

26/01/2021

The study of systems involving **moist air** (dry air + water vapour) is called psychrometrics.

0.1 Air-Water Vapour Mixtures

Moist air is found in:

- Nature;
- HVAC (heating, ventilation and air-conditioning) processes, where moisture (water vapour) is added or removed from the air;
- Steam turbines; and so on.

Moist air is a mixture of dry air and water vapour, where the **amount of dry air** is usually kept **constant** and the **amount of water vapour** may **change**. Two assumption are employed for air-water vapour properties :

1. **Dalton's law** applies for the mixture, i.e. partial pressure $\rightarrow p = p_a + p_v$
2. Air and water vapour are each treated as **ideal gases**, so is **their mixture**.

0.2 Moist Air

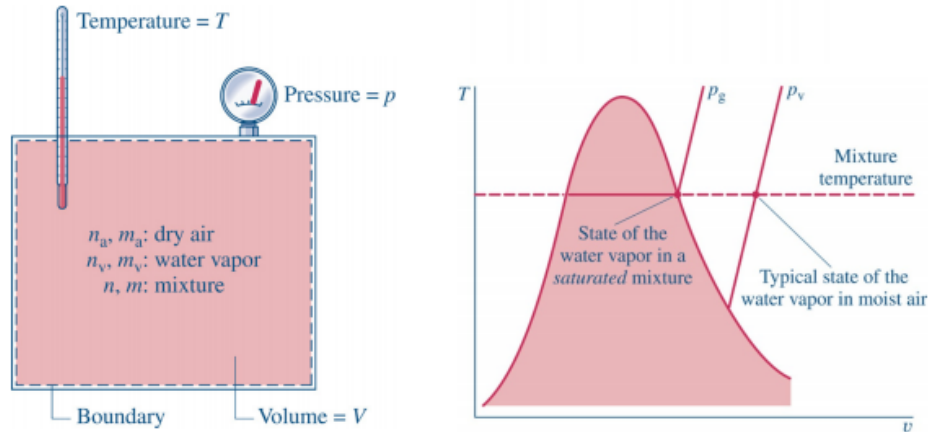


Figure 1:

Equation of state of ideal gases:

$$p = \frac{nR_u T}{V} = \frac{m(\frac{R_u}{M})T}{V} \quad (0.2.1)$$

$$p_a = \frac{n_a R_u T}{V} = \frac{m_a(\frac{R_u}{M_a})T}{V} \quad p_a = y_a p \quad (0.2.2)$$

$$p_v = \frac{n_v R_u T}{V} = \frac{m_v(\frac{R_u}{M_v})T}{V} \quad p_v = y_v p \quad (0.2.3)$$

p_v and T :

- Typical state: vapor is superheated;
- At $p_v = p_g$ (saturation pressure at T) \rightarrow Saturated Air (mixture of dry air and saturated water vapour); the mixture is said to be **saturated**.

0.2.1 Moist Air Properties

For a single component:

- **Two** independent, intrinsic properties are needed to define the state

For a two-component mixture (e.g. air-water vapor):

- **Three** independent, intrinsic properties are needed to define the state
 - Usually, two out of temperature, pressure and specific volume.
 - Plus humidity ratio ω or relative humidity ϕ

0.3 Specific Humidity

The **specific humidity** or **humidity ratio** ω is defined as:

$$\omega = \frac{m_v}{m_a} = \frac{\text{mass of water vapour}}{\text{mass of dry air}} \quad (0.3.1)$$

Using ideal gas law for both water vapour and dry air:

$$\omega = \frac{R_u T}{M_a p_a V} \frac{M_v p_v V}{R_u T} = \frac{M_v p_v}{M_a p_a} \quad (0.3.2)$$

Where, T is temperature of the mixture measured by a convetional thermometer, called the dry bulb temperature denoted by T_{db} .

The molecular weights:

$$M_v = 18 \quad M_a = 28.96 \quad (0.3.3)$$

$$\frac{M_v}{M_a} \approx 0.622 \quad (0.3.4)$$

Dalton's Law:

$$p = p_a + p_v \quad (0.3.5)$$

$$\therefore p_a = p - p_v \quad (0.3.6)$$

Specifiv Humidity:

$$\omega = 0.622 \frac{p_v}{p - p_v} \quad (0.3.7)$$

0.3.1 Moist Air Properties

The humidity ratio is a parameter for determining other quantities:

Mass:

$$m = m_m = m_a + m_v = m_a + \omega m_a \quad (0.3.8)$$

$$\therefore m = (1 + \omega)m_a \quad (0.3.9)$$

Energy:

$$H = H_m = m_a h_a + m_v h_v \quad (0.3.10)$$

$$\therefore H = m_a (h_a + \omega h_v) \quad (0.3.11)$$

$$h = h_m = \frac{H}{m_a} = h_a + \omega h_v \quad (0.3.12)$$

Humidity ratio is measured by a **hygrometer** in which a moist air sample is exposed to suitable chemicals until the moisture present is absorbed. The amount of water vapour is determined by weighing the chemicals.

