



UNIVERSITY COLLEGE LONDON

MENG MECHANICAL ENGINEERING

MECH0071 ELECTRICAL POWER SYSTEMS AND ELECTRICAL PROPULSION

## TOPIC NOTES

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# Contents

<b>List of Figures</b>	<b>6</b>
<b>List of Tables</b>	<b>9</b>
<b>1 Introduction</b>	<b>10</b>
1.1 Team . . . . .	10
1.2 Course Aim . . . . .	10
1.3 Student learning outcomes . . . . .	10
1.4 Assessment . . . . .	10
1.5 Textbooks . . . . .	11
1.6 Softwares . . . . .	11
<b>2 The Electrical Line Diagram</b>	<b>12</b>
2.1 Overview of electrical power systems . . . . .	12
2.1.1 Basic electrical power system . . . . .	12
2.1.2 What is an electrical power system? . . . . .	12
2.2 Components of electrical power systems . . . . .	12
2.2.1 Sources of electrical power include . . . . .	12
2.2.2 Sources of DC electrical power . . . . .	13
2.2.3 Generators ... single and multiphase AC . . . . .	13
2.2.4 Transmission systems . . . . .	13
2.2.5 Distribution systems . . . . .	13
2.2.6 Loads . . . . .	13
2.3 Representation by the electrical line diagram . . . . .	14
2.3.1 Electrical system representation . . . . .	14
2.3.2 Questions for you? . . . . .	15
2.3.3 The ‘Single Line Diagram’ (SLD) . . . . .	16
2.3.4 Some common features of SLDs . . . . .	18
2.3.5 Limitations of the electrical line diagram . . . . .	18
<b>3 Developing Impedance Diagram</b>	<b>19</b>
3.1 Three Phase Power . . . . .	19
3.1.1 Three-phase alternating voltages . . . . .	19
3.1.2 Three-phase emfs (or terminal voltages) can be expressed mathematically . . . . .	19
3.1.3 Three-phase, six-wire connection . . . . .	19
3.1.4 Three-phase current . . . . .	20
3.1.5 Three-phase alternating current . . . . .	20
3.1.6 Connecting Three-Phases . . . . .	20
3.1.7 Star and delta connections . . . . .	21
3.1.8 Phase and line voltages . . . . .	22
3.1.9 Relationships between star and delta . . . . .	22
3.1.10 Single-phase impedance triangle . . . . .	22
3.1.11 Single-phase power triangle . . . . .	23

---

3.1.12	Three-phase power . . . . .	23
3.1.13	Student Activity . . . . .	23
3.2	Per Unit (PU) System . . . . .	23
3.2.1	Electrical line diagram to Impedance diagram . . . . .	23
3.2.2	Simple equivalent impedances . . . . .	24
3.2.3	How manufacturers of electrical equipment specify ratings . . . . .	24
3.2.4	The per unit system . . . . .	24
3.2.5	Values in per unit system . . . . .	25
3.2.6	Three-Phase system PU conversion . . . . .	25
3.2.7	Example PU system conversion . . . . .	26
3.2.8	Reactance diagram . . . . .	28
3.2.9	Impedance and reactance diagrams . . . . .	29
3.3	Summary . . . . .	29
<b>4</b>	<b>Using the Impedance Diagram</b>	<b>31</b>
4.1	Load Flow Calculation . . . . .	31
4.1.1	Load flow . . . . .	31
4.1.2	Load flow analysis example . . . . .	32
4.1.3	Some thoughts . . . . .	34
4.2	Using impedance diagrams in short-circuit balanced faults . . . . .	34
4.2.1	Fault classification . . . . .	34
4.2.2	Types of faults . . . . .	34
4.2.3	Faults normally are due to: . . . . .	35
4.2.4	MVA method . . . . .	35
4.2.5	Balanced three-phase fault . . . . .	36
4.2.6	Solution . . . . .	37
4.2.7	Importance of MVA . . . . .	38
4.2.8	Power system symmetrical faults . . . . .	38
4.2.9	Conclusions . . . . .	38
<b>5</b>	<b>Faulted Networks</b>	<b>39</b>
5.1	Symmetrical faults recap . . . . .	39
5.2	Unbalanced faults . . . . .	39
5.2.1	Types of ‘unbalanced faults’ . . . . .	39
5.2.2	List of possible faults . . . . .	40
5.2.3	Method of analysis . . . . .	41
5.2.4	Fortescue’s Theorem . . . . .	41
5.2.5	Positive sequence components . . . . .	42
5.2.6	Negative sequence components . . . . .	42
5.2.7	Zero sequence components . . . . .	42
5.2.8	Summing sequence components . . . . .	44
5.2.9	Note about grounding/earthing . . . . .	44
5.2.10	The operator ‘a’ . . . . .	45
5.2.11	Expressing phasors $a^2$ and $a^3$ . . . . .	46
5.2.12	Representation using ‘a’ . . . . .	46
5.2.13	Representing all sequence components in terms of $V_a$ sequence components . . . . .	47
5.2.14	‘a’ matrix . . . . .	48
5.2.15	Inverse ‘a’ matrix . . . . .	48
5.2.16	Example . . . . .	48
5.2.17	Sequence components and faults . . . . .	49
5.2.18	Conclusions . . . . .	49
<b>6</b>	<b>Full Fault Analysis</b>	<b>50</b>
6.1	Unbalanced impedance . . . . .	50

---

6.1.1	Impedance and sequence components . . . . .	50
6.1.2	Unbalanced star and delta equivalence . . . . .	50
6.1.3	Good practice . . . . .	51
6.2	Impedance of sequences . . . . .	51
6.2.1	Sequence components and impedance . . . . .	51
6.2.2	The importance of sequence impedance . . . . .	51
6.2.3	Network elements . . . . .	52
6.2.4	Transmission lines and distribution cables . . . . .	52
6.2.5	Transmission line analysis . . . . .	52
6.2.6	Transmission line representation . . . . .	52
6.2.7	Transmission sequence representation . . . . .	52
6.2.8	Transmission line representation . . . . .	53
6.2.9	Lines and cables . . . . .	54
6.2.10	Synchronous machines (generators) . . . . .	54
6.2.11	Neutral connection . . . . .	55
6.2.12	Typical values of sequence impedances for synchronous generators . . . . .	55
6.2.13	Transformers . . . . .	55
6.3	Unbalanced faults . . . . .	56
6.3.1	Fortescue's symmetrical component process . . . . .	56
6.3.2	Standard fault sequence connections - single line to ground . . . . .	57
6.3.3	Standard fault sequence connections - line to line . . . . .	58
6.3.4	Standard fault sequence connections - double line to ground . . . . .	58
6.4	A full fault analysis study . . . . .	58
6.4.1	Breaker sizing method (most common approach) . . . . .	58
6.4.2	Breaker sizing example . . . . .	59
6.4.3	Sequence component arrangement . . . . .	59
6.4.4	Symmetrical fault current . . . . .	60
6.4.5	Single line to ground fault . . . . .	60
6.4.6	Singe line to ground fault . . . . .	61
6.4.7	Double line to ground fault . . . . .	61
6.4.8	Line to line fault . . . . .	62
6.4.9	Conversion to ampere ratings . . . . .	63
6.4.10	Practical sizing of breakers . . . . .	63
6.4.11	Conclusions . . . . .	63
<b>7</b>	<b>Network Analysis</b>	<b>64</b>
7.1	Electrical networks . . . . .	64
7.2	Split distribution system - high integrity . . . . .	64
7.3	Tree distribution . . . . .	65
7.4	Ring networks - grids . . . . .	66
7.5	Network analysis . . . . .	66
7.6	Techniques for power-flow studies . . . . .	66
7.7	Power flow calculations . . . . .	67
7.8	Basic techniques for power-flow studies . . . . .	67
7.9	Approach to analysis . . . . .	68
7.10	Constructing $Y_{bus}$ for power-flow analysis . . . . .	68
7.11	Power-flow analysis equations . . . . .	70
7.11.1	Gauss-Siedel iterative method . . . . .	70
7.12	Example 2 . . . . .	71
7.13	Conclusions . . . . .	73
<b>8</b>	<b>Marine Electric Propulsion</b>	<b>74</b>
8.1	Introduction . . . . .	74
8.1.1	The propulsion requirement . . . . .	74

---

8.1.2	Effective power . . . . .	74
8.1.3	The generalised resistance equation . . . . .	75
8.1.4	Propulsive power requirement . . . . .	75
8.1.5	Relationship between speed and power . . . . .	75
8.1.6	Shaft power and effective power . . . . .	75
8.1.7	Ship power/speed curves . . . . .	76
8.1.8	Power speed/curve - two shafts . . . . .	76
8.1.9	Main components of a marine propulsions system . . . . .	77
8.1.10	Efficiency of electrical propulsion . . . . .	77
8.2	Marine electric propulsion . . . . .	78
8.2.1	The early days . . . . .	78
8.2.2	Modern ship designs . . . . .	79
8.2.3	Summary . . . . .	87
<b>9</b>	<b>Seminar on Marine Propulsion</b>	<b>88</b>
9.0.1	Propulsion exercise . . . . .	88
9.1	Task 1 . . . . .	88
9.1.1	CODOG design issues . . . . .	90
9.1.2	Alternative propulsion arrangements . . . . .	90
9.2	Task 2 . . . . .	90
9.3	Task 3 . . . . .	91
9.4	Task 4 . . . . .	91
9.4.1	Calculations - Scenario 1 . . . . .	92
9.4.2	Calculations - Scenario 2 . . . . .	93
9.4.3	Observations of study . . . . .	93
9.5	Task 5 (formative) . . . . .	94
9.5.1	Task A . . . . .	95
9.5.2	Task B . . . . .	95
<b>10</b>	<b>Generators</b>	<b>96</b>
10.1	Synchronous machine . . . . .	96
10.1.1	Petrol/diesel generators . . . . .	96
10.1.2	Gas turbine generators . . . . .	96
10.1.3	Steam turbine generators . . . . .	97
10.1.4	Synchronous machine basics . . . . .	97
10.1.5	Concept of back emf and internal resistance . . . . .	97
10.1.6	Phasor diagram representation . . . . .	98
10.1.7	The speed of rotation of synchronous generators . . . . .	98
10.1.8	Frequency and voltage control . . . . .	99
10.1.9	Control of generators . . . . .	99
10.1.10	Single-generator operation - real power . . . . .	99
10.1.11	Automatic voltage regulator . . . . .	100
10.1.12	Circuit breaker and protector initiation . . . . .	100
10.2	Multi-synchronous generator operation . . . . .	100
10.2.1	Multi-generator operation . . . . .	100
10.2.2	Connection requirements . . . . .	100
10.2.3	Engine speed control . . . . .	101
10.2.4	Voltage and reactive power control . . . . .	103
10.2.5	Steady state performance . . . . .	103
10.3	Generator transient performance . . . . .	103
10.3.1	Transient performance . . . . .	103
10.3.2	Transient load response of a generator . . . . .	104
10.3.3	AVR arrangement for generator . . . . .	105
10.4	Generator faulted performance . . . . .	106

10.4.1 Synchronous machine - three-phase short circuit . . . . .	106
10.4.2 Synchronous machines short-circuit envelope . . . . .	106
10.4.3 Synchronous machine short-circuit . . . . .	107
10.4.4 Balanced three-phase component of the short-circuit current . . . . .	108
10.4.5 Class example 1 . . . . .	108
10.4.6 Worked example . . . . .	109
10.4.7 Class example 2 . . . . .	109
10.5 Summary . . . . .	110
<b>11 Electric / Hybrid RV Propulsion</b>	<b>111</b>
11.1 Introduction . . . . .	111
11.1.1 Reasons forcing change . . . . .	111
11.1.2 Global CO <sub>2</sub> emissions . . . . .	112
11.1.3 Typical driving energy losses (city use) . . . . .	112
11.1.4 Transport sector growth prediction . . . . .	113
11.1.5 Gasoline: the (almost) perfect fuel . . . . .	113
11.1.6 Towards zero emissions . . . . .	114

# List of Figures

2.1	Some types of electrical system representation. . . . .	14
2.2	Example of a ‘Single Line diagram’. . . . .	15
2.3	Symbols. . . . .	16
2.4	Marine SLD. . . . .	17
2.5	Naval SLD. . . . .	17
3.1	Three-phase, six-wire system. . . . .	20
3.2	Star and delta configurations. . . . .	21
3.3	Star generator and delta load. . . . .	21
3.4	Single-phase impedance triangle. . . . .	22
3.5	Single-phase power triangle. . . . .	23
3.6	Equivalent Impedance Representations. . . . .	24
3.7	Single Line Diagram. . . . .	26
3.8	Impedance Diagram. . . . .	28
3.9	Reactance Diagram. . . . .	29
4.1	Single Line Diagram. . . . .	32
4.2	Single Line Diagram. . . . .	33
4.3	Balanced three-phase fault. . . . .	36
4.4	Impedance diagram. . . . .	37
4.5	Impedance diagram circuit reduced. . . . .	37
5.1	Unsymmetrical/unbalanced faults. . . . .	40
5.2	Unsymmetrical/unbalanced fault graph. . . . .	41
5.3	Sequence components and phase relationship. . . . .	43
5.4	Sequence components 2. . . . .	43
5.5	Grounding/earthing. . . . .	44
5.6	Currents during grounded star point. . . . .	45
5.7	Currents during floating star point. . . . .	45
5.8	‘a’ operator. . . . .	45
5.9	‘a’ phasors. . . . .	46
5.10	List of ‘a’ phasors. . . . .	47
5.11	Phase voltages expressed in terms of $V_a$ . . . . .	47
6.1	Star and delta arrangements. . . . .	50
6.2	Transmission line mutual inductance and self-inductance. . . . .	52
6.3	Transmission line and cable arrangements. . . . .	54
6.4	Grounded star arrangement. . . . .	55
6.5	Line to ground fault. . . . .	56
6.6	Single line to ground connection. . . . .	57
6.7	Line to line connection. . . . .	58
6.8	Double line to ground connection. . . . .	58
6.9	Breaker sizing example. . . . .	59

