

# UCL Mechanical Engineering 2020/2021

## ENGF0004 Coursework 1

NCWT3

January 1, 2021

### 1 Question One

a

b

We are given:

$$f(x) = \frac{x}{\sqrt{1-x}} \quad (1.1)$$

$$= x(1-x)^{-\frac{1}{2}} \quad (1.2)$$

Differentiating three times yields:

$$f'(x) = (1-x)^{-\frac{1}{2}} + \frac{x}{2}(1-x)^{-\frac{3}{2}} \quad (1.3)$$

$$f''(x) = \frac{1}{2}(1-x)^{-\frac{3}{2}} + \frac{1}{2}(1-x)^{-\frac{3}{2}} + \frac{3x}{4}(1-x)^{-\frac{5}{2}} \quad (1.4)$$

$$= (1-x)^{-\frac{3}{2}} + \frac{3x}{4}(1-x)^{-\frac{5}{2}} \quad (1.5)$$

$$f'''(x) = \frac{3}{2}(1-x)^{-\frac{5}{2}} + \frac{3}{4}(1-x)^{-\frac{5}{2}} + \frac{15x}{8}(1-x)^{-\frac{7}{2}} \quad (1.6)$$

$$= \frac{9}{4}(1-x)^{-\frac{5}{2}} + \frac{15x}{8}(1-x)^{-\frac{7}{2}} \quad (1.7)$$

Inputting  $x = 0$ :

$$f(0) = 0 \cdot (1-0)^{-\frac{1}{2}} \quad (1.8)$$

$$= 0 \quad (1.9)$$

$$f'(0) = (1-0)^{-\frac{1}{2}} + \frac{0}{2}(1-0)^{-\frac{3}{2}} \quad (1.10)$$

$$= 1 \quad (1.11)$$

$$f''(0) = (1-0)^{-\frac{3}{2}} + \frac{3 \cdot 0}{4}(1-0)^{-\frac{5}{2}} \quad (1.12)$$

$$= 1 \quad (1.13)$$

$$f'''(0) = \frac{9}{4}(1-0)^{-\frac{5}{2}} + \frac{15 \cdot 0}{8}(1-0)^{-\frac{7}{2}} \quad (1.14)$$

$$= \frac{9}{4} \quad (1.15)$$

General form of Maclaurin series:

$$f(x) \approx f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots \quad (1.16)$$

Inputting the above variables into Eq.1.16:

$$f(x) \approx x + \frac{x^2}{2} + \frac{3x^3}{8} \quad (1.17)$$

**c**

**i**

We are given:

$$E = \frac{kq}{x^2} \quad (1.18)$$

Sum of electric fields due to both charged particles is:

$$E = \frac{ke}{(x-r)^2} - \frac{ke}{(x+r)^2} \quad (1.19)$$

$$= ke \left[ \frac{1}{x^2 \left(1 - \frac{r}{x}\right)^2} - \frac{1}{x^2 \left(1 + \frac{r}{x}\right)^2} \right] \quad (1.20)$$

$$= \frac{ke}{x^2} \left[ (1-y)^{-2} - (1+y)^{-2} \right] \quad (1.21)$$

Where  $y = \frac{r}{x}$ .

**ii**

Calculation of constants to be used in Maclaurin series expansion:

$$\begin{array}{ll} f(y) = (1-y)^{-2} & f(0) = 1 \\ f'(y) = 2(1-y)^{-3} & f'(0) = 2 \\ f''(y) = 6(1-y)^{-4} & f''(0) = 6 \\ f'''(y) = 24(1-y)^{-5} & f'''(0) = 24 \\ \\ g(y) = (1+y)^{-2} & g(0) = 1 \\ g'(y) = -2(1+y)^{-3} & g'(0) = -2 \\ g''(y) = 6(1+y)^{-4} & g''(0) = 6 \\ g'''(y) = -24(1+y)^{-5} & g'''(0) = -24 \end{array}$$

