University College London

MEng Mechanical Engineering

MECH0071 Electrical Power Systems and Electrical Propulsion

Topic Notes

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Chapter 1

Introduction

1.1 Team

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1.2 Course Aim

The aim of this course is to provide students with detailed knowledge and understanding of the design, performance and analysis of electrical power systems.

Students will increase their knowledge and understanding through face-to-face/synchronous lectures, asynchronous (including tutorials) tasks and a computer simulation workshop and demonstrate their learning through summative coursework and an examination.

1.3 Student learning outcomes

- Appreciate the components that make up electrical power systems and understand the similarities and differences between large, medium and small scale power systems.
- Develop skills needed to be able to design electrical power systems including analytical and computer based methods.
- Understand the behaviour of steady-state, transient and faulted networks and appreciate how such behaviour influences design.
- Understand the benefits of electrical propulsion for different vehicle types be able to undertake designs.
- Appreciate future developments and applications in electrical power and electrical propulsion systems.

1.4 Assessment

- Coursework - summative assessment exercise based around computer simulations
- Examination - two hour examination in January

1.5 Textbooks

Kirtley, James. *Electric Power Principles: Sources, Conversion, Distribution and Use.* Wiley. 2020. ISBN: 9781119585305.t

1.6 Softwares

- PSCAD

Chapter 2

The Electrical Line Diagram

2.1 Overview of electrical power systems

2.1.1 Basic electrical power system

Most electrical power systems contain:

- Generators to produce electrical energy (often coming from another store of energy e.g. chemical - oil, gas, coal)
- A means to transmit and distribute the electrical energy
- Loads that use the electrical energy for some purpose

2.1.2 What is an electrical power system?

An electric power system is a network or grid of electrical components that supply, transfer and use electric energy. Electrical power systems can be a:

- Large grids covering a wide area e.g. a continent
- Medium grid covering a large area e.g. a country
- Small network covering a small area e.g. a ship

2.2 Components of electrical power systems

2.2.1 Sources of electrical power include

Generators (rotating types AC and DC):

- Large AC generators e.g. 25 kV three-phase voltages
- Medium AC generators e.g. 440 V three-phase voltages
- Small AC generators e.g. e.g. single-phase 220 V voltages

Fuel cells:

- DC output voltage (typically 720 V DC)

Batteries (electro-chemical):

- DC output voltage (usually multiples of 12 V)

Photo-voltaic (solar) cells:

- DC output currents (usually mA/cell)

2.2.2 Sources of DC electrical power ...

A fuel cell in a car. Photovoltaics used in a solar farm. Battery energy store. DC systems are increasing in their popularity due to wider use of batteries, solar cells and fuel cells in grids and electrical propulsion.

2.2.3 Generators ... single and multiphase AC

AC generators:

- Large AC generators e.g. 25 kV 3 phase
- Medium AC generators e.g. 11 kV or 440 V 3 phase
- Small generators e.g. 220 V single-phase voltage

2.2.4 Transmission systems

HVAC often three-wire and three-phase e.g. 440 kV, 275 kV and 132 kV.

HVDC often two-wire and bipolar e.g. +/- 330 kV.

2.2.5 Distribution systems

AC distribution:

- 11 kV, 440 V three-phase
- 25 kV single-phase (rail)
- 240 V single-phase

DC distribution:

- 750 V (rail)
- 110 V (emergency lighting)

2.2.6 Loads

Three-phase loads:

- Induction motors to drive pumps, fans and compressors
- Propulsion drives

Single-phase loads:

- Lighting
- Heating
- Appliances e.g. domestic, electronics, small pumps

DC loads:

- DC motors
- Lighting and heating
- Battery charging

2.3 Representation by the electrical line diagram

2.3.1 Electrical system representation

Electrical systems are commonly represented as one of the following:

- Pictorial diagram
- Block diagram
- Wiring diagram
- Single line diagram
- Riser diagram
- Electrical floor plan
- Layout diagram

Of these the most useful to the *electrical power engineer* is the **Single line diagram.**

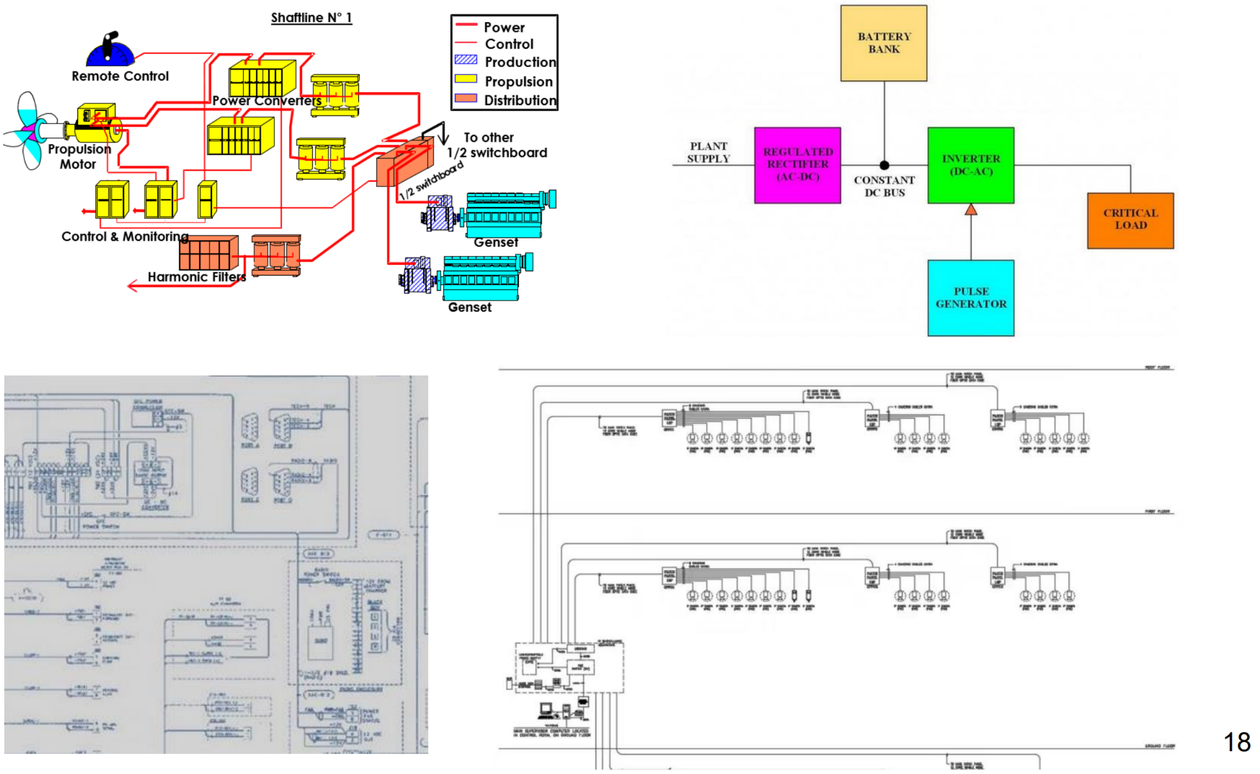


Figure 2.1: Some types of electrical system representation.

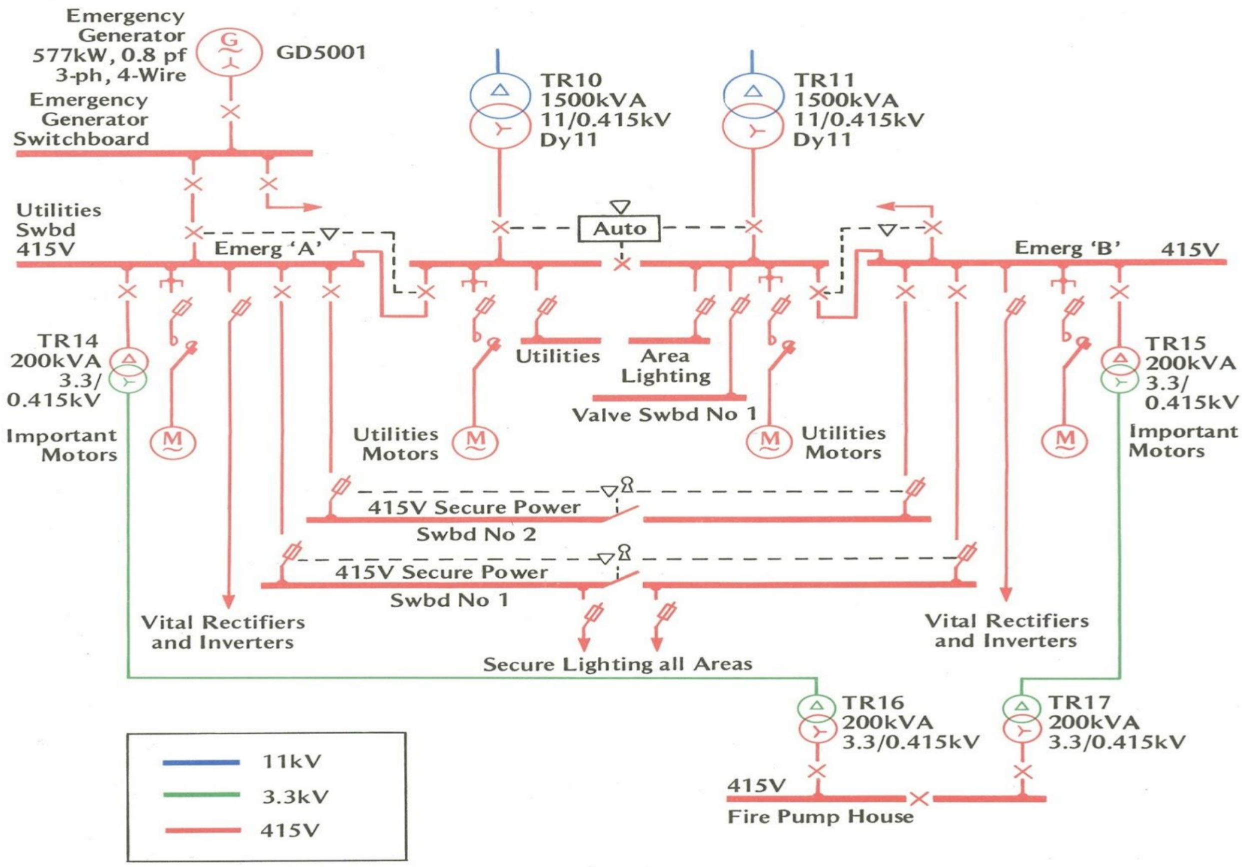


Figure 2.2: Example of a 'Single Line diagram'.

2.3.2 Questions for you?

1. The number of separate switchboards shown? 14 (each thick line is a separate switchboard)
2. Maximum current that will flow through the supply transformers? $I = \dfrac{\text{kV A}}{\text{kV} \times \sqrt{3}}$, (root 3 due to 3-phase)
3. How many different electrical sources supply the fire pump house? All three supplies can be connected to the fire pump house.

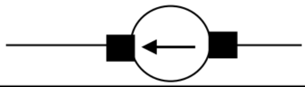
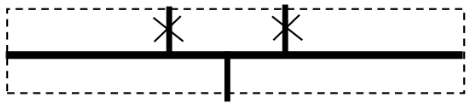
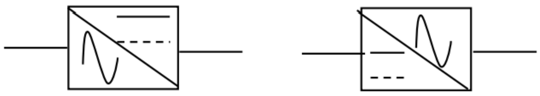
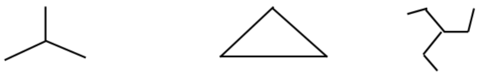
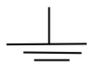
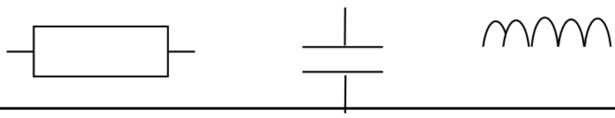
Equipment	Single Line Diagram Representation
AC Machine (Motor and Generator)	
DC Machine (Motor or Generator)	
Transmission Lines and Cables (With circuit breaker)	
Switchboards (with busbar, circuit breakers and feeders)	
Power Conversion (Rectifier AC-DC and Inverter DC-AC)	
Transformer (Two winding transformer, Three winding transformer)	
Star, Delta and Zig-Zag connections.	
Earth	
Passive Components (Resistance, Capacitance and inductance)	

Figure 2.3: Symbols.

2.3.3 The 'Single Line Diagram' (SLD)

The 'Single Line Diagram' (also known as the 'One Line Diagram') represents an electrical power system using single lines regardless of number of cables being used. It can be used to represent:

- Any type of electrical power system: DC, single-phase, three-phase or a mixed voltage electrical system.
- The interconnections between different electrical equipment including generators, switchboards, electrical distribution centres and loads.
- The types of electrical equipment and their main characteristics e.g. ratings of equipment such as voltage, power, power factor, and impedance.
- Emergency features such as reversionary modes, cross-connections and emergency generators. Sometimes these can be represented as single 'dotted line' connections rather than the usual solid single line.
- Other details such as 'earthing arrangements, arrangements of star/delta connections in three-phase systems and any autonomous operating systems such as circuit breakers.

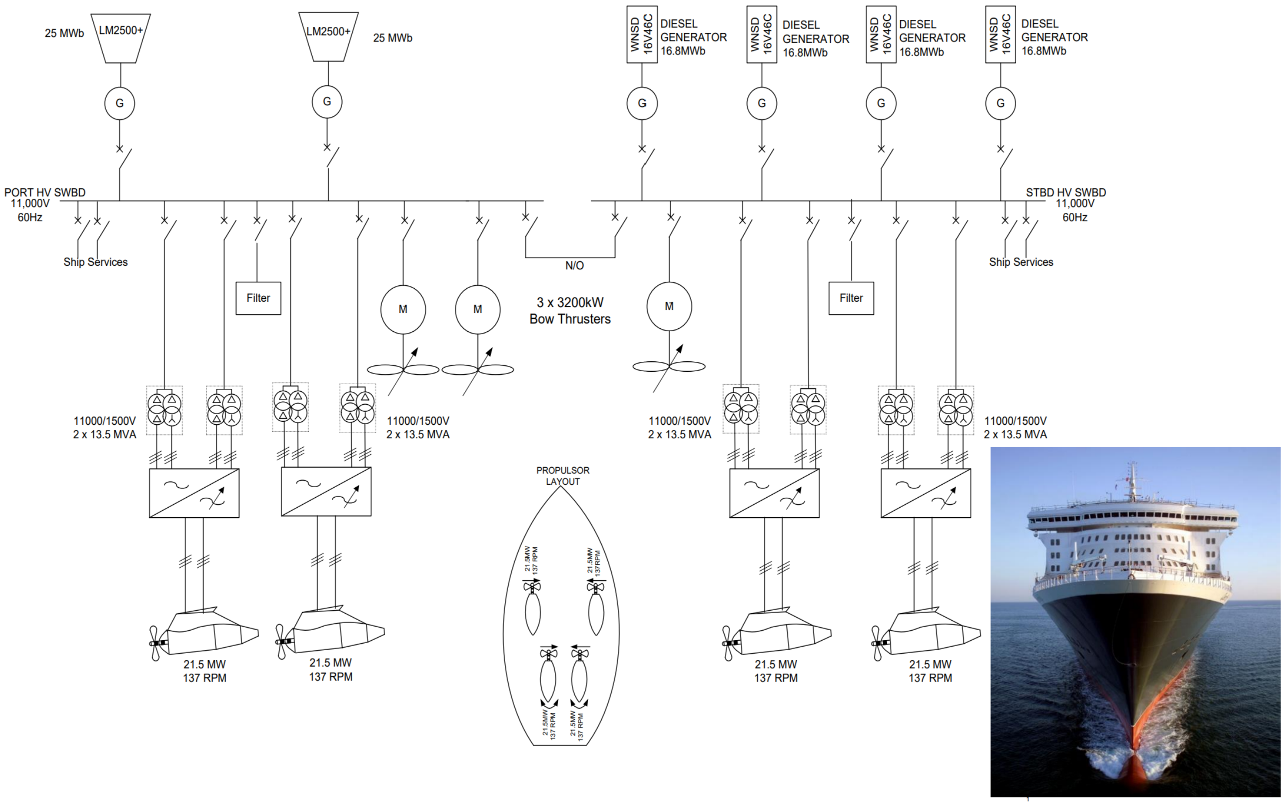


Figure 2.4: Marine SLD.

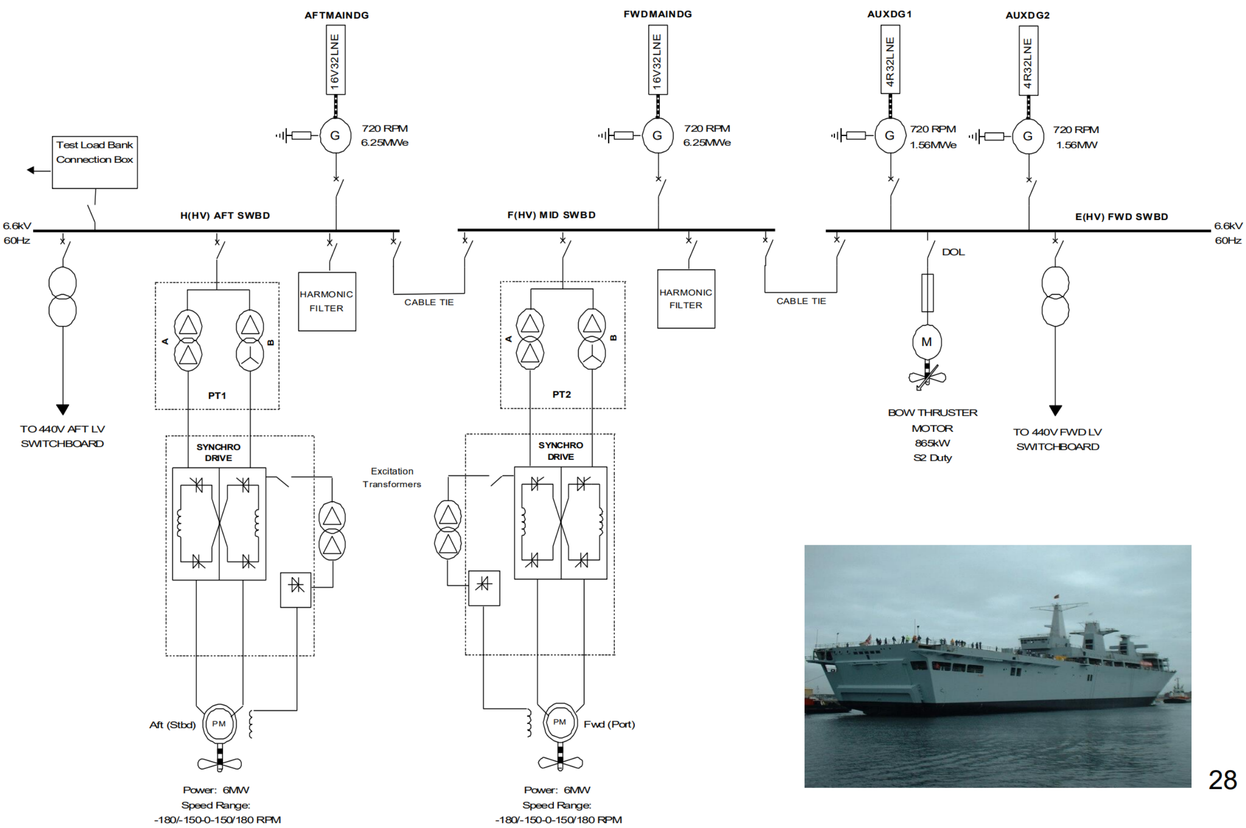


Figure 2.5: Naval SLD.