---

6.10 Sequence component arrangement. . . . .	59
6.11 Positive sequence impedance in symmetrical fault. . . . .	60
6.12 Positive sequence impedance in symmetrical fault. . . . .	60
6.13 Double line-ground fault configuration. . . . .	61
6.14 Line to line fault configuration. . . . .	63
7.1 Network at a works. . . . .	64
7.2 Split distribution system - high integrity. . . . .	65
7.3 Tree distribution. . . . .	65
7.4 Ring networks - grids. . . . .	66
7.5 Transmission line: only L and R are used in the $Y_{bus}$ . . . . .	68
7.6 Example 1 diagram. . . . .	69
7.7 Example 2 diagram. . . . .	71
8.1 Ship force diagram. . . . .	74
8.2 Ship power/speed curve. . . . .	76
8.3 Power speed/curve - two shafts. . . . .	76
8.4 Ship SLD efficiency. . . . .	77
8.5 Turbo-electric propulsion (Emmet) system. . . . .	78
8.6 Diesel-electric (DC) propulsion system. . . . .	79
8.7 Modern electric propulsion systems. . . . .	80
8.8 Electrical propulsion with CPPs. . . . .	80
8.9 Electrical propulsion with gearboxes. . . . .	81
8.10 Electrical propulsion with converters and CPPs. . . . .	81
8.11 Electrical propulsion with converters. . . . .	82
8.12 Main types of converters. . . . .	82
8.13 Electrical propulsion system arrangement. . . . .	83
8.14 Queen Elizabeth 2 electrical propulsion system arrangement. . . . .	84
8.15 T45 Frigate electrical line diagram (not exact). . . . .	84
8.16 Zonal power example architecture. . . . .	85
8.17 System configuration efficiencies. . . . .	85
8.18 Potential of fuel cell technology. . . . .	86
9.1 CODOG arrangement with CPP. . . . .	89
9.2 Engine 1 (low) power available for cruise speed. . . . .	89
9.3 Engine 2 (large) power available for top speed. . . . .	89
9.4 Task 5 Line diagram (use CODOG prime-movers). . . . .	94
10.1 Synchronous machine basics. . . . .	97
10.2 Generator diagram. . . . .	97
10.3 Phasor diagram. . . . .	98
10.4 Current magnitude and phase effects on $V_a$ . . . . .	98
10.5 Control of generators. . . . .	99
10.6 Brushless generator excitation system with PMG supply. . . . .	100
10.7 Synchronisation of generator to grid. . . . .	101
10.8 Single machine - governor droop characteristics (exaggerated). . . . .	101
10.9 Two machines operating in parallel. . . . .	102
10.10 Two machines operating in parallel - effects due to governor adjustment. . . . .	102
10.11 Two machines operating in parallel. . . . .	103
10.12 Transient load response of a generator. . . . .	104
10.13 Typical AVR controller showing time constants for PID for exciter, regulator and field. Response must be within certain limits by regulation. . . . .	105
10.14 Typical response to a sudden three-phase short circuit at the terminals of a generator. Note the asymmetrical arrangement of the waveforms. . . . .	106
10.15 Synchronous machine short-circuit envelope. . . . .	106

10.16 Synchronous machine short-circuit. . . . .	107
10.17 Balanced three-phase component of the short-circuit current. . . . .	108
10.18 Class example 2 SLD. . . . .	109
11.1 Global CO <sub>2</sub> emissions. . . . .	112
11.2 Typical driving energy losses (city use). . . . .	112
11.3 Transport sector growth prediction. . . . .	113
11.4 Transport sector growth prediction. . . . .	113
11.5 Drive trains for various vehicle types. . . . .	114

# List of Tables

6.1	Table to show typical value of sequence impedances for synchronous generators . . . . .	55
6.2	Table to show fault currents. . . . .	63
7.1	Example 1 series per unit admittances. . . . .	68
7.2	Example 1 table of busses. . . . .	68
7.3	Example 2 table of busses. . . . .	71
8.1	Example efficiencies of components in a marine propulsion system. . . . .	77
8.2	Current fuel cell technology. . . . .	86
9.1	Data on fuel consumption NOx emissions - Task 4. . . . .	91
9.2	Efficiency of generators and motors . . . . .	95
9.3	Data on fuel consumption NOx emissions - Task 5. . . . .	95

# **Chapter 1**

## **Introduction**

### **1.1 Team**

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- Mr Chris Greenough
- Mr Konrad Yearwood - Helpdesk email: k.yearwood@ucl.ac.uk

### **1.2 Course Aim**

The aim of this course is to provide students with detailed knowledge and understanding of the design, performance and analysis of electrical power systems.

Students will increase their knowledge and understanding through face-to-face / synchronous lectures, asynchronous (including tutorials) tasks and a computer simulation workshop and demonstrate their learning through summative coursework and an examination.

### **1.3 Student learning outcomes**

- Appreciate the components that make up electrical power systems and understand the similarities and differences between large, medium and small scale power systems.
- Develop skills needed to be able to design electrical power systems including analytical and computer based methods.
- Understand the behaviour of steady-state, transient and faulted networks and appreciate how such behaviour influences design.
- Understand the benefits of electrical propulsion for different vehicle types be able to undertake designs.
- Appreciate future developments and applications in electrical power and electrical propulsion systems.

### **1.4 Assessment**

- Coursework - summative assessment exercise based around computer simulations
- Examination - two hour examination in January

## 1.5 Textbooks

Kirtley, James. *Electric Power Principles: Sources, Conversion, Distribution and Use*. Wiley. 2020. ISBN: 9781119585305.t

## 1.6 Softwares

- PSCAD

# Chapter 2

## The Electrical Line Diagram

### 2.1 Overview of electrical power systems

#### 2.1.1 Basic electrical power system

Most electrical power systems contain:

- Generators to produce electrical energy (often coming from another store of energy e.g. chemical - oil, gas, coal)
- A means to transmit and distribute the electrical energy
- Loads that use the electrical energy for some purpose

#### 2.1.2 What is an electrical power system?

An **electric power system** is a network or grid of electrical components that supply, transfer and use electric energy. Electrical power systems can be a:

- Large grids covering a wide area e.g. a continent
- Medium grid covering a large area e.g. a country
- Small network covering a small area e.g. a ship

### 2.2 Components of electrical power systems

#### 2.2.1 Sources of electrical power include

Generators (rotating types AC and DC):

- Large AC generators e.g. 25 kV three-phase voltages
- Medium AC generators e.g. 440 V three-phase voltages
- Small AC generators e.g. e.g. single-phase 220 V voltages

Fuel cells:

- DC output voltage (typically 720 V DC)

Batteries (electro-chemical):

- DC output voltage (usually multiples of 12 V)

Photo-voltaic (solar) cells:

- DC output currents (usually mA/cell)

## 2.2.2 Sources of DC electrical power ...

A fuel cell in a car. Photovoltaics used in a solar farm. Battery energy store. DC systems are increasing in their popularity due to wider use of batteries, solar cells and fuel cells in grids and electrical propulsion.

## 2.2.3 Generators ...single and multiphase AC

AC generators:

- Large AC generators e.g. 25 kV 3 phase
- Medium AC generators e.g. 11 kV or 440 V 3 phase
- Small generators e.g. 220 V single-phase voltage

## 2.2.4 Transmission systems

HVAC often three-wire and three-phase e.g. 440 kV, 275 kV and 132 kV.

HVDC often two-wire and bipolar e.g. +/- 330 kV.

## 2.2.5 Distribution systems

AC distribution:

- 11 kV, 440 V three-phase
- 25 kV single-phase (rail)
- 240 V single-phase

DC distribution:

- 750 V (rail)
- 110 V (emergency lighting)

## 2.2.6 Loads

Three-phase loads:

- Induction motors to drive pumps, fans and compressors
- Propulsion drives

Single-phase loads:

- Lighting
- Heating
- Appliances e.g. domestic, electronics, small pumps

DC loads:

- DC motors
- Lighting and heating
- Battery charging

## 2.3 Representation by the electrical line diagram

### 2.3.1 Electrical system representation

Electrical systems are commonly represented as one of the following:

- Pictorial diagram
- Block diagram
- Wiring diagram
- Single line diagram
- Riser diagram
- Electrical floor plan
- Layout diagram

Of these the most useful to the *electrical power engineer* is the **Single line diagram**.

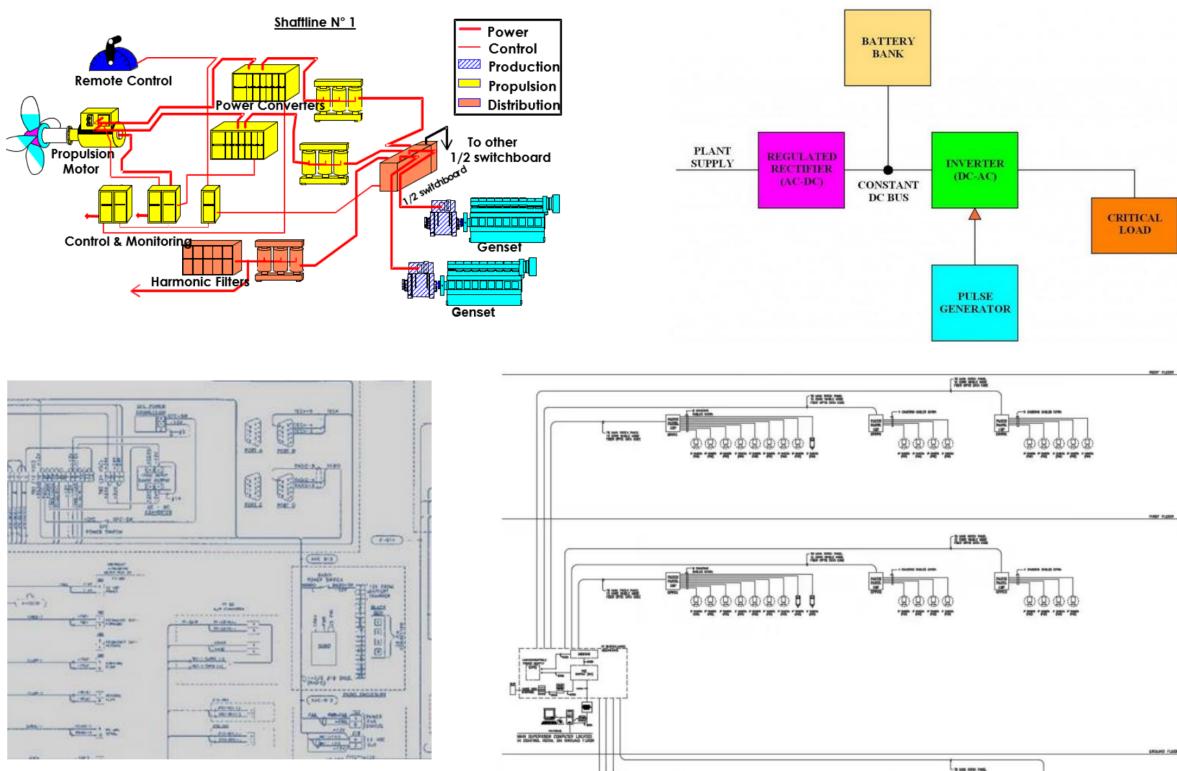


Figure 2.1: Some types of electrical system representation.

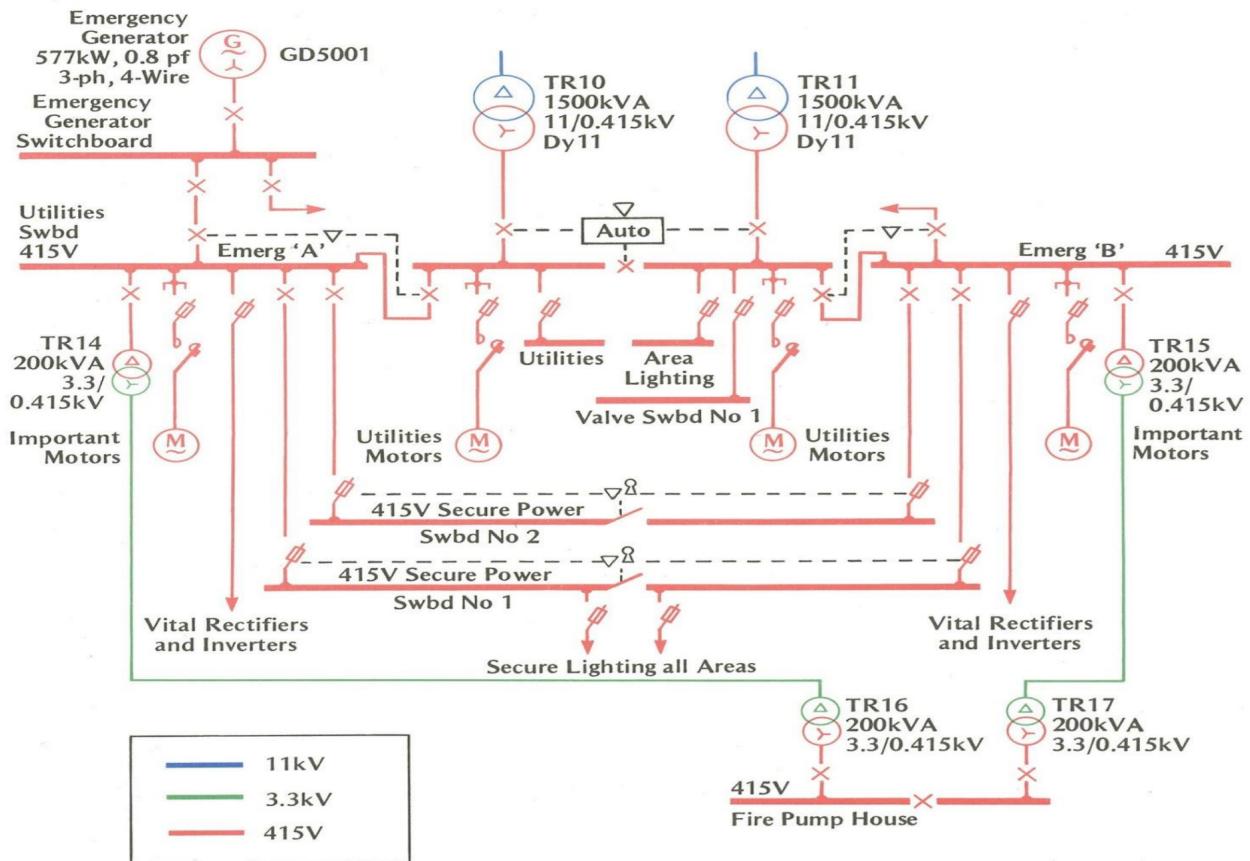


Figure 2.2: Example of a ‘Single Line diagram’.

### 2.3.2 Questions for you?

1. The number of separate switchboards shown? 14 (each thick line is a separate switchboard)
2. Maximum current that will flow through the supply transformers?  $I = \frac{kV A}{kV \times \sqrt{3}}$ , (root 3 due to 3-phase)
3. How many different electrical sources supply the fire pump house? All three supplies can be connected to the fire pump house.

Equipment	Single Line Diagram Representation		
AC Machine (Motor and Generator)			
DC Machine (Motor or Generator)			
Transmission Lines and Cables (With circuit breaker)			
Switchboards (with busbar, circuit breakers and feeders)			
Power Conversion (Rectifier AC-DC and Inverter DC-AC)			
Transformer (Two winding transformer, Three winding transformer)			
Star, Delta and Zig-Zag connections.			
Earth			
Passive Components (Resistance, Capacitance and inductance)			

Figure 2.3: Symbols.

