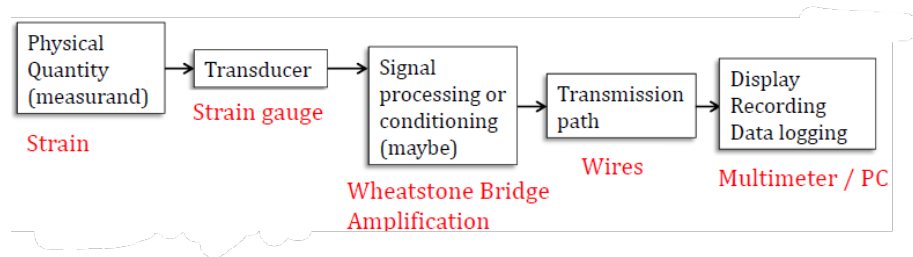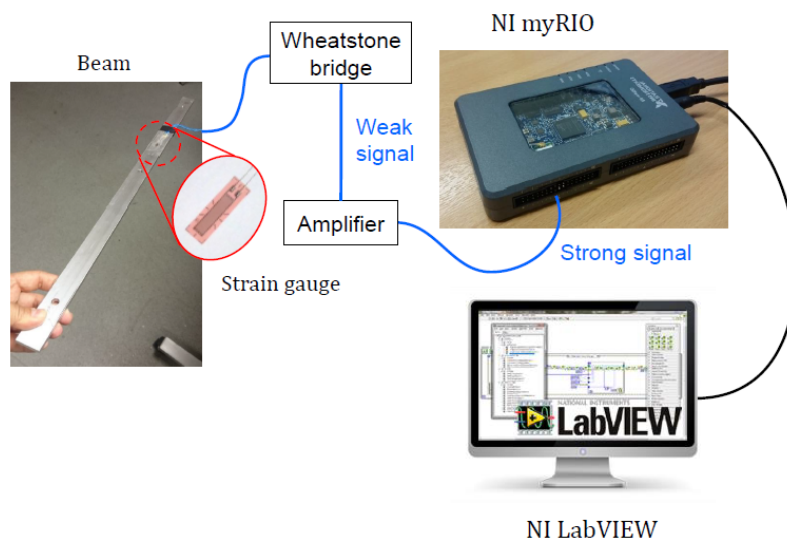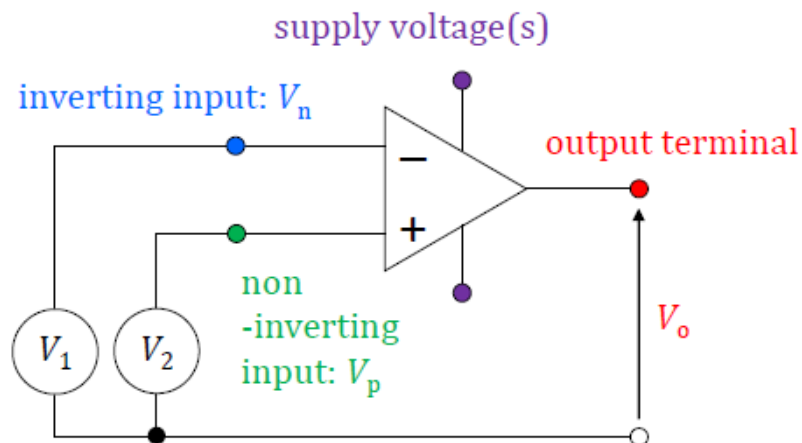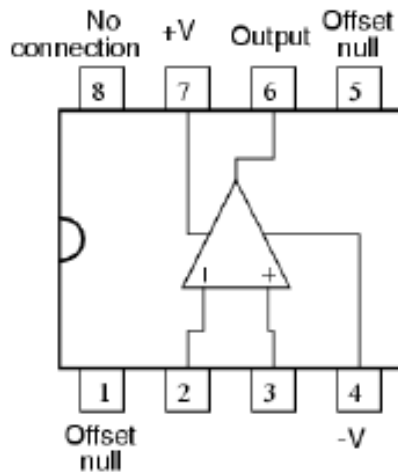## Stages in a measurement system

0.1 Amplifiers

The voltage output from a transducer tends to be weak and this needs to be amplified. Operational amplifiers are normally around 1cm in size.

