[X, Y] = meshgrid(-2:0.1:2,-2:0.1:2);
G1 = subs(g(1),[x y],{X,Y});
G2 = subs(g(2),[x y],{X,Y});
quiver(X,Y,G1,G2)
```

2 Question 2

2.1 a

In our series of equations, there are three unknown internal bar forces N_{12}, N_{23}, N_{13}, and three unknown reaction forces, R_{2x}, R_{2y}, R_{3y}. We also have two unknown angles, α and β, and the force F. Given that there are six unknowns that we would like to find and six equations with those variables, the conditions are fulfilled to solve this using matrices. Our answer would be in terms of the variables α, β and F. Values may be assumed, measured or calculated for these.

2.2 b

$$\begin{bmatrix} -\cos\alpha & 0 & \cos\beta & 0 & 0 & 0 \\ -\sin\alpha & 0 & -\sin\beta & 0 & 0 & 0 \\ \cos\alpha & 1 & 0 & 1 & 0 & 0 \\ \sin\alpha & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -\cos\beta & 0 & 0 & 0 \\ 0 & 0 & \sin\beta & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_{12} \\ N_{23} \\ N_{13} \\ R_{2x} \\ R_{2y} \\ R_{3y} \end{bmatrix} = \begin{bmatrix} 0 \\ F \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.1}$$

2.3 c

```
clc
clear
close all

alpha = pi/3;
beta = pi/6;
F = 1000;

A = [-cos(alpha) 0 cos(beta) 0 0 0;
     -sin(alpha) 0 -sin(beta) 0 0 0;
     cos(alpha) 1 0 1 0 0;
     sin(alpha) 0 0 0 1 0;
     0 -1 -cos(beta) 0 0 0;
     0 0 sin(beta) 0 0 1];
B = [0; F; 0; 0; 0; 0];

sol = A\B;
```

This returned the following:

$$\begin{bmatrix} N_{12} \\ N_{23} \\ N_{13} \\ R_{2x} \\ R_{2y} \\ R_{3y} \end{bmatrix} = \begin{bmatrix} -866.0254 \\ 433.0127 \\ -500.0000 \\ 0 \\ 750.0000 \\ 250.0000 \end{bmatrix} \tag{2.2}$$

2.4 d

```
clc
clear
close all

alpha = pi/3;
beta = pi/6;
```

```
F = 1000;

A = [-cos(alpha) 0 cos(beta) 0 0 0;
     -sin(alpha) 0 -sin(beta) 0 0 0;
     cos(alpha) 1 0 1 0 0;
     sin(alpha) 0 0 0 1 0;
     0 -1 -cos(beta) 0 0 0;
     0 0 sin(beta) 0 0 1];
B = [0; F; 0; 0; 0; 0];

[L,U] = lu(A); %splits matrix A such that A = L*U, L is lower triangular,
     U is upper triangular
y = L\B;
sol = U\y;
```

This returned the following:

$$\begin{bmatrix} N_{12} \\ N_{23} \\ N_{13} \\ R_{2x} \\ R_{2y} \\ R_{3y} \end{bmatrix} = \begin{bmatrix} -866.0254 \\ 433.0127 \\ -500.0000 \\ 0 \\ 750.0000 \\ 250.0000 \end{bmatrix} \tag{2.3}$$