### 2.3.3 The ‘Single Line Diagram’ (SLD)

The ‘Single Line Diagram’ (also known as the ‘One Line Diagram’) represents an electrical power system using single lines regardless of number of cables being used. It can be used to represent:

- Any type of electrical power system: DC, single-phase, three-phase or a mixed voltage electrical system.
- The interconnections between different electrical equipment including generators, switchboards, electrical distribution centres and loads.
- The types of electrical equipment and their main characteristics e.g. ratings of equipment such as voltage, power, power factor, and impedance.
- Emergency features such as reversionary modes, cross-connections and emergency generators. Sometimes these can be represented as single ‘dotted line’ connections rather than the usual solid single line.
- Other details such as ‘earthing arrangements, arrangements of star/delta connections in three-phase systems and any autonomous operating systems such as circuit breakers.

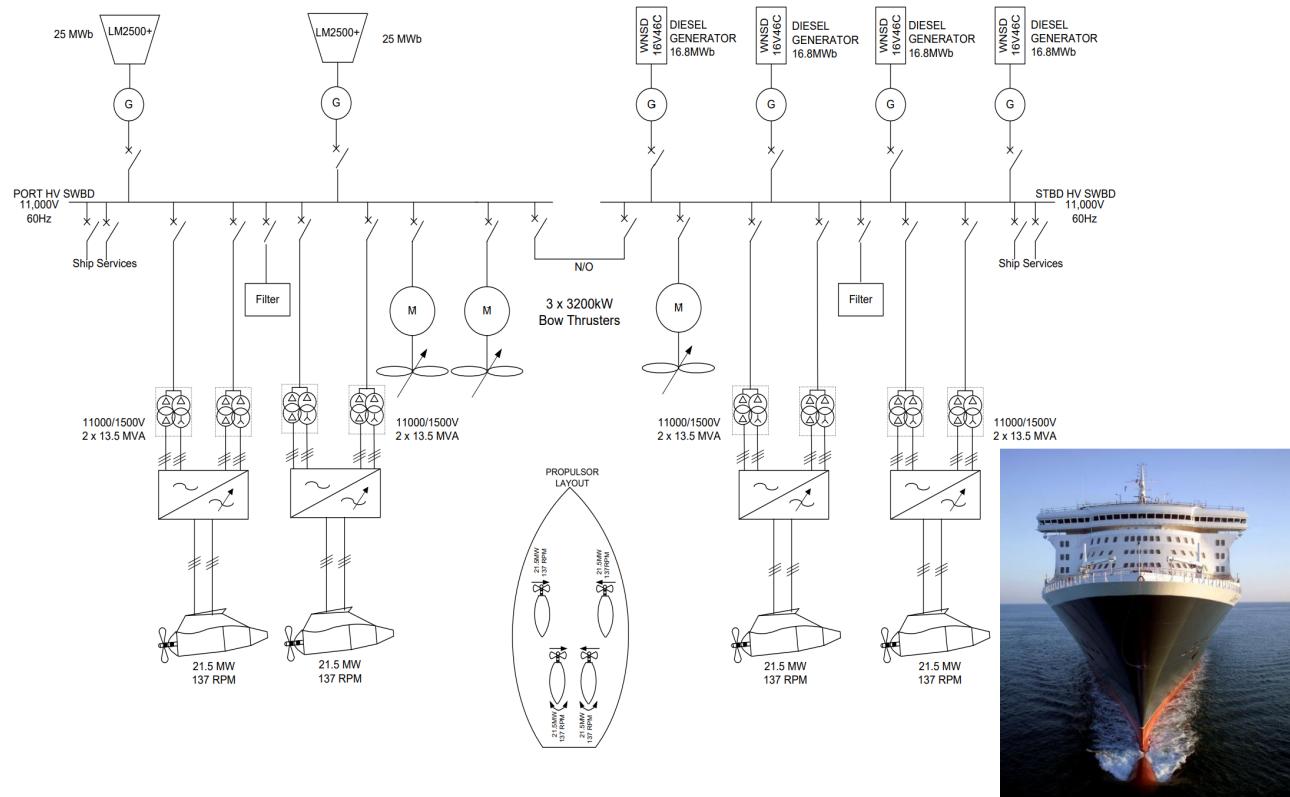


Figure 2.4: Marine SLD.

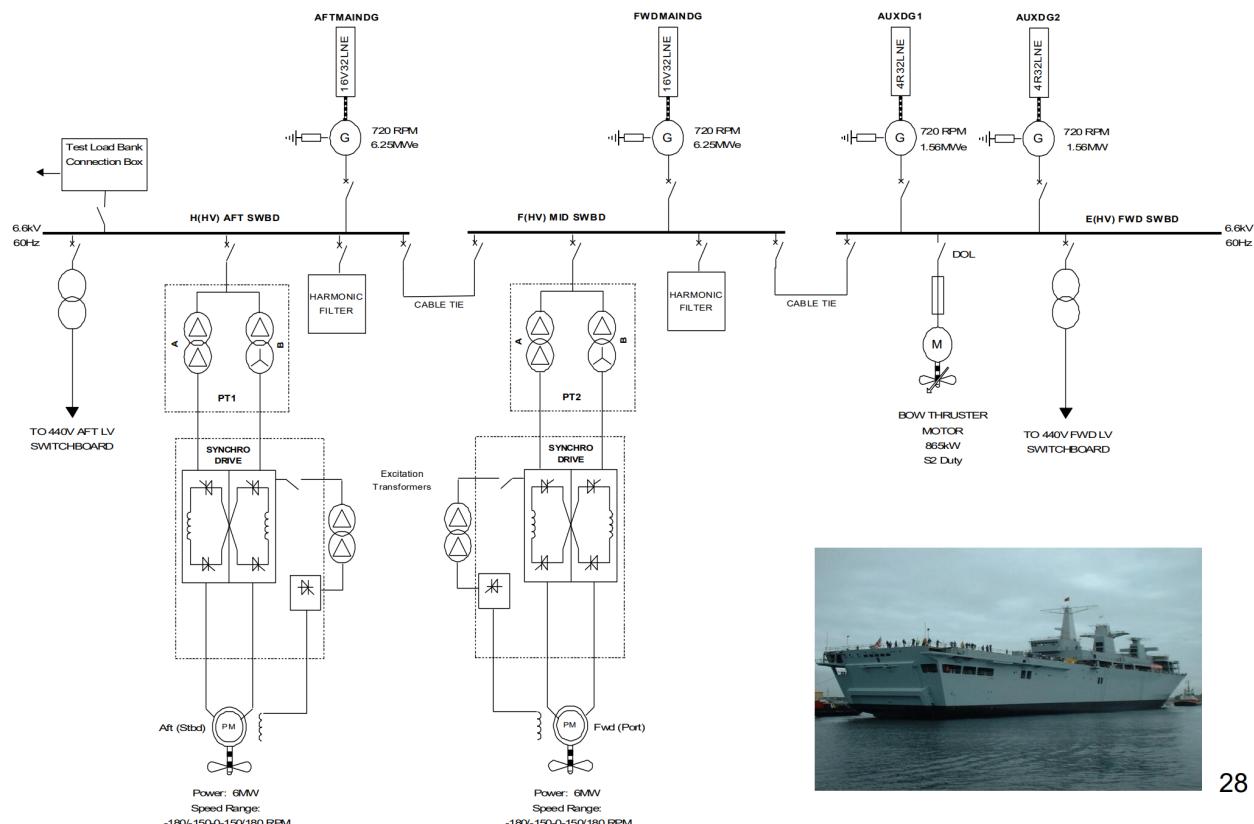


Figure 2.5: Naval SLD.

### 2.3.4 Some common features of SLDs

- Supplies (shore supplies, generators, incoming supply) are located at the top of the diagram
- The loads (motors, lighting, etc.) are located towards the bottom of the diagram.
- Switchboards are shown as thicker lines with interlocking switchgear being shown using dotted lines.
- Interconnections between equipment is a single-line representation regardless of number of phase (unless there is a good reason not to do so).
- Voltage, Frequency, Power, PF, revolutions, etc. are provided.

### 2.3.5 Limitations of the electrical line diagram

- The ‘Single Line Electrical Diagram’ is a very useful means of showing how electrical equipment is connected into a system using single lines (representing a three-phase system or some other electrical power system).
- It has very limited use when undertaking analysis. It is not an electrical circuit. To undertake analysis of electrical power systems then it is necessary to change the ‘Single Line Electrical Diagram’ into an ‘Impedance Diagram’.

# Chapter 3

## Developing Impedance Diagram

### 3.1 Three Phase Power

#### 3.1.1 Three-phase alternating voltages

A three-phase synchronous generator consists of a rotor and a stator.

- Adjusting excitation current on the rotating field will change the magnitude of the three AC phase emfs generated in the stator.
- Changing the rotational speed changes the frequency of the AC emfs
- The three phases generated are  $120^\circ$  displaced due to special arrangement

#### 3.1.2 Three-phase emfs (or terminal voltages) can be expressed mathematically

$$v_a(t) = V_m \sin(\omega t) \quad (3.1)$$

$$v_b(t) = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \quad (3.2)$$

$$v_c(t) = V_m \sin\left(\omega t - \frac{4\pi}{3}\right) \quad (3.3)$$

$V_m$  is the peak (maximum) voltage,  $\omega$  is the angular frequency,  $t$  is time. The phase displacement between the three-phase waveforms is  $120^\circ$  or  $\frac{2\pi}{3}$  radians.  $v_a$ ,  $v_b$  and  $v_c$  are the three phase voltages.

#### 3.1.3 Three-phase, six-wire connection

There are different arrangements for distributing three-phase electrical power. The three phases can be independent of each other as seen below and treated as three separate circuits. This is known as the *three-phase, six-wire system*.

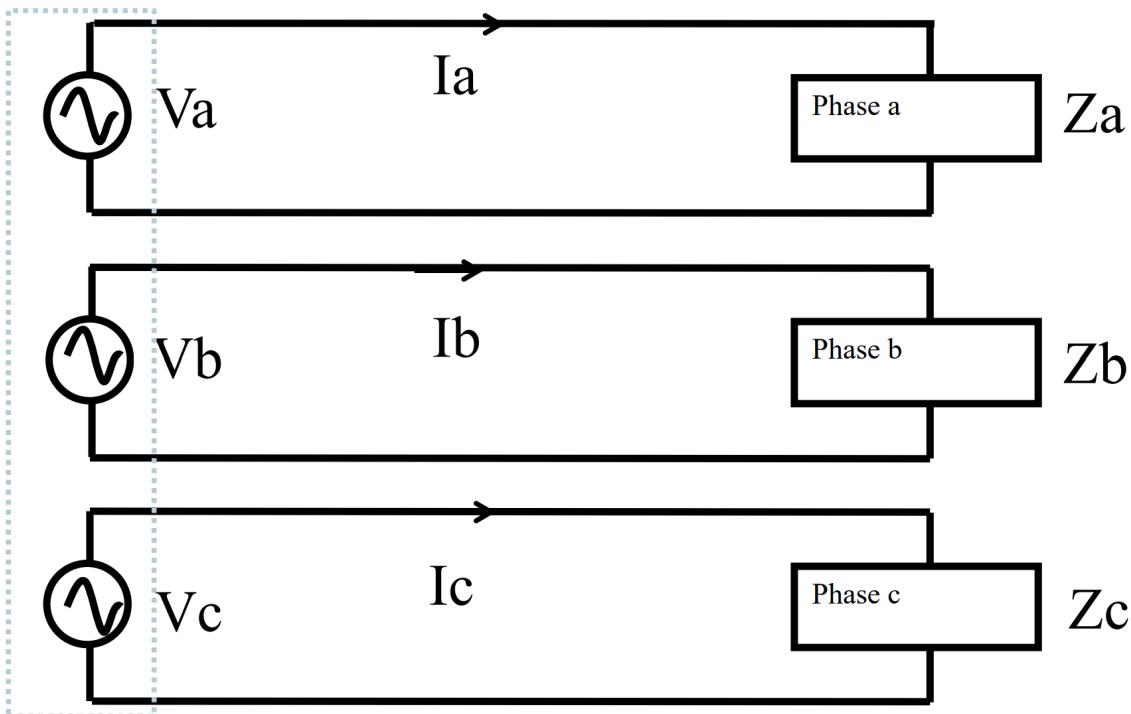


Figure 3.1: Three-phase, six-wire system.

### 3.1.4 Three-phase current

The currents flow in a three-phase circuit when there is a three-phase load. We will initially assume that the three-phase load is balanced i.e. the magnitude of voltage, current and the phase-angle is the same for each phase circuit. This is not true for three-phase circuits with unbalanced loads and the mathematical approach is different and more complex so we will examine this later.

### 3.1.5 Three-phase alternating current

The currents associated with a three-phase system that flow from the supply to the load may be described mathematically by:

$$i_a(t) = I_m \sin(\omega t + \theta) \quad (3.4)$$

$$i_b(t) = I_m \sin\left(\omega t - \frac{2\pi}{3} + \theta\right) \quad (3.5)$$

$$i_c(t) = I_m \sin\left(\omega t - \frac{4\pi}{3} + \theta\right) \quad (3.6)$$

Note: the phase displacement angle ( $\theta$ ) can be positive (leading PF) indicating a capacitive load or negative (lagging PF) indicating an inductive load. A zero phase displacement angle indicates a resistive circuit or a circuit at resonance ( $X_L = X_C$ ).

### 3.1.6 Connecting Three-Phases

A three-phase six wire system is generally expensive to install and is actually unnecessary due to an inherent balancing characteristic.

In the balanced three-phase system, the algebraic sum of voltage at any point where all three-phase voltages are connected is zero.

The zero voltage point is known as the ‘star point’ and this may be grounded or left isolated (floating). In most electrical systems the star point is grounded with exceptions being some ship types.

### 3.1.7 Star and delta connections

The number of transmission wires can be reduced by connecting the phases in either delta or star configuration.

