

# MECH0010 Assignment Report 1

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**Abstract**—This document is a model and instructions for L<sup>A</sup>T<sub>E</sub>X. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. \*CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

**Index Terms**—component, formatting, style, styling, insert

## I. QUESTION 1

In the real world, the operation of DC motor is the transfer from electrical energy to the mechanical rotation of motor. Therefore, the electric circuit of motor operation is composed by the input voltage ( $v$  (V)), terminal resistance ( $R$  ( $\Omega$ )), Inductance ( $L$  (H)), and the electromotive force ( $em$  (V)). In this case the formulas for the time domain and frequency domain are ( $i$  is the current of the circuit):

$$V = Ri + L \frac{di}{dt} + em \quad (1)$$

$$V(s) = RI(s) + LsI(s) + Em(s) \quad (2)$$

For the mechanical rotation, the relationship among torque ( $T$  (N m)), moment of inertia ( $Im$  (kg m<sup>2</sup>)), angular displacement ( $\theta$ ) and the friction coefficient ( $b$ ) in time domain and frequency domain are given:

$$T = Im \frac{d^2\theta}{dt^2} + b \frac{d\theta}{dt} \quad (3)$$

$$T(s) = Im s^2 \theta(s) + bs\theta(s) \quad (4)$$

In this case, angular velocity is proportional to the electromotive force and current is proportional to the torque of the motor. ( $K_e$  the electrical motor constant,  $K_t$  is the mechanical motor constant):

$$em = K_e \frac{d\theta}{dt} \quad (5)$$

$$Em(s) = K_e s \theta(s) \quad (6)$$

$$T = K_t i \quad (7)$$

$$T(s) = K_t I(s) \quad (8)$$

In order to find the response between angular position ( $\theta$ ) and input voltage ( $v$ ) in time domain, the Laplace transfer function by combination of Eq.2, 4, 7, 8 is shown below:

$$\frac{\theta(s)}{v(s)} = \frac{K_t}{bR_s(\frac{K_e K_t}{bR}) + \tau_e \tau_m s^2 + \tau_m s + \tau_e s + 1} \quad (9)$$

Note:  $\tau_m$  is the mechanical time constant of motor ( $\tau_m = \frac{Im}{b}$ ),  $\tau_e$  is the electrical time constant ( $\tau_e = \frac{L}{R}$ ).

Assumption: Because the electrical time constant ( $\tau_e$ ) is much smaller compared to the mechanical time constant of the motor ( $\tau_m$ ), term  $\tau_e \tau_m s^2$  and  $\tau_e s$  are approximately equal to zero compared with other terms. Therefore, the equation will become:

$$\frac{\theta(s)}{v(s)} = \frac{K}{s(\tau s + 1)} \left( K = \frac{K_t}{bR + K_t K_e}, \tau = \frac{\tau_m R b}{K K_t + R b} \right) \quad (10)$$

Due to the angular velocity ( $\omega$ ) is the first derivative of angular position related to time ( $\omega(s) = s \cdot \theta(s)$ ). In this case, the transfer function of angular velocity and input voltage is:

$$\frac{\omega(s)}{v(s)} = \frac{k}{\tau s + 1} \quad (11)$$

According to the specification data of Motor C42-L50 winding code 10

TABLE I  
SPECIFICATION DATA

Specification	Value
Torque sensitivity ( $K_t$ )	0.1412 N m A <sup>-1</sup>
Back e.m.f. ( $K_e$ )	0.1413 V rad <sup>-1</sup> s <sup>-1</sup>
Rotor inertia ( $Im$ )	6.3354 × 10 <sup>-4</sup> kg m <sup>-2</sup>
Mec. time constant ( $\tau_m$ )	0.0223 s
Terminal resistance ( $R$ )	0.7 $\Omega$
Friction coefficient ( $\frac{Im}{\tau_m}$ )	0.0285

Therefore, the constant  $K$  and overall time constant  $\tau$  can be calculated as:

$$K = \frac{K_t}{bR + K_t K_e} = 3.54 \quad (12)$$

$$\tau = \frac{\tau_m R b}{K K_t + R b} = 0.0112 \quad (13)$$

In this case, the transfer function of angular velocity and angular displacement of motor with voltage input is:

$$\frac{\omega(s)}{v(s)} = \frac{K}{(\tau s + 1)} = \frac{3.54}{0.0112s + 1} \quad (14)$$

$$\frac{\theta(s)}{v(s)} = \frac{K}{s(\tau s + 1)} = \frac{3.54}{0.0112s^2 + s} \quad (15)$$

