

UCL Mechanical Engineering 2020/2021

ENGF0004 48-hour Project

NCWT3

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1 PDEs, Matrix applications

1.1 Developing mathematical model

1.1.1 E1

Starting with:

$$(S_{t+\Delta t} - S_t) = (vS)_x \Delta t - (vS)_{x+\Delta x} \Delta t - gpS\Delta x\Delta t \quad (1.1)$$

Dividing by $\Delta x\Delta t$:

$$\frac{(S_{t+\Delta t} - S_t) \Delta x}{\Delta x \Delta t} = \frac{(vS)_x \Delta t}{\Delta x \Delta t} - \frac{(vS)_{x+\Delta x} \Delta t}{\Delta x \Delta t} - \frac{gpS\Delta x\Delta t}{\Delta x \Delta t} \quad (1.2)$$

Simplifying:

$$\frac{S_{t+\Delta t} - S_t}{\Delta t} = \frac{(vS)_x}{\Delta x} - \frac{(vS)_{x+\Delta x}}{\Delta x} - gpS \quad (1.3)$$

$$\frac{S_{t+\Delta t} - S_t}{\Delta t} = \frac{(vS)_x - (vS)_{x+\Delta x}}{\Delta x} - gpS \quad (1.4)$$

$$\frac{S_{t+\Delta t} - S_t}{\Delta t} + \frac{(vS)_{x+\Delta x} - (vS)_x}{\Delta x} + gpS = 0 \quad (1.5)$$

Applying our limits:

$$\lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \left[\frac{S_{t+\Delta t} - S_t}{\Delta t} + \frac{(vS)_{x+\Delta x} - (vS)_x}{\Delta x} + gpS \right] = 0 \quad (1.6)$$

We can see that in the first two terms of 1.6, we have the definition of a derivative by first principles. Hence:

$$\frac{\partial S}{\partial t} + \frac{\partial (vS)}{\partial x} + gpS = 0 \quad (1.7)$$

1.1.2 E2

Starting with:

$$\frac{\rho S \Delta x (\Delta v)}{\Delta t} = (pS)_x - (pS)_{x+\Delta x} - vrS\Delta x \quad (1.8)$$

Dividing by Δx :

$$\frac{\rho S \Delta x (\Delta v)}{\Delta x \Delta t} = \frac{(pS)_x}{\Delta x} - \frac{(pS)_{x+\Delta x}}{\Delta x} - \frac{vrS \Delta x}{\Delta x} \quad (1.9)$$

Simplifying:

$$\rho S \frac{\Delta v}{\Delta t} = -\frac{(pS)_{x+\Delta x} - (pS)_x}{\Delta x} - vrS \quad (1.10)$$

Applying our limits:

$$\lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \left[\rho S \frac{\Delta v}{\Delta t} \right] = \lim_{\Delta x \rightarrow 0, \Delta t \rightarrow 0} \left[-\frac{(pS)_{x+\Delta x} - (pS)_x}{\Delta x} - vrS \right] \quad (1.11)$$

We can see that in the first two terms of 1.11, we have the definition of a derivative by first principles. Hence:

$$\rho S \frac{\partial v}{\partial t} = -\frac{\partial (pS)}{\partial x} - vrS \quad (1.12)$$

1.1.3 E3 & E4

We know that:

$$c = \frac{1}{S} \frac{dS}{dp} \quad (1.13)$$

Given that S is only a function of the pressure p and p is a function of space and time, we can rewrite 1.13 as:

$$c = \frac{1}{S} \frac{\partial S}{\partial p} \quad (1.14)$$

Starting with:

$$\frac{\partial S}{\partial t} + \frac{\partial (vS)}{\partial x} + gpS = 0 \quad (1.15)$$

Using product rule on second term:

$$\frac{\partial S}{\partial t} + v \frac{\partial S}{\partial x} + S \frac{\partial v}{\partial x} + gpS = 0 \quad (1.16)$$

Dividing by S :

$$\frac{1}{S} \frac{\partial S}{\partial t} + \frac{v}{S} \frac{\partial S}{\partial x} + \frac{\partial v}{\partial x} + gp = 0 \quad (1.17)$$

Multiplying the first and second term by "1":

$$\frac{1}{S} \frac{\partial S}{\partial t} \frac{\partial p}{\partial p} + \frac{v}{S} \frac{\partial S}{\partial x} \frac{\partial p}{\partial p} + \frac{\partial v}{\partial x} + gp = 0 \quad (1.18)$$

Rearranging:

$$\frac{1}{S} \frac{\partial S}{\partial p} \frac{\partial p}{\partial t} + \frac{v}{S} \frac{\partial S}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial v}{\partial x} + gp = 0 \quad (1.19)$$

Substituting c :

$$c \frac{\partial p}{\partial t} + cv \frac{\partial p}{\partial x} + \frac{\partial v}{\partial x} + gp = 0 \quad (1.20)$$

Repeating again with:

$$\rho S \frac{\partial v}{\partial t} = -\frac{\partial (pS)}{\partial x} - vrS \quad (1.21)$$

Using product rule on second term:

$$\rho S \frac{\partial v}{\partial t} = -p \frac{\partial S}{\partial x} - S \frac{\partial p}{\partial x} - vrS \quad (1.22)$$

Dividing by S :

$$\rho \frac{\partial v}{\partial t} = -\frac{p}{S} \frac{\partial S}{\partial x} - \frac{\partial p}{\partial x} - vr \quad (1.23)$$

Multiplying second term by "1":

$$\rho \frac{\partial v}{\partial t} = -\frac{p}{S} \frac{\partial S}{\partial x} \frac{\partial p}{\partial p} - \frac{\partial p}{\partial x} - vr \quad (1.24)$$

Rearranging:

$$\rho \frac{\partial v}{\partial t} + \frac{p}{S} \frac{\partial S}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial p}{\partial x} + vr = 0 \quad (1.25)$$

Substituting c :

$$\rho \frac{\partial v}{\partial t} + cp \frac{\partial p}{\partial x} + \frac{\partial p}{\partial x} + vr = 0 \quad (1.26)$$

1.2 Assumption 4

1.2.1 Constant distensibility

Starting with:

$$c \frac{\partial p}{\partial t} = -\frac{\partial v}{\partial x} \quad (1.27)$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} \quad (1.28)$$

Differentiating 1.27 with respect to x and 1.28 with respect to y :

$$c \frac{\partial^2 p}{\partial x \partial t} = -\frac{\partial^2 v}{\partial x^2} \quad (1.29)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = -\frac{\partial^2 p}{\partial x \partial t} \quad (1.30)$$

Substituting:

$$c \left(-\rho \frac{\partial^2 v}{\partial t^2} \right) = -\frac{\partial^2 v}{\partial x^2} \quad (1.31)$$

$$c\rho \frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial x^2} \quad (1.32)$$

1.2.2 Solution to wave equation and plot

Starting with:

$$v = e^{-\left(x - \frac{1}{\sqrt{cp}}t\right)^2} \quad (1.33)$$

Differentiating:

$$\frac{\partial}{\partial x} \left(e^{-\left(x - \frac{1}{\sqrt{cp}}t\right)^2} \right) = -2 \left(x - \frac{1}{\sqrt{cp}}t \right) e^{-\left(x - \frac{1}{\sqrt{cp}}t\right)^2} \quad (1.34)$$

$$\frac{\partial}{\partial t} \left(e^{-\left(x - \frac{1}{\sqrt{cp}}t\right)^2} \right) = \frac{2}{\sqrt{cp}} \left(x - \frac{1}{\sqrt{cp}}t \right) e^{-\left(x - \frac{1}{\sqrt{cp}}t\right)^2} \quad (1.35)$$

Expanding and differentiating 1.34 with respect to x :

$$\begin{aligned} \frac{\partial}{\partial x} \left(-2xe^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} + \frac{2t}{\sqrt{cp}}e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \right) = \\ -2e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} + 4x \left(x - \frac{1}{\sqrt{cp}t} \right) e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - \frac{4t}{\sqrt{cp}} \left(x - \frac{1}{\sqrt{cp}}t \right) e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \end{aligned} \quad (1.36)$$

Factorising and simplifying:

$$\frac{\partial^2 v}{\partial x^2} = 4 \left(x - \frac{1}{\sqrt{cp}}t \right)^2 e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - 2e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \quad (1.37)$$

Expanding and differentiating 1.35 with respect to y :

$$\begin{aligned} \frac{\partial}{\partial x} \left(\frac{2x}{\sqrt{cp}}e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - \frac{2t}{cp}e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \right) = \\ -\frac{4x}{cp} \left(x - \frac{1}{\sqrt{cp}}t \right) e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - \frac{2}{cp}e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} + \frac{4t}{cp\sqrt{cp}} \left(x - \frac{1}{\sqrt{cp}}t \right) e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \end{aligned} \quad (1.38)$$