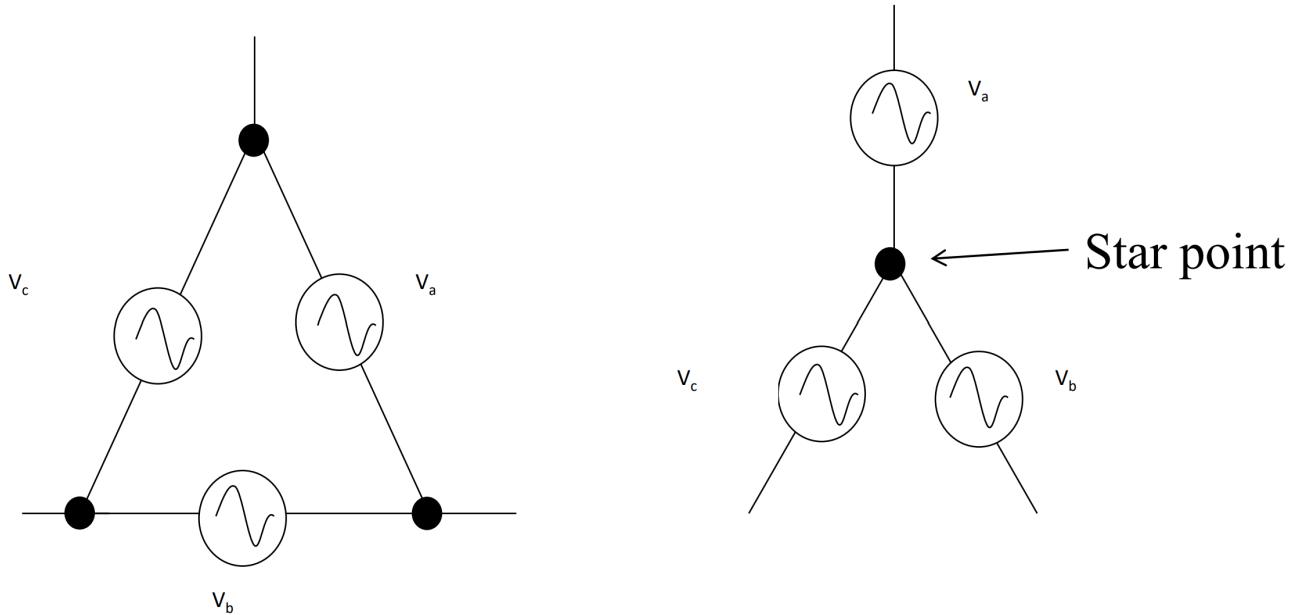


Figure 3.2: Star and delta configurations.

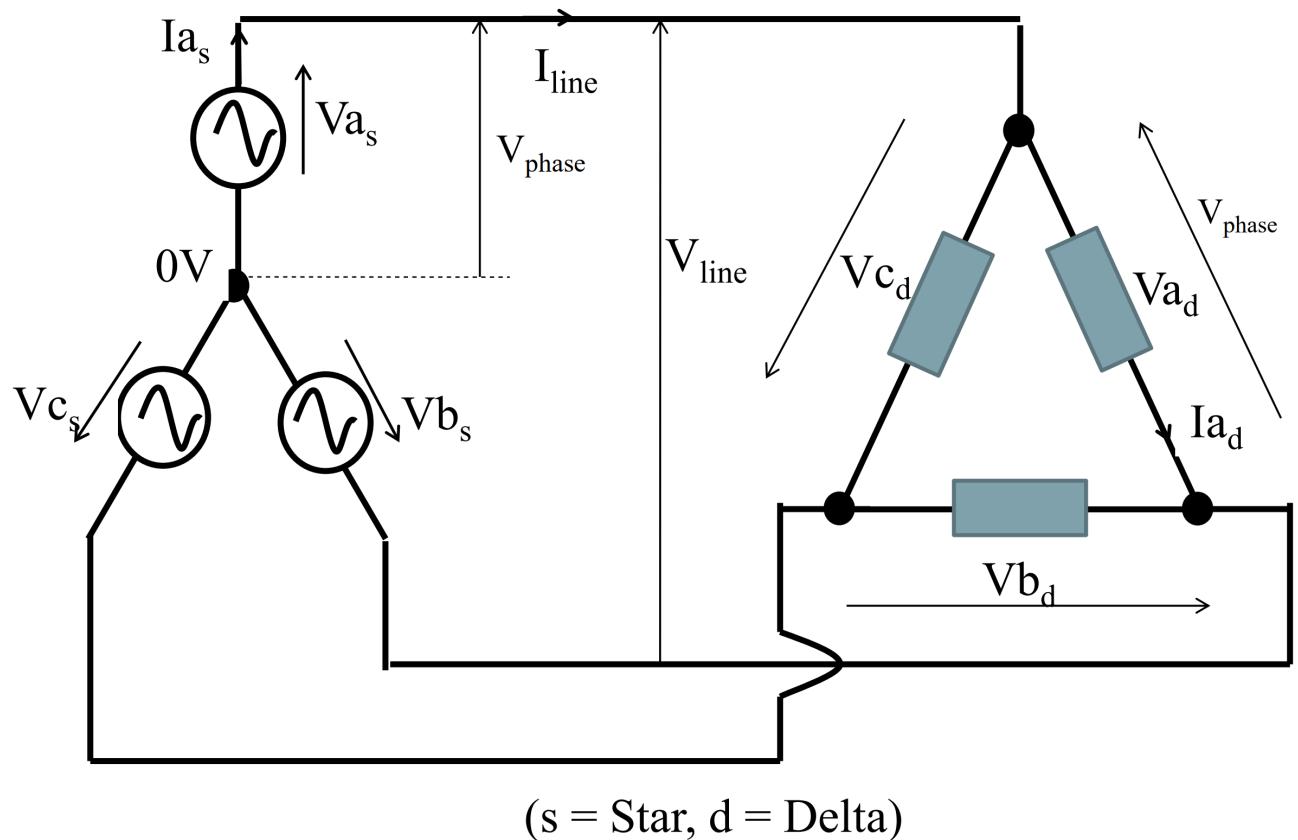


Figure 3.3: Star generator and delta load.

### 3.1.8 Phase and line voltages

There are therefore two voltage types (either generated as a potential difference) when considering three-phase circuits. These are commonly known as the *phase voltage* and *line voltage*.

The phase voltages in the star-delta circuit are as follows:

- $V_{as}, V_{bs}, V_{cs}$  for the star circuit
- $V_{ad}, V_{bd}, V_{cd}$  for the delta circuit

The line voltages can be measured as follows:

$$V_{ab} = V_{as} - V_{bs} = V_{ad} \quad (3.7)$$

$$V_{bc} = V_{bs} - V_{cs} = V_{bd} \quad (3.8)$$

$$V_{ca} = V_{cs} - V_{as} = V_{cd} \quad (3.9)$$

and if the line voltages measure is reversed:

$$V_{ba} = V_{bs} - V_{as} = -V_{ad} \quad (3.10)$$

$$V_{cb} = V_{cs} - V_{bs} = -V_{bd} \quad (3.11)$$

$$V_{ac} = V_{as} - V_{cs} = -V_{cd} \quad (3.12)$$

Which is why a three-phase system is known as a six-pulse system - (important in power electronic systems).

### 3.1.9 Relationships between star and delta

For the delta arrangement:

$$V_p = V_l \quad (3.13)$$

$$I_p = \frac{I_l}{\sqrt{3}} \quad (3.14)$$

For the star arrangement:

$$V_p = \frac{V_l}{\sqrt{3}} \quad (3.15)$$

$$I_p = I_l \quad (3.16)$$

Where  $I_p$  and  $V_p$  are the phase currents and voltages and  $I_l$  and  $V_l$  are the line currents and voltages respectively.  
Note: Delta is also known as ‘mesh’; Star is also known as ‘Y’.

### 3.1.10 Single-phase impedance triangle

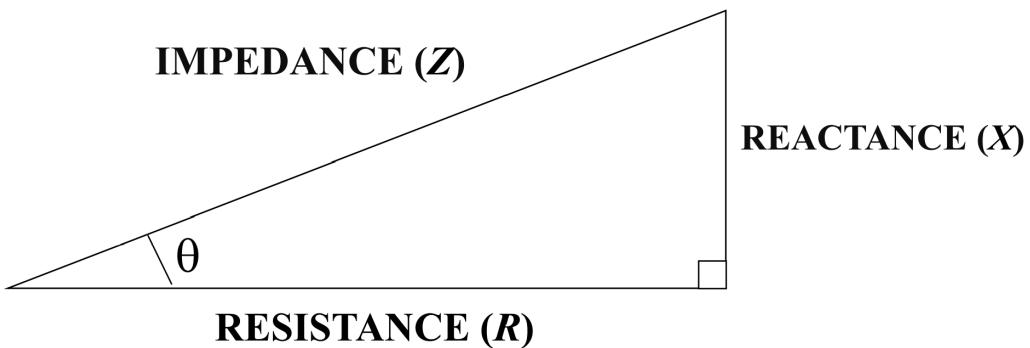


Figure 3.4: Single-phase impedance triangle.

$$Z = R + jX \quad (3.17)$$

$$= R + j(X_L - X_C) \quad (3.18)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (3.19)$$

Where,  $Z$  is impedance,  $R$  is resistance,  $X_L$  is inductive reactance,  $X_C$  is capacitive reactance,  $\omega$  is angular frequency ( $2\pi f$ ).

### 3.1.11 Single-phase power triangle

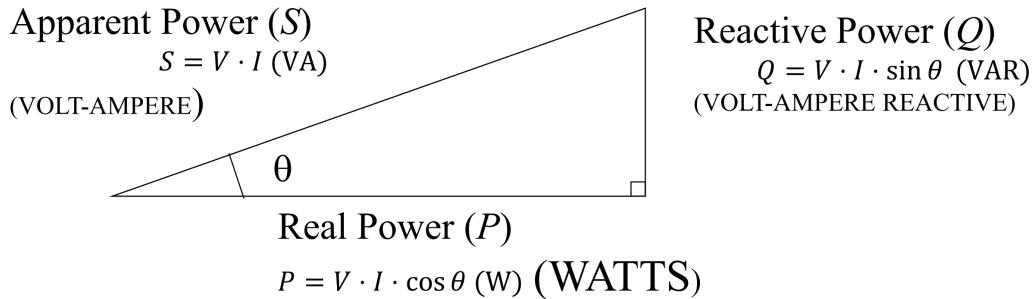


Figure 3.5: Single-phase power triangle.

- Real power ( $P$ ) is the power that can be put into or taken from the electrical system and is measured in Watts (W).
- Reactive power ( $Q$ ) is the power that circulates in the electrical system and is measured in Volt-Ampere-Reactive (VAR).
- Apparent power ( $S$ ) is what is apparent from the product of voltage and current and is measured in Volt-Amperes (VA).

### 3.1.12 Three-phase power

Since  $V$  in the star circuit and  $I$  in the delta circuit is subject to change simply by dividing by  $\sqrt{3}$ , whilst the other variable  $I$  and  $V$  in star and delta respectively remain unchanged. Hence we get:

$$P = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \cos \theta \quad (3.20)$$

For apparent power ( $S$ ) and reactive power ( $Q$ ) we have:

$$S = \sqrt{3} \cdot I_{line} \cdot V_{line} \quad (3.21)$$

$$Q = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \sin \theta \quad (3.22)$$

### 3.1.13 Student Activity

Three coils each of resistance  $5 \Omega$  and inductive reactance of  $10 \Omega$  are connected in (a) star and (b) delta across a 440 VRMS three-phase (line) supply.

If each coil has a capacitor connected in parallel having capacitive reactance of  $20 \Omega$  then calculate the line and phase currents and the total power absorbed.

## 3.2 Per Unit (PU) System

### 3.2.1 Electrical line diagram to Impedance diagram

- The ‘electrical line diagram’ - a schematic which allows an understanding of equipment and system arrangements.

- The ‘*impedance diagram*’ - a schematic which allows an understanding of the equipment and system impedances.
- The layout of both the ‘electrical line diagram’ and ‘impedance diagram’ should be similar but in the ‘*impedance diagram*’ all equipment and lines are replaced with impedances.
- All impedances will need to be calculated to a *common base* - hence use of a per unit system.

### 3.2.2 Simple equivalent impedances

For the purposes of steady-state analysis the Electrical Line Diagram is converted to an ‘*Impedance Line Diagram*’ where the equipment is represented as an ‘*Equivalent Impedance*’. Typical *simple* impedances representing equipment are: (note: not all  $R$ ,  $L$  and  $C$  values may be given).

Equipment	Equivalence Impedance Representation
AC Generator or Motor	
DC Machine (Motor or Generator)	
Transmission Lines and Cables	
Transformer	

Figure 3.6: Equivalent Impedance Representations.

### 3.2.3 How manufacturers of electrical equipment specify ratings

Manufacturers of electrical equipment would usually specify electrical equipment as follows:

e.g. A synchronous generator

- $S = 10 \text{ MVA}$  (value of apparent power)
- $V = 3.3 \text{ kV}$  (line voltage rating of the equipment)
- Phase = 3 (number of phases)
- $\text{PF} = 0.8$  (usual value of power factor of equipment)
- $N = 1500 \text{ rpm}$  (design speed of rotation)
- $F = 50 \text{ Hz}$  (frequency of the alternating current & voltage)
- $X = 0.14$  (Reactance given as a pu value or as a %)
- Connection = star (stator windings)

### 3.2.4 The per unit system

In Electrical Power System Analysis the per unit system is the preferred method for analysing circuit behaviour rather than the standard SI system of units (Watts, Volts, Amperes, etc.)

The advantages of the per unit system are:

- Computations for power systems have several voltage levels because of connected transformers is very cumbersome when using the SI system because values need to be referred across the transformer turns ratio. The per unit system (overcomes or simplifies) this problem.
- All powers, voltage, currents and impedances are expressed as per unit values of specified base values. This means they are easily compared with one another which is very helpful for equipment specification and selection and in power system design and its analysis.

### 3.2.5 Values in per unit system

In the per unit system five base values are needed. These are **power**, **current**, **voltage**, **impedance** and **power factor**. It is necessary to choose two base values and to calculate two base values.

Usually the base values defined are:

- the Apparent Power (Base\_VA)
- Voltage (Base\_V)

Power Factor is already expressed in per unit form. Once the base values are calculated then ‘actual values’ in the circuit can be expressed in per unit form.

### 3.2.6 Three-Phase system PU conversion

#### Step one

The per unit relationships for Base\_VA and Base\_V are define and Base\_I and Base\_Z are calculated:

$$\text{Base\_VA} = \text{Defined by Engineer} \quad (3.23)$$

$$\text{Base\_V} = \text{Defined by Engineer} \quad (3.24)$$

$$\text{Base\_I} = \frac{\text{Base\_VA}}{\sqrt{3} \cdot \text{Base\_V}} \quad (3.25)$$

$$\text{Base\_Z} = \frac{\text{Base\_V}}{\text{Base\_I}} \quad (3.26)$$

#### Step two

Having calculated the Base Values, these are then defined as being 1 per unit values:

- $\text{Base\_V} = 1$  per unit Voltage
- $\text{Base\_VA} = 1$  per unit Apparent Power
- $\text{Base\_I} = 1$  per unit Current
- $\text{Base\_Z} = 1$  per unit Impedance

#### Step three

In the circuit all apparent powers, voltage, currents and impedances are expressed as per unit values:

$$\text{Per\_Unit\_S} = \frac{\text{Actual\_Value\_S}}{\text{Base\_S}} \quad (3.27)$$

$$\text{Per\_Unit\_V} = \frac{\text{Actual\_Value\_V}}{\text{Base\_V}} \quad (3.28)$$

$$\text{Per\_Unit\_I} = \frac{\text{Actual\_Value\_I}}{\text{Base\_I}} \quad (3.29)$$

$$\text{Per\_Unit\_Z} = \frac{\text{Actual\_Value\_Z}}{\text{Base\_Z}} \quad (3.30)$$

## Step four

Sometimes parameters e.g. Z are already expressed in per unit form rather than as SI units but have been calculated to a different base (S and V). These can be converted as follows:

$$(Per\_Unit.Z)_{new\_base} = \frac{(Base\_VA)_{new\_base}}{(Base\_VA)_{old\_base}} \cdot \frac{(Base\_V)_{old\_base}^2}{(Base\_V)_{new\_base}^2} \cdot (Per\_Unit.Z)_{old\_base} \quad (3.31)$$

Some manufacturers and engineers prefer to work with the percentage system rather than the per unit system which of course is a simple matter of multiplying by 100/ Equipment manufacturers use a machine's own S and V to determine base values from which Z pu is then calculated.

