0.1 Thick wall cylinders

Lame's equations

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o)\, r_i^2 r_o^2}{\left(r_o^2 - r_i^2\right) r^2} \tag{1}$$

$$\sigma_\theta = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 r_i^2} + \frac{(p_i - p_o)\, r_i^2 r_o^2}{\left(r_o^2 - r_i^2\right) r^2} \tag{2}$$

We have seen that a thick cylinder subjected to internal pressure experiences very high peaks of stress at the inner surface.

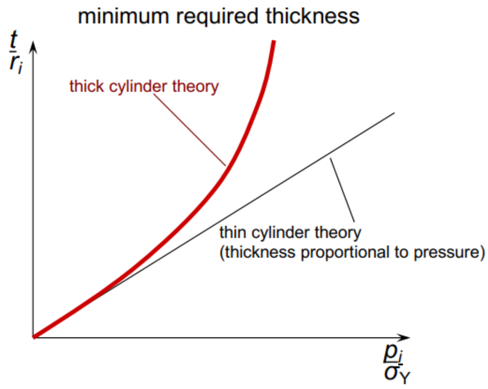

Figure 1: The required thickness increases non-linearly with the pressure.

The design of cylinders that have to maintain high levels of pressure requires specific strategies. aimed at reducing the hoop stress levels at the inside surface.

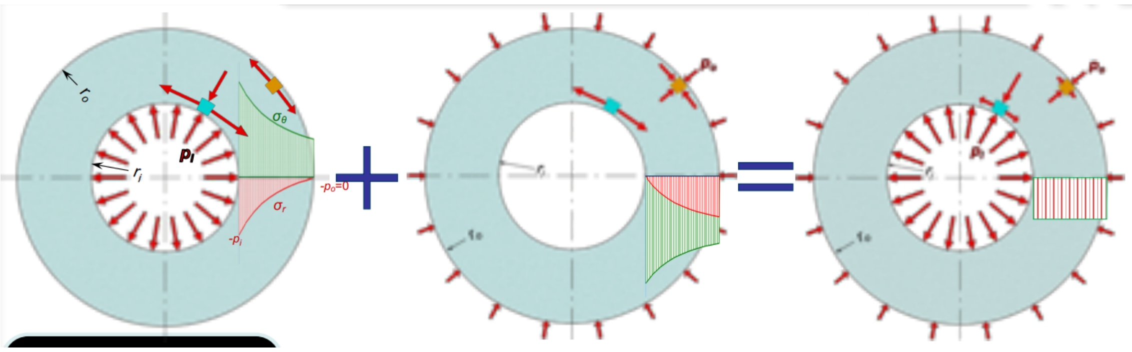

Figure 2:

A possible solution could be reducing the level of hoop stress by increasing the pressure at the outer surface.

0.2 Compound cylinders

0.2.1 Containing high pressures

This can be done by shrinking another tube on the outside of the original one:

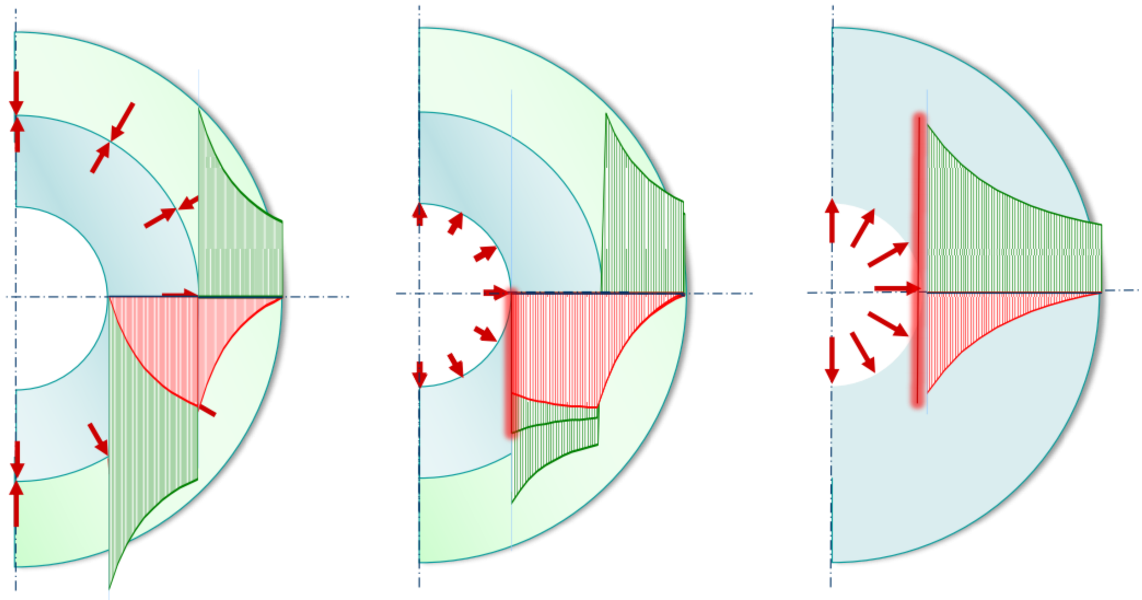

Figure 3:

0.2.2 Compound or 'built-up' cylinders

THis compound (or built-up) arrangement is obtained by assembling two cylinders with initial diametrical interference δ at room temperature. The outer cylinder is heated and the inner cylinder is cooled, in order to compensate for the interference by thermal expansion/contraction and allow assembly. Returning the components to room temperature, a set of residual radial and hoop stress is introduced, which locks the two components firmly together.
The interference between the two cylinders will produce an external pressure p_{int} acting on the internal cylinder, and an equal internal pressure p_{int} acting on the external cylinder, function of the interference δ.

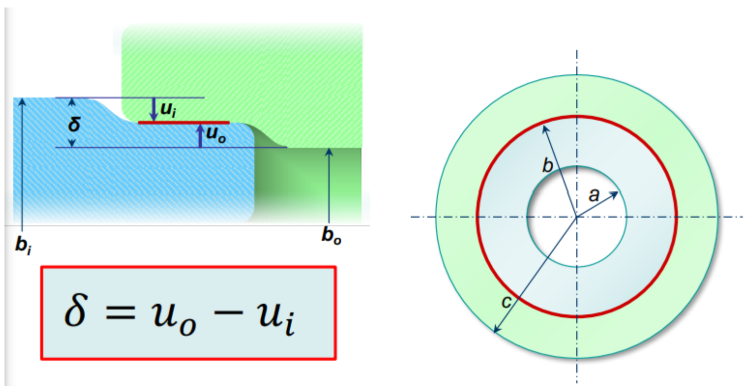

Figure 4:

Interface pressure can be expressed as a function of the interference, by using the solid mechanics equations.

