



UNIVERSITY COLLEGE LONDON

MENG MECHANICAL ENGINEERING

MECH0023 DYNAMICS AND CONTROL

TOPIC NOTES

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Contents

List of Figures	ii
List of Tables	iii
1 Introduction to Vibration	1
1.1 Introduction	1
1.1.1 What is the module about?	1
1.1.2 How to deal?	1
1.2 Fundamentals	2
1.2.1 Physical elements of vibrations	2
1.2.2 Degree of freedom	3
1.2.3 Continuous systems	5
1.2.4 Analysis procedure	5
1.2.5 Classification of vibration	8
1.2.6 Example: simple pendulum	8
1.2.7 Simple harmonic motion	9
1.2.8 Amplitude and phase	9
1.2.9 Springs	11
1.2.10 Modelling springs	11
1.2.11 Longitudinal motion of a bar	14
1.2.12 Torsional rotation of a shaft	14

List of Figures

1.1	Spring-mass system.	3
1.2	free body diagram.	3
1.3	SDOF systems.	4
1.4	Two degree of freedom systems.	4
1.5	Three degree of freedom systems.	5
1.6	Continuous degree of freedom.	5
1.7	Model of rider-motorbike-wheel system.	6
1.8	Various models of rider-motorbike-wheel system.	6
1.9	Complex model of rider-motorbike-wheel system.	7
1.10	Classification of vibrations.	8
1.11	Free body diagram of simple pendulum system.	8
1.12	Spring-mass system.	9
1.13	Free body diagram of spring-mass system.	10
1.14	Plots of SHM.	10
1.15	Analysis of SHM plots.	11
1.16	Spring system.	11
1.17	F vs x of spring.	11
1.19	Series springs.	12
1.18	Parallel springs.	12
1.20	Cantilever beam with mass at free end.	13
1.21	Longitudinal motion of a bar.	14
1.22	Torsional disc system.	14

List of Tables

Chapter 1

Introduction to Vibration

1.1 Introduction

1.1.1 What is the module about?

Dynamics: the study of forces and the resultant motion.

In this module we will study the oscillatory forces and the resulted motion of bodies, in other words "Mechanical Vibration."

- Harmful vibrations:
 - Vibrations can cause:
 - * Resonance
 - * Flutter
 - * Fatigue
 - It may cause discomfort and even be harmful to the human.
- Good vibrations:
 - Hearing
 - Loudspeakers
 - Musical instruments
 - Electric toothbrush
 - Clocks
 - Material handling
 - Sifting

1.1.2 How to deal?

Analysis:

- Mathematical modelling
- Derivation of governing equations
- Solution of the governing equations
- Interpretation of results

Measurements:

- Appropriate setup
- Interpretation of measurements
- Updating mathematical models

How would this module enable you to analyse dynamical systems?

- Mathematical modelling, week 1 also 2-11.
- Single degree of freedom systems
 - Free vibration
 - * Undamped, weeks 1 and 2
 - * Damped, week 3
 - Forced vibration of single degree of freedom system
 - * Harmonic excitation, weeks 4 and 5
 - * Arbitrary excitation, week 7
- Two degree of freedom system, week 8
- Multi degree of freedom systems, week 9
- The use of computer aided engineering to create a refined motor vehicle, invited talk, week 10
- Introduction to continuous systems, week 11
- Vibration measurements
- Vibration control

1.2 Fundamentals

1.2.1 Physical elements of vibrations

Vibration: A vibration or oscillation is a periodic motion, i.e. it repeats itself in all its particular after a certain interval of time.

In general in any mechanical oscillatory system there are:

- A **mass** that can store kinetic energy (accelerates when a load is applied upon)
- A **spring** that can store potential energy (constant displacement due to a constant force)
- A **damper** through which energy dissipates

What are the equivalent electrical elements?

- Force (F) - Voltage (V)
- Mass (M) - Inductance (L)
- Damping (B) - Resistance (R)
- Spring constant (K) - Reciprocal of Capacitance ($\frac{1}{C}$)
- Displacement (x) - Charge (q)
- Velocity (v) - Current (i)

In other words vibration is a result of the interaction of two forces. One a function of displacement (spring):

$$f_k = -kx(t) \quad (1.1)$$

One a function of acceleration (mass):

$$f_m = m\ddot{x}(t) \quad (1.2)$$

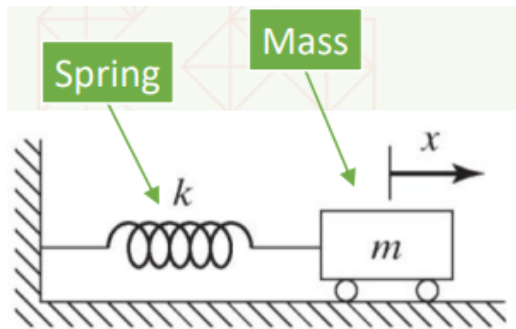


Figure 1.1: Spring-mass system.

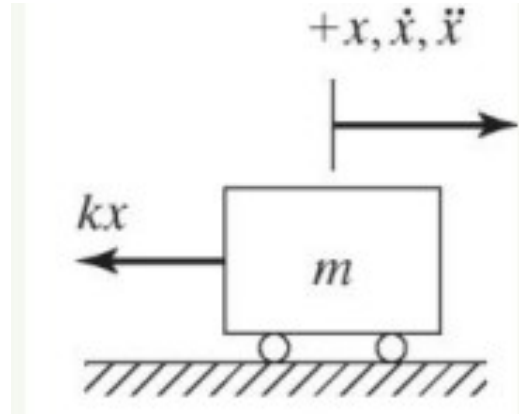


Figure 1.2: free body diagram.

