UCL Mechanical Engineering 2020/2021
MECH0013 Coursework 1

Hasha Dar
Benjamin Tan
Yu Lu

Deadline: 04/12/2020

Contents

1 Question 1

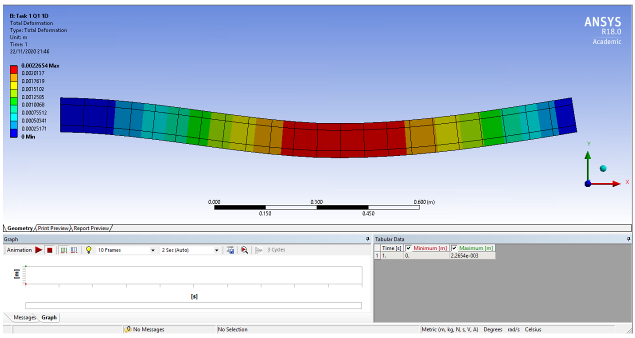


Figure 1: Total deformation in beam

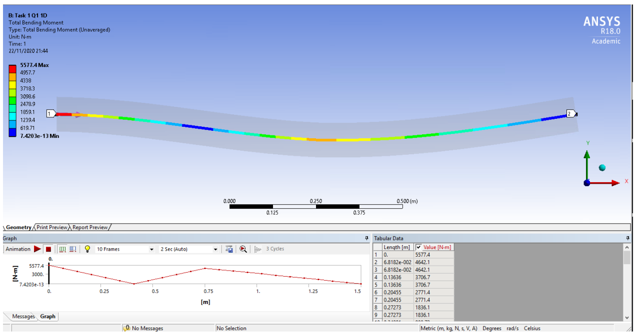


Figure 2: Bending moment in beam

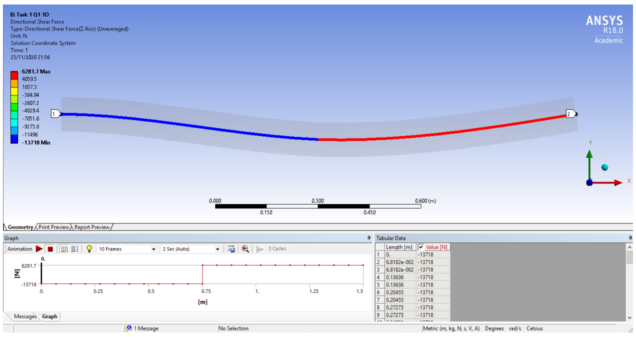


Figure 3: Directional shear in beam

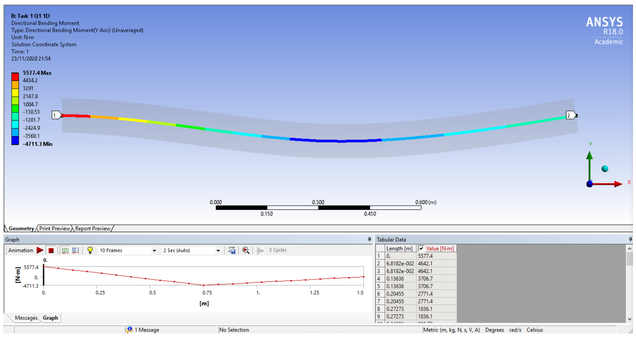
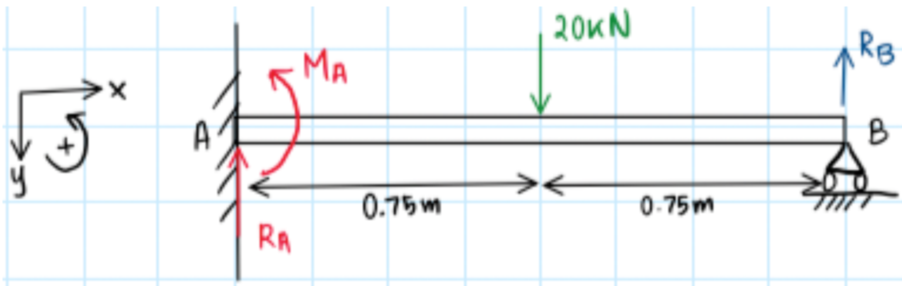


Figure 4: Directional bending in beam

FIX LINK to see numerical data of the directional deformation and the bending moment of the beam.

