

# UCL Mechanical Engineering 2020/2021

## MECH0011 Final Coursework

NCWT3

May 19, 2021

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## 1 Question 1

### 1.1 a

The data was imported into MATLAB and the shape of the hydrofoil, the chord line and the mean camber line were plotted for all four hydrofoils.

```

1  clc
2  clear
3  close all
4
5  %define vars
6  i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS
      930 Hydrofoil"]; %index hydrofoil names from sheets for ease
7  data = zeros(122,2,4); %initialise matrix
8  counter = 0; %initialise counter
9  x = linspace(0,1,100); %interpolation range initialisation
10
11 %import data
12 for j = 1:4 %index all data for plots
13     counter = counter + 1; %increment counter
14     data(:, :, counter) = readmatrix('suppFiles.xlsx', 'Sheet', i(j), 'Range',
        'A3:B124'); %loop through sheets and pull data
15 end
16
17 %camber line calculation
18 %pull positive and negative coordinate points
19 %eppler
20 dataPos1 = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'A3:B37');
21 dataNeg1 = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'A38:B70');
22
23 %naca
24 dataPos2 = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'A3:B28');
25 dataNeg2 = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'A29:B54');
26
27 %rg

```

```

28 dataPos3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A3:B34');
29 dataNeg3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A35:B64');
30
31 %ys
32 dataPos4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A3:B65');
33 dataNeg4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A66:B124');
34
35 %interpolate hydrofoil shape with 100 data points from 0 to 1
36 %eppler
37 dataIntPos1 = interp1(dataPos1(:,1), dataPos1(:,2), x);
38 dataIntNeg1 = interp1(dataNeg1(:,1), dataNeg1(:,2), x);
39
40 %naca
41 dataIntPos2 = interp1(dataPos2(:,1), dataPos2(:,2), x);
42 dataIntNeg2 = interp1(dataNeg2(:,1), dataNeg2(:,2), x);
43
44 %rg
45 dataIntPos3 = interp1(dataPos3(:,1), dataPos3(:,2), x);
46 dataIntNeg3 = interp1(dataNeg3(:,1), dataNeg3(:,2), x);
47
48 %ys
49 dataIntPos4 = interp1(dataPos4(:,1), dataPos4(:,2), x);
50 dataIntNeg4 = interp1(dataNeg4(:,1), dataNeg4(:,2), x);
51
52 %calculate camber line
53 %eppler
54 camber1 = (dataIntPos1 + dataIntNeg1)./2;
55
56 %naca
57 camber2 = (dataIntPos2 + dataIntNeg2)./2;
58
59 %rg
60 camber3 = (dataIntPos3 + dataIntNeg3)./2;
61
62 %ys
63 camber4 = (dataIntPos4 + dataIntNeg4)./2;
64
65 %plot data
66 subplot(4,1,1)
67 plot(dataPos1(:,1), dataPos1(:,2), 'b', dataNeg1(:,1), dataNeg1(:,2), 'b', x
    , camber1, 'r')
68 axis image
69 grid on
70 xlabel('Chord')
71 ylabel('Z(x)')
72 title('Plot of ' + i(1))
73 xlim([-0.05 1.05])
74 ylim([-0.05 0.1])
75 legend('Hydrofoil profile','Mean camber line')
76
77 subplot(4,1,2)
78 plot(dataPos2(:,1), dataPos2(:,2), 'b', dataNeg2(:,1), dataNeg2(:,2), 'b', x
    , camber2, 'r')

```

```

79 axis image
80 grid on
81 xlabel( 'Chord' )
82 ylabel( 'Z(x)' )
83 title( 'Plot of ' + i(2) )
84 xlim([ -0.05 1.05 ])
85 ylim([ -0.05 0.1 ])
86 legend( 'Hydrofoil profile', 'Mean camber line' )
87
88 subplot(4,1,3)
89 plot( dataPos3(:,1), dataPos3(:,2), 'b', dataNeg3(:,1), dataNeg3(:,2), 'b', x
      , camber3, 'r' )
90 axis image
91 grid on
92 xlabel( 'Chord' )
93 ylabel( 'Z(x)' )
94 title( 'Plot of ' + i(3) )
95 xlim([ -0.05 1.05 ])
96 ylim([ -0.05 0.1 ])
97 legend( 'Hydrofoil profile', 'Mean camber line' )
98
99 subplot(4,1,4)
100 plot( dataPos4(:,1), dataPos4(:,2), 'b', dataNeg4(:,1), dataNeg4(:,2), 'b', x
      , camber4, 'r' )
101 axis image
102 grid on
103 xlabel( 'Chord' )
104 ylabel( 'Z(x)' )
105 title( 'Plot of ' + i(4) )
106 xlim([ -0.05 1.05 ])
107 ylim([ -0.05 0.1 ])
108 legend( 'Hydrofoil profile', 'Mean camber line' )

