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0.1 Control-volume approach

Recall from thermodynamics th at $\dot{m}_{in} = \dot{m}_{out}$ for a control volume (A c.v. allows heat, work and mass transfer across its boundary). In fluid dynamics different terminology is usually used:

- Closed system = control mass = system (in fluid mechanics).
- Open system = control volume.

We also know that $\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out}$ from thermodynamics. For a *system*, $\left(\frac{dm}{dt}\right)_{sys} = 0$ since a system is a control mass.

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0.2 Laminar flow and boundary layers

0.2.1 Boundary conditions

You will generally only come across two types of boundary conditions:

1. No-slip condition: at the point where the fluid touches the boundary, the velocity of the fluid along the boundary is zero. This is the case where the boundary is a solid.
2. Free surface: there are no viscous forces on the fluid where it touches the boundary, so the speed can take any value. This is the case where there is a liquid-gas boundary and may be called (e.g. for a stream) *open-channel flow.*

0.2.2 Laminar flow calculations

Consider laminar flow between two flat plates. Split the flow into sheets of thickness d_{in}. The top plate moves at constant speed v and the lower plate is stationary. Here boundary condition (1) applies i.e. fluid at the boundary moves at the speed of the boundary. The top plate has an area A and the sideways force being applied to it is $F \therefore \tau = \frac{F}{A}$. If the flow is steady, the forces must balance for each layer. However, the liquid layers must transmit the stress so τ is constant throughout the depth. Let us apply our boundary conditions:

$$u_y(0) = 0 \tag{1}$$

$$u_y(D) = v \tag{2}$$

where D is the vertical distance between the upper and lower plate.

$$\therefore \tau = \mu \frac{du_y}{dx}$$

$$\left.\begin{aligned} \tfrac{\tau}{\mu} dx &= du_y \\ \tfrac{\tau}{\mu} \int dx &= \int du_y \\ k + \tfrac{\tau}{\mu} x &= u_y \end{aligned}\right\} \begin{array}{l} u_y = \frac{\tau}{\mu} x + k \\ 0 = \frac{\tau}{\mu} \cdot 0 + k \therefore k = 0 \\ \text{also } \frac{\tau}{\mu} = \frac{v}{D} \to \tau = \frac{v}{D}\mu \end{array}$$

$$\therefore u_y = \frac{v}{D} x$$

0.2.3 Laminar flow between solid boundaries

In laminar flow, individual particles of fluid follow paths that do not cross those of neighbouring particles. However, there is still a velocity gradient across the flow. Laminar flow is not normally found except in the neighbourhood of a solid boundary, the retarding effect of which causes the transverse velocity gradient.

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)$$

Where τ is the resultant shear stress, μ is the dynamic viscosity and $\left(\frac{\partial u}{\partial y} \right)$ is the rate at which the velocity u increases with the coordinate y perpendicular to the velocity.