Factorising and simplifying:

$$\frac{\partial^2 v}{\partial t^2} = \frac{4}{cp} \left(x - \frac{1}{\sqrt{cp}}t \right)^2 e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - \frac{2}{cp}e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \quad (1.39)$$

Multiplying by cp :

$$cp \frac{\partial^2 v}{\partial t^2} = 4 \left(x - \frac{1}{\sqrt{cp}}t \right)^2 e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} - 2e^{-\left(x-\frac{1}{\sqrt{cp}}t\right)^2} \quad (1.40)$$

Hence, equation 1.32 is satisfied. Plotting 1.33 in MATLAB for $cp = 1$ and at three discrete time points.

```

1 clc
2 clear
3 close all
4
5 v = zeros(3,1000);
6
7 for i = [1 2 3]
8 %define vars
9 x = linspace(0,8,1000);
10 c = 1;
11 p = 1;
12 t = 2*i;
13
14 %equation
15 v(i,:) = exp(-(x-(1/sqrt(c*p)).*t).^2);
16 end
17
18 %plots
19 subplot(3,1,1)
20 plot(x,v(1,:))
21 title('Graph to show flow velocity against distance along vessel with t=2 s')

```

```

22 xlabel('Distance along vessel/[L]')
23 ylabel('Flow velocity/[LS^{-1}]')
24 grid on
25 subplot(3,1,2)
26 plot(x,v(2,:))
27 title('Graph to show flow velocity against distance along vessel with t=4s')
28 xlabel('Distance along vessel/[L]')
29 ylabel('Flow velocity/[LS^{-1}]')
30 grid on
31 subplot(3,1,3)
32 plot(x,v(3,:))
33 title('Graph to show flow velocity against distance along vessel with t=6s')
34 xlabel('Distance along vessel/[L]')
35 ylabel('Flow velocity/[LS^{-1}]')
36 grid on

```

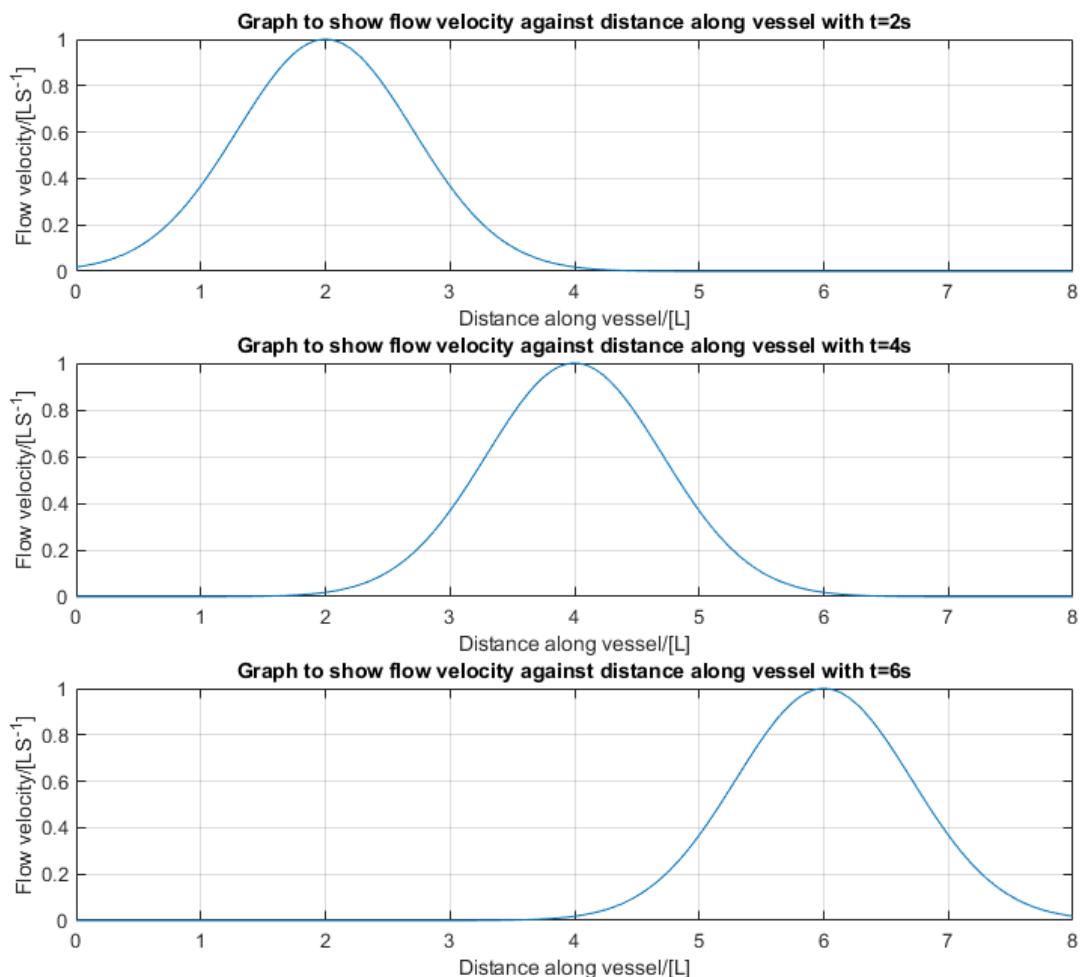


Figure 1: Graphs to show and compare the effect of varying t for flow velocity along the vessel.

1.3 Constant cross-sectional area

1.3.1 Effect on c

We know that:

$$c = \frac{1}{S} \frac{dS}{dP} \quad (1.41)$$

In the case where the cross-sectional area S is constant, its derivative will be 0:

$$c = \frac{1}{S} \cdot 0 \quad (1.42)$$

$$c = 0 \quad (1.43)$$

1.3.2 Substitution

We know that $c = 0$, hence:

$$\frac{\partial v}{\partial x} + gp = 0 \quad (1.44)$$

$$\rho \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + rv = 0 \quad (1.45)$$

Differentiating v with respect to t :

$$\frac{\partial}{\partial t} \left(V e^{-\frac{rt}{\rho}} \right) = \frac{\partial V}{\partial t} e^{-\frac{rt}{\rho}} - \frac{rV}{\rho} e^{-\frac{rt}{\rho}} \quad (1.46)$$

Differentiating v with respect to t :

$$\frac{\partial}{\partial x} \left(V e^{-\frac{rt}{\rho}} \right) = \frac{\partial V}{\partial x} e^{-\frac{rt}{\rho}} \quad (1.47)$$

Substituting 1.47 into 1.44:

$$\frac{\partial V}{\partial x} e^{-\frac{rt}{\rho}} + gp = 0 \quad (1.48)$$

$$p = -\frac{1}{g} e^{-\frac{rt}{\rho}} \frac{\partial V}{\partial x} \quad (1.49)$$

Differentiating with respect to x :

$$\frac{\partial p}{\partial x} = -\frac{1}{g} e^{-\frac{rt}{\rho}} \frac{\partial^2 V}{\partial x^2} \quad (1.50)$$

Substituting 1.46 and 1.50 into 1.45:

$$\rho \left(\frac{\partial V}{\partial t} e^{-\frac{rt}{\rho}} - \frac{rV}{\rho} e^{-\frac{rt}{\rho}} \right) - \frac{1}{g} e^{-\frac{rt}{\rho}} \frac{\partial^2 V}{\partial x^2} + r \left(V e^{-\frac{rt}{\rho}} \right) = 0 \quad (1.51)$$

Simplifying:

$$\rho \frac{\partial V}{\partial t} e^{-\frac{rt}{\rho}} - rV e^{-\frac{rt}{\rho}} - \frac{1}{g} e^{-\frac{rt}{\rho}} \frac{\partial^2 V}{\partial x^2} + rV e^{-\frac{rt}{\rho}} = 0 \quad (1.52)$$

$$\rho \frac{\partial V}{\partial t} e^{-\frac{rt}{\rho}} = \frac{1}{g} e^{-\frac{rt}{\rho}} \frac{\partial^2 V}{\partial x^2} \quad (1.53)$$

$$\frac{\partial V}{\partial t} = \frac{1}{g\rho} \frac{\partial^2 V}{\partial x^2} \quad (1.54)$$

1.4 Solving E6

1.4.1 Separation of variables

Starting with:

$$\frac{\partial V}{\partial t} = \frac{1}{\rho g} \frac{\partial^2 V}{\partial x^2} \quad (1.55)$$

We know that $\rho g = 1$:

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} \quad (1.56)$$

Assume that:

$$V(x, t) = X(x)T(t) \quad (1.57)$$

Substituting:

$$X(x)T'(t) = X''(x)T(t) \quad (1.58)$$

$$\frac{T'(t)}{T(t)} = \frac{X''(x)}{X(x)} = \lambda \quad (1.59)$$

where λ is a constant. Consider the case for T

$$\frac{1}{T(t)} \frac{dT(t)}{dt} = \lambda \quad (1.60)$$

$$\int \left(\frac{1}{T(t)} \frac{dT(t)}{dt} \right) dt = \int (\lambda) dt \quad (1.61)$$

$$\ln(T(t)) = \lambda t + c \quad (1.62)$$

$$T(t) = Ae^{\lambda t} \quad (1.63)$$

We can consider three cases for the above.

