UCL Mechanical Engineering 2020/2021
MECH0013 Coursework 1

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Contents

1 Question 1

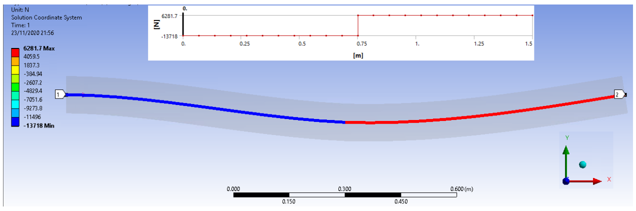


Figure 1: Directional shear in beam

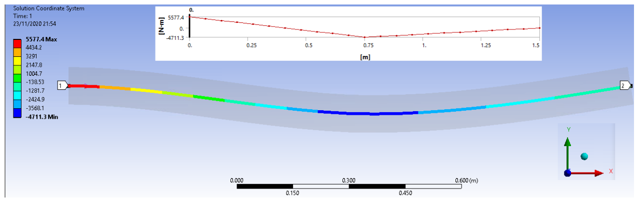
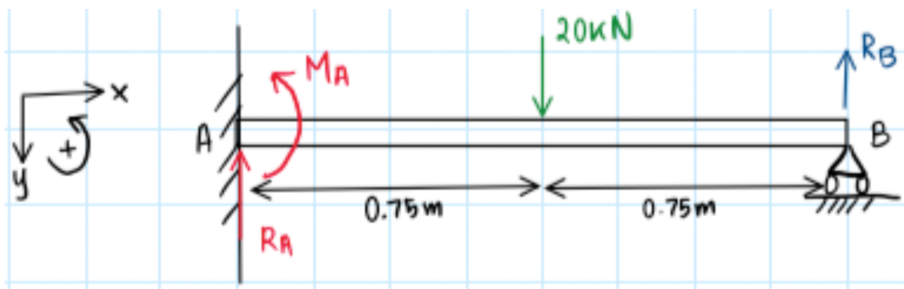


Figure 2: Directional bending in beam

FIX LINK to see numerical data of the directional deformation and the bending moment of the beam. We can see that the beam bends in the middle with a rotation in the right side of the beam due to free rotation of the joint. This leads to a higher shear force and bending moment in the left side of the beam where it has a fixed support. We see that the maximum bending moment occurs at the fixed support.

2 Question 2

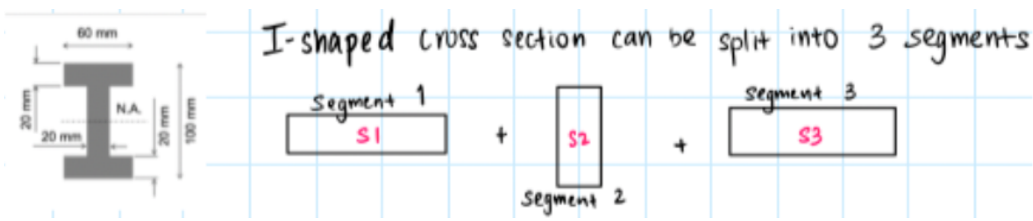


$$\sum F_y = 0 \rightarrow R_A + R_B = 20000 \tag{2.1}$$

$$\sum M_B = 0 \rightarrow M_B + 20000(0.75) - R_A(1.5) = 0 \tag{2.2}$$

$$M_A + 15000 - 1.5R_A = 0 \tag{2.3}$$

Determine second moment of area (I):



Segment 1

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \mathrm{m}^4 \tag{2.4}$$

Segment 2

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0)^2 = 3.6 \times 10^{-7}\ \text{m}^4 \tag{2.5}$$

Segment 3

$$I_x = \bar{I}_x + Ady^2 = \frac{1}{12}(0.06)(0.02)^3 + (0.06)(0.02)(0.04)^2 = 1.96 \times 10^{-6}\ \text{m}^4 \tag{2.6}$$

$$I_{\text{total}} = 4.28 \times 10^{-6}\ \text{m}^4 \tag{2.7}$$

Macaulay's Method

$$M = M_A + F(x - 0.75) - R_A(x) \tag{2.8}$$

$$\theta = -\frac{1}{EI}\int M\,\mathrm{d}x = -\frac{1}{EI}\left[M_A x + \frac{F(x-0.75)^2}{2} + \frac{R_A(x)^2}{2}\right] + \theta_0 \tag{2.9}$$

$$y = \int \theta\,\mathrm{d}x = -\frac{1}{EI}\left[\frac{M_A x^2}{2} + \frac{F(x-0.75)^3}{6} + \frac{R_A(x)^3}{6}\right] + \theta_0 x + y_0 \tag{2.10}$$

