MECH0010 Topic Notes

UCL

HD & MD

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Acknowledgments

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Part I

Instrumentation

Chapter 1

General Measurement Systems

A measurement system has to be devised such that the relationship between the real value of a variable and the value actually measured is unambiguously known. For example, when placing a weight on a scale, we must know the relationship between the **true** weight and the **measured** weight. The measurement system must allow:

- Easy interpretation of the measured data
- Provide high degree of confidence

1.1 Transducers

A transducer converts the sensed variable into a detectable signal form. Sometimes the device changes a mechanical quantity into a change in an electrical quantity. For example, a strain gauge converts a change in strain in the specimen to a change in electrical resistance in the gauge. Another example is a thermometer which converts thermal expansion of the liquid (due to a rise in temperature) into a mechanical translation.

1.2 Signal condition circuits

This takes the transducer signal and can convert, compensate or manipulate it into a more usable electrical quantity. This may include filters, compensators, modulators, demodulators, integrators or differentiators. For example, a Wheatstone bridge used with the strain gauge converts the change in the electrical resistance of the gauge to a change in voltage.

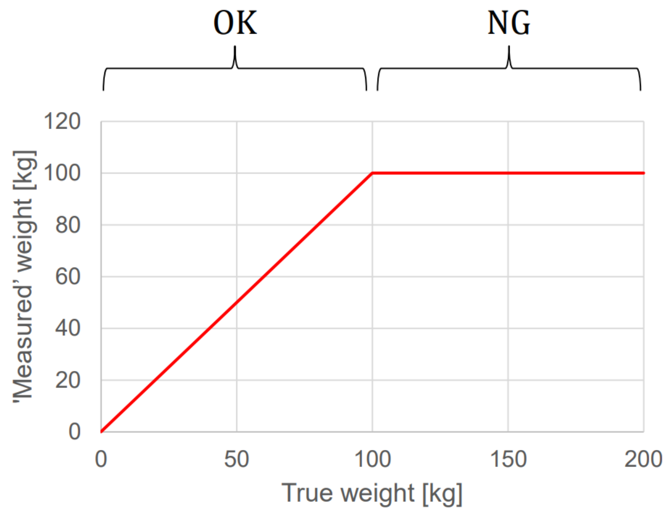


Figure 1.1: The scales work well up to 100kg, however it may not be able to measure heavier weights effectively.

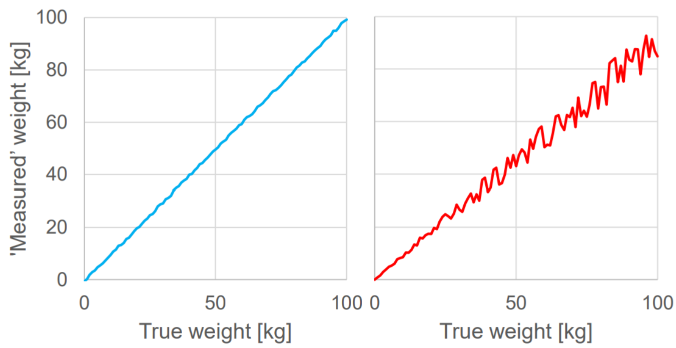


Figure 1.2: On the red graph we can see the effect that noise has on our measurement - a source of an inaccuracy. This must be dealt with to have an effective measurement system.

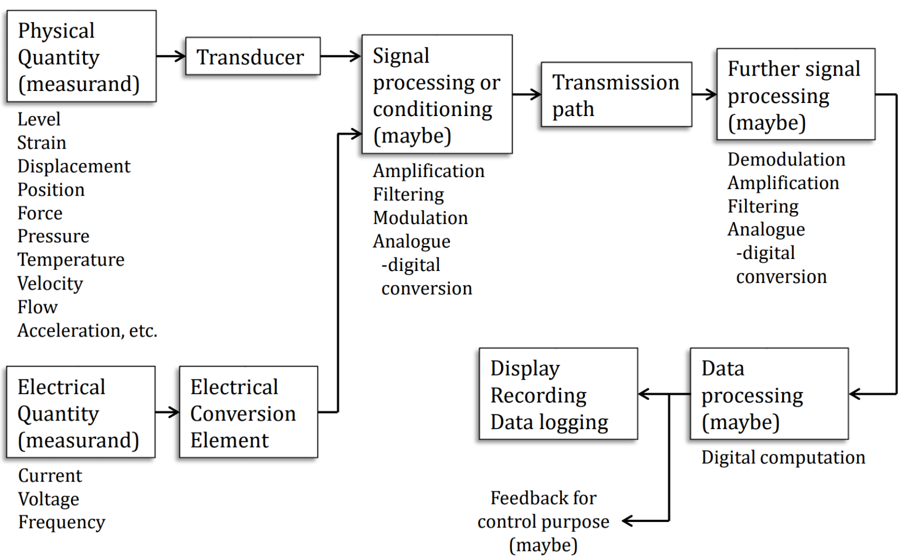


Figure 1.3: Block diagram to show a general measurement system.

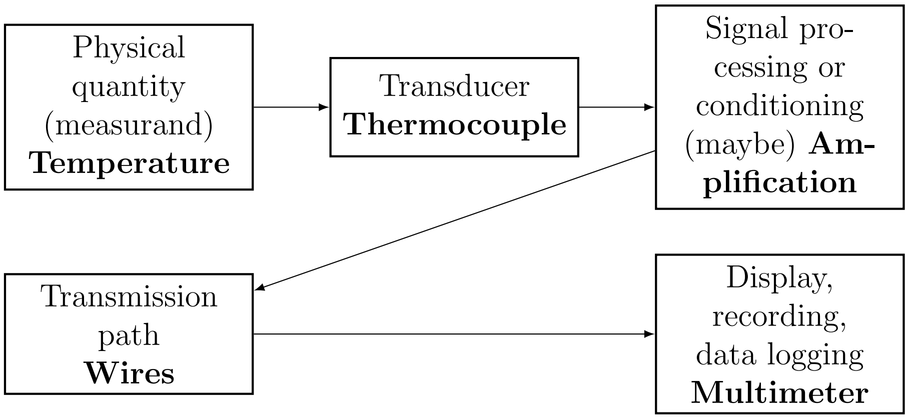


Figure 1.4: How the workflow of a measurement can be constructed.

1.3 Amplifiers

They are required in systems when the output from the transducer-signal conditions is small. Gains of 10 - 1000 are used to increase the levels of the signal, typically a millivolt or less, to what is compatible with the voltage-measuring devices used in the system. A negative-feedback amplifier circuit is an example of an amplifier.

1.4 Output stage

Generally, it is a voltage measurement device that is used to display the measurement in a form that can be read and interpreted. For example, digital voltmeters, self-balancing potentiometers, oscilloscopes, chart recorders and magnetic tape recorders.

1.5 Feedback-control stage

Used when the measurement system is employed in process control. The signal from the measurement system is compared with the command signal that reflects the required value of the quantity in the process. The process-controller forms the difference between these two and produces and error signal. The error signal is then used to automatically adjust the process. For example, the float system in a toilet (called a ballock) to control the water supply in the tank.

1.6 Calibration

How do we know whether one metre is one metre? We compare it with a reference, which is supposed to be reliable. This act of checking is called calibration. Our references are the measuring **standards**.

1.7 Standards

There are seven fundamental quantities of the International Measuring System.

- Length m - metre
- Time s - second
- Mass kg - kilogram
- Temperature K - kelvin
- Electric Current A - ampere
- Luminous Intensity - cd - candela
- Amount of Substance - mol - mole

Units and standard for all other quantities are derived from these.

We can look at how the definition of the metre has changed over the past 2 centuries for some insight into the accuracy of our reference.

- 1792 - a ten millionth of the quarter of a meridian ($0.5 - 0.1$mm of uncertainty)
- 1889 - a platinum-iridium bar at melting point of ice ($0.2 - 0.1$µm of uncertainty)
- 1983 - the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second (0.1nm of uncertainty)

Here are how the other fundamental quantities are defined.

- Metre - Length of the path travelled by light in a vacuum in 1/299792458 of a second
- Time - Duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom
- Mass - ~~Equal to the mass of the International Prototype of the Kilogram~~
  - Equal to h / (Metre standard^2 / Time standard) ... since 2019 h: Plank constant (in $e = hf,\ h = 6.626 \times 10^{-34}\text{kg}\,\text{m}^2\,\text{s}^{-1}$)

- Temperature - 1/273.16 of the thermodynamic temperature of the triple point of water (exactly 0.01 °C)
- Electric current - The flow of electric charges through a surface at the rate of one coulomb per second

The fundamental standards are protected by such organisations such organisations as the National Institute of Standard and Technology (NIST) in the US. They set up the National Reference Standards and below these in accuracy, the Working Standards (and more below these).

1.8 Units and dimensions

- Dimension - a physical variable that is used to describe some aspect of a physical system
- Unit - a measure of a dimension. Primary standards are used to form the exact definition of a unit

SI units are now commonly used worldwide. The fundamental dimensions are:

- Length [L]
- Mass [M]
- Force [F] ([MLS^{-2}])
- Time [τ]
- Temperature [T]

Chapter 2

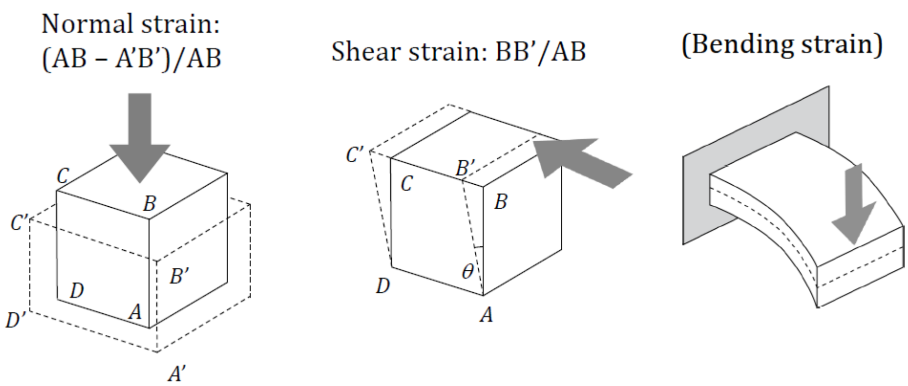
Strain Gauges

12/10/2020

2.1 Strain gauges

2.1.1 Strain

The changes in the value of a dimension of a body divided by the original value of the dimension is the relative change in the dimensions.



Information about strain is required in many engineering situations: an aircraft in flight, support pillars and spans of bridges, etc.

2.1.2 Stress

Density of the reactive (internal) forces distributed throughout the body (force per unit area), in response to external force. Types of stress:

- Tensile/compressive stress
- Shear stress
- Bending stress

Elastic modulus:

$$E = \frac{\text{stress}}{\text{strain}} \tag{2.1}$$

- Young's modulus - normal strain (and if relationship is linear)
- Shear modulus - shear strain
- Bulk modulus - volumetric strain

2.1.3 Strain gauges

A device which experiences a change of electric resistance when it is strained. A strain gauge is combined with other electrical components to obtain an electric voltage or current representing tensile, compressive or bending strain, together wit the means of displaying or recording its value. The total resistance of a block of conducting material of uniform cross section A and length l:

$$R = \rho \frac{l}{A} \tag{2.2}$$

Where ρ is resistivity with units $\Omega\,\mathrm{m}$.

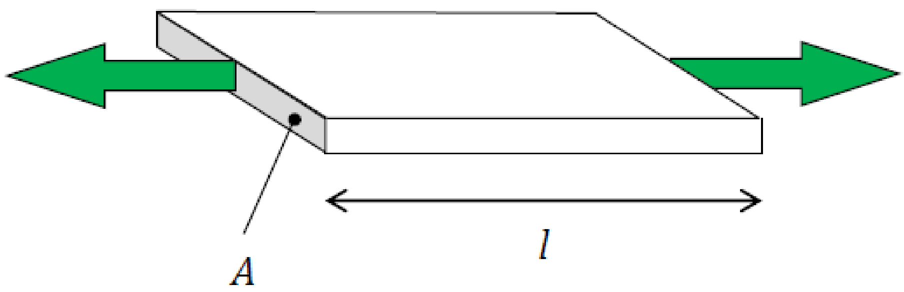


Figure 2.1: When the block is subjected to a tensile stress, its length will increase and cross-sectional area will decrease, both of which cause the resistance to increase according to equation (2.2)

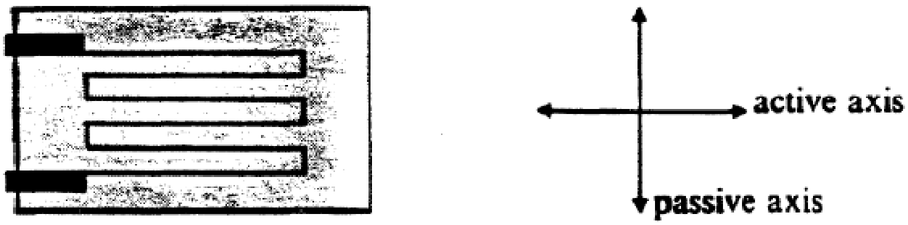
The resistivity of the material will also change because of the **piezo-resistive effect** (increase with tension and decrease with compression). The resistance of the block can thus be written as:

$$R' = (\rho + \delta\rho) \frac{l + \delta l}{A - \delta A} \tag{2.3}$$

Note: compressive stress will result in a decrease in total resistance.

2.1.4 Simple wire strain gauge

A long fine wire is folded to fit in a small area and then mounted on a flexible backing sheet, usually paper. When **firmly stuck** to the surface of a much more rigid body, any deformation of this body will cause an identical fractional change of the strain gauge wire. The change of resistance for any strain along the active axis will be much greater than for an equal strain occurring along the passive axis.



Single-active-axis strain gauge

2.1.5 Gauge factor

Defined as the fractional change of resistance of the gauge, divided by the fractional change in the length of the gauge along the active axis:

$$G = \frac{\frac{\Delta R}{R}}{\frac{\Delta l}{l}} \tag{2.4}$$

Since $\frac{\Delta l}{l}$ is the strain e in the body to which the gauge is fixed, this can be rewritten as:

$$\frac{\Delta R}{R} = eG \tag{2.5}$$

Most gauges have a G between **1.8 to 2.2**, depending on the gauge material and the magnitude of the piezo-resistive effect.

2.1.6 Foil strain gauge

Most modern strain gauges are formed by rolling out a thin foil of the resistive material and then cutting away parts of the foil by a photo-etching process, to create the required grid pattern.

- Usually supplied with think backing paper as electrical insulation
- Adhesive layer for the fixation to the specimen should be creep-free and allow heat dissipation

Advantages over simple wire strain gauges:

- Larger surface area results in a larger area of adhesion
- Accurate reproducibility due to photo-etching technique
- Small dimensions mean they are good for localised strain, fit well to curvature

2.1.7 How can we measure strain

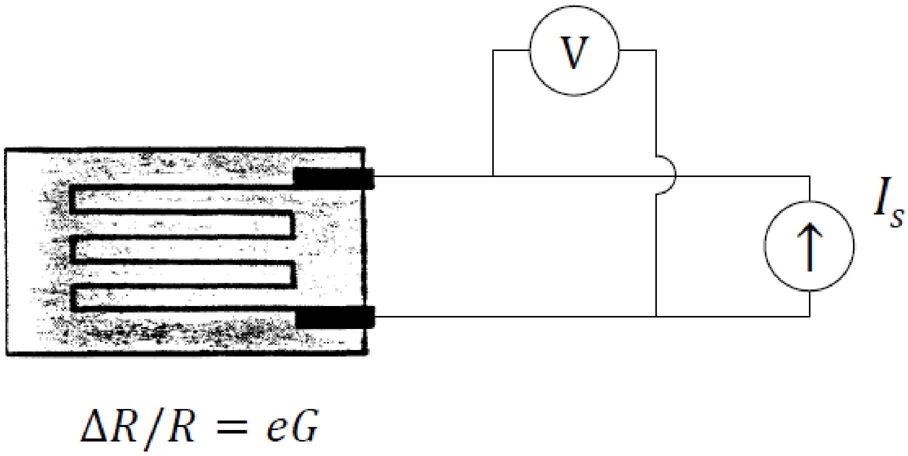


Figure 2.2: Is this good enough?

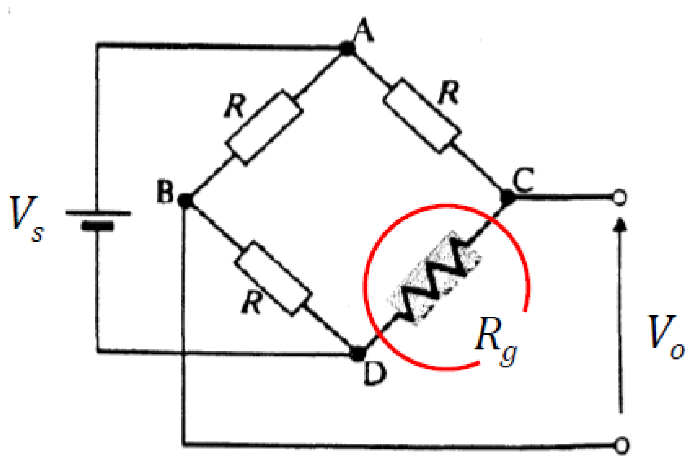
Typically $e = 10^{-3}$, $G = 2.1$, $R = 120\Omega$, $I_S < 50$mA.

$$V = 6\text{V},\ \Delta R = 0.252\Omega \tag{2.6}$$
$$\Delta V = 0.013\text{V} \tag{2.7}$$

The fractional change of the voltage is too small to detect comparing to the 'baseline' voltage ($= RI_S$).

2.2 Wheatstone bridge

Used to convert the change of resistance in the strain gauge into an electrical signal which could be used to indicate the value of strain. With this, **zero strain ↔ zero output voltage**.



Here,

$$V_C - V_D = \frac{V_S R_G}{R + R_G} \tag{2.8}$$

and then,

$$V_0 = V_C - V_B = \left(V_D + \frac{V_S R_G}{R + R_G}\right) - \left(V_D + \frac{V_S}{2}\right) = \frac{V_S R_G}{R + R_G} - \frac{V_S}{2} \tag{2.9}$$

R is chosen to have the same value as the unstrained gauge, i.e. $R = R_G$ when zero strain $\rightarrow V_0 = 0$ when zero strain. When the gauge is strained, $R_G = R + r$ and the output voltage is

$$V_0 = V_S \frac{R + r}{2R + r} - \frac{V_S}{2} = V_S \frac{r}{2(2R + r)} \tag{2.10}$$

Now using the gauge factor, $\frac{r}{R} = eG$ and the equation can be rewritten:

$$V_0 = V_S \frac{ReG}{2(2R + ReG)} = V_S \frac{eG}{2(2 + eG)} \approx \frac{1}{4} V_S eG \tag{2.11}$$

Because typically $eG < 0.02 << 2$

Sensitivity:

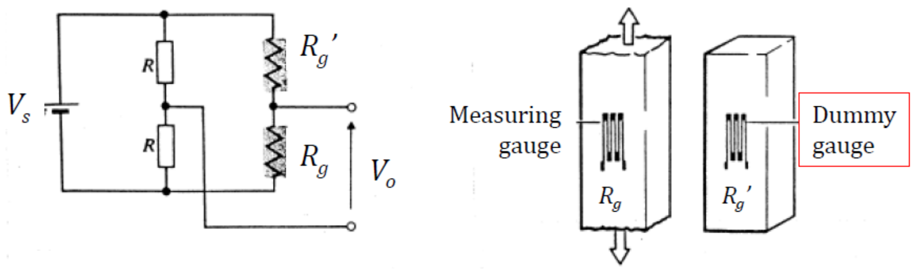
$$\frac{V_0}{e} = \frac{1}{4} V_S G \tag{2.12}$$

2.2.1 Temperature compensation

The output voltage from the strain gauge and the bridge also depends on the temperature. Changes in temperature will cause changes in:

- Dimensions of the specimen and hence the gauge due to thermal expansion
- Dimensions of the gauge itself (particularly thickness)
- Resistivity of the gauge material

Compensated with a dummy gauge:



The dummy gauge is:

- Identical to the measuring gauge
- Mounted on an unstressed specimen of the same material

The unstressed specimen should be placed as close to the strained member, in order to make it work accurately. Now, consider resistance of a strain gauge R_g:

- Changes due to tensile strain: $R_g \to R(1+x) = R(1+eG)$
- Changes due to temperature: $R_g \to R(1+y)$

The measuring gause and dummy gause resistances are then:

$$R_g = R(1+x)(1+y) \tag{2.13}$$
$$R_g' = R(1+y) \tag{2.14}$$

The output voltage of the bridge is:

$$V_0 = V_S \frac{R_G}{R_G + R_G'} - \frac{V_S}{2} \tag{2.15}$$

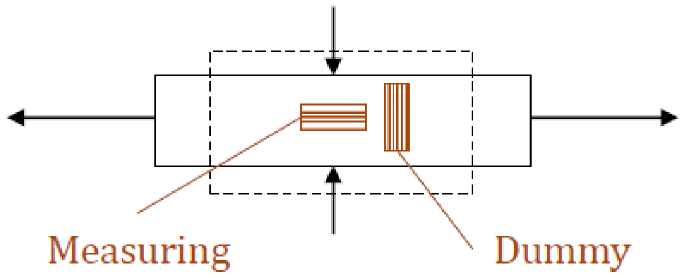
Substituting for R_G and R_G', we have:

$$V_0 = V_S \frac{R(1+x)(1+y)}{R(1+x)(1+y) + R(1+y)} - \frac{V_S}{2} \tag{2.16}$$
$$\frac{V_0}{V_S} = \frac{R(1+x)(1+y)}{R(1+x)(1+y) + R(1+y)} - \frac{1}{2} \leftarrow \text{no influence by y} \tag{2.17}$$

However, for this you must find the specimen of the same material and keep them under the same temperature - are these easy enough? The dummy gauge could be placed on the same member as the measuring gauge with its active axis in the direction normal to that of the strain.

2.2.2 Effect of Poisson's ratio on a dummy gauge

Consider both measuring and dummy gauges are attached to the same specimen but perpendicular to each other.



Deformation of the specimen elongates the measuring gauge and shortens the dummy gauge, and the ratio of strains in the two directions is Poisson's ratio, $v = e_{\text{lateral}}/e_{\text{longitudinal}}$, which is between 0.25 to 4 for most materials. Ignoring temperature changes, the measuring gauge and dummy gauge resistances are then

$$R_g = R(1+x) \tag{2.18}$$
$$R'_g = R(1-\nu x) \tag{2.19}$$

The bridge output is:

$$V_0 = V_S \frac{R_g}{R_g + R'_g} - \frac{V_S}{2} \tag{2.20}$$

Substituting for R_g and R'_g, we have:

$$V_0 = V_S \frac{R(1+x)}{R(1+x) + R_g(1-\nu x)} - \frac{V_S}{2} \tag{2.21}$$
$$\frac{V_0}{V_S} = \frac{R(1+x)}{R(1+x) + R_g(1-\nu x)} - \frac{1}{2} \tag{2.22}$$
$$\frac{V_0}{V_S} = \frac{(1+\nu)}{2[2+(1-\nu)x]} \approx \frac{1}{4}(1+\nu)eG \tag{2.23}$$

Because typically $(1-\nu)x << 2$. Comparing to the single gauge, $\frac{V_0}{V_S} = \frac{1}{4}eG$, sensitivity is increased by a factor $(1+\nu)$ due to the effect of Poisson's ratio on the dummy gauge.

