



UNIVERSITY COLLEGE LONDON

MENG MECHANICAL ENGINEERING

CEGE0016 FINANCIAL ASPECTS OF PROJECT ENGINEERING

## TOPIC NOTES

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October 30, 2022

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# Chapter 1

## Module Introduction

### 1.1 Teaching Team

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### 1.2 Assessment

Assessment Type	Description	Weighting
Quiz	Three multiple-choice quizzes <b>over the course of the module</b> , to be turned in within <b>24 hours</b>	15% (3 x 5%)
Course Project	Written report and recorded presentation on a project to be set <b>halfway through the module</b> , to be completed in <b>6 weeks</b>	40%
Individual Exercise	Written set of questions <b>completed in examination conditions</b>	45%

Figure 1.1: Module Assessment.

### 1.3 Timeline

Lecture Number	Date	Lecture Topic	Assessment Set	Assessment Due
1	3 <sup>rd</sup> October 2022	Introduction to Module; Public Economics		
2	10 <sup>th</sup> October 2022	Public Goods and Externalities		
3	17 <sup>th</sup> October 2022	Techniques for Project Evaluation	Quiz 1 set Wednesday 19 <sup>th</sup> October 12pm	Quiz 1 due Thursday 20 <sup>th</sup> October 12pm
4	24 <sup>th</sup> October 2022	Cost-Benefit Analysis	Quiz 2 set Wednesday 26 <sup>th</sup> October 12pm	Quiz 2 due Thursday 27 <sup>th</sup> October 12pm
5	31 <sup>st</sup> October 2022	Guest lectures on HS2 and the Lower Thames Crossing; Introduction to Course Project	Course project set Monday 31 <sup>st</sup> October in class	Course project presentation due Monday 12 <sup>th</sup> December in class; Course project report due Friday 16 <sup>th</sup> December at 5pm
-----Reading Week-----				
6	14 <sup>th</sup> November 2022	Companies and Financial Accounting	Quiz 3 set Wednesday 16 <sup>th</sup> November 12pm	Quiz 3 due Thursday 17 <sup>th</sup> November 12pm
7	21 <sup>st</sup> November 2022	Project Planning Financial Project Planning		
8	28 <sup>th</sup> November 2022	Risk Analysis		
9	5 <sup>th</sup> December 2022	Project Management		
10	12 <sup>th</sup> December 2022	Course Project Presentations	Course project presentation delivered in class	

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10	12 <sup>th</sup> December 2022	Course Project Presentations	Course project presentation delivered in class	

Figure 1.2: Module Timeline.



# Chapter 2

## Introduction to public economics

### 2.1 Introduction and overview of mixed economies

#### 2.1.1 Aims

1. Understand the roles of the private and public sector in mixed economies
2. Become (re)familiar with key microeconomic concepts of consumption and production, including Pareto optimality and market equilibrium
3. Understand the trade-offs between market efficiency and equitable distribution of resources
4. Be aware of the assumptions and limitations of fundamental theorems and associated neoclassical economics

#### 2.1.2 Economies

##### A simplified definition

An area of *production, trade* and *consumption* of goods and services by different *agents*.

##### Agents

- Individuals and households
- Businesses
- Government

#### 2.1.3 Mixed economies

**Economies today are predominantly *mixed economies***

Private sector:

profit-maximising firms operate in competitive markets

Public sector:

governments/other organisations make interventions in those markets

#### 2.1.4 Private sector

##### Welfare economics

"he is in this, as in many other cases, led by an invisible hand to promote an end which has no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing

his own interest he frequently promotes that of the society more effectively than when he really intends to promote it. (Smith 1776)

### 2.1.5 Public sector

#### Public sector aims to balance trade-offs

In particular

efficiency of competitive markets vs. improved equity of distribution of income from regulation

Understanding the role of public sector in mixed economies first requires us to understand operation of free-markets

### 2.1.6 Classical microeconomics

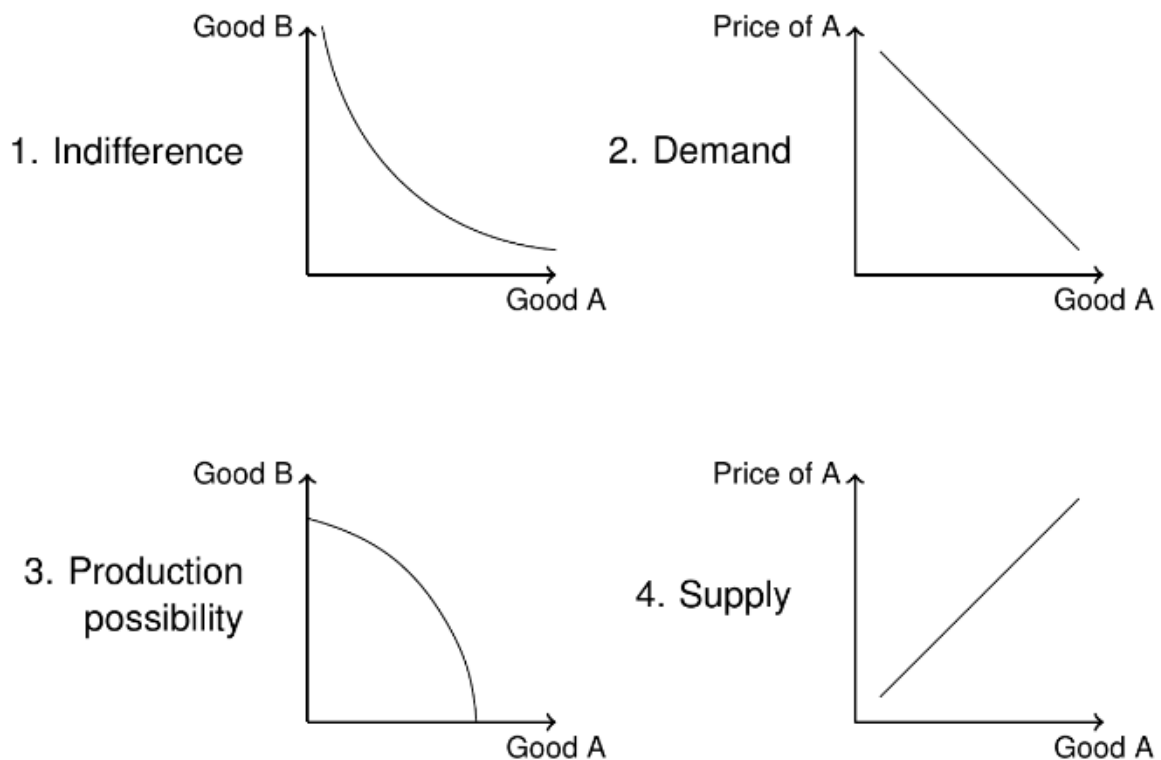


Figure 2.1: Classical microeconomic models.

### 2.1.7 Economic models

"Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful" (Box and Draper 1987)

## 2.2 Consumer theory

### 2.2.1 Consumer theory: continuous goods

- $J$  continuous goods, each good denoted by  $j = 1, \dots, J$
- Each good has associated price per unit  $p_j$ 
  - e.g.,  $j = 1$  corresponds to milk,  $p_j = 90$  pence/litre
  - $j = 2$  corresponds to eggs,  $p_j = 16$  pence/egg

- Consider choice of agent, with total budget  $I$ 
  - Assume agent represents individual
  - Individual chooses quantity  $q_j$  of each good  $j$ , subject to budget constraint

$$\sum_{j=1}^J (p_j q_j) \leq I \quad (2.1)$$

### 2.2.2 Utility

- Individual's choice of goods represented by consumption bundle  $Q$ 
  - i.e. vector of  $q_1, \dots, q_J$
- Individual gets utility  $U(Q)$  from  $Q$ 
  - Utility represents how individuals perceived benefit from consuming/owning  $Q$
  - Assume  $U(Q)$  increases monotonically with increasing  $q_i$

### 2.2.3 Choice between two continuous goods

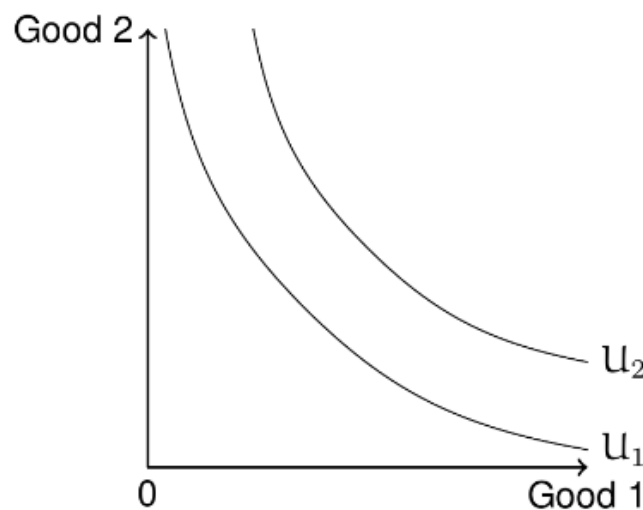


Figure 2.2:

Indifference curves:

Different combinations of each good that yield same level of utility

Marginal Rate of Substitution (MRS):

Gradient of indifference curve

- i.e. how many unites of good 2 individual would substitute for 1 unit of good 1
- Assumed to be convex

### 2.2.4 Utility maximisation with budget constraint

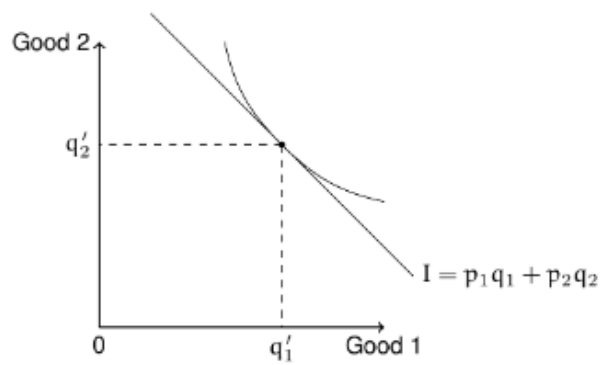


Figure 2.3:

- Assume agents try to maximise utility
- Under maximal utility assumption, optimal solution when indifference curve is tangent to budget line

$$MRS = \frac{p_1}{p_2} \quad (2.2)$$

### 2.2.5 Trade between two agents

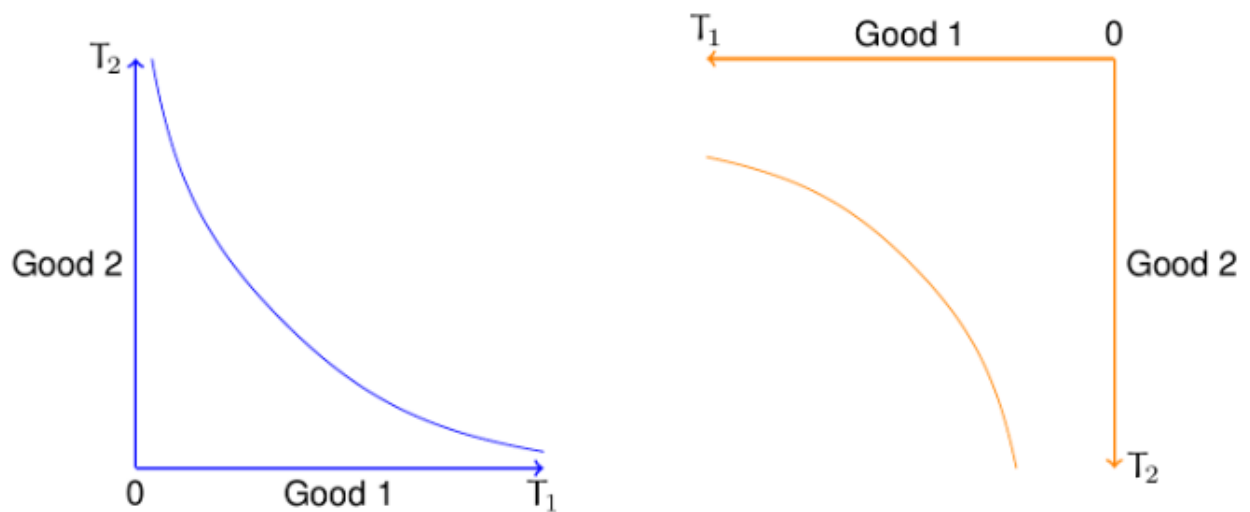


Figure 2.4:

## 2.2.6 Edgeworth box

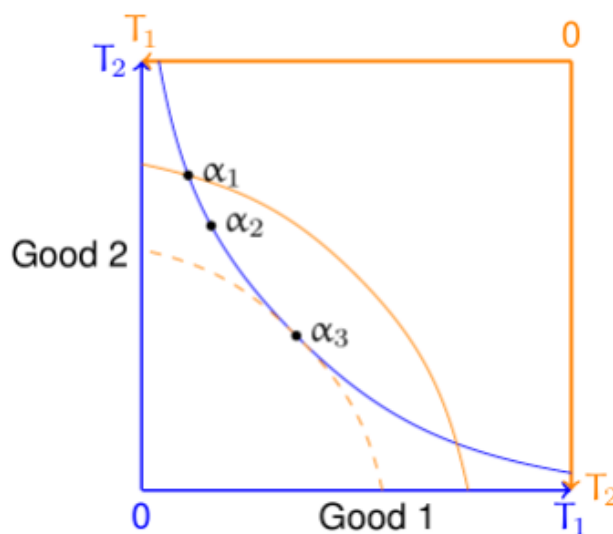


Figure 2.5:

Edgeworth box:

depicts distribution of commodities in closed economy between two agents

Pareto improvement:

A reallocation that improves utility of of one individual without reducing anyone else's utility

- $\alpha_2$  is Pareto improvement of  $\alpha_1$
- $\alpha_3$  is Pareto improvement of  $\alpha_2$

Pareto optimal/efficient:

An allocation from which no-one can improve utility without reducing someone else's

- $\alpha_3$  is Pareto optimal

## 2.2.7 Pareto efficiency

- Pareto efficient solutions happen when indifference curves have equal gradient
- i.e. each agent has equal MRS

Pareto frontier:

Set of all possible Pareto efficient allocations

## 2.3 Producer theory

### 2.3.1 Production of goods and services

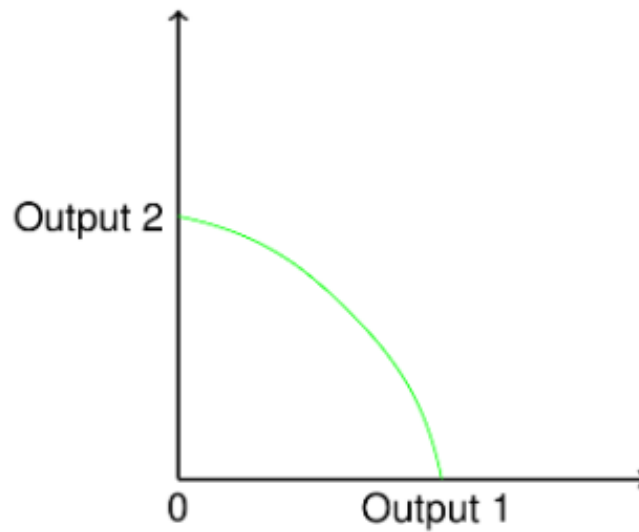


Figure 2.6:

Production Possibilities Frontier (PPF):

possible combinations of outputs (e.g. goods/services) that can be produced by economy with fixed inputs technology

- all points on PPF are production efficient: no more of one output can be produced without sacrificing the other

Marginal Rate of Transformation (MRT):

Gradient of PPF

- Measures amount of Output 2 that must be sacrificed to produced additional unit of Output 1

### 2.3.2 Marginal cost

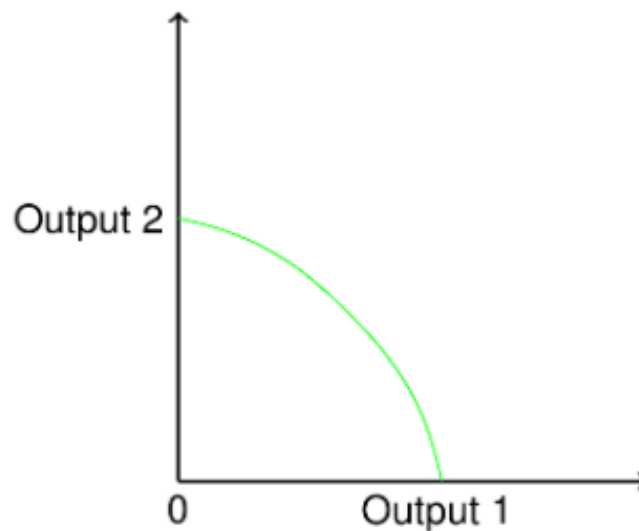


Figure 2.7:

Marginal cost:

Cost of producing one additional unit of output

$$MRT = \frac{MC_{output_1}}{MC_{output_2}} \quad (2.3)$$

- PPF often assumed to be concave under certain conditions (i.e. rewards diversity)
  - Easier to obtain low-hanging fruit

### 2.3.3 Pareto efficient production

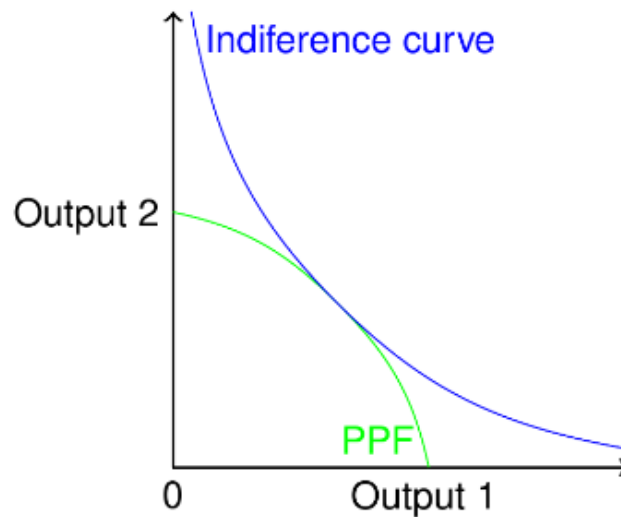


Figure 2.8:

Pareto efficiency only achieved when production of goods matches consumers' willingness to pay

- Gradient of PPF matches combined indifference curve of all consumers
- i.e.  $MRS = MRT$

### 2.3.4 Single market efficiency

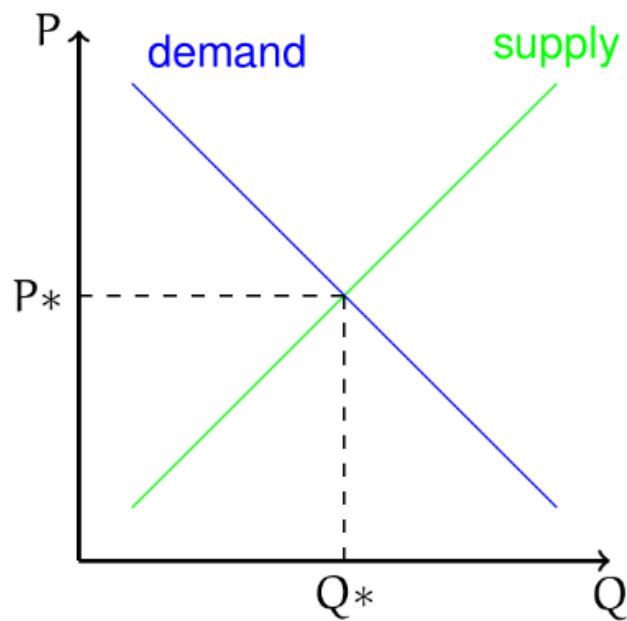


Figure 2.9:

Market equilibrium occurs when supply equals demand

- marginal benefit of consumption is equal to marginal cost of production

## 2.4 Fundamental theorems of welfare economics

### 2.4.1 Competitive economies

#### Fundamental theorems of welfare economics

If the economy is competitive, it is Pareto efficient

### 2.4.2 Efficiency vs equality

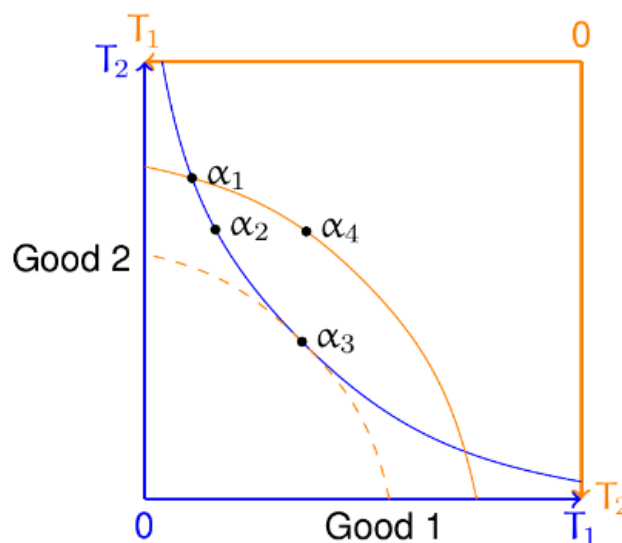


Figure 2.10:



- So far, considered only efficiency of allocations
  - $\alpha_3$  is Pareto efficient
- Social welfare also depends on equitable distribution of goods
- How do we choose between  $\alpha_3$  and  $\alpha_4$ 
  - Do we need to?

### 2.4.3 Wealth distribution

#### Fundamental theorems of welfare economics

- If the economy is competitive, it is Pareto efficient
- Every Pareto efficient resource allocation can be obtained with competitive market process with an appropriate initial redistribution of wealth

### 2.4.4 Efficiency and equality?

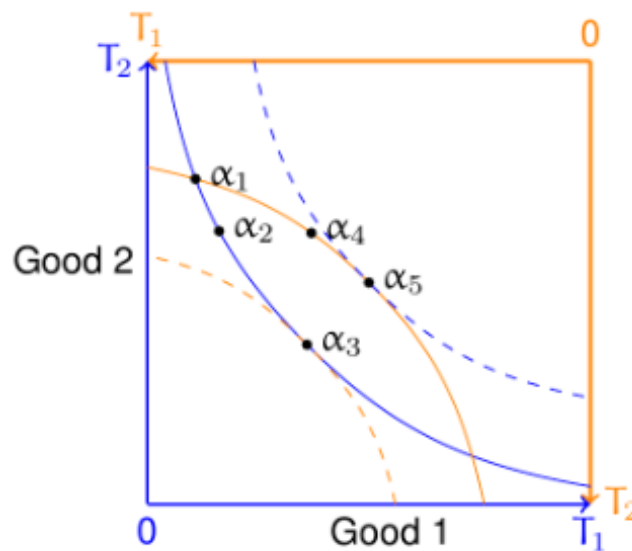


Figure 2.11:

According to second fundamental theorem:

more equitable allocation can be found through suitable assignment of initial endowments and free trade

## 2.5 Public sector

### 2.5.1 Role of government: the theory

#### When and how should governments make interventions in mixed economies?

According to first fundamental theorem:

government interventions that reduce competition make economies less efficient

Redistribute income and leave markets alone?

## **2.5.2 Role of government: reality**

Note... Governments play an active role in all major economies, including:

- Allocation
- Distribution
- Regulation
- Stabilisation

## **2.5.3 Market failures**

Several situations result in the failure of free markets to achieve optimal solutions. Causes include:

- existence and need for public goods
- existence of externalities
- imperfect competition
- incomplete information and uncertainty

## **2.6 Review and recap**

### **2.6.1 A need for better understanding?**

- Several strong assumptions
  - Individuals as rational utility maximisers
  - Equivalence of utility, value and price
  - Markets as continuous
  - Static tastes and preferences
  - Perfect competition
- Fundamental welfare economic theory does not capture
  - unpaid labour
  - social exchange
  - long-term resilience and sustainability

## Chapter 3

# Public Goods and Externalities

### 3.1 Introduction

#### 3.1.1 Aims

1. Recall the two dimensions of public good (rivalry and excludability) and understand how they lead to market failure
2. Identify and describe the occurrence and results of positive and negative externalities
3. Understand the role of the public sector in managing market failures arising from public goods and externalities
4. Be aware of the particular challenges related to climate externalities
5. Become familiar with the dimensions of the environmental ceiling and social foundations of the doughnut economic model

#### 3.1.2 Market equilibrium

Market equilibrium occurs when supply equals demand. Private marginal benefit of consumption is equal to private marginal cost of production.

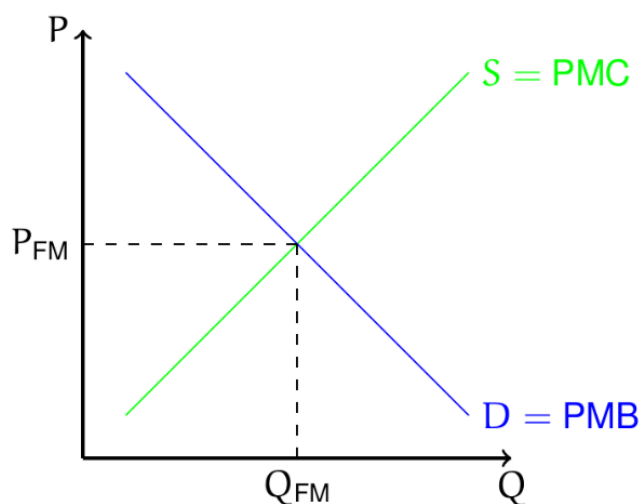


Figure 3.1: Market equilibrium.

### 3.1.3 Public goods and externalities

#### Definitions

Public goods:

Goods which are both non-excludable and have non rivalrous consumption.

Externalities:

Positive or negative effects on third parties arising from the production or consumption of goods, that are not reflected in the price

#### Market failures

Public goods and externalities cause market failures in the allocation of goods/services at the free-market equilibrium.

- i.e.  $Q_{\text{free-market}}$  is not optimal
- addressed through the allocative role of government

## 3.2 Public goods

### 3.2.1 Two dimensions of public good

**Excludability:** the degree to which access to a good, service or resource can be restricted.

- **Excludable:** agents can easily be prevented from using the good/service
- **Non-excludable:** preventing agents from consuming the good/service is impossible (or very expensive)

**Rivalry:** the degree to which consumption by one party affects another party's use of the good.

- **Rivalrous:** consumption by one agent prevents simultaneous consumption by other agents, or reduces the marginal benefit of other agents
- **Non-rivalrous:** once it is provided, the additional resource cost of another person consuming the good is zero (i.e.  $MC = 0$ ) and the marginal benefit does not decrease with number of users
- **(Anti-rivalrous:** marginal benefit increases with the number of users, e.g. social network)

### 3.2.2 Rivalry and capacity

Goods are often non-rivalrous up to a certain capacity, above which they are rivalrous e.g. public transport (bus/train), road bridge, internet bandwidth.

### 3.2.3 Continuous scale

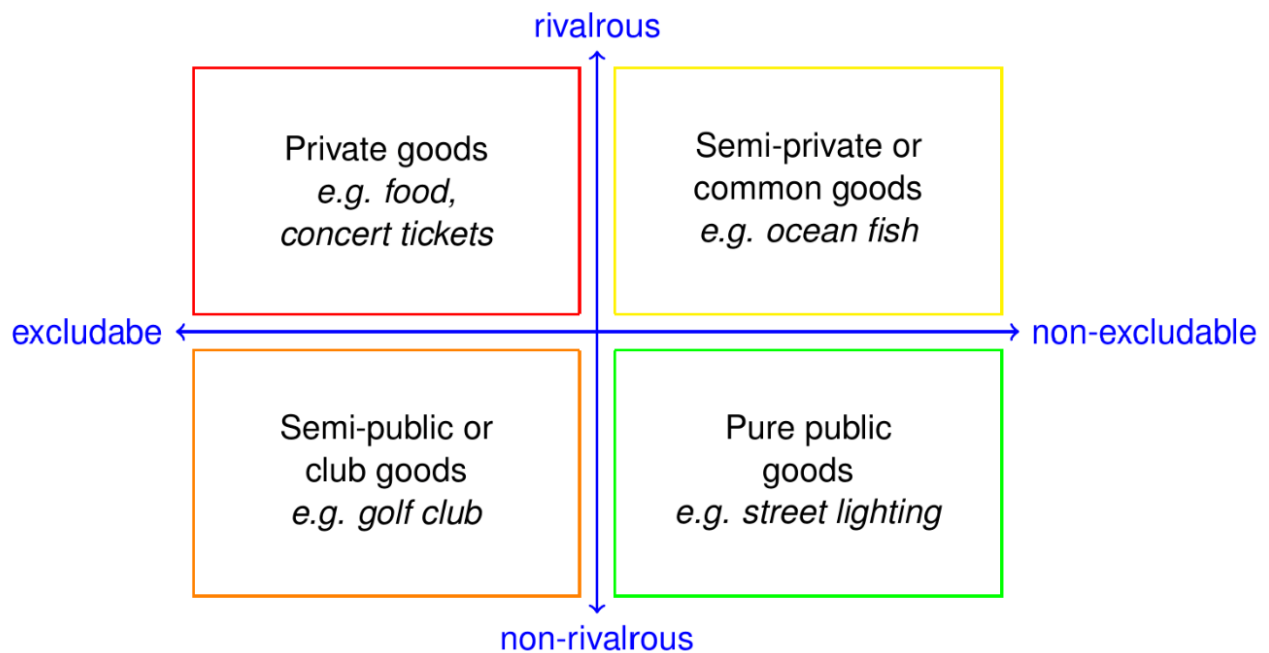


Figure 3.2: Continuous scale.