Boundary conditions. At $y = 0$, $x = 0$:

$$y(0) = 0 = \theta_0 \cdot (0) + y_0 \to y_0 = 0 \tag{2.11}$$

At $\theta = 0$, $x = 0$:

$$\theta(0) = 0 = \theta_0 \to \theta_0 = 0 \tag{2.12}$$

At $y = 0$, $x = 1.5$:

$$y(1.5) = 0 = -\frac{1}{EI}\left[\frac{M_A(1.5)^2}{2} + \frac{F(1.5-0.75)^3}{6} + \frac{R_A(1.5)^3}{6}\right] + 0 \cdot 1.5 + 0 \tag{2.13}$$

$$0 = \frac{9}{8}M_A + 1406.25 - \frac{9}{16}R_A \tag{2.14}$$

Multiply equation (2.3) by $\frac{9}{8}$:

$$\frac{9}{8}M_A + 16875 - \frac{27}{16}R_A = 0 \tag{2.15}$$

Equations (2.15) - (2.14):

$$15468.75 = \frac{9}{8}R_A \to R_A = 13\,750\ \text{N} \tag{2.16}$$

$$\therefore M_A = 1.5(13750) - 15000 \to M_A = 5625\ \text{N} \tag{2.17}$$

$$\therefore R_B = 20000 - 13750 \to R_B = 6250\ \text{N} \tag{2.18}$$

We know y_{max} occurs at $\theta = 0$

$$M_A x + \frac{F(x = 0.75)^2}{2} - \frac{R_A x^2}{2} = 3125x^2 - 9325x + 5625 = 0 \tag{2.19}$$

$$x \neq 2.171\ \text{m} \to x = 0.829\ \text{m}\ (3dp) \tag{2.20}$$

$$y_{max} = -\frac{1}{EI}\left[\frac{M_A(0.829)^2}{2} + \frac{F(0.829-0.75)^3}{6} + \frac{R_A(0.829)^3}{6}\right] = -2.099 \times 10^{-3}\ \text{m}\ (3dp) \tag{2.21}$$

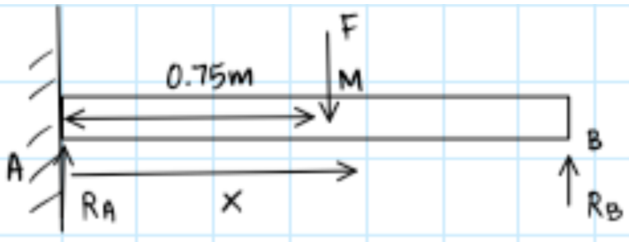


Figure 3: Shear force diagram

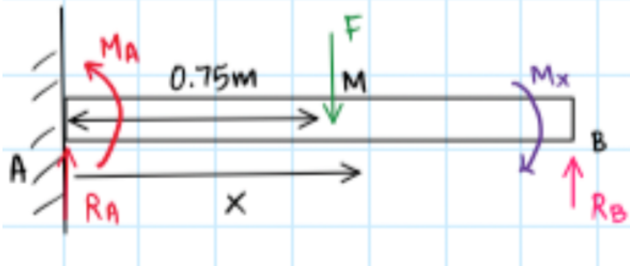


Figure 4: Bending moment diagram

Section at $0 \leq x < 0.75$.

$$Q_x = R_A = 13\,750\ \text{N} \tag{2.22}$$
$$M_x = M_A - R_A \cdot x = 5625 - 13750x \tag{2.23}$$
$$\tag{2.24}$$

Section at $0 \leq x < 1.5$.

$$\text{at } x = 0,\ M_x = 5626\ \text{N m} \tag{2.25}$$
$$\text{at } x = 0.75,\ M_x = -4687.5\ \text{N m} \tag{2.26}$$
$$\text{at } x = 1.5,\ M_x = 0\ \text{N m} \tag{2.27}$$

$$Q_x = R_A - F = 6250\ \text{N} \tag{2.28}$$
$$M_x = M_A - R_A \cdot x + F(x - 0.75) \tag{2.29}$$
$$M_x = 5625 - 13750x + 20000x - 15000 \tag{2.30}$$
$$M_x = 6250x - 9375 \tag{2.31}$$