2.3 Practical Aspects of Strain Gauge Measurement

2.3.1 Strain gauge rosette

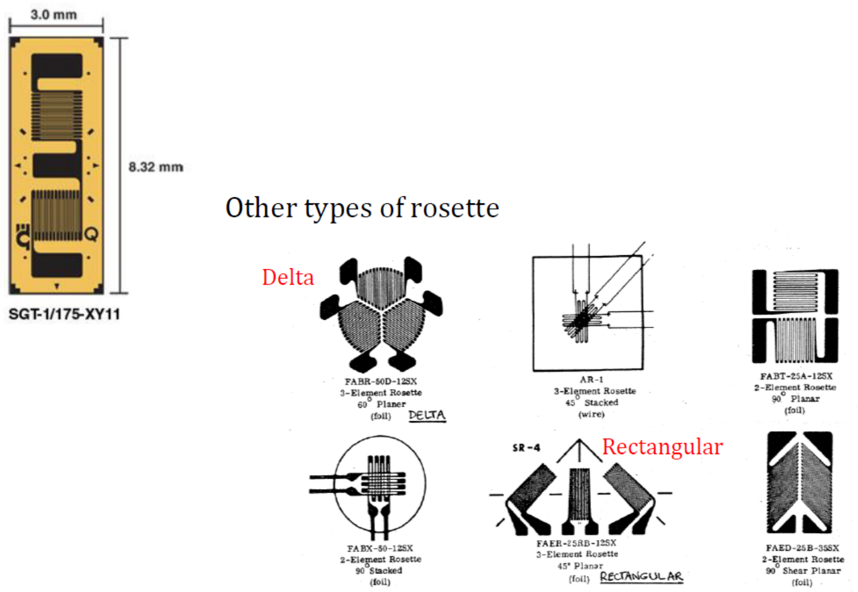
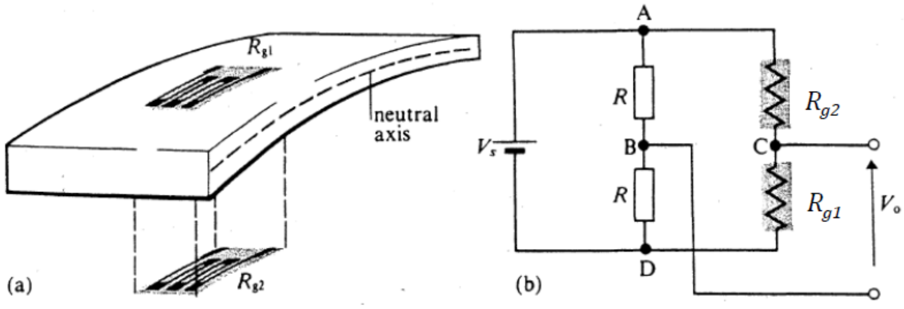


Figure 2.3: An example of a foil strain gauge having two elements with their active axis perpendicular to each other is shown in the top left (Omega Engineering Inc. USA). Some other types of rosette are also shown (figures courtesy of BLH Electronics, Waltham, Mass)

How can be these useful? Strain (stress) in real-world applications is, not uniformly distributed and/or the principal stress axis is unknown. We use strain gauge rosettes to measure strain in three orientations. Understand complex stress/strain state (principal stress calculation is a topic of MECH0013 in term 2).

2.3.2 Bending Strain

To measure the strain of a member in which only bending is occurring, i.e. no additional tensile or compressive stresses present, we attach two strain gauges one on either side of its neutral axis with their actives axes along the length of the member.



If the gauges are equidistant from the neutral axis, the tensile strain imposed on one will be equal to the compressive strain in the other, and t he change of the resistance is the same magnitude but in opposite directions. Similarly to the previous cases, consider the fractional change of resistance x and then output voltage.

$$V_0 = V_S \frac{R_{g1}}{R_{g1} + R_{g2}} - \frac{V_S}{2} \tag{2.24}$$

$$= V_S \frac{R(1+x)}{R(1+x) + R(1-x)} - \frac{V_S}{2} = \frac{1}{2} V_S x \tag{2.25}$$

Since $x = eG,\ V_0 = \frac{1}{2} V_S eG$ is a measure of bending occurring in the body.

2.3.3 Tensile or compressive strain in a bending member

The arrangement shown below allows to measure any tensile or compressive strain in the member, while ignoring any bending strain.

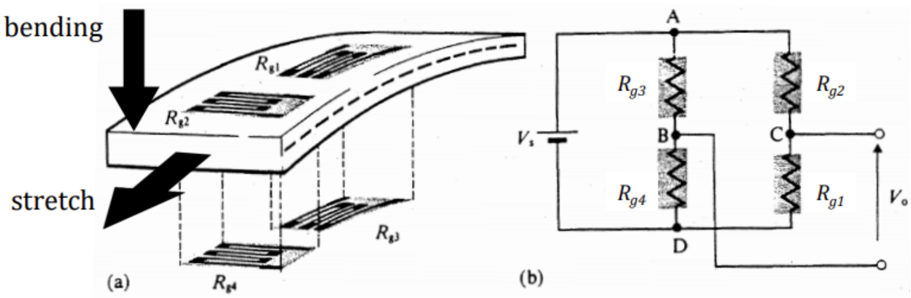


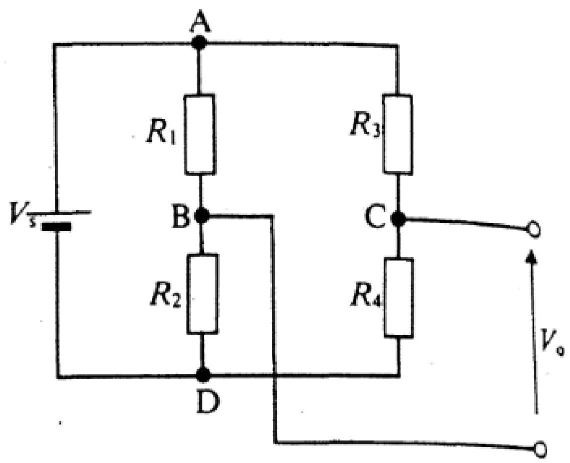
Figure 2.4: Variation of gauge resistance and output voltage in the configuration above.

	Rg_1	Rg_2	Rg_3	Rg_4	V_0
Under pure stretch	++	-	++	-	++
Under pure bending	++	-	- -	+	±0

2.3.4 Bridge balancing

In reality, resistance in the arms of the Wheatstone bridge slightly differ in value because of:

- Manufacturing variations
- Temperature differences between gauges and resistors
- Static strain occurring in a member

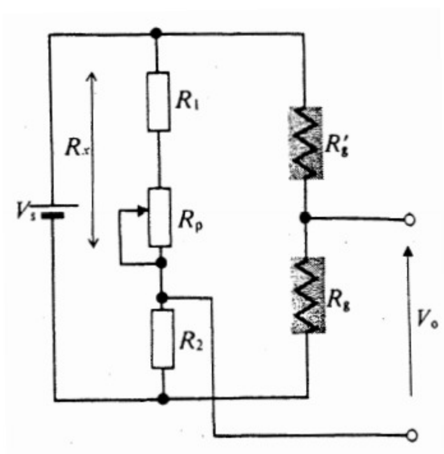


If the bridge is perfectly balanced:

$$\frac{R_4}{R_3 + R_4} = \frac{R_2}{R_1 + R_2} \tag{2.26}$$

$$\rightarrow \frac{R_3}{R_4} + 1 = \frac{R_1}{R_2} + 1 \rightarrow \frac{R_3}{R_4} = \frac{R_1}{R_2} \tag{2.27}$$

The imbalance needs to be adjusted before the measurement (adjustment of the output voltage to zero) → bridge balancing.



One possible way to achieve this is to connect a pot (potentiometer) in series with R_2. The total resistance $R_x = R_1 + R_p$ should be adjusted such that:

$$\frac{R_x}{R_2} = \frac{R'_g}{R_g} \tag{2.28}$$

2.3.5 Semiconductor strain gauge

In some crystalline materials such as germanium and silicon, the piezo-resistive effect is very large. If a slice of such a crystal is used as a strain gauge, very large gauge factors can be obtained, which could range from 100 to 300.

$$\text{Gauge factor: } G = \frac{1}{e}\frac{\Delta R}{R} \tag{2.29}$$

The magnitude of the piezo-resistive effect, which determines the sensitivity of the gauge, depends on the minute quantities of impurity introduced into the base material before formation of the crystal.

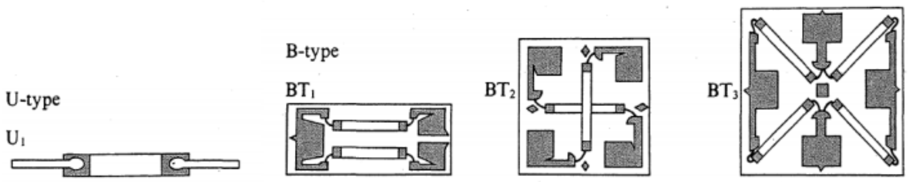


Figure 2.5: Some examples of semiconductor strain gauges

Characteristics of semiconductor strain gauges:

- The material of the strain gauge reaches its elastic limit at about 4000 microstrain (4000×10^{-6}), much smaller than that of metals (typically 20000 microstrain)
- The gauge factor G varies with high strain at high strain levels. i.e. the gauge is not linear
- The gauge factor varies with temperature
- The temperature coefficient of resistivity is large

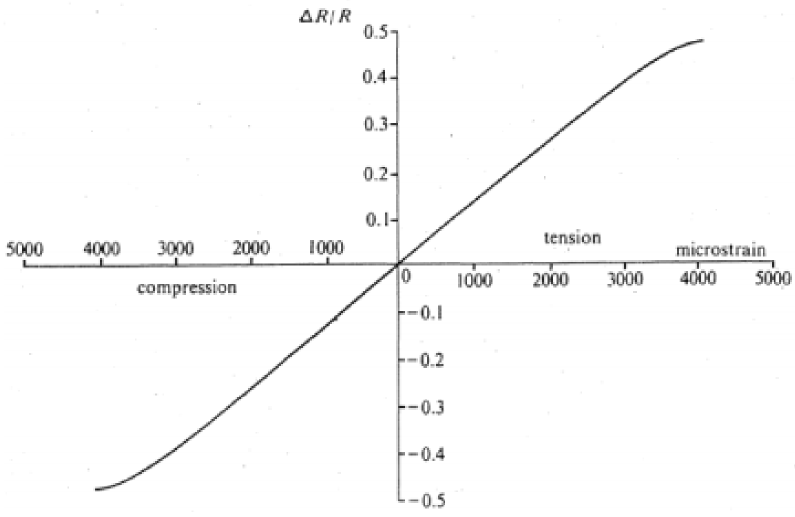
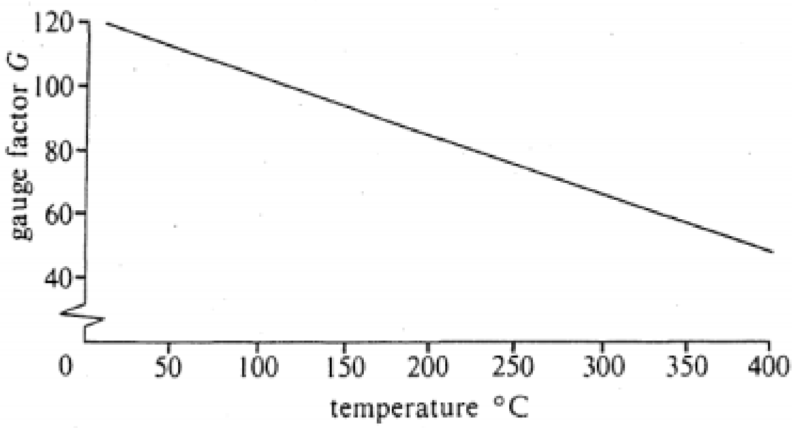


Figure 2.6: Change of resistance ratio with strain for a semiconductor strain gauge



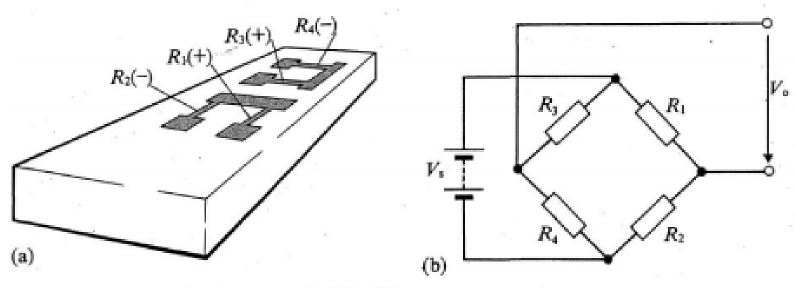
Negative effect of the temperature sensitivity of this type of strain gauge can be significantly reduced by using two gauges each consisting of two crystals connected in series. When two crystals are connected in series (but aligned in parallel), tensile of compressive strain applied to a gauge will produce an increase in the resistance of one crystal and a decrease in the other. Because the base material of each crystal is the same, their temperature coefficients of resistivity are approximately equal. In the configuration below, if the body experiences a tensile strain,

$$R_1 = R(1 + x) \tag{2.30}$$

$$R_2 = R(1 - x) \tag{2.31}$$

$$R_3 = R(1 - \nu x) \tag{2.32}$$

$$R_4 = R(1 + \nu x) \tag{2.33}$$



The bridge output voltage will thus be:

$$V_0 = \frac{1}{2} V_S x(1 + \nu) \tag{2.34}$$

Even if temperature increases, that affects all crystals equally. Also, changes in dimensions will cause $R_1 \uparrow$, $R_2 \downarrow$, $R_3 \uparrow$, $R_4 \downarrow$. Hence, no unbalance occurs.

2.4 Summary

1. Strain gauges utilise variability of resistance due to deformation
2. Strain gauges are used in Wheatstone bridges with various gauge arrangements
3. Bridge balance calculation is an essential part of the module

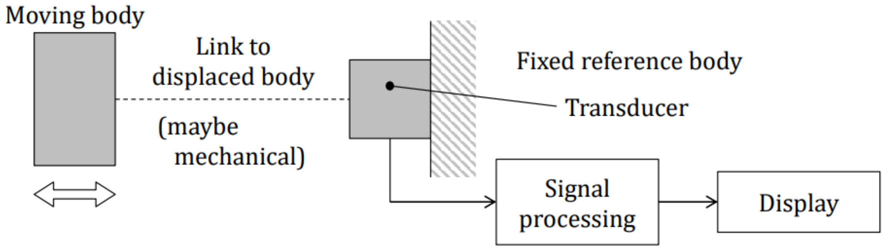
Chapter 3

Measurements of Displacement

26/10/2020

Displacement measurements:

- Most frequent in manufacturing and process-control applications
- Work as a secondary transduction process in many transducers for other measurements such as force, torque, pressure, flow and density.
- Mostly requires a stationary reference point.
- Utilises a wide variety of techniques and principles.



3.1 Capacitance-Based Displacement Transducers

3.1.1 Capacitive Displacement Transducers

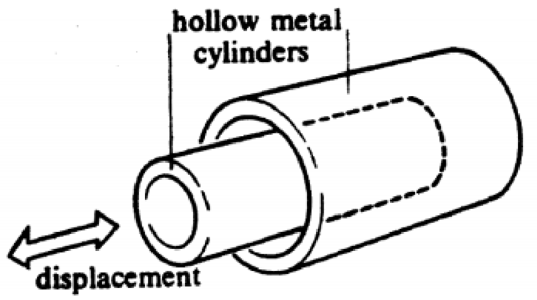
Displacement changes the capacitance of the transducer. The capacitance of a parallel-plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d} \tag{3.1}$$

Where:

- A is the area of the plates
- d is the separation between the plates
- ϵ_0 is the permittivity of free space $(8.854 \cdot 10^{-12} 4Fm^{-1})$

Example: The capacitance of the two metal cylinders is proportional to the area of overlap.



Variable capacitance can be introduced by an insulator between the plates. If the insulation is as thick as the plate separation, the overall capacitance is:

$$C = \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 \epsilon_r A_2}{d} \tag{3.2}$$

Where:

- ϵ_r is the relative permittivity of the insulator, or dialecetric as it is commonly known.

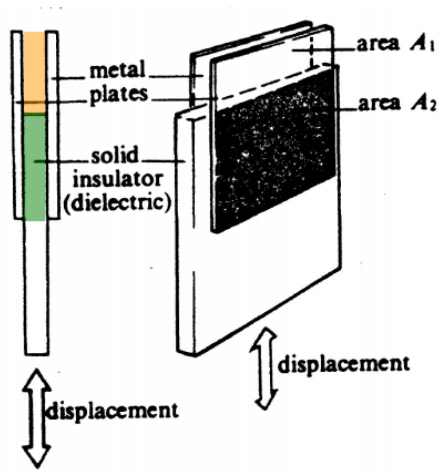


Figure 3.1: The orange part of the figure represents the 1st bit of the capacitance equation, when no insulator is inserted. The green part of the figure shows the 2nd part of the equation, where an insulator is introduced

3.1.2 Measurement of Capacitance

Suppose the capacitance of a capacitive transducer is a function of a displacement. When a constant voltage v is applied, charge is given as:

$$q = Cv \tag{3.3}$$

$$C = f(x) = \frac{q}{v} \tag{3.4}$$

It is impossible to measure q without disturbing it. If however, the voltage is made to vary, then q must vary too, and its rate of change, current i, can be measured:

$$\frac{\mathrm{d}q}{\mathrm{d}t}\left[\frac{C}{s}\right] = i[A] \tag{3.5}$$

Differentiating $q = Cv$ with respect to time gives:

$$\frac{\mathrm{d}q}{\mathrm{d}t} = i = C\frac{\mathrm{d}v}{\mathrm{d}t} \tag{3.6}$$

If the applied voltage is sinusoidal, i.e. $v = V_0 sin(\omega t)$, then:

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t} = V_0\omega cos(\omega t) \tag{3.7}$$

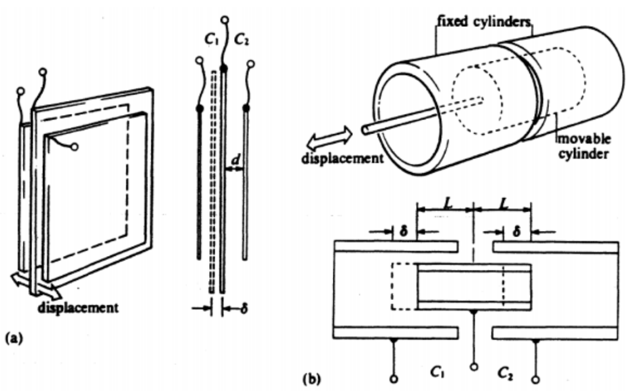
The amplitude of the current, i, equals to $V_0\omega C$ (proportional to the capacitance).

Changes in capacitance caused by changes in displacement can be measured by determining the AC current flowing through the capacitor. However, in some cases in practice, the capacitance change due to the displacement is only a small fraction of the total capacitance and difficult to be measured:

$$C_{total} = C_{transducer} + C_{cable} + C_{stray} \tag{3.8}$$

3.1.3 Differential Capacitance Displacement Transducers

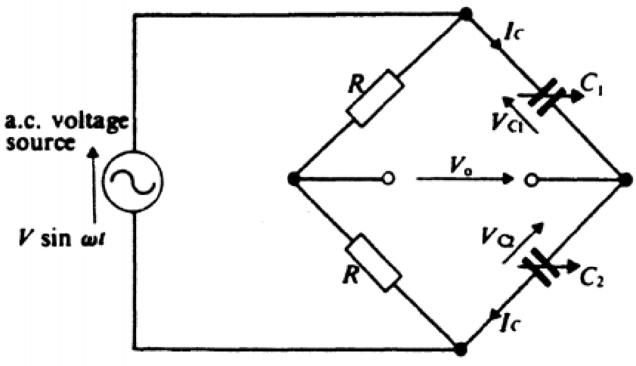
One way to get around the problem is to use either one of the differential forms of variable-separation or variable-area type transducer.



In each of the cases, two capacitors are formed which have equal capacitances when the transducer is geometrically centred.

3.1.4 Capacitive Bridge

The two capacitors are connected to a capacitive bridge. Here, difference in capacitance caused by the displacement unbalances the bridge and an AC output voltage V_o occurs.



To derive V_o, we start with the voltages across each capacitor:

$$V_{C1} = \frac{I_C}{\omega C_1} \tag{3.9}$$

$$V_{C2} = \frac{I_C}{\omega C_2} \tag{3.10}$$

This is because, if we consider voltage and currenct across/through a capacitor to be $v_C = V_C sin(\omega t)$ and $i_C = I_C cos(\omega t)$,

$$i_C = I_C cos(\omega t) = C\frac{\mathrm{d}v_C}{\mathrm{d}t} = C\frac{\mathrm{d}(V_C sin(\omega t))}{\mathrm{d}t} = \omega C V_C cos(\omega t) \tag{3.11}$$

$$\therefore V_C = \frac{I_C}{\omega C} \tag{3.12}$$

Now, knowing the voltages across each capacitor V_{C1} and V_{C2}, the voltage across C2 as a fraction of the bridge energising voltage, V is:

$$\frac{V_{C2}}{V} = \frac{V_{C2}}{V_{C1} + V_{C2}} \tag{3.13}$$

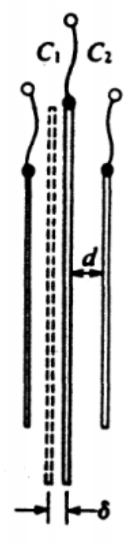
$$= \frac{\frac{I_C}{\omega C_2}}{\frac{I_C}{\omega C_1} + \frac{I_C}{\omega C_2}} \tag{3.14}$$

$$= \frac{C_1}{C_1 + C_2} \tag{3.15}$$

In the variable-separation type, the capacitances must be equal if the plate is centred. If the balance capacitance is C_0, then:

$$C_0 = \frac{\epsilon_0 A}{d} \tag{3.16}$$

where d is the balance separation. When the plate is displaced by a distance δ to the left as in the figure, we have:



$$C_1 = \frac{\epsilon_0 A}{d-\delta} = C_0 \frac{d}{d-\delta} \tag{3.17}$$

$$C_2 = C_0 \frac{d}{d+\delta} \tag{3.18}$$

Substituting these for the equation of the voltage across C_2 yields:

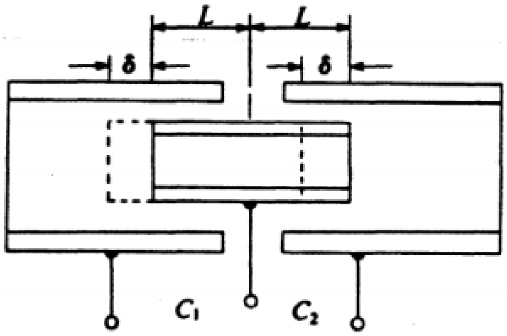
$$\frac{V_{C2}}{V} = \frac{d+\delta}{(d-\delta)+(d+\delta)} = \frac{d+\delta}{2d} \tag{3.19}$$

The voltage across each resistor is half the energising voltage, so the amplitude of the output voltage is:

$$V_o = V_{C2} - \frac{1}{2}V = \left(\frac{d+\delta}{2d} - \frac{1}{2}\right) V = \frac{\delta V}{2d} \tag{3.20}$$

There is a linear relationship between the voltage output and the displacement, despite the non-linear capacitance–displacement relationship.

For the variable-area type, the capacitances are proportional to their active areas, so if the movable cylinder is displaced to the left, we have:



$$C_1 = \frac{L + \delta}{L} C_0 \tag{3.21}$$

$$C_2 = \frac{L - \delta}{L} C_0 \tag{3.22}$$

Substituting these for the equation of the voltage across C_2 yields:

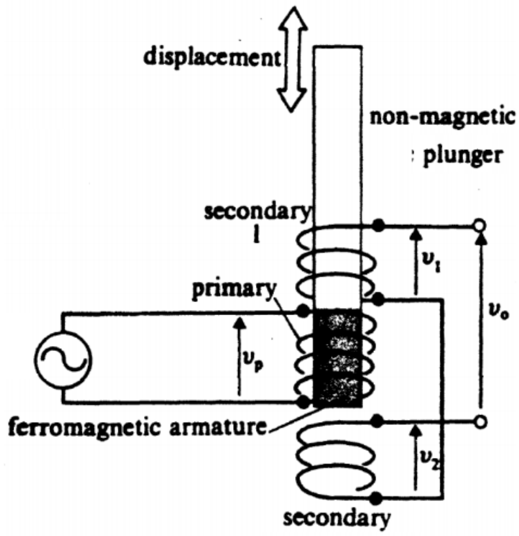
$$V_o = V_{C2} - \frac{1}{2} V = \frac{\delta V}{2L} \tag{3.23}$$

In both cases, the output voltage can be expressed as:

$$V_o = \frac{1}{2} x V \tag{3.24}$$

3.2 Inductance-Based Displacement Transducers

3.2.1 Linear Variable Differential Transformer (LVDT)



LVDT is one of the magnetic displacement transducers. An LVDT consists of:

- Primary coil
- Two secondary coils
- Ferromagnetic armature (iron core)
- An AC energising source
- Non-magnetic plunger

- and AC voltages are induced in the secondary coil by transformer action.