```

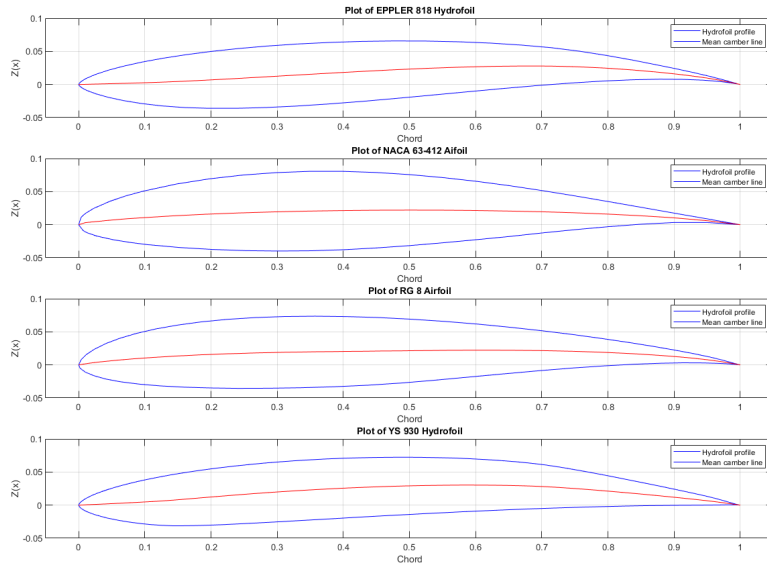


Figure 1: Graphs to show hydrofoil shape, chord line and mean camber line for four different hydrofoils.

## 1.2 b

MATLAB was used to calculate the lift-to-drag ratio for each hydrofoil.

```

1  clc
2  clear
3  close all
4
5  %define vars
6  i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS
      930 Hydrofoil"]; %index hydrofoil names from sheets for ease
7
8  %pull cl and cd from data
9  %eppler
10 epplerCL = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'E2:E79');
11 epplerCD = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'F2:F79');
12
13 %naca
14 nacaCL = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'E2:E107');
15 nacaCD = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'F2:F107');
16
17 %rg
18 rgCL = readmatrix('suppFiles.xlsx', 'Sheet', i(3), 'Range', 'E2:E101');
19 rgCD = readmatrix('suppFiles.xlsx', 'Sheet', i(3), 'Range', 'F2:F101');
20
21 %ys
22 ysCL = readmatrix('suppFiles.xlsx', 'Sheet', i(4), 'Range', 'E2:E78');
23 ysCD = readmatrix('suppFiles.xlsx', 'Sheet', i(4), 'Range', 'F2:F78');
24
25 %plot data

```

```

26 plot(epplerCL, epplerCD, 'r', nacaCL, nacaCD, 'g', rgCL, rgCD, 'b', ysCL, ysCD, '
    magenta')
27 legend(i(1), i(2), i(3), i(4))
28 axis square
29 grid on
30 xlabel('C_L')
31 ylabel('C_D')
32 title('Plot of lift-to-drag ratio for ' + i(4))