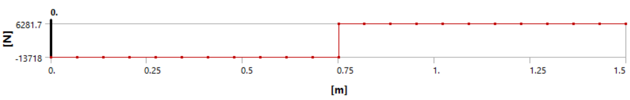


Figure 5: Ansys shear force graph

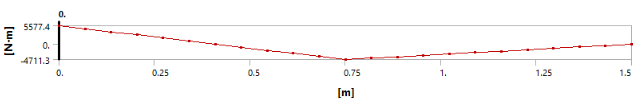


Figure 6: Ansys bending moment graph

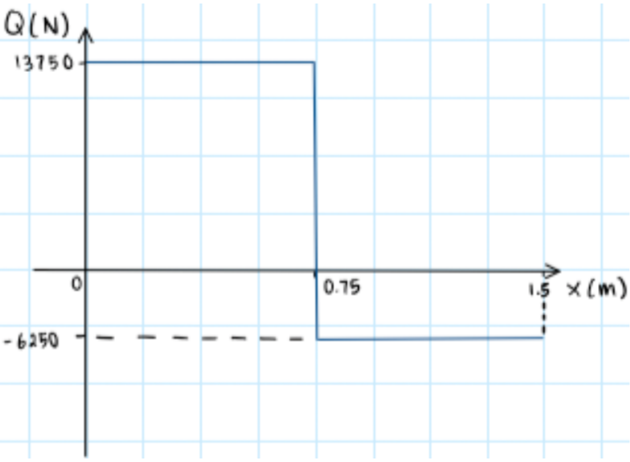


Figure 7: Shear force graph

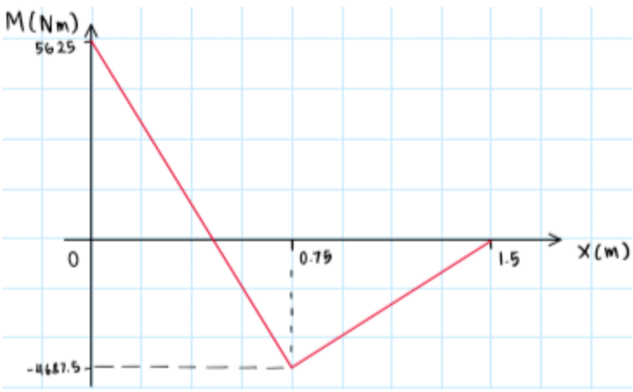


Figure 8: Bending moment graph

3 Question 3

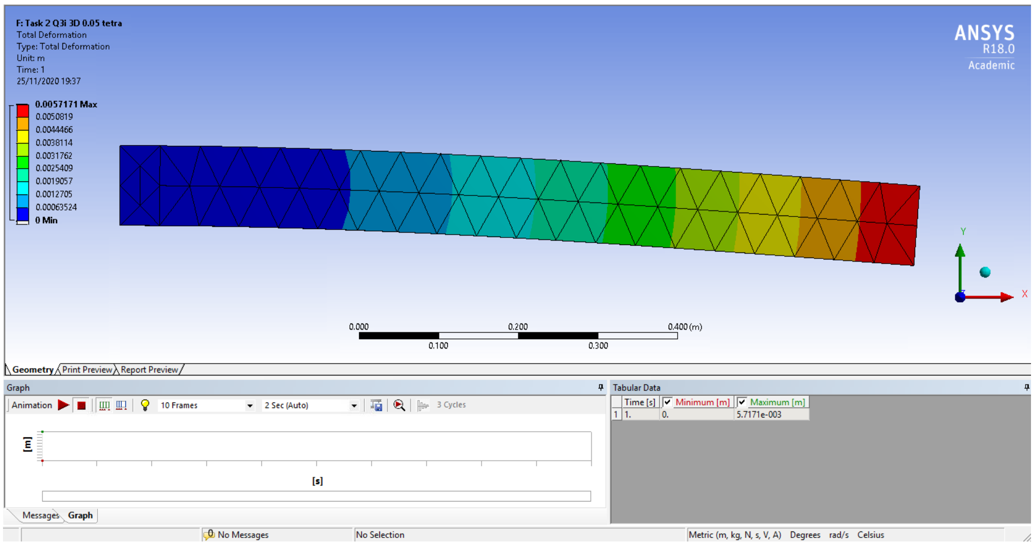


Figure 9: Tetrahedron mesh with size 0.05 m

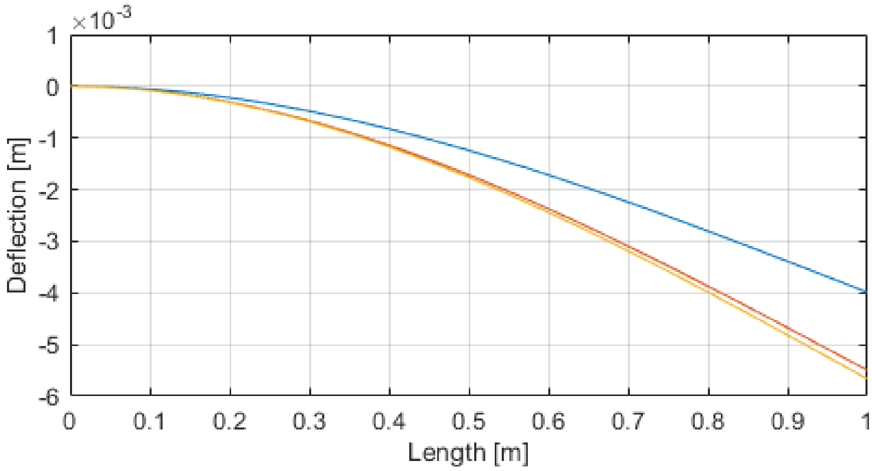


Figure 10: Tetrahedron mesh with size 0.05 m

4 Question 5

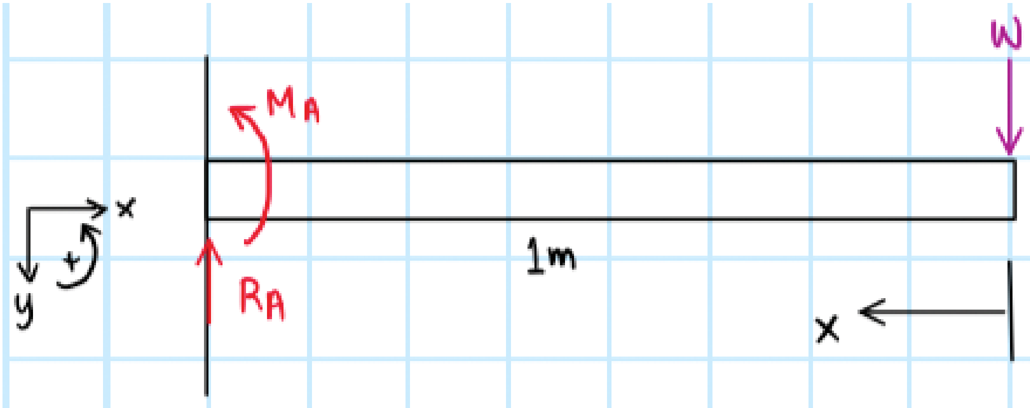


Figure 11: Square section cantilever beam

Determination of support reactions:

$$\sum F_y = 0,\ R_A = w,\ R_A = 1 \times 10^4\,\text{kN} \tag{4.1}$$

$$M_A = -w = -1 \times 10^4\,\text{kN} \tag{4.2}$$

Determine second moment of area (I). Given a square cross-section:

$$I = \frac{BH^3}{12} = \frac{(0.1)(0.1)^3}{12} = 8.33 \times 10^{-6}\,\text{m}^4 \tag{4.3}$$

Determination of deflection:

$$M = wx \tag{4.4}$$

$$\theta = -\frac{1}{EI}\int M\,\mathrm{d}x \tag{4.5}$$