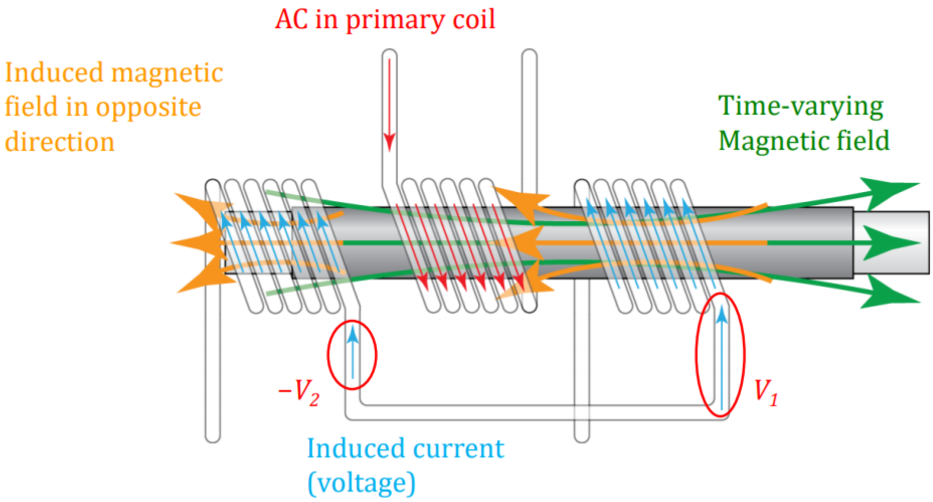


Figure 3.2: The working principle (with displacement to the right) of LVDTs

The working principle follows these steps:

1. AC in primary coil
2. Time-varying magnetic field
3. Induced magnetic field in opposite
4. Induced current (voltage)

If there is no displacement, the output voltage will be 0. When a displacement occurs, the electro-magnetic field around one of the secondary coils (the one the ferromagnetic core moves towards) will be stronger. Hence, the induced current and voltage on that coil will be larger than the other one.

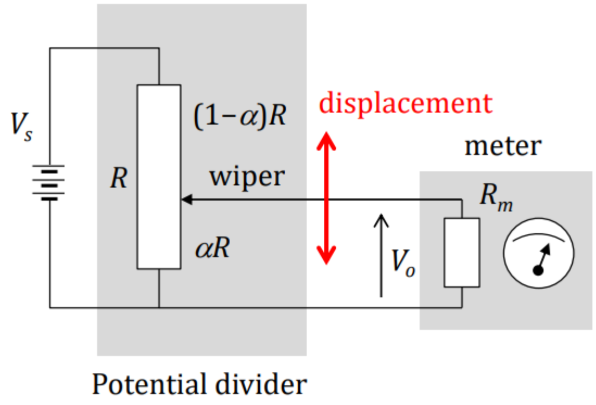
$$V_o = V_1 - V_2 \neq 0 \rightarrow \propto \textit{displacement} \tag{3.25}$$

The postivie of negative value of V_0 can indicate the displacement of the core.

- $V_o > 0 \longrightarrow$ Displacement to right
- $V_o < 0 \longrightarrow$ Displacement to left

3.3 Resistance-Based Displacement Transducers

A very common and simple form of displacement transducer.



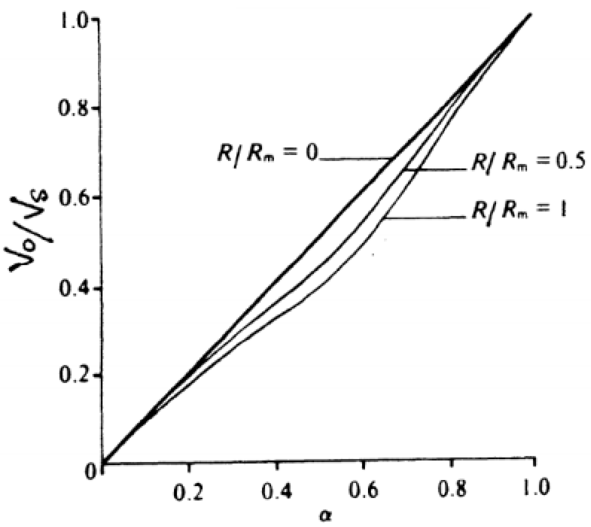
Resistor is typically a **coil winding**. Wiper displacement $\rightarrow$ change of α ($0 < \alpha < 1$). We assume the pot is wound uniformly with resistance wire and α varies linearly with the position of the wiper. With the internal resistance of the meter, the effective resistance R_{eff} to determine V_o is given as:

$$R_{eff} = \alpha R || R_m = \frac{\alpha R R_m}{\alpha R + R_m} \tag{3.26}$$

The two vertical lines $||$ indicate that αR and R_m are in parallel. Using potential divider theory, the meter voltage V_o is then:

$$V_o = \frac{R_{eff}}{(1-\alpha)R + R_{eff}} V_s \tag{3.27}$$

$$\frac{V_o}{V_s} = \frac{\alpha}{1 + \alpha(1-\alpha)\frac{R}{R_m}} \tag{3.28}$$



- If $R_m >> R$, then $\frac{V_o}{V_s}$ is approximated as linear.
- If $R_m > 10R$, then the maximum error is approximated as $Err_{max} = 15\frac{R}{R_m}(\%)$

Increasing V_s will increase the sensitivity ($= V_o$ / wiper travelling distance), but it also increases the power dissipated by the pot.

$$P = \frac{V_s^2}{R} \longrightarrow V_{s,max} = \sqrt{P_{max} R} \tag{3.29}$$

P_{max} is the rated maximum power dissipation (given in spec sheet). To increase the spatial resolution of pot:

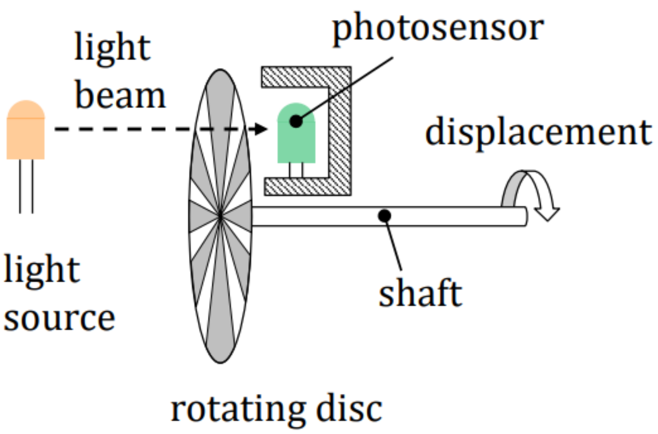
- carbon film
- conductive plastic
- ceramic-metal mix

could also be used. These also reduce the friction between the wiper and the resistive element. Linear resistance pots are made with spans ranging from about 10 mm to almost 1m.

3.4 Measuring Rotational Displacement

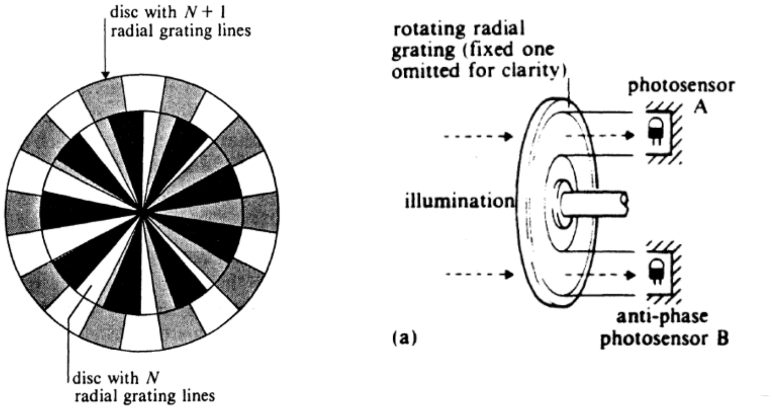
3.4.1 Optical Incremental Shaft Encoder

The output signals here, i.e. voltage output from the photosensor, depend on the angular displacement of the shaft.

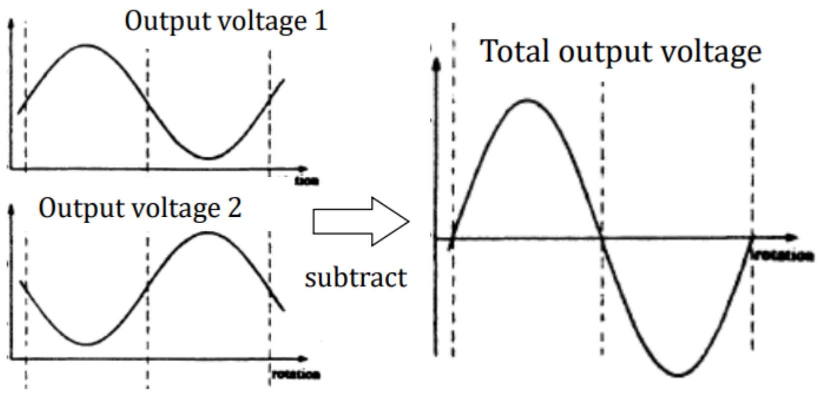


The pattern of the grating allow/disallow the light to reach the photosensor. The signal output varies depending on the **relative position** of the grating. Sinusoidal signal outputs are expected.

More elaborated version of incremental shaft encoder includes one with two discs (Moiré fringes) and two sensors.

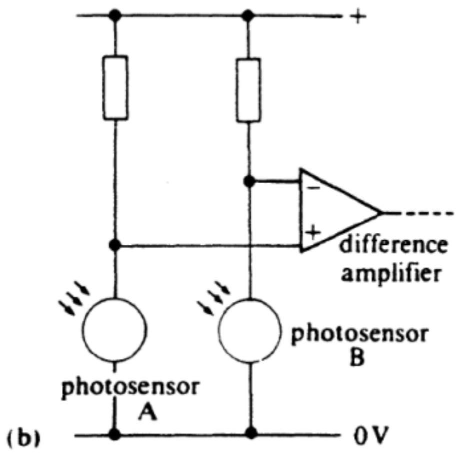


Differential output can be taken for clear signal.

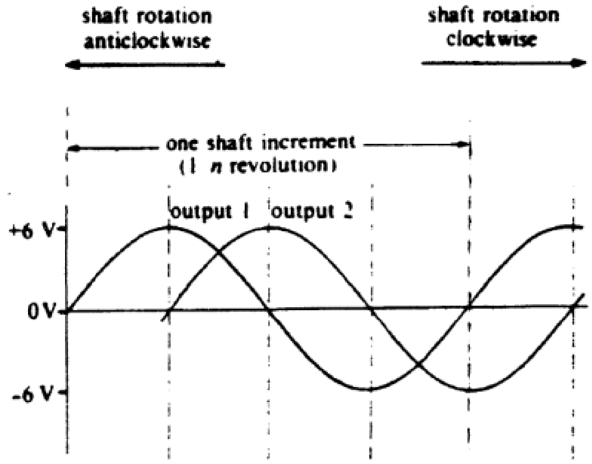


Advantage of the differential systems:

- Reduced influence by light variability
- Reduced influence by the environment

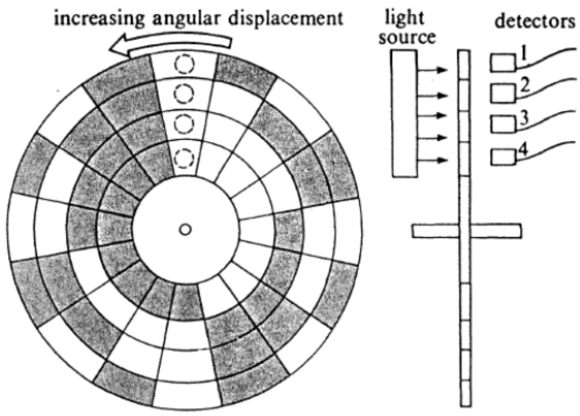


The pairs of light source and sensor may be placed 90° apart, in order to sense the direction of rotation .



3.4.2 Optical Absolute Shaft Encoder

Similar mechanism can be used as to measure an **absolute** rotary displacement by using binary code on the disc.



The four track disc above can have $2^4 = 16$ patterns $\longrightarrow$ 16 segments around the disc (resolution: $\frac{360}{16} = 22.5°$). In general, the resolution is:

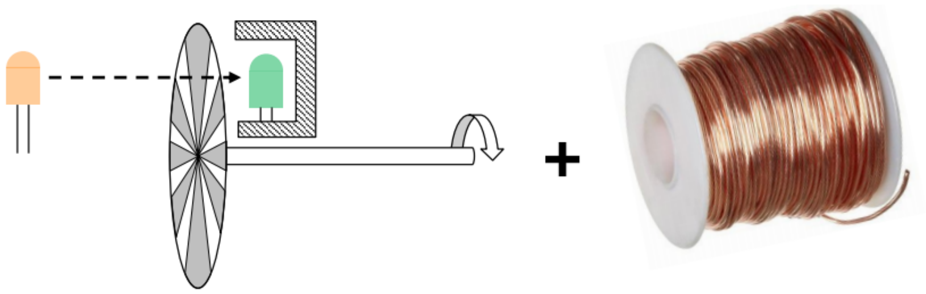
$$\frac{360}{2^{\ number\ of\ tracks}}(°) \tag{3.30}$$

and the detector output ranges from 0 to $2^{(number\ of\ tracks)} - 1$.

3.5 Measuring Large Linear Displacement

3.5.1 Measurement of Large Linear Displacements

The linear displacement transducers studied so far measure displacements of up to 1m. In order to measure larger displacements, other method are used. The simplest form of these is:

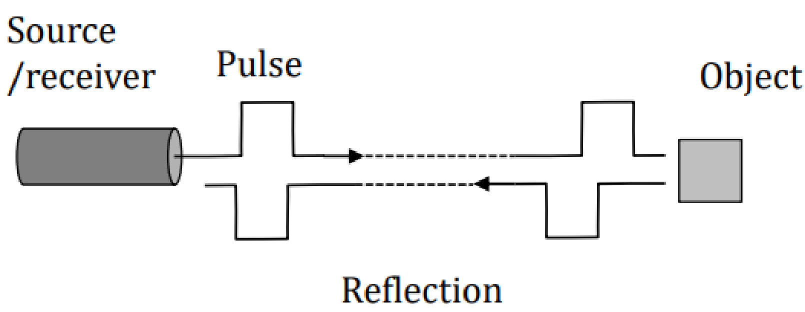


Commercial transducers of this type are available for displacements ranging from 50 mm to over 50m.

3.5.2 Wave Propagation Methods

Non-contact displacement measurements over long distances are possible by using ultrasonic waves, radio waves and light waves.

Pulse Reflection (Time of Flight)



$$D = V_{wave} \cdot \Delta t \tag{3.31}$$

Type of the pulse:

- Sonic/ultrasonic - 330 $\mathrm{m\,s^{-1}}$ (in the air)

- Radio/light wave - $3 \cdot 10^8\ \mathrm{m\,s^{-1}}$

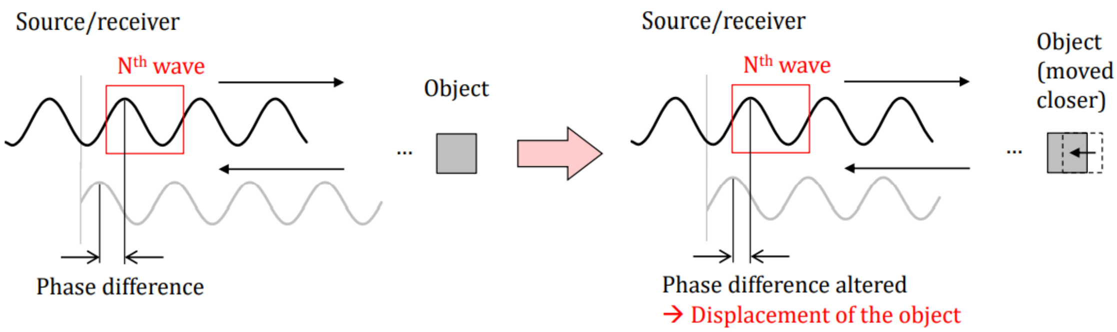
Accuracy of the measurement hinges on the measurement accuracy of time.

Other points to be considered:

- What is the medium
- Obstacles in the middle
- Diffraction
- ...

Phase Measurement

A continuous radio wave is emitted instead of pulses. The phase difference between the transmitted sine wave and the reflected sine wave is used to measure displacement.



Some Examples of Laser Distance Measurement

Laser range finders

- Distance: 15-400 m
- Error: 0.1% at 1 m

Laser scanner for 3D modelling

- Scan a 3D object with 4 lasers at about 127 μm accuracy to obtain its surface coordinates (used with tranangulation).

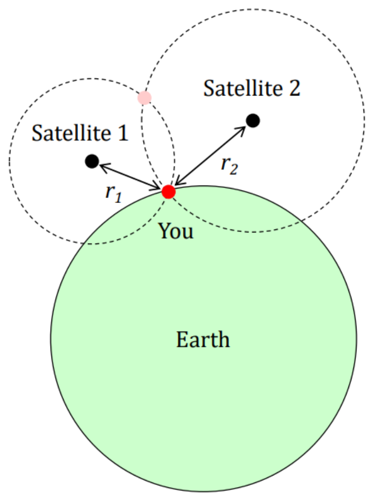
3.6 Other Methods of Measuring Position and/or Displacement

3.6.1 Projection Mapping

- Working principle of object/motion detection in Microsoft Kinect
- Detects objects shape or motion
- The projection from a light source is projected onto the object, and a number of cameras are used to detect and capture the image, based on the angle difference of the captured images.
- Applicable Range: 1.2-3.5 m (50 cm to 5 m)
- Depth Resolution:

3.6.2 Global Positioning System (GPS)

Location service with GPS



The distances between GPS satelites, r_1 and r_2, are measured by time of flight of radio waves. The location of the person (x, y, z) is determined by solving:

$$r_i^2 = (X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2 \tag{3.32}$$

with known locations of the satelites (X, Y, Z). To identify the position in 3D, 3 equations are needed, i.e. you need to communicate with 3 satellites.

3.7 Datasheet

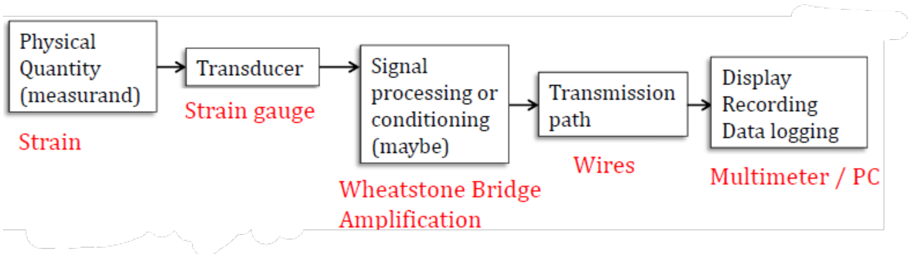
Datasheets are documents providing the specifications for a particular product. Some relevant examples from datasheets are:

- **Linearity:** how much the voltage-displacement relationship could go off the linear.
- **Temperature coefficient of sensitivity:** change of sensitivity due to the operating temperature
- **Zero shift with temperature:** how much the null position (location) could be shifted due to the operating temperature
- **Output ripple:** 'Hint' of AC input in DC output. Does not affect the mean DC output value.
- **Resistance range:** Range of available (total) pot resistance
- **Output smoothness:** Maximum resolution error
- **Tolerance:** Uncertainty in manufacturing of the resistance element

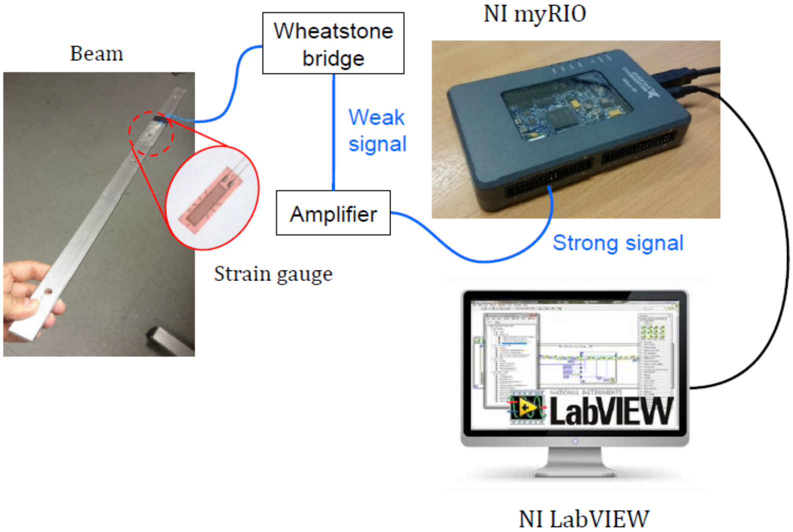
Chapter 4

Laboratory measurement

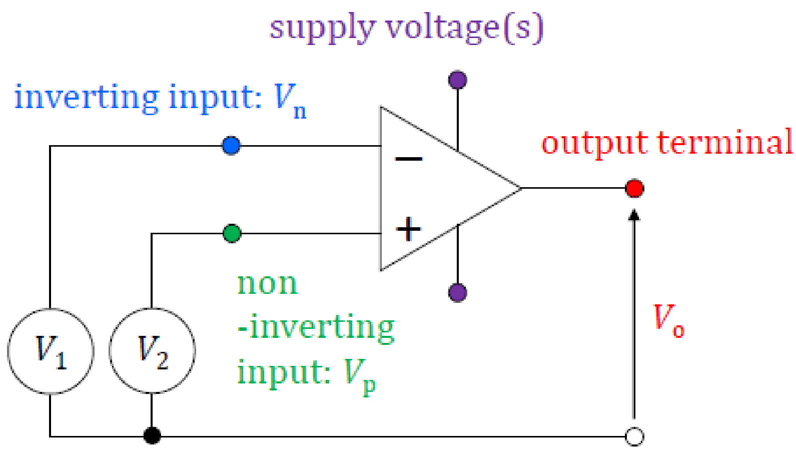
Stages in a measurement system



4.1 Amplifiers

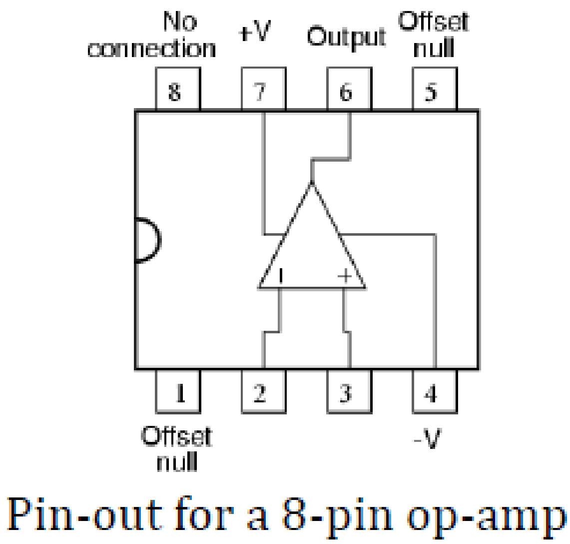


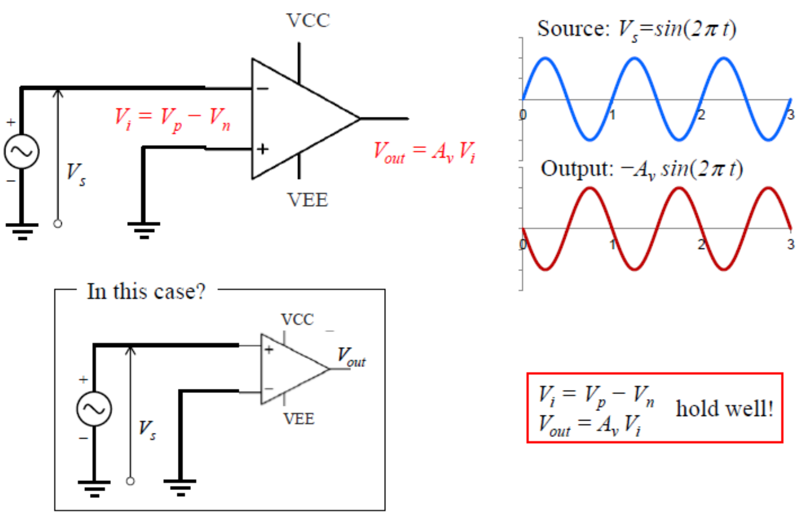
The voltage output from a transducer tends to be weak and this needs to be amplified. Operational amplifiers are normally around 1cm in size.