2 Question 2

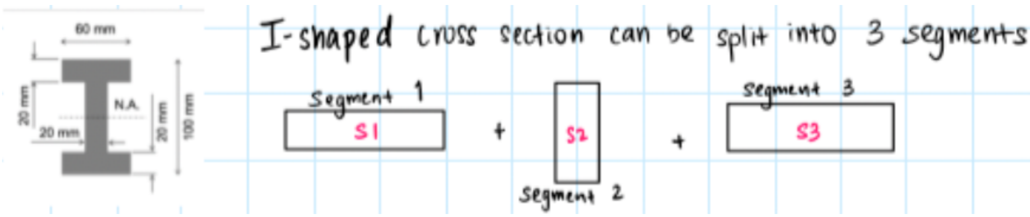


$$\sum F_y = 0 \rightarrow R_A + R_B = 20000 \tag{2.1}$$

$$\sum M_B = 0 \rightarrow M_B + 20000(0.75) - R_A(1.5) = 0 \tag{2.2}$$

$$M_A + 15000 - 1.5R_A = 0 \tag{2.3}$$

Determine second moment of area (I):



Segment 1

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \text{m}^4 \tag{2.4}$$

Segment 2

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0)^2 = 3.6 \times 10^{-7}\ \text{m}^4 \tag{2.5}$$

Segment 3

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \text{m}^4 \tag{2.6}$$

$$I_{\text{total}} = 4.28 \times 10^{-6}\ \text{m}^4 \tag{2.7}$$

Macaulay's Method

$$M = M_A + F(x - 0.75) - R_A(x) \tag{2.8}$$

$$\theta = -\frac{1}{EI}\int M\,\mathrm{d}x = -\frac{1}{EI}\left[M_A x + \frac{F(x-0.75)^2}{2} + \frac{R_A(x)^2}{2}\right] + \theta_0 \tag{2.9}$$

$$y = \int \theta\,\mathrm{d}x = -\frac{1}{EI}\left[\frac{M_A x^2}{2} + \frac{F(x-0.75)^3}{6} + \frac{R_A(x)^3}{6}\right] + \theta_0 x + y_0 \tag{2.10}$$

Boundary conditions. At $y = 0$, $x = 0$:

$$y(0) = 0 = \theta_0 \cdot (0) + y_0 \rightarrow y_0 = 0 \tag{2.11}$$

At $\theta = 0$, $x = 0$:

$$\theta(0) = 0 = \theta_0 \rightarrow \theta_0 = 0 \tag{2.12}$$

At $y = 0$, $x = 1.5$:

$$y(1.5) = 0 = -\frac{1}{EI}\left[\frac{M_A(1.5)^2}{2} + \frac{F(1.5-0.75)^3}{6} + \frac{R_A(1.5)^3}{6}\right] + 0 \cdot 1.5 + 0 \tag{2.13}$$

$$0 = \frac{9}{8}M_A + 1406.25 - \frac{9}{16}R_A \tag{2.14}$$

Multiply equation (2.3) by $\frac{9}{8}$:

$$\frac{9}{8}M_A + 16875 - \frac{27}{16}R_A = 0 \tag{2.15}$$

Equations (2.15) - (2.14):

$$15468.75 = \frac{9}{8}R_A \rightarrow R_A = 13\,750\ \text{N} \tag{2.16}$$

$$\therefore M_A = 1.5(13750) - 15000 \rightarrow M_A = 5625\ \text{N} \tag{2.17}$$

$$\therefore R_B = 20000 - 13750 \rightarrow R_B = 6250\ \text{N} \tag{2.18}$$

We know y_{max} occurs at $\theta = 0$

$$M_A x + \frac{F(x=0.75)^2}{2} - \frac{R_A x^2}{2} = 3125x^2 - 9325x + 5625 = 0 \tag{2.19}$$

$$x \neq 2.171\,\text{m} \rightarrow x = 0.829\,\text{m}\ (3dp) \tag{2.20}$$

$$y_{max} = -\frac{1}{EI}\left[\frac{M_A(0.829)^2}{2} + \frac{F(0.829-0.75)^3}{6} + \frac{R_A(0.829)^3}{6}\right] = -2.099 \times 10^{-3}\,\text{m}\ (3dp) \tag{2.21}$$

