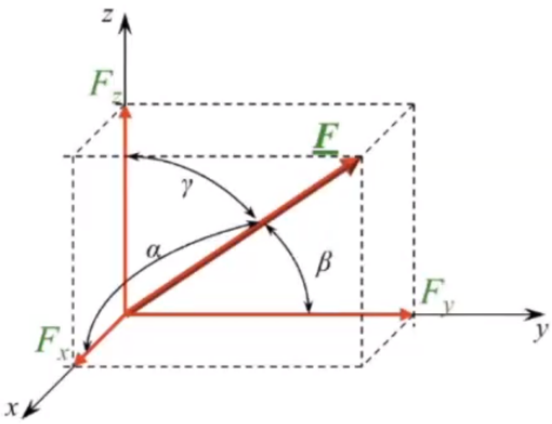
09/10/2020

0.1 Actions and Deformations

0.1.1 Vector Quantities

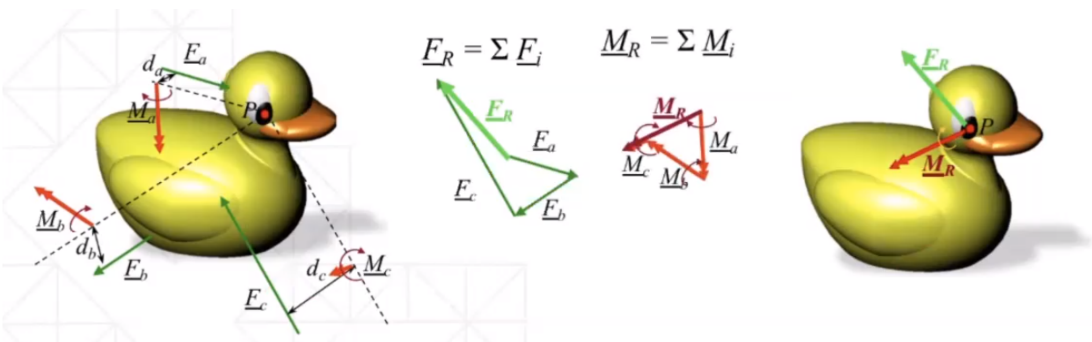
A vector is a quantity defined by **magnitude** and **direction**. Mechanical actions (forces and moments) can be represented as **vectors**.

Vector quantities can be decomposed in components, that can be conveniently oriented with the Cartesian reference system.



$$F_x = F \cdot \cos\alpha \qquad M_x = M \cdot \cos\alpha \tag{1}$$
$$F_y = F \cdot \cos\beta \qquad M_y = M \cdot \cos\beta \tag{2}$$
$$F_z = F \cdot \cos\gamma \qquad M_z = M \cdot \cos\gamma \tag{3}$$

On the other hand, a set of vector forces can be composed in a resultant force applied to any point P, and the moment they produce about P.



$$\vec{F_R} = \sum \vec{F_i} \qquad \vec{M_R} = \sum \vec{F_i} \tag{4}$$
$$F_R = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2} \tag{5}$$
$$M_R = \sqrt{(M_x)^2 + (M_y)^2 + (M_z)^2} \tag{6}$$

0.1.2 Equilibrium State

If a configuration is in equilibrium, the resultant of all external forces and moments is zero. This can be expressed mathematically in the following 6 equations:

$$\begin{array}{ccc} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array}$$

These equations have to be valid for the entire body, and for any of its portions.

0.1.3 Deformations

Mechanical actions produce **deformations** in the body. These can be:

- Tension
- Compression
- Bending
- Twisting

These deformations translate into local strains and are opposed and balanced by internal reaction forces (and stresses), that guarantee the structural congruence of the body.