### 3.2.4 Public goods in free markets

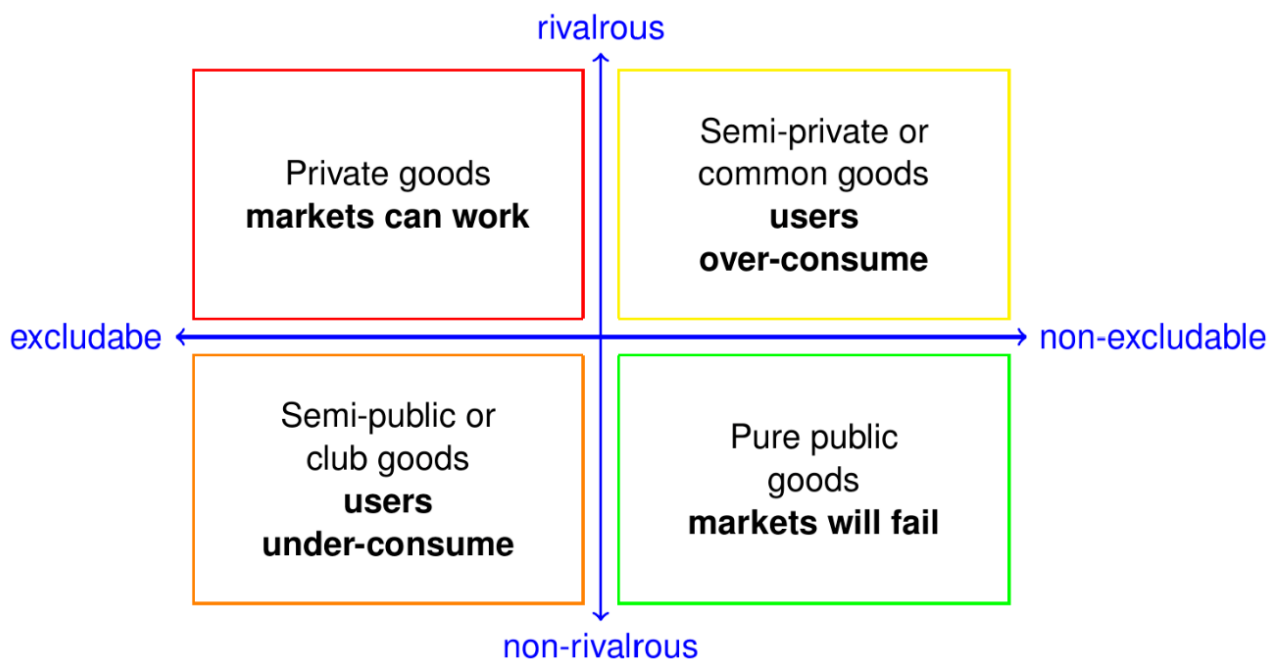


Figure 3.3: Public goods in free markets.

### 3.2.5 Public goods and market failure

Pure public goods are **non-excludable**

- Producers cannot exclude agents from consumption

- Unable to charge and therefore make profit
- Therefore (in theory) would not be produced through market action!

Possibility of funding via private cooperative, but...

### Free rider problem

as size of cooperative increases, possibility of avoiding contributing increases

### Public sector provision

Large group public goods supplied from public sector budget

- Allocative role of government

### 3.2.6 Privatisation in the public sector

Note... Public sector provision  $\neq$  equivalent public sector production.

The creation of markets in public services has been one of the great defining shifts in the way government has been run over the past 30 years (Gash and Roos 2012)

## 3.3 Externalities

### 3.3.1 Positive and negative externalities

Externalities

when the actions of one economic agent directly affect other agent(s) outside the market mechanism (production/consumption)

Externalities can arise from either production or consumption and have a net positive or negative effect.

### 3.3.2 Negative production externality

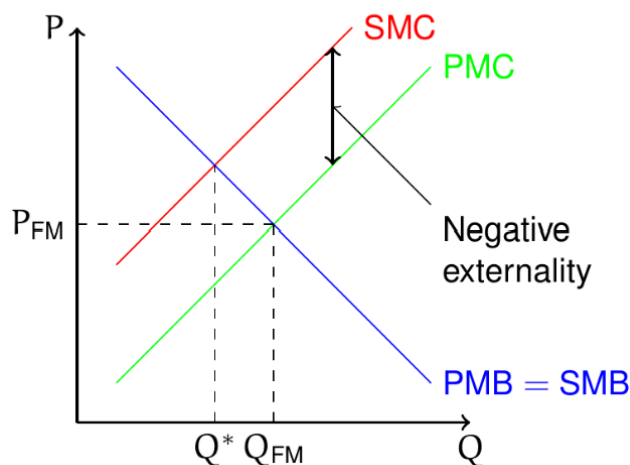


Figure 3.4: Negative production externality.

Production of output reduces well-being of third parties not involved in transaction,

- e.g. oil spills during fuel production pollute oceans and damage wildlife
- leads to overproduction

### 3.3.3 Negative consumption externality

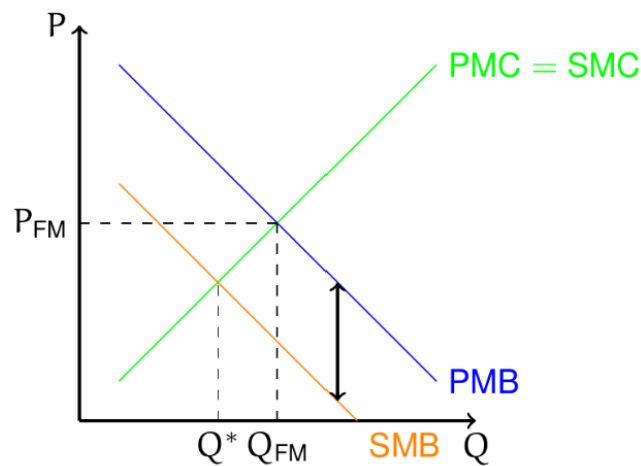


Figure 3.5: Negative consumption externality.

Consumption of output reduces well-being of third parties not involved in transaction,

- e.g. driving cars produces carbon emissions
- leads to overconsumption

### 3.3.4 Positive production externality

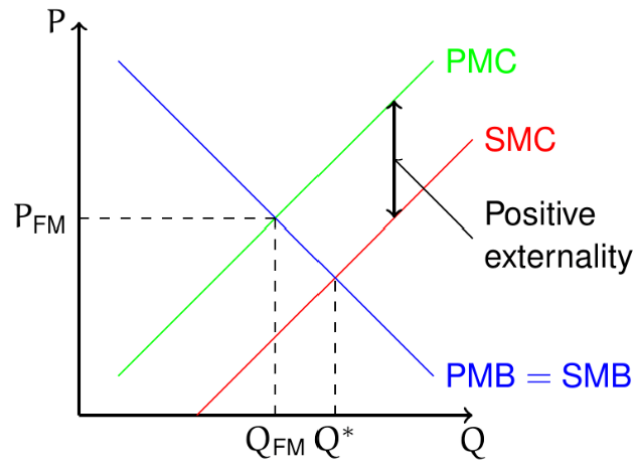


Figure 3.6: Positive production externality.

Production of output reduces well-being of third parties not involved in transaction,

- e.g. creating a new tourist attraction brings increases custom to local shops
- leads to underproduction

### 3.3.5 Positive consumption externality

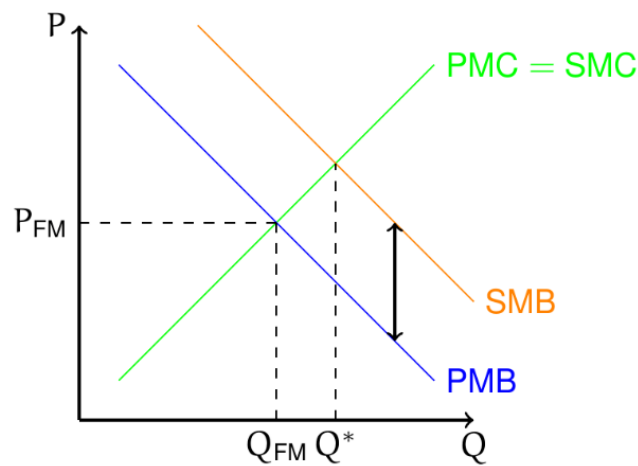


Figure 3.7: Positive consumption externality.

Consumption of output reduces well-being of third parties not involved in transaction,

- e.g. cycling improves people's general health, reducing pressure on public healthcare
- leads to under-consumption

### 3.3.6 Externalities and property rights

Externalities can be transferred where third party benefit/cost is clear i.e. where property rights are well defined.