Equation of motion:

$$\sum F = m\ddot{x}(t) \quad (1.3)$$

$$m\ddot{x}(t) = -kx(t) \quad (1.4)$$

Equation of motion:

$$m\ddot{x}(t) + kx(t) = 0 \quad (1.5)$$

Solution:

$$x(t) = A \sin(\omega_n t + \phi) \quad (1.6)$$

1.2.2 Degree of freedom

The minimum number of coordinates that are required to define the position of a system is called degree of freedom.

- Single degree of freedom (SDOF):
 - Only one coordinate is required to fully define the state of the system

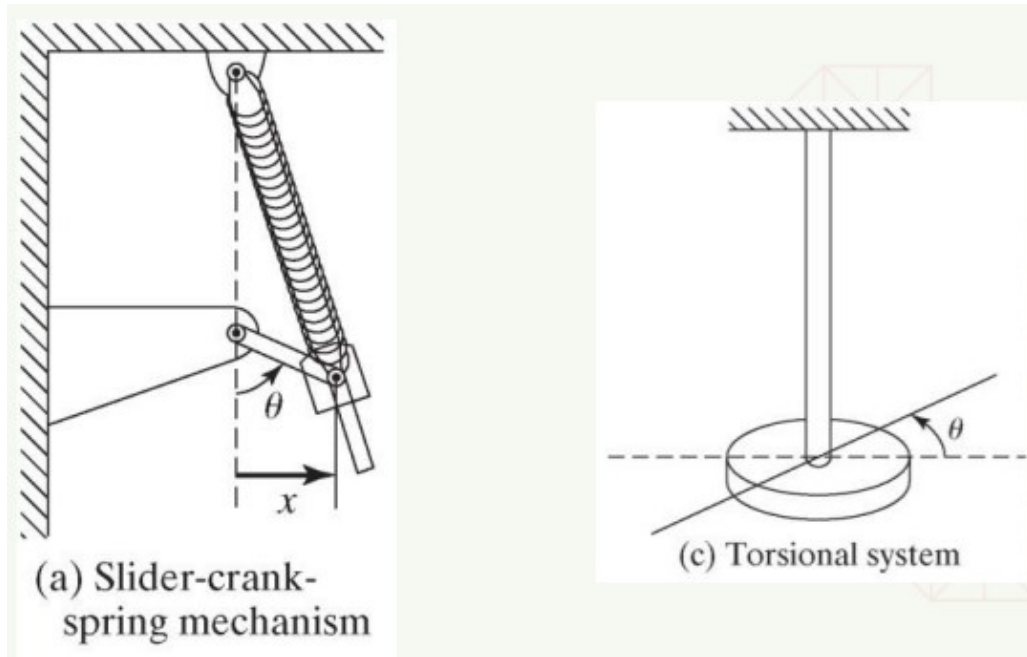


Figure 1.3: SDOF systems.

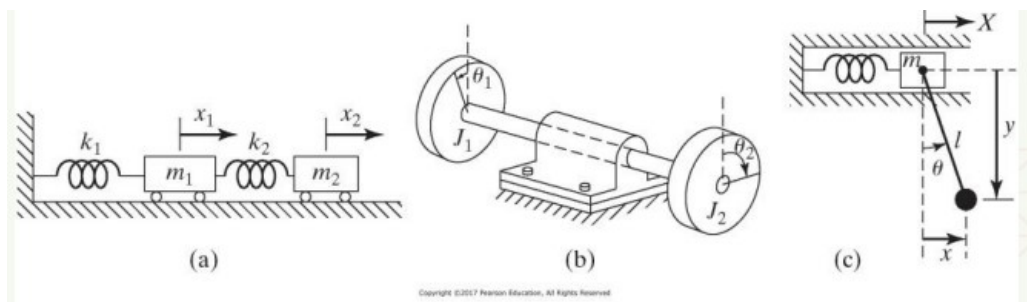


Figure 1.4: Two degree of freedom systems.

