Exercise 2 - Fluid tutorial group B

i

$$\psi = Ar^n \sin(n\theta) \tag{1}$$

$$u_\theta = -\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{2}$$

$$u_\theta = -nAr^{n-1} \sin(n\theta) \tag{3}$$

$$-nAr^{n-1} \sin(n\theta) = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{4}$$

$$\int \left(-nAr^n \sin(n\theta)\right) \, \mathrm{d}\theta = \int \left(\frac{\partial \phi}{\partial \theta}\right) \, \mathrm{d}\theta \tag{5}$$

$$\phi = Ar^n \cos(n\theta) + c \tag{6}$$

ii

$$\nabla \cdot \vec{V} = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \tag{7}$$

$$\frac{\partial \phi}{\partial r} = nAr^{n-1} \cos(n\theta) \tag{8}$$

$$\frac{\partial^2 \phi}{\partial r^2} = n(n-1)Ar^{n-2} \cos(n\theta) \tag{9}$$

$$\frac{\partial \phi}{\partial \theta} = -nAr^n \sin(n\theta) \tag{10}$$

$$\frac{\partial^2 \phi}{\partial \theta^2} = -n^2 Ar^n \cos(n\theta) \tag{11}$$

$$n(n-1)Ar^{n-2} \cos(n\theta) + nAr^{n-2} \cos(n\theta) - n^2 Ar^{n-2} \cos(n\theta) = 0 \tag{12}$$

$$Ar^{n-2} \cos(n\theta) \left[n(n-1) + n - n^2\right] = 0 \tag{13}$$

$$n^2 - n + n - n^2 = 0 \tag{14}$$

$$\therefore Ar^{n-2} \cos(n\theta) \times 0 = 0 \tag{15}$$

$$0 = 0 \tag{16}$$

Continuity equation satisfied.

iii

$$180 = 120n \tag{17}$$

$$n = \frac{3}{2} \tag{18}$$

iv

$$p + \frac{1}{2}\rho\left(u_r^2 + u_\theta^2\right) = c \tag{19}$$

$$u_r = \frac{\partial \phi}{\partial r} = nAr^{n-1}\cos(n\theta) \tag{20}$$

$$u_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -nAr^{n-1}\sin(n\theta) \tag{21}$$

$$\text{A: } 20000 + \frac{1}{2}(1000)\left(\frac{9}{4} + 0\right) = 21125 \tag{22}$$

$$\text{B: } p + \frac{1}{2}(1000)\left(\frac{9}{4} + \frac{9}{4}\right) = 21125 \tag{23}$$

$$p = 18875 \text{ Pa} \tag{24}$$

v

$$\psi = Ar^n \sin(n\theta) \tag{25}$$

$$\psi = 1 \times 2^{\frac{3}{2}} \sin(\frac{3}{2} \times 30) = 2 \tag{26}$$

$$\tag{27}$$