Case 1: $\lambda = 0$:

$$X''(x) = 0 \quad (1.64)$$

$$X'(x) = a_1 \quad (1.65)$$

$$X(x) = a_1 x + a_2 \quad (1.66)$$

Therefore:

$$V(x, t) = X(x)T(t) \quad (1.67)$$

$$V(x, t) = (a_1 x + a_2) (Ae^0) \quad (1.68)$$

$$V(x, t) = Aa_1 x + Aa_2 \quad (1.69)$$

Consider the third boundary condition:

$$V(0, t) = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.70)$$

$$Aa_2 = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.71)$$

This only occurs at $t = 0$, hence the solution is trivial and does not contribute to the solution.

Case 2: $\lambda = \mu^2 > 0$

$$\frac{X''(x)}{X(x)} = \mu^2 \quad (1.72)$$

$$X''(x) - \mu^2 X(x) = 0 \quad (1.73)$$

Solving the second order differential:

$$m^2 = \mu^2 \quad (1.74)$$

$$m_1 = \mu, m_2 = -\mu \quad (1.75)$$

$$X(x) = b_1 e^{\mu x} + b_2 e^{-\mu x} \quad (1.76)$$

$$T(t) = A e^{\mu^2 t} \quad (1.77)$$

$$X(x)T(t) = (b_1 e^{\mu x} + b_2 e^{-\mu x}) A e^{\mu^2 t} \quad (1.78)$$

Consider the third boundary condition:

$$V(0, t) = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.79)$$

$$(b_1 + b_2) A e^{\mu^2 t} = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.80)$$

$$\therefore \mu^2 t = -\frac{\pi^2 t}{4l^2} \quad (1.81)$$

$$\mu = \sqrt{-\frac{\pi^2 t}{4l^2}} \quad (1.82)$$

μ is complex and gives a trivial solution, which does not contribute to the solution.

Case 3: $\lambda = \mu^2 < 0$:

$$X''(x) + \mu^2 X(x) = 0 \quad (1.83)$$

Solving the second order differential equation:

$$m^2 = -\mu^2 \quad (1.84)$$

$$m_1 = \mu i, m_2 = -\mu i \quad (1.85)$$

Using Euler's Formula:

$$X(x) = c_3 \cos \mu x + c_4 \sin \mu x \quad (1.86)$$

where $c_3 = c_1 + c_2$ and $c_4 = i(c_1 - c_2)$. Consider the third boundary condition:

$$V(0, t) = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.87)$$

$$(c_3 \cos 0 + c_4 \sin 0) (A e^{\lambda t}) = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.88)$$

$$c_3 A e^{\lambda t} = V_0 e^{-\frac{\pi^2 t}{4l^2}} \quad (1.89)$$

$$\therefore \lambda t = -\frac{\pi^2 t}{4l^2} \quad (1.90)$$

$$(1.91)$$

We know that $\mu < 0$, hence:

$$\mu = \frac{\pi}{2l} \quad (1.92)$$

Therefore:

$$V(x, t) = X(x)T(t) \quad (1.93)$$

$$V(x, t) = (d_1 \cos \mu x + d_2 \sin \mu x) e^{\lambda t} \quad (1.94)$$

where $d_1 = Ac_1$ and $d_2 = Ac_2$. Consider the first boundary condition:

$$V(x, 0) = V_0 \cos\left(\frac{\pi x}{2l}\right) \quad (1.95)$$

$$(d_1 \cos \mu x + d_2 \sin \mu x) e^{\lambda \cdot 0} = V_0 \cos\left(\frac{\pi x}{2l}\right) \quad (1.96)$$

$$d_1 \cos \mu x + d_2 \sin \mu x = V_0 \cos\left(\frac{\pi x}{2l}\right) \quad (1.97)$$

If $d_1 = V_0$ then $d_2 \sin \mu x = 0$, but we know that $\sin \mu x \neq 0$, leading to $d_2 = 0$. Consider the second boundary condition:

$$V(l, t) = 0 \quad (1.98)$$

$$V(l, t) = V_0 \cos\left(\frac{\pi}{2}\right) e^{-\frac{\pi^2 t}{4l^2}} \quad (1.99)$$

$$V(l, t) = V_0 \cdot 0 \quad (1.100)$$

Hence, $V_0 \neq 0$ for a non-trivial solution. Hence:

$$V(x, t) = V_0 \cos\left(\frac{\pi x}{2l}\right) e^{-\frac{\pi^2 t}{4l^2}} \quad (1.101)$$

1.4.2 Plot of v

We know that:

$$v = V e^{-\frac{rt}{\rho}} \quad (1.102)$$

$\frac{r}{\rho} = 1$:

$$v = V e^{-t} \quad (1.103)$$

$$v = V_0 \cos\left(\frac{\pi x}{2l}\right) e^{-\frac{\pi^2 t}{4l^2}} \cdot e^{-t} \quad (1.104)$$

$$v = V_0 \cos\left(\frac{\pi x}{2l}\right) e^{-t\left(1+\frac{\pi^2}{4l^2}\right)} \quad (1.105)$$

```

1 clc
2 clear
3 close all
4
5 v = zeros(3,1000);
6
7 for i = [1 2 3]
8 %define vars
9 x = linspace(0,8,1000);
10 V0 = 1;
11 l = 8;
12 t = 2*i;
13
14 %equation
15 v(i,:) = V0.* (cos((pi.*x)./(2.*l))).*(exp(-t.*((1+(pi.^2)./(4.*l.^2))))) ;

```

```

16 end
17
18 %plots
19 plot(x,v(1,:),'red',x,v(2,:),'blue',x,v(3,:),'green')
20 title('Graph to show flow velocity against position along vessel')
21 xlabel('Distance along vessel/[L]')
22 ylabel('Flow velocity/[LS^{-1}]')
23 grid on
24 axis auto
25 legend('t = 2s', 't = 4s', 't = 6s')

```

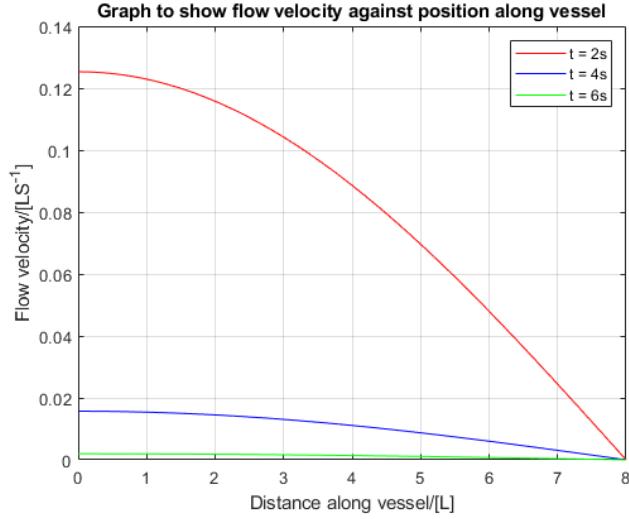


Figure 2: Graph of flow velocity against position in vessel for three values of t .

