UCL Mechanical Engineering 2020/2021
MECH0013 Final Assessment

NCWT3

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1 Question 1

1.1 i

Straight section analysis:

Equilibrium conditions:

$$\sum F_x : R_{Ax} = 0 \tag{1.1}$$

$$\sum F_y : R_{Ay} + R_B + P = W \tag{1.2}$$

$$\sum M_A : -M_A + M_C + 4WL = 2R_BL + 3PL \tag{1.3}$$

Using Macaulay's method:

$$M = -M_A + R_{Ay}x + R_B < x - 2L > +P < x - 3L > \tag{1.4}$$

Slope:

$$\theta = \frac{1}{EI} \int (M) \, \mathrm{d}x \tag{1.5}$$

$$\theta = \frac{1}{EI} \int \left(-M_A + R_{Ay}x + R_B < x - 2L > +P < x - 3L >\right) \mathrm{d}x \tag{1.6}$$

$$\theta = \frac{1}{EI} \left[-M_A x + \frac{R_{Ay}x^2}{2} + \frac{R_b < x - 2L >^2}{2} + \frac{P < x - 3L >^2}{2}\right] + \theta_0 \tag{1.7}$$

Deflection:

$$y = \int (\theta) \, \mathrm{d}x \tag{1.8}$$

$$y = \int \left(\frac{1}{EI} \left[-M_A x + \frac{R_{Ay}x^2}{2} + \frac{R_b < x - 2L >^2}{2} + \frac{P < x - 3L >^2}{2}\right] + \theta_0\right) \mathrm{d}x \tag{1.9}$$

$$y = \frac{1}{EI} \left[-\frac{M_A x^2}{2} + \frac{R_{Ay}x^3}{6} + \frac{R_B < x - 2L >^3}{6} + \frac{P < x - 3L >^3}{6}\right] + \theta_0 x + y_0 \tag{1.10}$$

Curved section analysis:

$$M(\theta) = WR\,(1 - \cos\theta) + H_0 R \sin\theta + M_0 \tag{1.11}$$

$$\frac{\partial M}{\partial M_0} = 1 \tag{1.12}$$

$$\varphi_A = \int_0^L \left(\frac{M}{EI}\frac{\partial M}{\partial M_0}\right) \mathrm{d}x = 0 \tag{1.13}$$

Converting to polar ($\mathrm{d}x = R\,\mathrm{d}\theta$):

$$\varphi_A = \frac{1}{EI} \int_0^{\frac{\pi}{2}} \Big(\big(WR\,(1 - \cos\theta) + H_0 R \sin\theta + M_0\big)\,(1)\,(R)\Big) \,\mathrm{d}\theta \tag{1.14}$$

M_0 and H_0 represent dummy loads and can be neglected:

$$\varphi_A = \frac{1}{EI} \int_0^{\frac{\pi}{2}} \Big(WR^2\,(1 - \cos\theta)\Big) \,\mathrm{d}\theta \tag{1.15}$$

$$\varphi_A = \frac{1}{EI} \Big[WR^2\,(\theta - \sin\theta)\Big]_0^{\frac{\pi}{2}} \tag{1.16}$$

$$\varphi_A = \frac{WR^2\left(\frac{\pi}{2} - 1\right)}{EI} \tag{1.17}$$

Boundary conditions:

$$x = 0,\ y = 0 \therefore y_0 = 0 \tag{1.18}$$

$$x = 0,\ \theta = 0 \therefore \theta_0 = 0 \tag{1.19}$$

$$x = 2L,\ y = 0 \tag{1.20}$$

$$x = 4l,\ \theta = \frac{WR^2\left(\frac{\pi}{2} - 1\right)}{EI} \tag{1.21}$$

From 1.20:

$$0 = \frac{1}{EI}\left[-\frac{M_A\left(2L\right)^2}{2} + \frac{R_{Ay}\left(2L\right)^3}{6} + \frac{R_B < 2L - 2L >^3}{6}\right] \tag{1.22}$$

$$0 = \frac{1}{EI}\left[-2M_AL^2 + \frac{4R_{Ay}L^3}{3} + 0\right] \tag{1.23}$$

$$0 = -2M_AL^2 + \frac{4R_{Ay}L^3}{3} \tag{1.24}$$

$$M_A = \frac{2R_{Ay}L}{3} \tag{1.25}$$

From 1.21:

$$\frac{WR^2\left(\frac{\pi}{2}-1\right)}{EI} = \frac{1}{EI}\left[-M_A\left(4L\right) + \frac{R_{Ay}\left(4L\right)^2}{2} + \frac{R_B < 4L - 2L >^2}{2} + \frac{P < 4L - 3L >^2}{2}\right] \tag{1.26}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = -4M_AL + 8R_{Ay}L^2 + 2R_BL^2 + \frac{PL^2}{2} \tag{1.27}$$