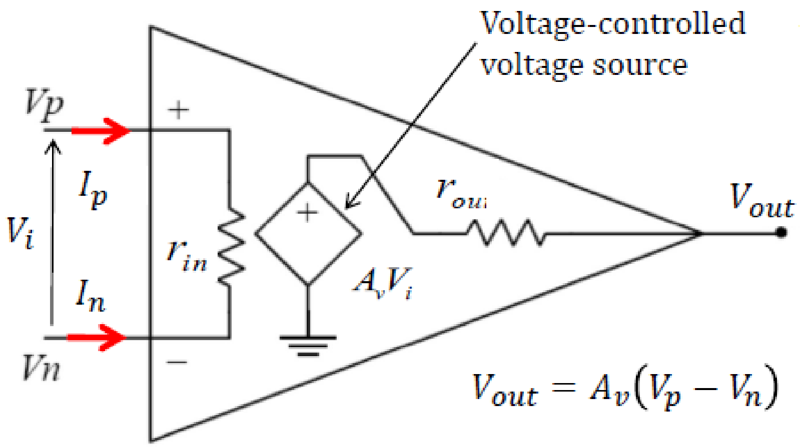
$$V_0 = A_v(V_p - V_n) = A_v(V_2 - V_1) \tag{4.1}$$

Where A_v is the gain and $(V_p - V_n)$ can be from a Wheatstone bridge. Operational amplifiers are quite an old technology (since WW2) and are still used because they are cheap and convenient.





4.1.1 Equivalent op-amp circuit and conditions of an ideal op-amp

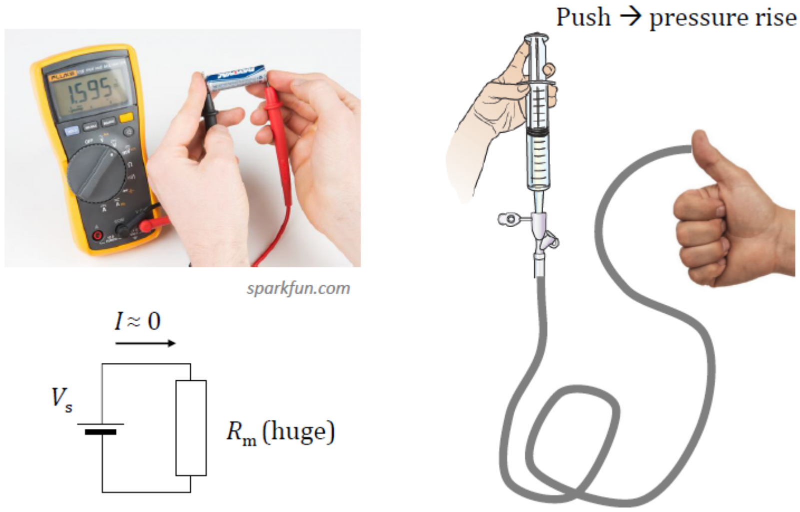


Terminology:

- Differential input voltage: $V_i = V_p - V_n$
- Input resistance: r_{in}
- Output resistance: r_{out}
- Open-circuit output voltage: V_{out}
- Differential voltage gain: A_v

Conditions of an ideal op-amp	**In reality**
No current into input terminals: $I_p = I_n = 0$	
Infinite input resistance: $r_{in} \to \infty$	$r_{in} > 200\text{k}\Omega$
Zero Output resistance $r_{out} = 0$	$r_{out} < 1\text{k}\Omega$
Infinite differential (or open-loop) gain $A_v \to \infty$	$A_v > 100000$
Zero common-mode voltage gain $A_{cm} = 0$	A_v also frequency dependant

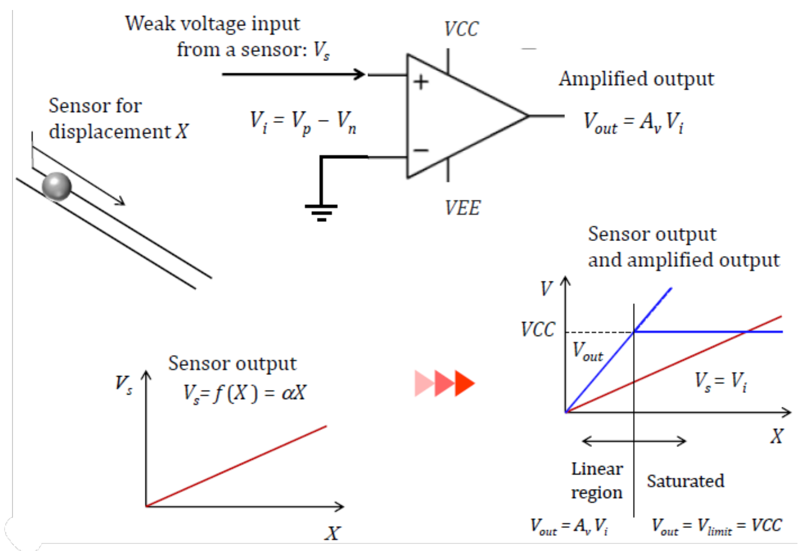
4.1.2 Non-zero voltage with zero current?



A question that is often asked is if the current is zero, how can we still have a voltage? When you connect a battery to a meter, where the meter resistance is normally huge. We can still measure the voltage. This is due to Ohm's law $I = \frac{V}{R_m}$. A good analogy to make is a plunger system, where the other end is covered. When you push down, increasing the pressure, this can still be felt on the tip of the thumb, despite there being no flow. In the same way, we can still 'feel' the voltage with 'no' current flow.

4.1.3 Saturation

In a practical scenario, we want to strengthen the weak signal from the sensor. Let us connect an input to the non-inverting terminal and the inverting input to the ground.

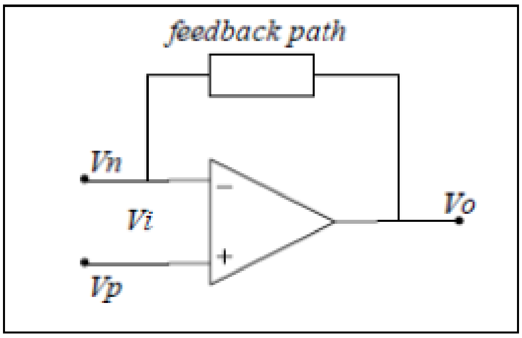


We can see that the signal is not infinitely amplified, as we are limited by the energy provided by our supply voltage. This is reflected in the graph above as the straight portion of the amplified line. This is called saturation. Due to A_v being quite large, our linear region is normally tiny. This may be a problem if we want to measure the linear region.

4.1.4 Negative feedback

So op-amps seem to be a useful device but also seem to be a bit tricky. They are much more useful when used as part of a larger circuit. Negative feedback is achieved by feeding a fraction of the output signal back to the **inverting** terminal as shown. By definition, the closed-loop gain of such a device is:

$$G = \frac{V_0}{V_i} \tag{4.2}$$



Negative feedback trades a reduction in gain with one that is smaller, stable and predictable, whilst giving the designer control over operational characteristics.

4.1.5 Basic feedback amplifier configurations

Only 2 resistors are required to construct a negative feedback amplifier. Note that output signal is always fed-back to the inverting input terminal.

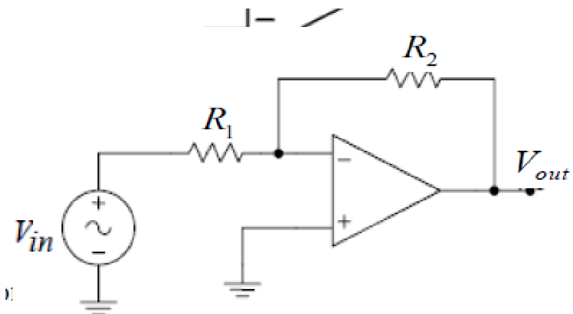


Figure 4.1: Inverting feedback amplifier arrangement.

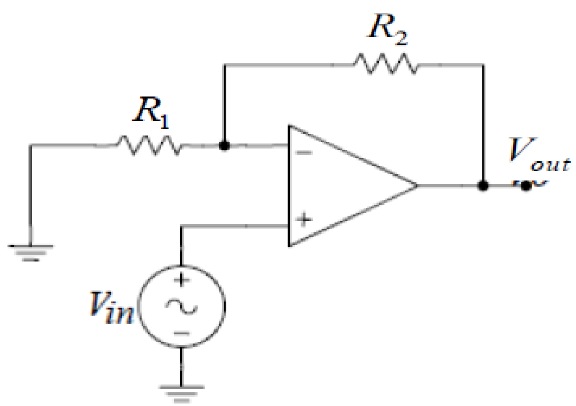
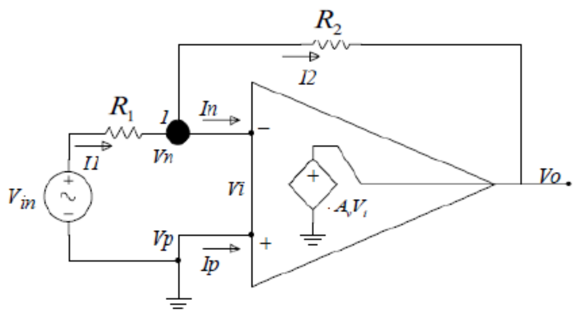


Figure 4.2: Non-inverting feedback amplifier arrangement.

We can see the difference between an inverting and non-inverting feedback amplifier is which input the source signal is connected to.

4.1.6 Basic inverting feedback amplifier



Let us derive its closed loop gain G. By definition:

$$V_0 = A_v(V_p - V_n) \tag{4.3}$$

$$V_p \to 0 \Longrightarrow V_n = -\frac{V_0}{A_v} \tag{4.4}$$

Apply KCL to Node 1:

$$\frac{V_{in} - V_n}{R_1} - I_n - \frac{V_n - V_0}{R_2} = 0 \Longrightarrow \frac{R_2}{R_1}V_{in} + v_n\left(1 + \frac{R_2}{R_1}\right) = V_0 \tag{4.5}$$

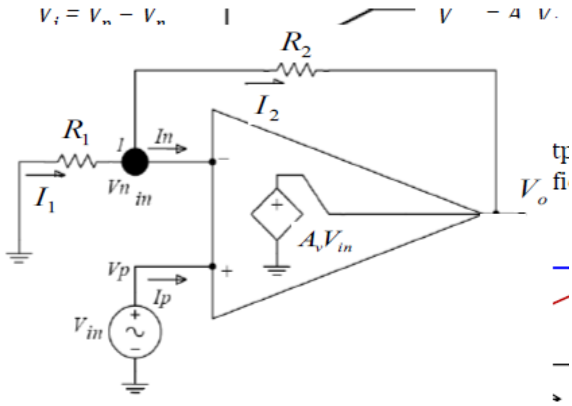
I_n is 0 from our model assumptions. Combining equations (4.4) and (4.5), we arrive at:

$$G \equiv \frac{V_0}{V_{in}} = -\frac{R_2}{R_1}\frac{1}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)} \tag{4.6}$$

For an ideal op-amp, $A_v \to \infty$:

$$G = \frac{V_0}{V_{in}} = -\frac{R_2}{R_1} \tag{4.7}$$

4.1.7 Basic non-inverting feedback amplifier



Derive its closed-loop gain G. By definition:

$$V_0 = A_v(V_p - V_n) \tag{4.8}$$

Apply KCL to node 1

$$\frac{0 - V_n}{R_1} = \frac{V_n - V_0}{R_2} \rightarrow \frac{V_0}{R_2} = V_n\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{4.9}$$

Combining equation (4.8) and (4.9), we arrive at:

$$G \equiv \frac{V_0}{V_{in}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_v}\left(1 + \frac{R_2}{R_1}\right)} \tag{4.10}$$

For an ideal amplifier, $A_v \rightarrow \infty$:

$$G = 1 + \frac{R_2}{R_1} \tag{4.11}$$

This is positive and always greater than 1.

4.1.8 Summary

Ideal feedback amplifier characteristics

	Inverting amplifier	**Non-inverting amplifier**
$G = \frac{V_0}{V_{in}}$	$-\frac{R_2}{R_1}$	$1 + \frac{R_2}{R_1}$
R_{in}	R_1	∞
R_{out}	0	0

4.1.9 Voltage amplification summary

Operational amplifier:

- The gain usually is too large (typically $> 100{,}000$) and not well defined. (V_0 cannot be larger than supply voltage)
- The gains for two op-amps of the same type can differ by a factor of two or more.
- The gain is not stable, can change with temperature, can change with changes of supply voltage.

To reduce these effects, we use negative feedback.

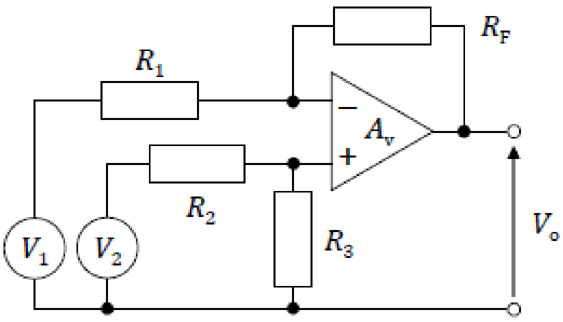


Figure 4.3: V_1 and V_2 could come from a Wheatstone bridge.

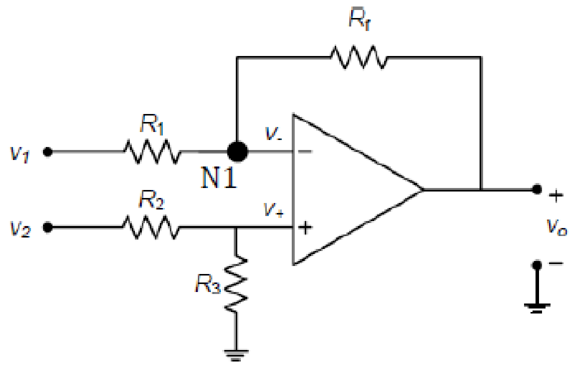
The output voltage V_0 is given as:

$$V_0 = \frac{R_F}{R_1}(V_2 - V_1) \tag{4.12}$$

Only when $\frac{R_3}{R_2} = \frac{R_F}{F_1}$. This is independent of A_v and the gain of this circuit, $G = \frac{R_F}{R_1}$ is well controllable.

4.1.10 Difference amplifier

Combining a non-inverting and inverting op-amp to take the difference between two inputs:



$$\frac{V_0}{A_v} = V_+ - V_- \to 0 \text{ as } A_v \to \infty \tag{4.13}$$

$$V_+ = \frac{R_3}{R_2 + R_3} \leftarrow \text{ voltage divider} \tag{4.14}$$

KCL at node N1

$$\frac{V_0 - V_-}{R_f} = \frac{V_- V_1}{R_1} \to \frac{V_0}{R_f} + \frac{V_1}{R_1} = \left(\frac{1}{R_f} + \frac{1}{R_1}\right) V_- \tag{4.15}$$

$$\frac{V_0}{R_f} + \frac{V_1}{R_1} = \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) V_2 \tag{4.16}$$

Combining equation (4.14) and (4.16), we arrive at:

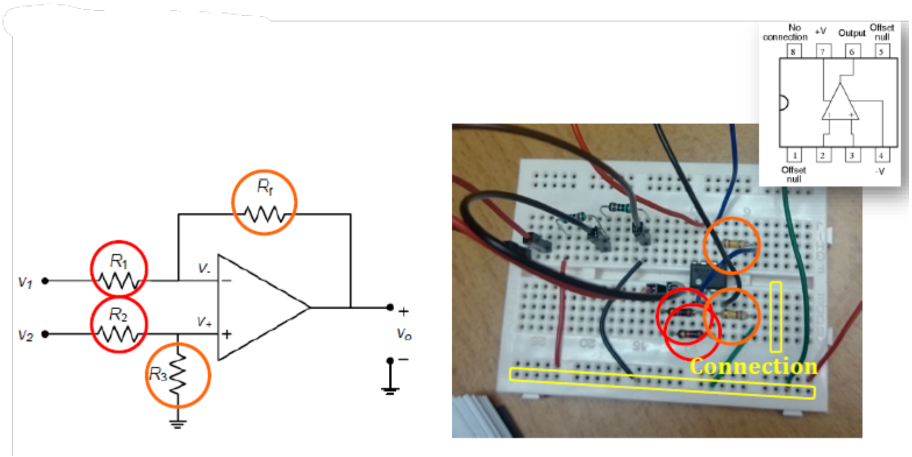
$$V_0 = R_f \left(\frac{R_f + R_1}{R_f R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) V_2 - \frac{R_f}{R_1} V_1 \tag{4.17}$$

$$V_0 = \frac{1 + \frac{R_f}{R_1}}{1 + \frac{R_2}{R_3}} V_2 - \frac{R_f}{R_1} V_1 \tag{4.18}$$

If we can make sure that $\frac{R_f}{R_1} = \frac{R_3}{R_2}$ is satisfied, we can further reduce our equation to:

$$V_0 = \frac{R_f}{R_i}(V_2 - V_1) \tag{4.19}$$

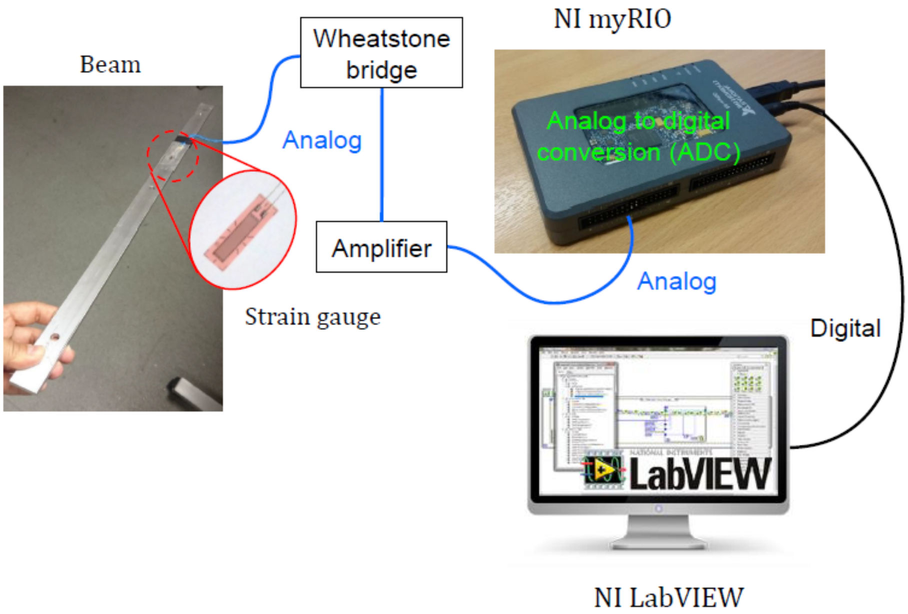
4.1.11 On a bread board



$$V_0 = \frac{R_f}{R_i}(R_2 - R_1) \tag{4.20}$$

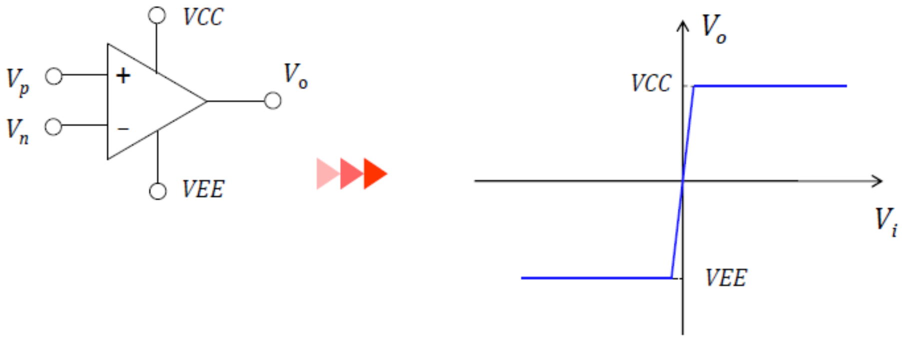
if $\frac{R_f}{R_1} = \frac{R_3}{R_2}$ can be satisfied.

4.2 Analogue-Digital (AD) conversion



4.2.1 Comparator

We are using an op-amp here, because when we have a high A_v, we have a very small linear region. Hence, when we look at a signal graph, we can see this as a sort of switching device:



With very large A_v, output V_0 would be: $V_0 = A_v(V_p - V_n)$. Hence:

$$V_0 \begin{cases} = VCC \text{ ... if } V_p > V_n \\ = VEE \text{ ... if } V_p < V_n \end{cases} \tag{4.21}$$

This essentially makes comparisons based on V_p and V_n

$$V_p > V_n \ ? \rightarrow \text{YES/NO} \tag{4.22}$$

4.2.2 Example ADC mechanism using comparators

A circuit to compare voltages:

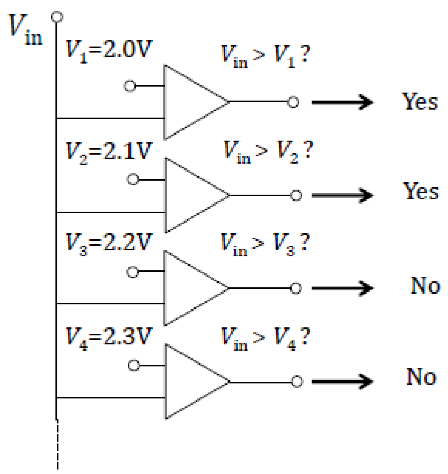


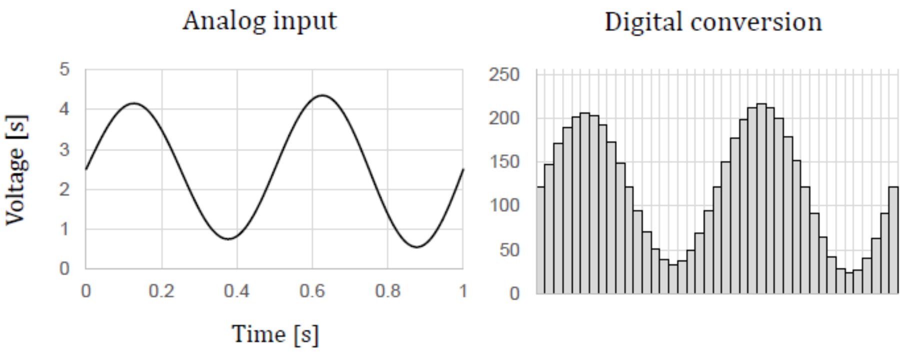
Figure 4.4: **Flash ADC:** number of comparisons done in parallel.

Setting up these comparators in parallel and setting a reference voltage for each, we can compare a single input and produce a digital binary signal, which tells us the range of voltages that V_{in} may occupy. In this case we can see that:

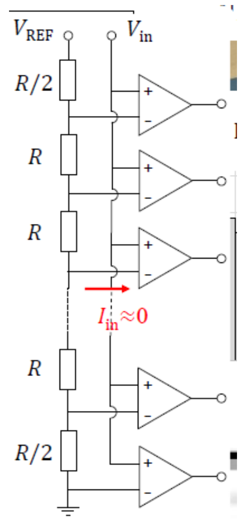
$$2.1\text{V} < V_{in} < 2.2\text{V} \tag{4.23}$$

Hence, V_{in} may be 2.1V, but other options are possible. When we store in a digital binary format this is called **quantisation**. When a voltage range 0-5V ($V_{REF} = 5$V) is quantised in 8-bit,

$$V_{in} = 2.14\text{V} \rightarrow V_{\text{in, quantised}} = int\left(\frac{V_{in}}{5[\text{V}]/2^8}\right) = 110 \tag{4.24}$$



A more practical form of a flash ADC circuit can be achieved using voltage dividers, for the k-th comparator:



$$V_{o,k} = A_v\left(V_{in} - \frac{\left(k+\frac{1}{2}\right)R}{nR}V_{REF}\right),\ k = 1,\ 2,\ ...,\ n-1 \tag{4.25}$$

n comparators allow judgement of YES/NO per every V_{REF}/n volt $\rightarrow$ **resolution** of conversion.