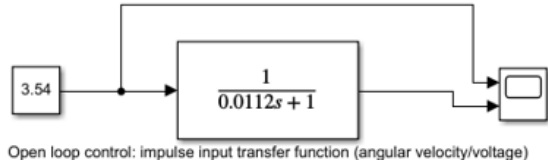


Fig. 1. Model of open loop transfer function of angular velocity.

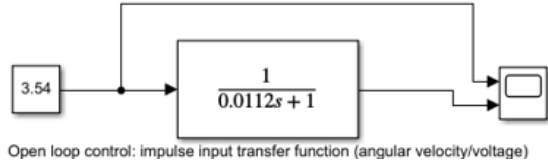


Fig. 2. Model of open loop transfer function of angular displacement.

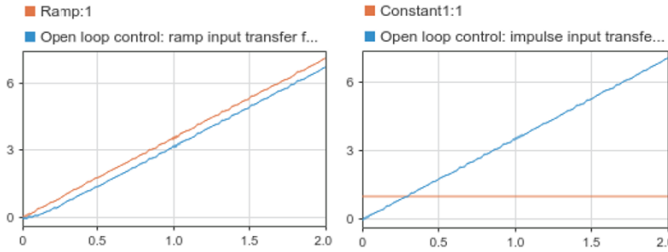
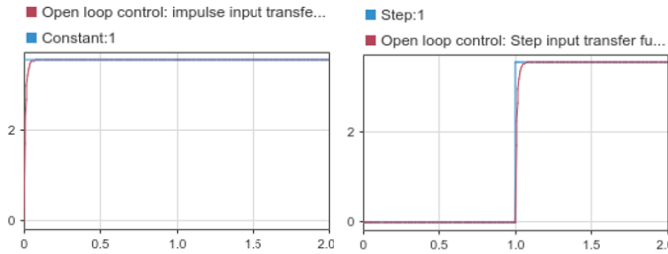


Fig. 5. Model of open loop transfer function of angular velocity with ramp input.

Fig. 6. Model of open loop transfer function of angular displacement with impulse input.

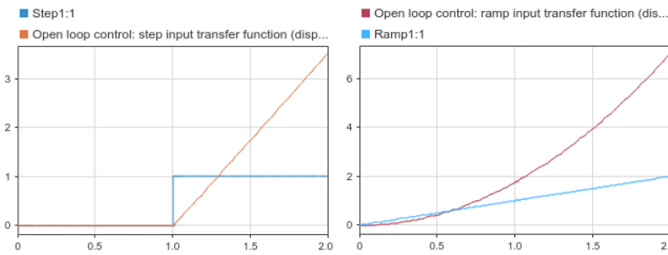


Fig. 7. Model of open loop transfer function of angular displacement with step input.

Fig. 8. Model of open loop transfer function of angular displacement with ramp input.

where 3.54 is the gain of the system.

According to the results show in Graphs.6, .7, .8, the response of output angular displacement will intersect with input and then overshoot to infinity (undesirable behaviour), which means these open loops cannot meet the requirement to control the angular displacement of the motor. Under this circumstance, the unit feedback closed loop with a gain of  $H$  is required to overcome this undesirable behaviour of open loop control system for angular velocity. The formula for the feedback loop is:

$$G(s) = \frac{\theta(s)}{v(s)} = \frac{3.54}{0.0112s^2 + s} \quad (16)$$

$$H(s) = H \quad (17)$$

$$G'(s) = \frac{G(s)}{1 + G(s) \cdot H(s)} \quad (18)$$

Therefore, the new transfer function is:

$$G'(s) = \gamma \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (19)$$

$$G'(s) = \frac{316.07H}{H(s^2 + 2 \cdot 44.64 \cdot \sqrt{\frac{1}{316.07H}} \cdot \sqrt{316.07H}s + 316.07H)} \quad (20)$$

Note:  $\gamma = H$ ,  $\zeta = 44.64 \sqrt{\frac{1}{316.07H}}$  and  $\omega_n = \sqrt{316.07H}$

In this case, we can use  $\zeta$  to determine the response wanted as overdamped, critically damped, underdamped or undamped to control the loop without undesired behaviour and according to the results and analysis of transfer function of angular velocity, step input is suitable to make the response reach the desired input compared to ramp input and impulse input. Therefore, the step input is the input for the new transfer function to overcome the undesirable behaviour. The results of the simulation for Simulink are shown in Fig.9, 10, 11.