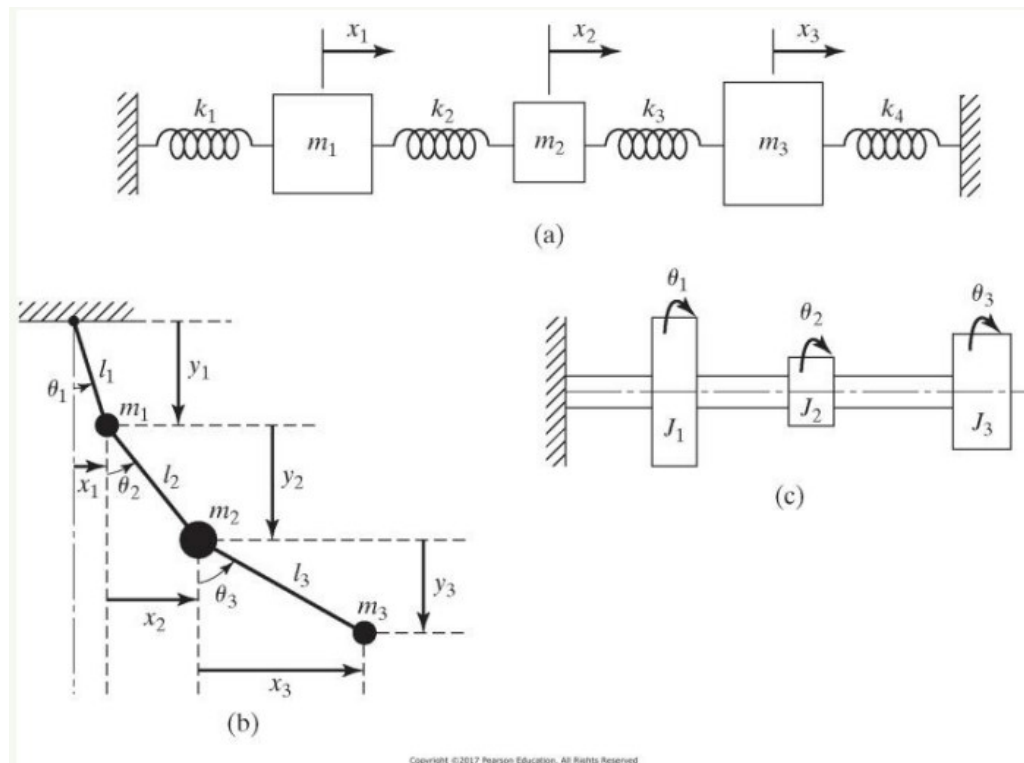


Figure 1.5: Three degree of freedom systems.

1.2.3 Continuous systems

Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom.

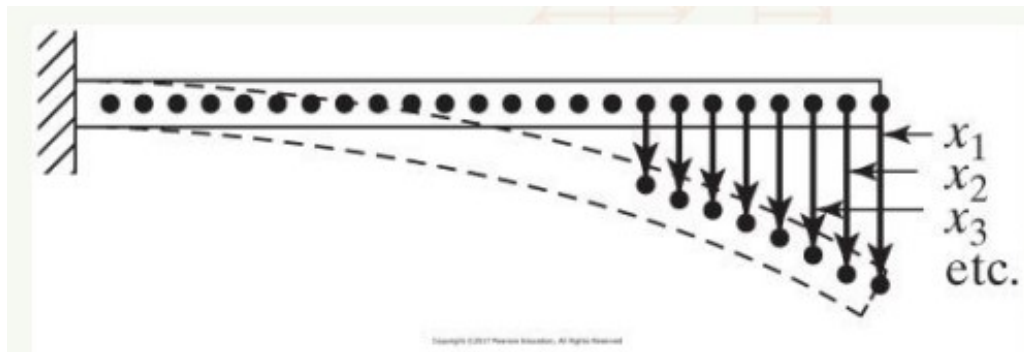


Figure 1.6: Continuous degree of freedom.

Discrete or lumped parameter systems: A system with finite number of degree of freedom. Continuous or distributed systems: an infinite number of degree of freedom.

1.2.4 Analysis procedure

Mathematical modelling

We need a mathematical model to obtain a solution for the vibrational problem. The model is a compromise between simplicity and accuracy. We make some assumption and our model is valid with the limitation of those assumptions only.

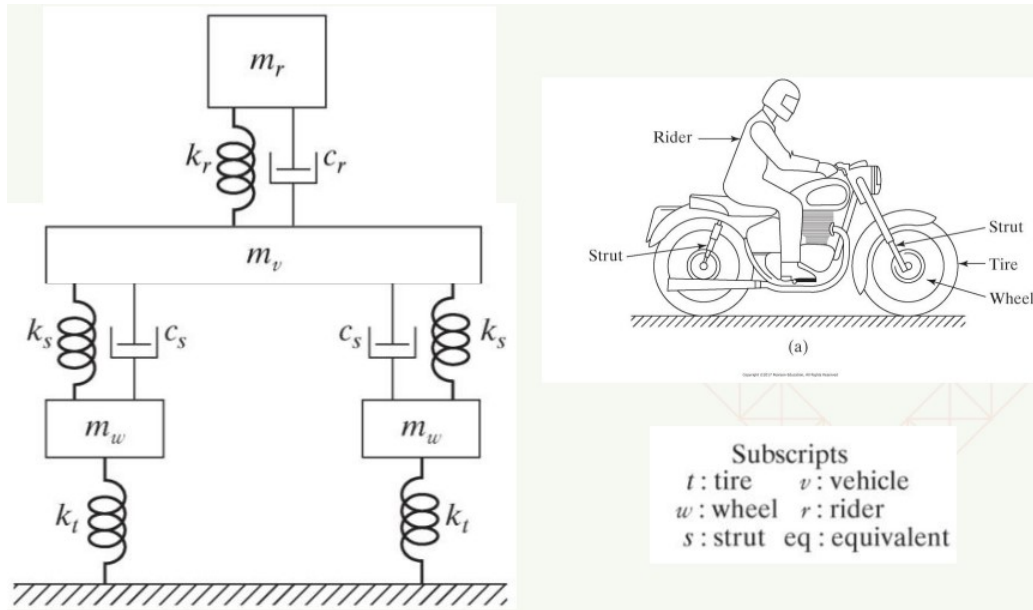


Figure 1.7: Model of rider-motorbike-wheel system.