### 3.3.7 Managing externalities

Where property rights are not clear, managing externalities relies on allocative role of government

#### Public sector interventions

Negative externalities:

- Corrective taxes
- Quantity restrictions
- Standards

Positive externalities

- Subsidies
- Tax benefits
- Direct production

### 3.3.8 Externalities and the environment

Note... Externalities related to climate change are critical to long term sustainability of the planet.

#### COP26

“Climate change is the single biggest health threat facing humanity. While no one is safe from the health impacts of climate change, they are disproportionately felt by the most vulnerable and disadvantaged.” (World Health Organisation 2021)



### 3.3.9 The doughnut economic model



Figure 3.8: Doughnut economic model.

## Chapter 4

# Techniques for Project Evaluation

### 4.1 Introduction to Capital and Interest

#### 4.1.1 Capital

- **Capital** is wealth in the form of money or property that can be used to produce more wealth
- A pound is worth more than a pound one or two years from now because of the **interest** it can earn
- Therefore money has a **time value**
- Often the riskiest thing a person can do with money is nothing

#### 4.1.2 Interest

- Interest pays the providers of capital for:
  - Forgoing its use during the time the capital is being used
  - The risk the investor takes in permitting another person or organisation to use their capital
- Investors must decide whether the return on their capital is sufficient to buy into a proposed project or venture
- The interest available from an alternative investment is the opportunity cost of using capital in the proposed undertaking

### 4.2 Simple interest

Interest earned or charged that is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods for which the principal is committed. Simple interest is not used frequently in modern commercial practice.

$$I = PNi \quad (4.1)$$

where:

- $I$  is total simple interest
- $P$  is principal amount lent or borrowed
- $N$  is number of interest periods
- $i$  is interest rate per interest period

The total amount repaid at the end of  $N$  interest periods is  $P + I$ . If £1000 were loaned for three years at a simple interest rate of 10% per year, the interest earned would be £300. The total amount owed at the end of three years would be £1300.

### 4.3 Compound interest

Interest earned or charged that is based on the remaining principal amount plus any accumulated interest charges up to the beginning of that period. Compound interest considers the time value of money, and is much more common than simple interest.

$$I = P(1 + i)^N - P \quad (4.2)$$

The total amount repaid at the end of  $N$  interest periods is  $P + I$ . If £1000 were loaned for three years at a compound interest rate of 10% per year, the interest earned would be £331. The total amount owed at the end of three years would be £1331.

#### 4.3.1 Compound vs simple interest

Assume that £1000 were loaned for three years at an interest rate of 10% per year.

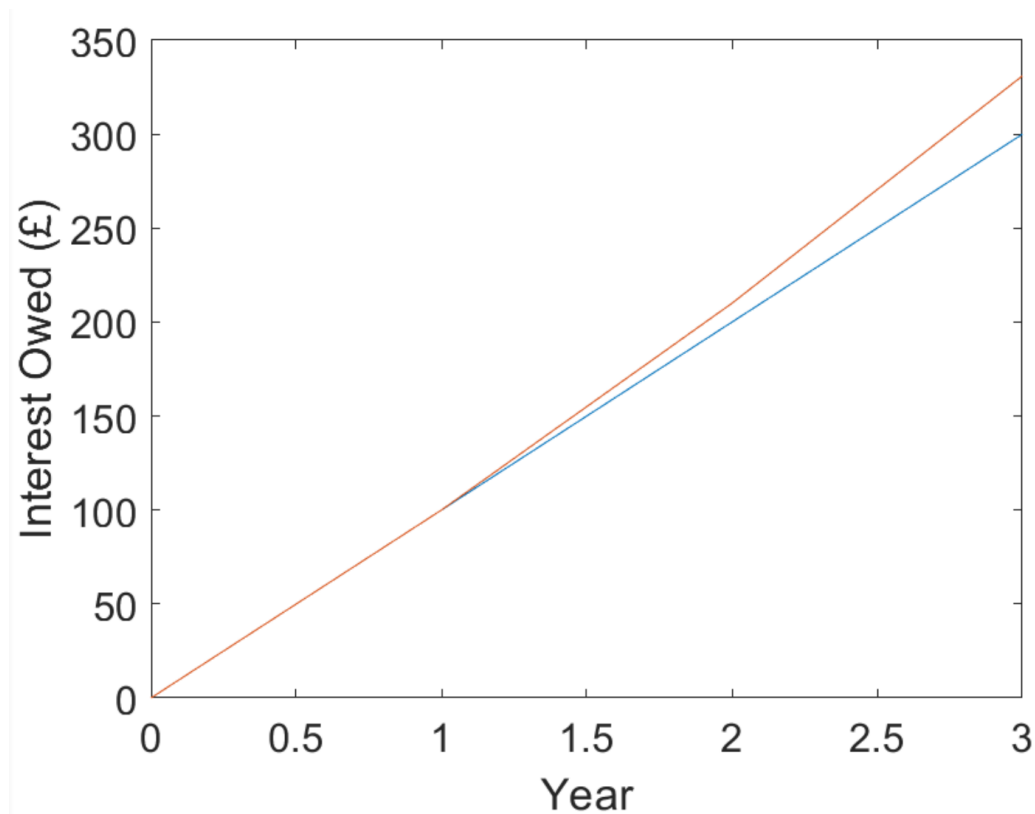


Figure 4.1: Blue: simple interest, Orange: compound interest

### 4.4 What is project evaluation

- Project evaluation considers the return that a given project will or should produce
- Project evaluation involves quantifying project profitability using various methods
- We address whether a proposed capital investment and its associated expenditures can be recovered by revenue (or savings) over a period of time, in addition to a return on the capital that is sufficiently attractive

## 4.5 Minimum attractive rate of return (MARR)

MARR is the **minimum rate of return** on a project that the top management of an organisation is willing to accept before starting a project. MARR depends on numerous factors:

- Amount of money available for investment (as well as the source and costs of funds)
- The number of projects available for investment and their purpose (i.e. whether they are essential or optional)
- The amount of perceived risk and the estimated cost of administering projects over different planning horizons
- The type of organisation involved (government, public utility, private industry)

## 4.6 Project evaluation using Net Present Value

### 4.6.1 Net present value (NPV)

The NPV method examines the equivalent worth of all cash flows relative to some base point in time i.e. the present. The future value ( $FV$ ) of a sum of money has a value today called the present value ( $PV$ ), which depends on the interest rate  $i$  that can be obtained (generally the MARR) - note that we are talking about a single sum of money in this case. The  $PV$  of a cashflow in  $n$  years' time as a function of  $i$  is:

$$PV = \frac{FV}{(1+i)^n} \quad (4.3)$$

Note that  $i$  is expressed as a decimal here. A series of uniform (annual) receipts ( $AV$ ) have a value today called the present value ( $PV$ ) which depends on the interest rate  $i$  that can be obtained (generally the MARR) - note that we are talking about multiple sums of money in this case. The  $PV$  of a series of cashflows that occur at the end of periods (years) 1 to  $n$  is:

$$PV = AV \frac{(1+i)^n - 1}{i(1+i)^n} = \sum_{k=1}^n \frac{AV}{(1+i)^k} \quad (4.4)$$

$NPV$  then accounts for all cash inflows and outflows:

$$NPV = PV_{\text{cash inflows}} - PV_{\text{cash outflows}} \quad (4.5)$$

To use the NPV method to determine project worthiness, we compute  $NPV$  using the MARR as the interest rate. The higher the interest rate ( $i$ ) and the farther into the future a cash flow occurs, the lower its  $PV$ .

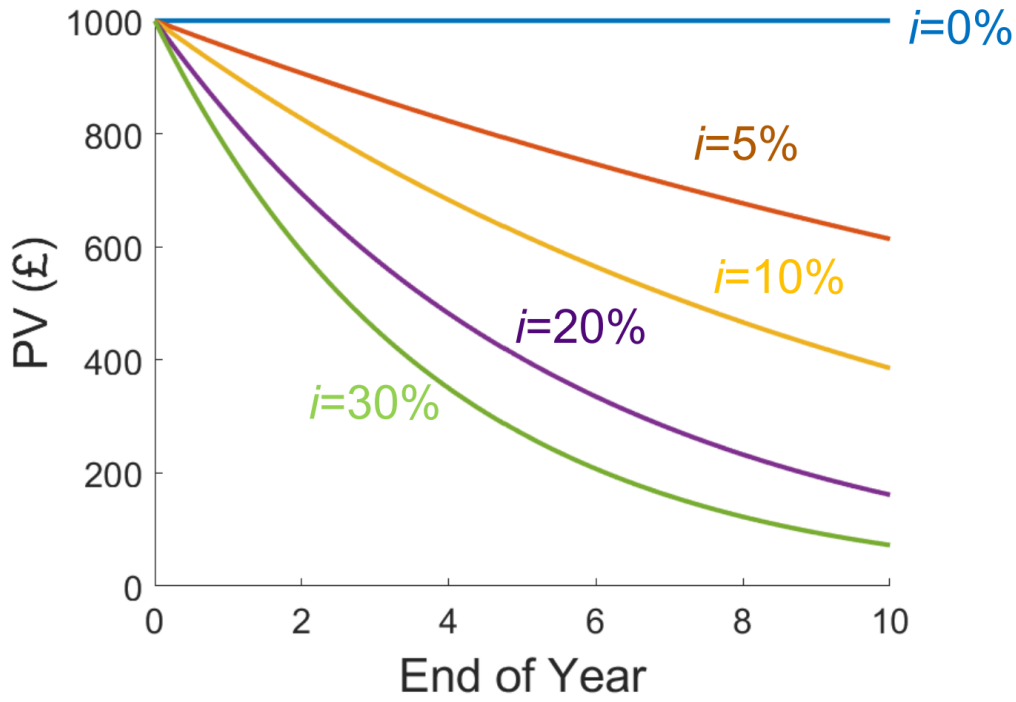


Figure 4.2: Effect of interest rate on PV.

