UCL Mechanical Engineering 2020/2021

MECH0013 Final Assessment

NCWT3

May 6, 2021

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1 Question 1

1.1 i

Straight section analysis:

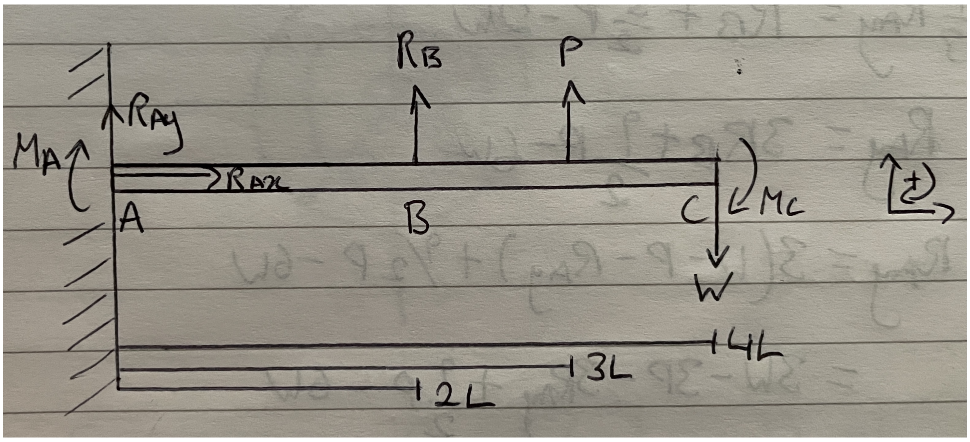

Figure 1: Diagram to show horizontal beam arrangement.

Equilibrium conditions:

$$\sum F_x : R_{Ax} = 0 \tag{1.1}$$

$$\sum F_y : R_{Ay} + R_B + P = W \tag{1.2}$$

$$\sum M_A : -M_A + M_C + 4WL = 2R_BL + 3PL \tag{1.3}$$

Using Macaulay's method:

$$M = -M_A + R_{Ay}x + R_B < x - 2L > +P < x - 3L > \tag{1.4}$$

Slope:

$$\theta = \frac{1}{EI}\int (M)\,\mathrm{d}x \tag{1.5}$$

$$\theta = \frac{1}{EI}\int \left(-M_A + R_{Ay}x + R_B < x - 2L > +P < x - 3L >\right)\mathrm{d}x \tag{1.6}$$

$$\theta = \frac{1}{EI}\left[-M_Ax + \frac{R_{Ay}x^2}{2} + \frac{R_b < x - 2L >^2}{2} + \frac{P < x - 3L >^2}{2}\right] + \theta_0 \tag{1.7}$$

Deflection:

$$y = \int (\theta)\,\mathrm{d}x \tag{1.8}$$

$$y = \int \left(\frac{1}{EI}\left[-M_Ax + \frac{R_{Ay}x^2}{2} + \frac{R_b < x - 2L >^2}{2} + \frac{P < x - 3L >^2}{2}\right] + \theta_0\right)\mathrm{d}x \tag{1.9}$$

$$y = \frac{1}{EI}\left[-\frac{M_Ax^2}{2} + \frac{R_{Ay}x^3}{6} + \frac{R_B < x - 2L >^3}{6} + \frac{P < x - 3L >^3}{6}\right] + \theta_0x + y_0 \tag{1.10}$$

Curved section analysis:

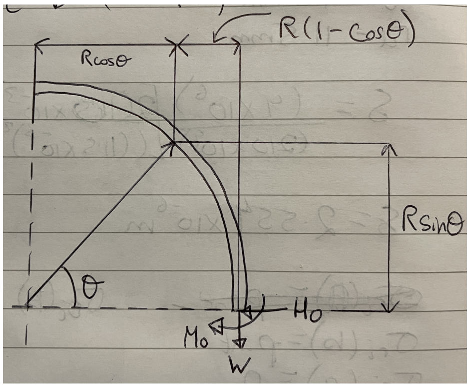

Figure 2: Diagram to show curved beam arrangement.

$$M(\theta) = WR\,(1-\cos\theta) + H_0 R\sin\theta + M_0 \tag{1.11}$$

$$\frac{\partial M}{\partial M_0} = 1 \tag{1.12}$$

$$\varphi_A = \int_0^L \left(\frac{M}{EI}\frac{\partial M}{\partial M_0}\right) \mathrm{d}x = 0 \tag{1.13}$$

Converting to polar ($\mathrm{d}x = R\,\mathrm{d}\theta$):

$$\varphi_A = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \Big(\big(WR\,(1-\cos\theta) + H_0 R\sin\theta + M_0\big)\,(1)\,(R)\Big)\,\mathrm{d}\theta \tag{1.14}$$