- 8-bit quantisation requires $2^8 = 256$ comparators.
- 16-bit quantisation requires $2^{16} = 25536$ comparators.

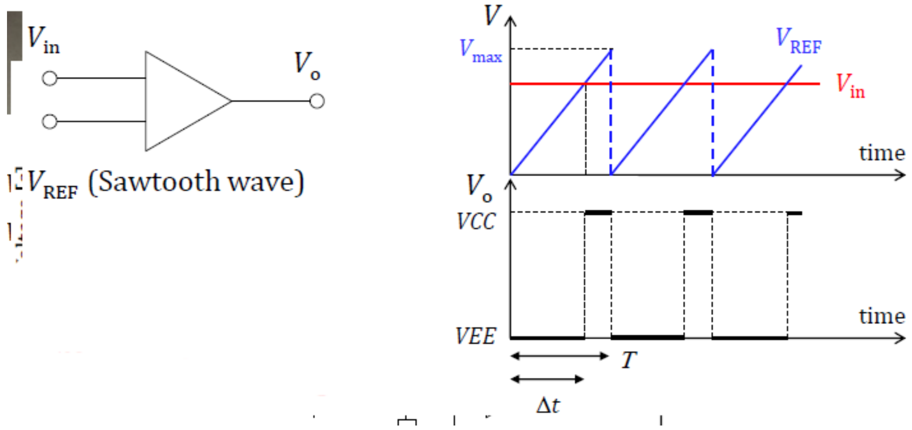
Flash ADC can work very quickly but becomes complex for high resolutions.

4.2.3 Other types of ADC

There are other types of conversion:

- Successive conversion - successively narrow down range of V_{in}
- Integrating - use input signal integration time to find V_{in}
- Ramp-compare

Ramp-comparison utilises a saw tooth wave and the time period of switching to work out what V_{in} is.



$$V_{in} = \frac{\Delta t}{T} V_{max} \tag{4.26}$$

Ramp-compare can be done with a simply system (only one comparator) and can be accurate but may required (relatively - you can make the frequency of the sawtooth very large) long processing time.

4.2.4 Typical issues to be considered

Resolution

Number of bits allocated to represent the input range (= interval of discrete data values).

Accuracy

How the digitised values are different from the original values. E.g. affected by accuracy of Δt measurement in ramp-compare. Though quantisation causes rounding error, it is not necessarily associated with accuracy.

Sampling rate

How many data points over time (how many ADC per time period). Affected by ADC time. Particular impact on time-dependent data/frequency (e.g. music) leading to aliasing/anti-aliasing.

Noise

Some methods (e.g. integrating) are good for noisy data.

4.3 Interface devices to PC and programming platform

4.3.1 Arduino

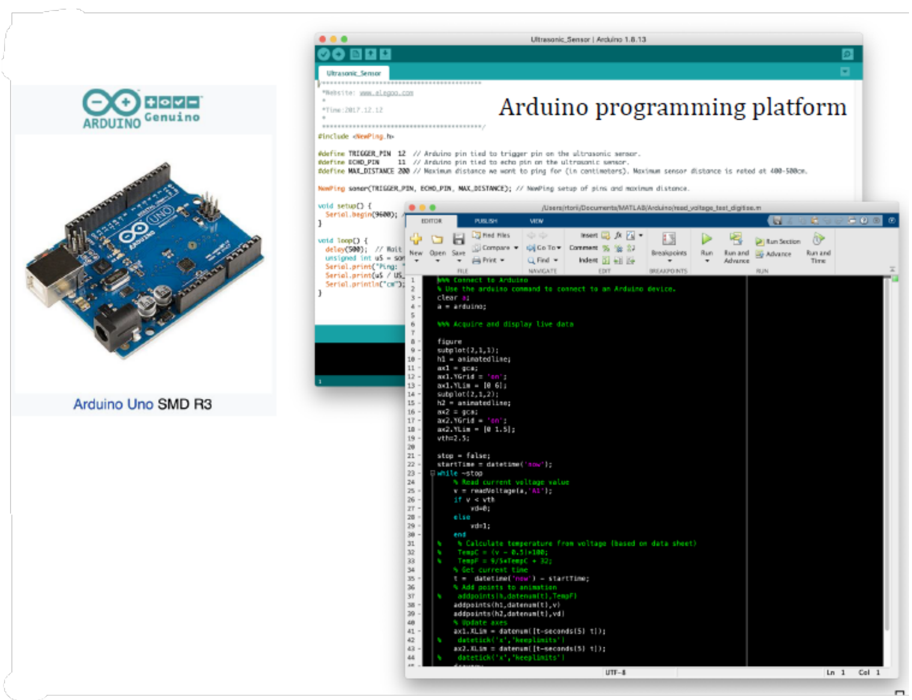
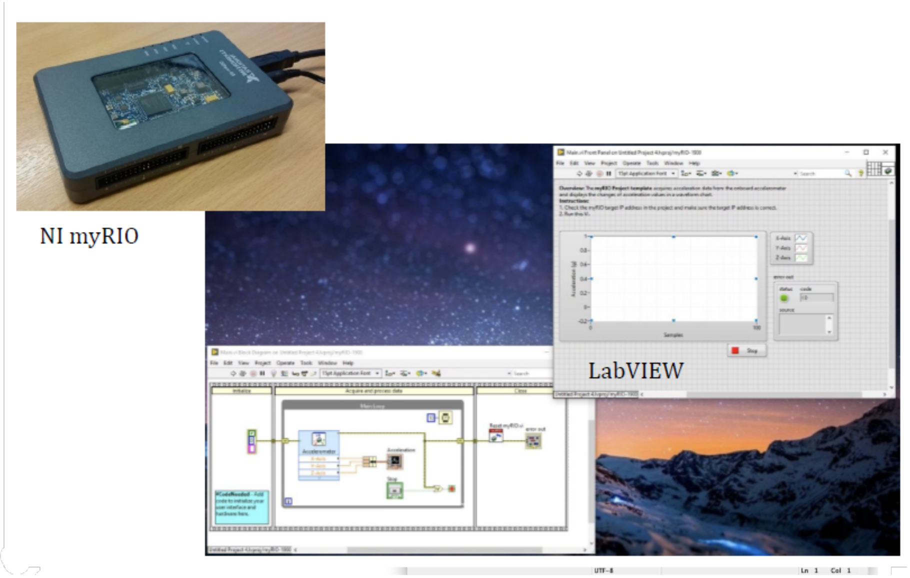


Figure 4.5: Can be linked up with MATLAB or any other language.

4.3.2 NI products and LabVIEW



4.4 Experimental Errors

Because of the inherent limitations of measurement equipment of techniques, there will always be some 'uncertainty' associated with experimental results. This uncertainty of error conveys significant information about the experiment and the result obtained, so you will be expected to assess the error in the measurements that you perform.

Example

We want to know the volume of a cylinder based on diameter and height measurement:

$$V = \pi r^2 h = \frac{\pi h d^2}{4} \tag{4.27}$$

We must ask how accurate are d and h, and also how they can be measured in the calculation of V? Suppose you are measuring a variable x:

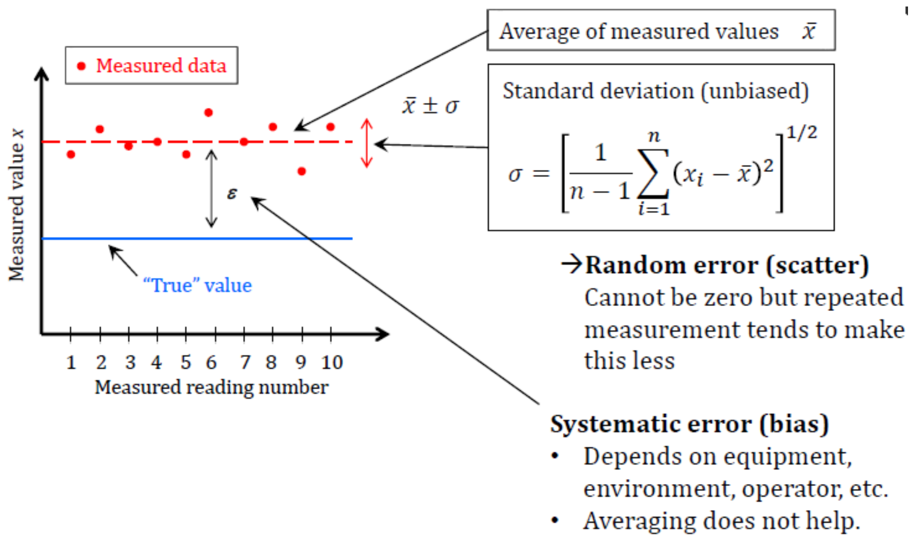


Figure 4.6: Actual error is mostly combination of both types of errors.

4.4.1 Systematic errors

Depends on:

- Equipment
- Environment
  - For example temperature may affect the measurement results, e.g. strain gauges. We deal with this by introducing temperature compensation.
- Operator
  - Consider measuring a sample with length L and mass M. These are well defined variables, however the thickness h will require sampling.

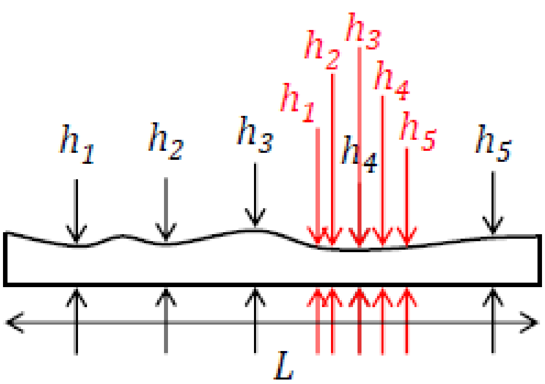


Figure 4.7: Sampling needs to be done carefully to ensure good representation.

Looping back to our cylinder example earlier, let us say:

$$d = 8.2 \pm 0.5\text{cm} \tag{4.28}$$
$$h = 9.5 \pm 0.2\text{cm} \tag{4.29}$$

4.4.2 Combining errors in a single quantity

Uncertainty analysis / error propagation - Estimating the overall error in the measurement system. For example, when two resistors are connected in series, what is the expected uncertainty of the total resistance?

$$R_1 = 1000 \pm 1.5\Omega \tag{4.30}$$
$$R_2 = 500 \pm 1.0\Omega \tag{4.31}$$

The straightforward option is to simply add them directly $\rightarrow 2.5\Omega$

$$R_{total} = R_1 + R_2 = 1500 \pm 2.5\Omega \tag{4.32}$$

The error in R_1 and R_2 are a statistical indication, whereby the error is assumed to follow a Gaussian (normal) distribution. However, simple addition of the errors is too pessimistic! The two errors are independent and unlikely to show their maximum at the same time. If the errors are **independent** and in the same **quantity**, they can be combined as:

$$E = \pm\sqrt{e_1^2 + e_2^2} \tag{4.33}$$

and for this resistor example, overall error is 1.8Ω. The rule above can be applied to any number of independent errors:

$$E = \pm\sqrt{e_1^2 + e_2^2 + e_e^2 + ...} \tag{4.34}$$

4.4.3 Various combinations of quantities

Assuming independent measurements A and B, which have total errors ΔA and ΔB associated with them, are combined t o give the result X, which has error ΔX, then:

$$\text{if } X = A + B \text{ or } X = A - B \longrightarrow \Delta X = \sqrt{(\Delta A)^2 + (\Delta B)^2} \tag{4.35}$$
$$\text{if } X = AB \text{ or } X = \frac{A}{B} \longrightarrow \frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2} \tag{4.36}$$
$$\text{if } X = A^n \longrightarrow \frac{\Delta X}{X} = n\frac{\Delta A}{A} \tag{4.37}$$
$$\tag{4.38}$$

Note for equation (4.36), we always use plus in our square root because the error only increases with addition or subtraction. When more than two quantities are involved, the equations are extended in a straightforward way:

$$\text{if } X = A + B - C + D... \longrightarrow \Delta X = \sqrt{(\Delta A)^2 + (\Delta B)^2 + (\Delta C)^2 + (\Delta D)^2} \tag{4.39}$$

$$\text{if } X = \frac{AB}{CD} \longrightarrow \frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2 + \left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta D}{D}\right)^2} \tag{4.40}$$

Adding errors in summation (A+B) and subtraction (A-B) are simpler because they must be the same physical quantity (mass, length, etc.) with the same unit. Products and divisions are usually in different quantity and the errors need to be added in normalised fashion. Let us use these in our cylinder example now:

$$V = \pi r^2 h = \frac{\pi h d^2}{4} \tag{4.41}$$

$$d = 8.2 \pm 0.5\text{cm} \tag{4.42}$$

$$h = 9.5 \pm 0.2\text{cm} \tag{4.43}$$

$$V = 387\text{cm}^3 \tag{4.44}$$

Using two of the error estimation rules in combination,

$$\frac{\Delta V}{V} = \sqrt{\left(\frac{\Delta h}{h}\right)^2 + \left(\frac{2\Delta d}{d}\right)^2} \longrightarrow \Delta V = 54\text{cm}^3 \tag{4.45}$$

Therefore the volume is $V = 387 \pm 54\text{cm}^3$

4.4.4 General rule for combining errors

Consider the problem of computing a quantity N, where N is a known function of the n independent variables $u_1,\ u_2,\ u_3,\ ...,\ u_n$. That is:

$$N = f(u_1,\ u_2,\ u_3,\ ...,\ u_n) \tag{4.46}$$

The us are the measured quantity with errors $\pm\Delta u_1,\ \pm\Delta u_2,\ \pm\Delta u_3,\ ...,\ \pm\Delta u_n$ which result in error in N. We could now write:

$$N \pm \Delta N = f(u_1 \pm \Delta u_1,\ u_2 \pm \Delta u_2,\ u_3 \pm \Delta u_3,\ ...,\ u_n \pm \Delta u_n) \tag{4.47}$$

Expanding f in Taylor series we get:

$$\begin{aligned} &f(u_1 \pm \Delta u_1,\ u_2 \pm \Delta u_2,\ u_3 \pm \Delta u_3,\ ...,\ u_n \pm \Delta u_n) = \\ &\qquad f(u_1,\ u_2,\ u_3,\ ...,\ u_n) + \Delta u_1 \frac{\partial f}{\partial u_1} + \Delta u_2 \frac{\partial f}{\partial u_2} + ... + \Delta u_n \frac{\partial f}{\partial u_n} \\ &\qquad\qquad + \text{higher order terms} \end{aligned} \tag{4.48}$$

Where $f(u_1,\ u_2,\ u_3,\ ...,\ u_n)$ is our N. We can neglect our higher order terms because $\Delta u_n << 1$ and what we are left with is our error.

$$\text{Error } = \Delta u_1 \frac{\partial f}{\partial u_1} + \Delta u_2 \frac{\partial f}{\partial u_2} + ... + \Delta u_n \frac{\partial f}{\partial u_n} \tag{4.49}$$

Considering Δus are not absolute limits of error but rather as statistical bounds of uncertainties the proper method of combing the error is according to the root-sum square formula:

$$E = \sqrt{\left(\Delta u_1 \frac{\partial f}{\partial u_1}\right)^2 + \left(\Delta u_2 \frac{\partial f}{\partial u_2}\right)^2 + ... + \left(\Delta u_n \frac{\partial f}{\partial u_n}\right)^2} \tag{4.50}$$

4.4.5 Example

The resistance of a certain size of copper wire is given as

$$R = R_0[1 + \alpha(T - 20)] \tag{4.51}$$

Where $R_0 = 6\Omega \pm 0.3\%$ is the resistance at 20°C, $\alpha = 0.004°\text{C}^{-1} \pm 1\%$ is the temperature coefficient of resistance, and the temperature of the wire is $T = 30 \pm 1°\text{C}$. Calculate the resistance of the wire and its uncertainty.

The nominal resistance is:

$$R = 6[1 + 0.004 \times (30 - 20)] = 6.24\Omega \tag{4.52}$$

The uncertainty is from the equation earlier,

$$\Delta R = \sqrt{\left(\Delta R_0 \frac{\partial R}{\partial R_0}\right)^2 + \left(\Delta \alpha \frac{\partial R}{\partial \alpha}\right)^2 + \left(\Delta T \frac{\partial R}{\partial T}\right)^2} \tag{4.53}$$

Here,

$$\frac{\partial R}{\partial R_0} = 1 + \alpha(T - 20) \tag{4.54}$$
$$= 1 + 0.004 \times (30 - 20) = 1.04 \tag{4.55}$$
$$\frac{\partial R}{\partial \alpha} = R_0(T - 20) \tag{4.56}$$
$$= 6 \times (30 - 20) = 60 \tag{4.57}$$
$$\frac{\partial R}{\partial T} = R_0 \alpha \tag{4.58}$$
$$= 6 \times 0.004 = 0.024 \tag{4.59}$$

And,

$$\Delta R_0 = 6 \times 0.003 = 0.018\Omega \tag{4.60}$$
$$\Delta \alpha = 0.004 \times 0.01 = 4 \times 10^{-5}°\text{C}^{-1} \tag{4.61}$$
$$\Delta T = 1°\text{C} \tag{4.62}$$

Thus, the uncertainty in the resistance is:

$$0.0305\Omega \text{ or } 0.49\% \tag{4.63}$$

4.4.6 What if we do not know the relationship between error sources and measured variables?

Example: load cell (force measurement)

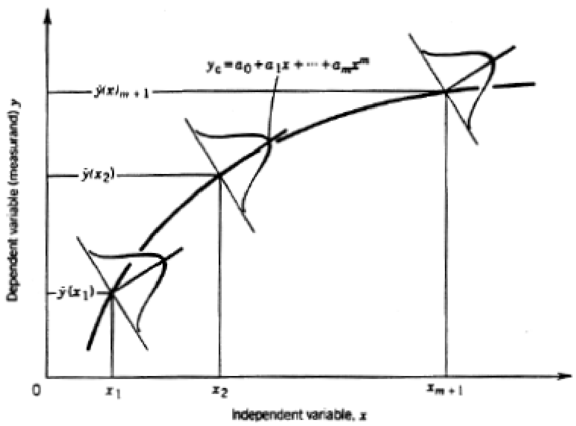
$$V_0 = f(\text{load, etc}) \tag{4.64}$$

Simply add relevant errors up based on the specification sheet as a safety measure.

4.5 Reference information about least-square curve fitting

4.5.1 Least square analysis

The least squares method is often used to establish the functional relationship between the dependent measured variable and the independent process variable. In most situations, the anticipated relationship is polynomial. The variation found in the measured variable is assumed to follow a normal (Gaussian) distribution about each fixed value of the independent variable.

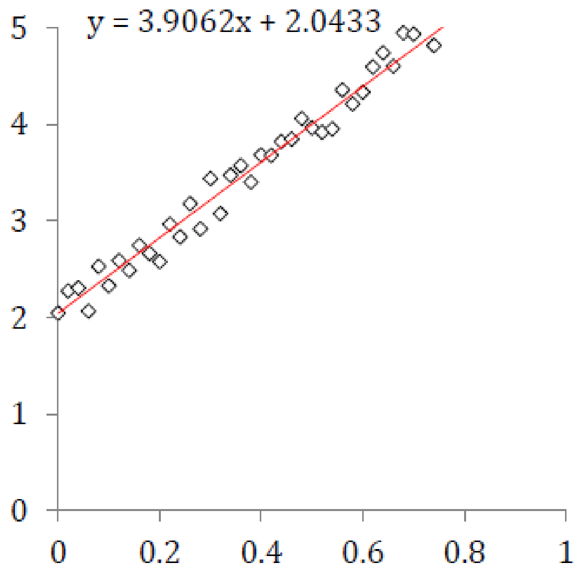


Suppose two variables x and y are measured over a range of values. We want to find their mathematical relationship. The simplest polynomial function is linear: $y = ax + b$. Least square analysis is done by minimising the quantity:

$$S = \sum_{i=1}^{n} [y_i - (ax_i + b)]^2 \tag{4.65}$$

This is the same as finding:

$$\frac{\partial S}{\partial a} = 0, \ \frac{\partial S}{\partial b} = 0 \tag{4.66}$$



$$\frac{\partial S}{\partial a} = \sum_{i=1}^{n}(2ax_i^2 + 2bx_i - 2x_iy_i) = 0 \rightarrow a\sum_{i=1}^{n} x_i^2 + b\sum_{i+1}^{n} x_i = \sum_{i=1}^{n} x_iy_i \tag{4.67}$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{n}(2ax_i + 2b - 2y_i) \rightarrow a\sum_{i=1}^{n} x_i + nb = \sum_{i=1}^{n} y_i \tag{4.68}$$

$$a = \frac{n\sum_{i=1}^{n} x_iy_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \tag{4.69}$$

$$b = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} x_iy_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \tag{4.70}$$

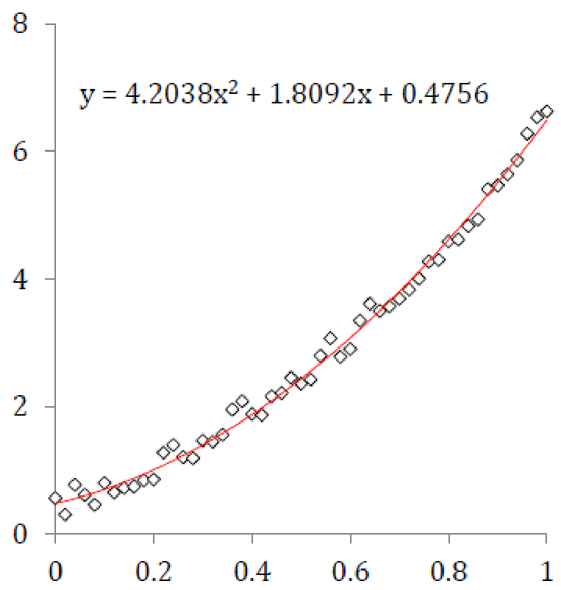
Even with higher order polynomials, for example

$$y = ax^2 + bx + c \tag{4.71}$$

We can define quantity

$$S = \sum_{i=1}^{n}[y_i - (ax_i^2 + bx_i + c)]^2 \tag{4.72}$$

for minimisation.


y = 4.2038x2 + 1.8092x + 0.4756
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1

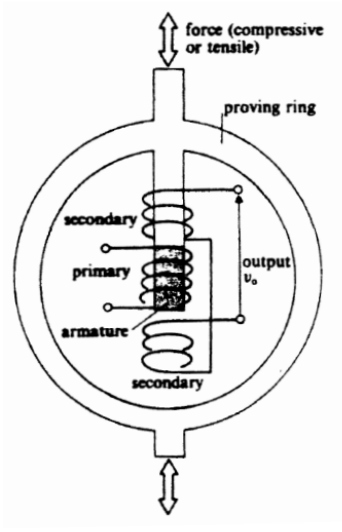
Chapter 5

Measurement of Force and Acceleration

16/11/2020

5.1 Measuring Forces

In the simplest form of force transducer, the force is applied to some form of **elastic member**, which compresses, expands or bends. The resulting displacement or strain is then sensed by a **secondary transducer** which converts it to an output signal.



The other forms of force transducers:

1. Using the force-balance principle

2. Using the piezoelectric effect

5.1.1 Elastic Sensing Elements

Elastic sensing elements:

- Are primary transducer (force $\rightarrow$ displacement)
- The force-displacement relationship should be definitive (mostly linear)

The deformation is then detected by either measuring the strain in the member using strain gauges or measuring the displacement of a point on the elastic member.

