0.1 Fluids formulas

0.1.1 Density

$$v = \frac{1}{\rho}$$

0.1.2 Specific weight

$$\gamma = \rho g$$

0.1.3 Specific gravity

$$SG = \frac{\rho}{\rho_{H_2O@4^\circ C}}$$

0.1.4 Ideal gas law

$$P = \rho RT$$
$$Pv = RT$$
$$PV = nR_uT$$

0.1.5 Kinematic viscosity

$$v = \frac{\mu}{\rho}$$

0.1.6 Surface tension

$$\sigma = \frac{F}{L}$$

0.1.7 Capillary action

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

0.1.8 Centre of mass

$$\begin{aligned}
\bar{x} &= \frac{\int_m (x) dm}{m} \\
&= \frac{\int_m (x) dm}{\int_m dm}
\end{aligned}$$

0.1.9 Stable equilibrium

The centre of gravity is directly below the centre of buoyancy.

0.1.10 Reynolds Number

$$Re = \frac{\rho L u}{\mu}$$

$$Re < 2000 \rightarrow \text{ Laminar}$$

$$Re > 2000 \rightarrow \text{ Turbulent}$$

0.1.11 Material derivative

$$\begin{aligned}
\frac{D}{Dt}() &= \frac{\partial}{\partial t}() + u\frac{\partial ()}{\partial x} + v\frac{\partial ()}{\partial y} + w\frac{\partial ()}{\partial z} \\
\underline{a} = \frac{D}{Dt}(\underline{v}) &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}
\end{aligned}$$

0.1.12 Streamline coordinates

$$\begin{aligned}
\underline{a} = \frac{Dv}{Dt} &= a_s \hat{s} + a_n \hat{n} \\
&= v\frac{\partial v}{\partial s}\hat{s} + \frac{v^2}{R}\hat{n} \\
a_s &= \frac{-v \sin\theta - \frac{\partial P}{\partial s}}{\rho} = v\frac{\partial v}{\partial s} \\
a_n &= \frac{-v \cos\theta - \frac{\partial P}{\partial n}}{\rho} = \frac{v^2}{R}
\end{aligned}$$

0.1.13 RTT

$$\left(\frac{DB}{Dt}\right)_{sys} = \frac{\partial}{\partial t}\int_{cv}(\rho b)d\forall + \int_{cs}(\rho b(\underline{v}\cdot\underline{\hat{n}}))dA$$
$$\text{When B=M} \rightarrow 0 = \frac{\partial}{\partial t}\int_{cv}(\rho)d\forall + \int_{cs}(\rho(\underline{v}\cdot\underline{\hat{n}}))dA$$
$$\text{When } \underline{\text{B}} = \text{m}\underline{\text{v}} \rightarrow \sum F_{sys} = \frac{\partial}{\partial t}\int_{cv}(\rho\underline{v})d\forall + \int_{cs}(\rho\underline{v}(\underline{v}\cdot\underline{\hat{n}}))dA$$

0.1.14 Propeller stages

1. Bernoulli equation between 1 and 2.
$$P_1 = P_2 + 0.5\rho(v_2^2 - v_1^2)$$

2. Bernoulli equation between 3 and 4.
$$P_4 = P_3 + 0.5\rho(v_3^2 - v_4^2)$$

3. $P_1 = P_4$.
$$\therefore \Delta P = P_3 - P_2 = 0.5\rho(v_4^2 - v_1^2)$$

4. $F = \Delta P \times A$.
$$\therefore F_{thrust} = \Delta P \times A = 0.5\rho A(v_4^2 - v_1^2)$$

5. Apply momentum equation.
$$\begin{aligned}\sum F_x &= \int_{inlet}\rho v_1(-v_1)dA + \int_{outlet}\rho v_4(v_4)dA \\ &= -\dot{m}_1 v_1 + \dot{m}_2 v_4 \\ &= \dot{m}(v_4 - v_1) \text{ (assume steady state, no viscous force, neglect g)}\end{aligned}$$

6. At the propeller, $\dot{m} = \rho A_P v_p = \rho A v_p$.

$$\therefore F_{\text{thrust}} = \rho A v_P (v_4 - v_1)$$

7. $0.5\rho A(v_4^2 - v_1^2) = \rho A v_p (v_4 - v_1)$.

$$\therefore v_P = 0.5(v_4 + v_1)$$

8. Work out v_P and use it to work out thrust.

9. $P = F \times v$.