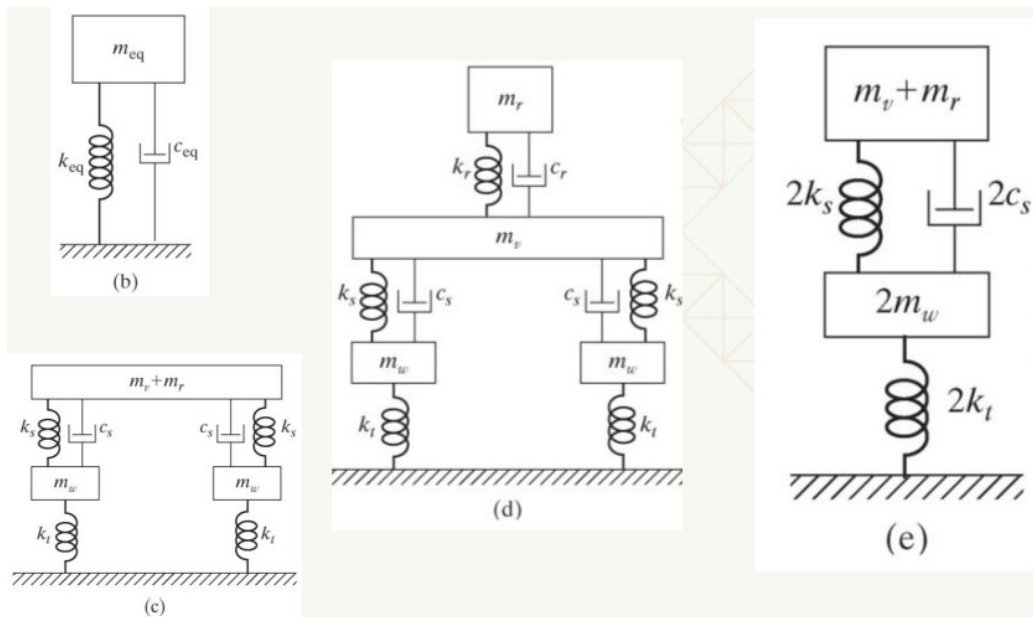


Figure 1.8: Various models of rider-motorbike-wheel system.

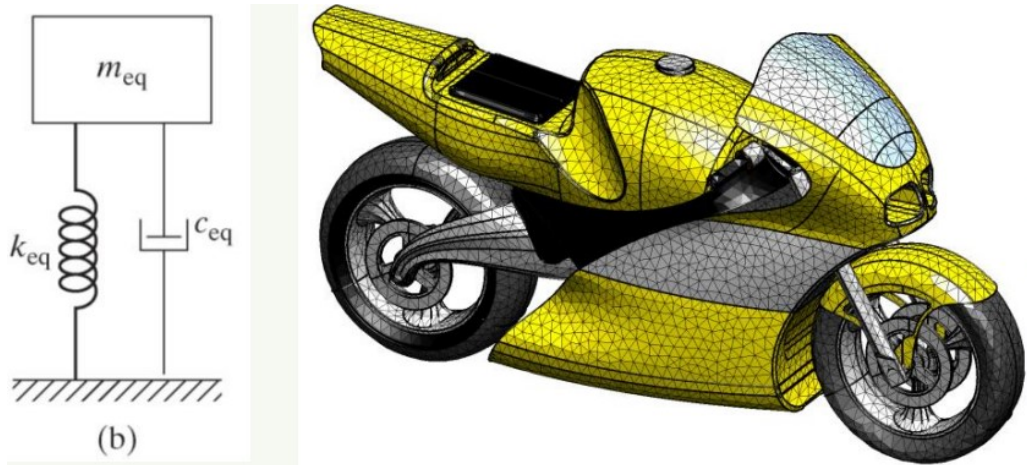


Figure 1.9: Complex model of rider-motorbike-wheel system.

Derivation of governing equations

Apply principle of dynamics to obtain the equations:

- Newton's second law of motion
- d'Alembert's principle
- The principle of conservation of energy

Usually in the form of a set of ordinary differential equations.

Solution of governing equations

- Standard method of solving differential equations
- Laplace transformation method
- Matrix methods
- Numerical methods

Interpretation of results

- Have a clear view of the purpose of the analysis
- Pay attention to the assumption made to obtain the results.

1.2.5 Classification of vibration

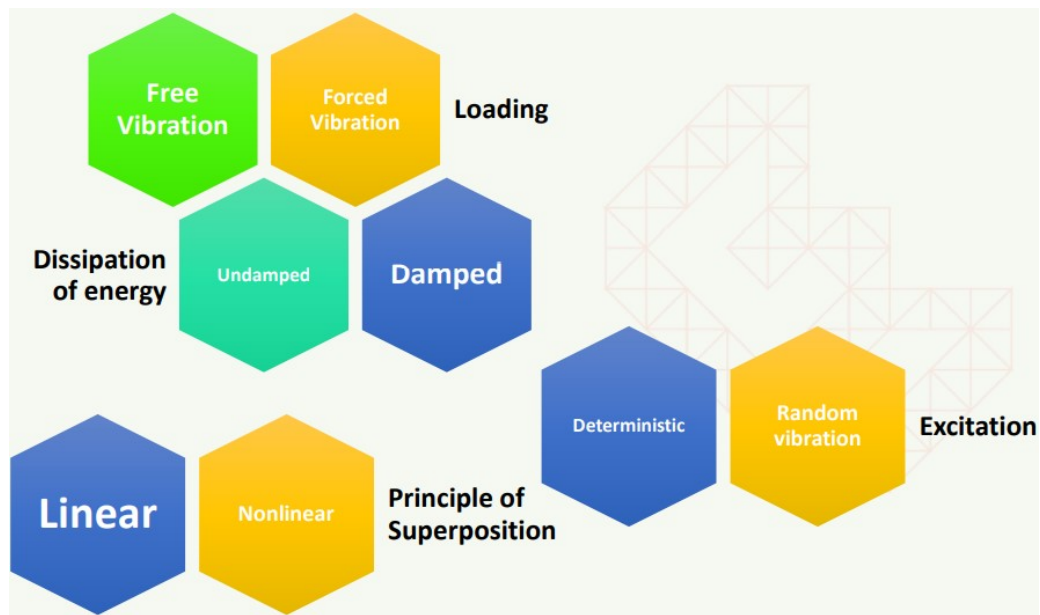


Figure 1.10: Classification of vibrations.