### 3.2.7 Example PU system conversion

#### Single Line Diagram

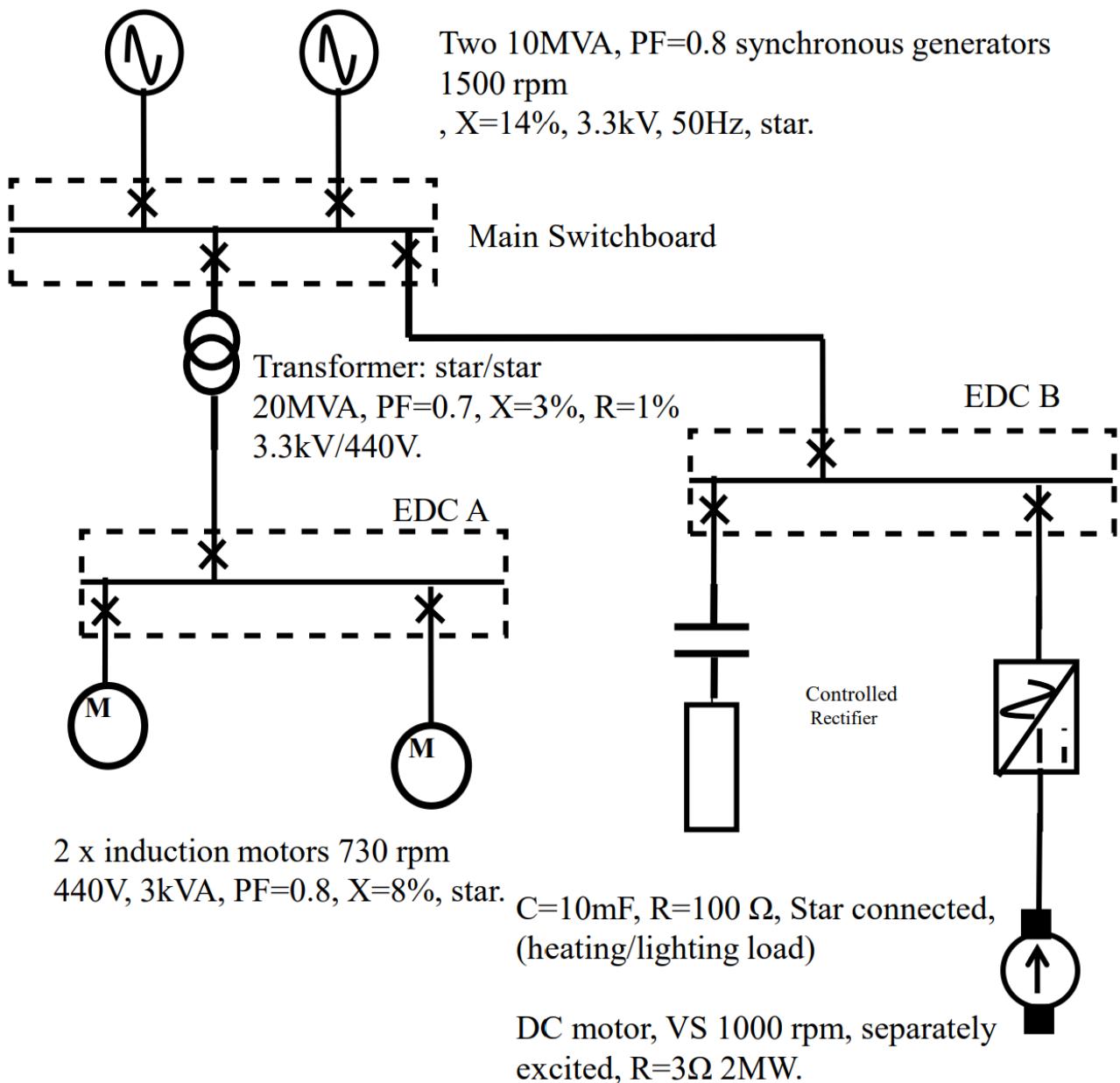


Figure 3.7: Single Line Diagram.

### Step one - calculating the base current and base impedance

Selecting 10 MVA as Base\_S and 3.3 kV as Base\_V (because it seems sensible considering the generators) then we have:

$$\text{Base\_I} = \frac{10^6}{\sqrt{3} \times 3.3 \times 10^6} = 1749.5 \text{ A} \quad (3.32)$$

$$\text{Base\_Z} = \frac{3.3 \times 10^3}{1749.5} = 1.886 \Omega \quad (3.33)$$

### Step two - defininng 1 p.u. values

- $3.3 \times 10^3 \text{ V} = 1 \text{ per unit Voltage} = 1 \text{ pu V}$
- $10 \times 10^6 \text{ VA} = 1 \text{ per unit Apparent Power} = 1 \text{ pu S}$
- $1749.5 \text{ A} = 1 \text{ per unit Current} = 1 \text{ pu A}$
- $1.886 \Omega = 1 \text{ per unit Impedance} = 1 \text{ pu Z}$

Sometimes % values are preferred by some engineers i.e. 1 pu = 100%

### Step three - converting impedances expressed in SI units to per unit form

The only ‘actual values’ i.e. expressed in SI units, are the heating load and the DC machine:

For the lighting/heating load:

$$-jXC = -j \left( \frac{1}{2\pi \cdot 50 \cdot 10 \times 10^{-3}} \right) = -j0.318 \quad (3.34)$$

$$-jXC = \frac{-j0.318}{1.886} = -j0.168 \text{ pu} \quad (3.35)$$

$$R = \frac{100}{1.886} = 53.022 \text{ pu} \quad (3.36)$$

For DC motor:

$$R = \frac{3}{1.886} = 1.591 \text{ pu} \quad (3.37)$$

$$S = P + \frac{2}{10} = 0.2 \text{ pu} \quad (3.38)$$

### Step four - converting impedances expressed in per unit form to another base

For the synchronous generators:

$$S = \frac{10}{10} = 1 \text{ pu} \quad (3.39)$$

$$V = 3.3 \text{ kV} = 1 \text{ pu} \quad (3.40)$$

$$X = \frac{14}{100} = 0.14 \text{ pu} \quad (3.41)$$

$$PF = 0.8 \text{ pu} \quad (3.42)$$

For the transformer:

$$S = \frac{20}{10} = 2 \text{ pu} \quad (3.43)$$

$$X = \frac{3}{100} \times \frac{10}{20} = 0.015 \text{ pu} \quad (3.44)$$

$$R = \frac{1}{100} \times \frac{10}{20} = 0.005 \text{ pu} \quad (3.45)$$

$$PF = 0.7 \text{ pu} \quad (3.46)$$

For the induction motors:

$$S = \frac{3}{10000} = 0.0003 \text{ pu} \quad (3.47)$$

$$X = \frac{8}{100} \times \frac{10000}{3} = 266.667 \text{ pu} \quad (3.48)$$

$$PF = 0.8 \quad (3.49)$$

#### Step five - drawing the impedance diagram

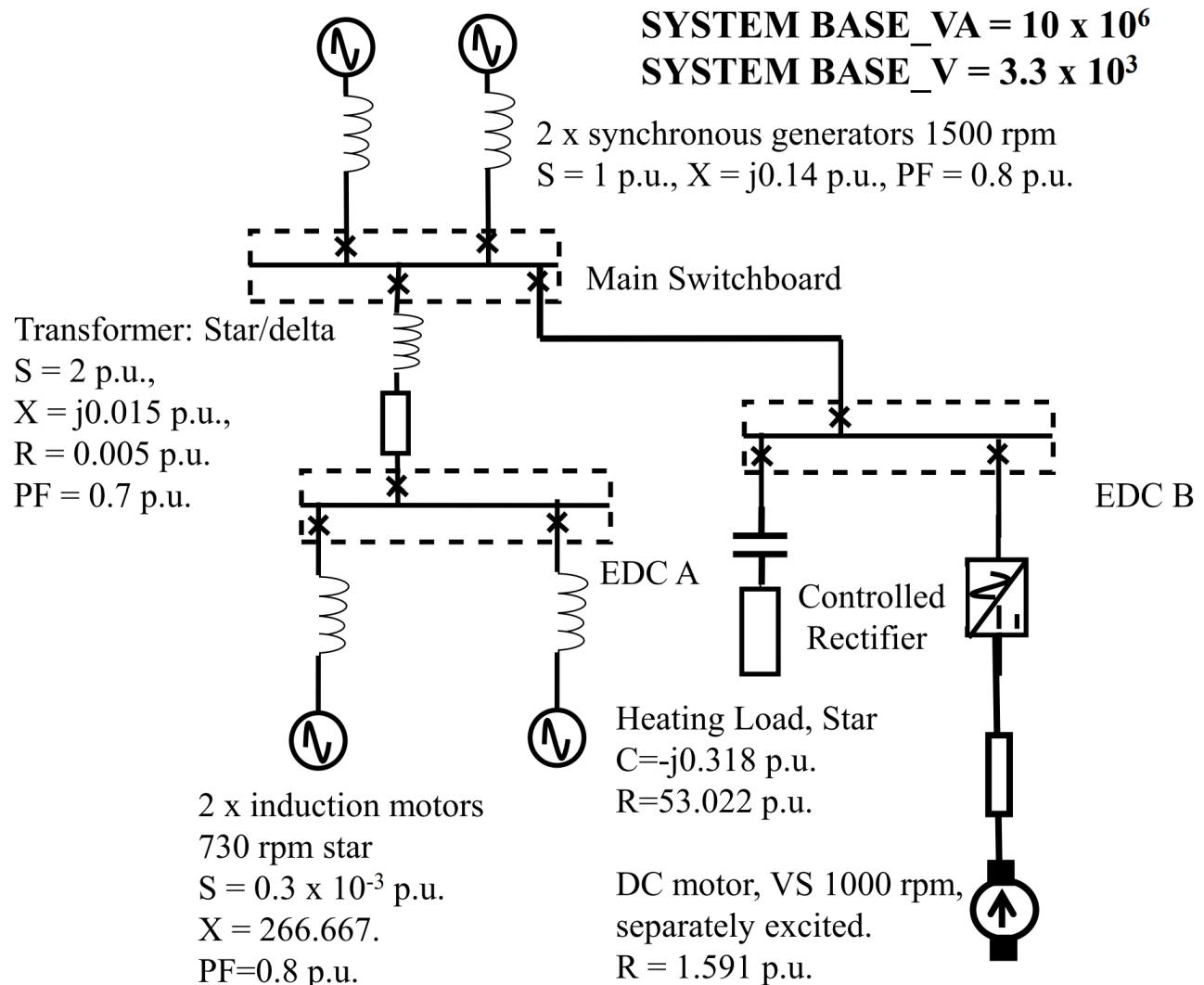


Figure 3.8: Impedance Diagram.

#### 3.2.8 Reactance diagram

The reactance diagram is a modification to the impedance diagram where only per unit reactances are shown. In a reactance diagram all resistances are ignored. The reactance diagram is useful because it allows ‘first pass’ calculations to be made in a power system without too much mathematical complexity due to having  $(R \pm jX)$ .

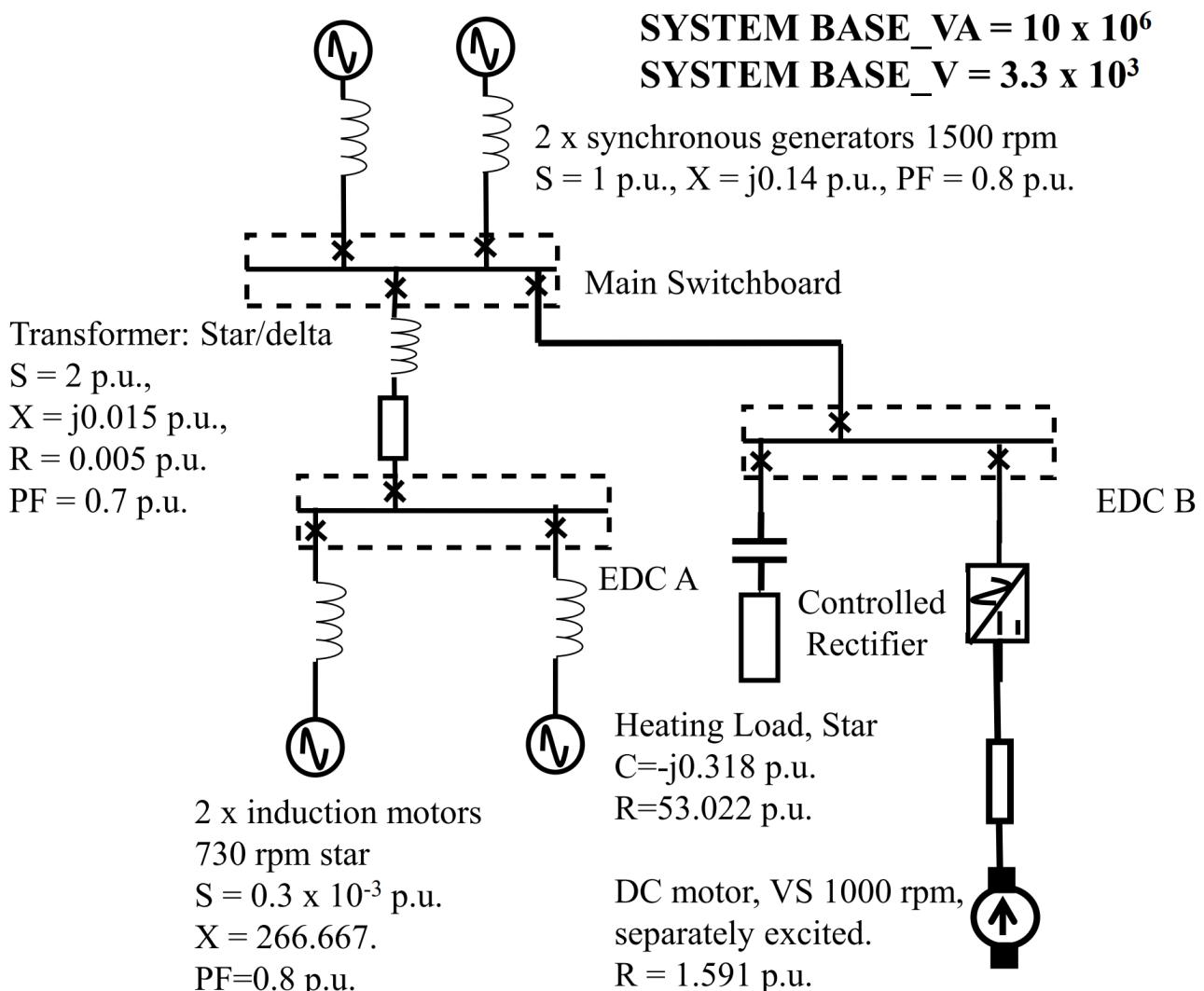


Figure 3.9: Reactance Diagram.