2.3.4 Some common features of SLDs

- Supplies (shore supplies, generators, incoming supply) are located at the top of the diagram
- The loads (motors, lighting, etc.) are located towards the bottom of the diagram.
- Switchboards are shown as thicker lines with interlocking switchgear being shown using dotted lines.
- Interconnections between equipment is a single-line representation regardless of number of phase (unless there is a good reason not to do so).
- Voltage, Frequency, Power, PF, revolutions, etc. are provided.

2.3.5 Limitations of the electrical line diagram

- The ‘Single Line Electrical Diagram’ is a very useful means of showing how electrical equipment is connected into a system using single lines (representing a three-phase system or some other electrical power system).
- It has very limited use when undertaking analysis. It is not an electrical circuit. To undertake analysis of electrical power systems then it is necessary to change the ‘Single Line Electrical Diagram’ into an ‘Impedance Diagram’.

Chapter 3

Developing Impedance Diagram

3.1 Three Phase Power

3.1.1 Three-phase alternating voltages

A three-phase synchronous generator consists of a rotor and a stator.

- Adjusting excitation current on the rotating field will change the magnitude of the three AC phase emfs generated in the stator.
- Changing the rotational speed changes the frequency of the AC emfs
- The three phases generated are 120° displaced due to special arrangement

3.1.2 Three-phase emfs (or terminal voltages) can be expressed mathematically

$$v_a(t) = V_m \sin(\omega t) \tag{3.1}$$

$$v_b(t) = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \tag{3.2}$$

$$v_c(t) = V_m \sin\left(\omega t - \frac{4\pi}{3}\right) \tag{3.3}$$

V_m is the peak (maximum) voltage, ω is the angular frequency, t is time. The phase displacement between the three-phase waveforms is 120° or $\frac{2\pi}{3}$ radians. v_a, v_b and v_c are the three phase voltages.

3.1.3 Three-phase, six-wire connection

The are different arrangements for distributing three-phase electrical power. The three phases can be independent of each other as seen below and treated as three separate circuits. This is known as the *three-phase, six-wire system.*

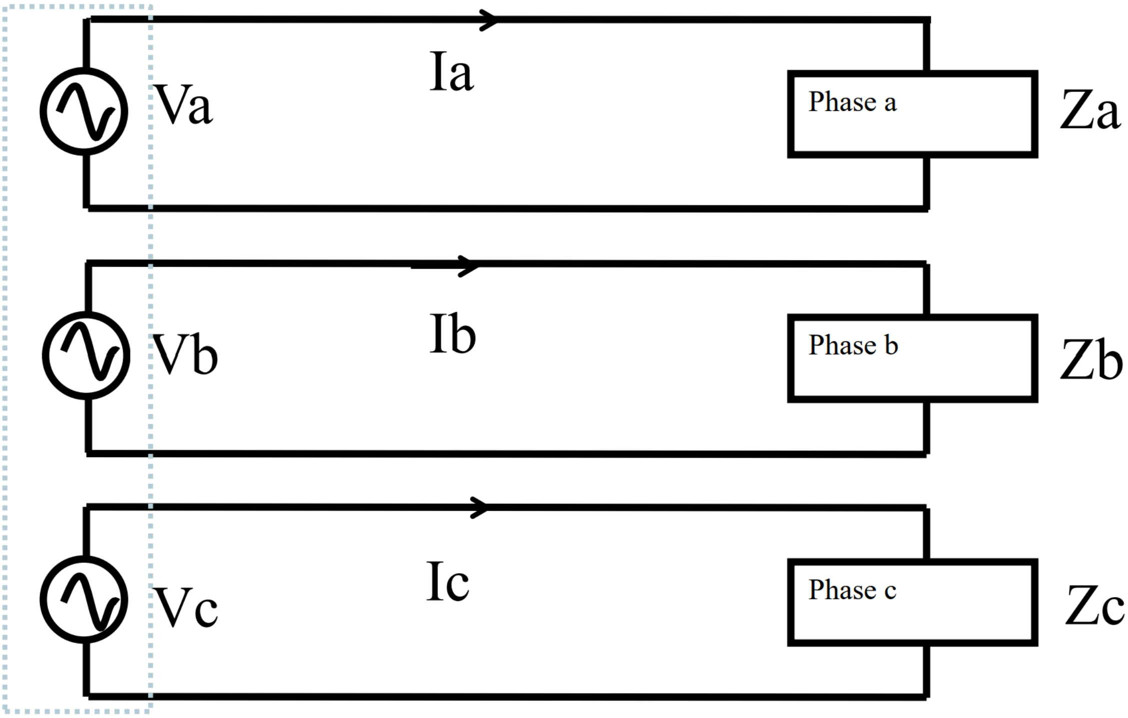


Figure 3.1: Three-phase, six-wire system.

3.1.4 Three-phase current

The currents flow in a three-phase circuit when there is a three-phase load. We will initially assume that the three-phase load is balanced i.e. the magnitude of voltage, current and the phase-angle is the same for each phase circuit. This is not true for three-phase circuits with unbalanced loads and the mathematical approach is different and more complex so we will examine this later.

3.1.5 Three-phase alternating current

The currents associated with a three-phase system that flow from the supply to the load may be described mathematically by:

$$i_a(t) = I_m \sin(\omega t + \theta) \tag{3.4}$$

$$i_b(t) = I_m \sin\left(\omega t - \frac{2\pi}{3} + \theta\right) \tag{3.5}$$

$$i_c(t) = I_m \sin\left(\omega t - \frac{4\pi}{3} + \theta\right) \tag{3.6}$$

Note: the phase displacement angle (θ) can be positive (leading PF) indicating a capacitive load or negative (lagging PF) indicating an inductive load. A zero phase displacement angle indicates a resistive circuit or a circuit at resonance ($X_L = X_C$).

3.1.6 Connecting Three-Phases

A three-phase six wire system is generally expensive to install and is actually unnecessary due to an inherent balancing characteristic.

In the balanced three-phase system, the algebraic sum of voltage at any point where all three-phase voltages are connected is zero.

The zero voltage point is known as the 'star point' and this may be grounded or left isolated (floating). In most electrical systems the star point is grounded with exceptions being some ship types.

3.1.7 Star and delta connections

The number of transmission wires can be reduced by connecting the phases in either delta or star configuration.

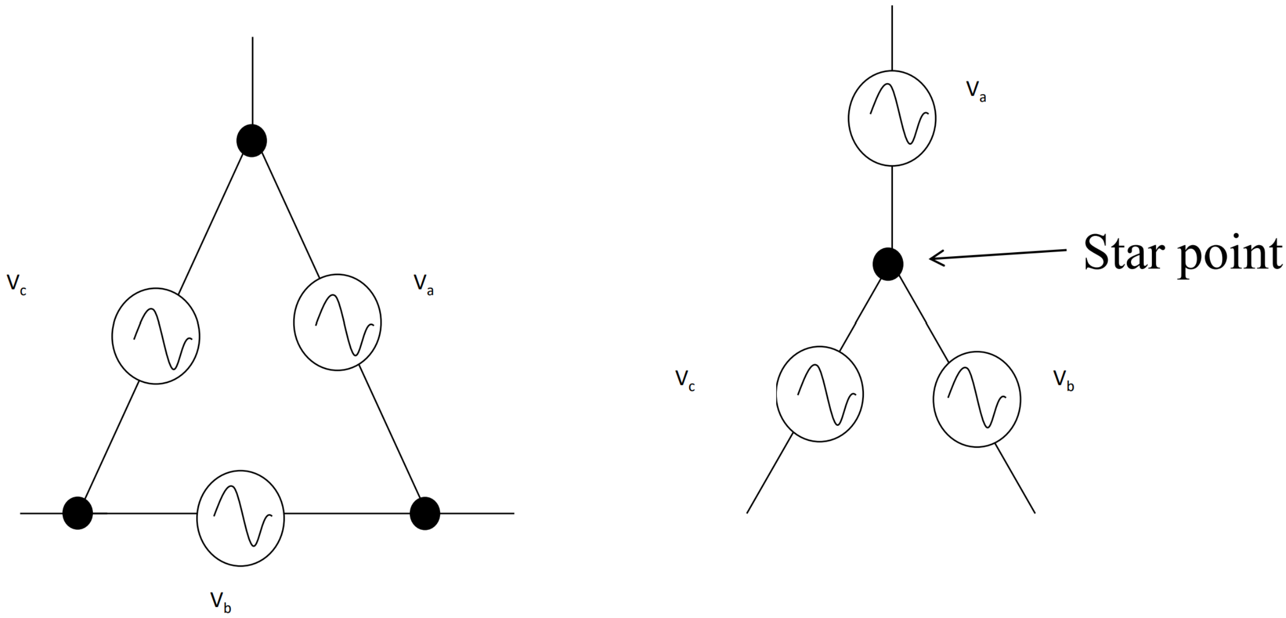


Figure 3.2: Star and delta configurations.

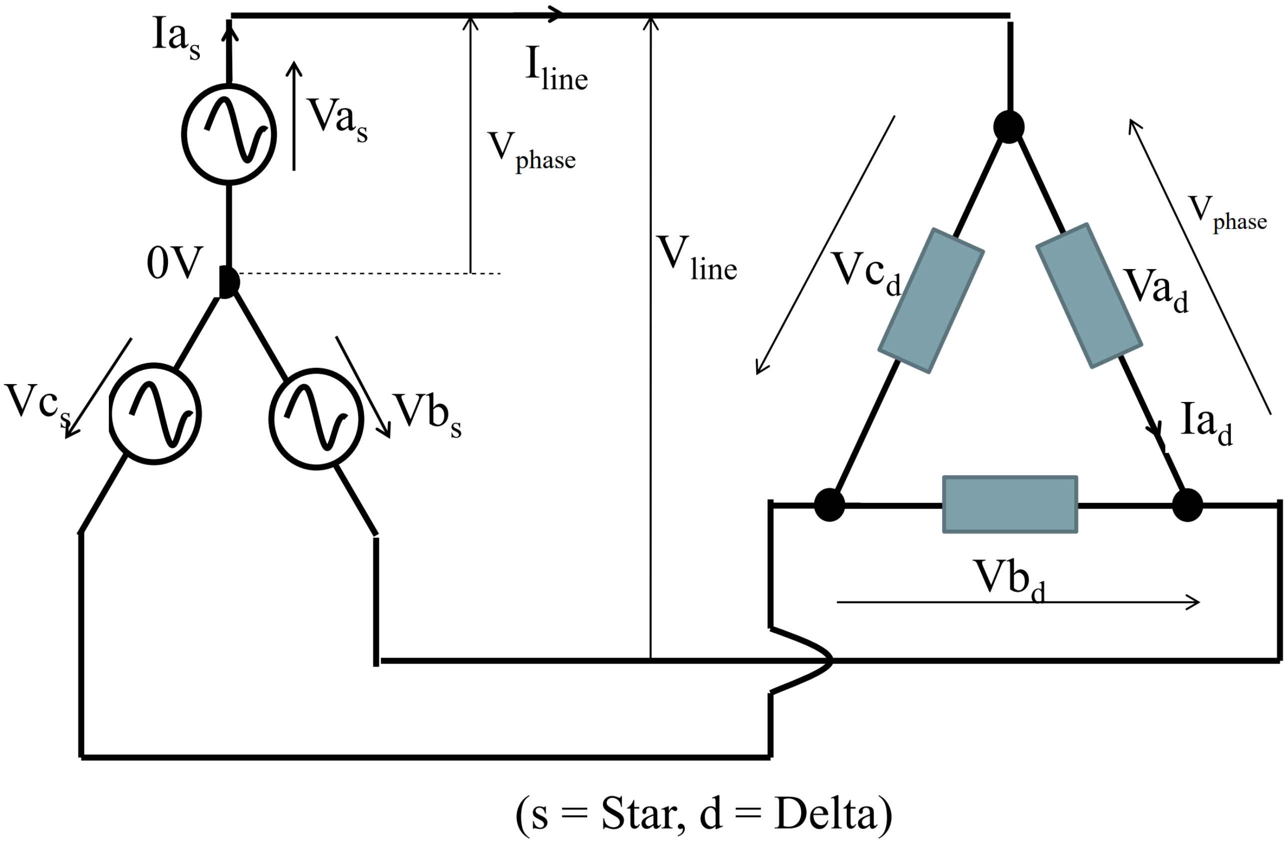


Figure 3.3: Star generator and delta load.

3.1.8 Phase and line voltages

There are therefore two voltage types (either generated as a potential difference) when considering three-phase circuits. These are commonly known as the *phase voltage* and *line voltage*.

The phase voltages in the star-delta circuit are as follows:

- Vas, Vbs, Vcs for the star circuit
- Vad, Vbd, Vcd for the delta circuit

The line voltages can be measured as follows:

$$Vab = Vas - Vbs = Vad \tag{3.7}$$
$$Vbc = Vbs - Vcs = Vbd \tag{3.8}$$
$$Vca = Vcs - Vas = Vcd \tag{3.9}$$

and if the line voltages measure is reversed:

$$Vba = Vbs - Vas = -Vad \tag{3.10}$$
$$Vcb = Vcs - Vbs = -Vbd \tag{3.11}$$
$$Vac = Vas - Vcs = -Vcd \tag{3.12}$$

Which is why a three-phase system is known as a six-pulse system - (important in power electronic systems).

3.1.9 Relationships between star and delta

For the delta arrangement:

$$V_p = V_l \tag{3.13}$$
$$I_p = \frac{I_l}{\sqrt{3}} \tag{3.14}$$

For the star arrangement:

$$V_p = \frac{V_l}{\sqrt{3}} \tag{3.15}$$
$$I_p = I_l \tag{3.16}$$

Where I_p and V_p are the phase currents and voltages and I_l and V_l are the line currents and voltages respectively. Note: Delta is also known as 'mesh'; Star is also known as 'Y'.

3.1.10 Single-phase impedance triangle

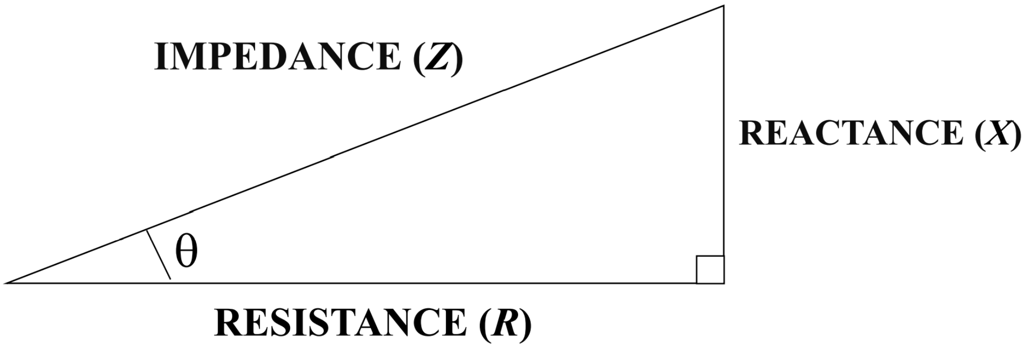


Figure 3.4: Single-phase impedance triangle.

$$Z = R + jX \tag{3.17}$$
$$= R + j\left(X_L - X_C\right) \tag{3.18}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{3.19}$$

Where, Z is impedance, R is resistance, X_L is inductive reactance, X_C is capacitive reactance, ω is angular frequency ($2\pi f$).

3.1.11 Single-phase power triangle

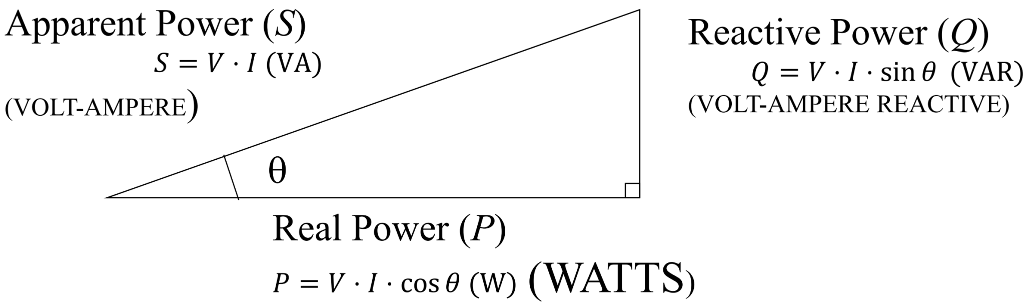


Figure 3.5: Single-phase power triangle.

- Real power (P) is the power that can be put into or taken from the electrical system and is measured in Watts (W).
- Reactive power (Q) is the power that circulates in the electrical system and is measured in Volt-Ampere-Reactive (VAR).
- Apparent power (S) is what is apparent from the product of voltage and current and is measured in Volt-Amperes (VA).

3.1.12 Three-phase power

Since V in the star circuit and I in the delta circuit is subject to change simply by dividing by $\sqrt{3}$, whilst the other variable I and V in star and delta respectively remain unchanged. Hence we get:

$$P = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \cos\theta \tag{3.20}$$

For apparent power (S) and reactive power (Q) we have:

$$S = \sqrt{3} \cdot I_{line} \cdot V_{line} \tag{3.21}$$
$$Q = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \sin\theta \tag{3.22}$$

3.1.13 Student Activity

Three coils each of resistance $5\,\Omega$ and inductive reactance of $10\,\Omega$ are connected in (a) star and (b) delta across a $440\,\mathrm{VRMS}$ three-phase (line) supply.

If each coil has a capacitor connected in parallel having capacitive reactance of $20\,\Omega$ then calculate the line and phase currents and the total power absorbed.

3.2 Per Unit (PU) System

3.2.1 Electrical line diagram to Impedance diagram

- The *'electrical line diagram'* - a schematic which allows an understanding of equipment and system arrangements.

- The *'impedance diagram'* - a schematic which allows an understanding of the equipment and system impedances.
- The layout of both the 'electrical line diagram' and 'impedance diagram' should be similar but in the 'impedance diagram' all equipment and lines are replaced with impedances.
- All impedances will need to be calculated to a *common base* - hence use of a per unit system.