1.2.6 Example: simple pendulum

Deriving the equation of motion. We have some assumptions:

- The mass of the rod is ignored
- Friction in the hinges is ignored
- The motion remains in a plane

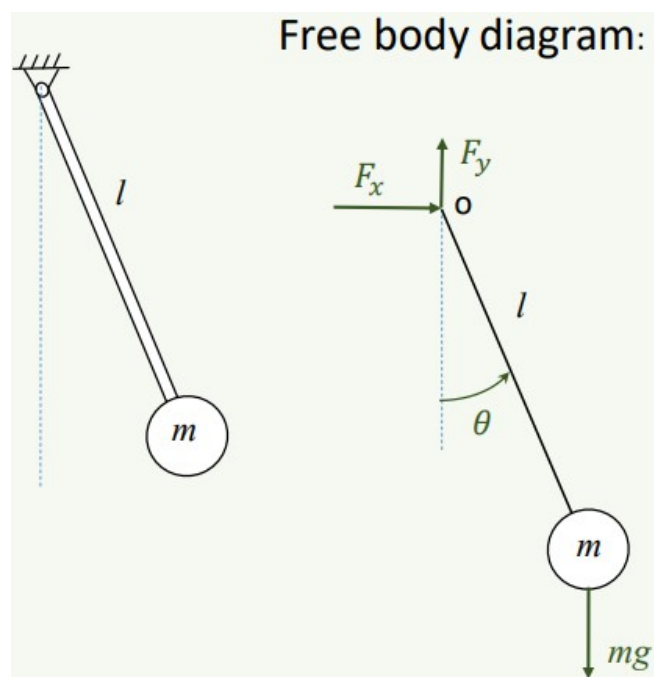


Figure 1.11: Free body diagram of simple pendulum system.

Euler's second law:

$$\sum M_o = J\alpha \quad (1.7)$$

$$J\alpha(t) = -mgl \sin \theta(t) \quad (1.8)$$

$$ml^2\ddot{\theta}(t) + mgl \sin \theta(t) = 0 \quad (1.9)$$

Linearisation:

$$\ddot{\theta}(t) + \frac{g}{l}\theta(t) = 0 \quad (1.10)$$

1.2.7 Simple harmonic motion

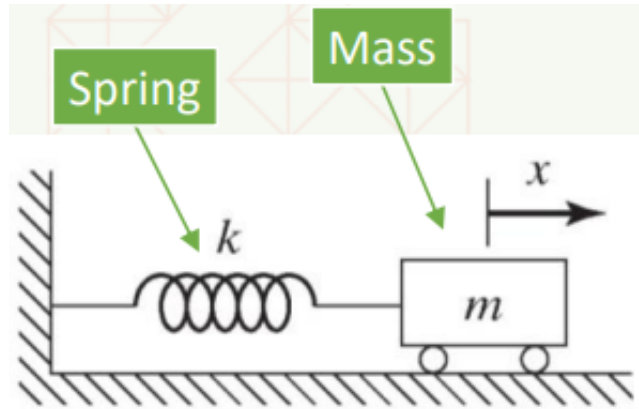


Figure 1.12: Spring-mass system.

$$x(t) = A \sin(\omega_n t + \phi) \quad (1.11)$$

$$\dot{x} = A\omega_n \cos(\omega_n t + \phi) \quad (1.12)$$

$$\ddot{x} = -A\omega_n^2 \sin(\omega_n t + \phi) \quad (1.13)$$

Substitute in equation of motion:

$$-mA\omega_n^2 \sin(\omega_n t + \phi) + kA \sin(\omega_n t + \phi) = 0 \quad (1.14)$$

$$\omega_n^2 = \frac{k}{m} \text{ or } \omega_n = \sqrt{\frac{k}{m}} \quad (1.15)$$

where 1.15 is the natural frequency.

1.2.8 Amplitude and phase

$$x(t) = A \sin(\omega_n t + \phi) \quad (1.16)$$

Initial conditions: initial displacement x_0 and initial velocity v_0 of the mass.

$$x_0 = x(t=0) = A \sin \phi \quad (1.17)$$

$$v_0 = \dot{x}(t=0) = A\omega_n \cos(\omega_n x_0 + \phi) = A\omega_n \cos \phi \quad (1.18)$$

$$\frac{v_0}{\omega_n} = A \cos \phi \quad (1.19)$$

$$x_0^2 + \frac{v_0^2}{\omega_n^2} = A^2 \sin^2 \phi + A^2 \cos^2 \phi = A^2 \quad (1.20)$$

$$\frac{\sin \phi}{\cos \phi} = \frac{x_0}{\frac{v_0}{\omega_n}} \quad (1.21)$$

Thus:

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \text{ and } \phi = \arctan \frac{\omega_n x_0}{v_0} \quad (1.22)$$

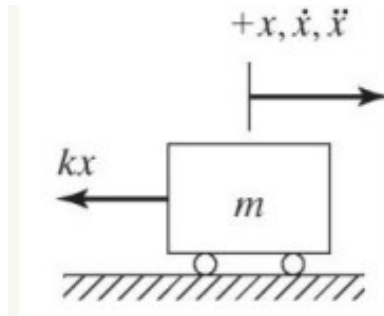


Figure 1.13: Free body diagram of spring-mass system.

$$m = 2 \text{ kg and } k = 200 \text{ N m}^{-1} \quad (1.23)$$

$$1. x_0 = -2 \text{ mm and } v_0 = 10 \text{ mm s}^{-1} \quad (1.24)$$

$$2. x_0 = 2 \text{ mm and } v_0 = -10 \text{ mm s}^{-1} \quad (1.25)$$

Solution:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 200 \text{ rad s}^{-1} \quad (1.26)$$