```

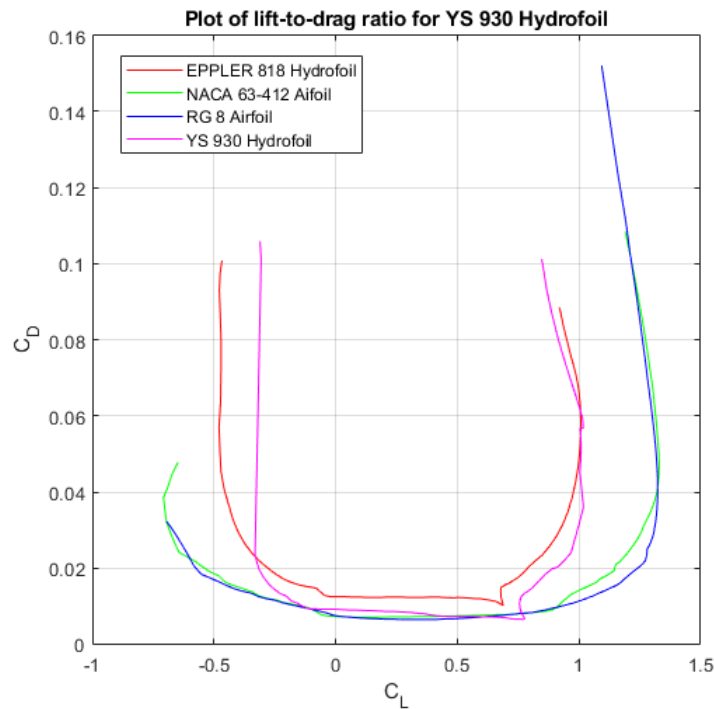


Figure 2: Graph to show lift-to-drag ratio for four different hydrofoils.

### 1.3 c

At a low angle of attack, the flow is attached to the hydrofoil and separates close to the trailing edge, leaving a small wake. The streamlined shape of the hydrofoil exerts a large shear force on the flow passing over it, leading to a large skin drag but low form drag. As we increase the angle of attack, the flow separation moves along the top of the hydrofoil. This decreases the shear force acting on the hydrofoil and the skin drag starts to reduce, with form drag increasing. At the stall angle, the pressure distribution along the top of the hydrofoil dramatically changes due to flow separation, suction pressure is mostly/all lost and a large wake with turbulent effects is present. The primary drag effect here is now form drag, as the flow is not attached along the length of the hydrofoil surface.

### 1.4 d

MATLAB was used to calculate each of the variables for each hydrofoil.

```

1 clc
2 clear

```

```

3  close all
4
5  %define vars
6  i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS
      930 Hydrofoil"]; %index hydrofoil names from sheets for ease
7  x = linspace(0,1,100); %interpolation range initialisation
8
9  %amax percentage camber line calculation
10 %camber line calculation
11 %pull positive and negative coordinate points
12 %eppler
13 dataPos1 = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'A3:B37');
14 dataNeg1 = readmatrix('suppFiles.xlsx', 'Sheet', i(1), 'Range', 'A38:B70');
15
16 %naca
17 dataPos2 = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'A3:B28');
18 dataNeg2 = readmatrix('suppFiles.xlsx', 'Sheet', i(2), 'Range', 'A29:B54');
19
20 %rg
21 dataPos3 = readmatrix('suppFiles.xlsx', 'Sheet', i(3), 'Range', 'A3:B34');
22 dataNeg3 = readmatrix('suppFiles.xlsx', 'Sheet', i(3), 'Range', 'A35:B64');
23
24 %ys
25 dataPos4 = readmatrix('suppFiles.xlsx', 'Sheet', i(4), 'Range', 'A3:B65');
26 dataNeg4 = readmatrix('suppFiles.xlsx', 'Sheet', i(4), 'Range', 'A66:B124');
27
28 %interpolate hydrofoil shape with 100 data points from 0 to 1
29 %eppler
30 dataIntPos1 = interp1(dataPos1(:,1), dataPos1(:,2), x);
31 dataIntNeg1 = interp1(dataNeg1(:,1), dataNeg1(:,2), x);
32
33 %naca
34 dataIntPos2 = interp1(dataPos2(:,1), dataPos2(:,2), x);
35 dataIntNeg2 = interp1(dataNeg2(:,1), dataNeg2(:,2), x);
36
37 %rg
38 dataIntPos3 = interp1(dataPos3(:,1), dataPos3(:,2), x);
39 dataIntNeg3 = interp1(dataNeg3(:,1), dataNeg3(:,2), x);
40
41 %ys
42 dataIntPos4 = interp1(dataPos4(:,1), dataPos4(:,2), x);
43 dataIntNeg4 = interp1(dataNeg4(:,1), dataNeg4(:,2), x);
44
45 %calculate camber line
46 %eppler
47 camber1 = (dataIntPos1 + dataIntNeg1)./2;
48
49 %naca
50 camber2 = (dataIntPos2 + dataIntNeg2)./2;
51
52 %rg
53 camber3 = (dataIntPos3 + dataIntNeg3)./2;
54