0.4 Relative Humidity

$$\phi = \frac{\text{partial pressure of water vapour at a temperature } T \text{ (in superheated state)}}{\text{saturation pressure of water vapour at the same temperature } T \text{ (or } T_{db})} \quad (0.4.1)$$

$$\phi = \frac{p_v}{p_g} \quad (0.4.2)$$

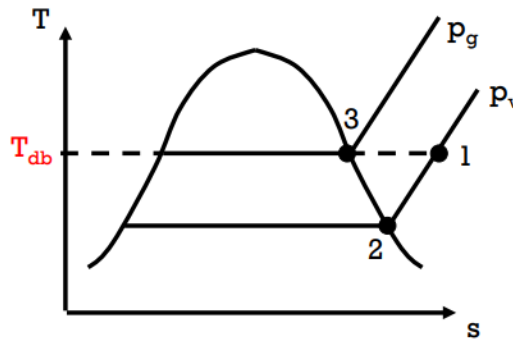


Figure 2:

Consider ideal gas mixture:

$$\phi = \frac{p_v}{p_g} = \frac{R_v T_v}{v_v} \frac{v_g}{R_v T_v} = \frac{v_g}{v_v} \quad (0.4.3)$$

$$\phi = \frac{v_g}{v_v} = \frac{p_v}{p_g} \leq 1.0 \quad \text{always} \quad (0.4.4)$$

0.4.1 An Alternative Definition

$$\phi = \frac{\text{mole fraction of water vapour at a given } T \text{ and } P \text{ (in superheated state)}}{\text{mole fraction of water vapour at saturation at the same } T \text{ and } P} \quad (0.4.5)$$

$$\phi = \frac{y_v}{y_{v,sat}} = \frac{y_v P}{y_{v,sat} P} = \frac{p_v}{p_g} \quad (0.4.6)$$

Relative humidity can be measured by **transducers** whose electrical characteristics change with relative humidity.

0.4.2 Condensation of Moist Air

Partial condensation of moist air can occur when temperature is reduced below a certain level:



Figure 3: Left: Condensate on windowpanes — Right: Dew on grass

The condensation process can be divided into the following stages:

- Superheated water vapour (state 1) is **cooled under constant system pressure** (thus the composition of moist air and partial pressure of vapour remain constant), until it reaches **the dew point** (state d);
 - The saturation temperature corresponding to p_v is called the dew point temperature (T_{dp}).
- Vapour starts to **condense when the moist air is cooled below T_{dp}** .
- Further cooling leads to more condensate until the dry air, saturated vapour and liquid water **reach equilibrium**.
- At the final state, the system consists of **saturated air** (state 2) and **saturated liquid** (state 3) at the final temperature. The saturated vapour (state 2) has a partial pressure of the saturation pressure p_{g2}

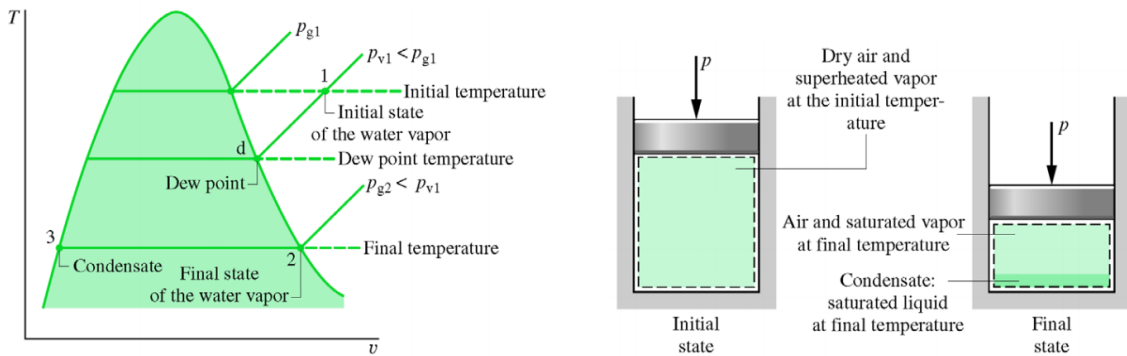


Figure 4:

0.5 Adiabatic Saturation Temperature T_{as}

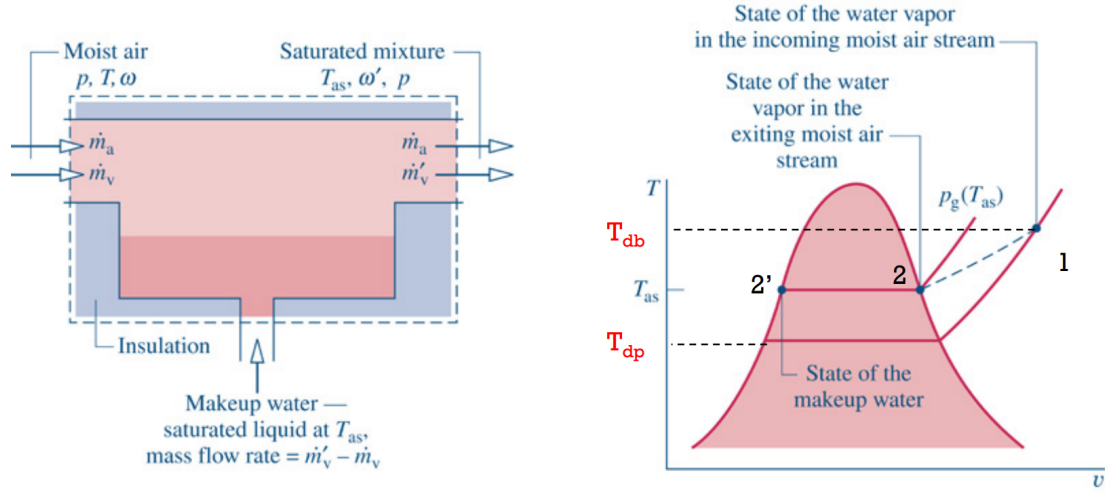


Figure 5:

The idea of the above **Adiabatic Saturator** is to determine the unknown humidity ratio at the inlet based on values of p , T and **Adiabatic Saturation Temperature** T_{as} .

0.5.1 The Adiabatic Saturation Analysis

Assumptions:

- Steady state
- $\Delta KE = \Delta PE = 0$
- Adiabatic process
- Constant pressure process
- Air and water vapor are ideal gases

Mass Balances:

$$\text{Dry air balance: } \dot{m}_{a,1} = \dot{m}_{a,2} = \dot{m}_a \quad (0.5.1)$$

$$\text{Water balance: } \dot{m}_{v,1} + \dot{m}_L = \dot{m}_{v,2} \quad (0.5.2)$$

$$\longrightarrow \omega \dot{m}_a + \dot{m}_L = \omega' \dot{m}_a \quad (0.5.3)$$

$$\longrightarrow \dot{m}_L = \dot{m}_a(\omega' - \omega) = \dot{m}_a(\omega_2 - \omega_1) \quad (0.5.4)$$

1st Law for Open System, energy balance:

$$\frac{dE_{CV}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m}_{in} \left(h + \frac{v^2}{2} + gz \right)_{in} - \sum_{out} \dot{m}_{out} \left(h + \frac{v^2}{2} + gz \right)_{out} \quad (0.5.5)$$

$$\frac{dE_{CV}}{dt} = \cancel{\dot{Q}} + \cancel{\dot{W}} + \sum_{in} \dot{m}_{in} \left(h + \frac{v^2}{2} + gz \right)_{in} - \sum_{out} \dot{m}_{out} \left(h + \frac{v^2}{2} + gz \right)_{out} \quad (0.5.6)$$

$$0 = \dot{m}_a h_{a,1} + \dot{m}_{v,1} h_{v,1} + \dot{m}_L h_L - \dot{m}_a h_{a,2} - \dot{m}_{v,2} h_{v,2} \quad (0.5.7)$$

$$0 = \dot{m}_a h_{a,1} + \omega_1 \dot{m}_a h_{v,1} + \dot{m}_a (\omega_2 - \omega_1) h_L - \dot{m}_a h_{a,2} - \omega_2 \dot{m}_a h_{v,2} \quad (0.5.8)$$

$$0 = h_{a,1} + \omega h_{v,1} + (\omega' - \omega) h_{f,2} - h_{a,2} - \omega' h_{v,2} \quad (0.5.9)$$