3.2.2 Simple equivalent impedances

For the purposes of steady-state analysis the Electrical Line Diagram is converted to an 'Impedance Line Diagram' where the equipment is represented as an 'Equivalent Impedance'. Typical *simple* impedances representing equipment are: (note: not all R, L and C values may be given).

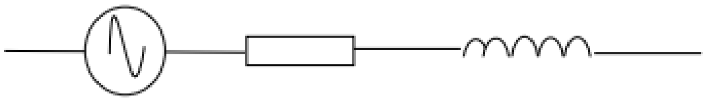
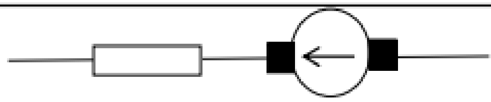
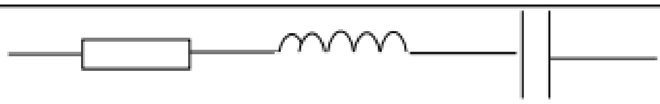
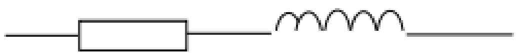
Equipment	Equivalence Impedance Representation
AC Generator or Motor	
DC Machine (Motor or Generator)	
Transmission Lines and Cables	
Transformer	

Figure 3.6: Equivalent Impedance Representations.

3.2.3 How manufacturers of electrical equipment specify ratings

Manufacturers of electrical equipment would usually specify electrical equipment as follows:

e.g. A synchronous generator

- S = 10 MVA (value of apparent power)
- V = 3.3 kV (line voltage rating of the equipment)
- Phase = 3 (number of phases)
- PF = 0.8 (usual value of power factor of equipment)
- N = 1500 rpm (design speed of rotation)
- F = 50 Hz (frequency of the alternating current & voltage)
- X = 0.14 (Reactance given as a pu value or as a %)
- Connection = star (stator windings)

3.2.4 The per unit system

In Electrical Power System Analysis the per unit system is the preferred method for analysing circuit behaviour rather than the standard SI system of units (Watts, Volts, Amperes, etc.)

The advantages of the per unit system are:

- Computations for power systems have several voltage levels because of connected transformers is very cumbersome when using the SI system because values need to be referred across the transformer turns ratio. The per unit system (overcomes or simplifies) this problem.
- All powers, voltage, currents and impedances are expressed as per unit values of specified base values. This means they are easily compared with one another which is very helpful for equipment specification and selection and in power system design and its analysis.

3.2.5 Values in per unit system

In the per unit system five base values are needed. These are **power, current, voltage, impedance** and **power factor**. It is necessary to choose two base values and to calculate two base values.

Usually the base values defined are:

- the Apparent Power (Base_VA)
- Voltage (Base_V)

Power Factor is already expressed in per unit form. Once the base values are calculated then 'actual values' in the circuit can be expressed in per unit form.

3.2.6 Three-Phase system PU conversion

Step one

The per unit relationships for Base_VA and Base_V are define and Base_I and Base_Z are calculated:

$$\text{Base\_VA} = \text{Defined by Engineer} \tag{3.23}$$

$$\text{Base\_V} = \text{Defined by Engineer} \tag{3.24}$$

$$\text{Base\_I} = \frac{\text{Base\_VA}}{\sqrt{3} \cdot \text{Base\_V}} \tag{3.25}$$

$$\text{Base\_Z} = \frac{\text{Base\_V}}{\text{Base\_I}} \tag{3.26}$$

Step two

Having calculated the Base Values, these are then defined as being 1 per unit values:

- Base_V = 1 per unit Voltage
- Base_VA = 1 per unit Apparent Power
- Base_I = 1 per unit Current
- Base_Z = 1 per unit Impedance

Step three

In the circuit all apparent powers, voltage, currents and impedances are expressed as per unit values:

$$\text{Per\_Unit\_S} = \frac{\text{Actual\_Value\_S}}{\text{Base\_S}} \tag{3.27}$$

$$\text{Per\_Unit\_V} = \frac{\text{Actual\_Value\_V}}{\text{Base\_V}} \tag{3.28}$$

$$\text{Per\_Unit\_I} = \frac{\text{Actual\_Value\_I}}{\text{Base\_I}} \tag{3.29}$$

$$\text{Per\_Unit\_Z} = \frac{\text{Actual\_Value\_Z}}{\text{Base\_Z}} \tag{3.30}$$

Step four

Sometimes parameters e.g. Z are already expressed in per unit form rather than as SI units but have been calculated to a different base (S and V). These can be converted as follows:

$$(\text{Per\_Unit\_Z})_{\text{new\_base}} = \frac{(\text{Base\_VA})_{\text{new\_base}}}{(\text{Base\_VA})_{\text{old\_base}}} \cdot \frac{(\text{Base\_V})^2_{\text{old\_base}}}{(\text{Base\_V})^2_{\text{new\_base}}} \cdot (\text{Per\_Unit\_Z})_{\text{old\_base}} \tag{3.31}$$

Some manufacturers and engineers prefer to work with the percentage system rather than the per unit system which of course is a simple matter of mulitplying by 100/ Equipment manufacturers use a machine's own S and V to determine base values from which Z pu is then calculated.

3.2.7 Example PU system conversion

Single Line Diagram

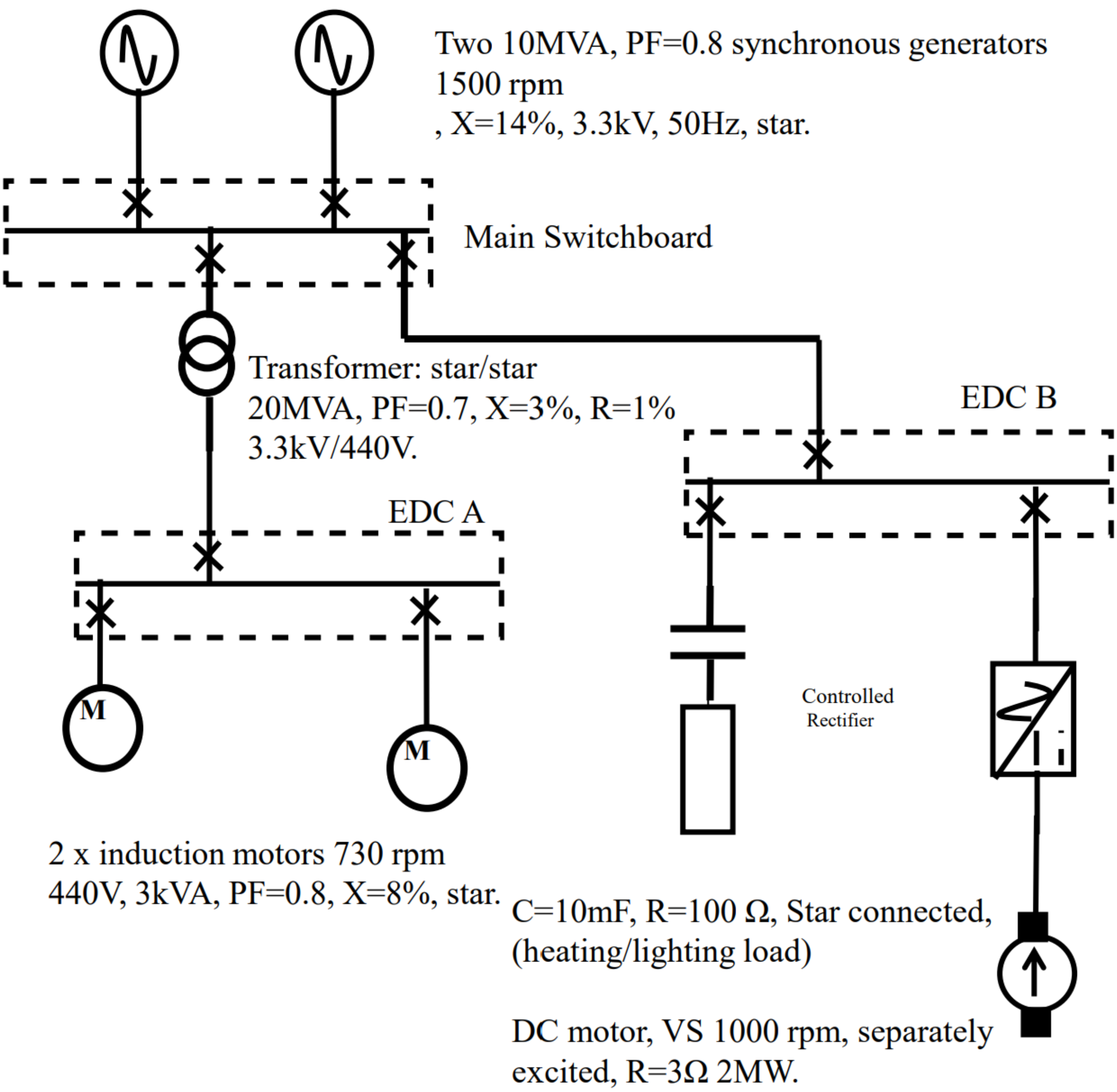


Figure 3.7: Single Line Diagram.

Step one - calculating the base current and base impedance

Selecting 10 MVA as Base_S and 3.3 kV as Base_V (because it seems sensible considering the generators) then we have:

$$\text{Base\_I} = \frac{10^6}{\sqrt{3} \times 3.3 \times 10^6} = 1749.5\,\text{A} \tag{3.32}$$

$$\text{Base\_Z} - \frac{3.3 \times 10^3}{1749.5} = 1.886\,\Omega \tag{3.33}$$

Step two - defininng 1 p.u. values

- 3.3×10^3 V = 1 per unit Voltage = 1 pu V
- 10×10^6 VA = 1 per unit Apparent Power = 1 pu S
- 1749.5 A = 1 per unit Current = 1 pu A
- 1.886 Ω = 1 per unit Impedance = 1 pu Z

Sometimes % values are preferred by some engineers i.e. 1 pu = 100%

Step three - converting impedances expressed in SI units to per unit form

The only 'actual values' i.e. expressed in SI units, are the heating load and the DC machine:

For the lighting/heating load:

$$-jXC = -j\left(\frac{1}{2\pi \cdot 50 \cdot 10 \times 10^{-3}}\right) = -j0.318 \tag{3.34}$$

$$-jXC = \frac{-j0.318}{1.886} = -j0.168\,\text{pu} \tag{3.35}$$

$$R = \frac{100}{1.886} = 53.022\,\text{pu} \tag{3.36}$$

For DC motor:

$$R = \frac{3}{1.886} = 1.591\,\text{pu} \tag{3.37}$$

$$S = P + \frac{2}{10} = 0.2\,\text{pu} \tag{3.38}$$

Step four - converting impedances expressed in per unit form to another base

For the synchronous generators:

$$S = \frac{10}{10} = 1\,\text{pu} \tag{3.39}$$

$$V = 3.3\,\text{kV} = 1\,\text{pu} \tag{3.40}$$

$$X = \frac{14}{100} = 0.14\,\text{pu} \tag{3.41}$$

$$PF = 0.8\,\text{pu} \tag{3.42}$$

For the transformer:

$$S = \frac{20}{10} = 2\,\text{pu} \tag{3.43}$$

$$X = \frac{3}{100} \times \frac{10}{20} = 0.015\,\text{pu} \tag{3.44}$$

$$R = \frac{1}{100} \times \frac{10}{20} = 0.005\,\text{pu} \tag{3.45}$$

$$PF = 0.7\,\text{pu} \tag{3.46}$$

For the induction motors:

$$S = \frac{3}{10000} = 0.0003\,\mathrm{pu} \tag{3.47}$$

$$X = \frac{8}{100} \times \frac{10000}{3} = 266.667\,\mathrm{pu} \tag{3.48}$$

$$PF = 0.8 \tag{3.49}$$

Step five - drawing the impedance diagram

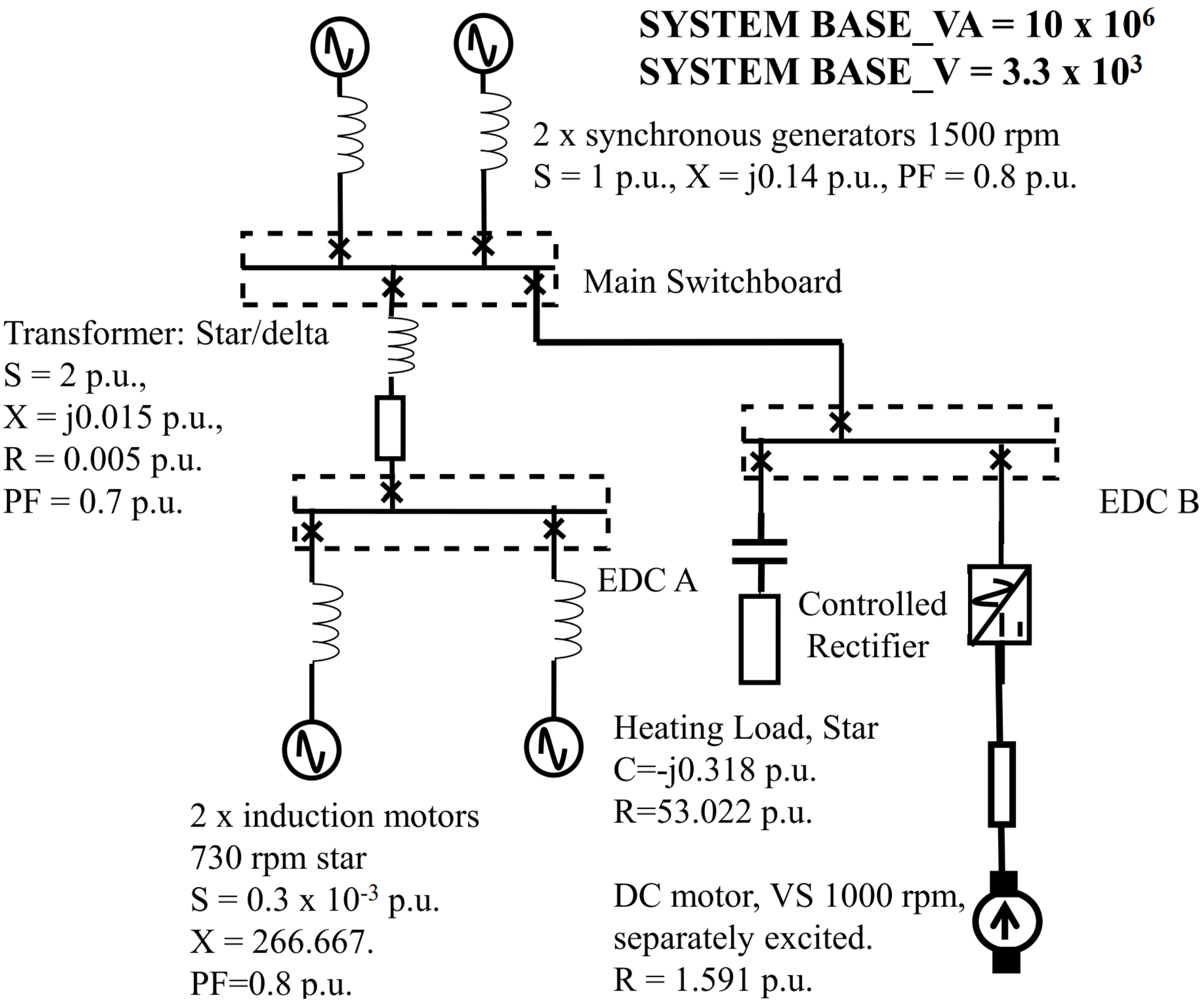


Figure 3.8: Impedance Diagram.

3.2.8 Reactance diagram

The reactance diagram is a modification to the impedance diagram where only per unit reactances are shown. In a reactance diagram all resistances are ignored. The reactance diagram is useful because it allows ‘first pass’ calculations to be made in a power system without too much mathematical complexity due to having $(R \pm jX)$.

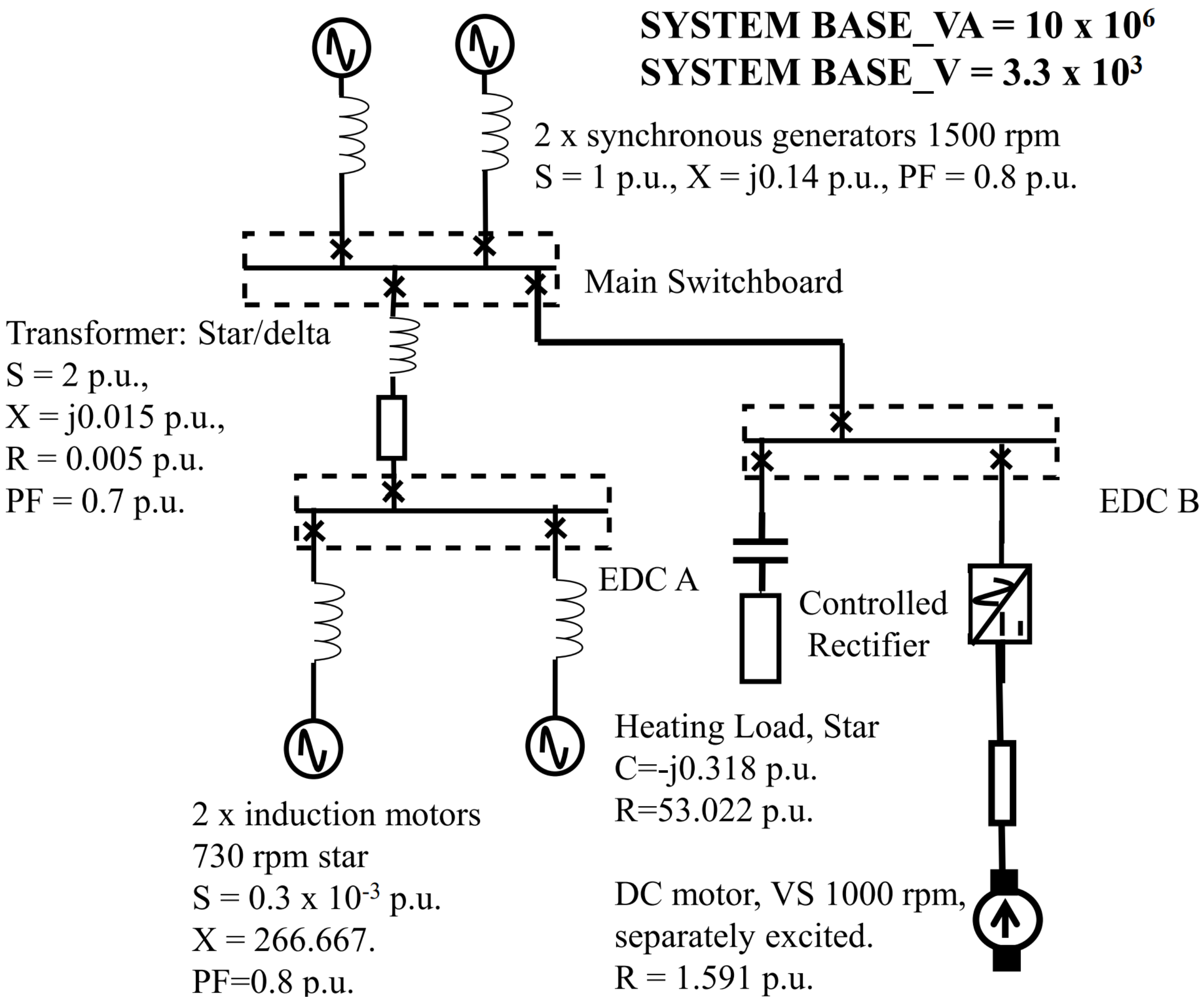


Figure 3.9: Reactance Diagram.