M_0 and H_0 represent dummy loads and can be neglected:

$$\varphi_A = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \Big(WR^2\,(1-\cos\theta)\Big)\,\mathrm{d}\theta \tag{1.15}$$

$$\varphi_A = \frac{1}{EI}\Big[WR^2\,(\theta - \sin\theta)\Big]_0^{\frac{\pi}{2}} \tag{1.16}$$

$$\varphi_A = \frac{WR^2\left(\frac{\pi}{2}-1\right)}{EI} \tag{1.17}$$

Boundary conditions:

$$x = 0,\ y = 0 \therefore y_0 = 0 \tag{1.18}$$

$$x = 0,\ \theta = 0 \therefore \theta_0 = 0 \tag{1.19}$$

$$x = 2L,\ y = 0 \tag{1.20}$$

$$x = 4l,\ \theta = \frac{WR^2\left(\frac{\pi}{2}-1\right)}{EI} \tag{1.21}$$

From 1.20:

$$0 = \frac{1}{EI}\left[-\frac{M_A\,(2L)^2}{2} + \frac{R_{Ay}\,(2L)^3}{6} + \frac{R_B < 2L - 2L >^3}{6}\right] \tag{1.22}$$

$$0 = \frac{1}{EI}\left[-2M_A L^2 + \frac{4R_{Ay}L^3}{3} + 0\right] \tag{1.23}$$

$$0 = -2M_A L^2 + \frac{4R_{Ay}L^3}{3} \tag{1.24}$$

$$M_A = \frac{2R_{Ay}L}{3} \tag{1.25}$$

From 1.21:

$$\frac{WR^2\left(\frac{\pi}{2}-1\right)}{EI} = \frac{1}{EI}\left[-M_A\left(4L\right)+\frac{R_{Ay}\left(4L\right)^2}{2}+\frac{R_B < 4L-2L >^2}{2}+\frac{P < 4L-3L >^2}{2}\right] \tag{1.26}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = -4M_AL+8R_{Ay}L^2+2R_BL^2+\frac{PL^2}{2} \tag{1.27}$$

Substituting 1.25:

$$WR^2\left(\frac{\pi}{2}-1\right) = -\frac{8R_{Ay}L^2}{3}+8R_{Ay}L^2+2R_BL^2+\frac{PL^2}{2} \tag{1.28}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3}+2R_BL^2+\frac{PL^2}{2} \tag{1.29}$$

From 1.2:

$$R_B = W-P-R_{Ay} \tag{1.30}$$

Substituting 1.30:

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3}+2\left(W-P-R_A\right)L^2+\frac{PL^2}{2} \tag{1.31}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{16R_{Ay}L^2}{3}+2WL^2-2PL^2-2R_AL^2+\frac{PL^2}{2} \tag{1.32}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{10R_{Ay}L^2}{3}+2WL^2-\frac{3PL^2}{2} \tag{1.33}$$

From 1.3, 1.2, 1.30, 1.25:

$$-\frac{2R_{Ay}L}{3}+WR+4WL = 2\left(W-P-R_{Ay}\right)+3PL \tag{1.34}$$

$$-\frac{2R_{Ay}L}{3}+2R_{Ay}L = 2WL-2PL+3PL-WR-4WL \tag{1.35}$$

$$\frac{4R_{Ay}L}{3} = PL-WR-2WL \tag{1.36}$$

$$R_{Ay} = \frac{3}{4}\left(P-\frac{WR}{L}-2W\right) \tag{1.37}$$

Substituting 1.37:

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{10L^2}{3}\left(\frac{3}{4}\left(P-\frac{WR}{L}-2W\right)\right)+2WL^2-\frac{3PL^2}{2} \tag{1.38}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = \frac{5PL^2}{2}-\frac{5WRL}{2}-5WL^2+2WL^2-\frac{3PL^2}{2} \tag{1.39}$$

$$WR^2\left(\frac{\pi}{2}-1\right) = PL^2-3WL^2-\frac{5WRL}{2} \tag{1.40}$$

Rearranging for P:

$$PL^2 = 3WL^2+\frac{5WRL}{2}+WR^2\left(\frac{\pi}{2}-1\right) \tag{1.41}$$

$$P = 3W+\frac{5WR}{2L}+\frac{WR^2}{L^2}\left(\frac{\pi}{2}-1\right) \tag{1.42}$$

$$P = 3(42.67)+\frac{5(42.67)(0.3)}{2(0.35)}+\frac{(42.67)(0.3)^2}{(0.35)^2}\left(\frac{\pi}{2}-1\right) \tag{1.43}$$

$$P = 237.34\,\mathrm{N} \tag{1.44}$$

1.2 ii

From Figure 2, the bending moment is:

$$M = W\left(R\left(1-\cos\theta\right)\right) + H_0\left(R\sin\theta\right) + M_0 \tag{1.45}$$

