# UCL Mechanical Engineering 2020/2021

MECH0010 Final Assessment

NCWT3

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## 1 Question 1

### 1.1 a

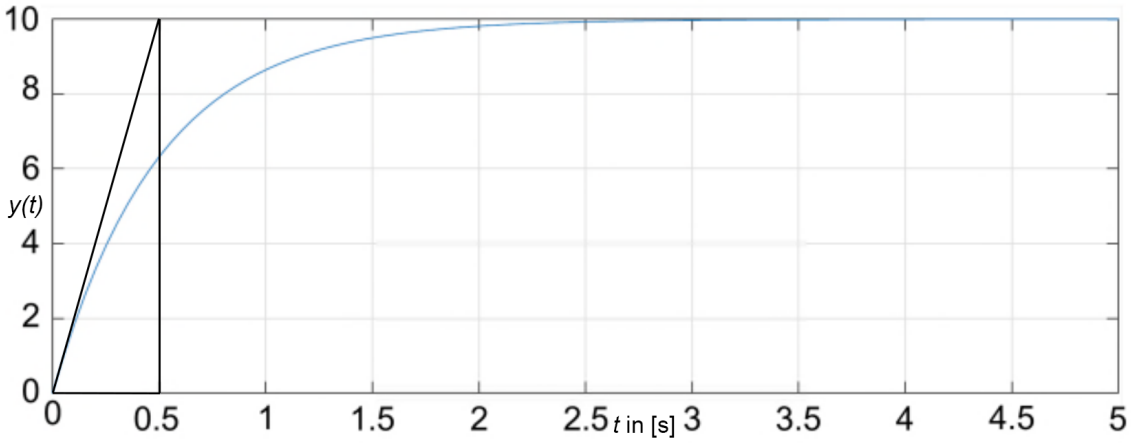


Figure 1: Diagram to show step response of a first-order system.

Let us look at 1.1

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = ay(t) + bu(t) \tag{1.1}$$

Transforming to Laplace domain:

$$sY(s) - y(0) = aY(s) + bU(s) \tag{1.2}$$

$y(0) = 0$:

$$Y(s)\,(s-a) = bU(s) \tag{1.3}$$

$$\frac{Y(s)}{U(s)} = \frac{b}{s-a} \tag{1.4}$$

For step input $U(s) = \frac{1}{s}$:

$$Y(s) = \frac{b}{s(s-a)} = \frac{k_1}{s} + \frac{k_2}{s-a} \tag{1.5}$$

Solving partial fraction:

$$k_1(s-a) + k_2 s = b \tag{1.6}$$

$$s = 0 \rightarrow -ak_1 = b \tag{1.7}$$

$$k_1 = -\frac{b}{a} \tag{1.8}$$

$$s = a \rightarrow ak_2 = b \tag{1.9}$$

$$k_2 = \frac{b}{a} \tag{1.10}$$

$$Y(s) = \frac{b}{a(s-a)} - \frac{b}{as} \tag{1.11}$$

$$Y(s) = \frac{b}{a}\left(\frac{1}{s-a} - \frac{1}{s}\right) \tag{1.12}$$

Returning to time domain (using tables):

$$y(t) = \frac{b}{a}\left(e^{at} - 1\right) \tag{1.13}$$

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = be^{at} \tag{1.14}$$

$$\left.\frac{\mathrm{d}y(t)}{\mathrm{d}t}\right|_{t=0} = b \tag{1.15}$$

From Figure 1, we can see that the gradient at 0 is:

$$b = \frac{10}{0.5} = 20 \tag{1.16}$$

Response settles at $y(t) = 10$, therefore:

$$e^{at} = 0 \tag{1.17}$$

$$\therefore 10 = \frac{20}{a}\,(-1) \tag{1.18}$$

$$a = -2 \tag{1.19}$$

Hence:

$$y(t) = -10\left(e^{-2t} - 1\right) \tag{1.20}$$

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = -2y(t) + 10u(t) \tag{1.21}$$