### 3.2.9 Impedance and reactance diagrams

Converting the electrical line diagram to an impedance diagram or reactance diagram is essential for:

- Potential difference (voltage drop) calculations
- Current flows in cables/lines
- Calculations of losses
- Power flows around an electrical system
- Understand transient effects
- Calculate fault level and fault currents
- Waveform distortion and its penetration
- Impacts when adding new equipment to the network

## 3.3 Summary

The per unit system allows powers, voltages, currents and impedances to be expressed relative to each other. This allows the designer to understand the relationships between different parts of the circuit.

Using the per-unit transformer model eliminates the need to scale quantities by the transformer turns ratio, thus eliminating a common source for error in electrical calculations.

# Chapter 4

## Using the Impedance Diagram

- Using impedance diagrams for load flows
- Using impedance diagrams for fault calculations

*By the end of this synchronous session you should be comfortable with how impedance diagrams can be used to calculate load flows and perform fault calculations in an electrical power system*

### 4.1 Load Flow Calculation

#### 4.1.1 Load flow

- In an electrical power system currents flow from generators to loads via a transmission/distribution system thereby permitting ‘load (power) flows’
- If a system is at steady-state then currents and power flows would be stable
- If there is a change in the system e.g. suddenly and additional load is connected, then there will be a change to currents and load flow
- An electrical system cannot change instantaneously from one state to another. The generators for example cannot instantaneously change the supply of power at the point load changes. There will be a transient period

#### 4.1.2 Load flow analysis example

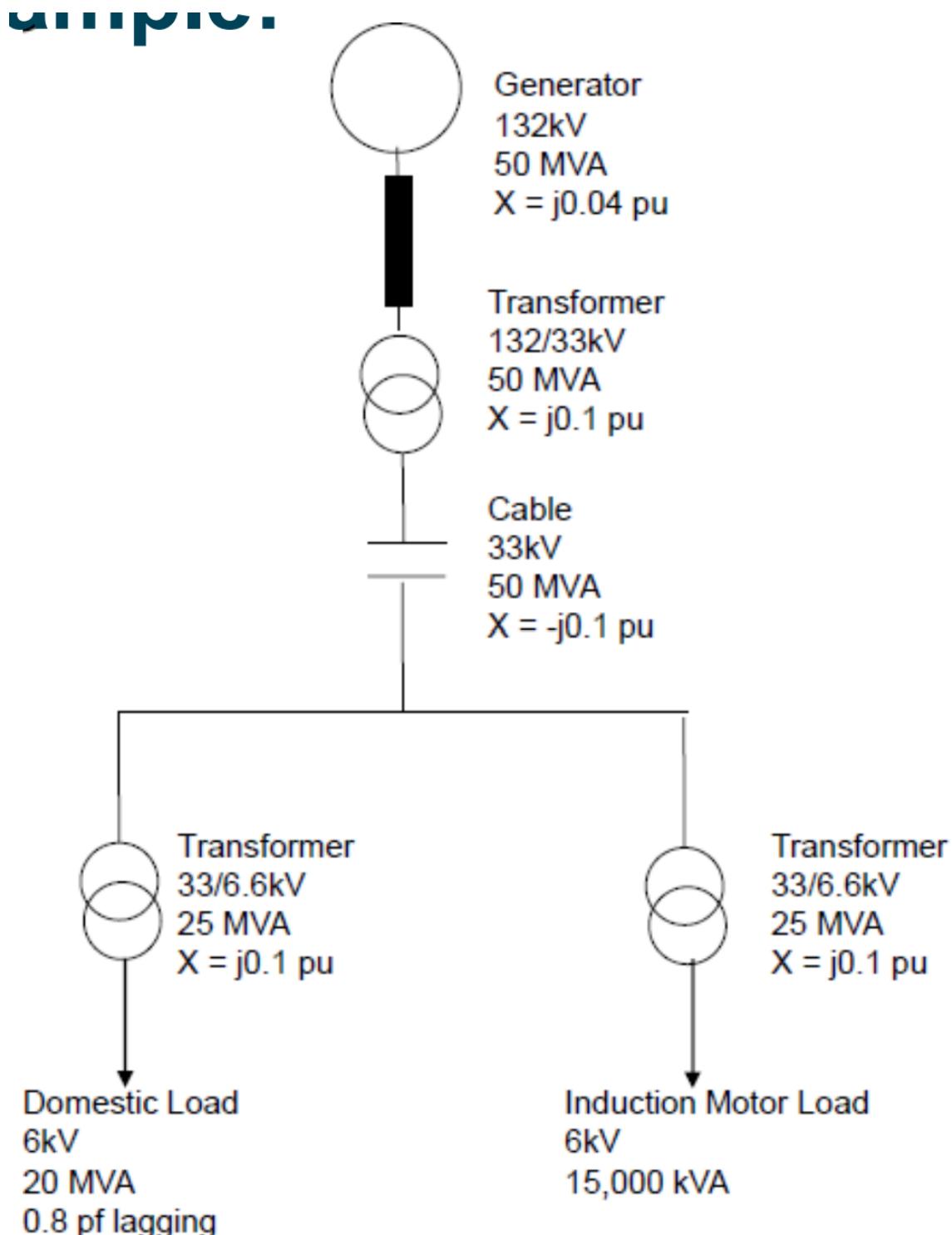


Figure 4.1: Single Line Diagram.

A 132 kV supply feeds two loads; a group of domestic consumers and a group of induction motors which on starting consume five times rated (or design) full load current at zero power factor lagging.

#### Part a

Convert the single line diagram into an impedance diagram. We will select a base S of 50 MVA and 33 kV as the base V. The values selected can be different but must be stated by the designer.

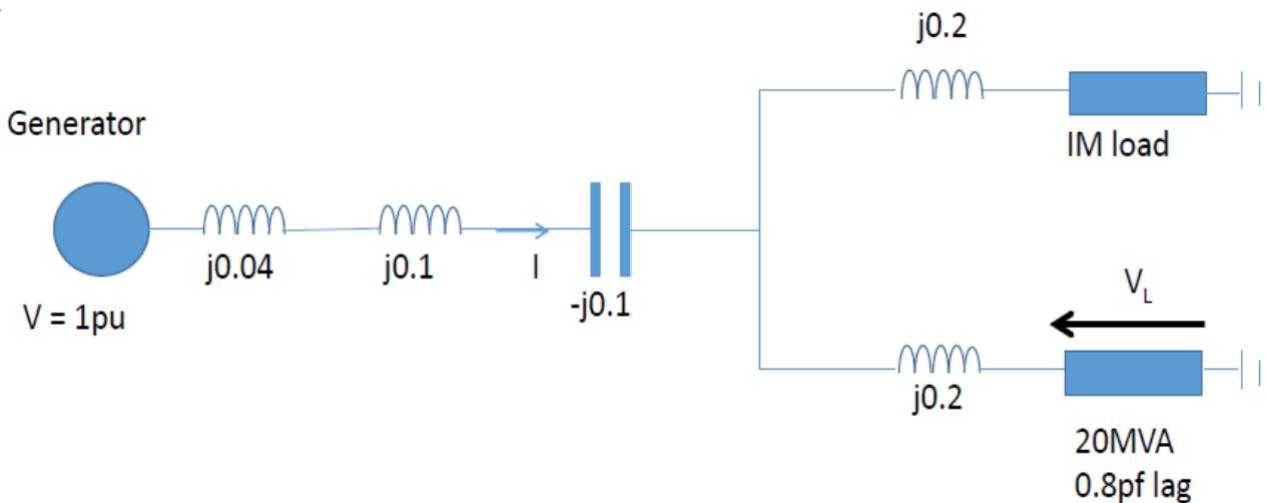


Figure 4.2: Single Line Diagram.

Here is the Impedance Diagram of the Single Line Diagram where all the impedances have been changed into per unit values on a 50 MVA base.

### Part b

Calculate the voltage at the domestic busbars prior to induction motor start.

- The induction motor load is open circuit so all current flowing from the generator will flow to the domestic load i.e. steady state
- To determine the voltage at the domestic busbar prior to the induction motor start then the equation for  $V_L = 1 - IZ$  can be used, where:
  - $V_L$  is the domestic voltage
  - 1 is the pu voltage at the generator
  - $I$  is the generator current
  - $Z$  is the system impedance between source and load

Calculating the impedance of the circuit then:

$$Z = j(0.04 + 0.1 - 0.1 + 0.2) = j0.24 \quad (4.1)$$

Calculating the current in the circuit then:

$$I = \frac{20 \times 10^6}{\sqrt{3} \times 6000} = 1925 \text{ A at } 0.8 \text{ pf lag} \quad (4.2)$$

Now defining base current related to domestic side (although the domestic side is rated at 6 kV, the transformer is rated at 6.6 kV and it is permissible to use this values as it is correct in the SLD and ID), we can say:

$$\text{Base current at } 6.6 \text{ kV} = \frac{50 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 4374 \text{ A} \quad (4.3)$$

$$\text{Domestic current pu} = \frac{1925}{4374} = 0.44 \text{ at } 0.8 \text{ pf lag} \quad (4.4)$$

$$V'_L = 1 - j0.04 [0.44 (0.8 - j0.6)] - j0.2 [0.44 (0.8 - j0.6)] = 0.94 \text{ pu} \quad (4.5)$$

### Part c

Calculate the maximum voltage dip that will occur when all the induction motors are started together at the same moment in time. The induction motor switch is now closed. The demand at this moment is five times normal full-load current. The induction motor load demands a substantial current:

$$\text{Starting current IM} = -j \frac{15000 \times 10^3}{\sqrt{3} \times 6000} \times 5 = -j7217 \text{ A} \quad (4.6)$$

$$\text{Starting current IM} = -j \frac{7217}{4374} = -j1.64 \text{ pu} \quad (4.7)$$

The induction motor load demands a substantial current which flows from the generator. Remember at IM start there is no real power so all power is reactive hence zero power factor. The voltage at the terminals will drop across the series connected devices:

$$V'_L = 1 - j0.04 [0.44 (0.8 - j0.6) - j1.64] - j0.2 [0.44 (0.8 - j0.6)] \quad (4.8)$$

$$= 0.871 - j0.084 = 0.875 \text{ pu} \quad (4.9)$$

Hence the voltage dips from 0.94 pu to 0.87 pu or alternatively from 6.204 kV to 5.78 pu. The voltage dip would be noticed temporarily as a light flicker or dimming. In practice, the generator would recover after a few seconds - transient response of the generator.

#### 4.1.3 Some thoughts

- Understand the initial conditions first and then calculate the impact of load changes
- The line series capacitor installed has partially neutralised the network inductance. Without this capacitance the dip would be much more severe
- Voltage flicker often occurs when there is a sudden demand for large power is demanded e.g. starting of large induction motors on ships or in grids e.g. near steel rolling mills or factories

## 4.2 Using impedance diagrams in short-circuit balanced faults

### 4.2.1 Fault classification

Faults may be classified as being:

- Open circuit faults
- Short circuit faults

Faults may occur in high voltage and low voltage systems meaning:

- Three-phase system faults
- Single-phase system faults
- DC system faults

For short circuit fault types then the engineer needs to appreciate its significance and protect against such events. Faults have two main characteristics: MVA fault level (MVA) and fault current ( $I_{fault}$ ).

### 4.2.2 Types of faults

Symmetrical fault (Fault currents are balanced in each phase)

- Three-phase short circuit
- Three-phase to ground fault
- (Three-phase open circuit)

Unsymmetrical fault (fault currents are **not** balanced in each phase)

- Single-phase to earth
- Double-phase to earth
- Two-phases short together
- Single-phase open circuit
- Double-phase open circuit

#### 4.2.3 Faults normally are due to:

- Wearing of insulation
- Aging
- Poor connections
- Fault due to lightning
- Tree limbs falling on the line
- Wind, weather impacts
- Impact/shock damage
- Vandalism
- Poor safety protocols or work on live equipment

#### 4.2.4 MVA method

- The MVA method is used to define the power at the point of a fault
- The accepted method is to calculate the Fault MVA as follows:

$$MVA_{fault} = \frac{\text{Base S (MVA)}}{\text{Impedance to fault (pu)}} \quad (4.10)$$

- Having calculated the  $MVA_{fault}$  the fault current can be calculated using the nominal voltage at the fault

$$I_{fault} = \frac{MVA_{fault}}{\sqrt{3} \times V_{base}} \quad (4.11)$$

#### 4.2.5 Balanced three-phase fault

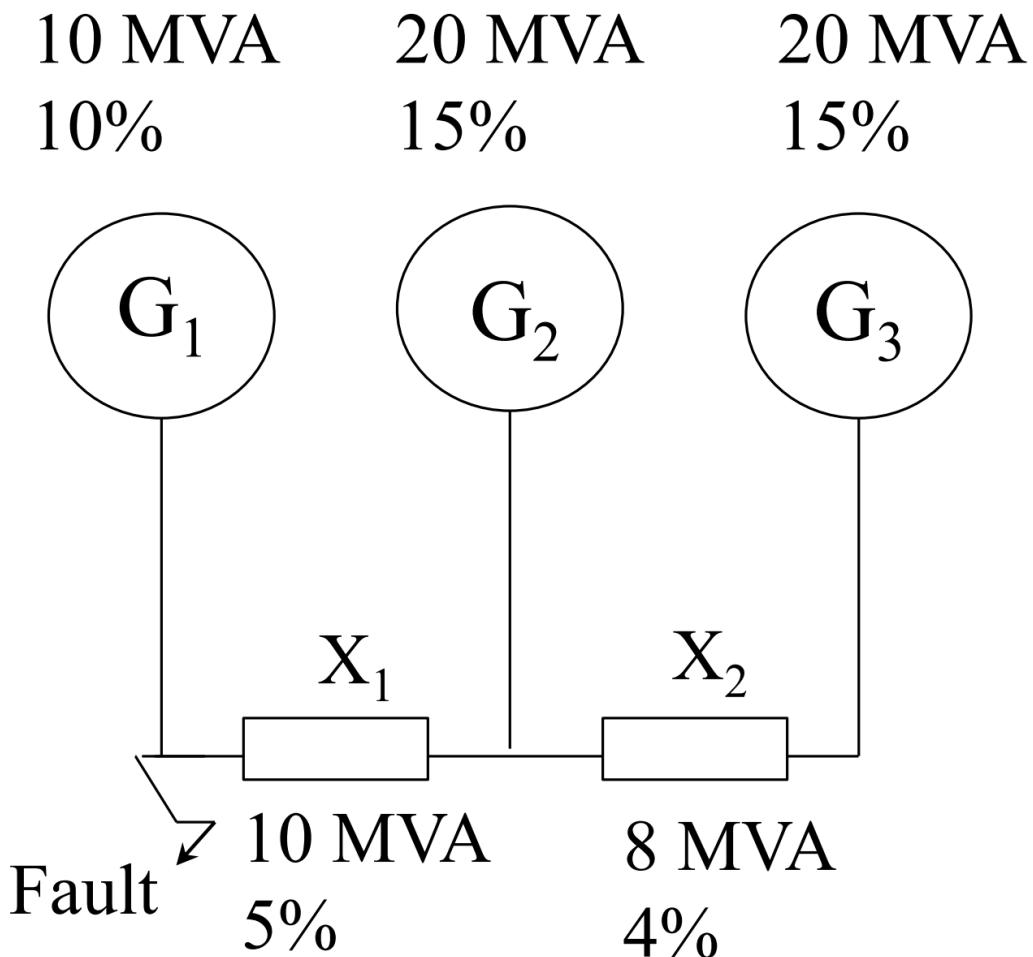


Figure 4.3: Balanced three-phase fault.