```

```

55 %ys
56 camber4 = (dataIntPos4 + dataIntNeg4)./2;
57
58 %maximum camber per hydrofoil
59 %eppler
60 percCamber1 = max(camber1);
61
62 %naca
63 percCamber2 = max(camber2);
64
65 %rg
66 percCamber3 = max(camber3);
67
68 %ys
69 percCamber4 = max(camber4);
70
71 %clean-up output
72 percCamber = 100.*[percCamber1 percCamber2 percCamber3 percCamber4];
73
74 %maximum percentage thickness calculation
75 %eppler
76 thickness1 = (abs(dataIntPos1) + abs(dataIntNeg1));
77
78 %naca
79 thickness2 = (abs(dataIntPos2) + abs(dataIntNeg2));
80
81 %rg
82 thickness3 = (abs(dataIntPos3) + abs(dataIntNeg3));
83
84 %ys
85 thickness4 = (abs(dataIntPos4) + abs(dataIntNeg4));
86
87 %max thickness per hydrofoil
88 %eppler
89 maxThick1 = max(thickness1);
90
91 %naca
92 maxThick2 = max(thickness2);
93
94 %rg
95 maxThick3 = max(thickness3);
96
97 %ys
98 maxThick4 = max(thickness4);
99
100 %clean-up output
101 maxThick = 100.*[maxThick1 maxThick2 maxThick3 maxThick4];
102
103 %maximum lift coefficient
104 %pull angle of attack, cl and cd from data
105 %eppler
106 epplerData = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','D2:F79');
107

```



```

108 %naca
109 nacaData = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','D2:F107');
110
111 %rg
112 rgData = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','D2:F101');
113
114 %ys
115 ysData = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','D2:F78');
116
117 %find max cl
118 %eppler
119 maxEpplerCL = max(epplerData(:,2));
120
121 %naca
122 maxNacaCL = max(nacaData(:,2));
123
124 %rg
125 maxRgCL = max(rgData(:,2));
126
127 %ys
128 maxYsCL = max(ysData(:,2));
129
130 %clean-up output
131 maxCL = [maxEpplerCL maxNacaCL maxRgCL maxYsCL];
132
133 %find angle of attack at max cl
134 %eppler
135 critEppler = epplerData(epplerData(:,2) == maxEpplerCL,1);
136
137 %naca
138 critNaca = nacaData(nacaData(:,2) == maxNacaCL,1);
139
140 %rg
141 critRg = rgData(rgData(:,2) == maxRgCL,1);
142
143 %ys SPECIAL CASE VALUE ASCERTAINED EMPIRICALLY
144 critYs = 9;
145
146 %clean-up output
147 crit = [critEppler critNaca critRg critYs];
148
149 %lift coefficient for alpha = 0
150 %eppler
151 liftAlpha0Eppler = epplerData(epplerData(:,1) == 0,2);
152
153 %naca
154 liftAlpha0Naca = nacaData(nacaData(:,1) == 0,2);
155
156 %rg
157 liftAlpha0Rg = rgData(rgData(:,1) == 0,2);
158
159 %ys
160 liftAlpha0Ys = ysData(ysData(:,1) == 0,2);

```

```

161
162 %clean-up output
163 liftAlpha0 = [liftAlpha0Eppler liftAlpha0Naca liftAlpha0Rg liftAlpha0Ys];
164
165 %angle of attack corresponding to cl = 0
166 %find min cl
167 %eppler
168 minEpplerCL = min(abs(epplerData(:,2)));
169
170 %naca
171 minNacaCL = min(abs(nacaData(:,2)));
172
173 %rg
174 minRgCL = min(abs(rgData(:,2)));
175
176 %ys
177 minYsCL = min(abs(ysData(:,2)));
178
179 %find angle of attack at min cl
180 %eppler
181 AOAMinEppler = epplerData(epplerData(:,2) == minEpplerCL,1);
182
183 %naca
184 AOAMinNaca = nacaData(nacaData(:,2) == -minNacaCL,1);
185
186 %rg
187 AOAMinRg = rgData(rgData(:,2) == -minRgCL,1);
188
189 %ys
190 AOAMinYs = ysData(ysData(:,2) == -minYsCL,1);
191
192 %clean-up output
193 AOAMin = [AOAMinEppler AOAMinNaca AOAMinRg AOAMinYs];
194
195 %generate table
196 T = table(i', percCamber', maxThick', maxCL', crit', liftAlpha0', AOAMin
    ');

```

Hydrofoil	Maximum		
	% camber	% thickness	lift coefficient
EPPLER 818 Hydrofoil	2.792	9.362	1.008
NACA 63-412 Aifoil	2.204	11.992	1.330
RG 8 Airfoil	2.226	10.795	1.323
YS 930 Hydrofoil	3.028	9.088	1.018

Table 1: Table to show maximum percentage camber and thickness and the maximum lift coefficient for four hydrofoils.