3.2.9 Impedance and reactance diagrams

Converting the electrical line diagram to an impedance diagram or reactance diagram is essential for:

- Potential difference (voltage drop) calculations
- Current flows in cables/lines
- Calculations of losses
- Power flows around an electrical system
- Understand transient effects
- Calculate fault level and fault currents
- Waveform distortion and its penetration
- Impacts when adding new equipment to the network

3.3 Summary

The per unit system allows powers, voltages, currents and impedances to be expressed relative to each other. This allows the designer to understand the relationships between different parts of the circuit.

Using the per-unit transformer model eliminates the need to scale quantities by the transformer turns ratio, thus eliminating a common source for error in electrical calculations.

Chapter 4

Using the Impedance Diagram

- Using impedance diagrams for load flows
- Using impedance diagrams for fault calculations

  By the end of this synchronous session you should be comfortable with how impedance diagrams can be used to calculate load flows and perform fault calculations in an electrical power system

4.1 Load Flow Calculation

4.1.1 Load flow

- In an electrical power system currents flow from generators to loads via a transmission/distribution system therby permitting 'load (power) flows'
- If a system is at steady-state then currents and power flows would be stable
- If there is a change in the system e.g. suddenly and additional load is connected, then there will be a change to currents and load flow
- An electrical system cannot change instantaneously from one state to another. The generators for example cannot instantaneously change the supply of power at the point load changes. There will be a transient period

4.1.2 Load flow analysis example

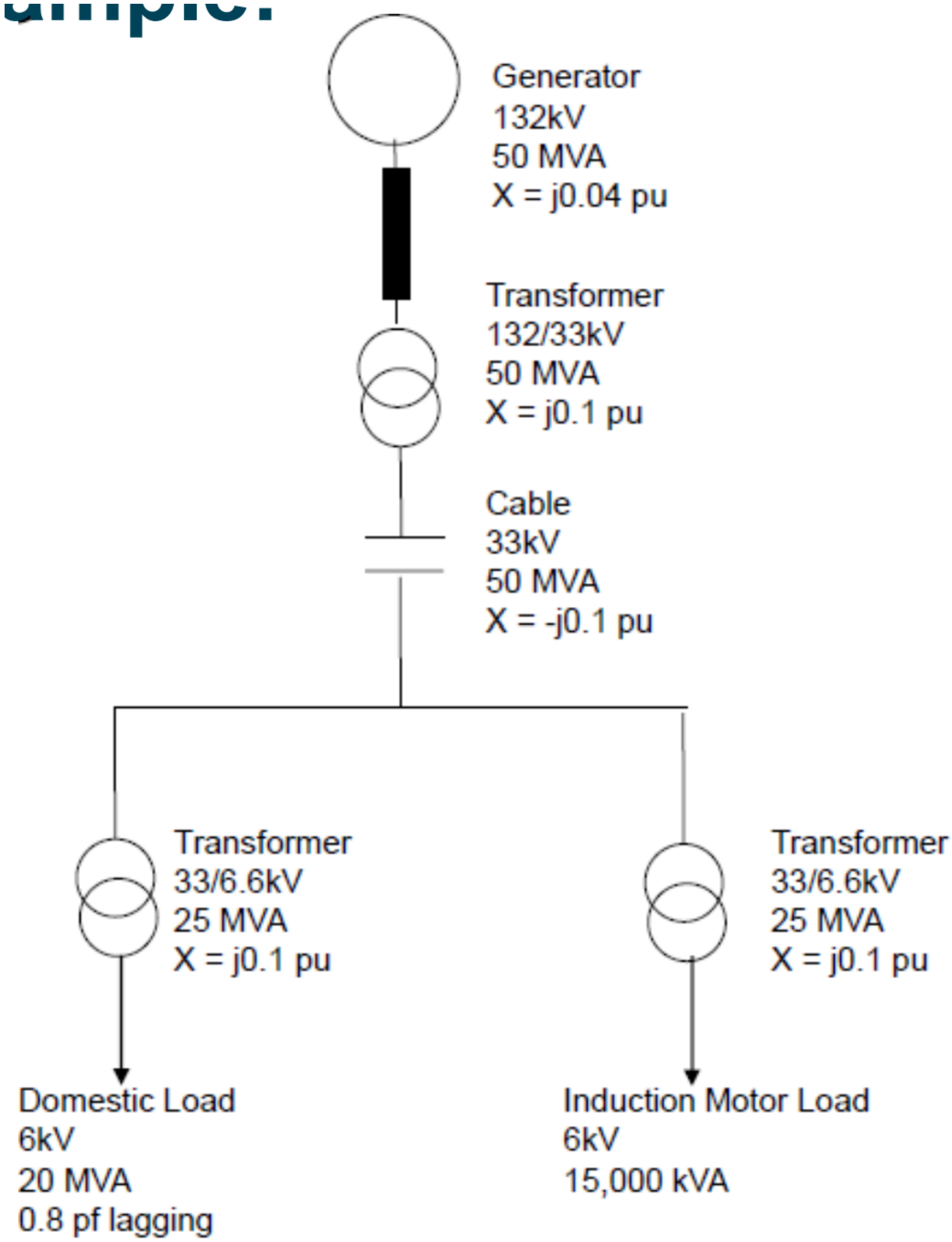


Figure 4.1: Single Line Diagram.

A 132 kV supply feeds two loads; a group of domestic consumers and a group of induction motors which on starting consume five times rated (or design) full load current at zero power factor lagging.

Part a

Convert the single line diagram into an impedance diagram. We will select a base S of 50 MVA and 33 kV as the base V. The values selected can be different but must be stated by the designer.

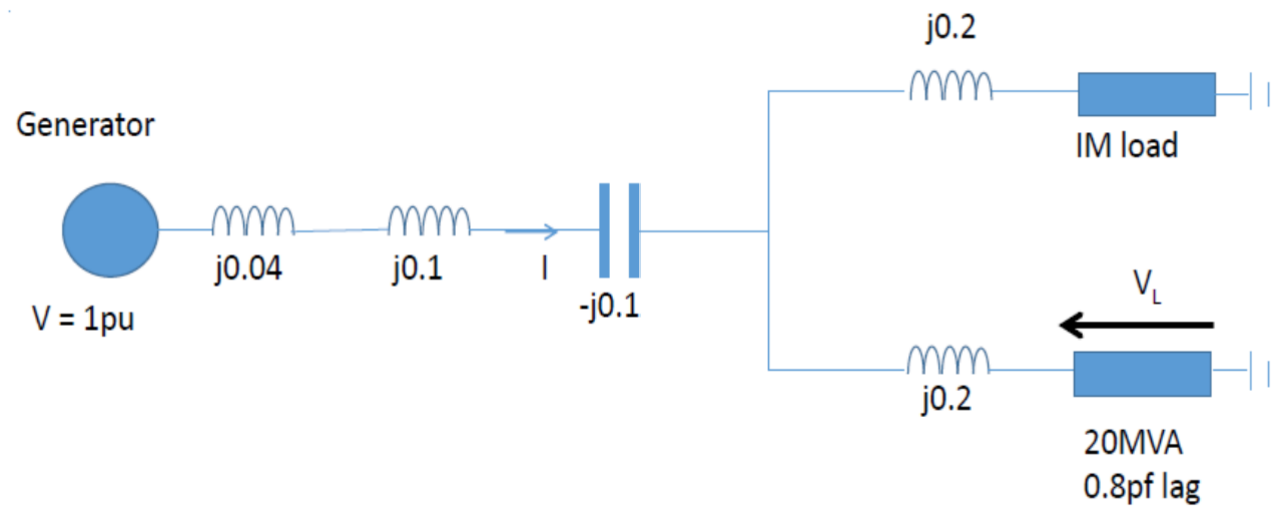


Figure 4.2: Single Line Diagram.

Here is the Impedance Diagram of the Single Line Diagram where all the impedances have been changed into per unit values on a 50 MVA base.

Part b

Calculate the voltage at the domestic busbars prior to induction motor start.

- The induction motor load is open circuit so all current flowing from the generator will flow to the domestic load i.e. steady state
- To determine the voltage at the domestic busbar prior to the induction motor start then the equation for $V_L = 1 - IZ$ can be used, where:
  - V_L is the domestic voltage
  - 1 is the pu voltage at the generator
  - I is the generator current
  - Z is the system impedance between source and load

Calculating the impedance of the circuit then:

$$Z = j\left(0.04 + 0.1 - 0.1 + 0.2\right) = j0.24 \tag{4.1}$$

Calculating the current in the circuit then:

$$I = \frac{20 \times 10^6}{\sqrt{3} \times 6000} = 1925 \text{ A at 0.8 pf lag} \tag{4.2}$$

Now defining base current related to domestic side (although the domestic side is rated at 6 kV, the transformer is rated at 6.6 kV and it is permissible to use this values as it is correct in the SLD and ID), we can say:

$$\text{Base current at } 6.6\,\text{kV} = \frac{50 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 4374\,\text{A} \tag{4.3}$$

$$\text{Domestic current pu } = \frac{1925}{4374} = 0.44 \text{ at 0.8 pf lag} \tag{4.4}$$

$$V_L' = 1 - j0.04\left[0.44\left(0.8 - j0.6\right)\right] - j0.2\left[0.44\left(0.8 - j0.6\right)\right] = 0.94\,\text{pu} \tag{4.5}$$

Part c

Calculate the maximum voltage dip that will occur when all the induction motors are started together at the same moment in time. The induction motor switch is now closed. The demand at this moment is five times normal full-load current. The induction motor load demands a substantial current:

$$\text{Starting current IM} = -j\frac{15000 \times 10^3}{\sqrt{3} \times 6000} \times 5 = -j7217\,\mathrm{A} \tag{4.6}$$

$$\text{Starting current IM} = -j\frac{7217}{4374} = -j1.64\,\mathrm{pu} \tag{4.7}$$

The induction motor load demands a substantial current which flows from the generator. Remember at IM start there is no real power so all power is reactive hence zero power factor. The voltage at the terminals will drop across the series connected devices:

$$V_L' = 1 - j0.04\left[0.44\,(0.8 - j0.6) - j1.64\right] - j0.2\left[0.44\,(0.8 - j0.6)\right] \tag{4.8}$$

$$= 0.871 - j0.084 = 0.875\,\mathrm{pu} \tag{4.9}$$

Hence the voltage dips from $0.94\,\mathrm{pu}$ to $0.87\,\mathrm{pu}$ or alternatively from $6.204\,\mathrm{kV}$ to $5.78\,\mathrm{pu}$. The voltage dip would be noticed temporarily as a light flicker or dimming. In practice, the generator would recover after a few seconds - transient response of the generator.

4.1.3 Some thoughts

- Understand the initial conditions first and then calculate the impact of load changes
- The line series capacitor installed has partially neutralised the network inductance. Without this capacitance the dip would be much more severe
- Voltage flicker often occurs when there is a sudden demand for large power is demanded e.g. starting of large induction motors on ships or in grids e.g. near steel rolling mills or factories

4.2 Using impedance diagrams in short-circuit balanced faults

4.2.1 Fault classification

Faults may be classified as being:

- Open circuit faults
- Short circuit faults

Faults may occur in high voltage and low voltage systems meaning:

- Three-ohase system faults
- Single-phase system faults
- DC system faults

For short circuit fault types then the engineer needs to appreciate its significance and protect against such events. Faults have two main characteristics: MVA fault level (MVA) and fault current (I_{fault}).

4.2.2 Types of faults

Symmetrical fault (Fault currents are balanced in each phase)

- Three-phase short circuit
- Three-phase to ground fault
- (Three-phase open circuit)

Unsymmetrical fault (fault currents are **not** balanced in each phase)

- Single-phase to earth
- Double-phase to earth
- Two-phases short together
- Single-phase open circuit
- Double-phase open circuit

4.2.3 Faults normally are due to:

- Wearing of insulation
- Aging
- Poor connections
- Fault due to lightning
- Tree limbs falling on the line
- Wind, weather impacts
- Impact/shock damage
- Vandalism
- Poor safety protocols or work on live equipment

4.2.4 MVA method

- The MVA method is used to define the power at the point of a fault
- The accepted method is to calculate the Fault MVA as follows:

$$MVA_{fault} = \frac{\text{Base S (MVA)}}{\text{Impedance to fault (pu)}} \tag{4.10}$$

- Having calculated the MVA_{fault} the fault current can be calculated using the nominal voltage at the fault

$$I_{fault} = \frac{\text{MVA}_{fault}}{\sqrt{3} times V_{base}} \tag{4.11}$$

4.2.5 Balanced three-phase fault

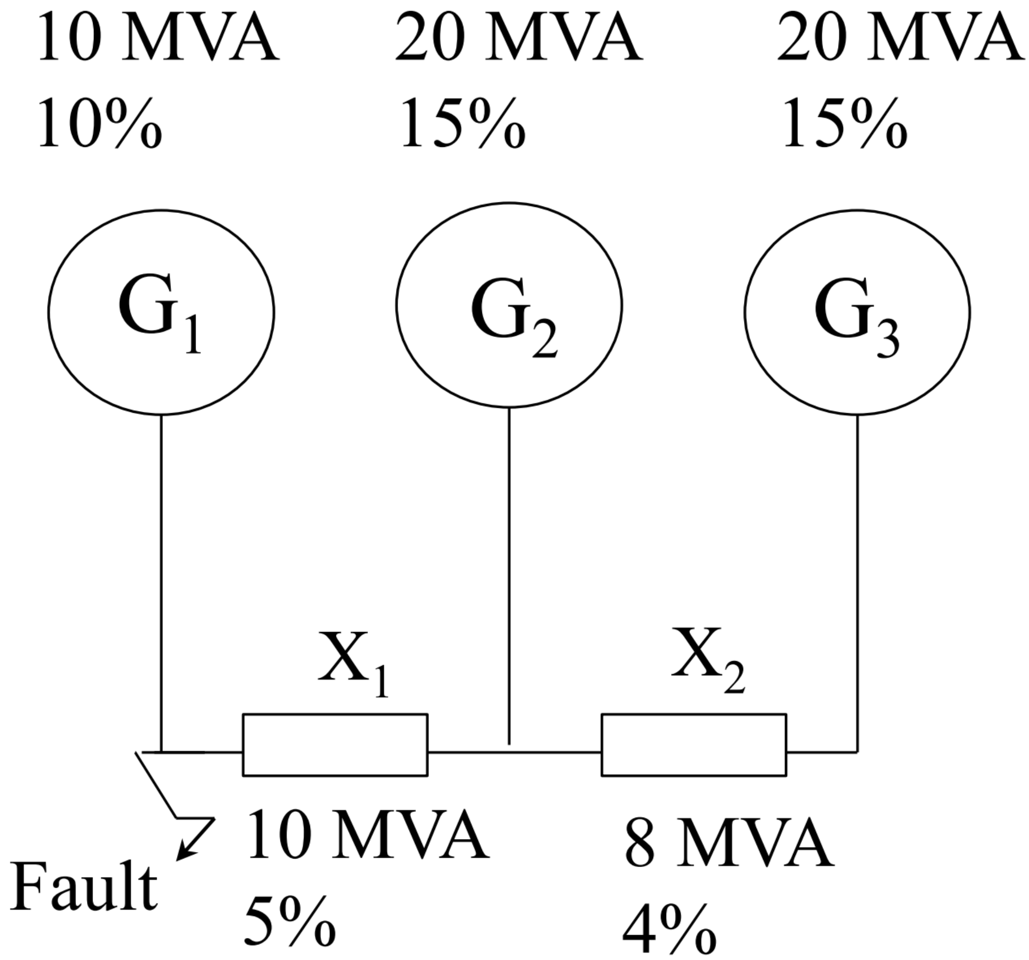


Figure 4.3: Balanced three-phase fault.

An interconnected generator-reactor system is active and suddenly incurs a balanced three-phase short circuit at the Fault indicated. Using a $50\,\mathrm{MVA}$ base then draw an impedance diagram and hence determine the Fault Level and Fault Current. It is an $11\,\mathrm{kV}$ three phase system.