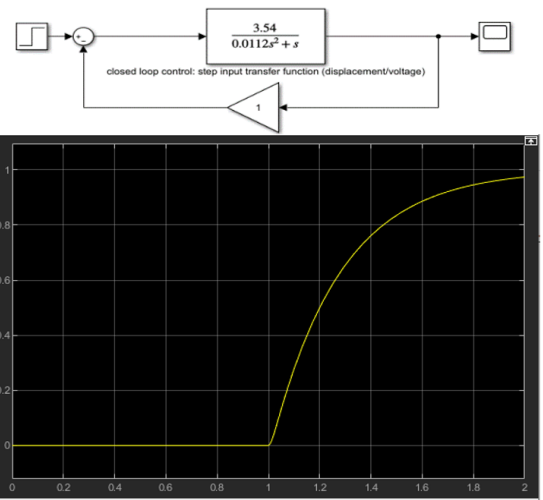


Fig. 9. Overdamped.  $\zeta > 1$ ,  $0 < H < 6.304 \therefore H = 1$

There is no undamped condition in this case as  $\omega_n$  also depends on  $H$ .

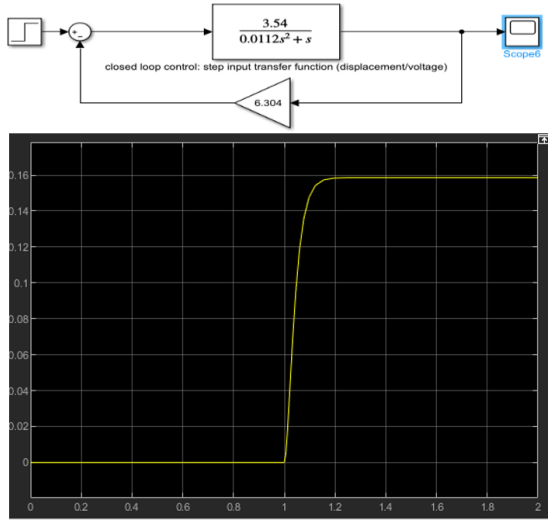


Fig. 10. Critically damped.  $\zeta = 1$ ,  $H = 6.304$

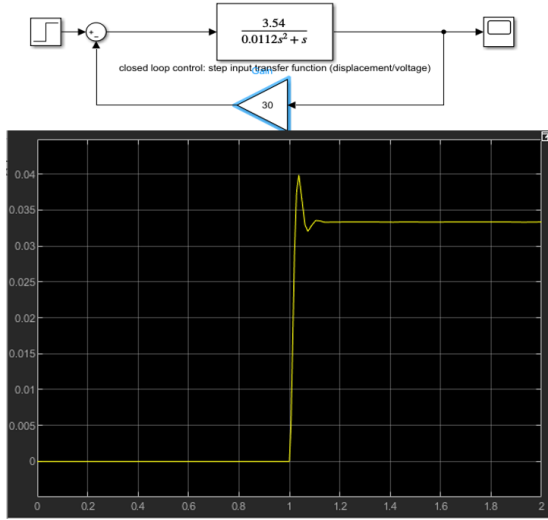


Fig. 11. Underdamped.  $0 < \zeta < 1$ ,  $H > 6.304$ . In this case,  $H = 30$

## II. QUESTION 2

### A. 4

Considering a FDM (fused deposition modelling) PLA (polyactic acid) 3D printer, typical printing speeds [1] for a medium end model are  $100 \text{ mm s}^{-1}$ . The accuracy for a good printer would be around  $\pm 0.2 \text{ mm}$ . The sampling rate of an Arduino's analogue input port is roughly  $9600 \text{ Hz}$ , as tested here. This would allow us to have a strip with a blocked-transparent pattern in  $1 \text{ mm}$  blocks (i.e.  $0.5 \text{ mm}$  of blocked out and then  $0.5 \text{ mm}$  of transparent). Assuming our sample rate is stable at  $9600 \text{ Hz}$ , if the head travels at  $100 \text{ mm s}^{-1}$ , we will traverse 96 patterned blocks. This relates to 96 samples per block traversed. This should provide adequate information to measure the intensity of light from the LED. When the printer head moves along the encoder, the intensity of the light reaching the LDR will form a sinusoidal intensity signal. We