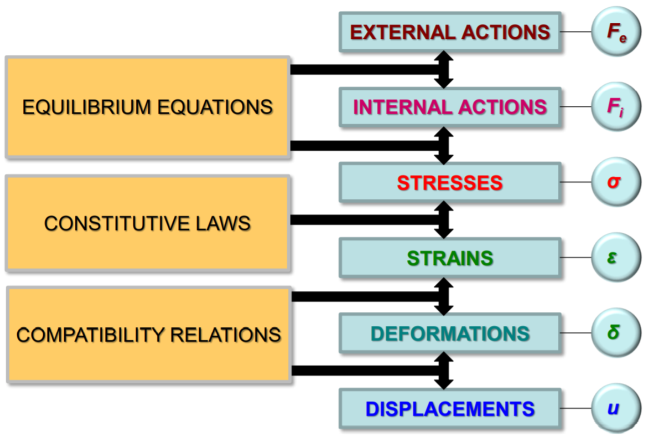
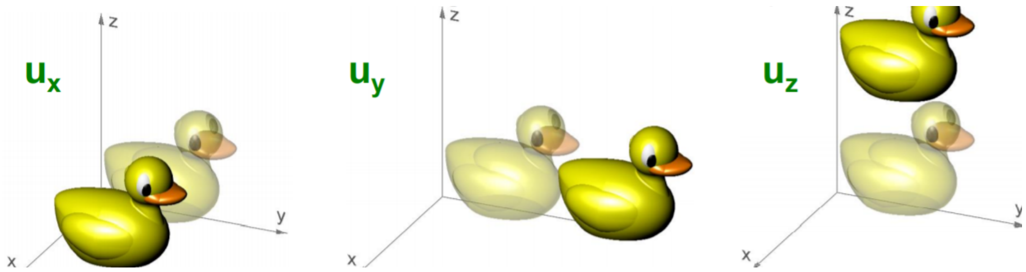


Figure 1: Solid Mechanics Equation: When dealing with mechanical action problems, the actions listed in the flowchart above occur, starting with external/internal forces and ending with displacements/deformations

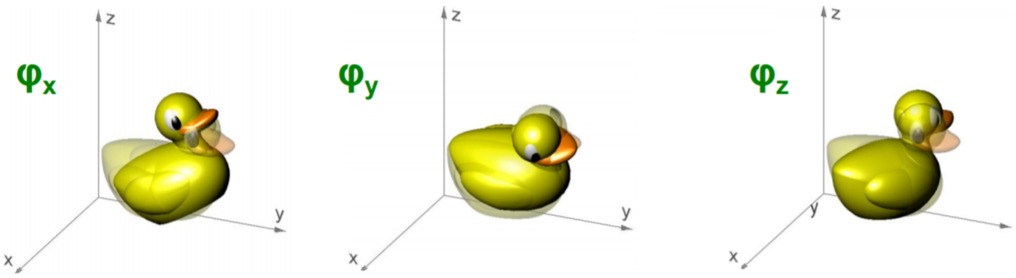
0.2 Degree of Freedom and Supports

We define **degree of freedom** of a system as all the basic kinematical parameters (or all the forms of movement) allowed. A rigid body in the space, in a coordinate system, has 6 degrees of freedom:

3 translations along the coordinate axes x, y and z



3 rotations about the coordinate axes x, y and z



The total translational and rotational movement of an object can be shown with the following expression:

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}$$

In a 2D plane, the degree of freedom reduces to pnly 3 variables:

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \\ \phi_z \end{bmatrix}$$

0.2.1 Constraint

We define **constraint** as a limitation of the degree of freedom of the system. The most common constraints are:

- Supports providing the required reacting forces to maintain overall equilibrium
- Connections providing reaction forces between two components of the system

The table below summarizes the different types of supports (constraints) that will be used throughout the course:

Fixed	Rotating	Roller	Sliding
			
$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ \phi \end{bmatrix}$	$\vec{u} = \begin{bmatrix} u_x \\ 0 \\ \phi \end{bmatrix}$	$\vec{u} = \begin{bmatrix} u_x \\ 0 \\ 0 \end{bmatrix}$
$\vec{R} = \begin{bmatrix} R_x \\ R_y \\ M_z \end{bmatrix}$	$\vec{R} = \begin{bmatrix} R_x \\ R_y \\ 0 \end{bmatrix}$	$\vec{R} = \begin{bmatrix} 0 \\ R_y \\ 0 \end{bmatrix}$	$\vec{R} = \begin{bmatrix} 0 \\ R_y \\ M \end{bmatrix}$