4.2.6 Solution

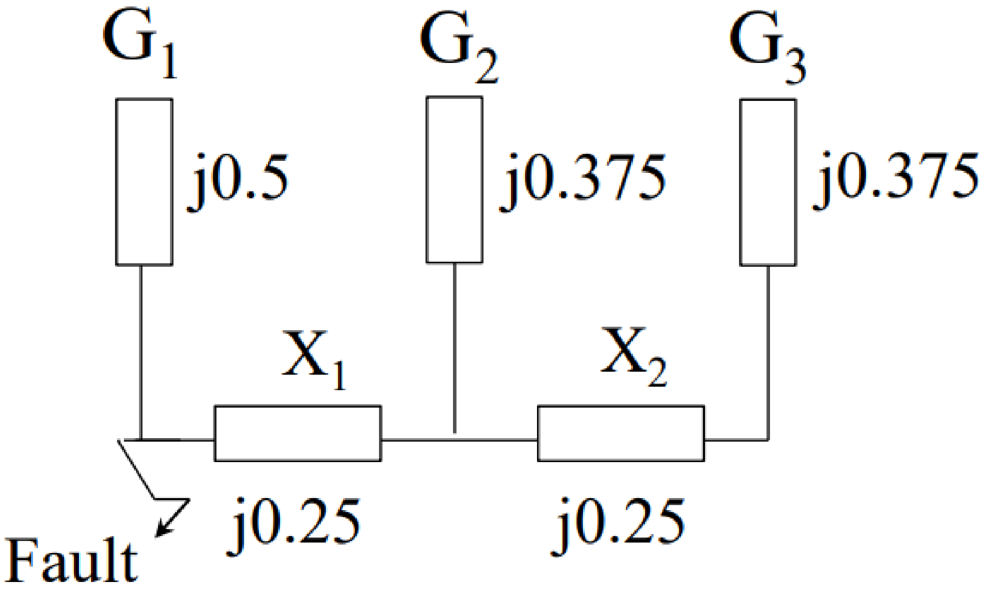


Figure 4.4: Impedance diagram.

$$X_{G1} = \frac{50}{10} \cdot 0.1 = 0.5\,\mathrm{pu} \tag{4.12}$$
$$X_{G2} = \frac{50}{20} \cdot 0.15 = 0.375\,\mathrm{pu} \tag{4.13}$$
$$X_{G3} = \frac{50}{20} \cdot 0.15 = 0.375\,\mathrm{pu} \tag{4.14}$$
$$X_1 = \frac{50}{10} \cdot 0.05 = 0.25\,\mathrm{pu} \tag{4.15}$$
$$X_1 = \frac{50}{8} \cdot 0.04 = 0.25\,\mathrm{pu} \tag{4.16}$$

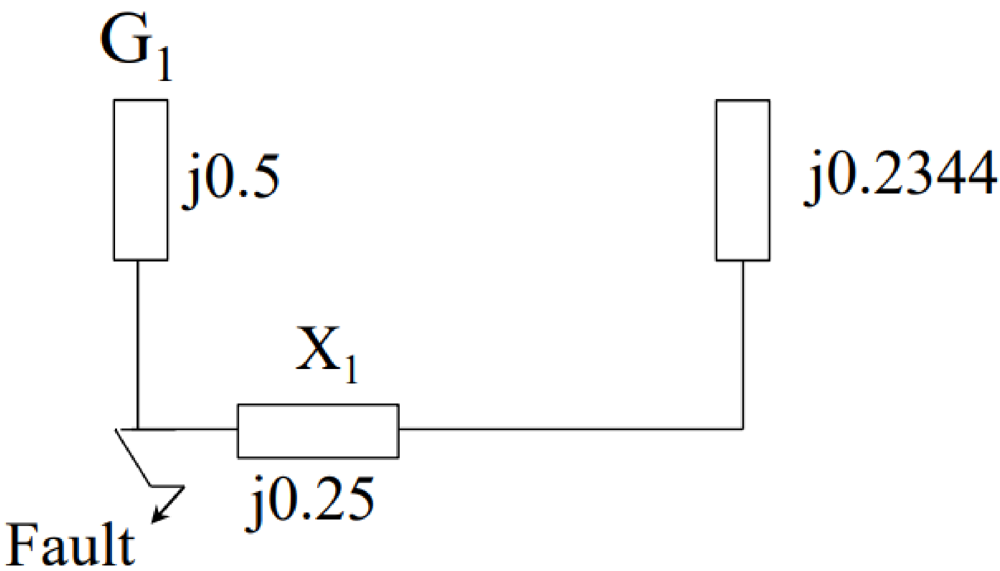


Figure 4.5: Impedance diagram circuit reduced.

$$\text{Per unit reactance} = \frac{0.5\,(0.2344 + 0.25)}{0.5 + (0.2344 + 0.25)} = j0.246 \tag{4.17}$$

$$\text{MVA Fault Level} = \frac{50 \times 10^3}{0.246} = 203.25\,\text{MVA} \tag{4.18}$$

$$\text{Fault current} = \frac{203.25 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 10\,668\,\text{A} \tag{4.19}$$

The MVA Fault Level provides information on the ‘power at the fault’. The Fault Current provides information on protection e.g. circuit breakers. This is known as the symmetrical fault current.

4.2.7 Importance of MVA

- The short circuit capacity (SCC) at the busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC.
- An infinitely strong bus (or infinite bus bar) has an infinite SCC, with a zero equivalent impedance and will maintain its voltage under all conditions
- Magnitude of short circuit current is time dependent due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax circuit breakers (CB) adversely so that current limiting reactors are sometimes used

4.2.8 Power system symmetrical faults

- In a power system, knowing the maximum MVA Fault Level and the Fault Current that could potentially flow into a zero impedance fault is necessary in order to rate switch gear correctly
- The MVA Fault Level defines the maximum MVA that is experience when a symmetrical fault event occurs. The fault level is usually expressed in MVA (or a corresponding per-unit value)
- The maximum fault current can be calculated using the MVA Fault Level and the nominal Voltage Rating at the fault location

4.2.9 Conclusions

- The analysis show in this session has explained how impedance diagrams can be used for system analysis for ‘load flows’ and ‘balanced faults’
- For larger or complex circuits then many more calculations are needed meaning computers are generally used to calculate load flows and faults
- Various computer programmes are available including MATLAB, Simulink Simpower Systems, PSCAD, ERACS, etc

Chapter 5

Faulted Networks

- Introducing the concept of unbalanced networks
- Using impedance diagrams for fault calculations

5.0.1 Symmetrical faults recap

In a power system the most significant fault that can occur is when all three-phases short together. This is a symmetrical or balanced fault. The MVA Fault Level defines the maximum MVA that the system is subjected to when a symmetrical fault event occurs. The fault level is usually expressed in MVA (or a corresponding per-unit value). The maximum fault current can be calculated using the MVA Fault Level and the nominal Voltage Rating at the fault location.

5.1 Unbalanced faults

5.1.1 Types of 'unbalanced faults'

Unsymmetrical faults - currents and voltages are not balanced in each phase:

- Single line to ground
- Line to line
- Double line to ground
- Single phase open circuit
- Dounle phase open circuit

For each short-circuit, the fault can be bolted (a zero impedance fault) or have a fault impedance known as Zf.

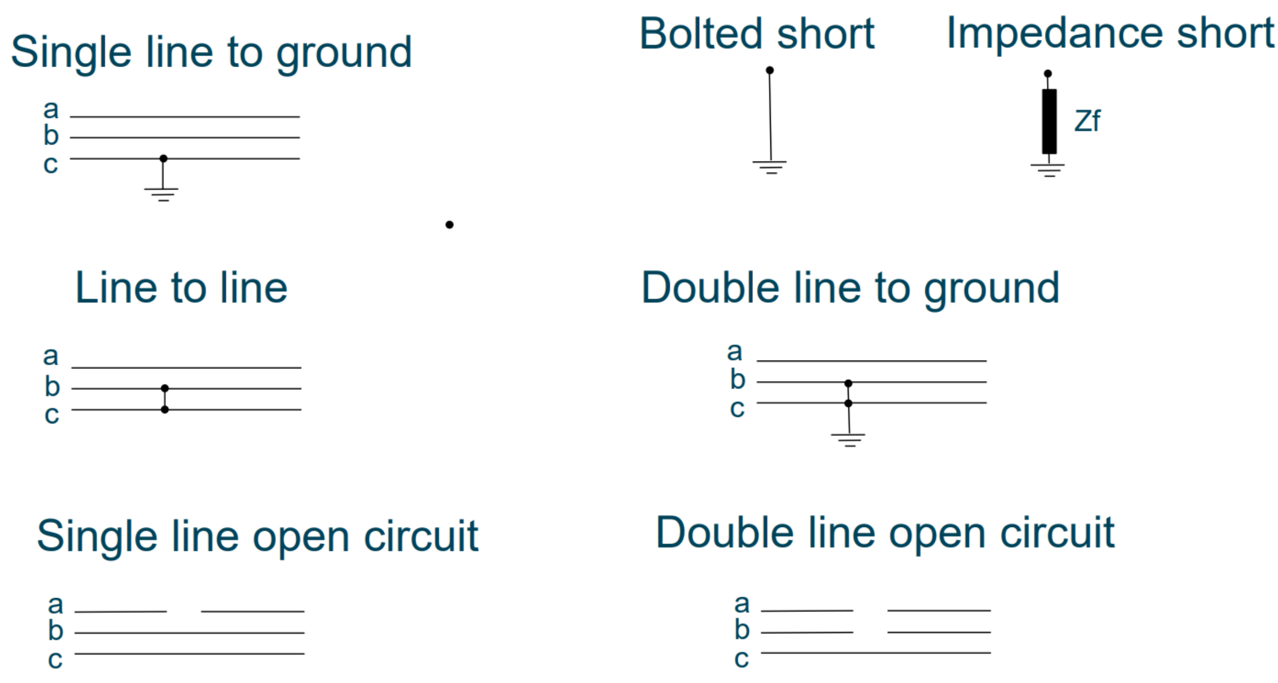


Figure 5.1: Unsymmetrical/unbalanced faults.

5.1.2 List of possible faults

- Three phase symmetrical fault L-L-L
- Three phase symmetrical fault L-L-L-G
- Line to line fault
- Double line to ground fault
- Single line to ground fault
- Single line open circuit
- Double line open circuit

The most common fault is the single line to ground fault. The worst fault is a three-phase to ground fault (L-L-L-G or L-L-L).

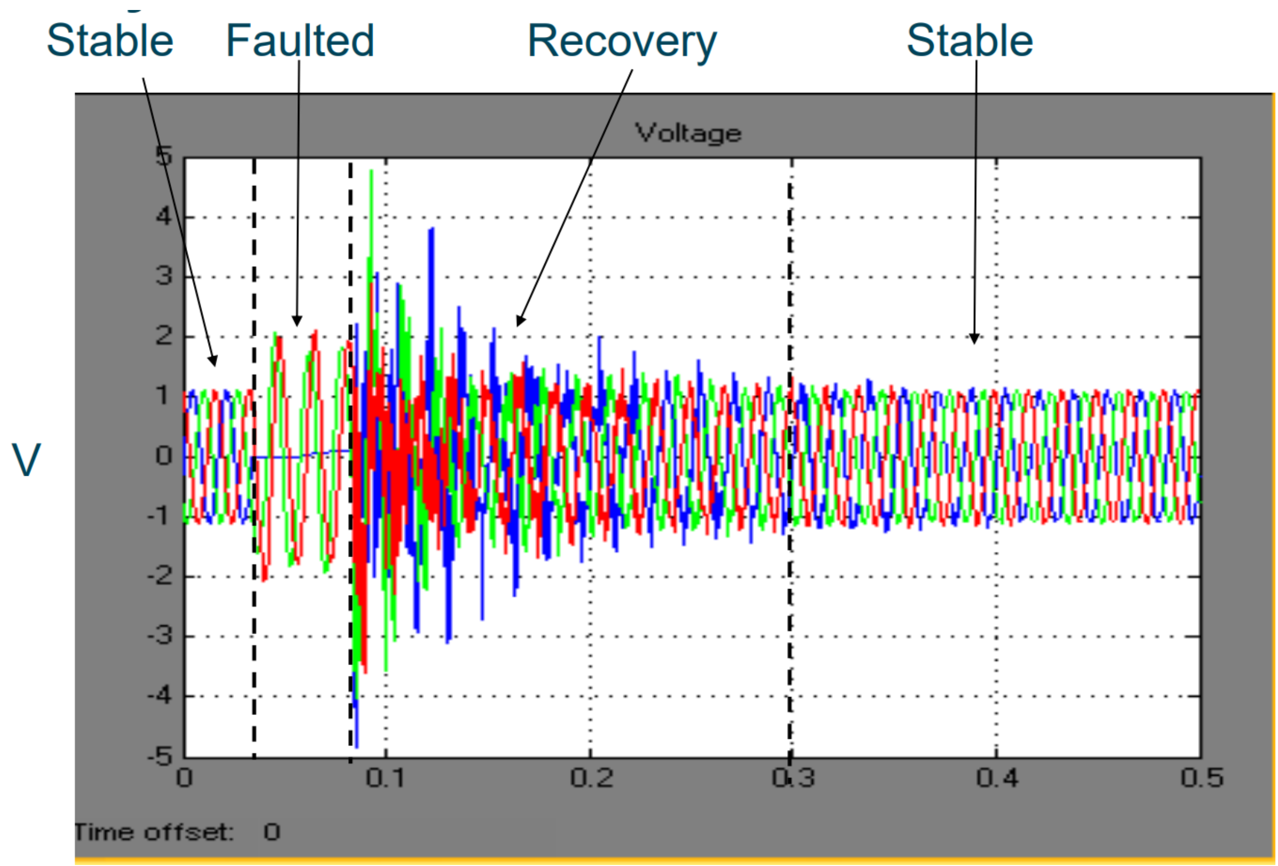


Figure 5.2: Unsymmetrical/unbalanced fault graph.

We see the blue phase go to ground (0V) and the other two phases increase in voltage and are no longer 120° out of phase with each other.

5.1.3 Method of analysis

Each phase is experiencing something different i.e. what is happening on one phase is not what is happening on the other. RMS voltages and currents are unbalanced.

$$V_a \neq V_b \neq V_c \text{ nor } I_a \neq I_b \neq I_c \tag{5.1}$$

The presumption that we used for symmetrical faults (the same equivalent circuit for each phase) is not valid in the unsymmetrical/unbalanced case. For the unbalanced case it is necessary to use a different method. We use 'Fortescues's Theorem'.

5.1.4 Fortescue's Theorem

Fortescue's Theorem says:

> Three unbalanced phasors in a multi-phase electrical system can be resolved into a set of balanced phasors consisting of:

- Positive-sequence components
- Negative-sequence components
- Zero sequence components

$$V_{line} = V_{positive} + V_{negative} + V_{zero} \tag{5.2}$$
$$I_{line} = I_{positive} + I_{negative} + I_{zero} \tag{5.3}$$

5.1.5 Positive sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Displaced from each other by $120°$
- Have phase sequence a-b-c
- Usually referred to as V_{a1}, V_{b1}, V_{c1}

5.1.6 Negative sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Displaced from each other by $120°$
- Have phase sequence a-c-b
- Usually referred to as V_{a2}, V_{b2}, V_{c2}

5.1.7 Zero sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Zero phase displacement
- No phase sequence
- Usually referred to as V_{a0}, V_{b0}, V_{c0}

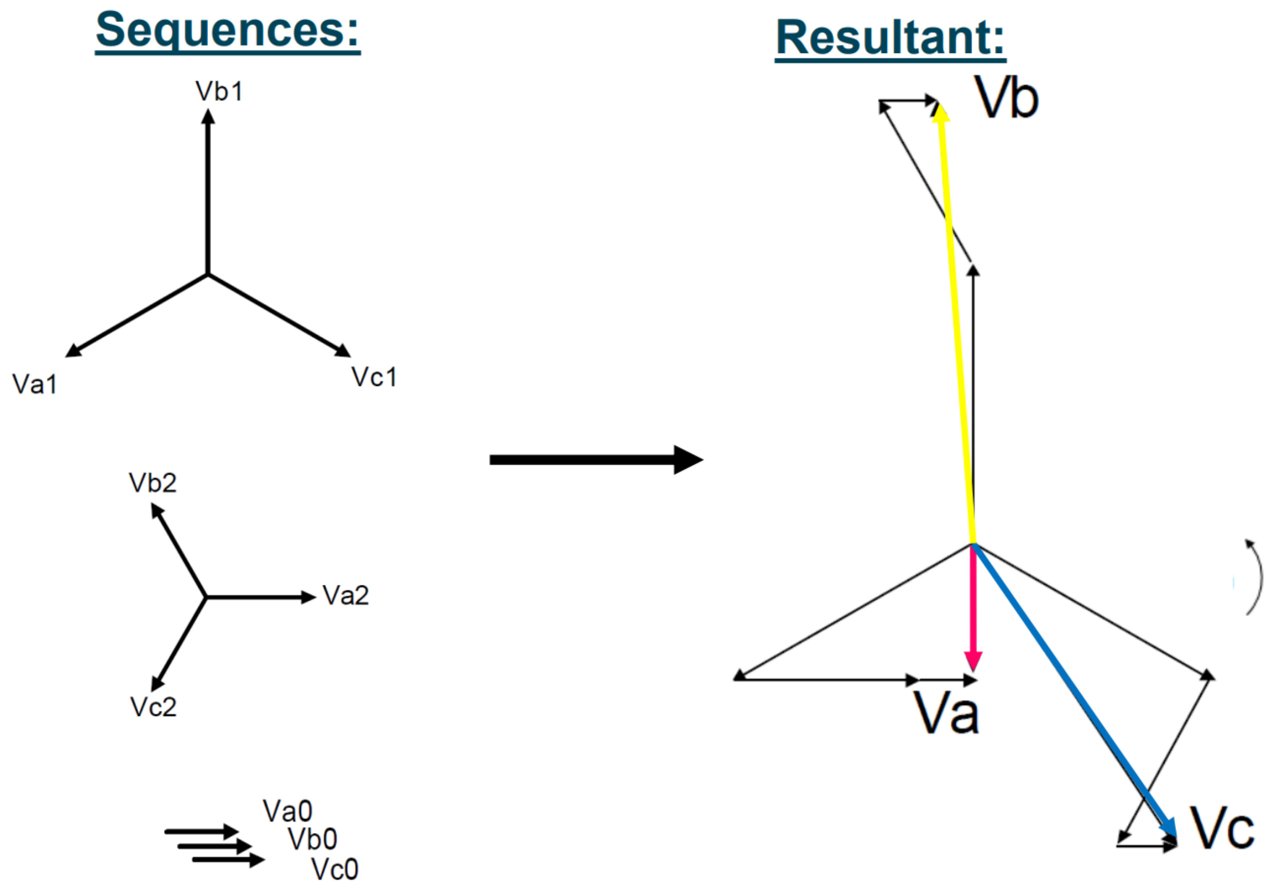


Figure 5.3: Sequence components and phase relationship.

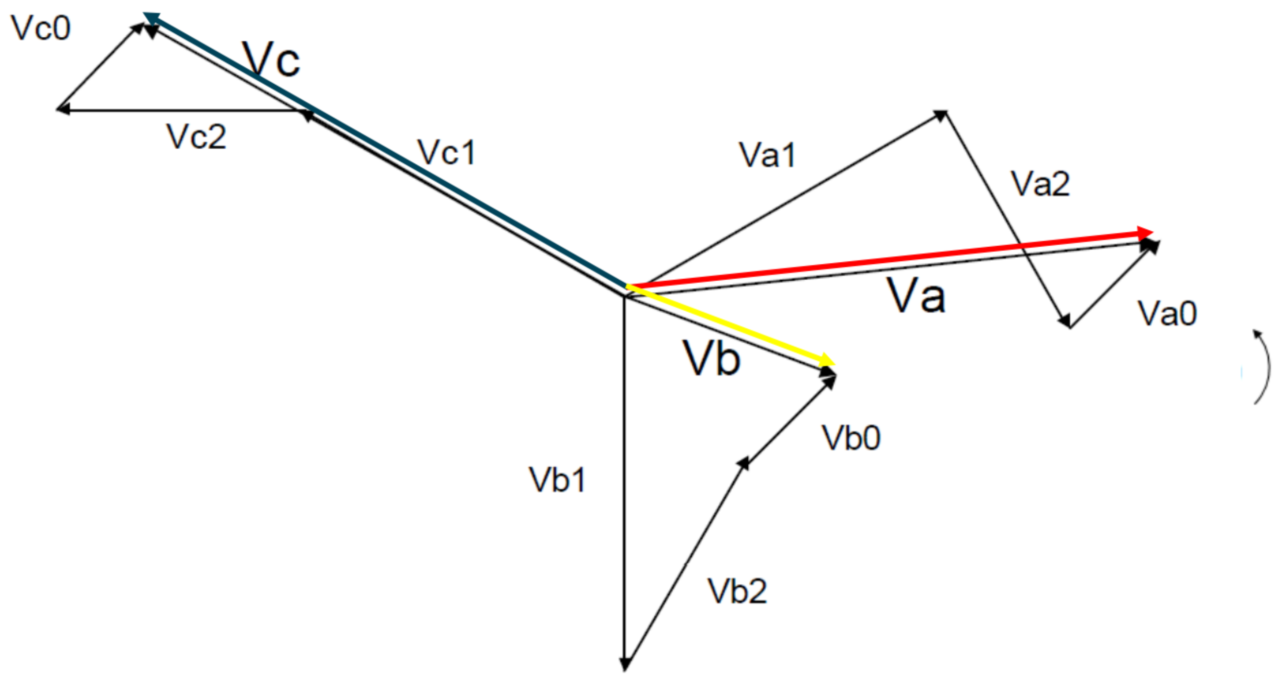


Figure 5.4: Sequence components 2.