#### 4.6.2 Example

A retrofitted heat-pump system is being considered for a small office building. The system can be installed and purchased for £110,000 and it will save an estimated 300,000 kilowatt-hours of electric power each year over a six-year period. A kilowatt-hour of electricity costs £0.10, and the company uses a MARR of 15% per year in its economic evaluations of refurbished systems. The market value of the system will be £8,000 at the end of six years, and additional annual operating and maintenance expenses are negligible. Use the NPV method to determine whether the system should be installed.

$$NPV = PV \text{ of estimated savings} + PV \text{ of market value} - PV \text{ of cost} \quad (4.6)$$

Estimated value:

$$PV_{ES} = 300000 \times 0.1 = 30000 \quad (4.7)$$

$$PV_{ES,y1} = \frac{30000}{(1 + 0.15)^1} \quad (4.8)$$

$$PV_{ES,y2} = \frac{30000}{(1 + 0.15)^2} \dots \quad (4.9)$$

$$PV_{ES,y6} = \frac{30000}{(1 + 0.15)^6} \quad (4.10)$$

$$\therefore \sum_{k=1}^6 \frac{30000}{(1 + 0.15)^k} \quad (4.11)$$

Market value:

$$PV_{MV} = \frac{8000}{(1 + 0.15)^6} \quad (4.12)$$

Cost:

$$PV_{cost} = 110000 \quad (4.13)$$

Therefore, NPV is:

$$NPV = \sum_{k=1}^6 \frac{30000}{(1 + 0.15)^k} + \frac{8000}{(1 + 0.15)^6} - 110000 \approx 6993 \quad (4.14)$$

### 4.6.3 Advantages and disadvantages of NPV

Advantages

- It accounts for the time value of money
- It accounts for uncertainties about future projections
- It accounts for all cash flows of interest

Disadvantages

- It is highly sensitive to the interest rate used
- It is not useful for comparing projects of different sizes
- It ignores costs that are incurred before the project starts

## 4.7 Project evaluation using Internal Rate of Return

### 4.7.1 Internal Rate of Return (IRR)

The IRR method solves for the interest rate that equates the present value of cash inflows (receipts or savings) to the present value of cash outflows (expenditures, e.g. investment costs). That is, the IRR provides the answer to the question: what interest rate provides an NPV of 0? This method is the most widely using rate-of-return method for performing engineering economic analyses.

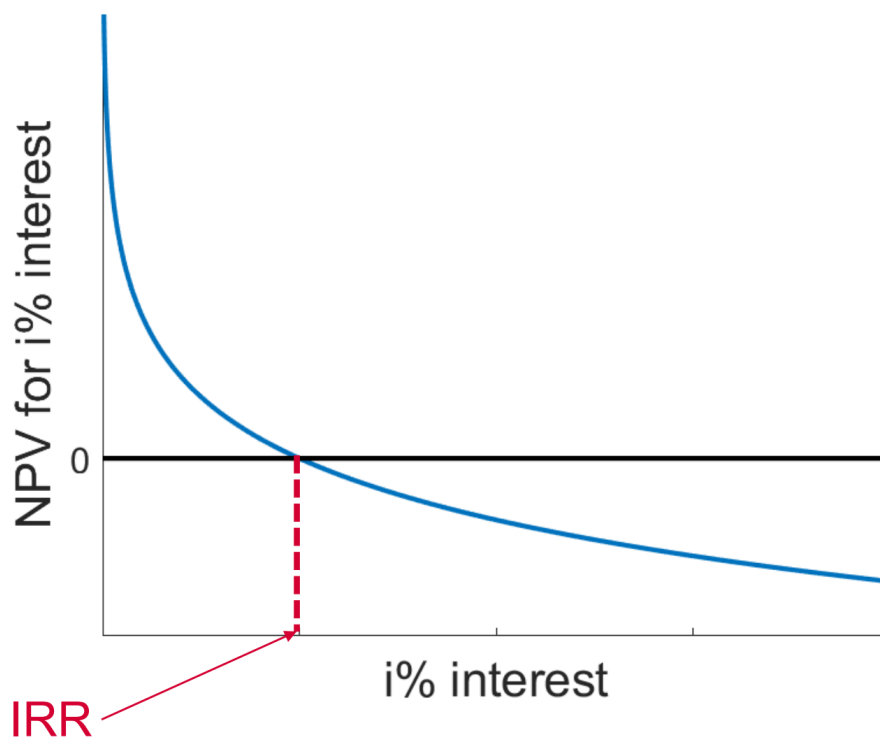


Figure 4.3: Internal Rate of Return.

### 4.7.2 Example

A company is considering the purchase of a digital camera for the maintenance of design specifications by feeding digital pictures directly into an engineering workstation where computer-aided design files can be superimposed over the digital pictures. Differences between the two images can be noted, and corrections as appropriate can then be made by design engineers. The capital investment requirement is £345,000 and the estimated market value of the system after a six-year study period is £115,000. Annual revenues attributable to the new system will be £120,000 and additional annual expenses will be £22,000. You have been asked by management to determine the IRR of this project and to make a recommendation. The corporation's MARR is 20% per year.

Denote IRR as  $i$ . First let's determine an equation for NPV:

$$NPV = PV \text{ of net annual revenue} + PV \text{ of market value} - PV \text{ of cost} \quad (4.15)$$

PV of net annual revenue:

$$PV_{NAR,y1} = \frac{120000 - 22000}{(1+i)^1} \quad (4.16)$$

$$PV_{NAR,y2} = \frac{120000 - 22000}{(1+i)^2} \dots \quad (4.17)$$

$$PV_{NAR,y6} = \frac{120000 - 22000}{(1+i)^6} \quad (4.18)$$

$$\therefore \sum_{k=1}^6 \frac{98000}{(1+i)^k} \quad (4.19)$$

Market Value:

$$PV_{MV} = \frac{115000}{(1+i)^6} \quad (4.20)$$

Cost:

$$PV_{cost} = 345000 \quad (4.21)$$

NPV:

$$NPV = \sum_{k=1}^6 \frac{98000}{(1+i)^k} + \frac{115000}{(1+i)^6} - 345000 \quad (4.22)$$

Lets try  $i = MARR = 20\% = 0.2$ :

$$NPV(i = 0.2) = +19,413 \quad (4.23)$$

However, this is not the IRR... We must calculate  $i$  using a solver to find which value of  $i$  gives and NPV of 0. Using Excel, we find that our IRR is 22%. Interpolation may also be used.

### 4.7.3 Advantages and disadvantages of IRR

Advantages:

- It has widespread acceptance in industry
- It is relatively simple to understand
- It accounts for the time value of money

Disadvantages

- It is difficult to compute
- It ignores the size and scope of projects
- It does not account for the actual reinvestment rate

## 4.8 Project evaluation using Payback Period

The payback period method evaluates the number of years  $\Theta$  it takes for cash inflows to equal cash outflows. Both of the previous evaluation methods focus on profitability. The payback period instead estimates a company's liquidity (i.e. how fast an investment can be recovered). There are two types of payback period methods:

1. Simple payback period - ignores the time value of money
2. Discounted payback period - accounts for the time value of money

### 4.8.1 Simple Payback Period Example

A public school is being renovated for £13.5 million. The building has geothermal heating and cooling, high-efficiency windows, and a solar array that permits the school to sell electricity back to the local electric utility. The annual value of these benefits is estimated to be £2.7 million. In addition, the residual value of the school at the end of its 40-year life is negligible. What is the simple payback period for the renovated school?

The simple payback period is:

$$SPP = \frac{13.5}{2.7} = 5 \text{ years} \quad (4.24)$$

### 4.8.2 Simple & Discounted Payback Period Example

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is £25,000 and the equipment will have a market value of £5,000 at the end of its expected life of 5 years. Increased productivity attributable to the equipment will amount to £8,000 per year after extra operating costs have been subtracted from the value of the additional production. MARR is 20% per year. Calculate the simple and the discounted payback periods.

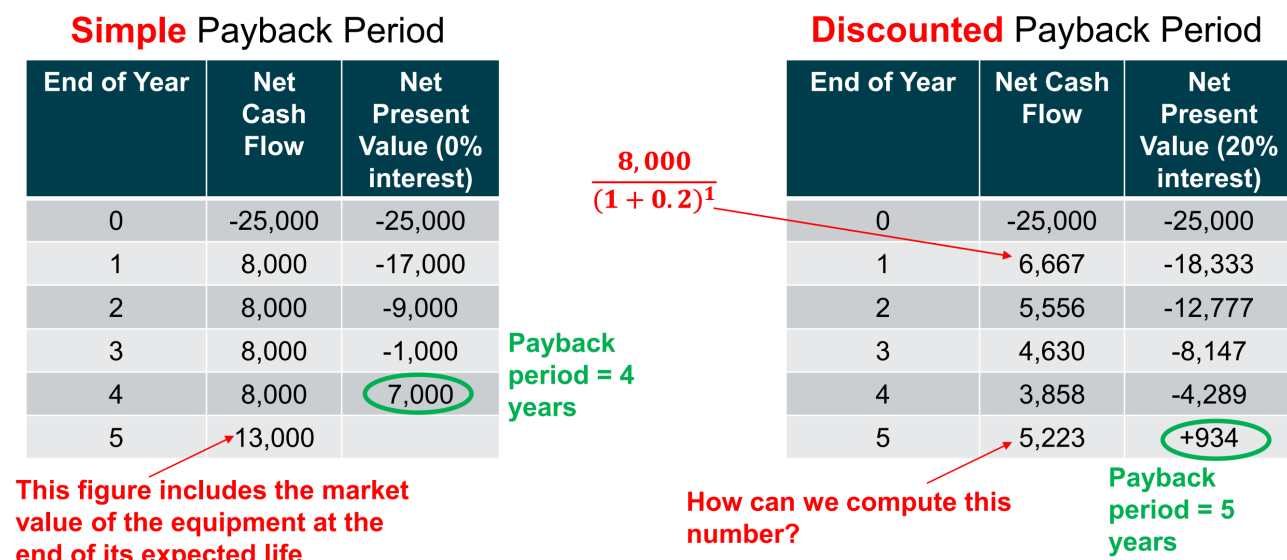


Figure 4.4: Simple and discounted payback periods.