0.3 Beams and Sign Conventions

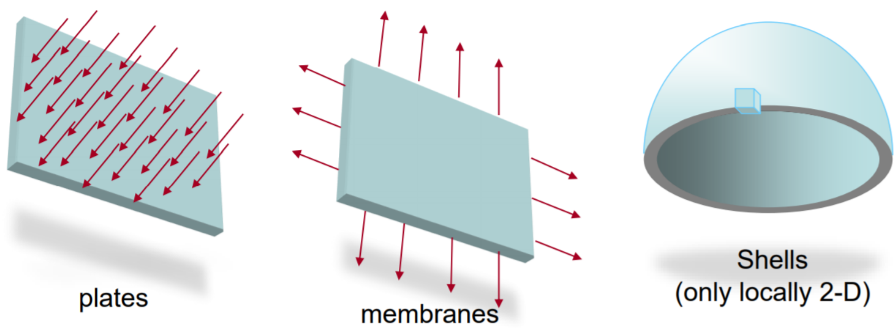
Structures are sets of solid bodies components with the function of carrying loads. All solid bodies are 3-dimensional, however, often it is possible to identify some dimension that is more relevant. Many structures can be analysed as bi-dimensional (2D) or mono-dimensional (1D).

0.3.1 Types of Structures

Bi-dimensional Structures

If one of the dimensions is negligible compared to the other two, the structure can be studied as bi-dimensional. Some examples are:

- Plates
- Membranes
- Shells (Only locally 2D)

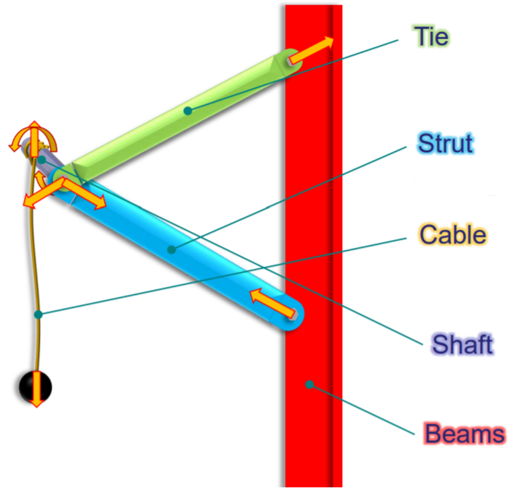


For shells to be considered bi-dimensional, they need to be looked at very closely where they can resemble a plate. The difference between plates and membranes is the bending rigidity; force is required to bend plates while membranes are really floppy.

Mono-dimensional Structures

If two of the dimensions are negligible compared to the other one, the structure can be studied as mono-dimensional. Some examples are:

- Tie - Prevents two parts of the structure from moving away
- Strut - Prevents two parts of the structure from moving forward
- Cable - Flexible string that stands only tensile loads
- Shaft - Is used for the transmission of torque
- Beams - Can carry also transverse loads



0.3.2 Beams

The generic mono-dimensional components of structures, able to carry also transverse loads are called **beams**. Beams are between the most common and important components in structures. In order to study beams as mono-dimensional structures, all mechanical actions have to act on the **centre of gravity (CG)** of the beam section. If there is a case where a force isn't acting on the CG, it will be converted to act on it, so that the beam can be analysed in a simple manner.