1.5 Finite difference numerical scheme

Starting with:

$$\frac{\partial v}{\partial t} = \frac{1}{\rho g} \frac{\partial^2 v}{\partial x^2} \quad (1.106)$$

$\rho g = 1$:

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} \quad (1.107)$$

We know that:

$$\frac{\partial v}{\partial t} = \frac{u_{i,j} - u_{i,j-1}}{k} \quad (1.108)$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \quad (1.109)$$

$$\therefore h^2 (u_{i,j} - u_{i,j-1}) = k (u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) \quad (1.110)$$

Expanding and simplifying:

$$h^2 u_{i,j} + 2ku_{i,j} = h^2 u_{i,j-1} + ku_{i+1,j} + ku_{i-1,j} \quad (1.111)$$

$$u_{i,j} = \frac{1}{h^2 + 2k} (h^2 u_{i,j-1} + ku_{i+1,j} + ku_{i-1,j}) \quad (1.112)$$

Using $h = k = 2$:

$$u_{i,j} = \frac{1}{8} (4u_{i,j-1} + 2u_{i+1,j} + 2u_{i-1,j}) \quad (1.113)$$

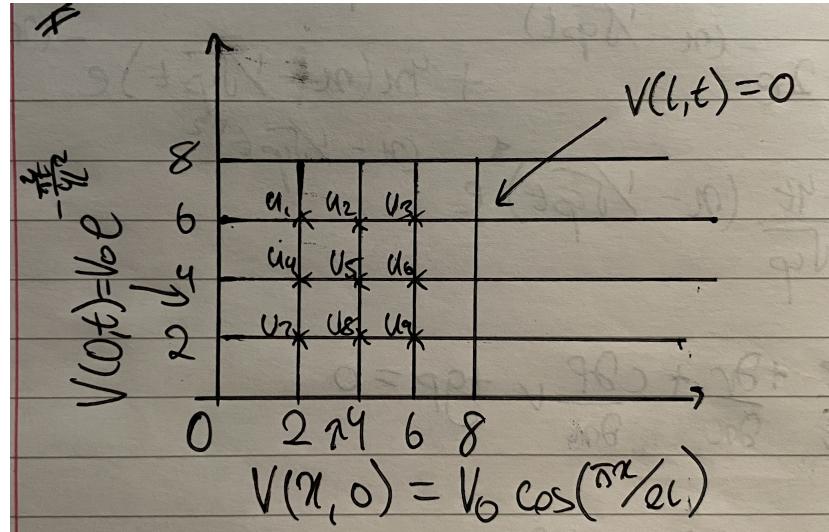


Figure 3: Sketch of domain.

Hence, our equations are:

$$u_1 = \frac{1}{8} \left(4u_4 + 2u_2 + 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \right) \quad (1.114)$$

$$u_2 = \frac{1}{8} (4u_5 + 2u_3 + 2u_1) \quad (1.115)$$

$$u_3 = \frac{1}{8} (4u_6 + 2(0) + 2u_2) \quad (1.116)$$

$$u_4 = \frac{1}{8} \left(4u_7 + 2u_5 + 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \right) \quad (1.117)$$

$$u_5 = \frac{1}{8} (4u_8 + 2u_6 + 2u_4) \quad (1.118)$$

$$u_6 = \frac{1}{8} (4u_9 + 2(0) + 2u_5) \quad (1.119)$$

$$u_7 = \frac{1}{8} \left(4V_0 \cos \left(\frac{\pi x}{2l} \right) + 2u_8 + 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \right) \quad (1.120)$$

$$u_8 = \frac{1}{8} \left(4V_0 \cos \left(\frac{\pi x}{2l} \right) + 2u_9 + 2u_7 \right) \quad (1.121)$$

$$u_9 = \frac{1}{8} \left(4V_0 \cos \left(\frac{\pi x}{2l} \right) + 2(0) + 2u_8 \right) \quad (1.122)$$

Matrix:

$$\begin{pmatrix} 8 & -2 & 0 & -4 & 0 & 0 & 0 & 0 & 0 \\ -2 & 8 & -2 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & -2 & 8 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & -2 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & -2 & 8 & -2 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & -2 & 8 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 8 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{pmatrix} = \begin{pmatrix} 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \\ 0 \\ 0 \\ 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \\ 0 \\ 0 \\ 4V_0 \cos\left(\frac{\pi x}{2l}\right) + 2V_0 e^{-\frac{\pi^2 t}{4l^2}} \\ 4V_0 \cos\left(\frac{\pi x}{2l}\right) \\ 4V_0 \cos\left(\frac{\pi x}{2l}\right) \end{pmatrix} \quad (1.123)$$

1.5.1 Implementation of numerical scheme in MATLAB

```

1 clc
2 clear
3 close all
4
5 %define vars
6 V0 = 1;
7 l = 8;
8 x = linspace(0,1,1000);
9
10 %create matrices
11 A = [8 -2 0 -4 0 0 0 0 0;
12     -2 8 -2 0 -4 0 0 0 0;
13     0 -2 8 0 0 -4 0 0 0;
14     0 0 0 8 -2 0 -4 0 0;
15     0 0 0 -2 8 -2 0 -4 0;
16     0 0 0 0 -2 8 0 0 -4;
17     0 0 0 0 0 0 8 -2 0;
18     0 0 0 0 0 0 -2 8 -2;
19     0 0 0 0 0 0 0 -2 8];
20
21 B = [2*V0*exp(-((pi^2)*6)/(4*l^2));
22     0;
23     0;
24     2*V0*exp(-((pi^2)*4)/(4*l^2));
25     0;
26     0;
27     4*V0*cos((pi*2)/(2*l)) + 2*V0*exp(-((pi^2)*2)/(4*l^2));
28     4*V0*cos((pi*4)/(2*l));
29     4*V0*cos((pi*6)/(2*l))];
30
31 %solve matrix
32 sol = A\B;
33
34 %exact answer check
35 solution = zeros(9,1); %initialise solution matrix
36 k = 1; %initialise counter
37

```

```

38 for j = [6 4 2] %y-values
39     for i = [2 4 6] %x-values
40         solution(k) = V0*(cos((pi*i)/(2*l)))*exp(-((pi^2)*j)/(4*l^2)); %
41             equation
42         k = k+1; %advance counter
43     end
44
45 %find percentage error
46 error = 100.*sqrt((solution - sol).^2)./solution;

```

This gave the following output:

Variable	Estimated value	Exact value	Percentage error
u_1	0.7376	0.7331	0.6212
u_2	0.5660	0.5611	0.8766
u_3	0.3066	0.3037	0.9769
u_4	0.7955	0.7918	0.4660
u_5	0.6099	0.6061	0.6289
u_6	0.3302	0.3280	0.6855
u_7	0.8576	0.8553	0.2663
u_8	0.6568	0.6546	0.3372
u_9	0.3556	0.3543	0.3576

Table 1: Table to show values of variables.

From our percentage errors, we can see that no value exceeds an error of 1%, hence we can say that the approximation is effective.

1.6 Implications on stiffening blood vessel walls in the ageing population

2 Vector calculus

2.1 Proof that divergence of velocity equals zero

Proof. If the fluid is incompressible, our total derivative is zero:

$$\frac{D\rho}{Dt} = 0 \quad (2.1)$$

We can start to derive the divergence of the velocity by rewriting the second term in 2.2:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0 \quad (2.2)$$

$$\frac{\partial \rho}{\partial t} + \rho (\nabla \cdot \underline{u}) + \underline{u} \cdot (\nabla \rho) = 0 \quad (2.3)$$

Looking at the $\nabla \rho$ term:

$$\nabla \rho = \left(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y}, \frac{\partial \rho}{\partial z} \right) \quad (2.4)$$

We know that all derivatives of ρ are zero as ρ is a constant, hence:

$$0 + \rho(\nabla \cdot \underline{u}) + 0 = 0 \quad (2.5)$$

$$\nabla \cdot \underline{u} = 0 \quad (2.6)$$

□

2.2 Acceleration of fluid element

Fluid element acceleration is given by:

$$\frac{D\underline{u}}{Dt} = \frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \quad (2.7)$$

Flow is steady, hence

$$\frac{D\underline{u}}{Dt} = 0 + (\underline{u} \cdot \nabla) \underline{u} \quad (2.8)$$

$$= -\omega y \frac{\partial \underline{u}}{\partial x} + -\omega x \frac{\partial \underline{u}}{\partial y} + 0 \frac{\partial \underline{u}}{\partial z} \quad (2.9)$$

$$= -\omega y \frac{\partial \underline{u}}{\partial x} + \omega x \frac{\partial \underline{u}}{\partial x} \quad (2.10)$$

$$= -\omega y \begin{pmatrix} 0 \\ \omega \\ 0 \end{pmatrix} + \omega x \begin{pmatrix} -\omega \\ 0 \\ 0 \end{pmatrix} \quad (2.11)$$

$$= \begin{pmatrix} -\omega^2 x \\ -\omega^2 y \\ 0 \end{pmatrix} \quad (2.12)$$

2.3 Integral

Considering the volume of an element V , where V is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$:

$$\iiint_V (xyz) dz dy dx \quad (2.13)$$

2.3.1 Area of integration

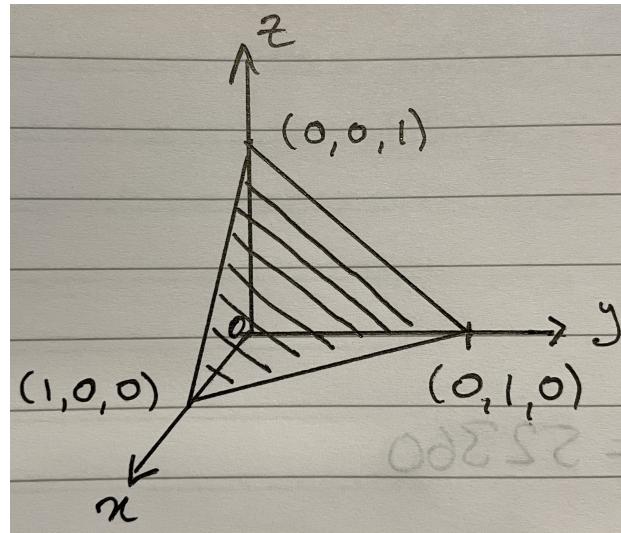


Figure 4: Graph to show area of integration of function.