The horizontal displacement of the curved beam can be found using 1.46:

$$\delta_H = \int_0^\theta \left(R\frac{M}{EI}\frac{\partial M}{\partial H_0}\right)\mathrm{d}\theta \tag{1.46}$$

From 1.45

$$\frac{\partial M}{\partial H_0} = R\sin\theta \tag{1.47}$$

Substituting 1.47 and 1.45 into 1.46

$$\delta_H = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \left(R\left(W\left(R\left(1-\cos\theta\right)\right) + H_0\left(R\sin\theta\right) + M_0\right)R\sin\theta\right)\mathrm{d}\theta \tag{1.48}$$

H_0 and M_0 are dummy forces and can be neglected:

$$\delta_H = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \left(R\left(W\left(R\left(1-\cos\theta\right)\right)\right)R\sin\theta\right)\mathrm{d}\theta \tag{1.49}$$

$$\delta_H = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(\left(1-\cos\theta\right)\left(\sin\theta\right)\right)\mathrm{d}\theta \tag{1.50}$$

$$\delta_H = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(\sin\theta - \sin\theta\cos\theta\right)\mathrm{d}\theta \tag{1.51}$$

$$\delta_H = \frac{R^3W}{EI}\left[-\cos\theta - \frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{2}} \tag{1.52}$$

$$\delta_H = \frac{R^3W}{EI}\left[-0 - \frac{1}{2} + 1 + 0\right] \tag{1.53}$$

$$\delta_H = \frac{R^3W}{2EI} \tag{1.54}$$

Substituting the values of R, W, E, $I = \frac{\pi}{4}\left(\frac{d_0^4}{2^4} - \frac{d_i^4}{2^4}\right)$:

$$\delta_H = \frac{0.3^3 \times 42.67}{2 \times 210 \times 10^9 \times \left(\frac{\pi}{4}\left(0.01^4 - 0.0085^4\right)\right)} \tag{1.55}$$

$$\delta_H = 7.307 \times 10^{-4}\,\mathrm{m} = 0.731\,\mathrm{mm}\ \text{(3sf)} \tag{1.56}$$

The vertical displacement of the curved beam can be found using 1.57:

$$\delta_V = \int_0^\theta \left(R\frac{M}{EI}\frac{\partial M}{\partial W}\right)\mathrm{d}\theta \tag{1.57}$$

From 1.45:

$$\frac{\partial M}{\partial W} = R\left(1-\cos\theta\right) \tag{1.58}$$

Substituting 1.58 and 1.45 into 1.57 and neglecting dummy loads:

$$\delta_V = \frac{1}{EI}\int_0^{\frac{\pi}{2}} \left(R\left(W\left(R\left(1-\cos\theta\right)\right)\right)R\left(1-\cos\theta\right)\right)\mathrm{d}\theta \tag{1.59}$$

$$\delta_V = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(1-\cos\theta\right)^2\mathrm{d}\theta \tag{1.60}$$

$$\delta_V = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(1-2\cos\theta+\cos^2\theta\right)\mathrm{d}\theta \tag{1.61}$$

$$\delta_V = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(1-2\cos\theta+\frac{1}{2}+\frac{\cos 2\theta}{2}\right)\mathrm{d}\theta \tag{1.62}$$

$$\delta_V = \frac{R^3W}{EI}\int_0^{\frac{\pi}{2}} \left(\frac{3}{2}-2\cos\theta+\frac{\cos 2\theta}{2}\right)\mathrm{d}\theta \tag{1.63}$$

$$\delta_V = \frac{R^3W}{EI}\left[\frac{3\theta}{2}-2\sin\theta+\frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}} \tag{1.64}$$

$$\delta_V = \frac{R^3W}{EI}\left[\frac{3\pi}{4}-2+0-0+2-0\right] \tag{1.65}$$

$$\delta_V = \frac{R^3W}{EI}\left(\frac{3\pi}{4}-2\right) \tag{1.66}$$

Substituting the values of R, W, E, $I = \frac{\pi}{4}\left(\frac{d_0^4}{2^4}-\frac{d_i^4}{2^4}\right)$:

$$\delta_V = \frac{0.3^3\times 42.67}{210\times 10^9\times\left(\frac{\pi}{4}\left(0.01^4-0.0085^4\right)\right)}\left(\frac{3\pi}{4}-2\right) \tag{1.67}$$

$$\delta_V = 5.205\times 10^{-4}\,\mathrm{m} = 0.521\,\mathrm{mm}\ \text{(3sf)} \tag{1.68}$$

From Figure 1 and 1.10:

$$y = \frac{1}{EI}\left[-\frac{M_Ax^2}{2}+\frac{R_{Ay}x^3}{6}+\frac{R_B < x-2L >^3}{6}+\frac{P < x-3L >^3}{6}\right]+\theta_0 x+y_0 \tag{1.69}$$