Substituting 1.25:

$$WR^2\left(\frac{\pi}{2}-1\right) = -\frac{8R_{Ay}L^2}{3} + 8R_{Ay}L^2 + 2R_BL^2 + \frac{PL^2}{2} \tag{1.28}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3} + 2R_BL^2 + \frac{PL^2}{2} \tag{1.29}$$

From 1.2:

$$R_B = W - P - R_{Ay} \tag{1.30}$$

Substituting 1.30:

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3} + 2\left(W - P - R_A\right)L^2 + \frac{PL^2}{2} \tag{1.31}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3} + 2WL^2 - 2PL^2 - 2R_AL^2 + \frac{PL^2}{2} \tag{1.32}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{10R_{Ay}L^2}{3} + 2WL^2 - \frac{3PL^2}{2} \tag{1.33}$$

$$R_{Ay} = \frac{3}{4}\left(P - \frac{WR}{L} - 2W\right) \tag{1.34}$$

Substituting 1.34:

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{10L^2}{3}\left(\frac{3}{4}\left(P - \frac{WR}{L} - 2W\right)\right) + 2WL^2 - \frac{3PL^2}{2} \tag{1.35}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{5PL^2}{2} - \frac{5WRL}{2} - 5WL^2 + 2WL^2 - \frac{3PL^2}{2} \tag{1.36}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = PL^2 - 3WL^2 - \frac{5WRL}{2} \tag{1.37}$$

Rearranging for P:

$$PL^2 = 3WL^2 + \frac{5WRL}{2} + WR^2\left(\frac{\pi}{2} - 1\right) \tag{1.38}$$

$$P = 3W + \frac{5WR}{2L} + \frac{WR^2}{L^2}\left(\frac{\pi}{2} - 1\right) \tag{1.39}$$

$$P = 3(42.67) + \frac{5(42.67)(0.3)}{2(0.35)} + \frac{(42.67)(0.3)^2}{(0.35)^2}\left(\frac{\pi}{2} - 1\right) \tag{1.40}$$

$$P = 237.34\,\mathrm{N} \tag{1.41}$$

1.2 ii

2 Question 2

2.1 ii

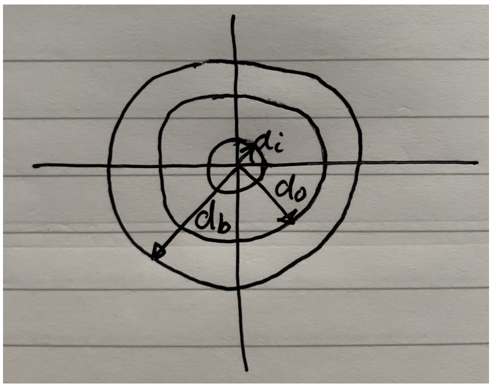

Figure 1: Sketch of cylinder arrangement.

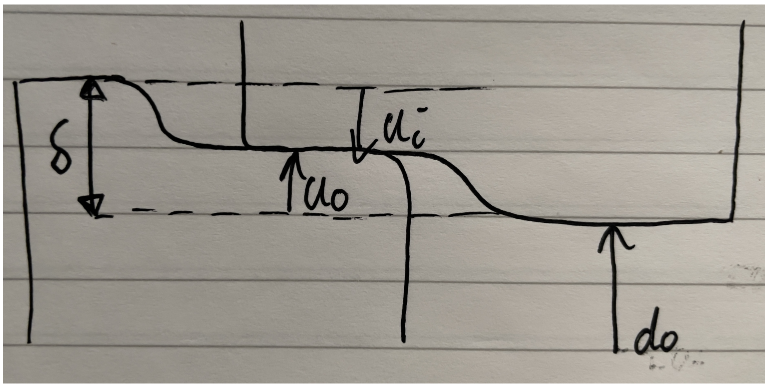

Figure 2: Diagram to show interference.