2.3.2 Find the limits of integration

We know the volume is bounded by the $x-y$, $x-z$ and $y-z$ planes. Hence, our lower limits are:

$$x = 0, y = 0, z = 0 \quad (2.14)$$

Our upper bound is $x + y + z \leq 1$. Hence, the upper bound for z is:

$$x + y + z \leq 1 \quad (2.15)$$

$$z \leq 1 - x - y \quad (2.16)$$

Upper bound for y ($x-y$ plane $\rightarrow z=0$):

$$x + y \leq 1 \quad (2.17)$$

$$y \leq 1 - x \quad (2.18)$$

Upper bound for x ($y=z=0$)

$$x \leq 1 \quad (2.19)$$

2.3.3 Calculation of triple integral

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} (xyz) dz dy dx \quad (2.20)$$

Computing the z integral:

$$= xy \int_0^{1-x-y} (z) dz \quad (2.21)$$

$$= xy \left[\frac{z^2}{2} \right]_0^{1-x-y} \quad (2.22)$$

$$= xy \left[\frac{(1-x-y)^2}{2} - \frac{0^2}{2} \right] \quad (2.23)$$

$$= \frac{xy}{2} (y^2 + x^2 + 2xy - 2x - 2y + 1) \quad (2.24)$$

$$= \frac{1}{2} (xy^3 + x^3y + 2x^2y^2 - 2x^2y - 2y^2x + xy) \quad (2.25)$$

Inputting 2.25 into 2.20:

$$\int_0^1 \int_0^{1-x} \left(\frac{1}{2} (xy^3 + x^3y + 2x^2y^2 - 2x^2y - 2y^2x + xy) \right) dy dx \quad (2.26)$$

Computing the y integral:

$$= \frac{1}{2} \int_0^{1-x} (xy^3 + x^3y + 2x^2y^2 - 2x^2y - 2y^2x + xy) dy \quad (2.27)$$

$$= \frac{1}{2} \left[\frac{xy^4}{4} + \frac{x^3y^2}{2} + \frac{2x^2y^3}{3} - x^2y^2 - \frac{2xy^3}{3} + \frac{xy^2}{2} \right]_0^{1-x} \quad (2.28)$$

$$= \frac{1}{2} \left[\frac{x(1-x)^4}{4} + \frac{x^3(1-x)^2}{2} + \frac{2x^2(1-x)^3}{3} - x^2(1-x)^2 - \frac{2x(1-x)^3}{3} + \frac{x(1-x)^2}{2} \right] \quad (2.29)$$

Expanding:

$$\begin{aligned} &= \frac{1}{2} \left[\frac{x - 4x^2 + 6x^3 - 4x^4 + x^5}{4} + \frac{x^3 - 2x^4 + x^5}{2} + \frac{2x^2 - 6x^3 + 6x^4 - 2x^5}{3} \right. \\ &\quad \left. - (x^2 - 2x^3 + x^4) - \frac{2x - 6x^2 + 6x^3 - 2x^4}{3} + \frac{x - 2x^2 + x^3}{2} \right] \end{aligned} \quad (2.30)$$

Simplifying

$$= \frac{x^5 - 4x^4 + 6x^3 - 4x^2 + x}{24} \quad (2.31)$$

$$= \frac{1}{24} (x^5 - 4x^4 + 6x^3 - 4x^2 + x) \quad (2.32)$$

Inputting 2.32 into 2.26:

$$\int_0^1 \left(\frac{1}{24} (x^5 - 4x^4 + 6x^3 - 4x^2 + x) \right) dx \quad (2.33)$$

Computing the x integral:

$$= \frac{1}{24} \left[\frac{x^6}{6} - \frac{4x^5}{5} + \frac{3x^4}{2} - \frac{4x^3}{3} + \frac{x^2}{2} \right]_0^1 \quad (2.34)$$

$$= \frac{1}{24} \left[\frac{1}{6} - \frac{4}{5} + \frac{3}{2} - \frac{4}{3} + \frac{1}{2} \right] \quad (2.35)$$

$$= \frac{1}{720} \quad (2.36)$$

3 Transforms

3.1 Plot of data

```
1 clc
2 clear
3 close all
4
5 %import data
6 data = readmatrix('Section3_data.txt');
7
8 %plot data
9 plot(data(:,1), data(:,2))
10 title('Graph to show variation in signal over a period of 100 seconds')
11 xlim([0 100])
12 ylim([-5 5])
13 xlabel('Time/s')
14 ylabel('Pulse oximeter signal/arbitrary units')
15 grid on
```

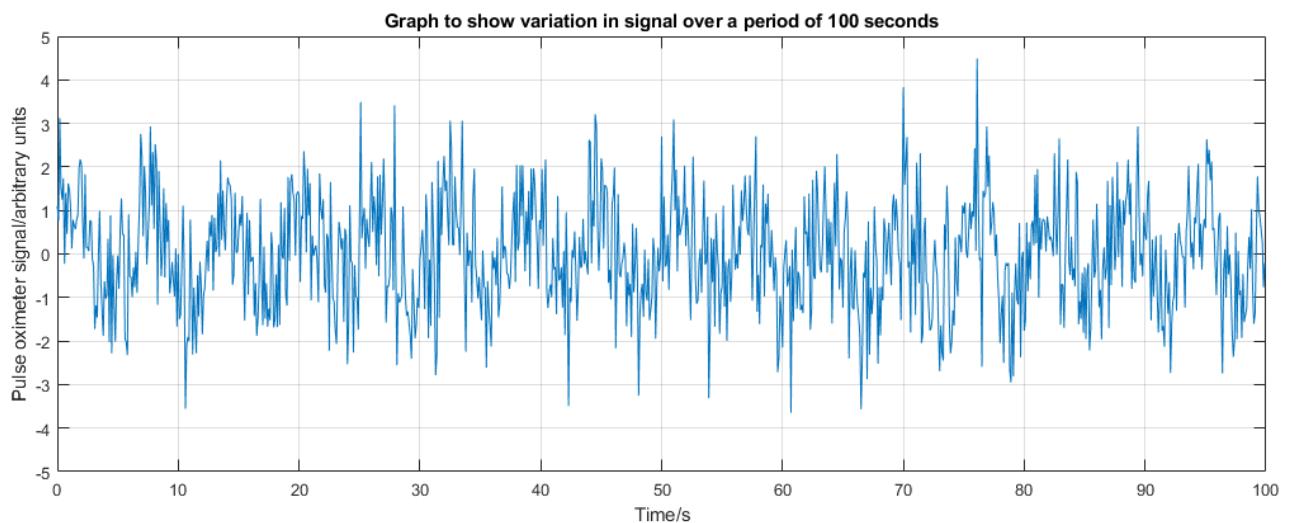


Figure 5: Graph to show variation in signal over a period of 100 seconds.

