0.1 Momentum equation

$$\sum F_{sys} = \frac{\partial}{\partial t}\int_{CV} \underline{V}\rho d\forall + \int_{CS} \rho\underline{V}(\underline{V}\cdot\underline{n})dA \tag{1}$$

0.1.1 Vane example:

A horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 3.048$ $\mathrm{m\,s^{-1}}$, strikes a vane and is turned through an angle θ. Determine the anchoring force needed to hold the vane stationary if gravity and visocus effects are negligible.

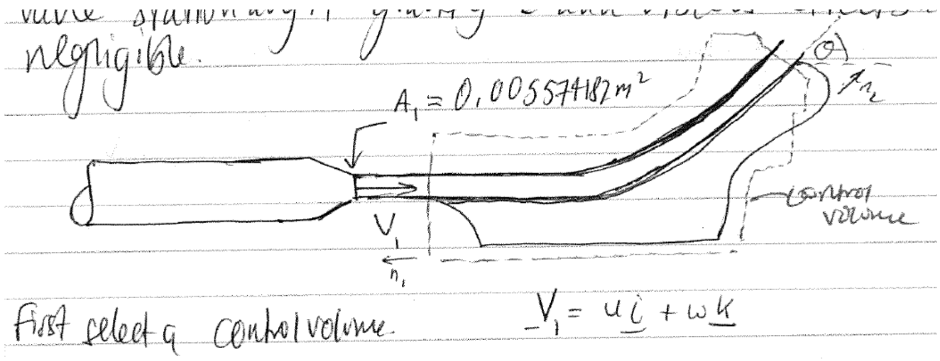

Figure 1: Water flow through vane system.

The only portions of the control surfrace across which fluid flows are section 1 (the entrance) and section 2 (the exit). Hence, the momentum equation becomes in the x and z components.

$$\sum F_x = \int_{inlet} u\rho(\underline{v}\cdot\underline{n})dA + \int_{outlet} u\rho(\underline{V}\cdot\hat{n})dA \tag{2}$$

$$\sum F_x = u_1\rho(-V_1)A_1 + u_2\rho(V_2)A_2 \tag{3}$$

$$\sum F_x = u_2\rho A_2V_2 - i_1\rho A_1V_1 \tag{4}$$

In the z direction:

$$\sum F_z = \int_{inlet} w\rho(\underline{V}\cdot\hat{n})dA + \int_{outlet} w\rho(\underline{V}\cdot n)dA \tag{5}$$

$$\sum F_z = w_2\rho A_2V_2 - w_1\rho A_1V_1 \tag{6}$$

We know that at inlet V_1 there is no vertical component, hence $w_1 = 0$ and $u_1 = V_1$. At the outlet:

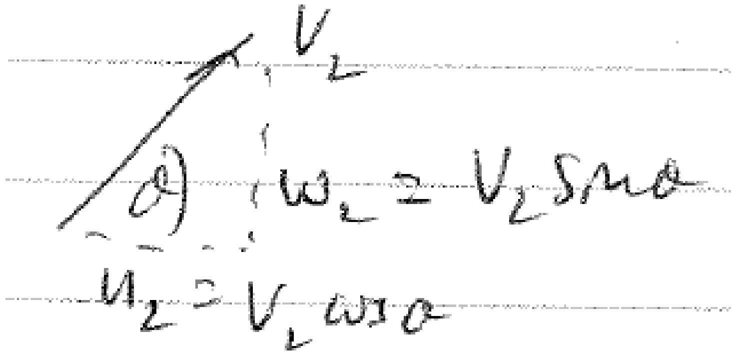

Figure 2: Velocity components at outlet.

Also lets find V_1 and V_2. From Bernoulli's equation (neglecting g, assuming incompressible and that $P_1 = P_2 = P_{atm}$), $V_1 = V_2$.

$$\therefore \sum F_z = V_2 \sin\theta \rho A_2 V_2 = V_1^2 A_2 \sin\theta \rho \tag{7}$$

$$\sum F_x = V_2 \cos\theta \rho A_2 V_2 - V_1 \rho A_1 V_1 \tag{8}$$

$$\sum F_x = V_1^2 A_2 \cos\theta \rho - V_1^2 \rho A_1 \tag{9}$$

$$\sum F_x = V_1^2 (A_2 \cos\theta \rho - \rho A_1) \tag{10}$$

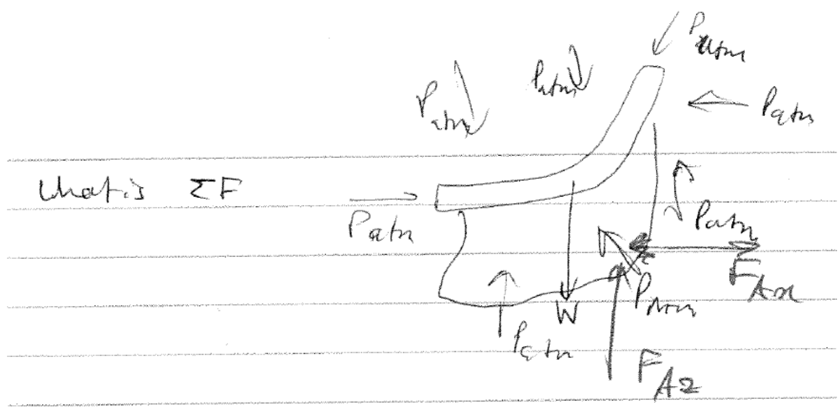

Figure 3: Forces acting on system.

Neglect w and the net force due to $P_{atm} = 0$

$$\therefore \sum F_x = F_{Ax} \tag{11}$$

$$\sum F_z = F_{Az} \tag{12}$$

$$F_{Ax} = V_1^2 A_2 \sin\theta\rho \tag{13}$$

$$F_{Az} = V_1^2 (A_z \cos\theta\rho - \rho A_1) \tag{14}$$

Also remember that,

$$\dot{m}_1 = \dot{m}_2 \tag{15}$$

$$V_1 A_1 = V_2 A_2 \text{ (incompressiblity)} \tag{16}$$

$$V_1 = V_2 \therefore A_1 = A_2 \tag{17}$$

$$\therefore F_{Ax} = V_1^2 A_1 \sin\theta\rho \tag{18}$$

$$F_{Az} = \rho V_1^2 (A_1 \cos\theta - A_1) = \rho V_1^2 A_1 (\cos\theta - 1) \tag{19}$$

Plug in data to find F_{Ax} and F_{Az}.