0.1.15 Moving control volume

$$\underline{v} = \underline{v}_{CV} + \underline{W}$$

Where $\underline{v}$ is the absolute velocity, $\underline{v}_{CV}$ is the velocity of the control volume and $\underline{W}$ is the velocity relative to the control volume.

Conservation of mass

$$0 = \frac{\partial}{\partial t} \int_{CV} (\rho) \, d\forall + \int_{CV} (\rho(\underline{W} \cdot \underline{\hat{n}})) \, dA$$

Conservation of momentum

$$\sum \underline{F}_{sys} = \int_{CV} (\rho \underline{W}(\underline{W} \cdot \underline{\hat{n}})) \, dA$$

When steady state and in an inertial reference frame.

0.1.16 Energy equation

$$\left[\sum \dot{Q}_{\text{net in}} + \sum \dot{W}_{\text{net in}}\right]_{CV} = \frac{\partial}{\partial t} \int_{CV} (e\rho) \, d\forall + \int (e\rho(\underline{v} \cdot \underline{\hat{n}})) \, dA$$

Or,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \frac{\partial}{\partial t} \int_{CV} (e\rho) \, d\forall + \int_{CS} \left(u + \frac{P}{\rho} + \frac{v^2}{2} + gz\right) \rho(\underline{v} \cdot \underline{\hat{n}}) dA$$

When steady flow and only one stream,

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} = \\ \dot{m}\left[u_{out} - u_{in} + \left(\frac{P}{\rho}\right)_{out} - \left(\frac{P}{\rho}\right)_{in} + \frac{v_{out}^2 - v_{in}^2}{2} + g(z_{out} - z_{in})\right]$$

In the Bernoulli form:

$$\frac{P_{out}}{\rho} + \frac{v_{out}^2}{2} + gz_{out} = \frac{P_{in}}{\rho} + \frac{v_{in}^2}{2} + gz_{in} + W_{\text{shaft net in}} - \text{loss, which is:}$$
$$\text{loss} = u_{out} - uin - q_{\text{net in}}$$

0.1.17 Shaft work efficiency

$$\eta = \frac{W_{\text{shaft net in}} - \text{loss}}{W_{shaftnetin}} \text{Where loss} \qquad = u_{out} - u_{in} - q_{\text{net in}}$$

0.1.18 Laminar flow and boundary layers

$$u_y(0) = 0$$
$$u_y(D) = v$$
$$\tau = \mu\frac{du_y}{dx}$$
$$\frac{\tau}{\mu}\int du = \int du_y$$
$$\frac{\tau}{\mu}x + k = u_y$$
$$\text{When } x = 0,\ u_y = 0 \rightarrow k = 0$$
$$\therefore u_y = \frac{\tau}{\mu}x$$
$$u_y = \frac{v}{D}x$$

0.1.19 Flow through porous media

$$\overline{u} = -C\frac{\partial P}{\partial x}$$

0.1.20 Newtonian fluids

$$\overline{u} \propto \frac{1}{\mu}$$

0.1.21 Hagen-Poiseulle law

$$\begin{aligned}
\Delta P &= \frac{dP}{dx} L \\
\tau &= \mu \frac{du_y}{dx} = \mu \frac{du_y}{dr} \\
F &= \tau \cdot 2\pi r \cdot L \\
F &= \left(\frac{dP}{dx} \cdot L \right) \pi r^2 \\
u &= -\frac{1}{4\mu} \frac{dP}{dx} (R^2 - r^2) \\
u_{mean} = \overline{u} &= \frac{Q}{\text{Area}} = -\frac{R^2}{8\mu} \frac{dP}{dx} \\
u_{mean} &= -\frac{1}{4\mu} \frac{dP}{dx} R^2 \\
Q &= -\frac{\pi R^4}{8\mu} \frac{dP}{dx} \\
\text{note } u_{mean} &= \overline{u} = \frac{u_{max}}{2}
\end{aligned}$$

0.1.22 Boundary layer

- Laminar region, thickness increases as $x^{0.5}$.
- Turbulent region, thickness increases as $x^{0.8}$
- Boundary is taken to be at the contour where the velocity is 99% of main flow.

0.1.23 Displacement thickness

$$S^* = \int_0^\infty \left(1 - \frac{u}{u_m} \right) dy$$

0.1.24 Momentum thickness

$$\theta = \int_0^\infty \frac{u}{u_m}\left(1 - \frac{u}{u_m}\right) dy$$