3.2 Plot of Fourier transform

```
1 clc
2 clear
3 close all
4
5 %import data
6 data = readmatrix('Section3_data.txt');
7
8 y = fft(data(:,2)); %compute discrete Fourier transform of data, (fast
%Fourier transform algorithim), indexing pulse oximeter data
9 n = length(data(:,2)); %find length of matrix
10 Fs = 10; % Sampling frequency (Hz)
```

```

11 f =(0:n-1)*(Fs/n); % Frequency range
12 fshift = (-n/2:n/2-1)*(Fs/n); %defines x-axis range for shifted transform
13 yshift = fftshift(y); %shifts zero-frequency component to centre of the
14 array, this swaps the left and the right halves of x
15 figure;
16 %plot data
17 plot(fshift, abs(yshift))
18 title('Graph to show absolute values of transform in the frequency domain')
19 xlabel('Frequency/Hz')
20 ylabel('Fourier transform of signal data/arbitrary units')
21 axis square
22 grid on

```

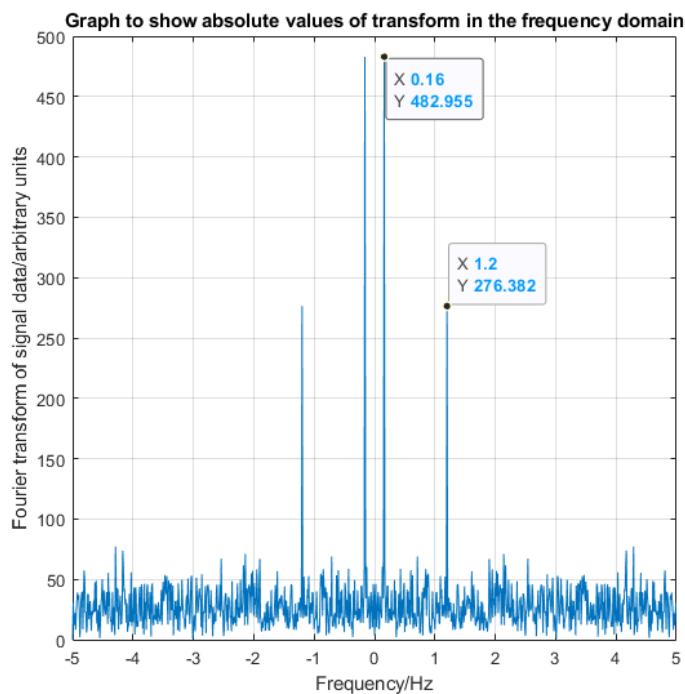


Figure 6: Graph to show absolute values of transform in the frequency domain.

3.3 Extraction of patient's cardiac and respiratory cycle

As seen from Figure 6, we can extract two values from our Fourier transform. The higher peak has a frequency of 0.16 Hz and a period of 6.25 s. This represents the breathing of the subject (9.6 breaths per minute). According to a Cleveland Clinic article on vital signs, the average human breathing rate for adults should be around 12-16 breaths per minute [1]. The lower peak has a frequency of 1.2 Hz and a period of 0.83 s. This represents the heartbeat of the subject (72 beats per minute). According to the British Heart Foundation, the average resting heart rate for adults is between 60-100 beats per minute [2].

3.4 Frequency filter

3.4.1 Gaussian functions

A Gaussian function is defined below as:

$$f(x, \sigma, \mu) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}} \quad (3.1)$$

Here μ determines where the peak of our curve is and σ determines the 'width' of our curve. We want to set μ to 1.2 and -1.2 to focus on the cardiac signal. We also want our standard deviation to be low, as to generate narrow peaks. We shall be utilising a special form of the Gaussian function for our filters which always has a maximum value of 1. This is called a Gaussian membership function and has the equation:

$$f(x, \sigma, \mu) = e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}} \quad (3.2)$$

A Gaussian membership function was generated using MATLAB's "gaussmf" function in the 'Fuzzy logic toolbox' [3]. $\mu = \pm 1.2$. $\sigma = 0.01$ was selected arbitrarily to de-noise the signal to an appropriate level.

```

1 clc
2 clear
3 close all
4 %import data
5 data = readmatrix('Section3_data.txt');
6
7 y = fft(data(:,2)); %compute discrete Fourier transform of data, (fast
        Fourier transform algorithm), indexing pulse oximeter data
8 n = length(data(:,2)); %find length of matrix
9 Fs = 10; % Sampling frequency (Hz)
10 fshift = (-n/2:n/2-1)*(Fs/n); %defines x-axis range for shifted transform
11 z = [gaussmf(fshift, [0.01 1.2])' + gaussmf(fshift, [0.01 -1.2])'];%%
        generate and add gaussians
12
13 %plot data
14 plot(fshift, z)
15 title('Graph to show filter, centred at positive and negative cardiac
        frequencies')
16 axis square;
17 grid on
18 xlabel('Frequency/Hz')
19 ylabel('Magnitude/arbitrary units')
```

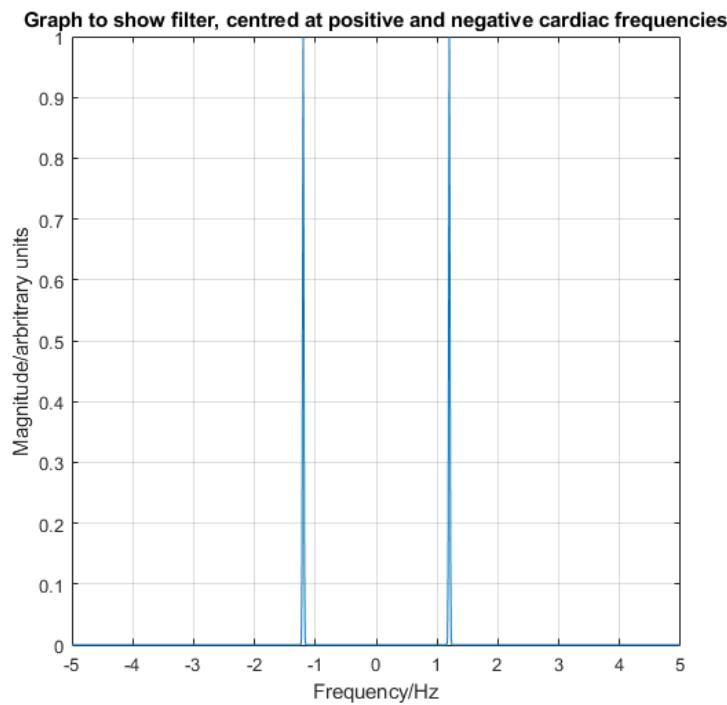


Figure 7: Graph to show filter, centred at positive and negative cardiac frequencies.

3.4.2 Filtered/unfiltered Fourier data comparison

```

1 clc
2 clear
3 close all
4
5 %import data
6 data = readmatrix('Section3_data.txt');
7
8 y = fft(data(:,2)); %compute discrete Fourier transform of data, (fast
    Fourier transform algorithim), indexing pulse oximeter data
9 n = length(data(:,2)); %find length of matrix
10 Fs = 10; % Sampling frequency (Hz)
11 f =(0:n-1)*(Fs/n); % Frequency range
12 fshift = (-n/2:n/2-1)*(Fs/n); %defines x-axis range for shifted transform
13 yshift = fftshift(y); %shifts zero-frequency component to centre of the
    array, this swaps the left and the right halves of x
14 z = [gaussmf(fshift, [0.01 1.2])' + gaussmf(fshift, [0.01 -1.2])'];%%
    generate and add gaussians
15 filtData = yshift.*z; %multiply FT signal data with gaussian
16 figure;
17
18 %plot data
19 plot(fshift, abs(yshift), fshift, abs(filtData))
20 title('Graph to show comparison between filtered and unfiltered FT signal
    ')
21 xlabel('Frequency/Hz')
22 ylabel('Fourier transform of signal data/arbitrary units')
23 legend('Unfiltered data','Filtered data')
```

```

24 axis square
25 grid on
26 figure(2);
27 plot(fshift, abs(yshift), fshift, abs(filtData))
28 xlim([0.7 1.7])
29 ylim([0 150])
30 title('Graph to show comparison between filtered and unfiltered FT signal
')
31 xlabel('Frequency/Hz')
32 ylabel('Fourier transform of signal data/arbitrary units')
33 legend('Unfiltered data','Filtered data')
34 axis square
35 grid on

```

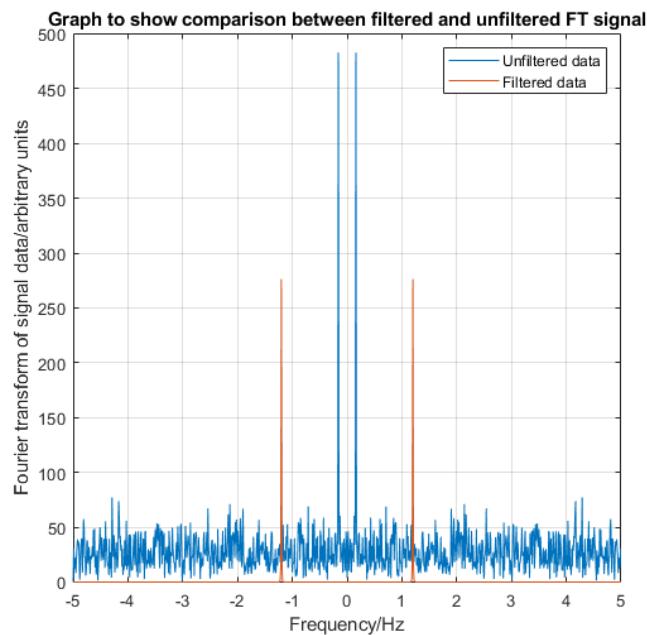


Figure 8: Graph to show comparison between filtered and unfiltered FT signal.