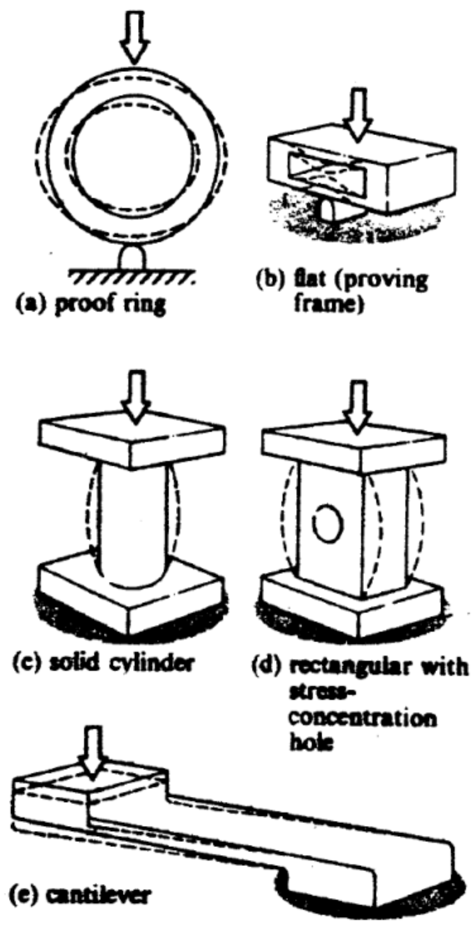


Figure 5.1: Common elastic sensing elements

Real World Example: Electronic Scale

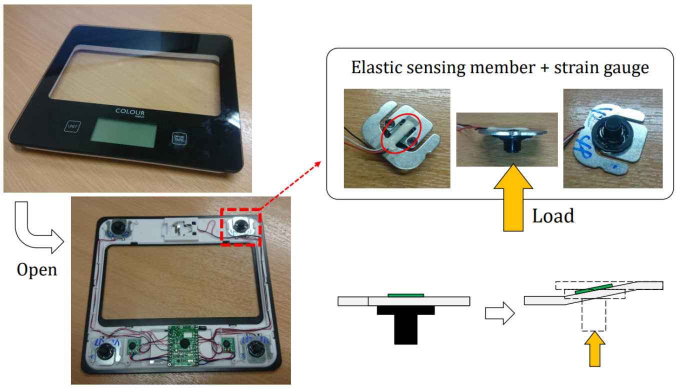
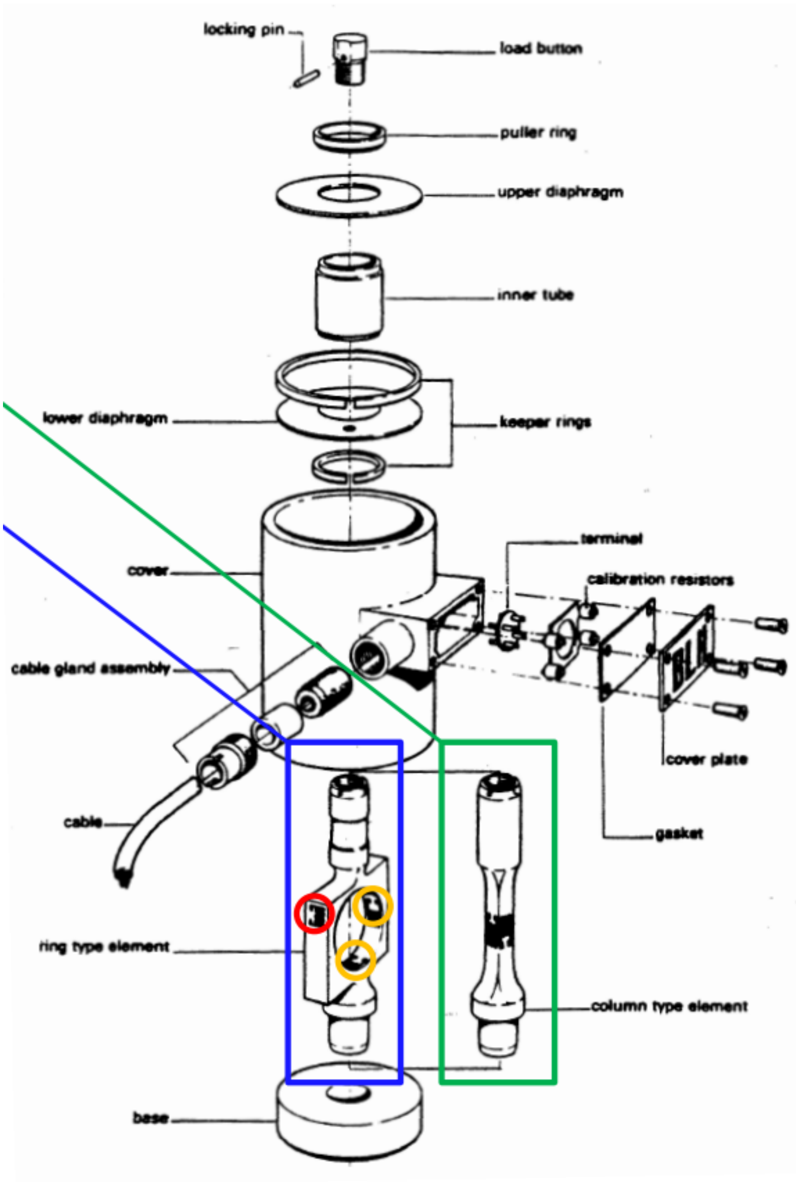


Figure 5.2: Electronic Kitchen Scale

Each transducer has a sensing member and a strain gauge. When a load is applied on the elastic member, it undergoes a displacement, which is sensed by the strain gauge. The output signal from all the transducers is used to determine a digital reading.

5.1.2 Strain Gauge Load Cell

The most common type of load sensing cell.



Sensing Element Types:

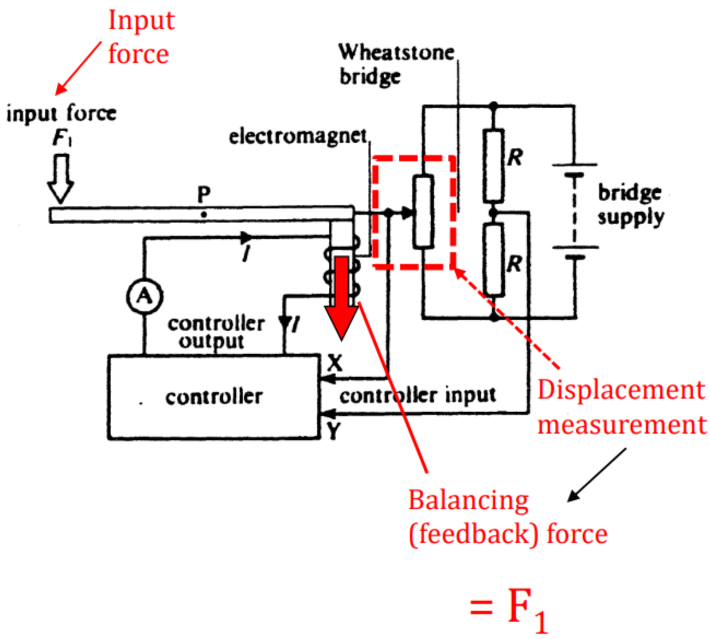
- Column type (green)
- Ring type (for smaller forces) (blue)

Each sensing element has multiple strain gauges mounted on it: typically, four measuring gauges (yellow) and two dummy gauges (red) for **span temperature compensation**.

- To account for the deformation of sensing element due to temperature.
- To compensate "load-displacement" relationship.

5.1.3 Force Transducer Using the Force-Balance Principle

Another type of force transducer makes use of a force balancing the input force provided by a feedback mechanism.



Advantages:

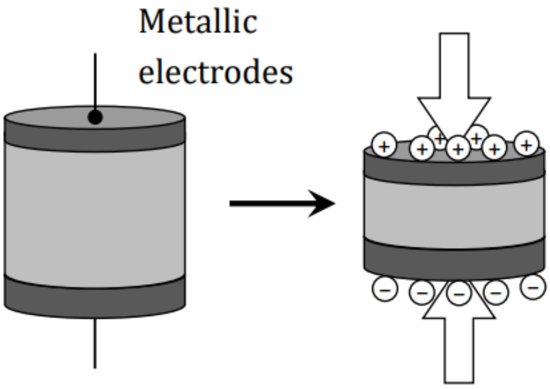
1. The relationship between the current I and the feedback force is linear.

2. The rotation of the beam is almost zero all the time and the force-deflection characteristics for the beam would not be present.

5.1.4 Piezoelectric Force Transducers

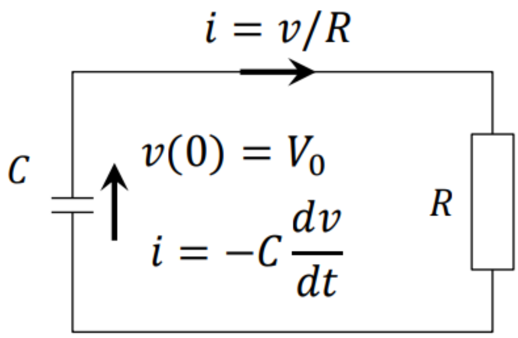
These transducers use a piezoelectric crystal (e.g. quartz) or ceramic (e.g. PZT – lead zirconate titanate [Has to be electrically polarised before acquiring piezoelectricity]) , where electric charges of opposite polarity appear on the parallel faces of the transducer when the faces are squeezed together.

The amount of charge depends linearly on the force applied, within a limited range, and its polarity depends on the directions of the crystallographic axes.



As a whole, this works as a capacitor as well. Step input force yields charge Q_o, and corresponding output voltage is $V_o = Q_o/C$.

If the force is held steady, no more charge is generated but the voltage across the transducer decreases because the charges are taken by the measuring circuit as a current (i.e. flow of charge).



From the current equations:

$$\frac{v}{R} = -C\frac{\mathrm{d}v}{\mathrm{d}t} \tag{5.1}$$

After rearranging:

$$\frac{1}{v}\,\mathrm{d}v = -\frac{1}{RC}\,\mathrm{d}t \tag{5.2}$$

Integration of the equation gives:

$$v = V_0 e^{-\frac{t}{RC}} \longrightarrow \text{Exponential decay} \tag{5.3}$$

Piezo**electric** transducers are **not suitable** for static or quasi-static force but are used for measuring rapidly changing forces, such as those in mechanical vibrations. (Note: Piezo**resistive** transducers are used for static measurements)

5.2 Measuring Acceleration

Acceleration transducers are called accelerometer.

A measure g is widely used, indicating the acceleration experienced by a freely falling body due to gravity ($\approx 9.81 m/s^2$).

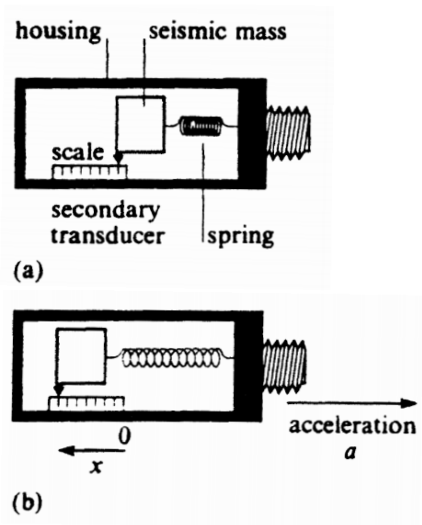
This is a convenient standard for the calibration and in many transducers specifications, the acceleration is stated as multiples of g.

Several different types of accelerometers (all of them are making use of seismic-mass):

- Seismic-mass accelerometer
- Strain-gauge accelerometer
- Potentiometric accelerometer
- Servo accelerometer

5.2.1 Seismic-Mass Accelerometer

A known mass, called seismic mass (or proof mass), is connected to some form of spring.



Ignoring friction, the seismic mass displacement along the sensing axis of the accelerometer is related to the force exerted by the spring as $F = kx$
(k is the spring constant in N/m).

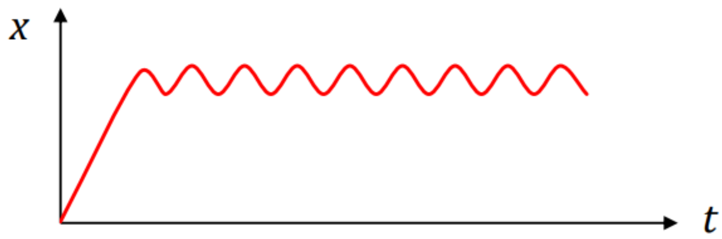
Considering also the acceleration force $F = ma$, we get:

$$ma = kx \tag{5.4}$$

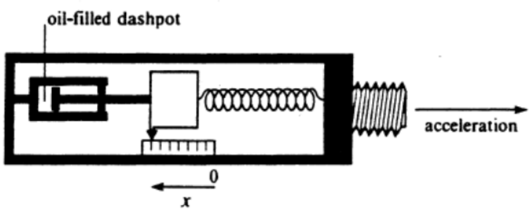
$$a = \frac{kx}{m} \tag{5.5}$$

However, this only applies to **static response** of accelerometer.

Question: What is the expected motion of the seismic mass when the accelerometer is turned upright (measuring rapidly changing / dynamic acceleration)? (assuming the motion is limited in the sensing axis)



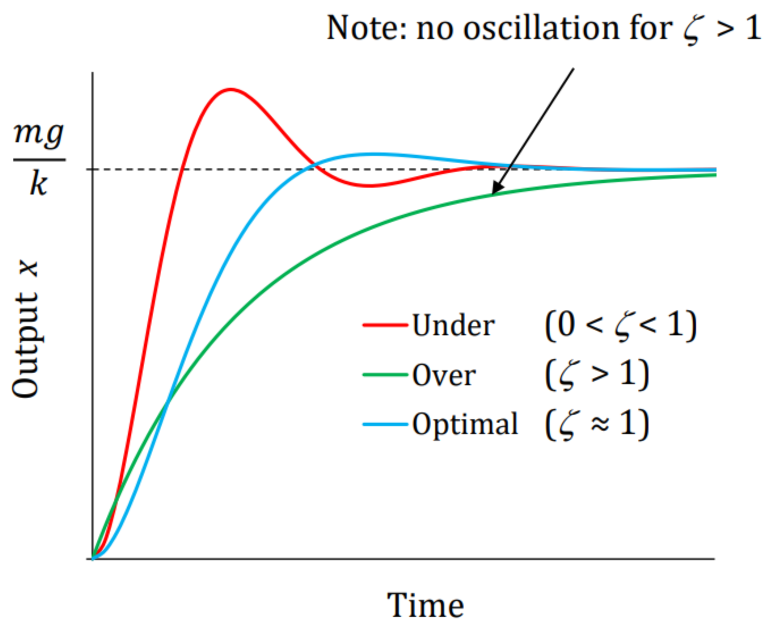
In order to obtain a more acceptable dynamic response, a controlled amount of extra damping is added via an oil-filled dashpot.



As the result, the position of the seismic mass x varies in time as shown in the figure below, in three different modes:

- Underdamped (Red)
- Overdamped (Green)
- Optimal (Blue)

where the steady state is reached relatively rapidly.



The amount of damping in a system of this sort is expressed by means of a damping factor ζ (zeta). Without damping, the system would oscillate at its natural frequency:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{5.6}$$

As the damping increase, the oscillating frequency changes as:

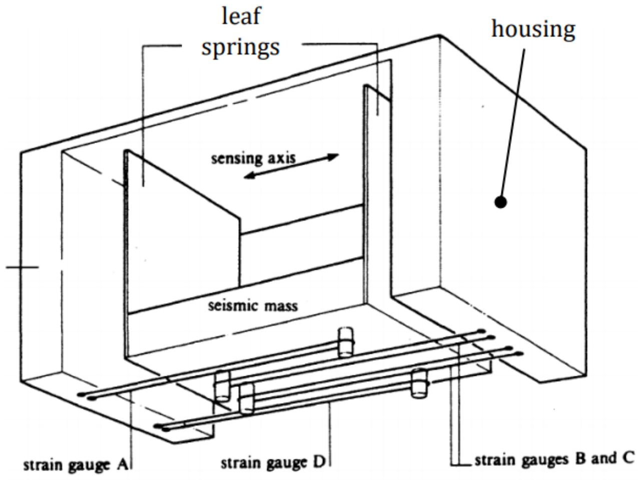
$$f = f_n\sqrt{1-\zeta^2} \tag{5.7}$$

5.2.2 Strain Gauge Accelerometer

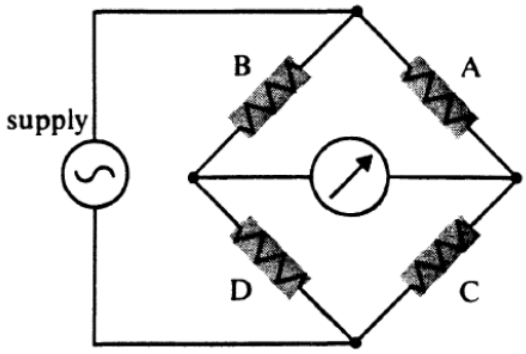
- The seismic mass is suspended from the housing by a pair of leaf springs.

- The closed housing contains silicon fluid for the damping.

- Four unbonded strain gauges are used (that is, no backing material).

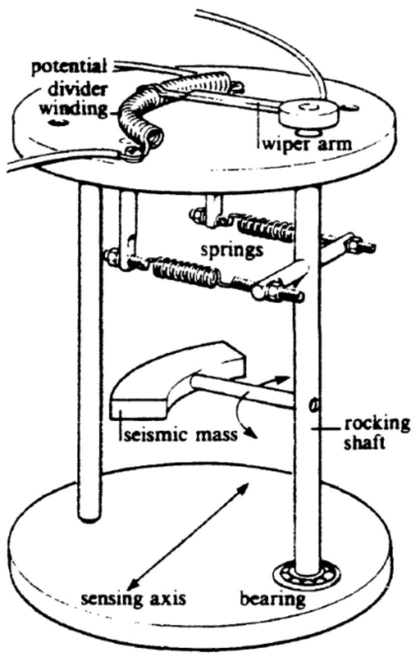


For this type of accelerometers, the seismic mass can move in any direction, so an estimate for **cross-axis sensitivity** is important.
→ Sensitivity to the motion in the direction normal to the sensing axis.
→ Often mentioned in specifications, expressed in percentage of the full-scale output.

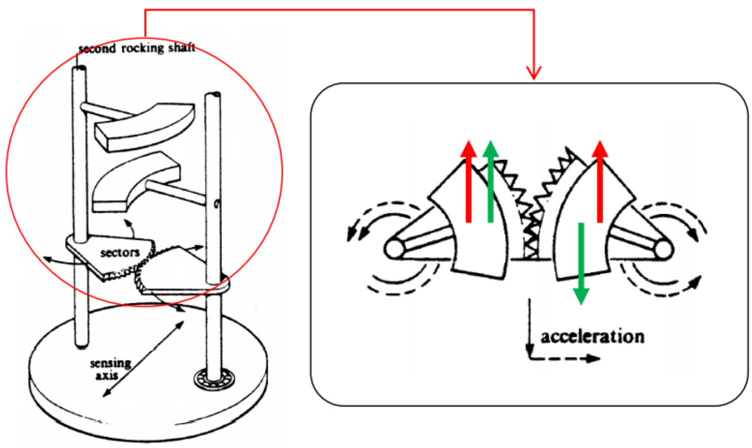


5.2.3 Potentiometric Accelerometer

This is again a type of seismic accelerometer, has been designed for use in the flight recorder of an aircraft. It measures the vertical accelerations.



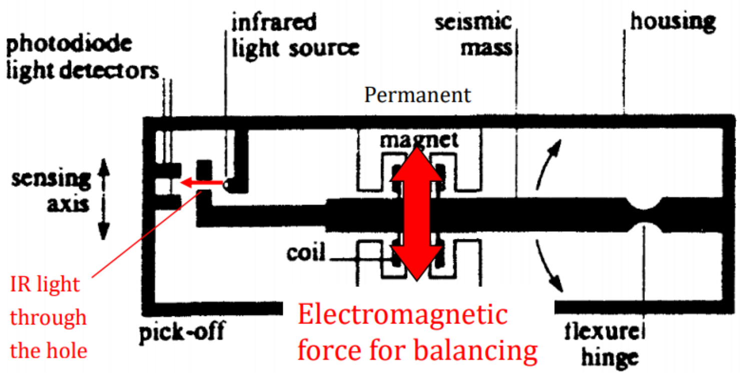
A large cross-axis sensitivity is expected and is a problem. $\longrightarrow$ Can be avoided by a mechanism with a second shaft as shown below.



5.2.4 Servo Accelerometer

The spring force of the previously introduced accelerometers is replaced here by an electromechanical force provided by what is called a **forcer**.

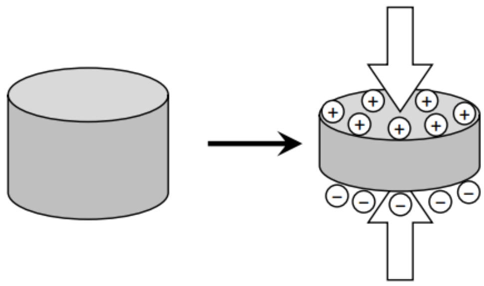
The system is similar to the force transducer using force-balance principle; the deflection of the sensing element (the beam) is detected by the light source and photodiodes (on the left) whose output is fed back to the current through the forcer coil to maintain zero deflection position via the electromechanical force generated between the coil and permanent magnet.



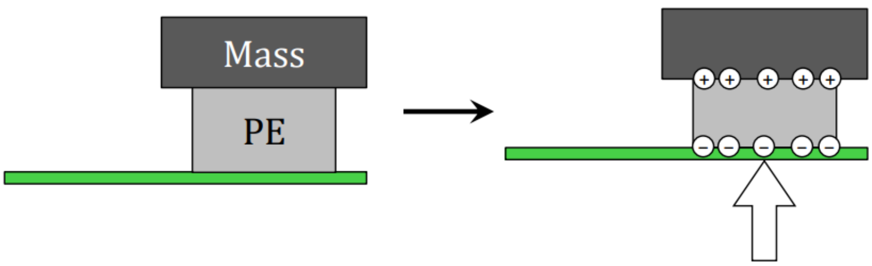
5.2.5 Piezoelectric Accelerometer

Piezoelectric effect

- Electric charge appearing on the surface of certain solid materials in response to mechanical stress applied



Use as accelerometer with a mass on top



Advantages:

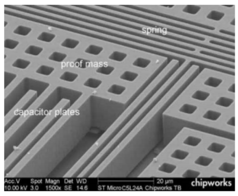
- Small, simple and cheap
- Durable (little degradation in time)

Disadvantages:

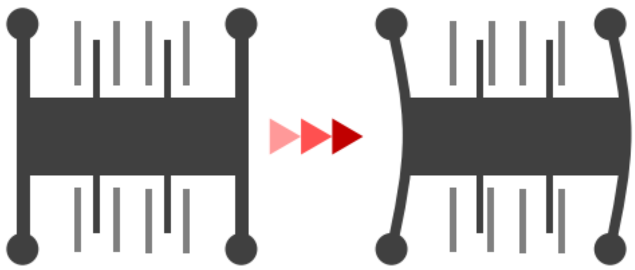
- Mass of the measurement system may alter the dynamic characteristics of whole system
- Difficult to measure static variables

5.2.6 MEMS Accelerometer

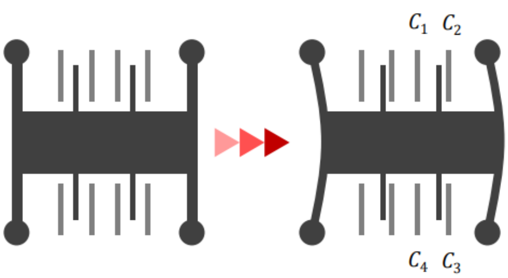
In many nowadays applications such as mobile phones, same principles as we learned are used but in a smaller scale, in MEMS (micro-electromechanical system).

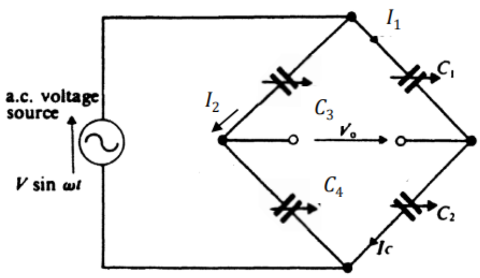


A simplified example of MEMS accelerometer is shown below – this design is making use of a seismic mass and capacitance displacement transducer.