$$\longrightarrow \omega h_{v,1} - \omega h_{f,2} = (h_{a,2} - h_{a,1}) + \omega' (h_{v,2} - h_{f,2}) \quad (0.5.10)$$

$$\longrightarrow \omega = \frac{(h_{a,2} - h_{a,1}) + \omega' (h_{v,2} - h_{f,2})}{h_{v,1} - h_{f,2}} \quad (0.5.11)$$

$$h_v \approx h_g(T) \quad (0.5.12)$$

$$\omega = \frac{[h_a(T_{as}) - h_a(T)] + \omega' [h_g(T_{as}) - h_f(T_{as})]}{h_g(T) - h_f(T_{as})} \quad (0.5.13)$$

$$\omega' = 0.622 \frac{p_g(T_{as})}{p - p_g(T_{as})} \quad (0.5.14)$$

0.6 Wet-bulb Temperature T_{wb}

Dry-bulb Temperature T_{db}

- Measured by a normal thermometer placed in the mixture.

Wet-bulb Temperature T_{wb}

- Measured by a wet-bulb thermometer, which is an ordinary liquid-in-glass thermometer whose bulb is enclosed by a wick moistened with water.
- T_{wb} is a close approximation of the adiabatic saturation temperature.
- T_{wb} is used in place of T_{as} to determine ω in the adiabatic saturator.

$$\omega = \frac{[h_a(T_{wb}) - h_a(T_{db})] + \omega' [h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T_{db}) - h_f(T_{wb})} \quad (0.6.1)$$

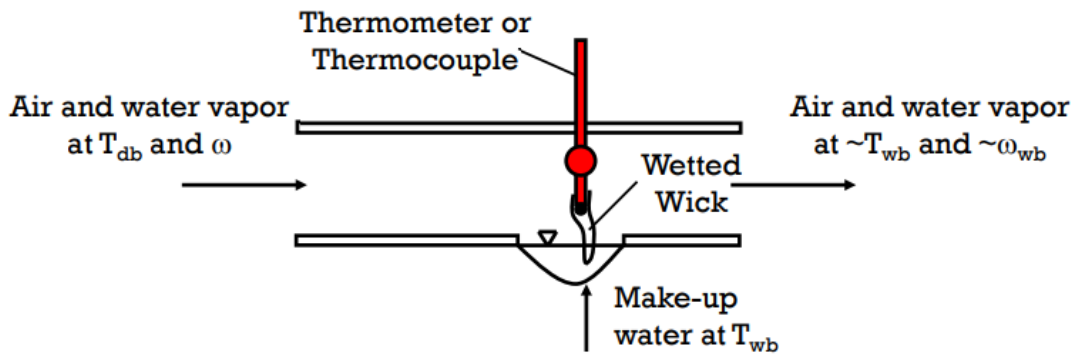


Figure 6:

0.7 Psychrometers

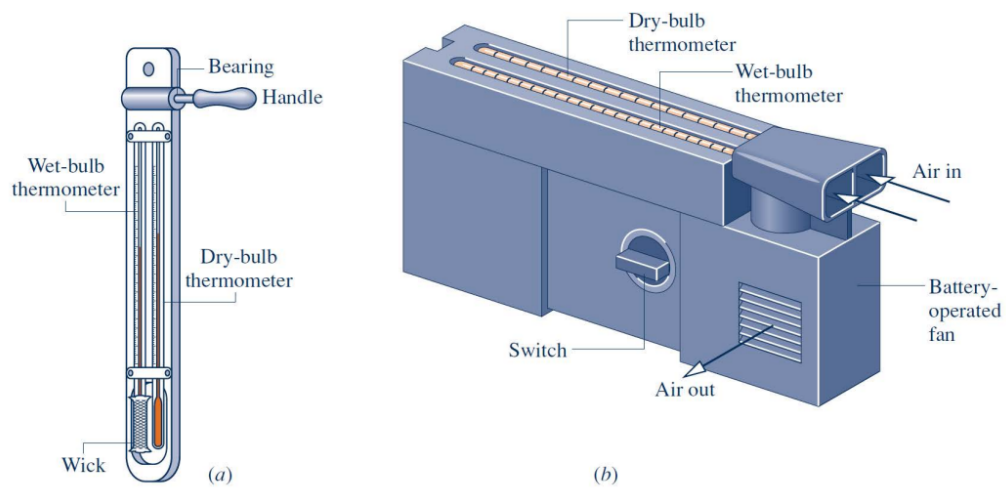


Figure 7: (a) Sling Psychrometers — (b) Aspirating Psychrometers

These are the devices to measure temperature and provide the humidity ratio.