5.1.8 Summing sequence components

Original phasors are the sum of their components

$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{5.4}$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \tag{5.5}$$
$$V_c = V_{c0} + V_{c1} + V_{c2} \tag{5.6}$$

Hence:

$$\text{Line} = \sum \text{sequence components} \tag{5.7}$$

In balanced/symmetrical networks in multi-phase systems then only positive sequence components are present.

5.1.9 Note about grounding/earthing

How a system is grounded has a major impact on fault current. Zero sequence current can only flow when the start point of the source is tied to ground / earth directly or via an impedance Ze.

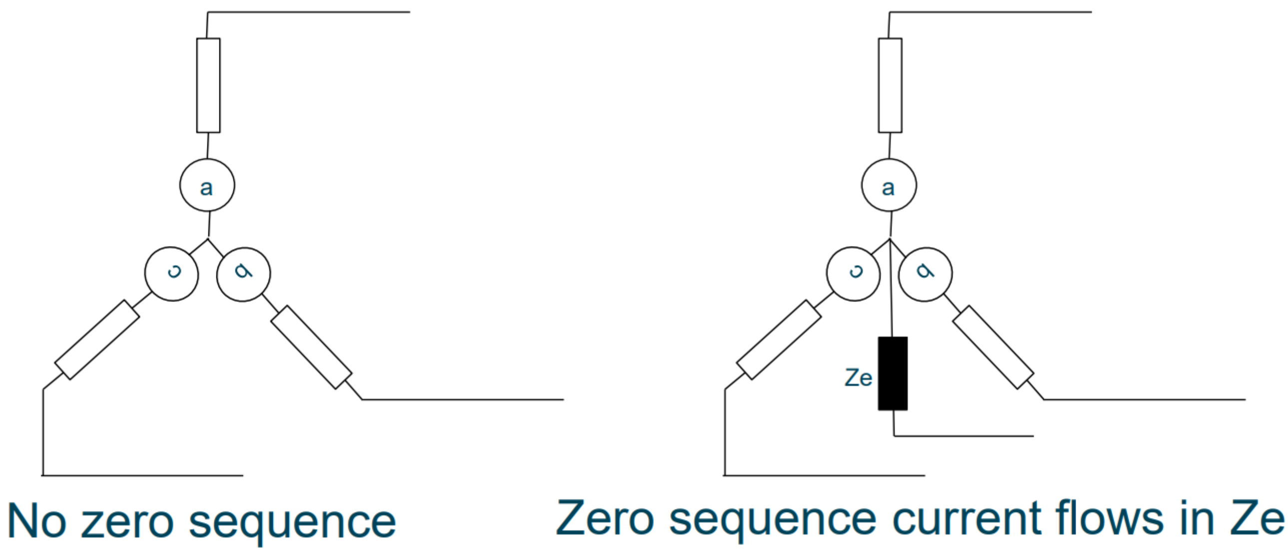


Figure 5.5: Grounding/earthing.

We can see the virtual/floating star point on the sequence on the left. Normally, this is left floating on ship systems for example. The star point can be connected to ground (unusual for generators) or we can add an impedance to the star point connection. This is because the star point is not always 0V under a fault condition. Hence, by including an earth impedance, we can limit current flow.

Zero sequence current flows can only happen if we have a connection to ground. In floating star point connections, we cannot have zero sequence current flows.

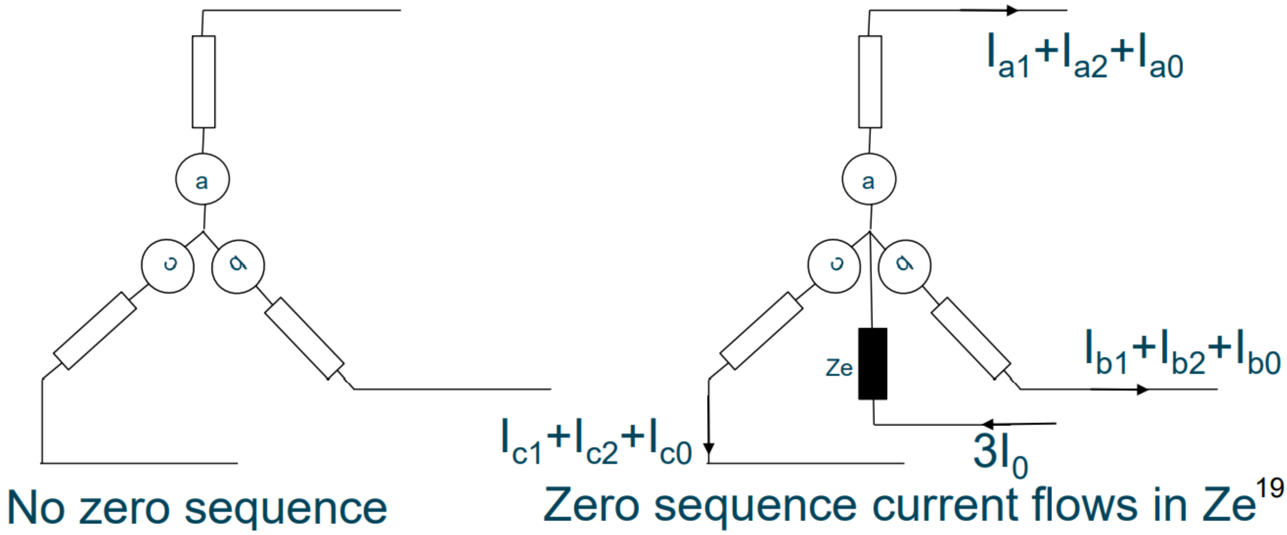


Figure 5.6: Currents during grounded star point.

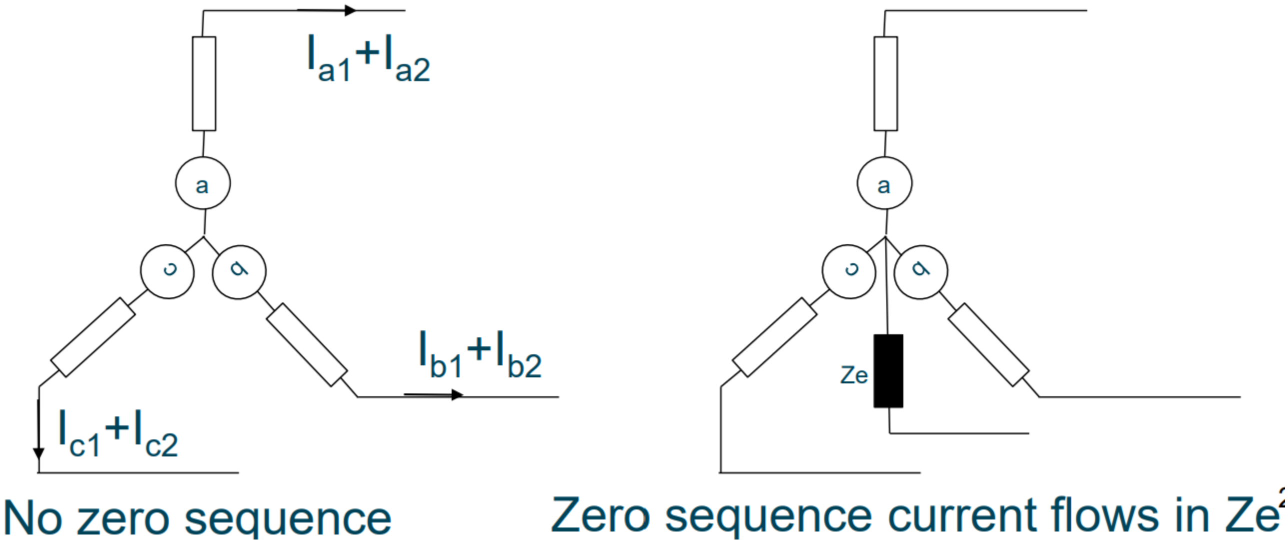


Figure 5.7: Currents during floating star point.

5.1.10 The operator ‘a’

Let us define an operator that rotates a phasor by 120°:

$$a = 1\angle 120^\circ = (-0.5 + j0.8666) \tag{5.8}$$

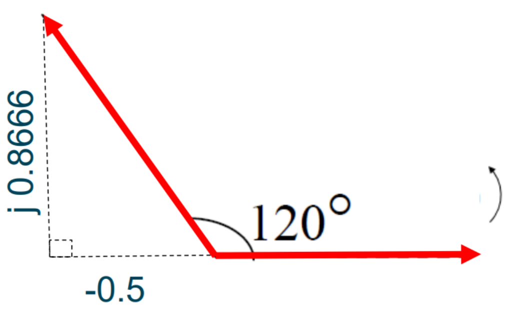


Figure 5.8: ‘a’ operator.

5.1.11 Expressing phasors a^2 and a^3

$$a^2 = a \times a = (1\angle 240°) = 1\angle{-120°} \tag{5.9}$$

Similarly:

$$a^3 = (1\angle 360°) = 1\angle 0° \tag{5.10}$$

Therefore:

$$a + a^2 + a^3 = 0 \tag{5.11}$$
$$1 + a + a^2 = 0 \tag{5.12}$$

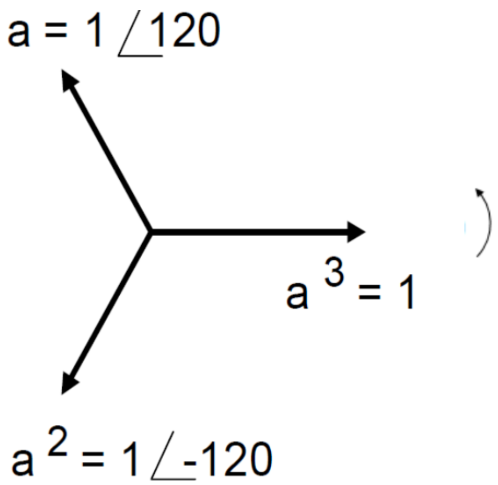


Figure 5.9: ‘a’ phasors.

The value of the star point changes with fault conditions.

5.1.12 Representation using ‘a’

Using the ‘a’ operator then the positive sequence components can be written:

$$V_{a1} = 1 \tag{5.13}$$
$$V_{b1} = (1\angle - 120°) = a^2 V_{a1} \tag{5.14}$$
$$V_{c1} = (1\angle 120°) = aV_{a1} \tag{5.15}$$

In other words we have used the ‘a’ operator to express V_{b1} and V_{c1} in terms of V_{a1}. Similarly for the negative sequence, we have:

$$V_{a2} = 1 \tag{5.16}$$
$$V_{b2} = (1\angle 120°)\, V_{a2} = aV_{a2} \tag{5.17}$$
$$V_{b2} = (1\angle{-120°})\, V_{a2} = a^2 V_{a2} \tag{5.18}$$

In other words we have used the ‘a’ operator to express V_{b2} and V_{c2} in terms of V_{a2}. For the zero sequence:

$$V_{a0} = V_{b0} = V_{c0} \tag{5.19}$$

No need for the operator ‘a’ here as all zero sequence components are in phase!

Function	*Polar*	*Rectangular*
a	$1\angle 120^\circ$	$-0.5 + j0.866$
a^2	$1\angle 240^\circ$	$-0.5 - j0.866$
a^3	$1\angle 0^\circ$	$1.0 + j0$
a^4	$1\angle 120^\circ$	$-0.5 + j0.866$
$1 + a = -a^2$	$1\angle 60^\circ$	$0.5 + j0.866$
$1 + a^2 = -a$	$1\angle -60^\circ$	$0.5 - j0.866$
$1 - a$	$\sqrt{3}\angle -30^\circ$	$1.5 - j0.866$
$1 - a^2$	$\sqrt{3}\angle 30^\circ$	$1.5 + j0.866$
$a - 1$	$\sqrt{3}\angle 150^\circ$	$-1.5 + j0.866$
$a^2 - 1$	$\sqrt{3}\angle -150^\circ$	$-1.5 - j0.866$
$a - a^2$	$\sqrt{3}\angle 90^\circ$	$0.0 + j1.732$
$a^2 - a$	$\sqrt{3}\angle -90^\circ$	$0.0 - j1.732$
$a + a^2$	$1\angle 180^\circ$	$-1.0 + j0$
$1 + a + a^2$	0	0

Figure 5.10: List of ‘a’ phasors.

5.1.13 Representing all sequence components in terms of V_a sequence components

$$\text{Line} = \sum \text{sequence components} \tag{5.20}$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = V_{a0} + V_{a1} + V_{a2} \tag{5.21}$$

$$V_b = V_{b0} + V_{b1} + V_{b2} = V_{a0} + a^2 V_{a1} + aV_{a2} \tag{5.22}$$

$$V_c = V_{c0} + V_{c1} + V_{c2} = V_{a0} + aV_{a1} + a^2 V_{a2} \tag{5.23}$$

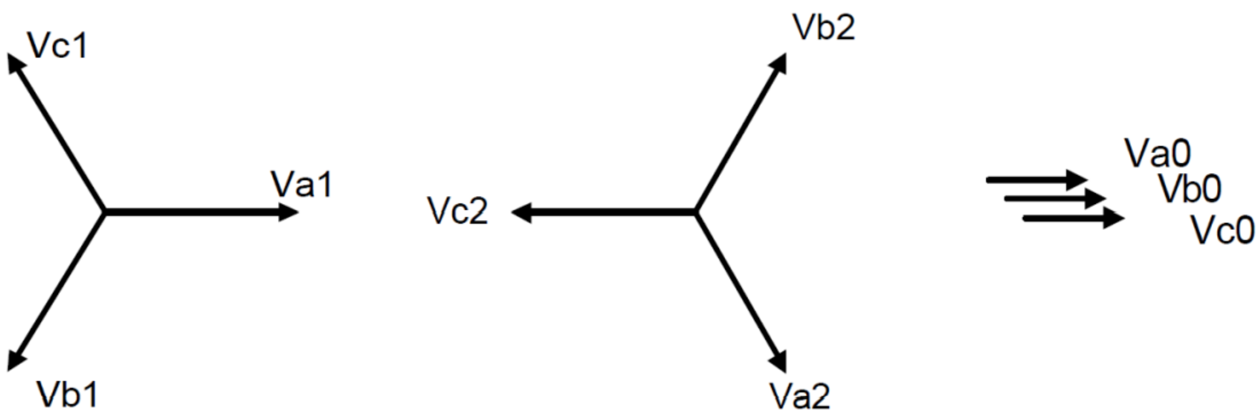


Figure 5.11: Phase voltages expressed in terms of V_a.

5.1.14 'a' matrix

$$\text{Line} = \sum \text{sequence components} \tag{5.24}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{5.25}$$

5.1.15 Inverse 'a' matrix

The sequences may be described by the 'inverse a matrix' and phasors:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ b_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{5.26}$$

Where:

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{5.27}$$

5.1.16 Example

A three-phase star connected load is connected across a three-phase balanced supply system. Obtain a set of equations relating the symmetrical components of a line and its phase voltages. Assuming:

$$V_{ab} = V_a - V_b \tag{5.28}$$

We will do this for one line voltage...

Zero sequence. Since:

$$V_{ab} + V_{bc} + V_{ca} = 0 \tag{5.29}$$

then

$$V_{ab0} + V_{bc0} + V_{ca0} = 0 \tag{5.30}$$

In other words there is no change in the zero sequence relationships. Assume balance

Positive sequence: Choosing V_{ab} then:

$$V_{ab1} = \frac{1}{3}\left(V_{ab} + aV_{bc} + a^2 V_{ca}\right) \text{ from inverse 'a' matrix} \tag{5.31}$$

$$= \frac{1}{3}\left[(V_a - V_b) + a\,(V_b - V_c) + a^2\,(V_c - V_a)\right] \tag{5.32}$$

$$\ldots \tag{5.33}$$

$$= \frac{1}{3}\left[\left(1 - a^2\right)\left(V_a + aV_b + a^2 V_c\right)\right] \tag{5.34}$$

$$= \left(1 - a^2\right) V_{a1} \text{ from table} \tag{5.35}$$

$$= \sqrt{3} V_{a1} e^{j30} \text{ using exp form} \tag{5.36}$$

Negative sequence:

$$
\begin{aligned}
V_{ab2} &= \frac{1}{3}\left(V_{ab} + a^2 V_{bc} + a V_{ca}\right) \text{ from inverse `a' matrix} && (5.37)\\
&= \frac{1}{3}\left[\left(V_a - V_b\right) + a^2\left(V_b - V_c\right) + a\left(V_c - V_a\right)\right] && (5.38)\\
\ldots & && (5.39)\\
&= \frac{1}{3}\left[\left(1-a\right)\left(V_a + a^2 V_b + a V_c\right)\right] && (5.40)\\
&= \left(1-a\right) V_{a2} \text{ from table} && (5.41)\\
&= \sqrt{3} V_{a2} e^{-j30} \text{ using exp form} && (5.42)
\end{aligned}
$$

5.1.17 Sequence components and faults

- This lecture started by considering unsymmetrical faults
- The lecture has introduced the method of sequence components and has provided a method analysis of unsymmetrical faults based on Fortescue's theorem
- Manipulation of the voltages and currents using the 'a' matrix is an important step since this provides the analytical means to analyse unsymmetrical faults from sequence, phase and line perspectives
- In the next lecture we will look at unsymmetrical faults by applying this methodology

5.1.18 Conclusions

- The analysis shown in this session has explained the system analysis methods for 'unbalanced faults'
- The introuction to the 'a' matrix which will be used for relationships between phase and line values and also introuced sequence components
- Appreciate the need for positive, negative and zero sequence impedances of different components that make up a power system

Chapter 6

Full fault analysis

6.1 Unbalanced impedance

6.1.1 Impedance and sequence components

We have established that in a three-phase unbalanced network there are line and phase voltages and currents that deviate in their relationships from the balanced case. Furthermore, any unbalance can be described as a set of sequence components consisting positive, negative and zero sequence phasors. Now considering impedances in unbalanced networks then we need to ensure that we understand:

- How to change between star and delta arrangements
- Appreciate how sequence impedance is calculated

6.1.2 Unbalanced star and delta equivalence

$Z_{delta} = 3 \cdot Z_{star}$ for all three phases when the loads in the three-phase system were balanced. This was helpful when looking at symmetrical faults since we normally convert delta connections into star connections and consider an impedance diagram as representing one phase. When loads are unbalanced then we need to consider each phase independently because they are subjected to different voltages, currents and impedances. Consider the two circuits below. The star and delta equivalence must result in the same line voltages and currents. In other words the impedance between any two impedances must be equivalent.