### 4.8.3 Advantages and disadvantages of payback period

Advantages

- It provides a new perspective on performance (by focusing on liquidity)
- It is relatively simple to understand and compute
- It requires relatively few inputs



#### Disadvantages

- It does not account for cash flows that occur after the payback period
- It may not consider the time value of money
- It ignores profitability, and should only be used as a secondary evaluation measure.

# Chapter 5

## Cost-Benefit Analysis

### 5.1 What is cost-benefit analysis?

“Assessing costs and benefits across all affected groups or places matters because even a proposal with a relatively low public sector cost such as new regulation, may have significant effects on specific groups in society, places or business”

- A cost-benefit analysis is the process used to measure the benefits of a decision or taking action relative to the associated costs
- If benefits > costs, the decision or action is a good one to take
- If costs > benefits, the proposed action or decision should be reconsidered
- Cost benefit analysis can also be used to compare alternate decisions or actions

Both costs and benefits are required to be expressed in monetary terms, accounting for the time value of money. Costs may be categorised as:

- Direct - e.g. labour costs, manufacturing costs, material costs
- Indirect - e.g. utilities, rent
- Intangible - e.g. reduced productivity because of a new process
- Opportunity - lost benefits (opportunities) when pursuing one strategy over another

Benefits may be categorised as:

- Direct - e.g. increased revenue and sales
- Indirect - e.g. increased consumer interest
- Intangible - e.g. improved employee morale
- Competitive - e.g. being an industry leader

### 5.2 The Benefit-Cost Ratio method

The Benefit-Cost Ratio (BCR) is defined as the ratio of the equivalent value of benefits to the equivalent value of costs. The equivalent-value measure can be:

- Annual value (AV)
- Present value (PV)
- Future value (FV)

The BCR method has been the accepted procedure for making decisions and comparing projects in the public sector for many decades.

if  $BCR \geq 1$ , the project is acceptable

Several different formulations of the BCR method have been developed. We examine two formulations of the BCR method that are commonly used by government agencies:

- Conventional BCR method
- Modified BCR method

Both formulations will lead to identical project acceptability decisions (i.e.  $BCR \geq 1$  or  $BCR < 1$ ).

### 5.2.1 Conventional BCR method

Present value (PV) formulation:

$$BCR_{PV} = \frac{PV_{benefits}}{PV_{costs}} \quad (5.1)$$

$$BCR_{PV} = \frac{PV_{benefits}}{I - PV_{MV} + PV_{O\&M}} \quad (5.2)$$

Annual value (AV) formulation:

$$BCR_{AV} = \frac{AV_{benefits}}{AV_{costs}} \quad (5.3)$$

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} \quad (5.4)$$

Both ratios lead to identical numerical results. Where:

- $I$  is initial investment in the proposed project
- $MV$  is market value at the end of useful life
- $O\&M$  is operating and maintenance costs
- $CR$  is capital-recovery amount (i.e. equivalent cost of  $I$ , including an allowance for market or salvage value)

### 5.2.2 Modified BCR method

Present value (PV) formulation:

$$BCR_{PV} = \frac{PV_{benefits}}{PV_{costs}} \quad (5.5)$$

$$BCR_{PV} = \frac{PV_{benefits} - PV_{O\&M}}{I - PV_{MV}} \quad (5.6)$$

Annual value (AV) formulation:

$$BCR_{AV} = \frac{AV_{benefits}}{AV_{costs}} \quad (5.7)$$

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} \quad (5.8)$$

Both ratios lead to identical numerical results.

### 5.2.3 Conventional and modified BCRs

Remember the formula for PV:

$$PV = \frac{FV}{(1+i)^n} \quad (5.9)$$

Present value (PV) of a future value (FV) in  $n$  years, for an interest rate  $i$ . Remember the formula for AV:

$$AV = PV \frac{i(1+i)^n}{(1+i)^n - 1} \quad (5.10)$$

Value of a series of uniform (annual) receipts (AV) that occur at the end of periods (years) 1 to  $n$ , given their present value (PV) and an interest rate  $i$ .

### 5.2.4 What value of $i$ to use?

There are three main considerations when it comes to what interest rate to use for engineering economy studies of public-sector projects:

- the interest rate on borrowed capital
- The opportunity cost of capital to the government agency
- The opportunity cost of capital to the taxpayers

### 5.2.5 Why do conventional and modified BCRs lead to the same decision?

Conventional BCR formulation:

$$BCR_V = \frac{V_{benefits}}{I - V_{MV} + V_{O\&M}} = \frac{B}{C} \quad (5.11)$$

Where subscript  $V$  denotes either PV or AV. Modified BCR formulation:

$$BCR_V = \frac{V_{benefits} - V_{O\&M}}{I - V_{MV} + V_{O\&M} - V_{O\&M}} = \frac{B - X}{C - X} \quad (5.12)$$

Both the numerator and denominator differ by the same constant.

$$\frac{B}{C} > 1 \rightarrow B > C \rightarrow B - X > C - X \rightarrow \frac{B - X}{C - X} > 1 \quad (5.13)$$

leading to the same decision.

### 5.2.6 Example 1

The Greater London Authority is considering extending the runways of Stansted Airport so that larger commercial airplanes can use the facility. The land necessary for the runway extension is currently a farmland that can be purchased for £350,000. Construction costs for the runway extension are projected to be £600,000, and the additional annual maintenance costs for the extension are estimated to be £22,500. If the runways are extended, a small terminal will be constructed at a cost of £250,000. The annual operating and maintenance costs for the terminal are estimated at £75,000. Finally, the projected increase in flights will require the addition of two air traffic controllers at an annual cost of £100,000. Annual benefits of the runway extension have been estimated as follows: Apply the BCR method with a study of 20 years and a MARR of 10% per year to determine whether the runways at Stansted airport should be extended.

Description	Annual benefit
Leasing fee receipts from airlines	£325,000
Passenger airport tax receipts	£65,000
Convenience benefit for residents near Stansted	£50,000
Additional tourism money for London	£50,000
<b>Total</b>	<b>£490,000</b>

Table 5.1: Example 1.

Information provided:

$$i = 0.1 \quad (5.14)$$

$$n = 20 \text{ years} \quad (5.15)$$

$$I = £350000 + £600000 + £250000 = £1200000 \quad (5.16)$$

$$AV_{benefits} = £490000 \quad (5.17)$$

$$PV_{MV} = AV_{MV} = £0 \quad (5.18)$$

$$AV_{O\&M} = £22500 + £75000 + £100000 = £197500 \quad (5.19)$$

First, we need to determine PVs and AVs using:

$$PV = AV \frac{(1+i)^n - 1}{i(1+i)^n} \quad (5.20)$$

$$AV = PV \frac{i(1+i)^n}{(1+i)^n - 1} \quad (5.21)$$

$$PV_{benefits} = £4171646 \quad (5.22)$$

$$PV_{O\&M} = £1681429 \quad (5.23)$$

$$AV_I = CR = £140951 \quad (5.24)$$

Conventional BCRs:

$$BCR_{PV} = \frac{PV_{benefits}}{I - PV_{MV} + PV_{O\&M}} = 1.448 \quad (5.25)$$

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.448 \quad (5.26)$$

Modified BCRs:

$$BCR_{PV} = \frac{PV_{benefits} - PV_{O\&M}}{I - PV_{O\&M}} = 2.075 \quad (5.27)$$

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} = 2.075 \quad (5.28)$$

$BCR \geq 1$  in all cases, so runway should be extended.

### 5.2.7 Issues of concern using BCRs

- The treatment of disbenefits
  - Negative consequences to the public resulting from the implementation of a public sector project
- The treatment of certain cash flows as additional benefits or reduced costs

### 5.2.8 Treatment of disbenefits

Disbenefits can be incorporated in BCR calculations by:

- Reducing benefits accordingly (traditional approach) or
- Increasing costs accordingly

How do these approaches affect the BCR? How do these approaches affect the final decision?

### 5.2.9 Example 2

Refer back to Example 1. Suppose that there are disbenefits associated with the runway extension project. Specifically, the increased noise level from commercial jet traffic will be a serious nuisance to homeowners living along the approach path Stansted Airport. The annual disbenefit to these citizens is estimated to be £100,000.

Reapply the conventional BCR method, with equivalent annual worth, to determine whether this disbenefit affects your recommendation on the desirability of this project.

Disbenefit treated as a reduced benefit:

$$BCR_{AV} = \frac{AV_{benefits} - 100000}{CR + AV_{O\&M}} = 1.152 \quad (5.29)$$

Disbenefit treated as an increased cost:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M} + 100000} = 1.118 \quad (5.30)$$

$BCR \geq 1$  in both cases, so runway should be extended. The treatment of disbenefits affects the magnitude of the BCR, but not the decision.

### 5.2.10 Treatment of certain cash flows

Certain cash flows can be incorporated in BCR calculations by:

1. Increasing benefits accordingly
2. Reducing costs accordingly

How do these approaches affect the BCR? How do these approaches affect the final decision?