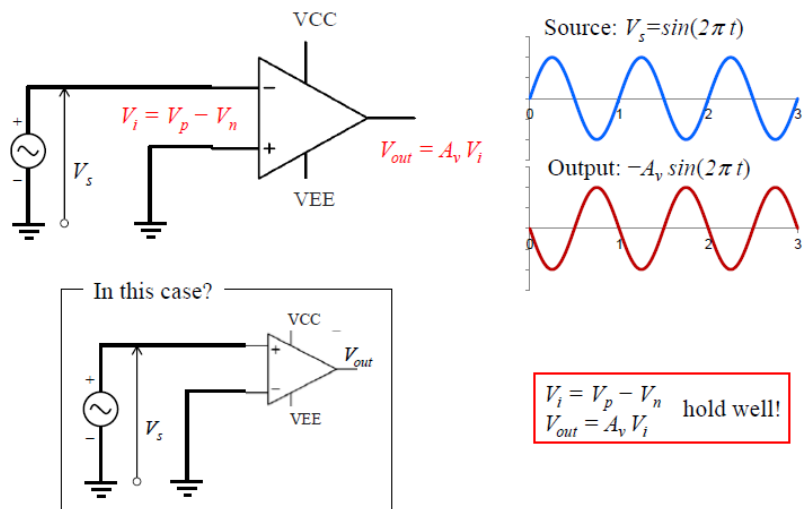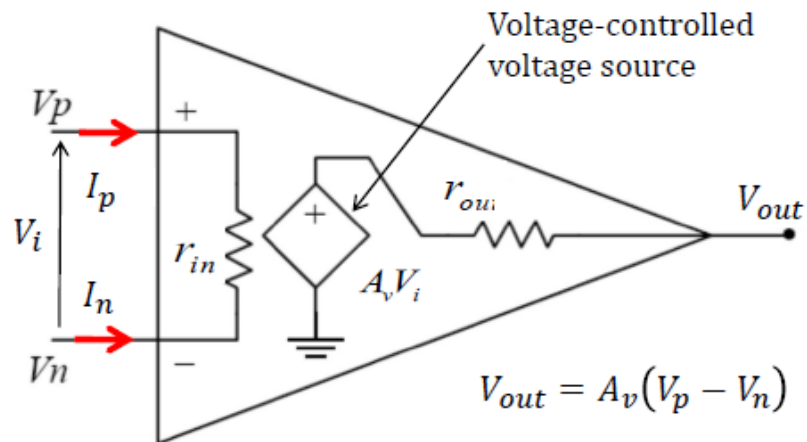
$$V_0 = A_v(V_p - V_n) = A_v(V_2 - V_1) \tag{1}$$

Where A_v is the gain and $(V_p - V_n)$ can be from a Wheatstone bridge. Operational amplifiers are quite an old technology (since WW2) and are still used because they are cheap and convenient.

No connection +V Output Offset null

| 8 | 7 | 6 | 5 |

| 1 | 2 | 3 | 4 |

Offset null -V

Pin-out for a 8-pin op-amp

VCC

$V_i = V_p - V_n$

V_s

$V_{out} = A_v V_i$

VEE

Source: $V_s = sin(2\pi t)$

Output: $-A_v \, sin(2\pi t)$

In this case?

VCC

V_{out}

V_s

VEE

$V_i = V_p - V_n$
$V_{out} = A_v V_i$ hold well!

0.1.1 Equivalent op-amp circuit and conditions of an ideal op-amp

Voltage-controlled voltage source

Vp

I_p

V_i

r_{in}

I_n

Vn

r_{oui}

$A_v V_i$

V_{out}

$$V_{out} = A_v(V_p - V_n)$$

Terminology:

- Differential input voltage: $V_i = V_p - V_n$

2

- Input resistance: r_{in}

- Output resistance: r_{out}

- Open-circuit output voltage: V_{out}

- Differential voltage gain: A_v

| Conditions of an ideal op-amp	In reality
No current into input terminals: $I_p = I_n = 0$	
Infinite input resistance: $r_{in} \to \infty$	$r_{in} > 200\text{k}\Omega$
Zero Output resistance $r_{out} = 0$	$r_{out} < 1\text{k}\Omega$
Infinite differential (or open-loop) gain $A_v \to \infty$	$A_v > 100,000$
Zero common-mode voltage gain $A_{cm} = 0$	A_v also frequency dependant

