UCL Mechanical Engineering 2020/2021

ENGF0004 Coursework 2

NCWT3

April 7, 2021

1 Question 1

1.1 a

For the line integral to be independent from the path of integration, the following conditions must be fulfilled:

$$I = \int_A^B \left(\frac{\partial u}{\partial x} \, \mathrm{d}x + \frac{\partial u}{\partial y} \, \mathrm{d}y \right) \tag{1.1}$$

$$P\left(x,\, y\right) = \frac{\partial u}{\partial x} \text{ and } Q\left(x,\, y\right) = \frac{\partial u}{\partial y} \tag{1.2}$$

$$\frac{\partial P\left(x,\, y\right)}{\partial y} = \frac{\partial Q\left(x,\, y\right)}{\partial x} \tag{1.3}$$

Considering the integral:

$$I = \int_A^B \left[e^{-\alpha x y} \left(\frac{\alpha - 2}{x} \right) \mathrm{d}x - \frac{1}{\alpha y} \left(e^{-\alpha x y} - 1 \right) \mathrm{d}y \right] \tag{1.4}$$

$$P\left(x,\, y\right) = e^{-\alpha x y} \left(\frac{\alpha - 2}{x} \right) \text{ and } Q\left(x,\, y\right) = -\frac{1}{\alpha y} \left(e^{-\alpha x y} - 1 \right) \tag{1.5}$$

$$\frac{\partial P\left(x,\, y\right)}{\partial y} = -\alpha x \left(\frac{\alpha - 2}{x} \right) e^{-\alpha x y} = \left(2\alpha - \alpha^2 \right) e^{-\alpha x y} \tag{1.6}$$

$$\frac{\partial Q\left(x,\, y\right)}{\partial x} = -\frac{1}{\alpha y} \left(-\alpha y \right) e^{-\alpha x y} = e^{-\alpha x y} \tag{1.7}$$

$$\therefore 2\alpha e^{-\alpha x y} - \alpha^2 e^{-\alpha x y} = e^{-\alpha x y} \tag{1.8}$$

$$e^{-\alpha x y} \left(\alpha^2 - 2\alpha + 1 \right) = 0 \tag{1.9}$$

$$e^{-\alpha x y} = 0 \rightarrow \text{no solutions} \tag{1.10}$$

$$\left(\alpha - 1 \right)^2 = 0 \tag{1.11}$$

$$\alpha = 1 \tag{1.12}$$

1.2 b

Calculating the line integral of 1.13 from $O\,(0,\,0)$ to $A(1,\,e-1)$ along $y = e^x - 1$:

$$I = \int_O^A \left(ye^{-2x}\right)(\mathrm{d}x + \mathrm{d}y) \tag{1.13}$$

$$y = e^x - 1 \tag{1.14}$$

$$\mathrm{d}y = e^x\,\mathrm{d}x \tag{1.15}$$

$$I = \int_0^1 \left((e^x - 1)\left(e^{-2x}\right) + (e^x - 1)\left(e^{-2x}\right)(e^x)\right)\mathrm{d}x \tag{1.16}$$

$$I = \int_0^1 \left(e^{-x} - e^{-x} - e^{-2x} + 1\right)\mathrm{d}x \tag{1.17}$$

$$I = \int_0^1 \left(1 - e^{-2x}\right)\mathrm{d}x \tag{1.18}$$

$$I = \left[x + \frac{e^{-2x}}{2}\right]_0^1 \tag{1.19}$$

$$I = 1 + \frac{e^{-2}}{2} - 0 - \frac{1}{2} \tag{1.20}$$

$$I = \frac{1}{2}\left(e^{-2} + 1\right) \tag{1.21}$$

1.3 c

$$\underline{F}\,(x,\,y,\,z) = \begin{pmatrix} \frac{y}{x^2} \\ \frac{x}{y^2} \end{pmatrix} \tag{1.22}$$