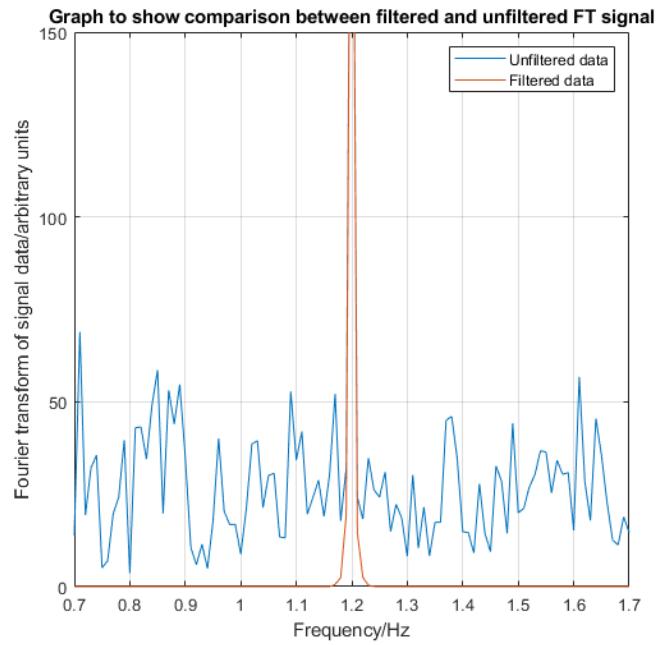


Figure 9: Graph to show comparison between filtered and unfiltered FT signal (close-up).

3.5 Filtered data

```

1 clc
2 clear
3 close all
4
5 %import data
6 data = readmatrix('Section3_data.txt');
7
8 y = fft(data(:,2)); %compute discrete Fourier transform of data, (fast
    %Fourier transform algorithm), indexing pulse oximeter data
9 n = length(data(:,2)); %find length of matrix
10 Fs = 10; % Sampling frequency (Hz)
11 f =(0:n-1)*(Fs/n); % Frequency range
12 fshift = (-n/2:n/2-1)*(Fs/n); %defines x-axis range for shifted transform
13 yshift = fftshift(y); %shifts zero-frequency component to centre of the
    %array, this swaps the left and the right halves of x
14 z = [gaussmf(fshift, [0.01 1.2])' + gaussmf(fshift, [0.01 -1.2])'];%%
    %generate and add gaussians
15 filtData = yshift.*z; %multiply FT signal data with gaussian
16 y2 = ifftshift(filtData); %inverse zero frequency shift
17 x2 = ifft(y2); %inverse fourier
18 figure;
19
20 %plot data
21 plot(data(:,1), x2)
22 title('Graph to show filtered data from pulse oximeter')
23 xlabel('Time/s')
24 ylabel('Pulse oximeter signal/arbitrary units')
25 axis auto
26 grid on

```

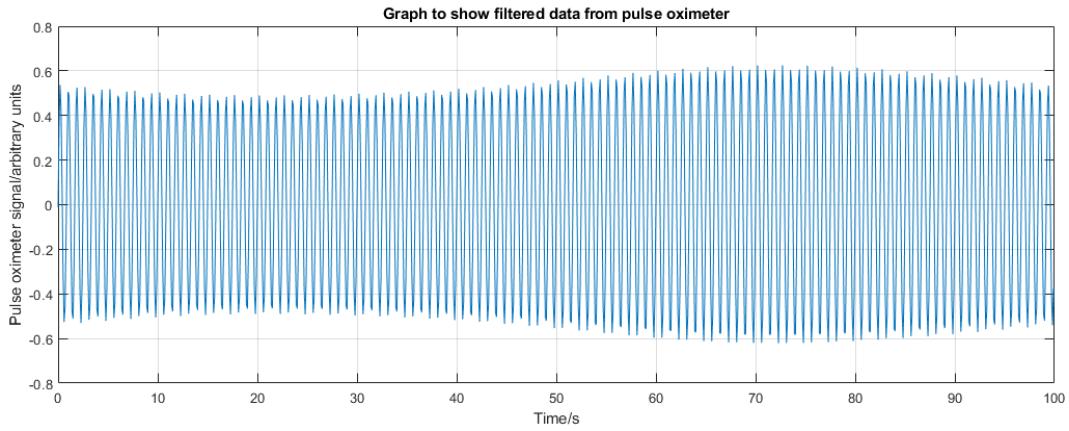


Figure 10: Graph to show filtered data from pulse oximeter.

3.6 Effect of varying the width of Gaussian function

The code was adjusted to created two additional cases, to make four in total:

- Unfiltered data
- Gaussian filter with $\sigma = 0.1$
- Gaussian filter with $\sigma = 0.01$
- Gaussian filter with $\sigma = 0.001$

```

1 clc
2 clear
3 close all
4
5 %import data
6 data = readmatrix('Section3_data.txt');
7
8 y = fft(data(:,2)); %compute discrete Fourier transform of data, (fast
    %Fourier transform algorithim), indexing pulse oximeter data
9 n = length(data(:,2)); %find length of matrix
10 Fs = 10; % Sampling frequency (Hz)
11 f =(0:n-1)*(Fs/n); % Frequency range
12 fshift = (-n/2:n/2-1)*(Fs/n); %defines x-axis range for shifted transform
13 yshift = fftshift(y); %shifts zero-frequency component to centre of the
    %array, this swaps the left and the right halves of x
14 z1 = [gaussmf(fshift, [0.01 1.2])' + gaussmf(fshift, [0.01 -1.2])'];%
    %generate and add gaussians
15 z2 = [gaussmf(fshift, [0.1 1.2])' + gaussmf(fshift, [0.1 -1.2])'];%
    %generate and add gaussians
16 z3 = [gaussmf(fshift, [0.001 1.2])' + gaussmf(fshift, [0.001 -1.2])'];%
    %generate and add gaussians
17 filtData1 = yshift.*z1; %multiply FT signal data with gaussian 0.1
18 filtData2 = yshift.*z2; %multiply FT signal data with gaussian 0.01
19 filtData3 = yshift.*z3; %multiply FT signal data with gaussian 0.001
20 y21 = ifftshift(filtData1); %inverse zero frequency shift 0.1

```

```

21 x21 = ifft(y21); %inverse fourier
22 y22 = ifftshift(filtData2); %inverse zero frequency shift 0.01
23 x22 = ifft(y22); %inverse fourier
24 y23 = ifftshift(filtData3); %inverse zero frequency shift 0.001
25 x23 = ifft(y23); %inverse fourier
26 figure;
27
28 %plot data
29 subplot(2,2,1)
30 plot(fshift, abs(yshift))
31 title('unfiltered')
32 xlim([0.7 1.7])
33 ylim([0 150])
34 xlabel('Magnitude')
35 ylabel('Frequency/Hz')
36 axis square
37 grid on
38 subplot(2,2,2)
39 plot(fshift, abs(filtData2))
40 title('stdev = 0.1')
41 xlim([0.7 1.7])
42 ylim([0 150])
43 xlabel('Magnitude')
44 ylabel('Frequency/Hz')
45 axis square
46 grid on
47 subplot(2,2,3)
48 plot(fshift, abs(filtData1))
49 xlim([0.7 1.7])
50 ylim([0 150])
51 xlabel('Magnitude')
52 ylabel('Frequency/Hz')
53 title('stdev = 0.01')
54 axis square
55 grid on
56 subplot(2,2,4)
57 plot(fshift, abs(filtData3))
58 xlim([0.7 1.7])
59 ylim([0 150])
60 xlabel('Magnitude')
61 ylabel('Frequency/Hz')
62 title('stdev = 0.001')
63 axis square
64 grid on
65
66 figure(2)
67 subplot(4,1,1)
68 plot(data(:,1), data(:,2))
69 title('unfiltered')
70 xlabel('Time/s')
71 ylabel('Magnitude')
72 axis auto
73 grid on

```

```
74 subplot(4,1,2)
75 plot(data(:,1), x22)
76 title('stdev = 0.1')
77 xlabel('Time/s')
78 ylabel('Magnitude')
79 axis auto
80 grid on
81 subplot(4,1,3)
82 plot(data(:,1), x21)
83 title('stdev = 0.01')
84 xlabel('Time/s')
85 ylabel('Magnitude')
86 axis auto
87 grid on
88 subplot(4,1,4)
89 plot(data(:,1), x23)
90 title('stdev = 0.001')
91 xlabel('Time/s')
92 ylabel('Magnitude')
93 axis auto
94 grid on
```