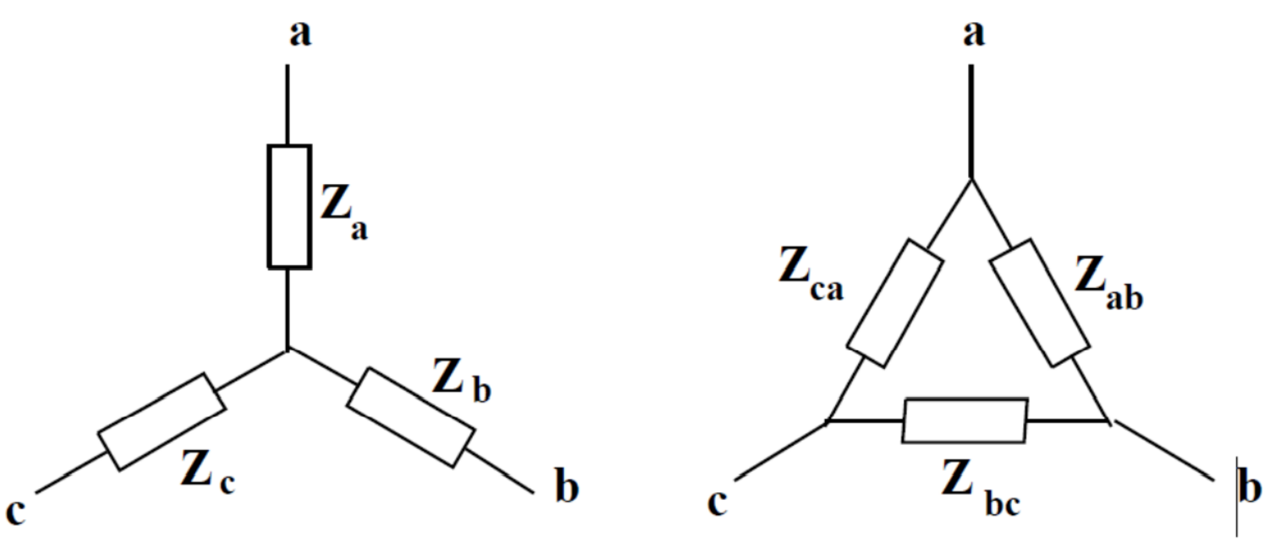


Figure 6.1: Star and delta arrangements.

For example considering phase a and phase b, the impedance equivalence must be:

$$Z_a + Z_b = Z_{ab} // (Z_{ca} + Z_{bc}) \text{ similarly,} \tag{6.1}$$
$$Z_b + Z_c = Z_{bc} // (Z_{ab} + Z_{ca}) \tag{6.2}$$
$$Z_c + Z_a = Z_{ca} // (Z_{bc} + Z_{ab}) \tag{6.3}$$

By manipulation and substitution then it is possible to derive the following relationships:

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c} \tag{6.4}$$
$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a} \tag{6.5}$$
$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b} \tag{6.6}$$

and

$$Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{6.7}$$
$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{6.8}$$
$$Z_b = \frac{Z_{bc} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{6.9}$$

These relationships are needed when considering impedance in unbalanced loads.

6.1.3 Good practice

In many fault calculations it is handy to convert delta impedances into star impedances because:

- It ensures that all balanced arrangements are related to ground or virtual group (floating star point)
- When calculating faults then it is apparent that such calculations are made for one phase and then 'phase shifted' to determine impact on other phases. Having everything as a star arrangement (mathematically and circuit wise) assits in ensuring that right values are obtained.

6.2 Impedance of sequences

There are positive, negative and zero phase sequence components: In voltage these are represented by V_0, V_1 and V_2. In current these are represented by I_0, I_1 and I_2.

6.2.1 Sequence components and impedance

Since $V = IZ$, it follows if there are sequence components in both voltage and current then there must be a sequence impedance too:

- $V_1 = I_1 Z_1$ where Z_1 is the positive sequence impedance
- $V_2 = I_2 Z_2$ where Z_2 is the negative sequence impedance
- $V_0 = I_0 Z_0$ where Z_0 is the zero sequence impedance

6.2.2 The importance of sequence impedance

The impedance of a network is important for calculating currents for an applied voltage. Remembering earlier work on balanced networks, we established that the impedance limited the fault current i.e. the further from the source you were the greater the impedance. the lower the fault current. Now considering the sequence components, it is apparent that the sequence impedances Z_0, Z_1, Z_2 will limit sequence currents I_0, I_1, I_2 for the applied sequence voltages V_0, V_1, V_2.

6.2.3 Network elements

Different network equipment exhibit different sequence impedances:

- Typically, transmission lines and cables have one impedance value for positive and negative sequence, but an entirely different impedance value for zero sequence
- Typically, rotating machines e.g. generators and mots have different impedance values for all three sequences
- Typically, transformers positive, negative and zero sequence components depend upon connection by positive and negative are often the same value

Appreciating these different impedances is important for accurate calculation of unsymmetrical faults.

6.2.4 Transmission lines and distribution cables

Power cables and transmission linse are used to carry power from the source to the load. Typically (over short distances) they can be represented as resistance and inductance. The inductance is comprised of its own self-inductance and mutual inductance between each line or cable.

6.2.5 Transmission line analysis

In a three-phase system interconnected between a three-phase generator and three-phase load the lines/cables usually run close to each so there is always mutual inductance and self-inductance of the lines.

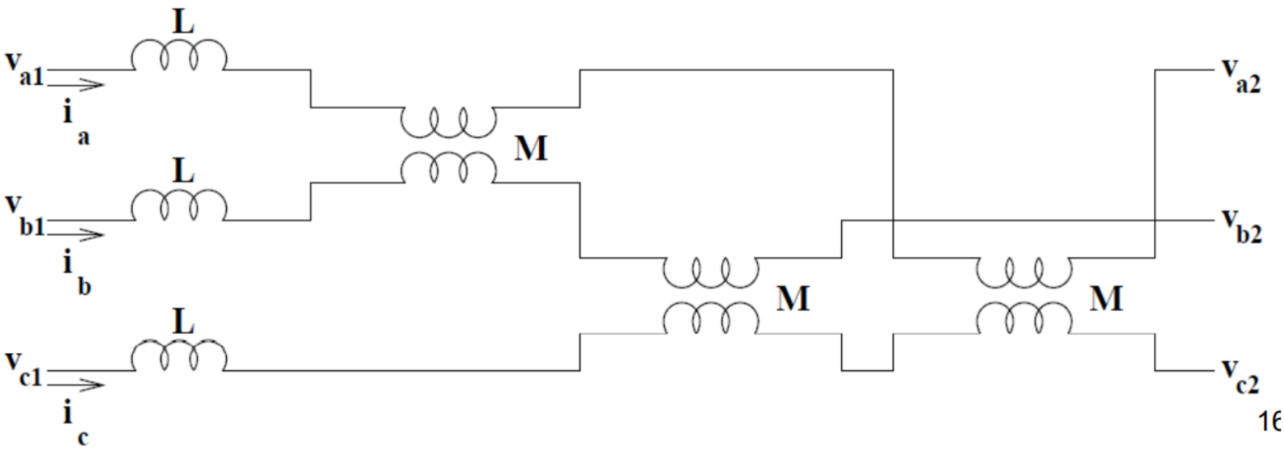


Figure 6.2: Trasmission line mutual inductance and self-inductance.

6.2.6 Transmission line representation

Hence, we can write the relationship ($V = XI$):

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{6.10}$$

It is reasonable to say that the line and mutual inductances are the same for each transmission line or cable under steady-state balanced conditions. This is not necessarily the case for transient or unbalanced case.

6.2.7 Transmission sequence representation

Bringing in the relationship between phase and sequence components we have (ignoring 1/3):

$$I_{sequence} = [A]^{-1} \cdot I_{phase} \tag{6.11}$$

$$V_{sequence} = [A]^{-1} \cdot V_{phase} \tag{6.12}$$

Hence:

$$\begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{6.13}$$

$$[A] \left[V_{sequence}\right] = j\omega \, [LM] \cdot [A] \left[I_{sequence}\right] \tag{6.14}$$

6.2.8 Transmission line representation

Hence by transformation we obtain:

$$\left[V_{sequence}\right] = j\omega \, [A] \cdot [LM] \cdot [A]^{-1} \left[I_{sequence}\right] \tag{6.15}$$

The part ($[A] \cdot [LM] \cdot [A]^{-1}$) provides the inductance sequence relationship for the transmission line or distribution cable. Resolving gives:

$$\begin{pmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{pmatrix} \tag{6.16}$$

The relationship between sequence components becomes:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{6.17}$$

The sequence component relationships become:

$$V_1 = j\omega \left(L - M\right) I_1 \tag{6.18}$$

$$V_2 = j\omega \left(L - M\right) I_2 \tag{6.19}$$

$$V_0 = j\omega \left(L - M\right) I_0 \tag{6.20}$$

The positive, negative and zero sequence reactances of the balanced transmission line are then:

$$Z_1 = Z_2 = j\omega \left(L - M\right) \tag{6.21}$$

$$Z_0 = j\omega \left(L + 2M\right) \tag{6.22}$$

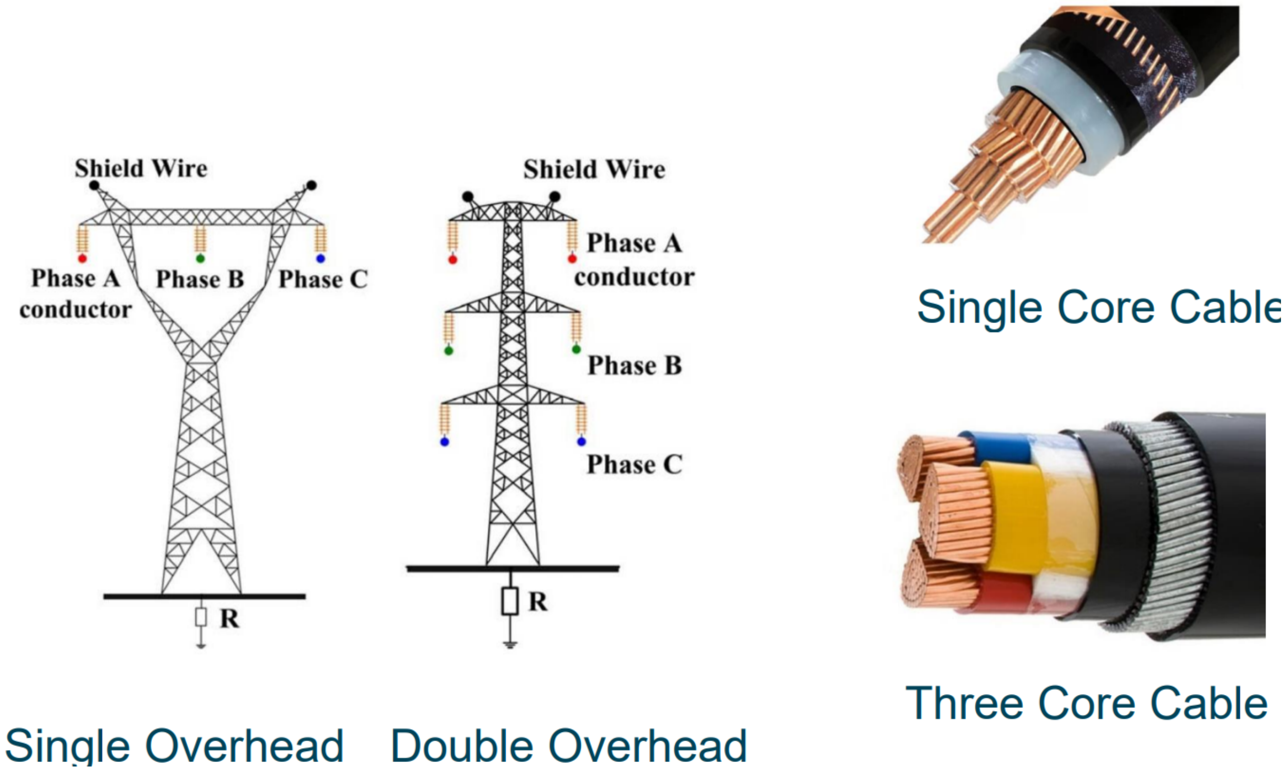


Figure 6.3: Transmission line and cable arrangements.

6.2.9 Lines and cables

The positive and negative sequence impedances are normally balanced i.e. $Z_1 = Z_2$. The zero sequence impedance depends upon the nature of the return path through the earth. Typical relative values of Z_0 during faults are

Overhead:

- For a single-circuit arrangement (Z_0/Z_1) = 3.5
- For a double-circuit arrangement (Z_0/Z_1) = 5.5

Cable arrangements:

- For a single-core arrangement (Z_0/Z_1) = 1.25
- For a three-core arrangement (Z_0/Z_1) = 4

6.2.10 Synchronous machines (generators)

The positive sequence reactance Z_1 is the value used under balanced operation due to positive sequence currents flowing in the windings of the machine in steady-state and transient. The negative sequence reactance Z_2 is due to negative sequence currents which give rise to fluxes in the air gap of the machine that rotate in the opposite direction during unbalance. Z_2 is different to Z_1 in most designs. The zero sequence reactance Z_0 depends upon the nature of the connection of the star point. Zero sequence currents will not flow when the star point is floating but will flow when there is.

Type of machine	+ve sequence	-ve sequence	zero sequence
440 V 50 Hz 1 MVA	0.16 pu	0.11 pu	0.05 pu
11 kV 50 Hz 75 MVA	0.18 pu	0.14 pu	0.07 pu
16 kV 50 Hz 275 MVA	0.21 pu	0.18 pu	0.08 pu
22 kV 50 Hz 575 MVA	0.28 pu	0.21 pu	0.12 pu

Table 6.1: Table to show typical value of sequence impedances for synchronous generators

6.2.11 Neutral connection

The symmetrical components are independent with the voltage-current relationships:

$$V_1 = ZI_1 \tag{6.23}$$
$$V_2 = ZI_2 \tag{6.24}$$
$$V_0 = \left(Z + 3Z_g\right) I_0 \tag{6.25}$$

In many generators that are tied to ground at the star point will have additional impedance separately added to reduce the level of zero sequence currents.

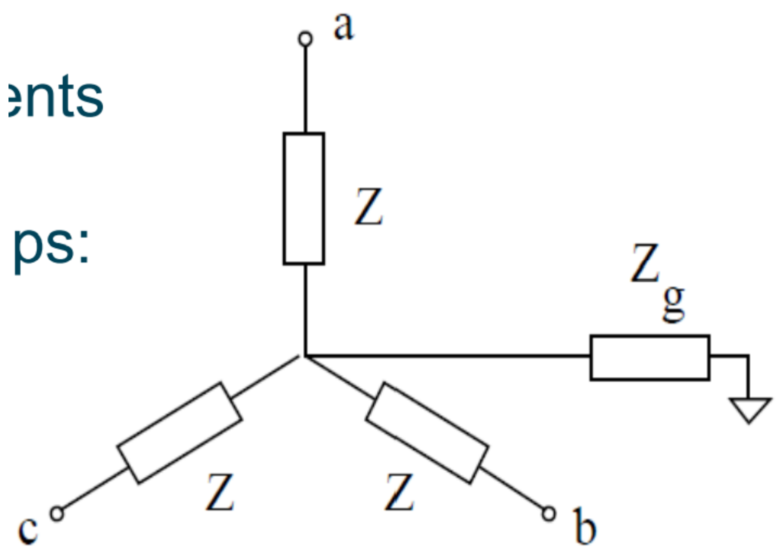


Figure 6.4: Grounded star arrangement.

6.2.12 Typical values of sequence impedances for synchronous generators

Manufacturers will test their machines to obtain the relevant data/ The value of the sequence components may differ from country to country, manufacturer to manufacturer.

6.2.13 Transformers

The positive and negative sequence sequence impedances are the normal values obtained from the per-phase equivalent circuit. ($Z_1 = Z_2$). The zero sequence components depend upon the connection of the windings. Zero sequence currents in the windings on one side of the transformer must produce the corresponding ampere-turns in the other. In delta windings the zero-sequence currents circulate through the three-phase windings but do not leave the transformer.

6.3 Unbalanced faults

6.3.1 Fortescue's symmetrical component process

Symmetrical components are used extensively for fault study calculations. in these calculations the positive, negative and zero-sequence impedance networks are either given by the manufacturer or are calculated by the user using base voltages and base power for the system of interest. Each of the sequence networks are then connected together to calculate fault currents and voltages depending upon the type of fault. Standard circuit arrangements have been derived in this course to keep variation reasonable.

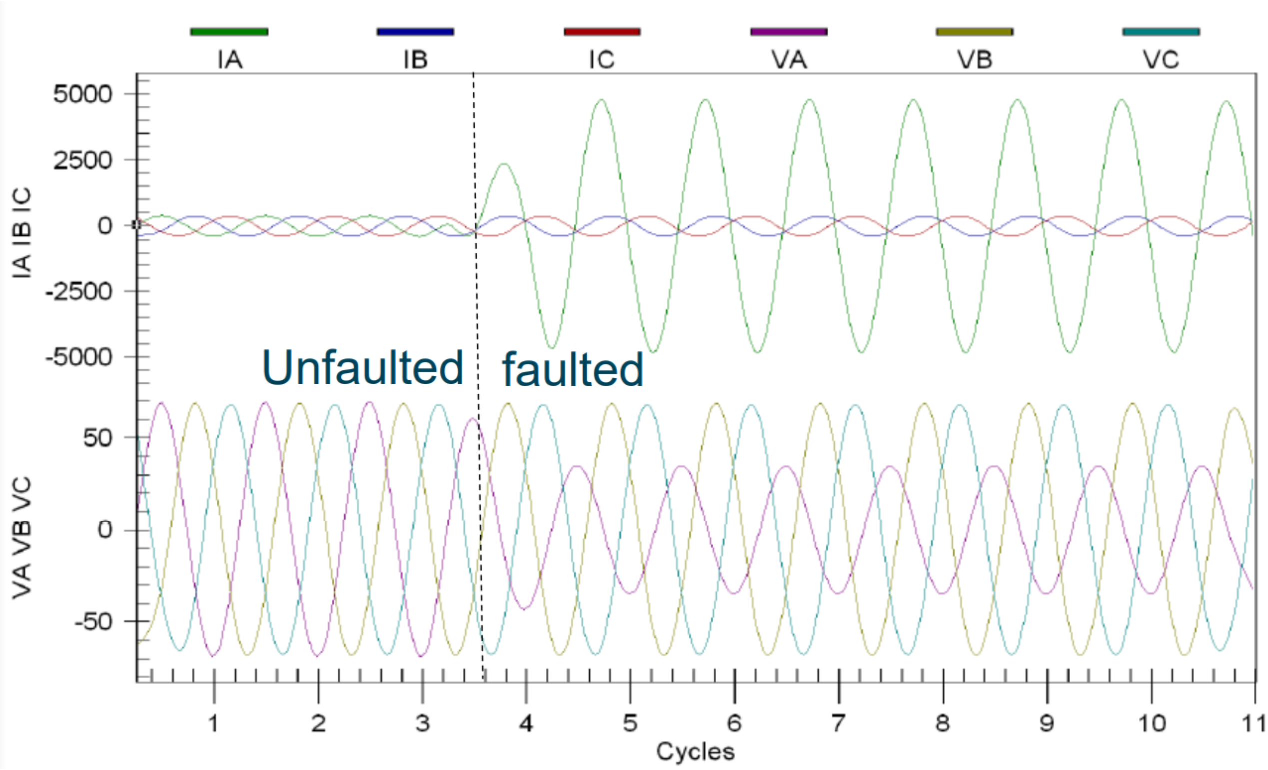


Figure 6.5: Line to ground fault.