Damping is provided by **squeeze film damping**.





Following the same derivation as in the displacement measurement:

$$V_{c1} = \frac{I_1}{\omega C_1} \tag{5.8}$$

$$V_{c2} = \frac{I_1}{\omega C_2} \tag{5.9}$$

$$V_{c3} = \frac{I_2}{\omega C_3} \tag{5.10}$$

$$V_{c4} = \frac{I_2}{\omega C_4} \tag{5.11}$$

The voltage across C_2 as a fraction of the bridge energising voltage, V is:

$$\frac{V_{c2}}{V} = \frac{V_{c2}}{V_{c1} + V_{c2}} = \frac{\frac{I_C}{\omega C_2}}{\frac{I_C}{\omega C_1} + \frac{I_C}{\omega C_2}} = \frac{C_1}{C_1 + C_2} \tag{5.12}$$

$$\frac{V_{c4}}{V} = \frac{C_3}{C_3 + C_4} \tag{5.13}$$

The capacitance in balance:

$$C_0 = \frac{\epsilon_0 A}{d} \tag{5.14}$$

where d is the balance separation. This varies with displacement as:

$$C_1 = \frac{\epsilon_0 A}{d + \delta} = C_0 \frac{d}{d + \delta} \tag{5.15}$$

$$C_2 = C_0 \frac{d}{d - \delta} \tag{5.16}$$

$$C_3 = C_0 \frac{d}{d - \delta} \tag{5.17}$$

$$C_4 = C_0 \frac{d}{d + \delta} \tag{5.18}$$

Substituting these for the equation of the voltage across C_2 and C_4 yields:

$$\frac{V_{c2}}{V} = \frac{d - \delta}{(d - \delta) + (d + \delta)} = \frac{d - \delta}{2d} \tag{5.19}$$

$$\frac{V_{c4}}{V} = \frac{d + \delta}{2d} \tag{5.20}$$

The amplitude of the output voltage is thus:

$$V_o = V_{c4} - V_{c2} = \left(\frac{d + \delta}{2d} - \frac{d - \delta}{2d} \right) V = \frac{\delta V}{d} \tag{5.21}$$

Acceleration can then be calculated from (ignoring damping):

$$F = ma = kx \tag{5.22}$$

$$a = \frac{k\delta}{m} = \frac{kdV_o}{mV} \tag{5.23}$$

Chapter 6

Measurement of Velocity, Torque and other Variables

23/11/2020

6.1 Measurement of Velocity

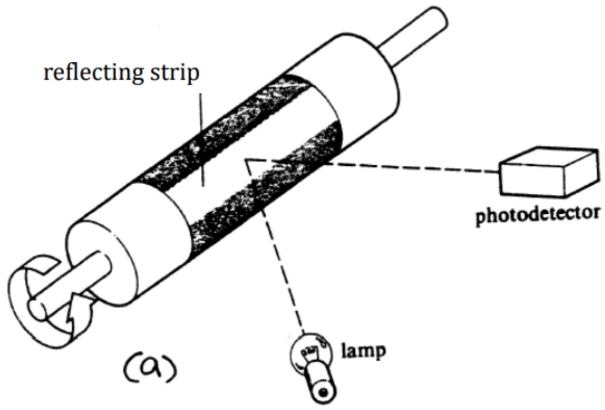
Linear- and angular-velocity transducers are available based on many different principles. Some of the more common types are introduced here. Linear velocity is often measured with an angular-velocity transducer by using a wheel to convert the linear motion into rotation. Angular-velocity transducers are given different names depending on their use:

- Revolution counters – measure rates of rotation of a body by counting the rotation
- Tachogenerators – generate an output (typically voltage) proportional to the angular velocity of an element of the system

Mechanical rotation is coupled to an angular-velocity transducer via 'sensing shaft' which rotates at the angular velocity to be measured.

6.1.1 Velocity transducers with a frequency output

Similar principle to the angular position transducers, such as optical incremental shaft encoders, can be used to measure rotational frequency.



Simple examples include a rotating shaft with a reflecting strip on non-reflecting background colour band (figure). Here, one electrical pulse per revolution is received by the detector.

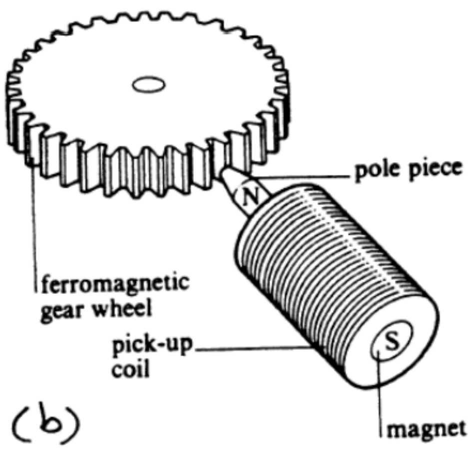
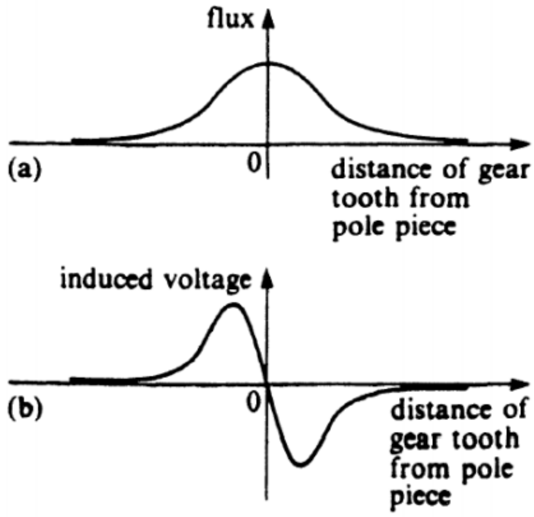


Figure 6.1: A variable reluctance tachometer. A pick-up coil surrounds a permanent magnet, one pole of which is placed close to a ferromagnetic gear wheel mounted on the sensing shaft.

The reluctance of the magnetic circuit varies depending on the distance between the pole and gear tooth, and hence the magnetic flux inside the coil changes which induces the electromotive force (voltage).



The induced EMF (electromotive force) and voltage is proportional to the rate of change of flux, hence the distance between the gear and pole is related to the induced voltage via:

$$\epsilon = -\frac{\mathrm{d}}{\mathrm{d}t}(\text{flux}) \tag{6.1}$$

6.1.2 Tachogenerator

The tachogenerator generates a voltage (usually DC) proportional to angular velocity, using electromagnetic induction. The transducer uses the energy of rotation of the sensing shaft to generate an electrical output and requires no external power supply.

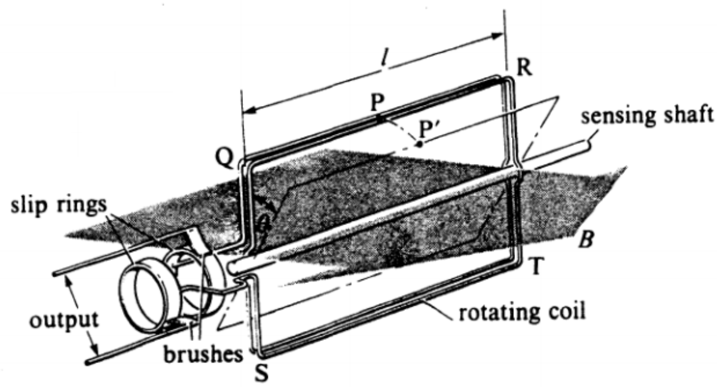


Figure 6.2: Basic AC tachogenerator

Any conductor moving through a magnetic field at an angle θ to the direction of the field has an EMF induced as:

$$\epsilon = Blv\sin(\theta) \tag{6.2}$$

Where:

- B is the magnetic flux density
- l is the length of the conductor
- v is the velocity of the conductor in the field

If the conductor is moving parallel to the field, $\theta = 0$ and no EMF is induced, whereas if it is moving normal to the field, $\theta = \pi/2$ and $\epsilon = Blv$.

In the tachogenerator, the induced voltage is picked up via two spring-loaded brushes, rubbing on two conducting rings called 'slip rings'. The slip rings are connected to the two ends of the coil and rotate with it. The voltages generated in sides QR and ST are in the correct direction to add. The other two sides (QS and RT) do not contribute to the voltage.

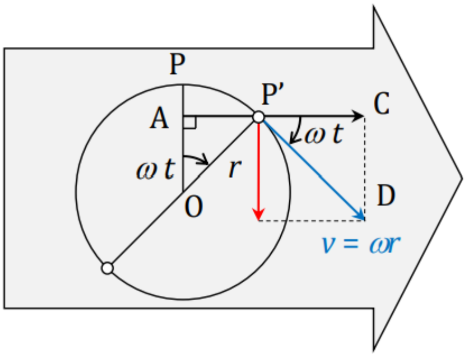


Figure 6.3: Red arrow: $v \sin(\omega t) \longrightarrow$ Component crossing the magnetic field

From the figure, the induced EMF in the side QR can be calculated as:

$$\epsilon = Blv\sin(\theta) = Bl\omega r\sin(\omega t) \tag{6.3}$$

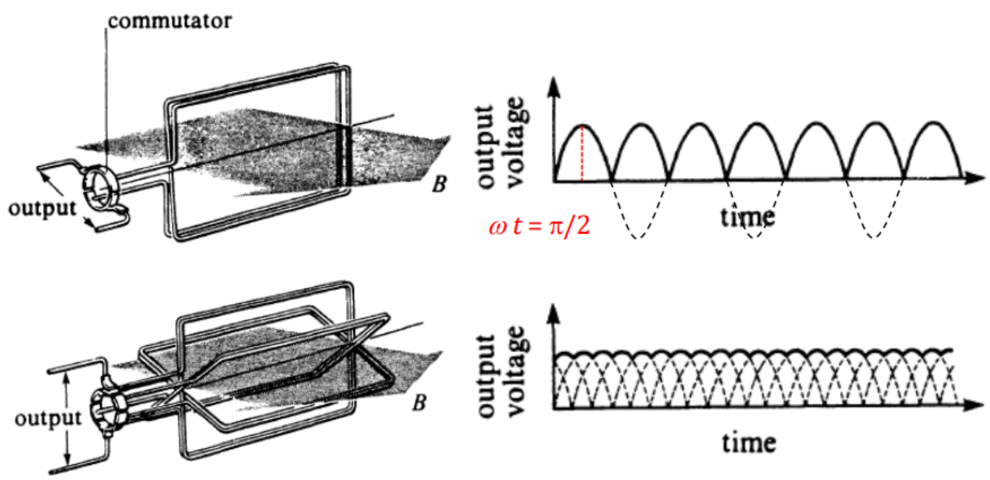
With the contribution from the other side (ST), for n turns of the coil:

$$\text{Total induced voltage} = 2Bl \cdot n \cdot \omega r \cdot \sin(\omega t) \tag{6.4}$$

From the equation, we can see that the output voltage is sinusoidal with amplitude proportional to the angular velocity ω (or frequency $f = \omega/2\pi$).

However, it is difficult to deal with an output where both the amplitude and frequency vary with angular velocity. It is more convenient to have DC output with its magnitude proportional to the angular velocity.

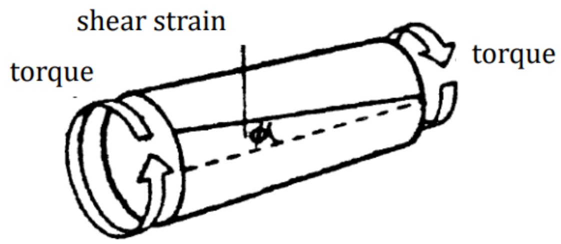
Modifications using **commutator** which switches over the two (n) output terminals from the coil every half ($1/n$) turn.



6.2 Measurement of Torque

6.2.1 Torque measurement with strain gauges

Torque, a twisting force, can be measured by detecting the angular displacement or surface strain of a shaft that is subjected to a torque.



For the use of strain gauge, it is necessary to know how much strain is present on the surface of the shaft for a given torque.

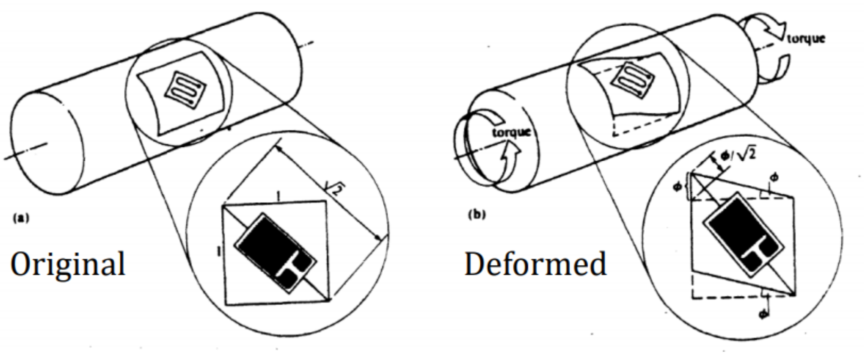
The relationship between torque T, shear strain (angle) ϕ, shear modulus of the shaft material G and radius r is given as:

$$T = \frac{1}{2}\pi G r^3 \phi \tag{6.5}$$

The shear sensitivity can be considered as the shear strain in the surface per unit applied torque:

$$\text{Shear sensitivity} = \frac{\phi}{T} = \frac{2}{\pi G r^3} \tag{6.6}$$

A strain gauge will measure linear strain in the direction of the gauge axis. To maximise this for a given shear strain in the shaft, the gauge is mounted with its active axis 45° to the axis of the shaft.



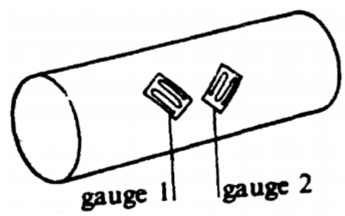
Considering a unit square area on the surface and when ϕ is small, the strain in its diagonal is given as:

$$e = \frac{\frac{\phi}{\sqrt{2}}}{\sqrt{2}} = \frac{\phi}{2} \tag{6.7}$$

The gauge sensitivity can then be:

$$\frac{e}{T} = \frac{\phi}{2T} = \frac{1}{\pi G r^3} \tag{6.8}$$

Temperature compensation can be achieved by mounting two strain gauges with their active axes perpendicular to each other.

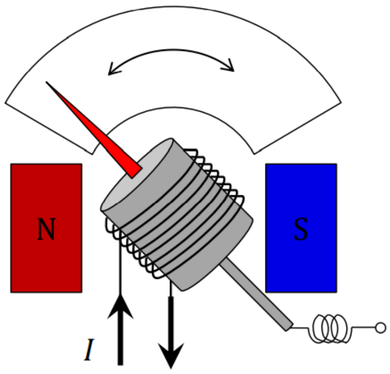


6.3 Measurement of Other Variables

6.3.1 Voltage – voltmeter

Analogue voltmeter and ammeter are making use of electromagnetic force. Digital ones could be made with comparators.

- Analogue Voltmeter - A coil is wrapped around an electromagnet, which is placed inbetween 2 poles of a magnet. The current passing through the coil generates an EMF, which moves the dial across, giving a reading of the voltage.
- Digital Voltmeter - A comparator is used. The input voltage is passed through a series of comparators and through yes-no logic gates, the correct voltage is measured and displayed on a digital reader.



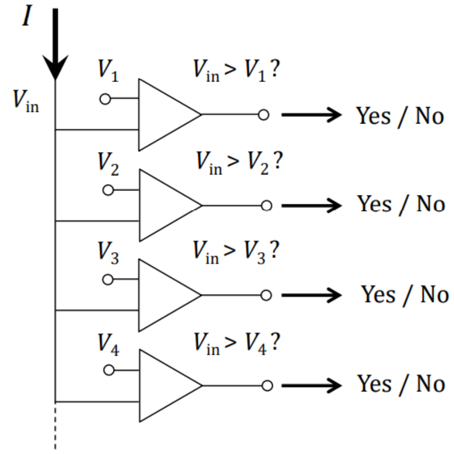


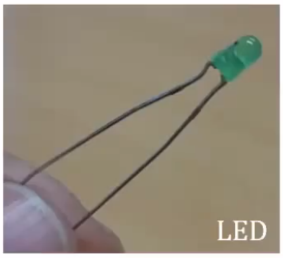
Figure 6.4: Left-Analogue Voltmeter, Right-Digital Voltmeter

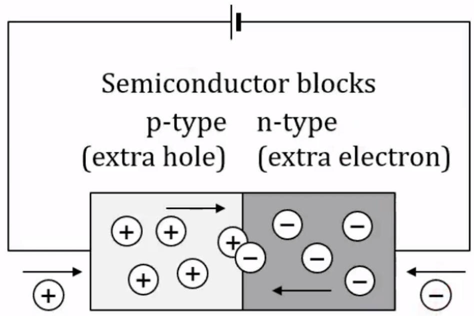
Advantages of digital voltage meters (DVMs) over analogue:

- Read out of DVMs is easy as it eliminates observational errors in measurement committed by operators.
- Error on account of parallax and approximation is entirely eliminated.
- Reading can be taken very fast.
- Output can be fed to memory devices for storage and future computations.
- Versatile and accurate
- Compact and cheap
- Low power requirements
- Portability increased

6.3.2 Light – photodiode

LEDs or light emitting diodes make use of semiconductor blocks. Semiconductor blocks usually have a p-type and an n-type block. The n-type usually has an excess of electrons, whereas the p-type has extra capacity to hold electrons. When a current is passed through a circuit with semiconductor LED, the electrons move from the n-type to the p-type block, hence completing the circuit.



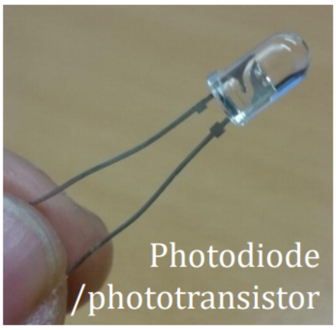


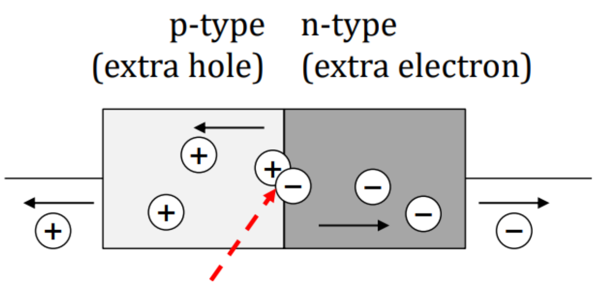
LED: Energy → Light

- Type of semiconductors determines the amount of energy
- The amount of energy determines wavelength of the light ($E = hv$)

What is the difference between and LED and a Photodiode/Phototransistor?

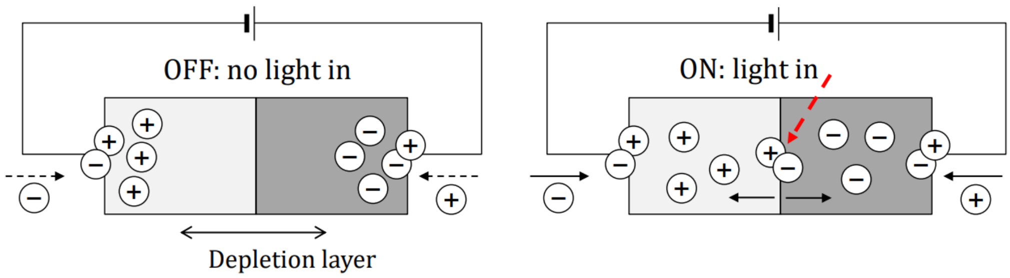
A photodiode or a phototransistor work on opposite principles to an LED. When light is shone on the diode, it gets converted into energy and completes the circuit. So in this case, the light energy is used to push the electrons from the n-type semiconductor block to the p-type semiconductor block.





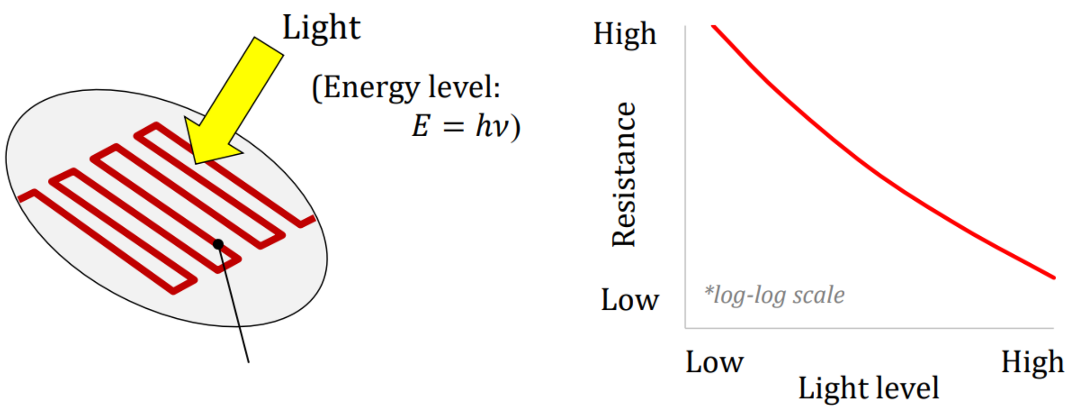
Photodiode: Light → Energy (photoelectric effect)

- Pattern 1: Working as a power source ⟶ photovoltaic mode – same principle in solar cells
- Pattern 2: Working as a switch ⟶ photoconductive mode



6.3.3 Light – LDR (light dependent resistor)

A light dependant resistor or an LDR changes its resistance depending on the intensity of light being shone on the semiconductor.



Semiconductor (CdS: cadmium sulfide) track:

- Light energy excites electrons from the valence band to the conductive band
  - Low resistance for high light level
- Required energy level to "free up" electrons depends on the track material
  - Semiconductor requires lower excitation energy than metal

6.3.4 Other Important Measurements

- Pressure Measurement
  - Pitot tube
  - Pressure probes (for healthcare applications)
- Velocity/Flow Measurement
  - Rotor
  - Ultrasound doppler

- Optical approach
  - Dye visualization
  - Particle image velocimetry

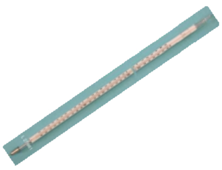
Chapter 7

Measurement of Temperature

30/11/2020

There are many ways to measure temperature, including changes of:

1. Pressure/volume (thermal expansion) – generic thermometer;
2. Contact potential between different metals – thermocouple;
3. Resistance of a metal or a semiconductor – thermistor;
4. Energy radiated by a hot object – radiation pyrometer.




Note: None of the properties of materials used for temperature sensing varies strictly linearly with temperature. Thus, calibration is very important for temperature transducers.

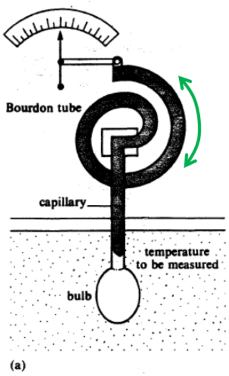
7.1 Standard Temperature for Calibration

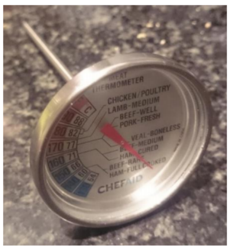
In the IPTS (the International Practical Temperature Scale), values of temperature are assigned to the eleven reproducible fixed points shown below.

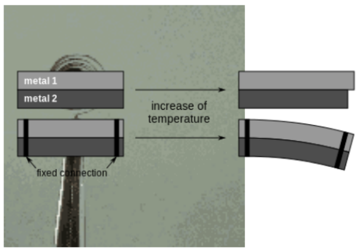
Fixed point	Assigned temperature		Standard instrument
			Optical pyrometer (above 1337.58 K)
Freezing point of gold	1337.58 K	1064.43 °C	Thermocouple
Freezing point of silver	1235.08 K	961.93 °C	(90 – 1337.58 K)
Freezing point of zinc	692.73 K	419.58 °C	
Boiling point of water	373.15 K	100 °C	
Triple point of water	273.16 K	0.01 °C	
Boiling point of oxygen	90.188 K	−182.962 °C	
Triple point of oxygen	54.361 K	−218.789 °C	Platinum resistance
Boiling point of neon	27.102 K	−246.048 °C	thermometer
Boiling point of equilibrium hydrogen	20.28 K	−252.87 °C	(13.81 – 903.89 K)
Equilibrium between the liquid and vapour phases of equilibrium hydrogen at 33330.6 Pa	17.042 K	−256.108 °C	
Triple point of equilibrium hydrogen	13.81 K	−259.34 °C	

7.2 Measurement Using Changes in Volume/Pressure

Thermal expansion is the key.