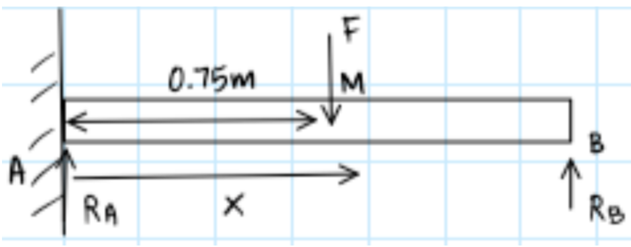


Figure 5: Shear force diagram

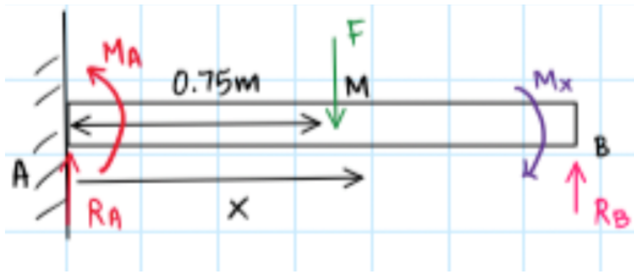


Figure 6: Bending moment diagram

Section at $0 \leq x < 0.75$.

$$Q_x = R_A = 13\,750\,\text{N} \tag{2.22}$$

$$M_x = M_A - R_A \cdot x = 5625 - 13750x \tag{2.23}$$

$$\tag{2.24}$$

Section at $0 \leq x < 1.5$.