Applying boundary conditions and finding slope at location $x = 4L$:

$$y = \frac{1}{EI}\left[-\frac{M_A(4L)^2}{2}+\frac{R_{Ay}(4L)^3}{6}+\frac{R_B(2L)^3}{6}+\frac{PL^3}{6}\right] \tag{1.70}$$

Substituting values of M_A, R_{Ay}, R_B, P, E, I and L:

$$y = \frac{1}{\left(\frac{\pi}{4}\left(0.01^4-0.0085^4\right)\right)}\left[-\frac{20.1995(4(0.35))^2}{2}+\frac{86.5694(4(0.35))^3}{6} + \frac{-281.2393(2(0.35))^3}{6}+\frac{237(0.35)^3}{6}\right] \tag{1.71}$$

$$y = 6.864\times 10^{-3}\,\mathrm{m} = 6.86\,\mathrm{mm}\ \text{(3sf)} \tag{1.72}$$

1.3 iii

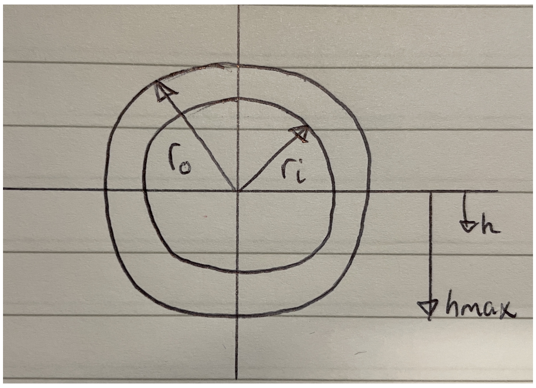

Figure 3: Diagram to show cross-section of horizontal beam.

Where $r_i = 8.5\,\text{mm}$, $r_o = 10\,\text{mm}$. The yield moment can be found using 1.73:

$$M_Y = \frac{\sigma_Y I}{h_{max}} \tag{1.73}$$

Where $I = \frac{\pi}{4}\left(r_o^4 - r_i^4\right)$ and $h_{max} = r_o$. Substituting:

$$M_Y = \frac{480 \times 10^6 \times \frac{\pi}{4}\left(0.01^4 - 0.0085^4\right)}{0.01} \tag{1.74}$$

$$M_Y = 180.199\,\text{N m} = 180\,\text{N m (3sf)} \tag{1.75}$$

We now want to plot the bending moment as a function of x. MATLAB was used to plot Figure 4.

```
clc
clear
close all

%define vars
P = 237.34; %value of corrective load
W = 42.67; %camera weight
R = 0.3; %radius of curvature of curved section
L = 0.35; %length of a quarter of the beam

%calc vars
R_Ay = 0.75*(P-((W*R)/L) - 2*W); %support reaction at A
R_B = W - P - R_Ay; %support reaction at B
M_A = (2/3)*R_Ay*L; %moment at A

%bending moment equation
%x = 0
M0 = -M_A;
%x = L
M1 = -M_A + R_Ay*L;
%x = 2L
M2 = -M_A + 2*R_Ay*L;
%x = 3L
M3 = -M_A + 3*R_Ay*L + R_B*L;
%x = 4L
M4 = -M_A + 4*R_Ay*L + 2*R_B*L + P*L;
```

```
%concatenate
x = [0 L 2*L 3*L 4*L];
M = [M0 M1 M2 M3 M4];

%plot
plot(x,M,'color','red')
xlim([0 4*L])
ylim([-30 45])
grid on
hline = refline(0,0);
hline.Color = 'b';
title('Graph to show bending moment of beam')
xlabel('Length along beam/m')
ylabel('Magnitude of bending moment/Nm')
```