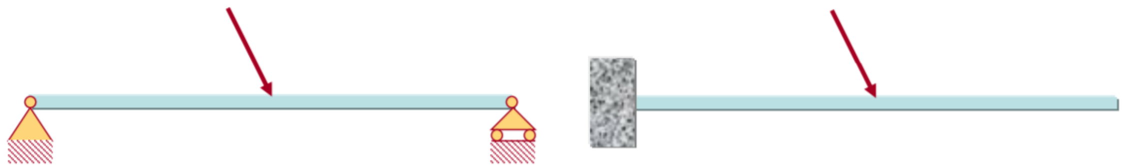
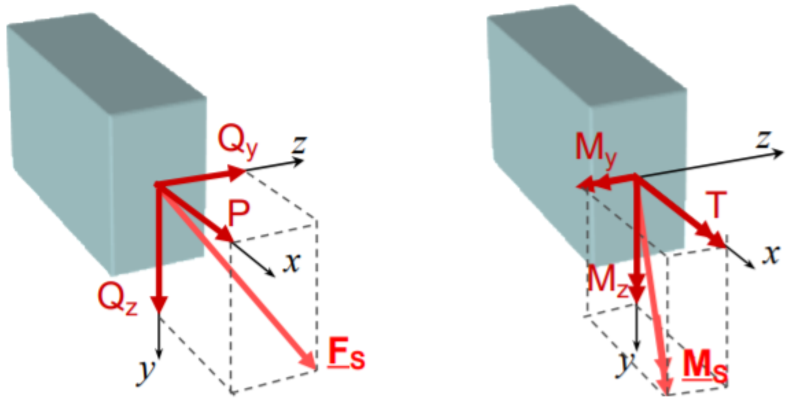


Figure 2: On the left: Simply supported beam (rotating support + roller support), On the right: Cantilever beam (with a fixed end)

0.3.3 Internal Forces

Each point of the beam is characterised by a specific set of internal forces. We consider a cross section of the beam to investigate these forces.

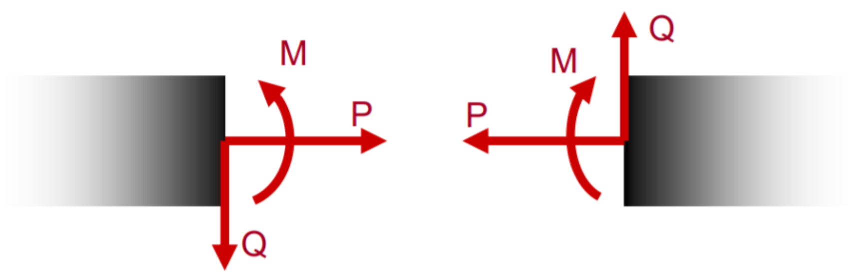


- P - Longitudinal force
- Q_y - y shear forces
- Q_z - z shear forces
- T - Torque
- M_y - y bending moment
- M_z - z bending moment

The internal forces at every point of the beam can be characterised with the following expression:

$$\vec{F} = \begin{bmatrix} P \\ Q_y \\ Q_z \\ \\ T \\ M_y \\ M_z \end{bmatrix}$$

In a 2D plane, it simplifies to:



- P - Longitudinal force
- Q - Shear forces
- M - Bending moment

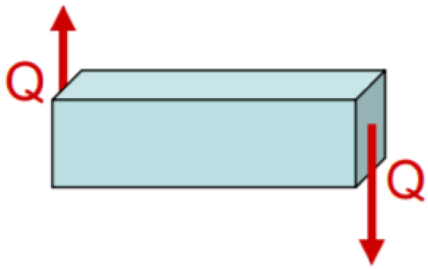
$$\vec{F} = \begin{bmatrix} P \\ Q \\ M \end{bmatrix}$$

0.3.4 Sign Conventions

Longitudional Force

The direction of pulling is considered to be positive.
The direction of compression is considered to be negative.

Shear Force

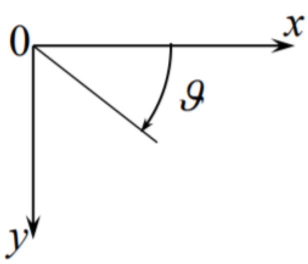


The left side pointing upwards and right side pointing downwards are taken as positive.
The left side pointing downwards and right side pointing upwards are taken as negative.

Bending Moment

If the bending moment is making the beam a upwards concave shape (like U), it is considered to be positive.
If the bending moment is making the beam a downwards concave shape, it is considered to be negative.

Reference System



x axis (horizontal direction) to the right is taken as positive.
y axis (vertical direction) downwards is taken as positive.