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} = \frac{\sqrt{10^2 \cdot 2^2 + 10^2}}{10} = 2.2 \text{ mm} \quad (1.27)$$

$$1. \phi = \arctan \left(\frac{\omega_n x_0}{v_0} \right) = \arctan \left(\frac{10 \cdot -2}{10} \right) = -1.107 \text{ rad or } -63.4^\circ \quad (1.28)$$

$$2. \phi = \arctan \left(\frac{\omega_n x_0}{v_0} \right) = \arctan \left(\frac{10 \cdot -2}{10} \right) = (-1.107 + \pi) \text{ rad or } 116.6^\circ \quad (1.29)$$

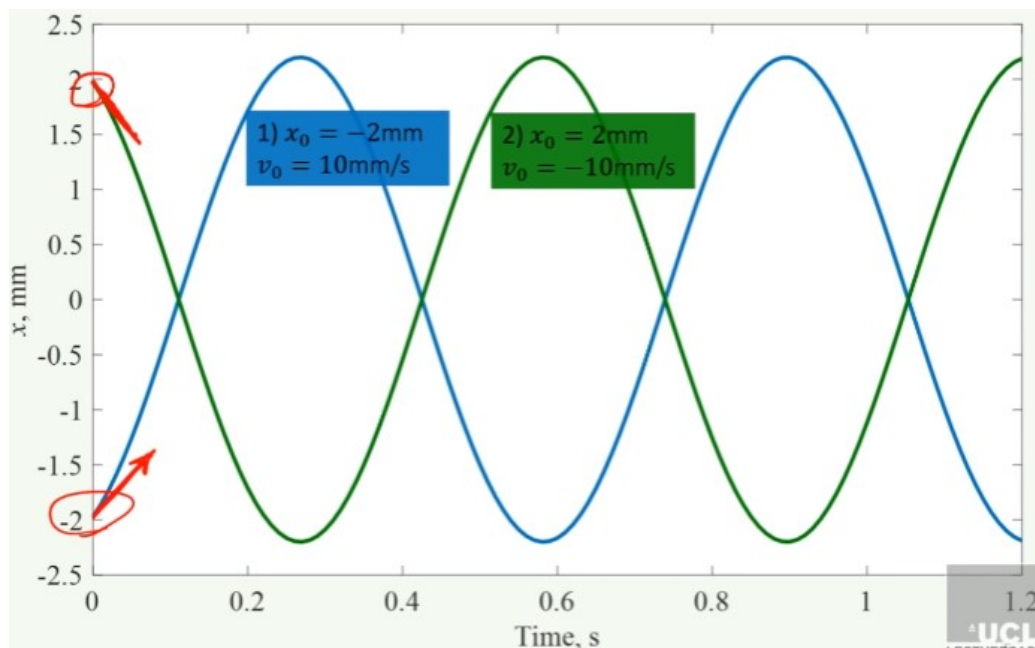


Figure 1.14: Plots of SHM.

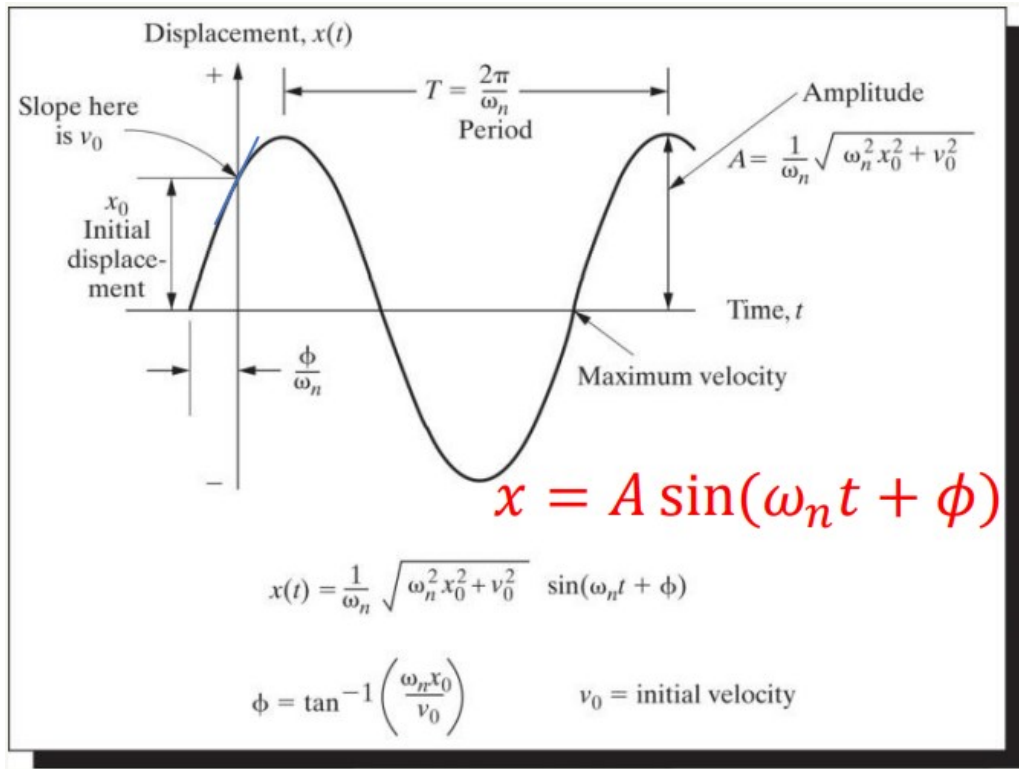


Figure 1.15: Analysis of SHM plots.