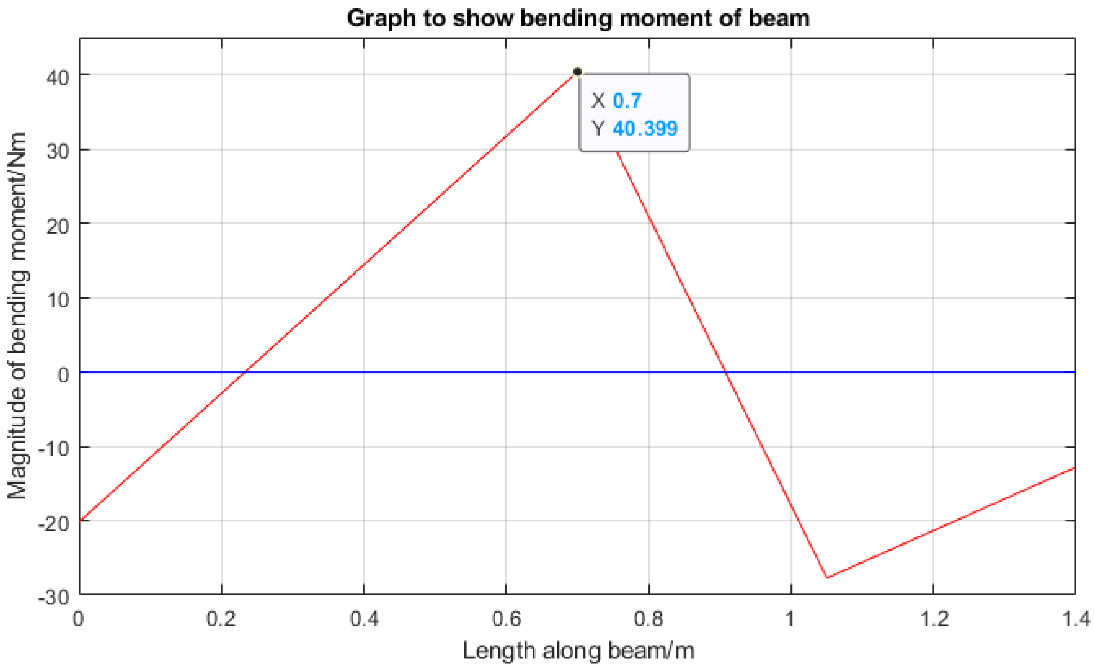

Figure 4: Graph to show bending moment along the horizontal beam.

Variable	Value
M_A	$20.1995\,\mathrm{N\,m}$
R_{Ay}	$86.5694\,\mathrm{N}$
R_B	$-281.2393\,\mathrm{N}$

Table 1: Table to show values of support reactions from MATLAB.

Hence, we know that the maximum bending moment occurs at $2L$. Let us find the yielding force at this location.

$$M(x = 2L) = -M_A + 2R_{Ay}L \tag{1.76}$$

Substituting in M_A and R_{Ay} (1.25 and 1.37):

$$M_A = \frac{2}{3} R_{Ay} L \tag{1.77}$$

$$R_{Ay} = \frac{3}{4}\left(P - \frac{WR}{L} - 2W\right) \tag{1.78}$$

$$M_A = \frac{2}{3}\left(\frac{3}{4}\left(P - \frac{WR}{L} - 2W\right)\right) L \tag{1.79}$$

$$M_A = \frac{L}{2}\left(P - \frac{WR}{L} - 2W\right) \tag{1.80}$$

$$M(x = 2L) = -\frac{1}{2}PL + \frac{1}{2}WR + WL + \frac{3}{2}PL - \frac{3}{2}WR - 3WL \tag{1.81}$$

$$M = PL - WR - 2WL \tag{1.82}$$

Let $M = M_Y$ and $P = P_Y$:

$$M_Y = P_Y L - WR - 2WL \tag{1.83}$$

$$P_Y = \frac{1}{L}\left(M_Y + WR + 2WL\right) \tag{1.84}$$

Substituting the values of M_Y, W, R, L:

$$P_Y = \frac{1}{0.35}\left(180.199 + 42.67 \times 0.3 + 2 \times 42.67 \times 0.35\right) \tag{1.85}$$

$$P_Y = 636.769\,\mathrm{N} = 637\,\mathrm{N}\ \text{(3sf)} \tag{1.86}$$

From 1.44:

$$P = 237\,\mathrm{N}\ \text{(3sf)} \tag{1.87}$$

Comparing P and P_Y:

$$P < P_Y \tag{1.88}$$

$$237\,\mathrm{N} < 637\,\mathrm{N} \tag{1.89}$$

Therefore, the corrective load will not result in yielding.

1.4 iv

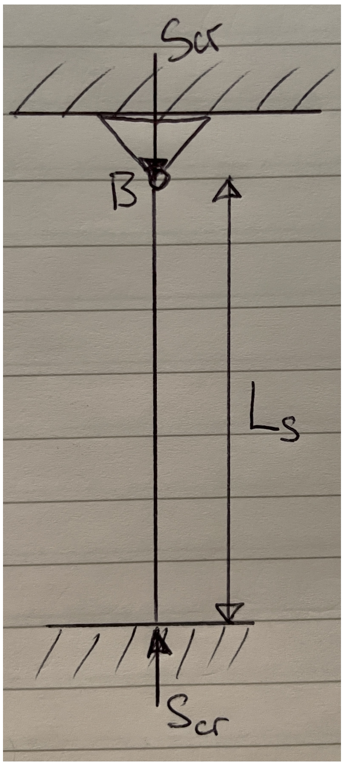

Figure 5: Graph to show arrangement of vertical strut at B.