$$\text{at } x = 0,\ M_x = 5626\,\text{N m} \tag{2.25}$$

$$\text{at } x = 0.75,\ M_x = -4687.5\,\text{N m} \tag{2.26}$$

$$\text{at } x = 1.5,\ M_x = 0\,\text{N m} \tag{2.27}$$

$$Q_x = R_A - F = 6250\,\text{N} \tag{2.28}$$

$$M_x = M_A - R_A \cdot x + F(x - 0.75) \tag{2.29}$$

$$M_x = 5625 - 13750x + 20000x - 15000 \tag{2.30}$$

$$M_x = 6250x - 9375 \tag{2.31}$$

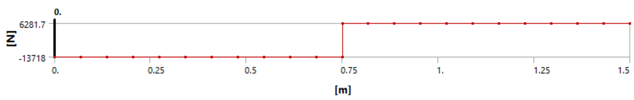


Figure 7: Ansys shear force graph

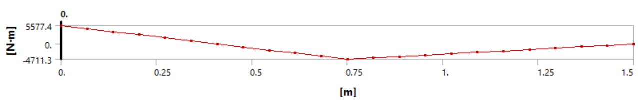


Figure 8: Ansys bending moment graph

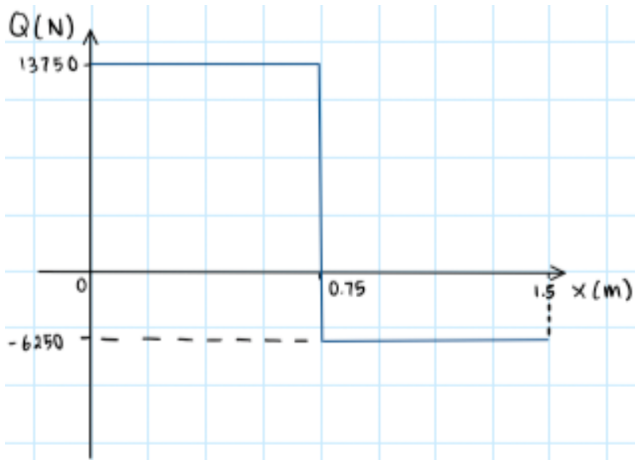


Figure 9: Shear force graph

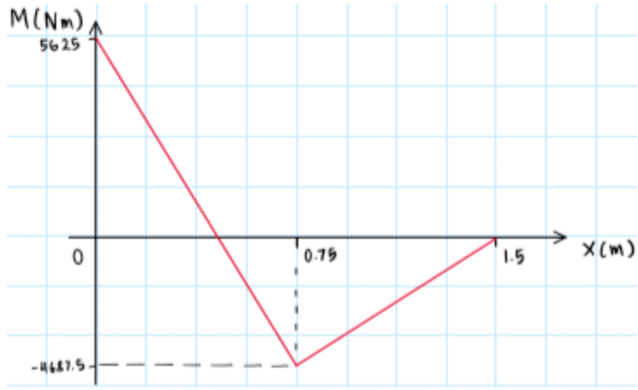


Figure 10: Bending moment graph

3 Question 3

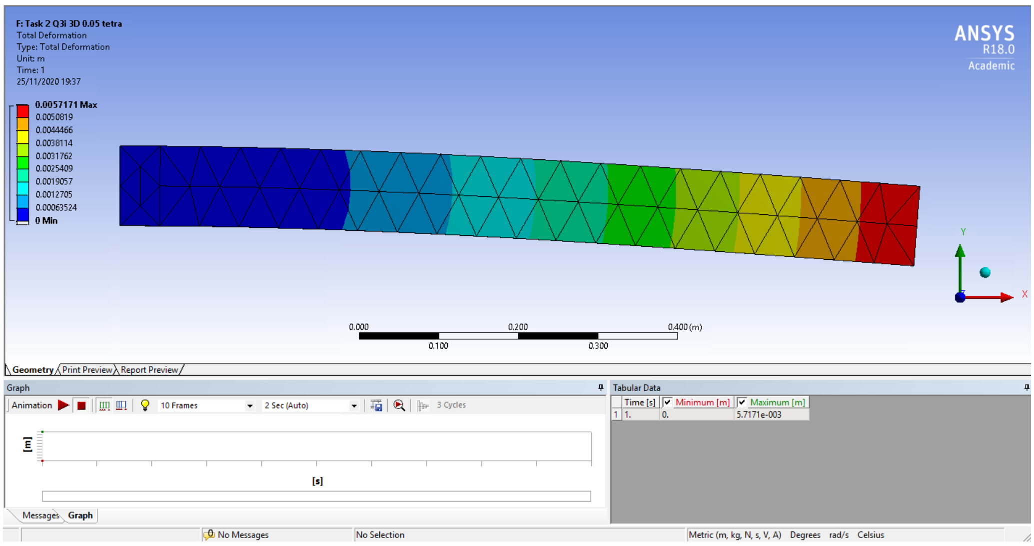


Figure 11: Tetrahedron mesh with size 0.05 m

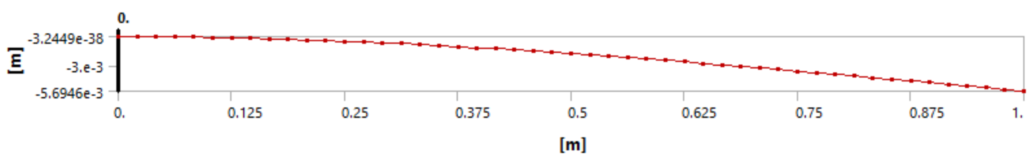


Figure 12: Tetrahedron mesh with size 0.05 m

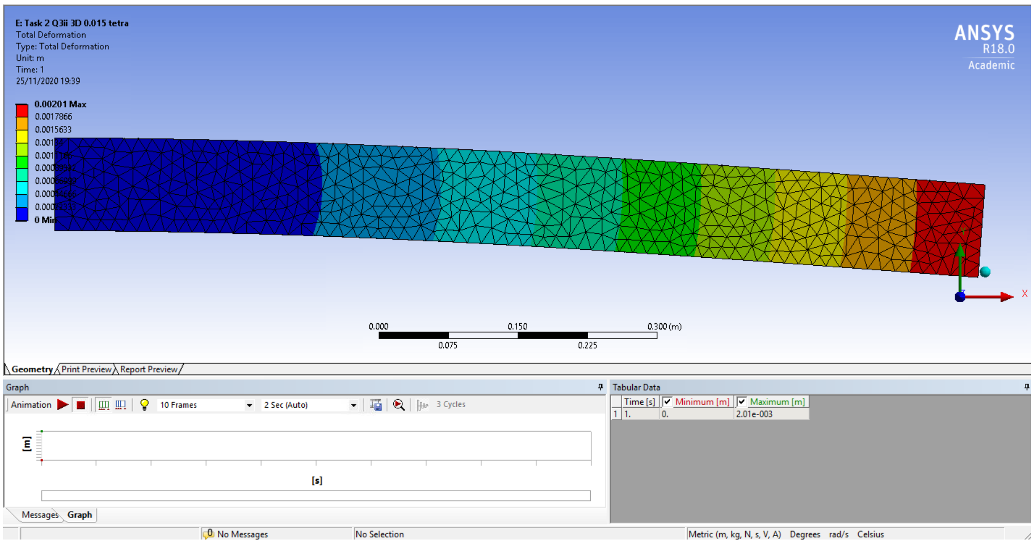