0.1.2   Non-zero voltage with zero current?
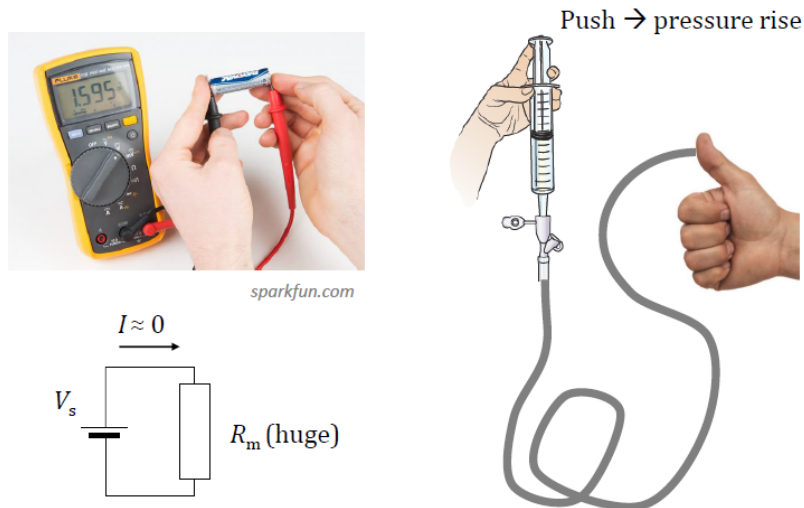
A question that is often asked is if the current is zero, how can we still have a voltage? When you connect a battery to a meter, where the meter resistance is normally huge. We can still measure the voltage. This is due to Ohm's law $I = \frac{V}{R_m}$. A good analogy to make is a plunger system, where the other end is covered. When you push down, increasing the pressure, this can still be felt on the tip of the thumb, despite there being no flow. In the same way, we can still 'feel' the voltage with 'no' current flow.

0.1.3   Saturation

In a practical scenario, we want to strengthen the weak signal from the sensor. Let us connect an input to the non-inverting terminal and the inverting input to the ground.
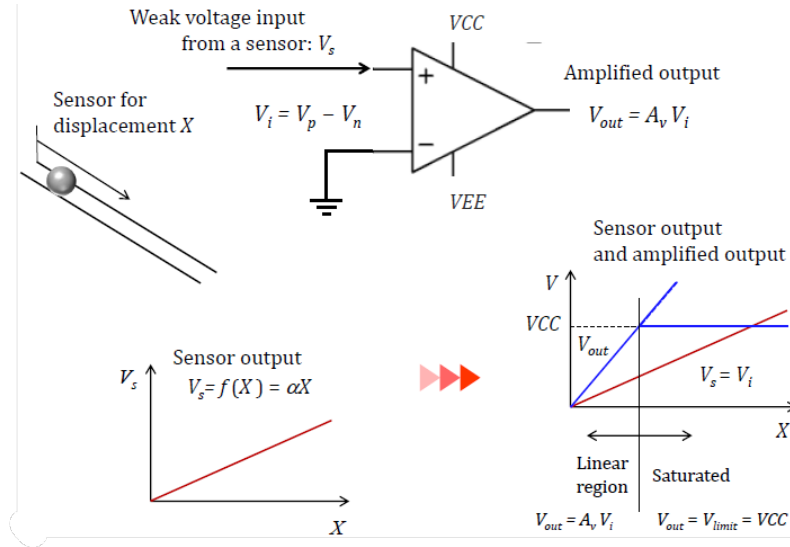
We can see that the signal is not infinitely amplified, as we are limited by the energy provided by our supply voltage. This is reflected in the graph above as the straight portion of the amplified line. This is called saturation. Due to A_v being quite large, our linear region is normally tiny. This may be a problem if we want to measure the linear region.

0.1.4 Negative feedback

So op-amps seem to be a useful device but also seem to be a bit tricky. They are much more useful when used as part of a larger circuit. Negative feedback is achieved by feeding a fraction of the output signal back to the **inverting** terminal as shown. By definition, the closed-loop gain of such a device is:

$$G = \frac{V_0}{V_i} \tag{2}$$
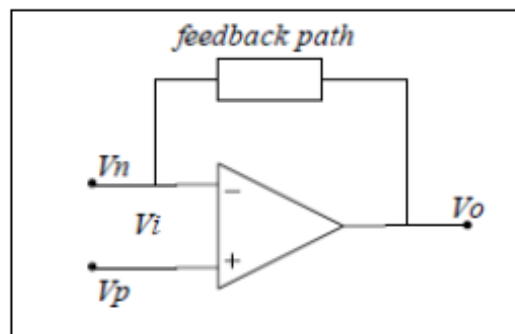
Negative feedback trades a reduction in gain with one that is smaller, stable and predictable, whilst giving the designer control over operational characteristics.