Hence:

$$\delta = u_o - u_i \tag{2.1}$$

Finding u_i. Let us start with the general equation for u:

$$u = \frac{r}{E}\left(\sigma_\theta - v\sigma_r\right) \tag{2.2}$$

$$\rightarrow u_i = \frac{d_0}{E_i}\left[\sigma_{\theta,i}\left(d_0\right) - v_i\sigma_{r,i}(d_0)\right] \tag{2.3}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{2.4}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{2.5}$$

Boundary conditions:

$$\sigma_{r,i}(d_i) = 0 = A - \frac{B}{d_i^2} \tag{2.6}$$

$$\sigma_{r,i}(d_0) - -p_{int} = A - \frac{B}{d_0^2} \tag{2.7}$$

$$A = -p_{int}\frac{d_0^2}{d_0^2 - d_i^2} \tag{2.8}$$

$$B = -p_{int}\frac{d_i^2 \cdot d_0^2}{d_0^2 - d_i^2} \tag{2.9}$$

Substituting:

$$\sigma_{\theta,i} = A + \frac{B}{r^2} = -p_{int}\frac{d_0^2}{d_0^2 - d_i^2}\left(1 + \frac{d_i^2}{r^2}\right) \tag{2.10}$$

Therefore:

$$\sigma_{r,i}(d_0) = -p_{int} \qquad \sigma_{\theta,i}(d_0) = -p_{int}\frac{d_0^2 + d_i^2}{d_0^2 - d_i^2} \tag{2.11}$$

$$u_i = -p_{int}\frac{d_0}{E_i}\left(\frac{d_0^2 + d_i^2}{d_0^2 - d_i^2} - v_i\right) \tag{2.12}$$

Repeating to find u_o:

$$u_o = \frac{d_0}{E_o}\left[\sigma_{\theta,o}(d_0) - v_o\sigma_{r,o}(d_0)\right] \tag{2.13}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{2.14}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{2.15}$$

Boundary conditions:

$$\sigma_{r,o}(d_0) = -p_{int} = A - \frac{B}{d_0^2} \tag{2.16}$$

$$\sigma_{r,o}(d_b) = 0 = A - \frac{B}{d_b^2} \tag{2.17}$$

$$A = -p_{int}\frac{d_0^2}{d_0^2 - d_b^2} \tag{2.18}$$

$$B = -p_{int}\frac{d_0^2 \cdot d_b^2}{d_b^2 - d_0^2} \tag{2.19}$$

Substituting:

$$\sigma_{\theta,o} = A + \frac{B}{r^2} = -p_{int}\frac{d_0^2}{d_b^2 - ^2}\left(1 + \frac{d_b^2}{r^2}\right) \tag{2.20}$$

Therefore:

$$\sigma_{r,o}(d_b) = 0 \qquad \sigma_{\theta,o}(d_0) = p_{int}\frac{d_b^2 + d_0^2}{d_b^2 - d_0^2} \tag{2.21}$$

$$u_o = p_{int}\frac{d_0}{E_o}\left(\frac{d_b^2 + d_0^2}{d_b^2 - d_0^2} + v_o\right) \tag{2.22}$$

Finding δ:

$$u_i = -p_{int}\frac{d_0}{E_i}\left(\frac{d_0^2 + d_i^2}{d_0^2 - d_i^2} - v_i\right) \tag{2.23}$$

$$u_o = p_{int}\frac{d_0}{E_o}\left(\frac{d_b^2 + d_0^2}{d_b^2 - d_0^2} + v_o\right) \tag{2.24}$$

$$\delta = p_{int}d_0\left[\frac{1}{E_o}\left(\frac{d_b^2 + d_0^2}{d_b^2 - d_0^2} + v_o\right) + \frac{1}{E_i}\left(\frac{d_0^2 + d_i^2}{d_0^2 - d_i^2} - v_i\right)\right] \tag{2.25}$$

For a single material: $E_i = E_o = E$, and $v_i = v_o = v$:

$$\delta = p_{int}\frac{d_0}{E}\left[\frac{2d_0^2\left(d_b^2 - d_i^2\right)}{\left(d_b^2 - d_0^2\right)\left(d_0^2 - d_i^2\right)}\right] \tag{2.26}$$

$$p_{int} = E\delta\left[\frac{\left(d_b^2 + d_0^2\right)\left(d_0^2 - d_b^2\right)}{2d_0^3\left(d_b^2 - d_i^2\right)}\right] \tag{2.27}$$