Figure 13: Tetrahedron mesh with size 0.015 m

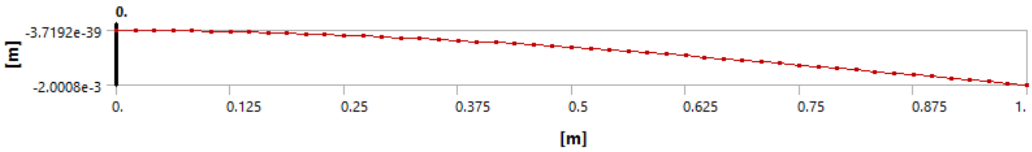


Figure 14: Tetrahedron mesh with size 0.015 m

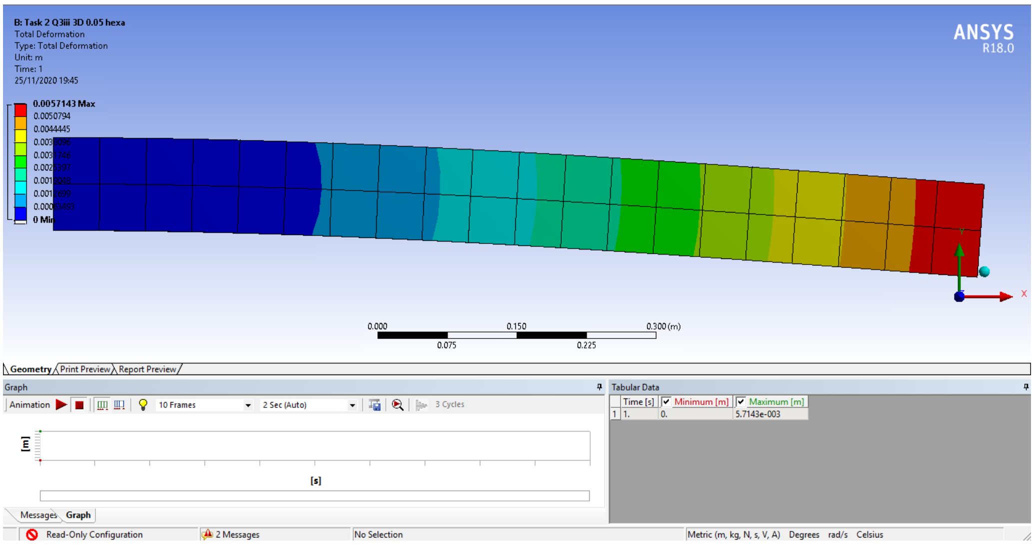


Figure 15: Hexahedron mesh with size 0.05 m

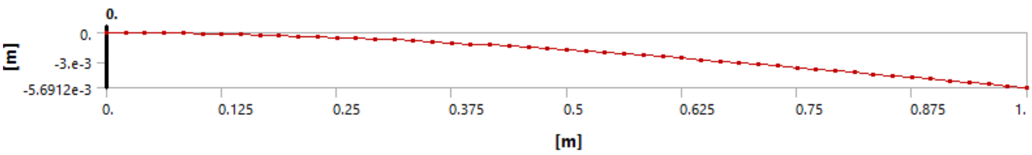


Figure 16: Hexahedron mesh with size 0.05 m

4 Question 5

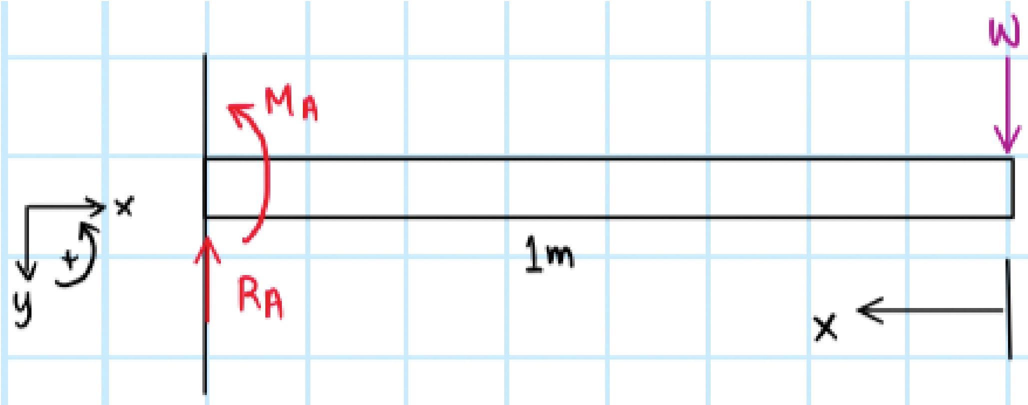


Figure 17: Square section cantilever beam

Determination of support reactions:

$$\sum F_y = 0,\ R_A = w,\ R_A = 1 \times 10^4\ \text{kN} \tag{4.1}$$

$$M_A = -w = -1 \times 10^4\ \text{kN} \tag{4.2}$$

Determine second moment of area (I). Given a square cross-section:

$$I = \frac{BH^3}{12} = \frac{(0.1)(0.1)^3}{12} = 8.33 \times 10^{-6}\ \text{m}^4 \tag{4.3}$$

Determination of deflection:

$$M = wx \tag{4.4}$$

$$\theta = -\frac{1}{EI}\int M\,\mathrm{d}x \tag{4.5}$$

$$\theta = -\frac{1}{EI}\left[\frac{wx^2}{2}\right] + \theta_0 \tag{4.6}$$