Inputting the above variables into Eq.1.16:

$$f(y) \approx 1 + \frac{2y}{1!} + \frac{6y^2}{2!} + \frac{24y^3}{3!} + \dots \quad (1.22)$$

$$f(y) \approx 1 + 2y + 3y^2 + 4y^3 \quad (1.23)$$

$$g(y) \approx 1 - \frac{2y}{1!} + \frac{6y^2}{2!} - \frac{24y^3}{3!} + \dots \quad (1.24)$$

$$g(y) \approx 1 - 2y + 3y^2 - 4y^3 \quad (1.25)$$

Substitution:

$$E \approx \frac{ke}{x^2} [f(y) - g(y)] \quad (1.26)$$

$$E \approx \frac{ke}{x^2} [1 + 2y + 3y^2 + 4y^3 - 1 + 2y - 3y^2 + 4y^3] \quad (1.27)$$

$$E \approx \frac{ke}{x^2} [4y + 8y^3] \quad (1.28)$$

$$E \approx \frac{4ke}{x^2} [y + 2y^3] \quad (1.29)$$

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$y = 0.01$ . Exact:

$$E_E = \frac{ke}{x^2} [(1 - 0.01)^{-2} - (1 + 0.01)^{-2}] \quad (1.30)$$

$$E_E = \frac{ke}{x^2} [0.0400080012] \quad (1.31)$$

$$(1.32)$$

Approximation:

$$E_A = \frac{ke}{x^2} [0.01 + 2(0.01)^3] \quad (1.33)$$

$$E_A = \frac{ke}{x^2} [0.010002] \quad (1.34)$$

Percentage error:

$$\frac{E_A}{E_E} \cdot 100 = \frac{0.010002}{0.0400080012} \cdot 100 = 25\% \text{ error (2sf)} \quad (1.35)$$

d

We are given:

$$y'' - 2y' + y = te^t \quad (1.36)$$

$$y(0) = 0, \quad y'(0) = 1 \quad (1.37)$$

Laplace transformation (from tables):

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = \mathcal{L}\{te^t\} \quad (1.38)$$

$$s^2Y(s) - sy(0) - y'(0) - 2(sY(s) - y(0)) + Y(s) = \frac{1!}{(s-1)^2} \quad (1.39)$$

$$s^2Y(s) - 1 - 2(sY(s) - 1) + Y(s) = \frac{1}{(s-1)^2} \quad (1.40)$$

$$Y(s) [s^2 - 2s + 1] = \frac{1}{(s-1)^2} \quad (1.41)$$

$$Y(s) = \frac{1}{(s-1)^2 (s^2 - 2s + 1)} \quad (1.42)$$

$$Y(s) = \frac{1}{(s-1)^2 (s-1)^2} \quad (1.43)$$

$$Y(s) = \frac{1}{(s-1)^4} \quad (1.44)$$

e

i

$a = 1 \therefore -3 \leq t \leq 3$ . Sketch:

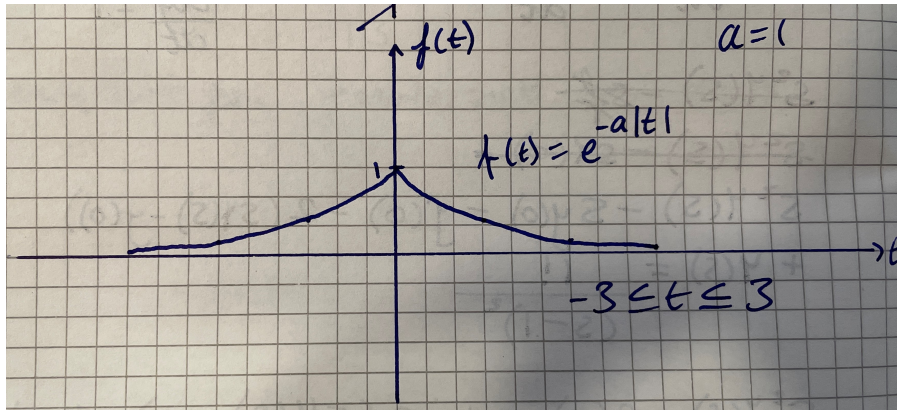


Figure 1:

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$$F(u) = \int_{-\infty}^0 e^{at} e^{-jut} dt + \int_0^{\infty} e^{-at} e^{-jut} dt \quad (1.45)$$

$$F(u) = \int_{-\infty}^0 e^{t(a-ju)} dt + \int_0^{\infty} e^{-t(a+ju)} dt \quad (1.46)$$

$$F(u) = \frac{1}{(a-ju)} e^{-t(a-ju)} \Big|_0^{\infty} + \frac{1}{-(a+ju)} e^{-t(a+ju)} \Big|_0^{\infty} \quad (1.47)$$

$$F(u) = \frac{1}{a-ju} + \frac{1}{a+ju} \quad (1.48)$$

$$F(u) = \frac{2a}{a^2 + u^2} \quad (1.49)$$

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v

## 2 Question Two

a

i

ii

iii

b

i

ii

iii

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