0.1.5 Basic feedback amplifier configurations

Only 2 resistors are required to construct a negative feedback amplifier. Note that output signal is always fed-back to the inverting input terminal.
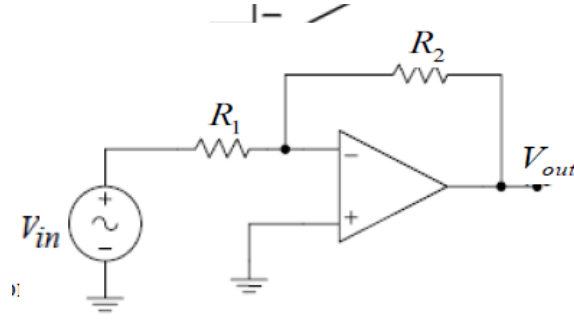
4

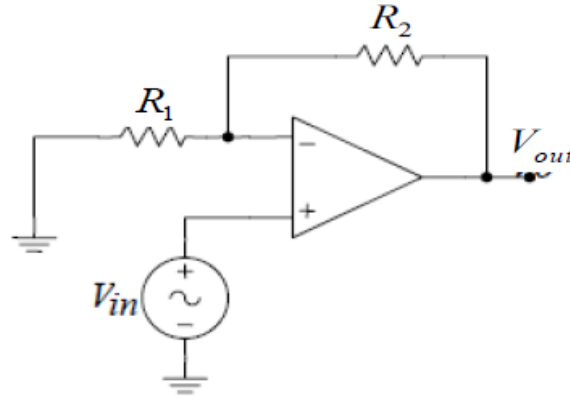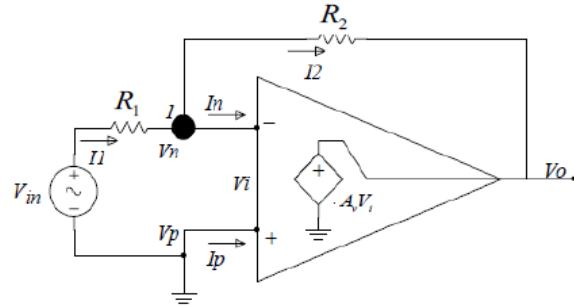Figure 1: Inverting feedback amplifier arrangement.

Figure 2: Non-inverting feedback amplifier arrangement.

We can see the difference between an inverting and non-inverting feedback amplifier is which input the source signal is connected to.

0.1.6 Basic inverting feedback amplifier

Let us derive its closed loop gain G. By definition:

$$V_0 = A_v(V_p - V_n) \tag{3}$$

$$V_p \to 0 \Longrightarrow V_n = -\frac{V_0}{A_v} \tag{4}$$

Apply KCL to Node 1:

$$\frac{V_{in} - V_n}{R_1} - I_n - \frac{V_n - V_0}{R_2} = 0 \Longrightarrow \frac{R_2}{R_1}V_{in} + v_n\left(1 + \frac{R_2}{R_1}\right) = V_0 \tag{5}$$
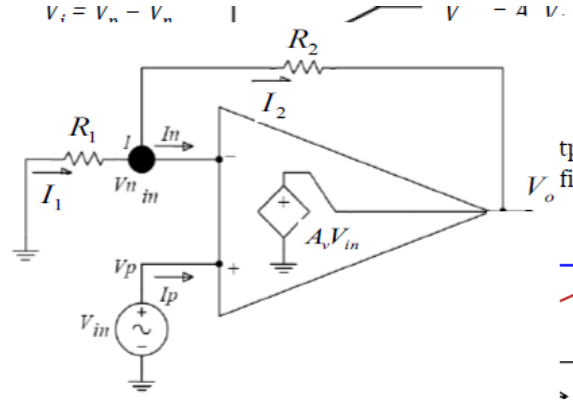
5

I_n is 0 from our model assumptions. Combining equations (4) and (5), we arrive at:

$$G \equiv \frac{V_0}{V_{in}} = -\frac{R_2}{R_1}\frac{1}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)} \tag{6}$$

For an ideal op-amp, $A_v \to \infty$:

$$G = \frac{V_0}{V_{in}} = -\frac{R_2}{R_1} \tag{7}$$

0.1.7   Basic non-inverting feedback amplifier

Derive its closed-loop gain G. By definition:

$$V_0 = A_v(V_p - V_n) \tag{8}$$

Apply KCL to node 1

$$\frac{0 - V_n}{R_1} = \frac{V_n - V_0}{R_2} \rightarrow \frac{V_0}{R_2} = V_n\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{9}$$

Combining equation (8) and (9), we arrive at:

$$G \equiv \frac{V_0}{V_{in}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)} \tag{10}$$

For an ideal amplifier, $A_v \to \infty$:

$$G = 1 + \frac{R_2}{R_1} \tag{11}$$

This is positive and always greater than 1.