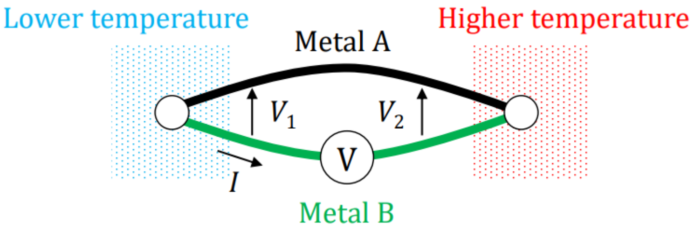






7.3 Thermoelectric Temperature Sensors (Thermocouples)

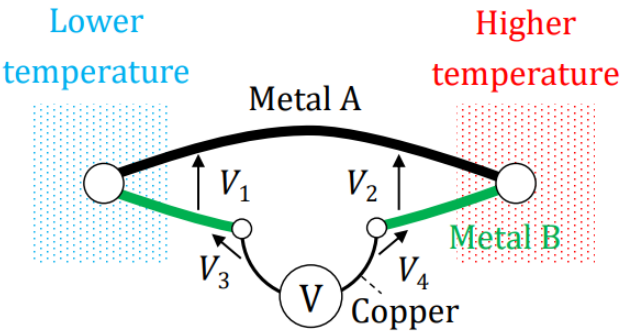
A thermocouple consists of two wires of different metals joined at both ends. If the junctions between the metals are at different temperatures, an electric current will flow around the circuit. This phenomenon is called **thermoelectric or Seebeck effect**.



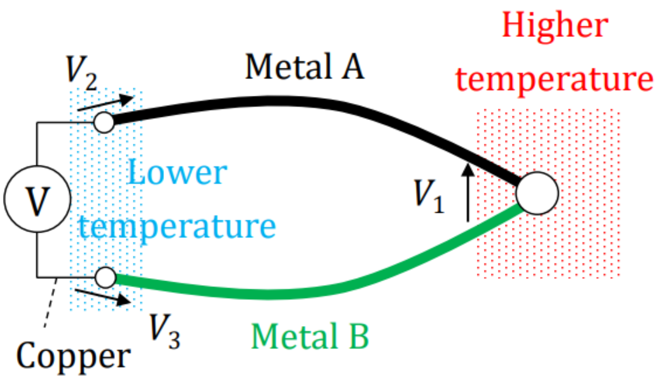
Across the junction of any two dissimilar metals, there always appear a difference in electric potential called the 'contact potential'. The contact potential varies with the temperature at the junction between the two metals, increasing in magnitude with an increase in temperature.

7.3.1 Thermocouples in practice

Contact potential could also occur at the junction between metal B and the wires leading to the voltmeter, unless the wire is made of metal B (very unlikely), generating contact potential V_3 and V_4.



In practice, it does not matter as long as the junctions are kept under the same temperature (i.e. $V_3 = V_4$).



An alternative configuration is the one in the bottom panel. Here, the net contact potential is $V_1 + V_3 - V_2$.

7.3.2 Thermocouple materials and construction

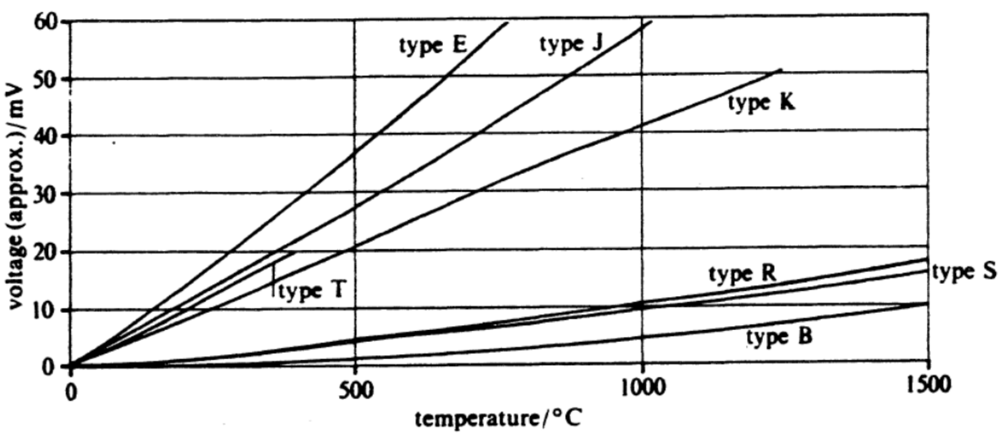
Various standards have been developed for thermocouples, with the relevant British Standard being BS4937, part of which is shown below.

Designated type	Metal or alloy For 1st wire	For 2nd wire	Useful temp range (°C)	Environment for use
Type S	Platinum	90% platinum, 10% rhodium	0 to 1400	A
Type R	Platinum	87% platinum, 13% rhodium	0 to 1400	A
Type J	Iron	Copper nickel	-200 to 850	B
Type K	Nickel chromium	Nickel aluminium	-200 to 1100	A
Type T	Copper	Copper nickel	-250 to 400	C
Type E	Nickel chromium	Copper nickel	-200 to 850	C
Type B	70% platinum, 30% rhodium	94% platinum, 6% rhodium	0 to 1500	A

A: In oxidising or inert atmospheres
B: In reducing or inert atmospheres
C: In most non-corrosive atmospheres

The choice of the right thermocouple for any application depends on:

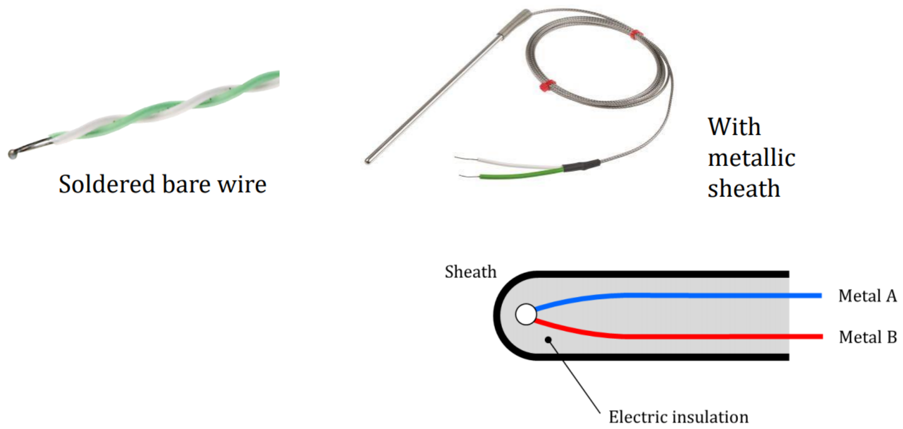
- The sensitivity as well as the temperature range (shown in the figure)
- Applicability in corrosive environments
- Whether it can withstand thermal and mechanical stresses
- Accuracy and reproducibility
- Price



7.3.3 Thermocouple structure

Thermocouples can have 2 different types of probe ends:

1. Soldered bare wire
2. With metallic sheath



The main differences between the two different types:

- With a metallic sheath, the probe can touch conductive surfaces
  - Sheath has electric insulation inside, so the reading won't be affected
- Longer response time
  - The metallic sheath and the electric insulation must be heated up first before reaching the joint inside
- Insulation material needs to be high electric resistance + high thermal conductivity

7.4 Basic Metallic Resistance Thermometers

The resistance of certain metals varies with temperature. The resistance R_T at temperature T is approximately shown relative to the resistance R_0 at 0°C, using the following equation:

$$R_T = R_0(1 + \alpha T) \tag{7.1}$$

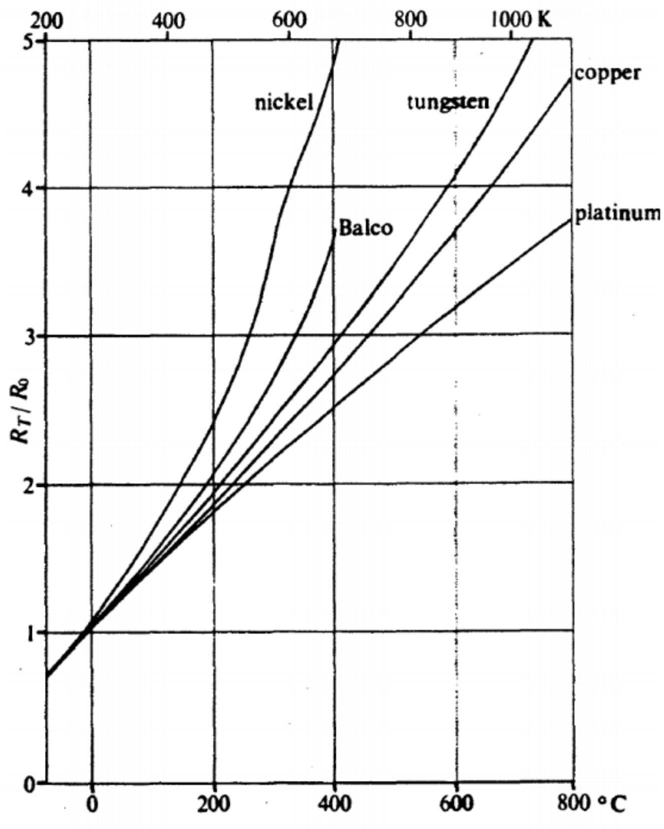
Where:

- α is called the temperature coefficient of resistance

This could be described more generally:

$$R_T = R_0(1 + \alpha(T - T_0)) \tag{7.2}$$

taking T_0 as the reference temperature fro R_0.



The figure shows the relative resistance variation of certain metals over temperature (using R_0 at 0°C). Although the sensitivity of platinum is the smallest, it has advantage of:

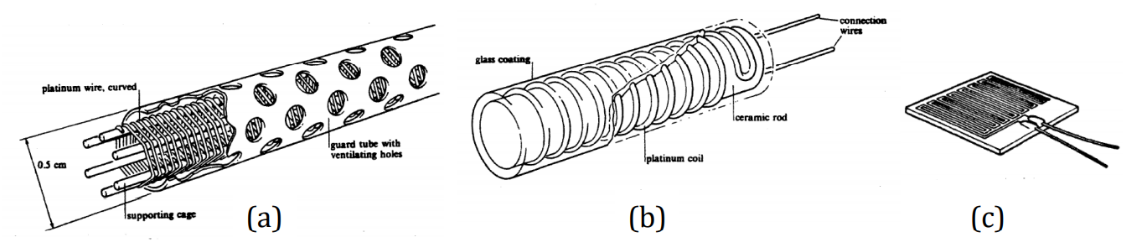
- Most stable and reproducible (linear),
- Chemically inert and resistant to contamination.

Typical resistance thermometer construction includes:

- A type with its platinum coil of wire directly exposed to the fluid whose temperature is to be measured.

- The resistance wire is isolated from the fluid, so that the resistance wire does not become contaminated with other materials which could change the relationship of its resistance to temperature. The ceramic rod is to help the sensor to withstand mechanical shock.

- The resistance metal is deposited as a thin film pattern on an insulating substrate. This type of sensor can be attached to a flat surface.



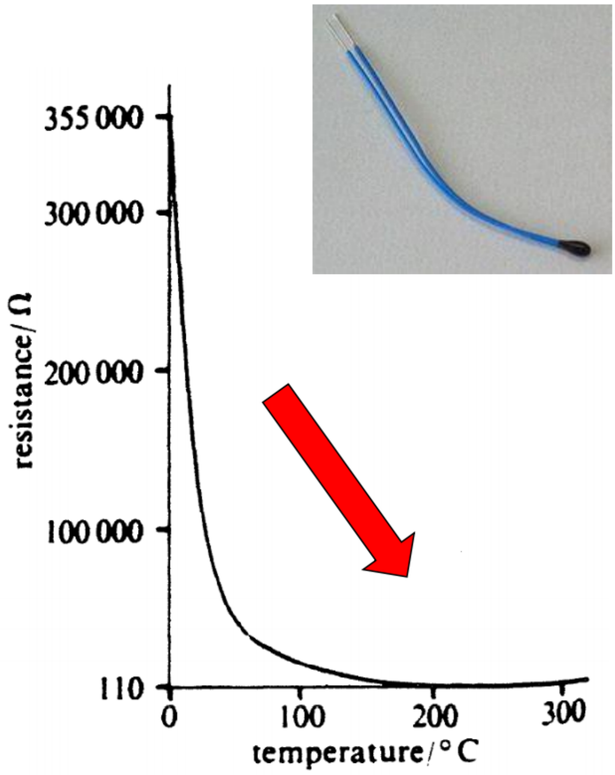
Errors can be caused by:

- Self-heating effect – electric current flows through the resistance wire heats the resistor up because of energy dissipation.

- Thermoelectric effect – likewise thermocouple, if the resistance wire and the lead wire are made of different metal, an additional electric current will flow due to thermoelectric effect.

7.5 Thermistors

THERM-ally sensitive res-ISTOR



The resistance against temperature curve for a thermistor (semiconductor) is very different from that for metallic resistance thermometers. It decreases with temperature rather than increase, and the negative temperature coefficient of resistance in this case is much larger than that of metals.

The relationship between R_T and temperature T for thermistor can be modelled as:

$$R_T = A \cdot e^{\frac{B}{T}} \tag{7.3}$$

Where:

- A and B are constants.
- B is called the characteristic temperature of the thermistor (material constant) whose value is normally between 2000 K and 4000 K.

If the resistance at a temperature T_0 is known (for example, base resistance at room temperature),

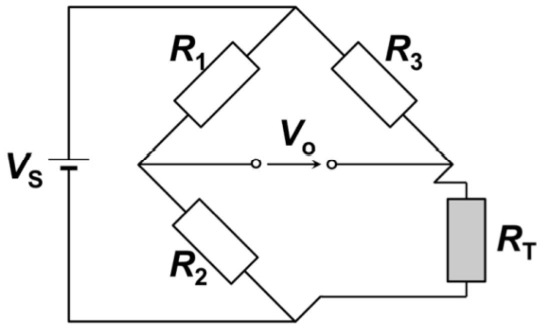
$$R_0 = A \cdot e^{\frac{B}{T_0}} \tag{7.4}$$

and combining the two equation:

$$R_T = R_0 \exp\left(B\left[\frac{1}{T} - \frac{1}{T_0}\right]\right) \tag{7.5}$$

Thermistors can be made **much smaller** than metallic resistance thermometers. This enables them to **respond to temperature variation more quickly**. However, this also means that the **self-heating effect is greater** for the same current than metallic sensors, and so they must be operated at smaller current levels.

7.5.1 Measuring circuits for resistive sensors



We use Wheatstone bridge for resistive sensors and the output voltage from the bridge is:

$$V_o = \left(\frac{R(1+x)}{R(1+x)+R} - \frac{1}{2} \right) V_S = \frac{x}{2(2+x)} V_S \tag{7.6}$$

Where x is increase in the resistance due to the measurement and all resistors are assumed to have the same resistance at reference temperature. **If x is small enough**, this can be approximated as:

$$V_o = \frac{x}{4} V_S \tag{7.7}$$

But this does not apply to resistive temperature sensors, that is thermistors (x is large) and the bridge output voltage becomes highly nonlinear against T.

A simple workaround is not to measure V_o but to vary R_2 (by use of potentiometer) such that the bridge is balanced. If $R_1 = R_3$, $R_2 = R_T$ allows calculation of T, based on the following relationships.

Resistance thermometer:

$$R_T = R_0(1 + \alpha(T - T_0)) \tag{7.8}$$

Thermocouple:

$$R_T = R_0 \exp \left(B \left[\frac{1}{T} - \frac{1}{T_0} \right] \right) \tag{7.9}$$

7.6 Pyrometer and Radiation

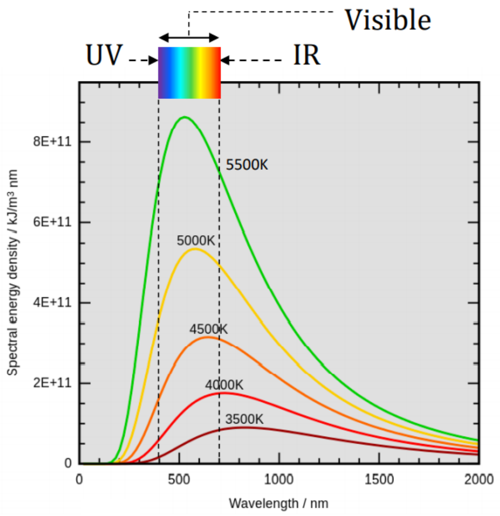
Radiation pyrometers detect the infrared radiation given off by the object whose temperature is being measured. Radiation pyrometers can be used to measure temperatures as low as −50°C, so infrared radiation is not only emitted by hot bodies. Any object whose temperature is not at absolute zero (0 K) emits radiation.

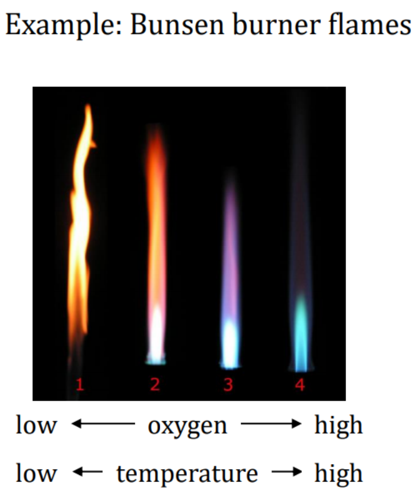
Radiation = travel of energy wave (like X-ray)

- Does not require medium to propagate
- ≠ sound (which requires medium to travel)

7.6.1 Black body

A body at a certain temperature emits a certain amount of energy with a certain spectrum (see figure below). Note: An ideal body, blackbody, emits radiation in all wavelengths. This is used to measure temperature in approximate manner.





7.6.2 Radiation pyrometers

Working principle:

Temperature transducer is used in the detector (e.g. thermopile), the detected temperature is converted to the radiation energy using Stefan-Boltzmann law.

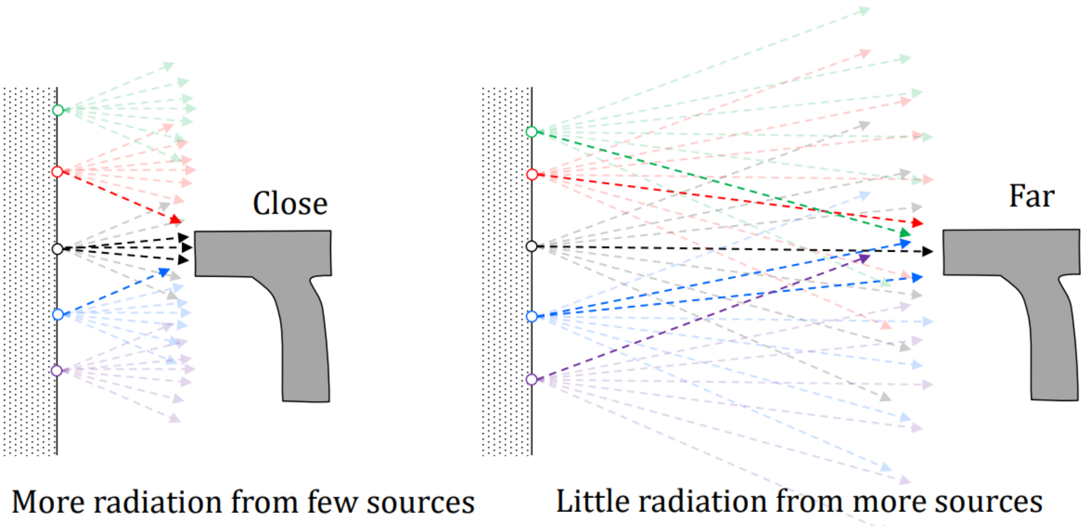
$$M = \sigma T_{detected}^4 \tag{7.10}$$

- Note: $T_{detected}$ is the temperature due to radiation and not the temperature of the surface

The energy is then converted to temperature of the measured surface using the black body energy plot.

Some features

- Very short response time
- Reflective or transparent/translucent surface is likely to lead to error
- Distance independent measurement



7.7 Temperature Response

A challenge of temperature measurement $\longrightarrow$ **no thermometer can respond instantly to a change in temperature**. The response is expressed mathematically as:

$$\frac{\mathrm{d}T}{\mathrm{d}t} \propto (T_f - T) \quad \text{or} \quad \frac{\mathrm{d}T}{\mathrm{d}t} + KT = KT_f \tag{7.11}$$

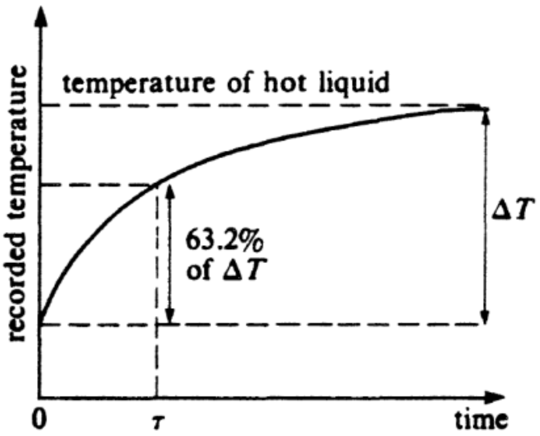
Where:

- K is a constant
- T_f is the temperature to be measured

The general solution is:

$$T = A \cdot e^{-Kt} + T_f = (T_0 - T_f) \cdot e^{-Kt} + T_f \tag{7.12}$$

The speed of change is characterised by the time constant $\tau(= 1/K)$ which is the time interval in which the temperature varies 63.2% of the difference between initial and final temperatures.



(a)

7.7.1 Transient temperature response

If the transducer is subjected to a sinusoidally varying input temperature, the differential equation becomes:

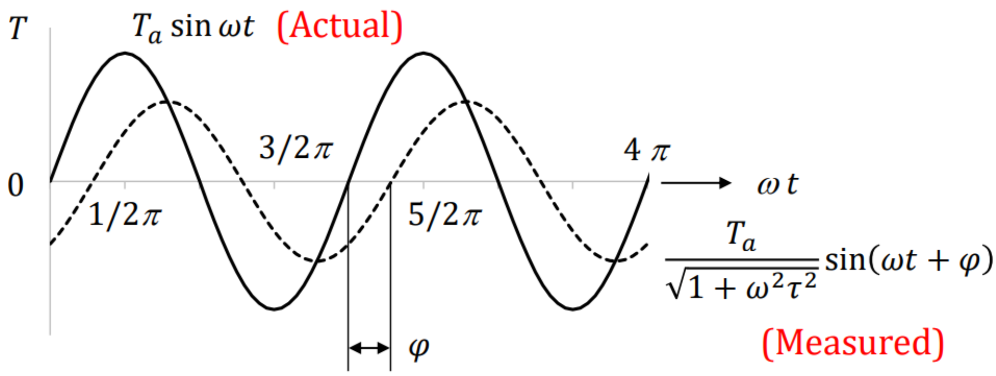
$$\frac{\mathrm{d}T}{\mathrm{d}t} + KT = KT_f = KT_a \sin(\omega t) \tag{7.13}$$

The solution of this equation in terms of the time-constant τ is:

$$y = A \cdot e^{\frac{-t}{\tau}} + \frac{T_a}{\sqrt{1 + \omega^2 \tau^2}} \sin(\omega t + \varphi) \tag{7.14}$$

Where:

- $\varphi = \tan^{-1}(\omega/k)$. The figure below shows the two temperature variations.


T
$T_a \sin \omega t$ (Actual)
0
$1/2\pi$
$3/2\pi$
$5/2\pi$
$4\ \pi$
ωt
$\frac{T_a}{\sqrt{1+\omega^2\tau^2}} \sin(\omega t + \varphi)$
(Measured)
φ