6.3.2 Standard fault sequence connections - single line to ground

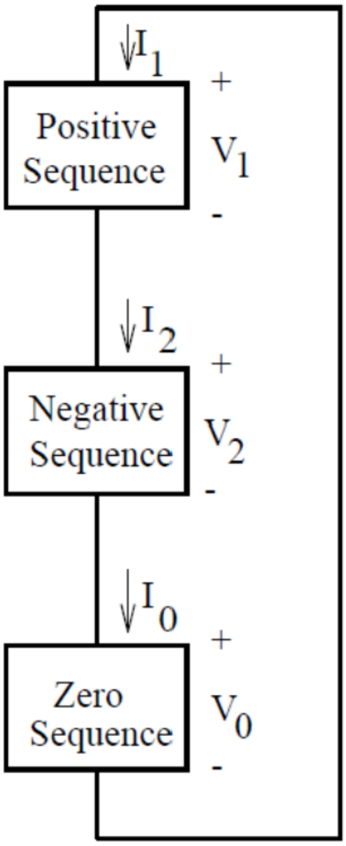


Figure 6.6: Single line to ground connection.

Assumptions:

- $V_a = 0$; I_a = very large value (faulted line)
- $I_b = 0$ (small in comparision to fault current)
- $I_c = 0$ (small in comparision to fault current)

Hence for phae voltage 'a' we can say:

$$V_0 + V_1 + V_2 = 0 \tag{6.26}$$

And for the current we can say:

$$I_0 + I_1 + I_2 = \frac{1}{3} I_a \tag{6.27}$$

Together, these two expressions describe the sequence network connection.

6.3.3 Standard fault sequence connections - line to line

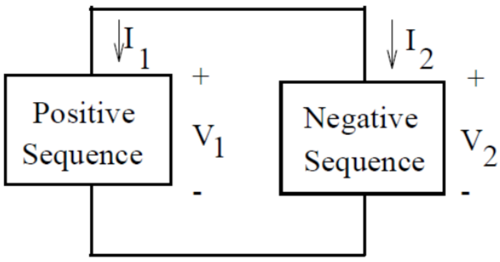


Figure 6.7: Line to line connection.

Assumptions. If the fault occurs between phase b and c then we can say:

- $V_b = V_c$
- $I_b = -I_c$
- $I_a = 0$ (since it is small in comparison with the fault current)

Hence, we can use the phase sequence relationships to say:

$$V_1 = V_2 \text{ and also } I_a = I_1 + I_2 \text{ since } I_0 = 0 \tag{6.28}$$

6.3.4 Standard fault sequence connections - double line to ground

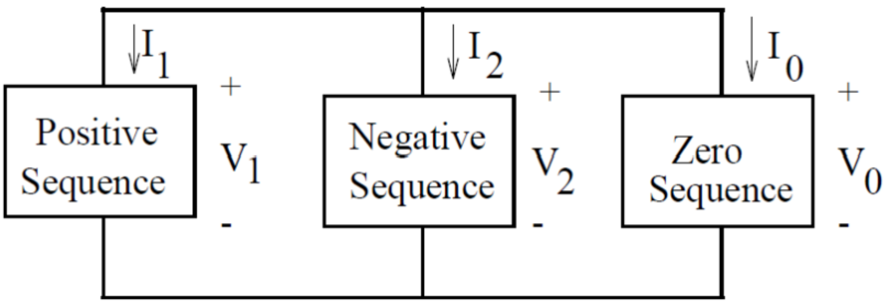


Figure 6.8: Double line to ground connection.

Assumptions. If the fault involves phases b and c to ground then we can say:

- $I_a = 0$ (small in comparison to fault current)
- $V_b = 0$ (faulted line)
- $V_c = 0$ (faulted line)

Hence using phase-sequence relationships we can further say that:

$$V_0 + V_1 + V_2 = 0 \tag{6.29}$$
$$I_a = I_0 + I_1 + I_2 = 0 \tag{6.30}$$

6.4 A full fault analysis study

6.4.1 Breaker sizing method (most common approach)

One of the main purposes of circuit breakers is to arrest large currents that flow when there is a fault. Breaker sizing is achieved by understanding currents flowing under both symmetrical and non-symmetrical fault conditions (to be calculated). Calculations are carried out using symmetrical components i.e. positive, negative and zero sequence. Only one phae needs to be considered . . . but all fault types need to be calculated.

6.4.2 Breaker sizing example

Determine the maximum current through the breaker B due to a fault at the location X. Calculate all three types of unbalanced fult and the balanced fault currents.

- System base: voltage 138 kV (1 pu), Power 100 MVA (1 pu)
- Transformer T_1 leakage reactance j0.1 pu
- Transformer T_2 leakage reactance j0.1 pu
- Line L1: positive and negative sequence reactance j0.05 pu, zero sequence reactance j0.1 pu
- Line L2: positive and negative sequence reactance j0.02 pu, zero sequence reactance j0.1 pu

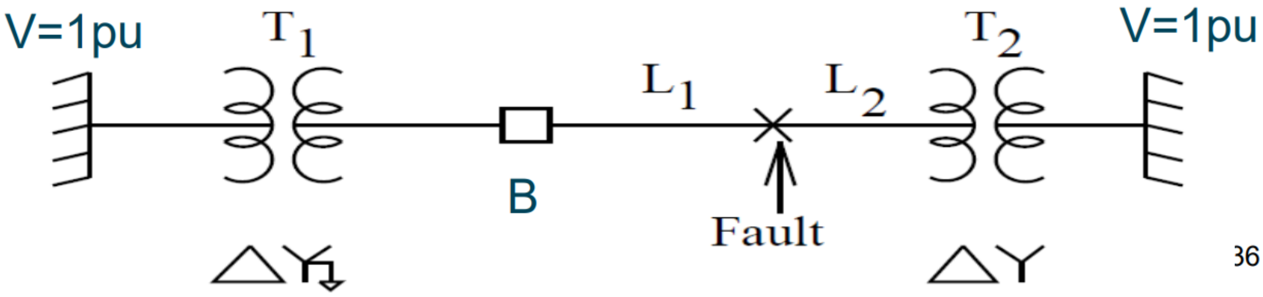


Figure 6.9: Breaker sizing example.

6.4.3 Sequence component arrangement

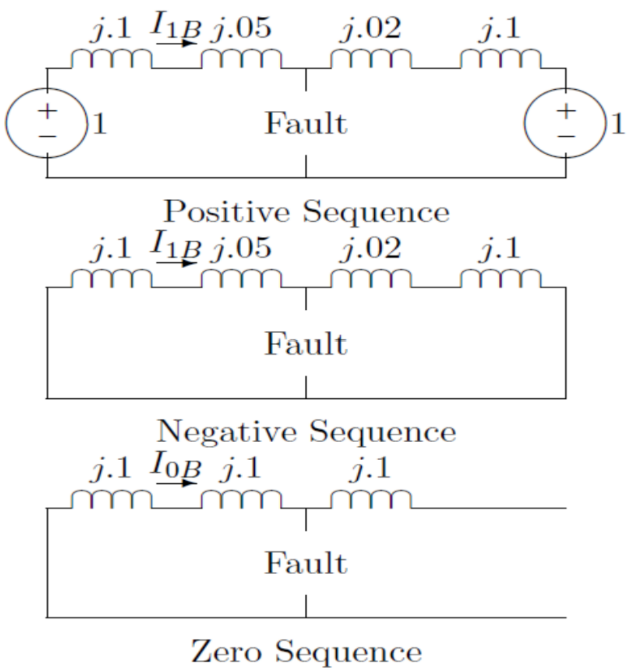


Figure 6.10: Sequence component arrangement.

The sequence networks are exactly like what we would expect to have drawn for equivalent single phase networks. A positive, negative and zero sequence arrangmenet has been shown for one phase. Only the positive sequence network has sources, because the infinite bus supplies only positive sequence voltage. The zero sequence network is open at the right hand side because of the delta-wye transformer connection.

6.4.4 Symmetrical fault current

For a symmetrical (three-phase) fault, only the positive sequence network is involved. The fault shorts the network at its position, so that the current is:

$$I_1 = \frac{1}{j0.15} - j6.67 \text{ per unit from LHS} \tag{6.31}$$

$$(I_1 = \frac{1}{j0.12} - j8.33 \text{ per unit from RHS}) \tag{6.32}$$

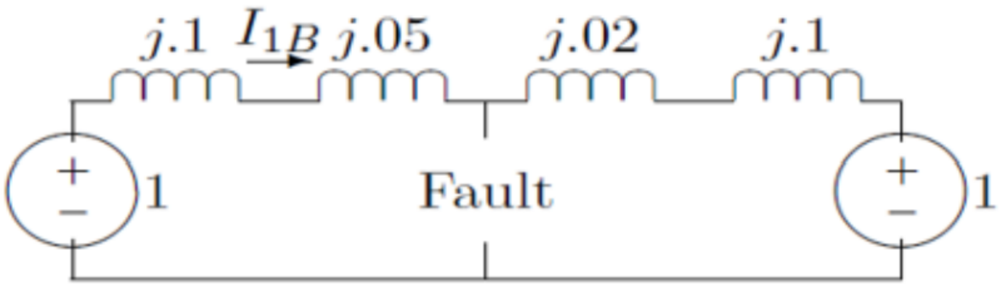


Figure 6.11: Positive sequence impedance in symmetrical fault.

6.4.5 Single line to ground fault

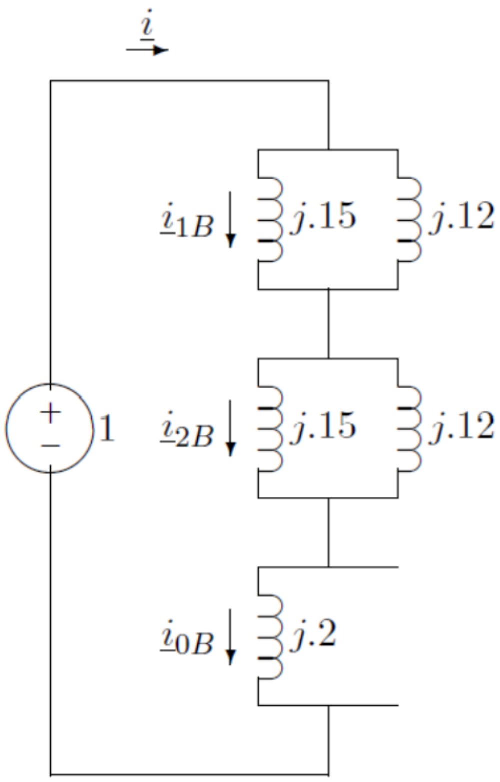


Figure 6.12: Positive sequence impedance in symmetrical fault.

The three networks are in series and the situation is as shown with the total current given by:

$$\underline{i} = \frac{1}{2 \times \left(j0.15||j0.12\right) + j0.2} = -j3.0 \tag{6.33}$$

The sequence currents are:

$$\underline{i}_{1B} = \underline{i}_{2B} \tag{6.34}$$
$$= \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \tag{6.35}$$
$$= -j1.33\underline{i}_{0B} \qquad = \underline{i} \tag{6.36}$$
$$= -j3.0 \tag{6.37}$$

6.4.6 Singe line to ground fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = \underline{i}_{1B} + \underline{i}_{2B} + \underline{i}_{0B} \tag{6.38}$$
$$\underline{i}_b = \underline{a^2 i}_{1B} + \underline{a i}_{2B} + \underline{i}_{0B} \tag{6.39}$$
$$\underline{i}_c = \underline{a i}_{1B} + \underline{a^2 i}_{2B} + \underline{i}_{0B} \tag{6.40}$$

Hence:

$$\underline{i}_a = -j5.66\,\text{pu} \tag{6.41}$$
$$\underline{i}_b = -j1.67\,\text{pu} \tag{6.42}$$
$$\underline{i}_c = -j1.67\,\text{pu} \tag{6.43}$$

6.4.7 Double line to ground fault

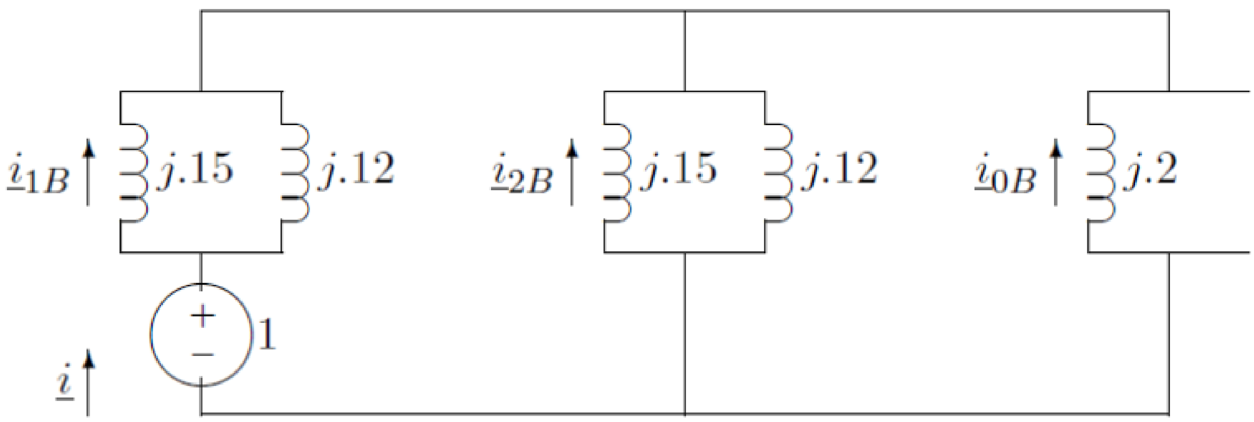


Figure 6.13: Double line-ground fault configuration.

For the double line-ground fault, the networks are in parallel.

$$\underline{i} = \frac{1}{j\left(0.15||0.12\right) + j\left(0.15||0.12||0.2\right)} \tag{6.44}$$
$$= -j8.57 \tag{6.45}$$
$$\underline{i}_{1B} = \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \tag{6.46}$$
$$= -j3.81 \tag{6.47}$$
$$\underline{i}_{2B} = -\underline{i} \times \frac{j0.12||j0.2}{j0.12||j0.2 + j0.15} \tag{6.48}$$
$$= j2.86 \tag{6.49}$$
$$\underline{i}_{0B} = \underline{i} \times \frac{j0.12||j0.15}{j0.2 + j0.12||j0.15} \tag{6.50}$$
$$= j2.14 \tag{6.51}$$

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = j1.19 \tag{6.52}$$
$$\underline{i}_b = \underline{i}_{0B} - \frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) - \frac{\sqrt{3}}{2}j\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{6.53}$$
$$= j2.67 - 5.87 \tag{6.54}$$
$$\underline{i}_c = \underline{i}_{0B} - \frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) + \frac{\sqrt{3}}{2}j\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{6.55}$$
$$= j2.67 + 5.87 \tag{6.56}$$

Hence:

$$|\underline{i}_a| = 1.19\,\text{pu} \tag{6.57}$$
$$|\underline{i}_b| = 6.43\,\text{pu} \tag{6.58}$$
$$|\underline{i}_c| = 6.43\,\text{pu} \tag{6.59}$$

6.4.8 Line to line fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = 0 \tag{6.60}$$
$$\underline{i}_b = -\frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) - j\frac{\sqrt{3}}{2}\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{6.61}$$

Hence:

$$|\underline{i}_b| = 5.77\,\text{pu} \tag{6.62}$$
$$|\underline{i}_c| = 5.77\,\text{pu} \tag{6.63}$$

There are only two networks at play - positive and negative sequence.

	Phase A	Phase B	Phase C
Three-phase fault	2791	2791	2791
Single line-ground, ϕ_a	2368	699	699
Double line-ground, ϕ_b, ϕ_c	498	2690	2690
Line-line, ϕ_b, ϕ_c	0	2414	2414

Table 6.2: Table to show fault currents.

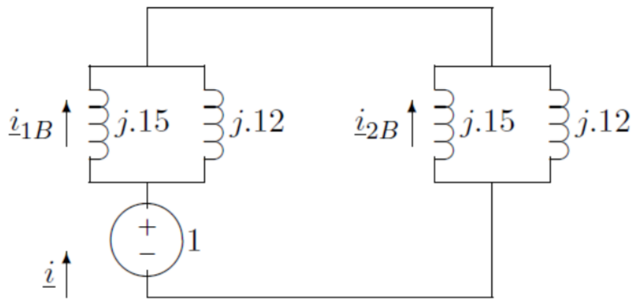


Figure 6.14: Line to line fault configuration.

6.4.9 Conversion to ampere ratings

Having calculated the fault currents then the values in per unit can be expressed as amperes. The value of I_B is:

$$I_B = \frac{P_B}{\sqrt{3}V_{Bl-l}} = 418.8\,\text{A} \tag{6.64}$$

Hence the fault currents are calculated as being: The worst fault is the balanced three-phase fault.

6.4.10 Practical sizing of breakers

Key information needed for sizing a circuit breaker include:

- Voltage rating
- Normal current rating
- MVA fault level
- Fault current levels
- Withstand voltage levels

There are three main types of circuit breakers: Air, vacuum and SF6.

6.4.11 Conclusions

- Appreciated the need for positive, negative and zero sequence impedances of different components to make up a power system
- Introduced the concept of positive, negative and zero sequence impedance. Examined this at a component level
- A system analysis method has been applied for ‘unbalanced faults’ in a transmission system and fault current table produced