### 5.2.11 Example 3

Transport for London is considering upgrading an ageing bridge across the Thames. The existing two-lane bridge is expensive to maintain and creates a traffic bottleneck because the road is four lanes wide on either side of the bridge. The new bridge can be constructed at a cost of £300,000, and estimated annual maintenance costs are £10,000. The existing bridge has annual maintenance costs of £18,500. The annual benefit of the new four-lane bridge to motorists, due to the removal of the traffic bottleneck, has been estimated to be £25,000.

Conduct a cost-benefit analysis based on equivalent annual worth, using a MARR of 8% and a study period of 25 years, to determine whether the new bridge should be constructed.

Information provided:

$$i = 0.08 \quad (5.31)$$

$$n = 25 \text{ years} \quad (5.32)$$

$$I = £300000 \quad (5.33)$$

$$AV_{benefits} = £25000 \quad (5.34)$$

$$PV_{MV} = AV_{MV} = £0 \quad (5.35)$$

$$AV_{O\&M} = £10000 - £18500 = -£8500 \quad (5.36)$$

Cost is negative because it represents a reduction with respect to the current cost.

Required information:

$$AV_I = CR = £28104 \quad (5.37)$$

Reduced cost treated as a reduced cost (conventional BCR approach):

$$BCR_{AV} = \frac{BCR_{AV}}{CR + AV_{O\&M}} = 1.275 \quad (5.38)$$

Reduced cost treated as an increased benefit (modified BCR approach):

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} = 1.192 \quad (5.39)$$

$BCR \geq 1$  in both cases, so bridge should be constructed. The classification of the cash-flow items affects the magnitude of the BCR, but not the decision.

### 5.2.12 Treatment of disbenefits/certain cash flows

Arbitrary decisions on the classification of benefits and costs has no bearing on project acceptability because if X is classified as an added benefit:

$$BCR = \frac{B + X}{C} \quad (5.40)$$

and

$$BCR > 1 \rightarrow B + X > C \rightarrow B > C - X \rightarrow \frac{B}{C - X} > 1 \quad (5.41)$$

leading to the same decision.

## 5.3 Evaluating independent projects using the BCR method

### 5.3.1 What are independent projects?

Independent projects are categorised as groupings of projects for which the choice to select any particular project in the group is **independent** of choices regarding all other projects within the group.

It is therefore acceptable to select:

1. None of the projects
2. A combination of the projects
3. All of the projects

Formal comparisons of independent projects is unnecessary. The only criterion for selecting each independent project is  $BCR \geq 1$ .

### 5.3.2 Example 4

Uncontrolled water flow has increased flow conditions along a river. You have independent options to alleviate the problem of building a reservoir and/or improving the channel. Relevant information is as follows:

	Reservoir construction	Channel improvement
$CR + AV_{O\&M}$	£1,642,200	£1,815,100
$AV_{benefits}$	£1,742,200	£2,856,300

Table 5.2: Example 4 information.

Conduct a cost-benefit analysis using the conventional BCR method and equivalent annual worth to determine the best course of action.

Reservoir construction:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.061 \quad (5.42)$$

Channel improvement:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.574 \quad (5.43)$$

$BCR \geq 1$  in both cases, so both options should be pursued (the fact that the channel improvement has a higher BCR is irrelevant).

## 5.4 Evaluating mutually exclusive projects using the BCR method

### 5.4.1 What are mutually exclusive projects

Mutually exclusive projects are a group of projects from which, **at most, one project may be selected**. Each mutually exclusive project can be viewed as a feasible design alternative. Because the BCR method provides a ratio of benefits to costs rather than a direct measure of a project's profit potential, **selecting the project that maximises the BCR does not guarantee that the best project is selected**.

### 5.4.2 Procedure for evaluating mutually exclusive projects

1. Calculate equivalent value (PV, AV or FV) of costs for each mutually exclusive project
2. Rank-order mutually exclusive projects by increasing equivalent value of costs (note: the rank-order will be the same for all equivalent value types)
3. Calculate the BCR for the project with the lowest equivalent cost ( $BCR_L$ )
  - If  $BCR_L \geq 1 \rightarrow$  baseline = project with the lowest equivalent cost
  - Else  $\rightarrow$  baseline = "do-nothing"
4. Follow the flow chart in Figure 5.1

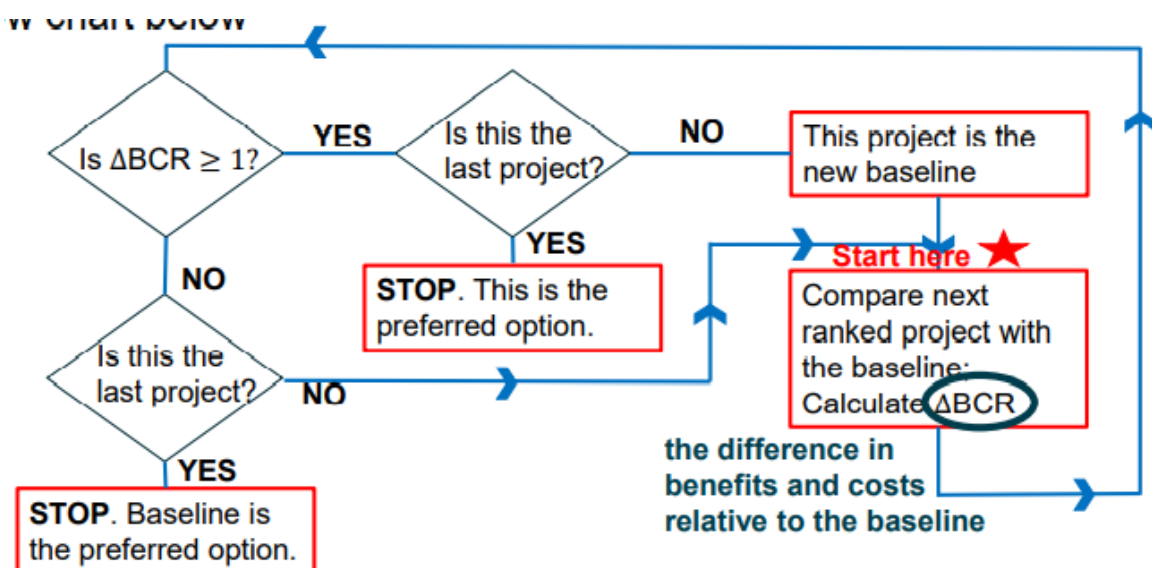


Figure 5.1: Flow chart for evaluating mutually exclusive projects



### 5.4.3 Example 5

Three mutually exclusive public-works projects are currently under consideration. Their respective costs and benefits are included in the table that follows. Each of the projects has a useful life of 50 years, and MARR is 10% per year.

	Project A	Project B	Project C
Capital investment	£8,500,000	£10,000,000	£12,000,000
Annual O&M costs	£750,000	£725,000	£700,000
Market value	£1,250,000	£1,750,000	£2,000,000
Annual benefit	£2,150,000	£2,265,000	£2,500,000

Table 5.3: Example 5 information.

Which, if any of these projects, should be selected?

Information provided and required information:

$$i = 0.1 \quad (5.44)$$

$$n = 50 \text{ years} \quad (5.45)$$

Step 1: convert to PV.

	Project A	Project B	Project C
I	£8,500,000	£10,000,000	£12,000,000
$PV_{O\&M}$	£7,436,111	£7,188,241	£6,940,370
$PV_{MV}$	£10,648	£14,907	£17,037
$PV_{costs}$	£15,925,463	£17,173,333	£18,923,333
$PV_{benefits}$	£21,316,851	£22,457,055	£24,787,036

Table 5.4: Example 5 required information.

Step 2

$$BCR_A = \frac{21316851}{15925463} = 1.339 \geq 1 \rightarrow \text{Project A is the baseline} \quad (5.46)$$

Step 3: establish baseline.

Step 4:

$$\Delta BCR_B = \frac{22457055 - 21316851}{17173333 - 15925463} = 0.914 < 1 \rightarrow \text{Project A is still the baseline} \quad (5.47)$$

Proceed to Project C

$$\Delta BCR_C = \frac{24787036 - 21316851}{18923333 - 15925463} = 1.158 \geq 1 \rightarrow \text{Project C is the preferred option} \quad (5.48)$$

### 5.4.4 Mutually exclusive projects with unequal lives

It is not uncommon for public projects to have different useful lives. How can we conduct BCR analyses in these cases? In these cases, annual values (AVs) should be used to conduct incremental cost-benefit analyses.

### 5.4.5 Example 6

Two mutually exclusive alternative public-works projects are under consideration. Their respective costs and benefits are included in the table that follows. Project A has an anticipated life of 35 years, and the useful life of Project B has been estimated to be 25 years. The effect of inflation is negligible.

	Project A	Project B
Capital investment	£750,000	£625,000
Annual O&M costs	£120,000	£110,000
Annual benefit	£245,000	£230,000

Table 5.5: Example 6 information.

If the MARR is 9% per year, which, if either, of these projects should be selected?

Information provided and required information:

$$i = 0.09 \quad (5.49)$$

$$n = 35 \text{ or } 25 \text{ years} \quad (5.50)$$

Step 1: convert to AV.

	Project A	Project B
$AV_I$	£70,977	£63,629
$AV_{O\&M}$	£120,000	£110,000
$AV_{MV}$	£0	£0
$AV_{costs}$	£190,977	£173,629
$AV_{benefits}$	£245,000	£230,000

Table 5.6: Example 6 required information.

Step 2:

$$BCR_B = \frac{230000}{173629} = 1.325 \geq 1 \rightarrow \text{Project B is the baseline} \quad (5.51)$$

Step 3: establish baseline.

Step 4:

$$\Delta BCR_A = \frac{245000 - 230000}{190977 - 173629} = 0.865 < 1 \rightarrow \text{Project B is the preferred option} \quad (5.52)$$