Other applications

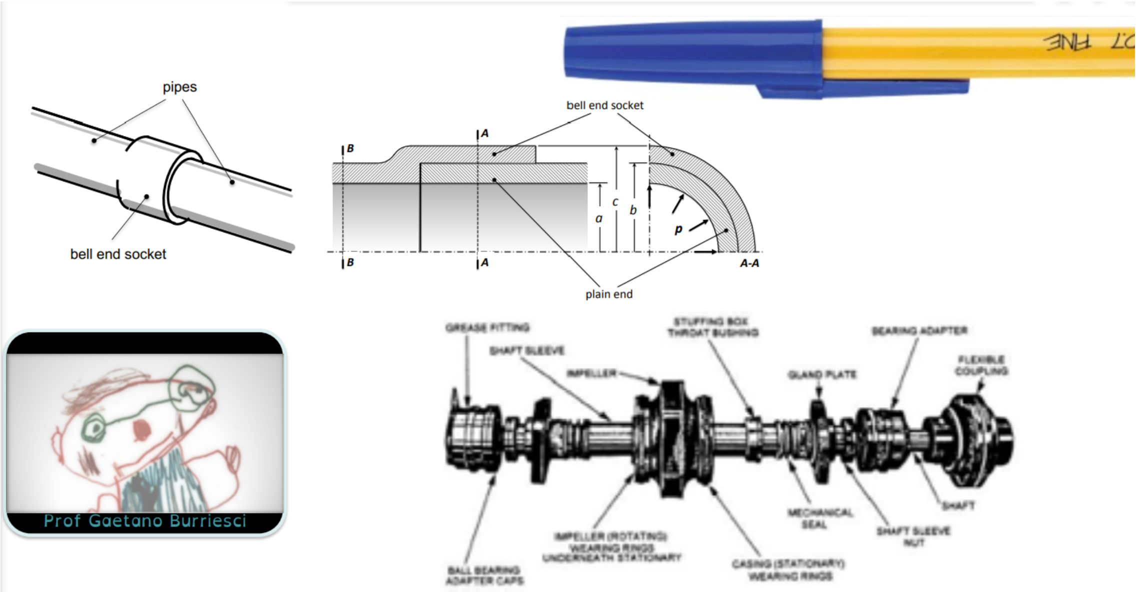

Figure 5:

0.2.3 Stress state at the interface

$$u = \frac{r}{E}\left(\sigma_\theta - v\sigma_r\right) \tag{3}$$

$$\rightarrow u_i = \frac{b}{E_i}\left[\sigma_{\theta,i}\left(b\right) - v_i\sigma_{r,i}(b)\right] \tag{4}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{5}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{6}$$

Boundary conditions:

$$\sigma_{r,i}(a) = 0 = A - \frac{B}{a^2} \tag{7}$$

$$\sigma_{r,i}(b) - -p_{int} = A - \frac{B}{b^2} \tag{8}$$

$$A = -p_{int}\frac{b^2}{b^2 - a^2} \tag{9}$$

$$B = -p_{int}\frac{a^2 \cdot b^2}{b^2 - a^2} \tag{10}$$

Substituting:

$$\sigma_{\theta,i} = A + \frac{B}{r^2} = -p_{int}\frac{b^2}{B^2 - a^2}\left(1 + \frac{a^2}{r^2}\right) \tag{11}$$

Therefore:

$$\sigma_{r,i}(b) = -p_{int} \qquad \sigma_{\theta,i}(b) = -p_{int}\frac{b^2+a^2}{b^2-a^2} \tag{12}$$

$$u_i = -p_{int}\frac{b}{E_i}\left(\frac{b^2+a^2}{b^2-a^2} - v_i\right) \tag{13}$$

Repeating:

$$u_o = \frac{b}{E_o}\left[\sigma_{\theta,o}(b) - v_o\sigma_{r,o}(b)\right] \tag{14}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{15}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{16}$$

Boundary conditions:

$$\sigma_{r,o}(b) = -p_{int} = A - \frac{B}{b^2} \tag{17}$$

$$\sigma_{r,o}(c) = 0 = A - \frac{B}{c^2} \tag{18}$$

$$A = -p_{int}\frac{b^2}{b^2-c^2} \tag{19}$$

$$B = -p_{int}\frac{b^2 \cdot c^2}{c^2-b^2} \tag{20}$$

Substituting:

$$\sigma_{\theta,o} = A + \frac{B}{r^2} = -p_{int}\frac{b^2}{c^2-b^2}\left(1 + \frac{c^2}{r^2}\right) \tag{21}$$

Therefore:

$$\sigma_{r,o}(c) = 0 \qquad \sigma_{\theta,o}(b) = p_{int}\frac{c^2+b^2}{c^2-b^2} \tag{22}$$

$$u_o = p_{int}\frac{b}{E_o}\left(\frac{c^2+b^2}{c^2-b^2} + v_o\right) \tag{23}$$