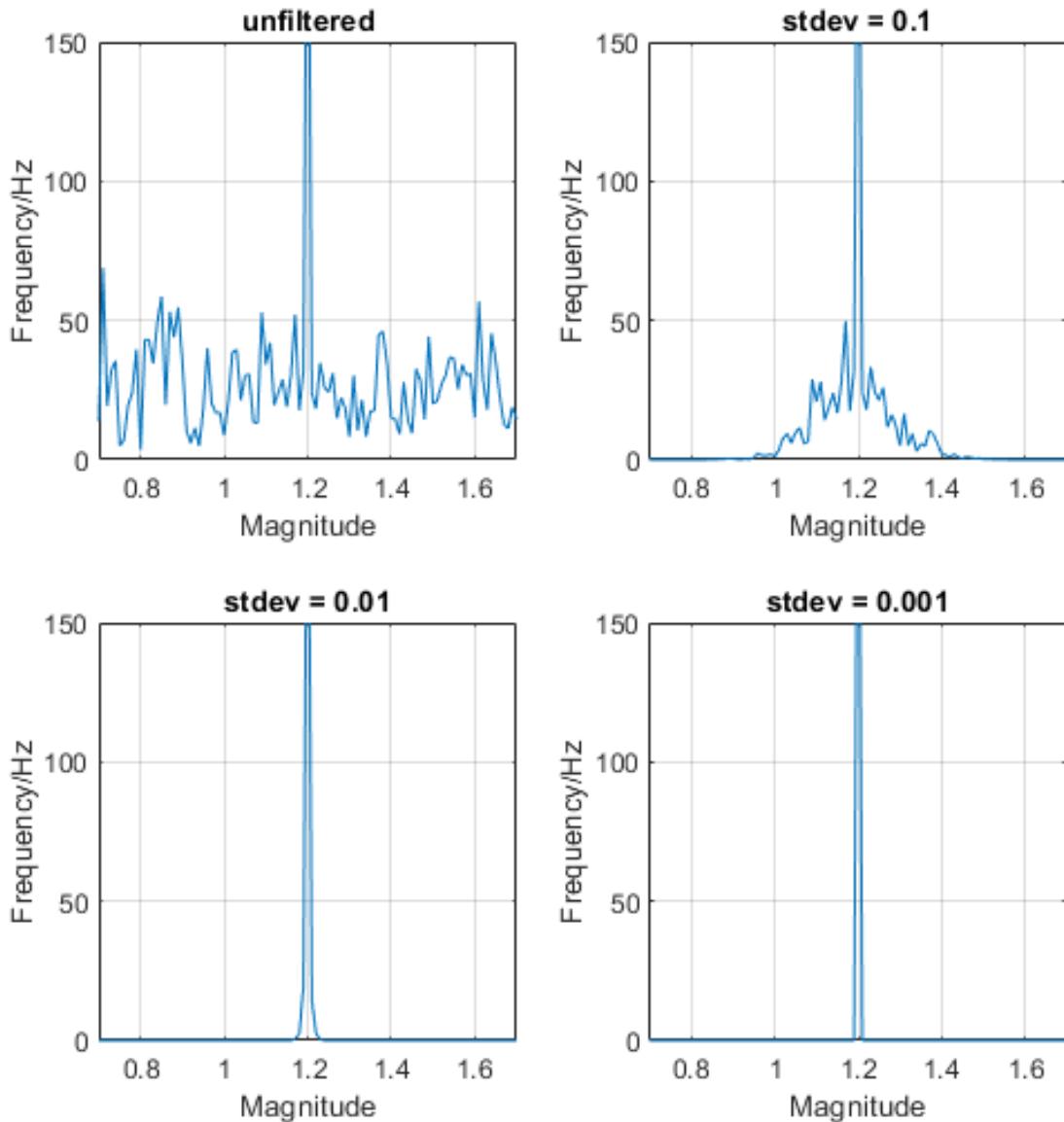


Figure 11: Graphs to compare the effect of varying Gaussian filter width on FT signal.

Here we can see that adjusting the value of σ effects the amount of noise that appears at the base of the peak in the Fourier transformed data. For $\sigma = 0.1$, there is still quite a bit of residual noise. $\sigma = 0.01$ and $\sigma = 0.001$ both do not exhibit any noise at the base, but we can see that for $\sigma = 0.01$, there is a slight flaring at the base.

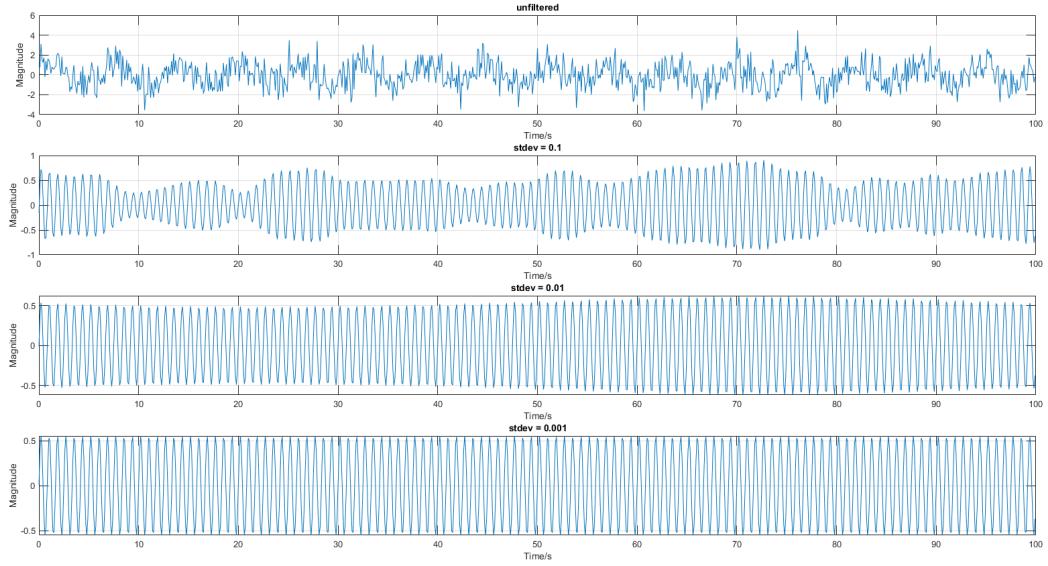


Figure 12: Graphs to compare the effect of varying Gaussian filter width on signal from pulse oximeter.

Here we can see the effect of the residual noise in the $\sigma = 0.1$ case, with relatively large variations in the amplitude of the signal. We can also see the effect of the flared base in the $\sigma = 0.01$ case as a smooth decrease and then increase in the amplitude of the signal. The $\sigma = 0.001$ case represents a virtually perfect signal with a frequency of 1.2 Hz.

4 Statistics

4.1 Confidence interval

Formula for calculating mean:

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} \quad (4.1)$$

Formula for sample standard deviation:

$$\sigma = \sqrt{\sum_{i=1}^n \frac{(\bar{x} - x_i)^2}{n-1}} \quad (4.2)$$

```

1 clc
2 clear
3 close all
4
5 %import data
6 rest = readmatrix('Section4_data.xlsx', 'Range', 'A2:A39');
7 anti = readmatrix('Section4_data.xlsx', 'Range', 'B2:B43');
8
9 nRest = numel(rest); %number of elements

```

```

10 nAnti = numel(anti);
11
12 muRest = mean(rest); %mean
13 muAnti = mean(anti);
14
15 sigmaRest = std(rest); % sample standard deviation
16 sigmaAnti = std(anti);

```

	Rest	Anticipation
<i>n</i>	38	42
Mean	86.7368	92.4048
Standard deviation	11.2842	16.6177

Table 2: Table to show values of number of elements, means and standard deviations of heart rate data.

A 95% confidence interval can be found using 4.3:

$$CI = \bar{x}_1 - \bar{x}_2 \pm z_{crit} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad (4.3)$$

$z_{crit} = 1.96$ for a 95% confidence interval in a two-tailed test, hence:

$$CI = 86.7368 - 92.4048 \pm 1.96 \sqrt{\frac{11.2842^2}{38} + \frac{16.6177^2}{42}} \quad (4.4)$$

$$CI_L = -11.84 \quad CI_H = 0.51 \quad (4.5)$$

We can say with 95% confidence that the true population mean's difference is in the interval [-11.85, 0.51]. This interval contains 0, hence we can deduce that there is not a significant indication of the means being different in this case.

However, if we reduce our z_{crit} value o

4.2 Reasoning for choice of test statistics

We have assumed that a normal distribution applies in this case. Our sample sizes are larger than 30 in both cases, making the normal distribution a good approximation. This allows us to approximate the sample's variance to the population's. Hence, using a z score is justified in this case. The Central Limit Theorem also tells us that our samples are normally distributed around 0, allowing for ease of calculation.

4.3 Hypothesis test

References

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