can utilise a comparator to remove noise from our signal by inverting one of the outputs and running both signals to the inputs of an op-amp. This will be beneficial in reducing error signals, a necessary requirement for a high-accuracy system.

We can measure which direction the head is moving in and the position of the head by using a quadrature sine/cosine signal. This is where we take the voltage signal from our LDR (a sinusoid) and compare it with a signal with a  $\frac{\pi}{2}$  phase shift. Plotting these on an xy oscilloscope will produce a plot called a Lissajous figure. Under perfect conditions, our Lissajous figure will be a circle centred on the origin. The radius of the Lissajous is based on the amplitude and the direction in which the point is traced relates to whether our linear encoder is being read in the positive or negative direction. However, we may see that our Lissajous is not a perfect circle and this can be amended with trimming the signal and calibrations. The Lissajous figure tells us the position of the printer head, but we will need an absolute reference in order for our printer head to know where it is. For example, setting the zero point at the top right corner, we can count the number of times our Lissajous traverses  $2\pi$  times in the oscilloscope, when the head moves. We can use the axis' crossed as a reference as to whether the number should be increased or decreased. As we also know the size of our strip pattern, we can derive the position of the head.

## III. INTRODUCTION

This document is a model and instructions for  $\text{\LaTeX}$ . Please observe the conference page limits.

## IV. EASE OF USE

### A. Maintaining the Integrity of the Specifications

The IEEEtran class file is used to format your paper and style the text. All margins, column widths, line spaces, and text fonts are prescribed; please do not alter them. You may note peculiarities. For example, the head margin measures proportionately more than is customary. This measurement and others are deliberate, using specifications that anticipate your paper as one part of the entire proceedings, and not as an independent document. Please do not revise any of the current designations.

## V. PREPARE YOUR PAPER BEFORE STYLING

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections V-A–V-E below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads— $\text{\LaTeX}$  will do that for you.

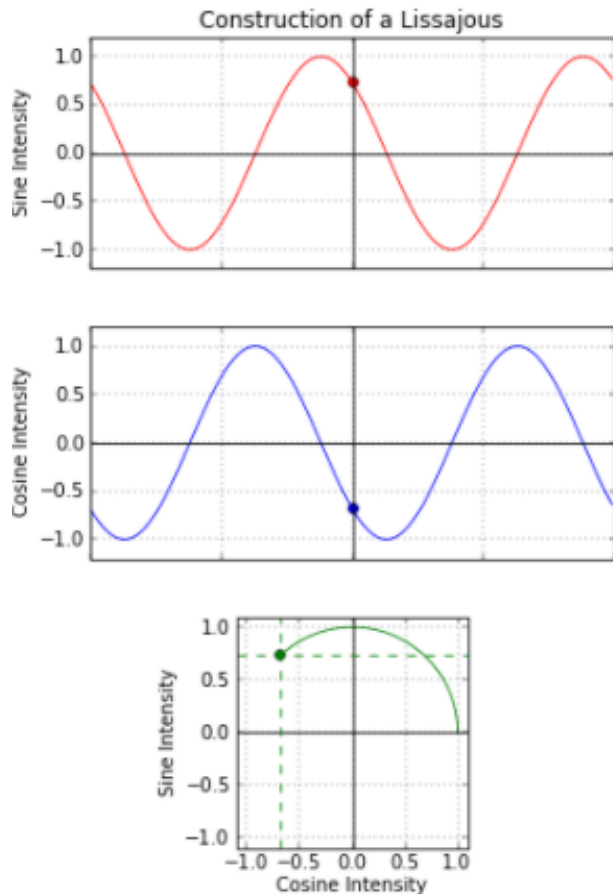


Fig. 12. Lissajous plot

#### A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

#### B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as “3.5-inch disk drive”.
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: “Wb/m<sup>2</sup>” or “webers per square meter”, not “webers/m<sup>2</sup>”. Spell out units when they appear in text: “. . . a few henries”, not “. . . a few H”.