We can find the critical load using 1.90:

$$S_{cr} = \frac{n^2\pi^2 EI}{L_e^2} \tag{1.90}$$

Where $L_e = 0.7L_s$ for pinned-fixed columns. The second moment of inertia for the strut can be found using 1.91:

$$I = \frac{bh^3}{12} \tag{1.91}$$

Where $b = 0.04$ and $h = 0.0045$. Calculating I for each plane:

$$I_x = \frac{bh^3}{12} = \frac{0.04 \times 0.0045^3}{12} = 3.0375 \times 10^{-10}\,\mathrm{m}^4 \tag{1.92}$$

$$I_y = \frac{b^3h}{12} = \frac{0.04^3 \times 0.0045}{12} = 2.4 \times 10^{-8}\,\mathrm{m}^4 \tag{1.93}$$

Buckling occurs in the plane with the lowest I, hence buckling may occur in the x-plane. Testing:

$$S_{cr} = \frac{1 \times \pi^2 \times 71 \times 10^9 \times \frac{0.04 \times 0.0045^3}{12}}{(0.7 \times 0.96)^2} \tag{1.94}$$

$$S_{cr} = 471.342\,\mathrm{N} = 471\,\mathrm{N}\ (3\mathrm{sf}) \tag{1.95}$$

From Table 1, we can compare the value of R_B to our S_{cr} value.

$$R_B < S_{cr} \tag{1.96}$$

$$281\,\mathrm{N} < 471\,\mathrm{N} \tag{1.97}$$

Critical load is not reached, hence the strut is suitable and will not fail under these conditions.

2 Question 2

2.1 i

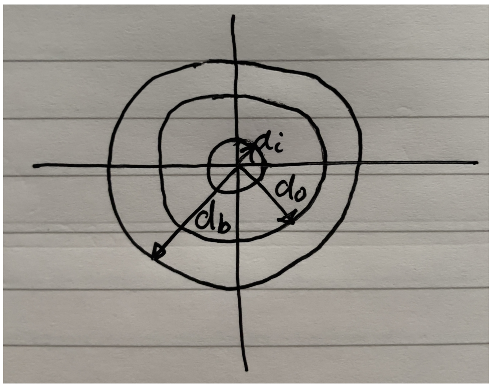

Figure 6: Sketch of cylinder arrangement.

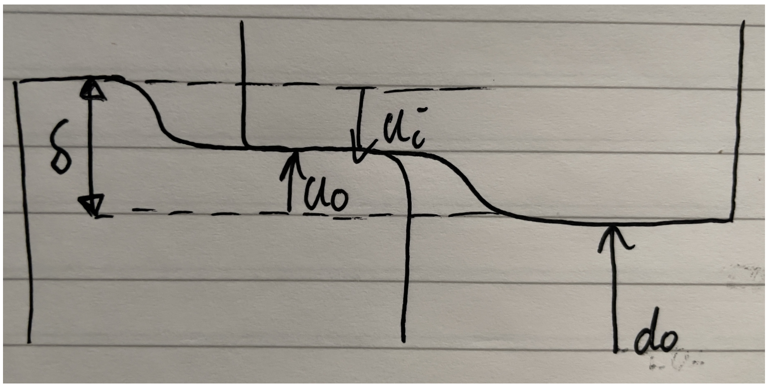

Figure 7: Diagram to show interference.

Hence:

$$\delta = u_o - u_i \tag{2.1}$$

Finding u_i. Let us start with the general equation for u:

$$u = \frac{r}{E}\left(\sigma_\theta - v\sigma_r\right) \tag{2.2}$$

$$\rightarrow u_i = \frac{r_o}{E_i}\left[\sigma_{\theta,i}\left(r_o\right) - v_i\sigma_{r,i}(r_o)\right] \tag{2.3}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{2.4}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{2.5}$$

Boundary conditions:

$$\sigma_{r,i}(r_i) = 0 = A - \frac{B}{r_i^2} \tag{2.6}$$

$$\sigma_{r,i}(r_o) - -p_{int} = A - \frac{B}{r_o^2} \tag{2.7}$$

$$A = -p_{int}\frac{r_o^2}{r_o^2 - r_i^2} \tag{2.8}$$

$$B = -p_{int}\frac{r_i^2 \cdot r_o^2}{r_o^2 - r_i^2} \tag{2.9}$$

Substituting:

$$\sigma_{\theta,i} = A + \frac{B}{r^2} = -p_{int}\frac{r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right) \tag{2.10}$$

Therefore:

$$\sigma_{r,i}(r_o) = -p_{int} \qquad \sigma_{\theta,i}(r_o) = -p_{int}\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \tag{2.11}$$

$$u_i = -p_{int}\frac{r_o}{E_i}\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} - v_i\right) \tag{2.12}$$

Repeating to find u_o:

$$u_o = \frac{r_o}{E_o}\left[\sigma_{\theta,o}(r_o) - v_o\sigma_{r,o}(r_o)\right] \tag{2.13}$$

