0.1 Dimensionless Numbers

0.1.1 Notation

We denote a dimension by using a capital letter in square brackets. Here are some common dimensions.

- [M] - mass (unit: kilogram).
- [T] - time (unit: second).
- [L] - length (unit: metre).
- [Θ] - temperature (unit: kelvin).

Thus, we can derive that the dimensions of acceleration (which has units $\mathrm{m\,s^{-2}}$) $[\mathrm{L}][\mathrm{T}]^{-2}$. Some dimensions of common measurements are shown below:

- Force - $[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}$.
- Energy - $[\mathrm{M}][\mathrm{L}]^2[\mathrm{T}]^{-2}$.

We use dimensional analysis to check derivations. The dimensions of both sides of any equation must match. Physical constants also often have units associated with them - these must also be considered. Some variables are dimensionless such as the Reynolds number.

$$Re = \frac{\rho l u}{\mu}$$

$$[Re] = \frac{[M][L]^{-3} \cdot [L] \cdot [L][T]^{-1}}{[L][M][T]^{-2} \cdot [T][L]^{-2}} = \frac{[M][L]^{-1}[T]^{-1}}{[M][L]^{-1}[T]^{-1}}$$

Hence, we can see that the Reynolds number is a dimensionless quantity as all the dimensions cancel.

0.1.2 Example

We can use dimensional analysis to derive basic forms of equations. We want to work out the pressure drop as oil flows though a pipe. Let us consider the parameters this may depend on.

- Viscosity - $[M][L]^{-1}[T]^{-1}$.
- Pipe length - [L].
- Pipe diameter - [L].
- Velocity - $[L][T]^{-1}$.
- Pressure - $[M][L]^{-1}[T]^{-2}$.

Next we can assume that the pressure is a function of the other four. Some combination of the others must have the same dimension as the quantity we want.

$$[M][L]^{-1}[T]^{-2} = ([M][L]^{-1}[T]^{-1})^{\alpha} \cdot ([L])^{\beta} \cdot ([L])^{\gamma} \cdot ([L][T]^{-1})^{\delta}$$

$$[L] : -1 = -\alpha + \beta + \gamma + \delta$$

$$[M] : 1 = \alpha$$

$$[T] : -2 = -\alpha - \delta$$

$$\alpha = 1,\ \delta = 1,\ \beta + \gamma = -1$$

So it must be true that:

$$\Delta P = \mu \cdot v \cdot I^{\beta} \cdot D^{\gamma}$$

Where $\beta + \gamma = -1$ The actual answer for laminar flow is:

$$\Delta P = \frac{2\mu L v}{D^2}$$

This sort of analysis is useful for checking on the functional form of relationships, but it won't give you the exact relationship, or the value of any dimensionless constants involved.

0.1.3 Similarity

- Geometrical similarity: fixed ratio of lengths.
- Kinematic similarity: fixed ratio of velocities.
- Dynamic similarity - fixed ratio of forces.

Note on inertia: Inertia is not a force. However, for considering its importance to dynamic similarity, we can use the force needed to slow down a moving object. So we quantify inertia for these purposes as ma, from $F = ma$. Since the forces on flow change fluid motion, we use this often.

Dynamic similarity: viscosity

Compare the inertia "force" and the viscous force for a fluid:

$$\frac{[Inertia\ force]}{[Viscous\ force]} = \frac{\rho L^2 u^2}{\mu u L} = \frac{\rho L u}{\mu}$$

The Reynolds number is something very specific - it allows us to calculate the ratio of inertial and viscous forces in order to check for dynamical similarity.

- Honey: Re $\approx 1.3 \times 10^{-4}$
- Tea: Re ≈ 1100

Therefore, they are not dynamically similar with respect to viscosity.

0.1.4 Dimensionless groups

We have identified some dimensionless groups such as Reynolds number and Froude number. There are many more such as:

- Bond number: ratio of gravitational to surface tension forces.
- Capillary number: ratio of surface tension to viscous forces.
- Euler number: ratio of pressure force to inertial force.
- Grashof number: ratio of buoyancy to viscous forces.
- Cauchy number: ratio of inertial to elastic forces.
- Weber number: ratio of inertial to surface tension forces.

0.2 Buckhingham Pi

insert theory here