0.1 Exercise 2 - Fluid tutorial Group B

Question a

When using Bernoulli and potential function, we have the following assumptions

- No temporal changes (steady flow)
- Inviscid flow
- Incompressible flow
- Irrotational flow

Irrotational proof:

$$\omega_z = 0 \tag{1}$$

$$0 = \frac{1}{r}\left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta}\right) \tag{2}$$

$$ru_r = -V_\infty r\sin(\theta-\alpha)\left(1+\frac{R^2}{r^2}\right) - \frac{\Gamma}{2\pi} \tag{3}$$

$$= -V_\infty r\sin(\theta-\alpha) - V_\infty\frac{R^2}{r}\sin(\theta-\alpha) - \frac{\Gamma}{2\pi} \tag{4}$$

$$\frac{\partial(ru_\theta)}{\partial r} = V_\infty\frac{R^2}{r^2}\sin(\theta-\alpha) - V_\infty\sin(\theta-\alpha) \tag{5}$$

$$u_r = V_\infty\cos(\theta-\alpha)\left(1-\frac{R^2}{r^2}\right) \tag{6}$$

$$\frac{\partial u_r}{\partial \theta} = -V_\infty\sin(\theta-\alpha)\left(1-\frac{R^2}{r^2}\right) \tag{7}$$

$$\omega_z = V_\infty\sin(\theta-\alpha)\left[\left(\frac{R^2}{r^2}-1\right) - \left(\frac{R^2}{r^2}-1\right)\right] \tag{8}$$

$$= 0 \tag{9}$$

Incompressible proof:

$$u_r = \frac{\partial\phi}{\partial r} \tag{10}$$

$$= V_\infty\cos(\theta-\alpha)\left(1-\frac{R^2}{r^2}\right) \tag{11}$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} \tag{12}$$

$$= -V_\infty\sin(\theta-\alpha)\left(1+\frac{R^2}{r^2}\right) - \frac{\Gamma}{2\pi r} \tag{13}$$

$$\text{Bernoulli: } 0 = \tag{14}$$

0.1.1 Question b

Pressure coefficient:

$$c_p = \frac{p - p_\infty}{\frac{1}{2} p V_\infty^2} = 1 - \frac{u_r^2 + i_\theta^2}{V_\infty^2} \tag{15}$$