Lame's equations:

$$\sigma_r = A - \frac{B}{r^2} \tag{2.14}$$

$$\sigma_\theta = A + \frac{B}{r^2} \tag{2.15}$$

Boundary conditions:

$$\sigma_{r,o}(r_o) = p_{int} = A - \frac{B}{r_o^2} \tag{2.16}$$

$$\sigma_{r,o}(r_b) = 0 = A - \frac{B}{r_b^2} \tag{2.17}$$

$$A = p_{int}\frac{r_o^2}{r_o^2 - r_b^2} \tag{2.18}$$

$$B = p_{int}\frac{r_o^2 \cdot r_b^2}{r_b^2 - r_o^2} \tag{2.19}$$

Substituting:

$$\sigma_{\theta,o} = A + \frac{B}{r^2} = p_{int}\frac{r_o^2}{r_b^2 - r_o^2}\left(1 + \frac{r_b^2}{r^2}\right) \tag{2.20}$$

Therefore:

$$\sigma_{r,o}(r_b) = 0 \qquad \sigma_{\theta,o}(r_o) = p_{int}\frac{r_b^2 + r_o^2}{r_b^2 - r_o^2} \tag{2.21}$$

$$u_o = p_{int}\frac{r_o}{E_o}\left(\frac{r_b^2 + r_o^2}{r_b^2 - r_o^2} + v_o\right) \tag{2.22}$$

Finding δ:

$$u_i = -p_{int}\frac{r_o}{E_i}\left(\frac{r_o^2+r_i^2}{r_o^2-r_i^2}-v_i\right) \tag{2.23}$$

$$u_o = p_{int}\frac{r_o}{E_o}\left(\frac{r_b^2+r_o^2}{r_b^2-r_o^2}+v_o\right) \tag{2.24}$$

$$\delta = p_{int}r_o\left[\frac{1}{E_o}\left(\frac{r_b^2+r_o^2}{r_b^2-r_o^2}+v_o\right)+\frac{1}{E_i}\left(\frac{r_o^2+r_i^2}{r_o^2-r_i^2}-v_i\right)\right] \tag{2.25}$$

For a single material: $E_i = E_o = E$, and $v_i = v_o = v$:

$$\delta = p_{int}\frac{r_o}{E}\left[\frac{2r_o^2\left(r_b^2-r_i^2\right)}{\left(r_b^2-r_o^2\right)\left(r_o^2-r_i^2\right)}\right] \tag{2.26}$$

$$p_{int} = E\delta\left[\frac{\left(r_b^2-r_o^2\right)\left(r_o^2-r_b^2\right)}{2r_o^3\left(r_b^2-r_i^2\right)}\right] \tag{2.27}$$

Converting radius to diameters:

$$\delta = p_{int}\frac{d_o}{E}\left[\frac{d_o^2\left(d_b^2-d_i^2\right)}{\left(d_b^2-d_o^2\right)\left(d_o^2-d_i^2\right)}\right] \tag{2.28}$$

$$p_{int} = E\delta\left[\frac{\left(d_b^2-d_o^2\right)\left(d_o^2-d_b^2\right)}{d_o^3\left(d_b^2-d_i^2\right)}\right] \tag{2.29}$$

Where δ is the radial interference.

2.2 ii

We can find the internal pressure through the following equation:

$$F \leq \pi d_o L_b p_{int}\mu_s \tag{2.30}$$

$$p_{int} \geq \frac{F}{\pi d_o L_b \mu_s} \tag{2.31}$$

$$p_{int} \geq \frac{10000}{\pi(20\times10^{-3})(50\times10^{-3})(0.8)} \tag{2.32}$$

$$p_{int} \geq 3\,978\,873.577\,\mathrm{Pa} \tag{2.33}$$

Using a value of 4×10^6 Pa for p_{int}, we can now substitute into 2.28:

$$\delta = \frac{(4\times10^6)(20\times10^{-3})}{210\times10^9}\left[\frac{(20\times10^{-3})^2\left((23\times10^{-3})^2-(17\times10^{-3})^2\right)}{\left((23\times10^{-3})^2-(20\times10^{-3})^2\right)\left((20\times10^{-3})^2-(17\times10^{-3})^2\right)}\right] \tag{2.34}$$

$$\delta = 2.554\times10^{-6}\,\mathrm{m} = 0.002\,55\,\mathrm{mm} \tag{2.35}$$

Where δ is the radial interference. We need to make sure that this interference value does not result in the engineering failure of the internal or external cylinder. Looking at the internal cylinder, we can find the stress by utilising Lame's equations (2.10 and 2.20):

$$\sigma_{ri}\left(r_o\right) = -p_{int} = -4\times10^6\,\mathrm{Pa} \tag{2.36}$$

$$\sigma_{\theta i}\left(r_o\right) = -p_{int}\frac{r_o^2+r_i^2}{r_o^2-r_i^2} = -4\times10^6\cdot\frac{0.01^2+0.0085^2}{0.01^2-0.0085^2} = -2.483\times10^7\,\mathrm{Pa} \tag{2.37}$$

$$\sigma_{ro}\left(r_o\right) = p_{int} = 4\times10^6\,\mathrm{Pa} \tag{2.38}$$

$$\sigma_{\theta o}\left(r_o\right) = p_{int}\frac{r_b^2+r_o^2}{r_b^2-r_o^2} = 4\times10^6\cdot\frac{0.0115^2+0.01^2}{0.0115^2-0.01^2} = 2.881\times10^7\,\mathrm{Pa} \tag{2.39}$$