Hydrofoil	Stall angle	Lift coefficient for $\alpha = 0^\circ$	Angle of attack $\alpha_0$ corresponding to $C_L = 0$
EPPLER 818 Hydrofoil	$7.75^\circ$	0.361	$-3^\circ$
NACA 63-412 Aifoil	$13^\circ$	0.338	$-3^\circ$
RG 8 Airfoil	$12.75^\circ$	0.382	$-3^\circ$
YS 930 Hydrofoil	$9^\circ$	0.391	$-3.75^\circ$

Table 2: Table to show the stall angle, lift coefficient for  $\alpha = 0^\circ$  and the angle of attack  $\alpha_0$  corresponding to  $C_L = 0$  for four hydrofoils.

The hydrofoils both have a higher percentage camber than the airfoils. They also have a lower percentage thickness than the airfoils. We can see that this leads to a reduction in the stall angle and subsequently their maximum lift coefficients are lower. However, we can see that they perform better at  $\alpha = 0^\circ$ . This can be attributed to the fact that at small  $\alpha$ , larger camber generates more lift, at the expense of a smaller stall angle, as the boundary layer is more prone to separation at higher  $\alpha$ . However, increasing either percentage camber or thickness will increase  $C_D$ .

## 1.5 e

The RG 8 Airfoil was selected as it has a high stall angle, with a subsequently large maximum lift coefficient, the lift coefficient at  $\alpha = 0^\circ$  is also the second highest. We can calculate the lift on the hydrofoil using 1.1:

$$L = \frac{1}{2} \rho C_L V_w^2 A \quad (1.1)$$

where  $\rho$  is the density of seawater,  $C_L$  is the lift coefficient for a given angle of attack,  $V_w$  is the velocity of the fluid relative to the wing and  $A$  is the projected surface area of the wing. The surface area  $A$  can be found by integration:

$$A = \int_{x_0}^{b+x_0} (-0.07x + c_0) dx \quad (1.2)$$

$$A = \left[ -0.035x^2 + c_0x \right]_{x_0}^{b+x_0} \quad (1.3)$$

$x_0 = 0.1$  as given in the material. I have also selected  $b = 2$  m, hence:

$$A = -0.035(2.1)^2 + c_0(2.1) + 0.035(0.1)^2 - c_0(0.1) \quad (1.4)$$

$$A = 2c_0 - 0.154 \quad (1.5)$$

Substituting 1.5 into 1.1:

$$L = \frac{1}{2} \rho C_L V_w^2 (2c_0 - 0.154) \quad (1.6)$$

Rearranging for  $c_0$ :

$$(2c_0 - 0.154) = \frac{L}{\frac{1}{2} \rho C_L V_w^2} \quad (1.7)$$

$$c_0 = \frac{L}{\rho C_L V_w^2} + 0.077 \quad (1.8)$$

The following values were used:  $\rho = 1036 \text{ kg m}^{-3}$  [1],  $V_w = 11 \text{ m s}^{-1}$  ( $39.6 \text{ km h}^{-1}$ ),  $L = 2000 \times 9.81 \approx 20000 \text{ N}$ . MATLAB was used to plot the chord length against the angle of attack. As  $V_w$  represents the upper speed limit of the boat, we want to select a low angle of attack, to reduce the amount of drag on the foil. Hence, the value for  $C_L$  at  $\alpha = 0$  will be used.

```

1  clc
2  clear
3  close all
4
5  %define vars
6  i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS
      930 Hydrofoil"]; %index hydrofoil names from sheets for ease
7  Vw = 11; %39.6km/h
8  VwLower = 6; %21.6km/h
9
10 %pull angle of attack, cl and cd from data
11 %rg
12 rgData = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','D2:F101');
13
14 %chord length equation
15 RGChord = ((20000)./(1036.*(Vw^2).*(rgData(rgData(:,1) == 0,2)))) +
      0.077;%finds CL at alpha = 0 and inputs into equation and solves
16 L = 0.5*1036.*(Vw^2).*(rgData(rgData(:,1) == 0,2)).*((2.*RGChord)-0.154)
      ; %check
17 LLower = 0.5*1036.*(VwLower^2).*(rgData(rgData(:,1) == 11,2)).*((2.*
      RGChord)-0.154);%alpha = 11 degrees