$$y = \int \theta\,\mathrm{d}x \tag{4.7}$$

$$y = -\frac{1}{EI}\left[\frac{wx^3}{6}\right] + \theta_0 x + y_0 \tag{4.8}$$

Boundary conditions: At $x = L$, $\theta = 0$:

$$0 = -\frac{wL^2}{2EI} + \theta_0 \rightarrow \theta_0 = \frac{wL^2}{2EI} \tag{4.9}$$

At $x = L$, $y = 0$:

$$0 = -\frac{wL^3}{6EI} + \frac{wL^3}{2EI} + y_0 \rightarrow y_0 = -\frac{wL^3}{3EI} \tag{4.10}$$

Thus,

$$\theta = -\frac{1}{EI}\left[\frac{wx^2}{2}\right] + \frac{wL^2}{2EI} \tag{4.11}$$

$$y = -\frac{1}{EI}\left[\frac{wx^3}{6}\right] + \frac{wL^2}{2EI}x - \frac{wL^3}{3EI} \tag{4.12}$$

y_{max} occurs at free end, $x = 0$ m

$$y_{max} = -\frac{1}{EI}\left[\frac{w(0)^3}{6}\right] + \frac{wL^2}{2EI}(0) - \frac{wL^3}{3EI} \tag{4.13}$$

$$y_{max} = -\frac{wL^3}{3EI} \tag{4.14}$$

$$y_{max} = -\frac{10 \times 10^3 \times 1^3}{70 \times 10^9 \times 3 \times 8.33 \times 10^{-6}} \tag{4.15}$$

$$y_{max} = -5.714 \times 10^{-3}\text{ m} \tag{4.16}$$