We can use the distortion-energy theory to calculate the strain energy in the tubes. This theory is justified as it is the most reliable method for calculating the stress. The strain energy associated with distortion can be found using 2.41 and this must be lower than the yield stress σ_Y:

$$\sigma_i < \sigma_Y \tag{2.40}$$

$$\sqrt{\frac{\sigma_{ri}^2 + \sigma_{\theta i}^2 + (\sigma_{ri} - \sigma_{\theta i})^2}{2}} < \sigma_Y \tag{2.41}$$

$$\sqrt{\frac{(-4\times10^6)^2 + (-2.483\times10^7)^2 + \left((-4\times10^6) - (-2.483\times10^7)\right)^2}{2}} < 4.8\times10^8 \tag{2.42}$$

$$2.309\times10^7 < 4.8\times10^8 \tag{2.43}$$

Hence, inner tube is not subject to yielding stress. Repeating for outer tube:

$$\sigma_o < \sigma_Y \tag{2.44}$$

$$\sqrt{\frac{\sigma_{ro}^2 + \sigma_{\theta o}^2 + (\sigma_{ro} - \sigma_{\theta o})^2}{2}} < \sigma_Y \tag{2.45}$$

$$\sqrt{\frac{(4\times10^6)^2 + (2.881\times10^7)^2 + \left((4\times10^6) - (2.881\times10^7)\right)^2}{2}} < 4.8\times10^8 \tag{2.46}$$

$$2.703\times10^7 < 4.8\times10^8 \tag{2.47}$$

Hence, outer tube is not subject to yielding stress.

2.3 iii

```
clc
clear
close all

%vars
pint = 4e6;
a = 8.5e-3;
b = 10e-3;
c = 11.5e-3;
ri = 0.0085;
ro = 0.01;

%internal
sigmaRI = -pint;
sigmaThetaI = -pint.*((b^2 + a^2)/(b^2 - a^2));

%external
sigmaRO = 0;
sigmaThetaO = pint.*((c^2 + b^2)/(c^2 - b^2));

%sigmaThetaO circle
p1 = nsidedpoly(1000,'Center',[sigmaThetaO/2 0],'Radius',abs(sigmaThetaO
    /2));
%sigmaRO circle
p2 = nsidedpoly(1000,'Center',[sigmaRI/2 0],'Radius',abs(sigmaRI/2));
%third circle
```

```
p3 = nsidedpoly(1000,'Center',[(sigmaRI + sigmaThetaO)/2 0],'Radius',abs
    ((sigmaThetaO - sigmaRI)/2));

plot(p1)
hold on
xline(0);
yline(0);
xlabel('\sigma')
ylabel('\tau')
title('Graph to show Mohrs circle')
grid on
axis image
plot(p2)
plot(p3)
hold off
```

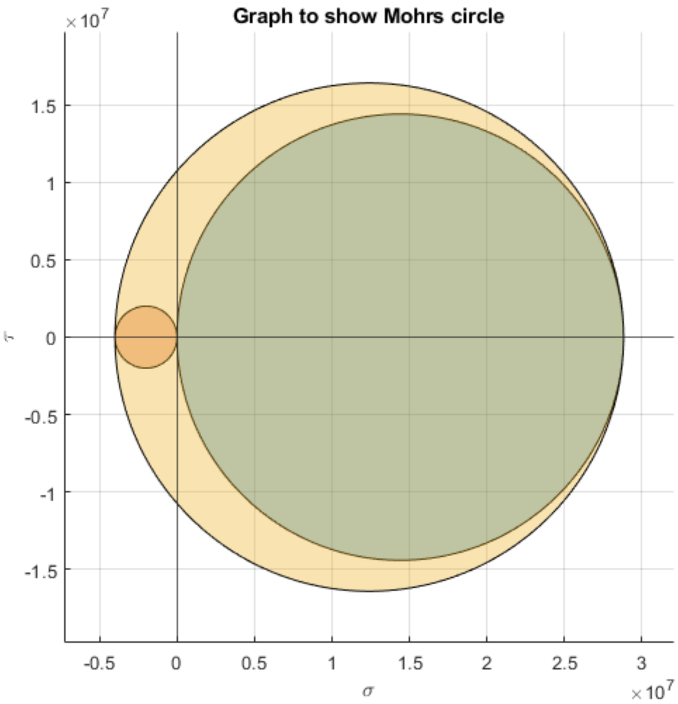

Figure 8: Graph to show Mohr's circle.