An interconnected generator-reactor system is active and suddenly incurs a balanced three-phase short circuit at the Fault indicated. Using a 50 MVA base then draw an impedance diagram and hence determine the Fault Level and Fault Current. It is an 11 kV three phase system.

#### 4.2.6 Solution

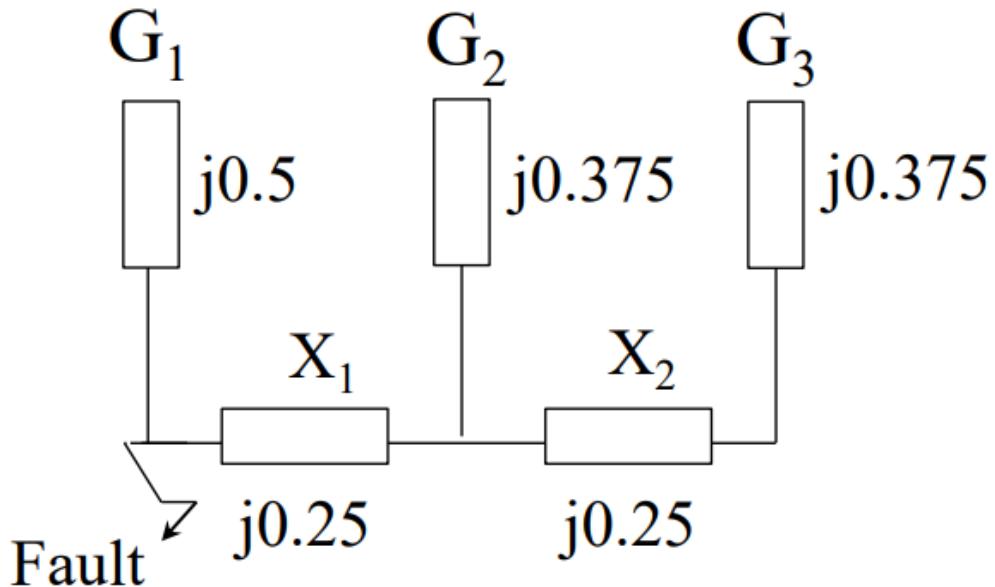


Figure 4.4: Impedance diagram.

$$X_{G1} = \frac{50}{10} \cdot 0.1 = 0.5 \text{ pu} \quad (4.12)$$

$$X_{G2} = \frac{50}{20} \cdot 0.15 = 0.375 \text{ pu} \quad (4.13)$$

$$X_{G3} = \frac{50}{20} \cdot 0.15 = 0.375 \text{ pu} \quad (4.14)$$

$$X_1 = \frac{50}{10} \cdot 0.05 = 0.25 \text{ pu} \quad (4.15)$$

$$X_1 = \frac{50}{8} \cdot 0.04 = 0.25 \text{ pu} \quad (4.16)$$

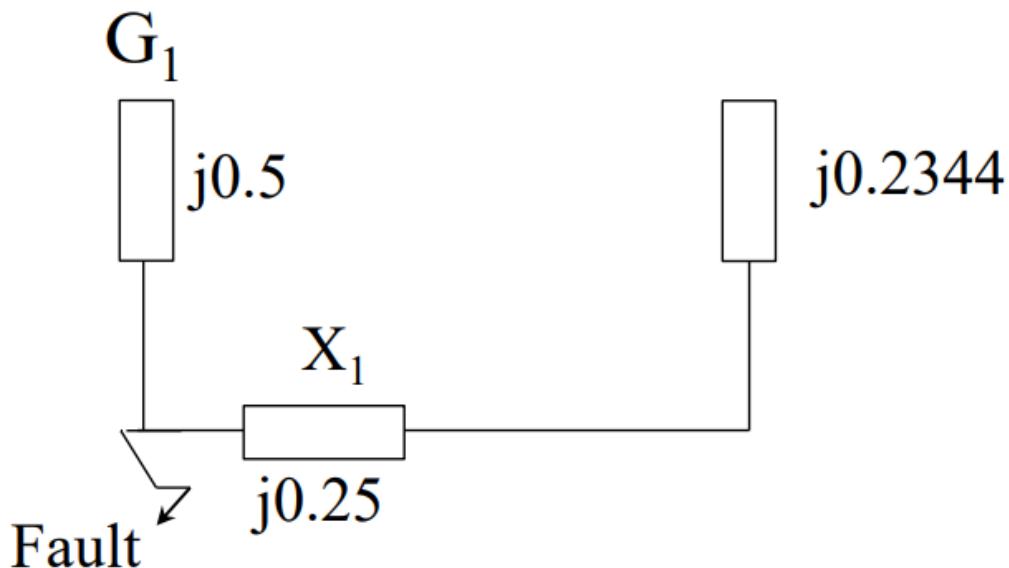


Figure 4.5: Impedance diagram circuit reduced.

$$\text{Per unit reactance} = \frac{0.5(0.2344 + 0.25)}{0.5 + (0.2344 + 0.25)} = j0.246 \quad (4.17)$$

$$\text{MVA Fault Level} = \frac{50 \times 10^3}{0.246} = 203.25 \text{ MVA} \quad (4.18)$$

$$\text{Fault current} = \frac{203.25 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 10668 \text{ A} \quad (4.19)$$

The MVA Fault Level provides information on the ‘power at the fault’. The Fault Current provides information on protection e.g. circuit breakers. This is known as the symmetrical fault current.

#### 4.2.7 Importance of MVA

- The short circuit capacity (SCC) at the busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC.
- An infinitely strong bus (or infinite bus bar) has an infinite SCC, with a zero equivalent impedance and will maintain its voltage under all conditions
- Magnitude of short circuit current is time dependent due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax circuit breakers (CB) adversely so that current limiting reactors are sometimes used

#### 4.2.8 Power system symmetrical faults

- In a power system, knowing the maximum MVA Fault Level and the Fault Current that could potentially flow into a zero impedance fault is necessary in order to rate switch gear correctly
- The MVA Fault Level defines the maximum MVA that is experienced when a symmetrical fault event occurs. The fault level is usually expressed in MVA (or a corresponding per-unit value)
- The maximum fault current can be calculated using the MVA Fault Level and the nominal Voltage Rating at the fault location

#### 4.2.9 Conclusions

- The analysis shown in this session has explained how impedance diagrams can be used for system analysis for ‘load flows’ and ‘balanced faults’
- For larger or complex circuits then many more calculations are needed meaning computers are generally used to calculate load flows and faults
- Various computer programmes are available including MATLAB, Simulink Simpower Systems, PSCAD, ERACS, etc

# Chapter 5

## Faulted Networks

- Introducing the concept of unbalanced networks
- Using impedance diagrams for fault calculations

### 5.1 Symmetrical faults recap

In a power system the most significant fault that can occur is when all three-phases short together. This is a symmetrical or balanced fault. The MVA Fault Level defines the maximum MVA that the system is subjected to when a symmetrical fault event occurs. The fault level is usually expressed in MVA (or a corresponding per-unit value). The maximum fault current can be calculated using the MVA Fault Level and the nominal Voltage Rating at the fault location.

### 5.2 Unbalanced faults

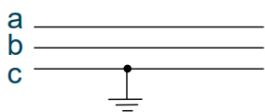
#### 5.2.1 Types of ‘unbalanced faults’

Unsymmetrical faults - currents and voltages are not balanced in each phase:

- Single line to ground
- Line to line
- Double line to ground
- Single phase open circuit
- Double phase open circuit

For each short-circuit, the fault can be bolted (a zero impedance fault) or have a fault impedance known as  $Z_f$ .

Single line to ground



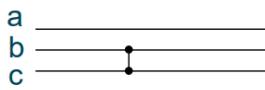
Bolted short



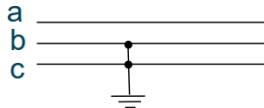
Impedance short



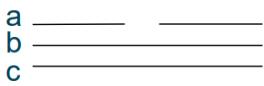
Line to line



Double line to ground



Single line open circuit



Double line open circuit

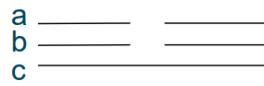


Figure 5.1: Unsymmetrical/unbalanced faults.

### 5.2.2 List of possible faults

- Three phase symmetrical fault L-L-L
- Three phase symmetrical fault L-L-L-G
- Line to line fault
- Double line to ground fault
- Single line to ground fault
- Single line open circuit
- Double line open circuit

The most common fault is the single line to ground fault. The worst fault is a three-phase to ground fault (L-L-L-G or L-L-L).

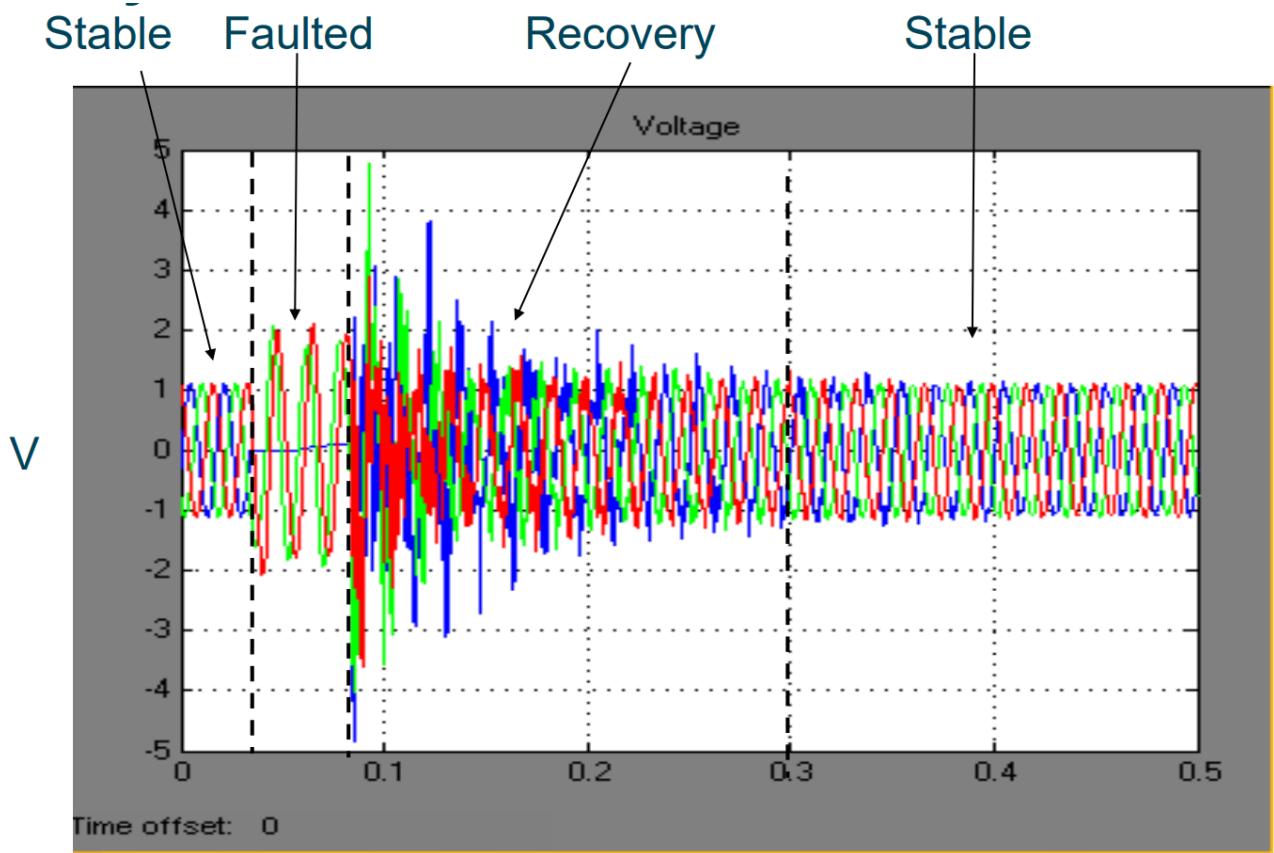


Figure 5.2: Unsymmetrical/unbalanced fault graph.

We see the blue phase go to ground (0V) and the other two phases increase in voltage and are no longer  $120^\circ$  out of phase with each other.

### 5.2.3 Method of analysis

Each phase is experiencing something different i.e. what is happening on one phase is not what is happening on the other. RMS voltages and currents are unbalanced.

$$V_a \neq V_b \neq V_c \text{ nor } I_a \neq I_b \neq I_c \quad (5.1)$$

The presumption that we used for symmetrical faults (the same equivalent circuit for each phase) is not valid in the unsymmetrical/unbalanced case. For the unbalanced case it is necessary to use a different method. We use ‘Fortescue’s Theorem’.