0.1.8   Summary

Ideal feedback amplifier characteristics

	Inverting amplifier	Non-inverting amplifier
$G = \frac{V_0}{V_{in}}$	$-\frac{R_2}{R_1}$	$1 + \frac{R_2}{R_1}$
R_{in}	R_1	∞
R_{out}	0	0

0.1.9   Voltage amplification summary

Operational amplifier:

- The gain usually is too large (typically ¿ 100,000) and not well defined. (V_0 cannot be larger than supply voltage)

- The gains for two op-amps of the same type can differ by a factor of two or more.

- The gain is not stable, can change with temperature, can change with changes of supply voltage.

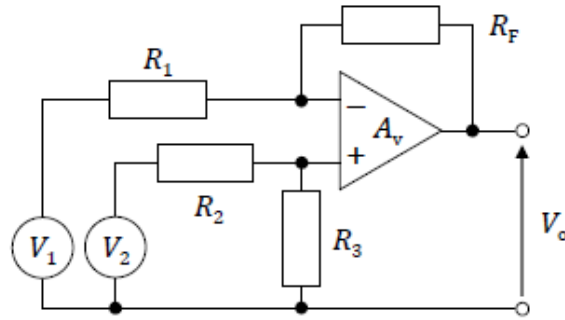To reduce these effects, we use negative feedback.

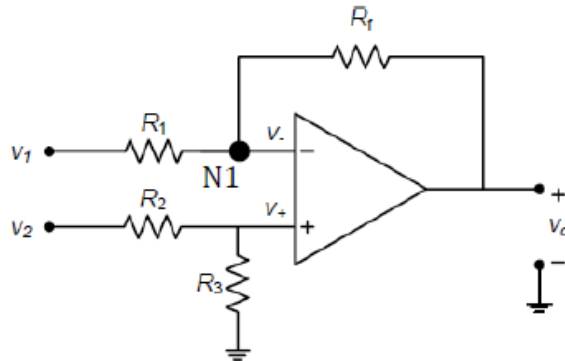Figure 3: V_1 and V_2 could come from a Wheatstone bridge.

The output voltage V_0 is given as:

$$V_0 = \frac{R_F}{R_1}(V_2 - V_1) \tag{12}$$

Only when $\frac{R_2}{R_3} = \frac{R_F}{F_1}$. This is independent of A_v and the gain of this circuit, $G = \frac{R_F}{R_1}$ is well controllable.

0.1.10   Difference amplifier

Combining a non-inverting and inverting op-amp to take the difference between two inputs:

$$\frac{V_0}{A_v} = V_+ - V_- \to 0 \text{ as } A_v \to \infty \tag{13}$$

$$V_+ = \frac{R_3}{R_2 + R_3} \leftarrow \text{ voltage divider} \tag{14}$$

KCL at node N1

$$\frac{V_0 - V_-}{R_f} = \frac{V_- V_1}{R_1} \to \frac{V_0}{R_f} + \frac{V_1}{R_1} = \left(\frac{1}{R_f} + \frac{1}{R_1}\right) V_- \tag{15}$$

$$\frac{V_0}{R_f} + \frac{V_1}{R_1} = \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) V_2 \tag{16}$$

Combining equation (14) and (16), we arrive at:

$$V_0 = R_f \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) V_2 - \frac{R_f}{R_1} V_1 \tag{17}$$

$$V_0 = \frac{1 + \frac{R_f}{R_1}}{1 + \frac{R_2}{R_3}} V_2 - \frac{R_f}{R_1} V_1 \tag{18}$$

If we can make sure that $\frac{R_f}{R_1} = \frac{R_3}{R_2}$ is satisfied, we can further reduce our equation to:

$$V_0 = \frac{R_f}{R_i}(V_2 - V_1) \tag{19}$$