1.2.9 Springs

Hooke's law (linear spring):

$$F_k = kx \quad (1.30)$$

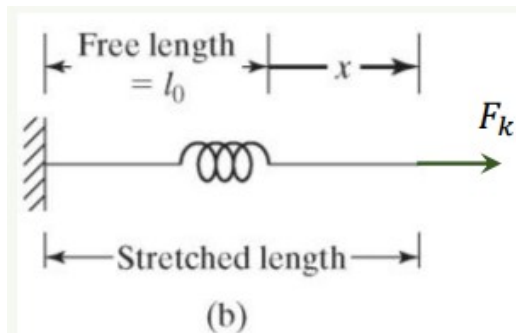


Figure 1.16: Spring system.

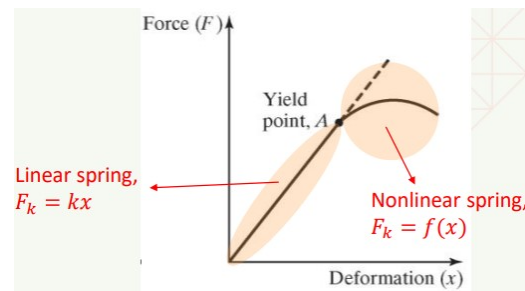


Figure 1.17: F vs x of spring.

1.2.10 Modelling springs

Springs in parallel:

$$W = k_1 \delta_{st} + k_2 \delta_{st} \quad (1.31)$$

$$= (k_1 + k_2) \delta_{st} \quad (1.32)$$

$$W = k_{eq} \delta_{st} \quad (1.33)$$

$$k_{eq} = (k_1 + k_2) \quad (1.34)$$

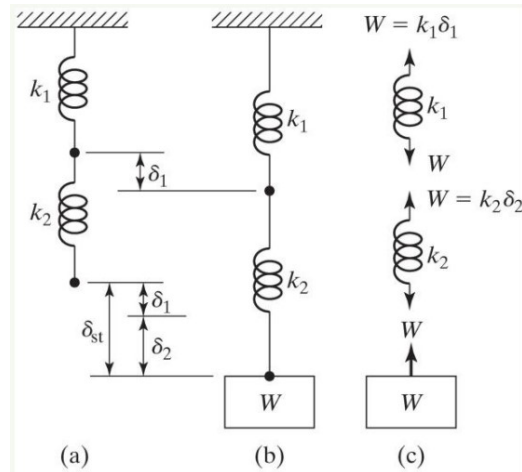


Figure 1.19: Series springs.

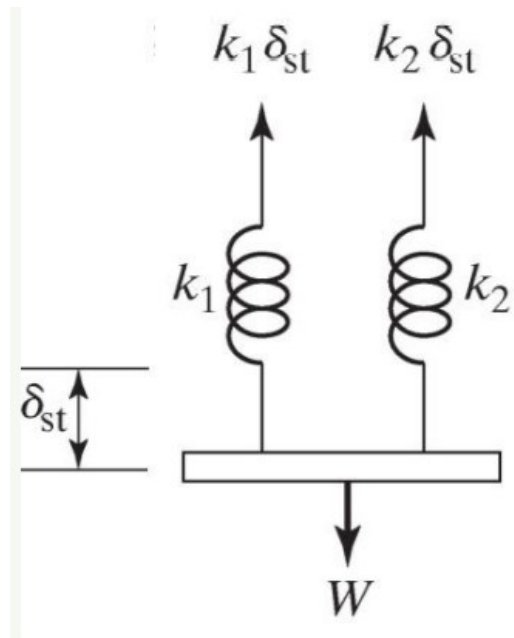


Figure 1.18: Parallel springs.

Springs in series:

$$\delta_{st} = \delta_1 + \delta_2 \quad (1.35)$$

$$w = k_1 \delta_1 \quad (1.36)$$

$$w = k_2 \delta_2 \quad (1.37)$$

or

$$\delta_{st} = \frac{w}{k_1} + \frac{w}{k_2} \quad (1.38)$$

$$\delta_{st} = \frac{w}{k_{eq}} \quad (1.39)$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \quad (1.40)$$

A cantilever beam with a mass at the free end:

$$\delta_{st} = \frac{Wl^3}{3EI} \quad (1.41)$$

$$k_{eq} = \frac{W}{\delta_{st}} \quad (1.42)$$

$$k_{eq} = \frac{3EI}{l^3} \quad (1.43)$$

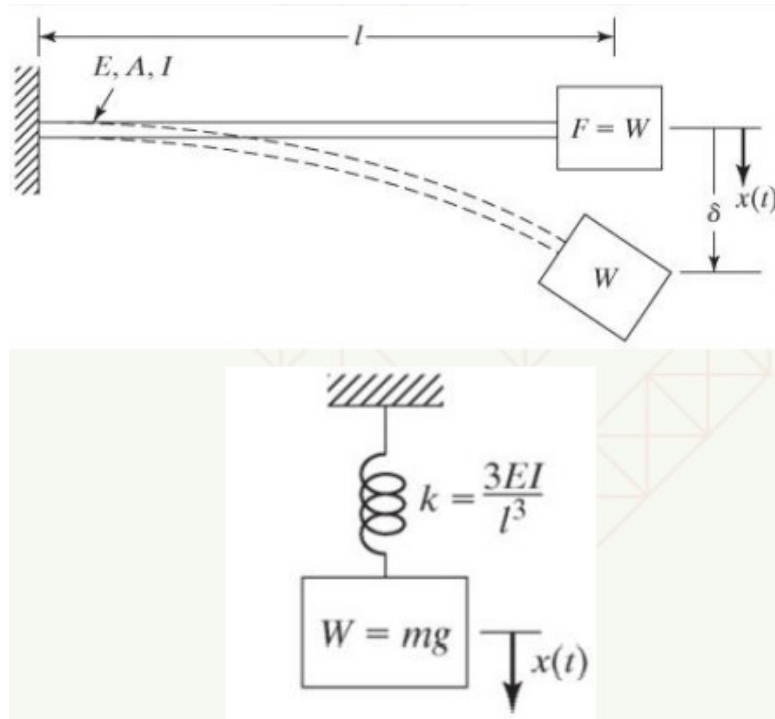


Figure 1.20: Cantilever beam with mass at free end.