```

Our chord length is calculated to be  $c_0 = 0.4948$  m. We must now check whether this chord length is sufficient to lift the boat at lower speeds. In order to do this we require a high angle of attack, to generate more lift. The value for  $C_L$  at  $11^\circ$  was used to test the lift at  $6 \text{ m s}^{-1}$  ( $21.6 \text{ km h}^{-1}$ ) and MATLAB gives a lift value of 20215 N, allowing the boat to be out of the water at lower speeds as well.

## 2 Question 2

## 3 Question 3

### 3.1 a

MATLAB was used to plot the boundary layer velocity profile.

```

1  clc
2  clear
3  close all
4
5  %import data
6  data = readmatrix('suppFiles.xlsx','Sheet','Boundary Layer','Range','A2:
      B102');
7
8  %plot data
9  plot(data(:,2), data(:,1))
10 axis image
11 grid on
12 xlim([0 1])
13 ylim([0 1])

```

```

14 xlabel('y/\delta')
15 ylabel('u/U')
16 title('Graph to show the boundary layer velocity profile')

```

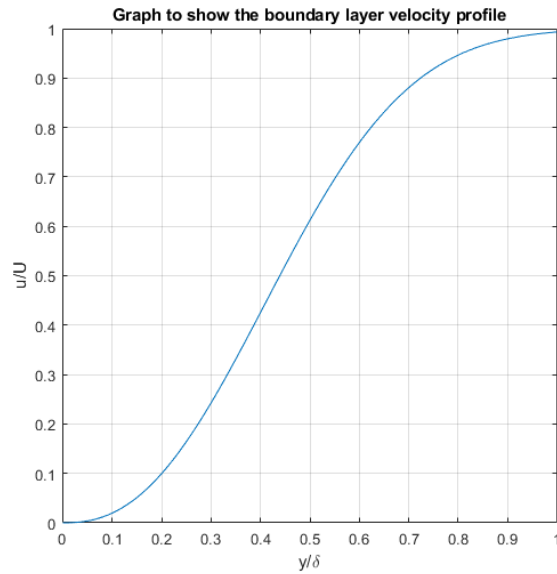
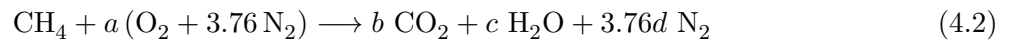
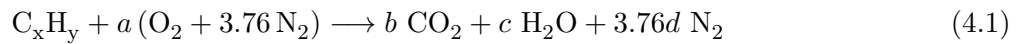


Figure 3: Graph to show boundary layer velocity profile.

## 4 Question 4

### 4.1 a

The balanced reaction equation takes the general form:



where  $x = 1$  and  $y = 4$ . Balancing atoms:

$$\text{C} : x = b \quad (4.3)$$

$$\text{H} : y = 2c \quad (4.4)$$

$$\text{O} : 2a = 2b + c \quad (4.5)$$

$$\text{N} : 2 \times 3.76a = 2 \times 3.76d \quad (4.6)$$

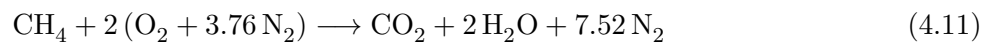
$$\therefore a = x + \frac{y}{4} = 2 \quad (4.7)$$

$$b = x = 1 \quad (4.8)$$

$$c = \frac{y}{2} = 2 \quad (4.9)$$

$$d = a = 2 \quad (4.10)$$

Therefore our balanced reaction equation is:



## 4.2 b

The air-fuel ratio on a mass basis can be found using 4.12

$$AF = \bar{A}F \times \frac{M_{air}}{M_{fuel}} \quad (4.12)$$

Where  $\bar{A}F$  is volumetric air ratio and  $M$  is the molar mass. Assuming that the air only consists of oxygen and nitrogen, we can input molar masses from the property tables into the 4.12:

$$AF = \frac{2.2(1 + 3.76)}{1} \times \left( \frac{32 + 3.76(28.01)}{4.76} \right) \quad (4.13)$$

$$AF = 18.8 \text{ kg}_{air} \text{ kg}_{fuel}^{-1} \quad (4.14)$$

## 4.3 c

I selected a temperature of 450 K (176.85 °C).