- Use a zero before decimal points: “0.25”, not “.25”. Use “cm<sup>3</sup>”, not “cc”).

#### C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus ( / ), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \quad (21)$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use “(21)”, not “Eq. (21)” or “equation (21)”, except at the beginning of a sentence: “Equation (21) is . . .”

#### D. L<sup>A</sup>T<sub>E</sub>X-Specific Advice

Please use “soft” (e.g., `\eqref{Eq}`) cross references instead of “hard” references (e.g., (1)). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don’t use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in L<sup>A</sup>T<sub>E</sub>X will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you’ve discovered a new method of counting.

BIB<sub>T</sub>E<sub>X</sub> does not work by magic. It doesn’t get the bibliographic data from thin air but from .bib files. If you use BIB<sub>T</sub>E<sub>X</sub> to produce a bibliography you must send the .bib files.

L<sup>A</sup>T<sub>E</sub>X can’t read your mind. If you assign the same label to a subsection and a table, you might find that Table I has been cross referenced as Table IV-B3.

L<sup>A</sup>T<sub>E</sub>X does not have precognitive abilities. If you put a `\label` command before the command that updates the counter it’s supposed to be using, the label will pick up the last counter to be cross referenced instead. In particular, a `\label` command should not go before the caption of a figure or a table.

Do not use `\nonumber` inside the `{array}` environment. It will not stop equation numbers inside `{array}` (there won’t be any anyway) and it might stop a wanted equation number in the surrounding equation.

#### E. Some Common Mistakes

- The word “data” is plural, not singular.
- The subscript for the permeability of vacuum  $\mu_0$ , and other common scientific constants, is zero with subscript formatting, not a lowercase letter “o”.

- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- Do not use the word “essentially” to mean “approximately” or “effectively”.
- In your paper title, if the words “that uses” can accurately replace the word “using”, capitalize the “u”; if not, keep using lower-cased.
- Be aware of the different meanings of the homophones “affect” and “effect”, “complement” and “compliment”, “discreet” and “discrete”, “principal” and “principle”.
- Do not confuse “imply” and “infer”.
- The prefix “non” is not a word; it should be joined to the word it modifies, usually without a hyphen.
- There is no period after the “et” in the Latin abbreviation “et al.”.
- The abbreviation “i.e.” means “that is”, and the abbreviation “e.g.” means “for example”.

An excellent style manual for science writers is [7].

#### F. Authors and Affiliations

**The class file is designed for, but not limited to, six authors.** A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

#### G. Identify the Headings

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is “Heading 5”. Use “figure caption” for your Figure captions, and “table head” for your table title. Run-in heads, such as “Abstract”, will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head

because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

#### H. Figures and Tables

a) *Positioning Figures and Tables:* Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation “Fig. 13”, even at the beginning of a sentence.

TABLE II  
TABLE TYPE STYLES

Table Head	Table Column Head		
	Table column subhead	Subhead	Subhead
copy	More table copy <sup>a</sup>		

<sup>a</sup>Sample of a Table footnote.



Fig. 13. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity “Magnetization”, or “Magnetization, M”, not just “M”. If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write “Magnetization (A/m)” or “Magnetization {A[m(1)]}”, not just “A/m”. Do not label axes with a ratio of quantities and units. For example, write “Temperature (K)”, not “Temperature/K”.

#### ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g”. Avoid the stilted expression “one of us (R. B. G.) thanks ...”. Instead, try “R. B. G. thanks...”. Put sponsor acknowledgments in the unnumbered footnote on the first page.

#### REFERENCES

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use “Ref. [3]” or “reference [3]” except at the beginning of a sentence: “Reference [3] was the first ...”

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

#### REFERENCES

- [1] 3D printing, how to obtain the best possible speeds, Chris Joel, <https://www.3dprintersonlinestore.com/how-to-obtain-best-3d-printing-speed> Accessed at 28-11-20 16:14:32
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