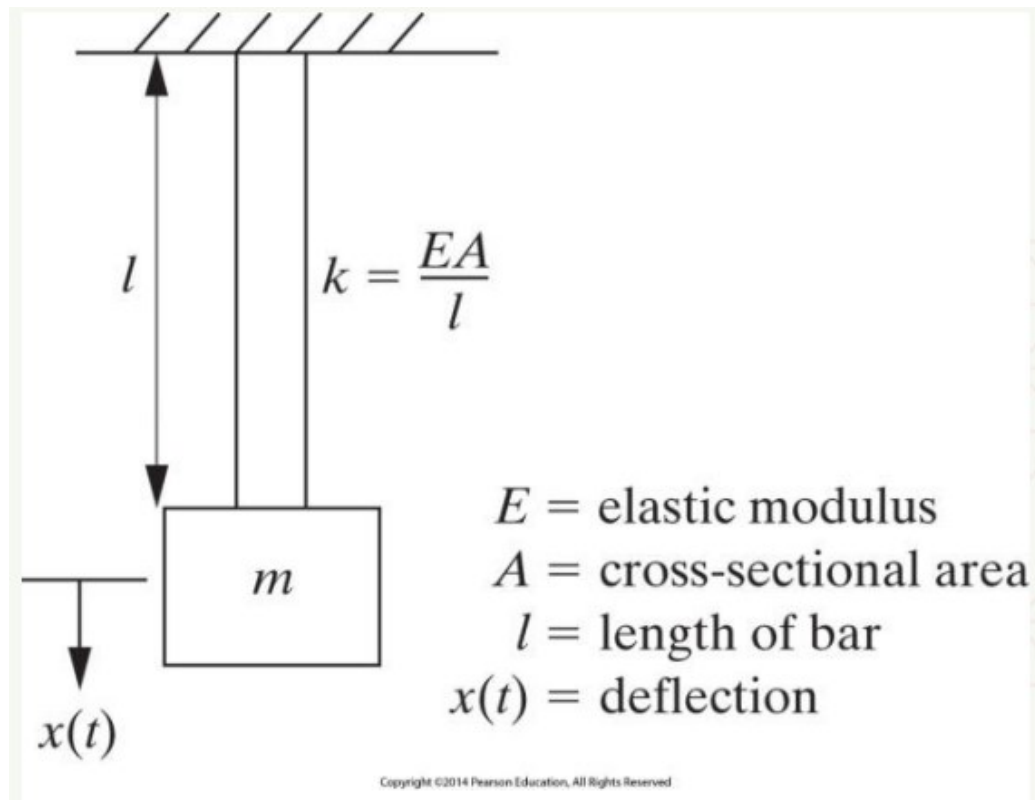
1.2.11 Longitudinal motion of a bar

Figure 1.21: Longitudinal motion of a bar.

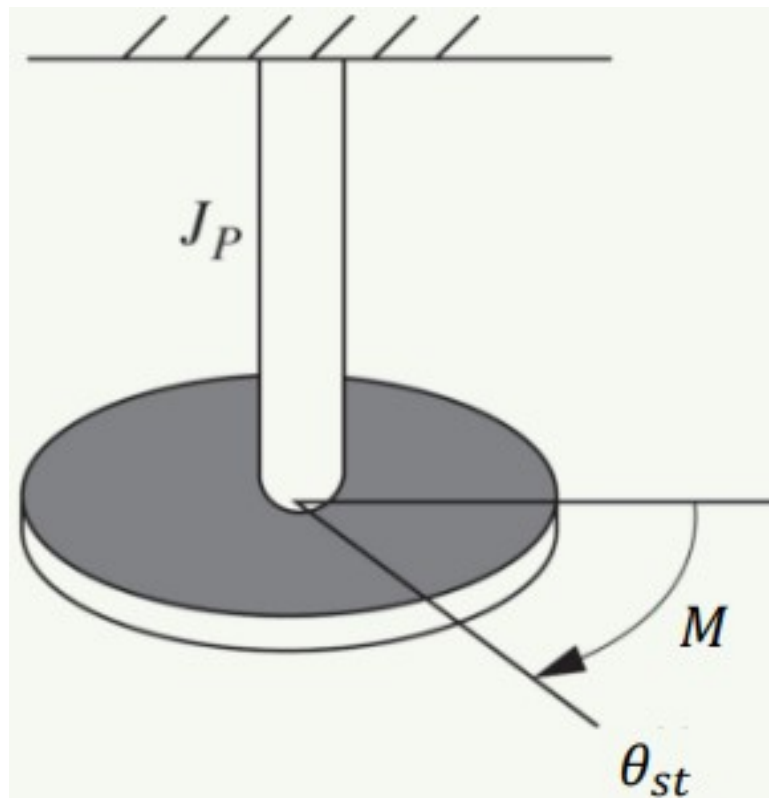
1.2.12 Torsional rotation of a shaft

Figure 1.22: Torsional disc system.

$$M = k_{eq}\theta_{st} \quad (1.44)$$

$$k_{eq} = \frac{GJ_p}{l} \quad (1.45)$$

- G : Shear modulus of rigidity
- J_p : Polar second moment of area
- l : Length of the shaft