## 5 Question 5

### 5.1 a

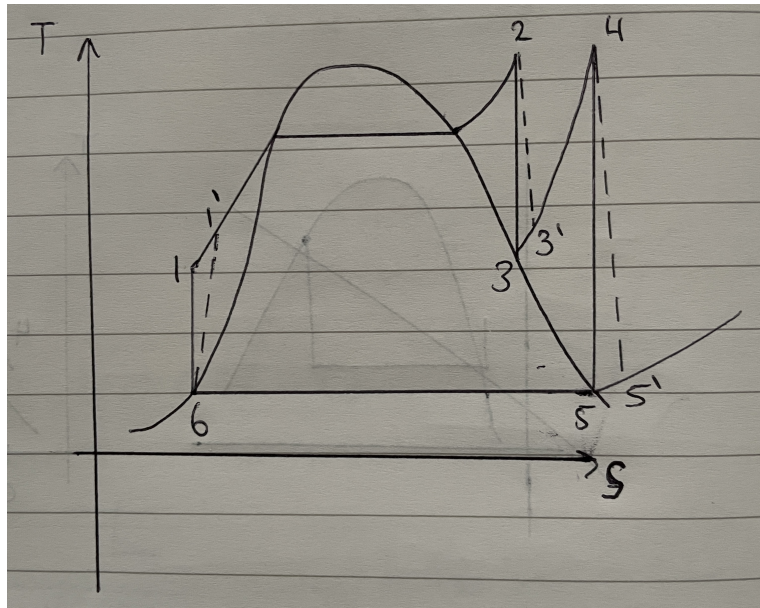


Figure 4: Graph to show Rankine cycle on T-s diagram.

In an ideal Rankine reheat cycle, we would see that the expansion and compression processes are reversible and isentropic. These are shown as straight vertical lines from 6 - 1 (pump compression), 2 - 3 and 4 - 5 (turbine expansion). However, in an actual Rankine reheat cycle, we will have irreversible losses present. For a pump, work must be done to overcome frictional forces. For a turbine, heat transfer from the turbine to the surroundings represents a loss. This process of transfer of energy as work results in entropy being produced within the system, hence we would see the entropy increase for these processes. These have been represented as the curved lines 6 - 1', 2 - 3' and 4 - 5'.

## 5.2 b

All components in the Rankine reheat cycle are steady flow devices. Neglect kinetic and potential energy changes as they are usually small relative to the work and heat transfer terms. Starting with SFEE formula:

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_1 h_1 = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_2 h_2 \quad (5.1)$$

$\dot{m}_1 = \dot{m}_2 = \dot{m}$ . Dividing by  $\dot{m}$ :

$$q_{in} + w_{in} + h_1 = q_{out} + w_{out} + h_2 \quad (5.2)$$

The thermal efficiency is given by:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{turbines} - w_{pump}}{q_{in}} \quad (5.3)$$

We know that the total heat input will come from the primary heating stage and the reheating stage, hence:

$$q_{in} = q_{primary} + q_{reheat} = (h_2 - h_1) + (h_4 - h_3) \quad (5.4)$$

Total power output from the turbines can be calculated as:

$$w_{turbines} = w_{turb1} + w_{turb2} = (h_2 - h_3) + (h_4 - h_5) \quad (5.5)$$

Pump power can be calculated as:

$$w_{pump} = (h_1 - h_6) \quad (5.6)$$

Therefore, the thermal efficiency is:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} \quad (5.7)$$

## 5.3 c

If the HPT and LPT do not expand isentropically, our thermal efficiency will become:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_2 - h_{3'}) + (h_4 - h_{5'}) - (h_{1'} - h_6)}{(h_2 - h_{1'}) + (h_4 - h_{3'})} \quad (5.8)$$

As the enthalpies at the actual points is more than those at the ideal points, we see a net decrease in the thermal efficiency.

## 5.4 d

## References

- [1] Pawlowicz, R. (2013) Key Physical Variables in the Ocean: Temperature, Salinity, and Density. Nature Education Knowledge 4(4):13 <https://www.nature.com/scitable/knowledge/library/key-physical-variables-in-the-ocean-temperature-102805293/> Accessed: 19/05/2021 00:00