$$\theta = -\frac{1}{EI}\left[\frac{wx^2}{2}\right] + \theta_0 \tag{4.6}$$

$$y = \int \theta\,\mathrm{d}x \tag{4.7}$$

$$y = -\frac{1}{EI}\left[\frac{wx^3}{6}\right] + \theta_0 x + y_0 \tag{4.8}$$

Boundary conditions: At $x = L$, $\theta = 0$:

$$0 = -\frac{wL^2}{2EI} + \theta_0 \rightarrow \theta_0 = \frac{wL^2}{2EI} \tag{4.9}$$

At $x = L$, $y = 0$:

$$0 = -\frac{wL^3}{6EI} + \frac{wL^3}{2EI} + y_0 \rightarrow y_0 = -\frac{wL^3}{3EI} \tag{4.10}$$

Thus,

$$\theta = -\frac{1}{EI}\left[\frac{wx^2}{2}\right] + \frac{wL^2}{2EI} \tag{4.11}$$

$$y = -\frac{1}{EI}\left[\frac{wx^3}{6}\right] + \frac{wL^2}{2EI}x - \frac{wL^3}{3EI} \tag{4.12}$$

y_{max} occurs at free end, $x = 0$ m

$$y_{max} = -\frac{1}{EI}\left[\frac{w(0)^3}{6}\right] + \frac{wL^2}{2EI}(0) - \frac{wL^3}{3EI} \tag{4.13}$$

$$y_{max} = -\frac{wL^3}{3EI} \tag{4.14}$$

$$y_{max} = -\frac{10 \times 10^3 \times 1^3}{70 \times 10^9 \times 3 \times 8.33 \times 10^{-6}} \tag{4.15}$$

$$y_{max} = -5.714 \times 10^{-3}\ \text{m} \tag{4.16}$$