### 5.2.4 Fortescue’s Theorem

Fortescue’s Theorem says:

*Three unbalanced phasors in a multi-phase electrical system can be resolved into a set of balanced phasors consisting of:*

- Positive-sequence components
- Negative-sequence components
- Zero sequence components

$$V_{line} = V_{positive} + V_{negative} + V_{zero} \quad (5.2)$$

$$I_{line} = I_{positive} + I_{negative} + I_{zero} \quad (5.3)$$

### 5.2.5 Positive sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Displaced from each other by  $120^\circ$
- Have phase sequence a-b-c
- Usually referred to as  $V_{a1}, V_{b1}, V_{c1}$

### 5.2.6 Negative sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Displaced from each other by  $120^\circ$
- Have phase sequence a-c-b
- Usually referred to as  $V_{a2}, V_{b2}, V_{c2}$

### 5.2.7 Zero sequence components

For a three-phase system there are three balanced phasors:

- Equal in magnitude
- Zero phase displacement
- No phase sequence
- Usually referred to as  $V_{a0}, V_{b0}, V_{c0}$

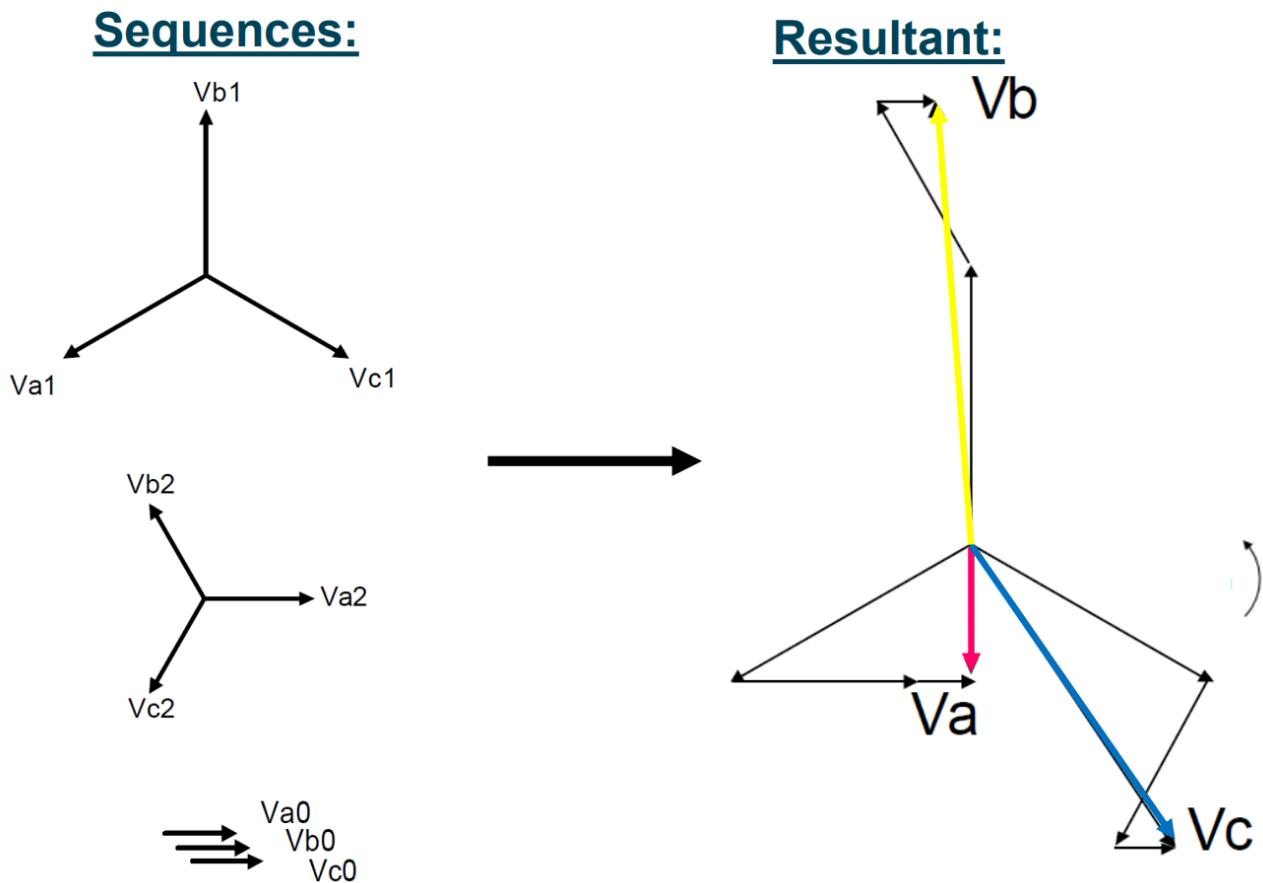


Figure 5.3: Sequence components and phase relationship.

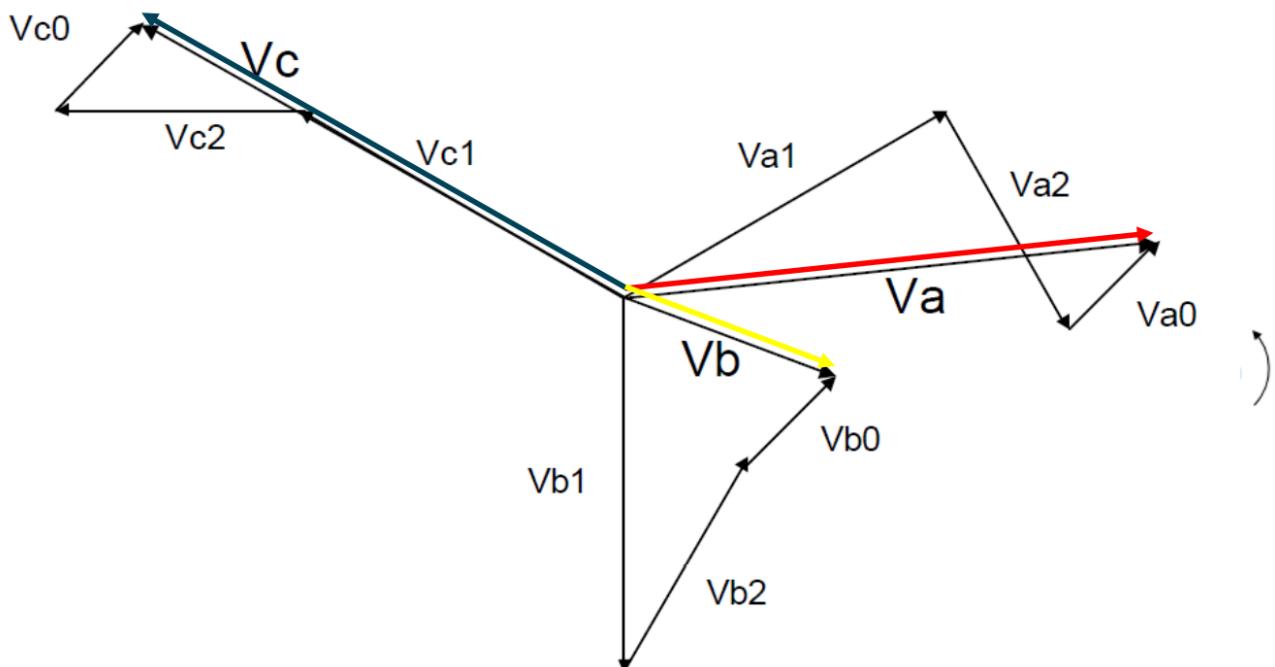


Figure 5.4: Sequence components 2.

### 5.2.8 Summing sequence components

Original phasors are the sum of their components

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad (5.4)$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \quad (5.5)$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \quad (5.6)$$

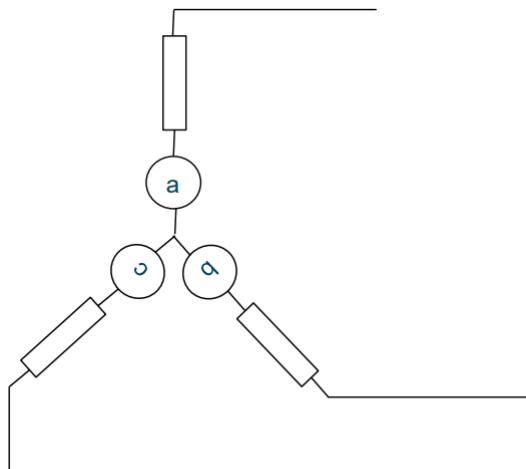
Hence:

$$\text{Line} = \sum \text{sequence components} \quad (5.7)$$

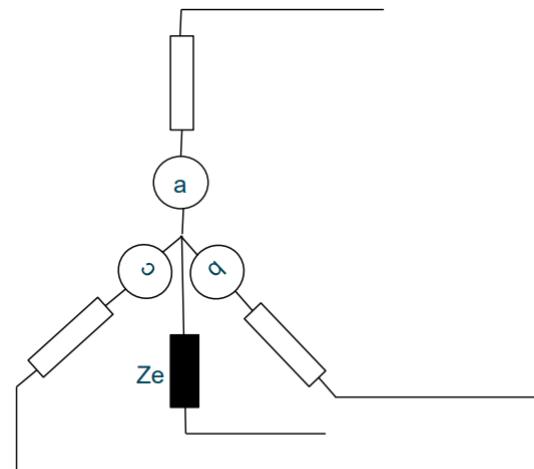
In balanced/symmetrical networks in multi-phase systems then only positive sequence components are present.

### 5.2.9 Note about grounding/earthing

How a system is grounded has a major impact on fault current. Zero sequence current can only flow when the start point of the source is tied to ground / earth directly or via an impedance  $Z_e$ .



No zero sequence



Zero sequence current flows in  $Z_e$

Figure 5.5: Grounding/earthing.

We can see the virtual/floating star point on the sequence on the left. Normally, this is left floating on ship systems for example. The star point can be connected to ground (unusual for generators) or we can add an impedance to the star point connection. This is because the star point is not always 0V under a fault condition. Hence, by including an earth impedance, we can limit current flow.

Zero sequence current flows can only happen if we have a connection to ground. In floating star point connections, we cannot have zero sequence current flows.

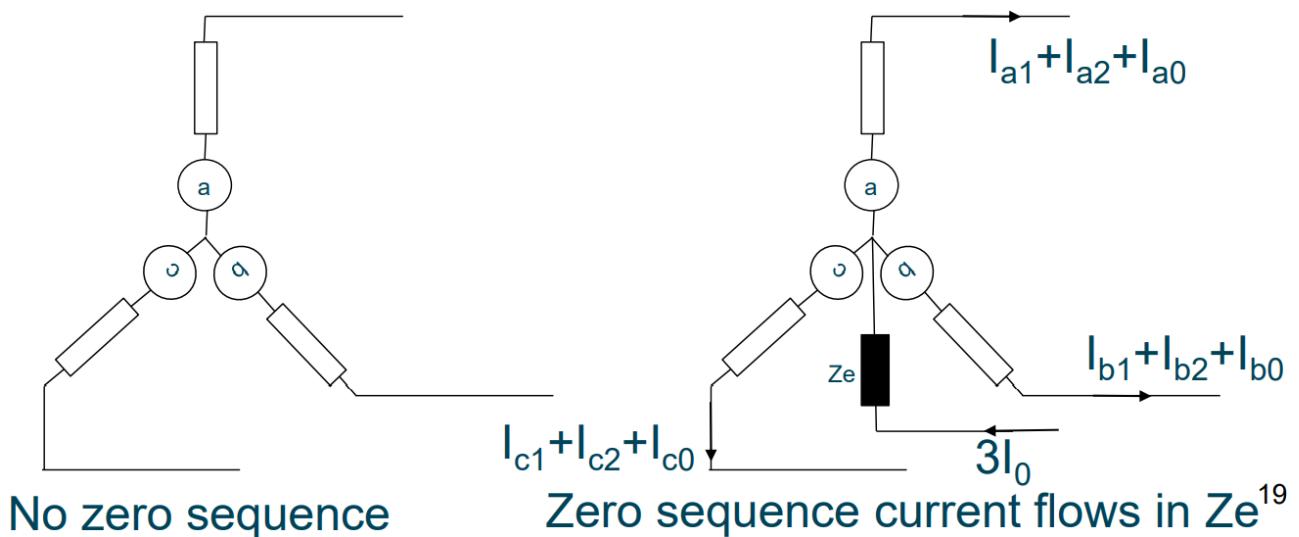


Figure 5.6: Currents during grounded star point.

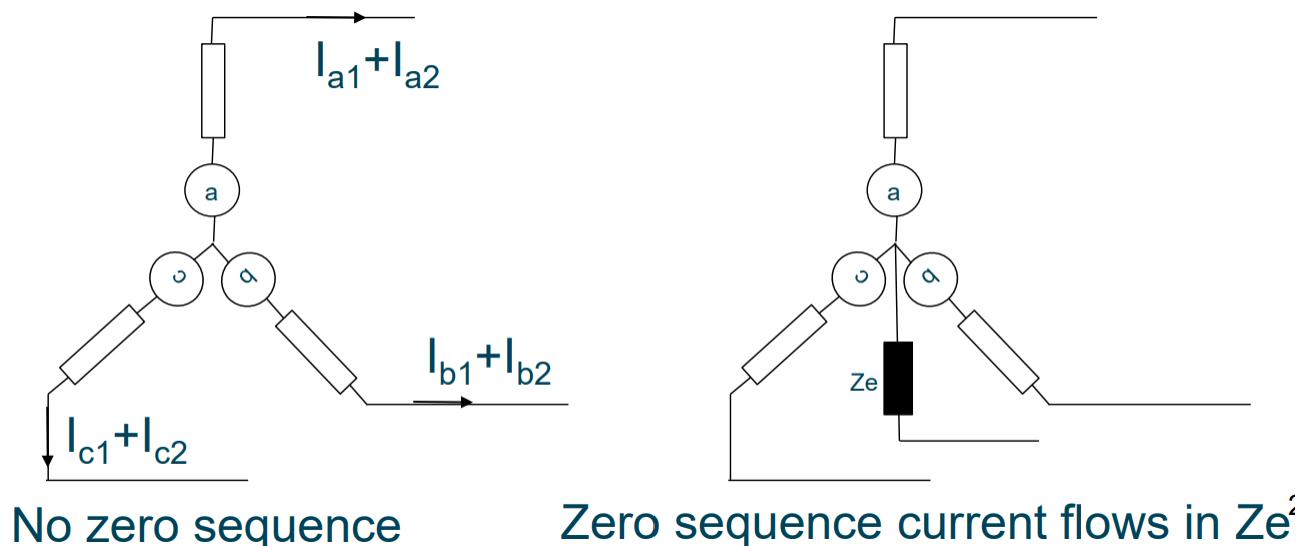


Figure 5.7: Currents during floating star point.

### 5.2.10 The operator ‘a’

Let us define an operator that rotates a phasor by  $120^\circ$ :

$$a = 1\angle 120^\circ = (-0.5 + j0.8666) \quad (5.8)$$

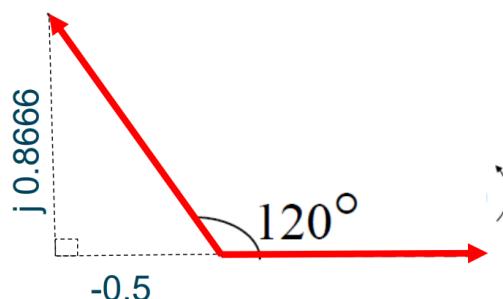


Figure 5.8: ‘a’ operator.

### 5.2.11 Expressing phasors $a^2$ and $a^3$

$$a^2 = a \times a = (1\angle 240^\circ) = 1\angle -120^\circ \quad (5.9)$$

Similarly:

$$a^3 = (1\angle 360^\circ) = 1\angle 0^\circ \quad (5.10)$$

Therefore:

$$a + a^2 + a^3 = 0 \quad (5.11)$$

$$1 + a + a^2 = 0 \quad (5.12)$$

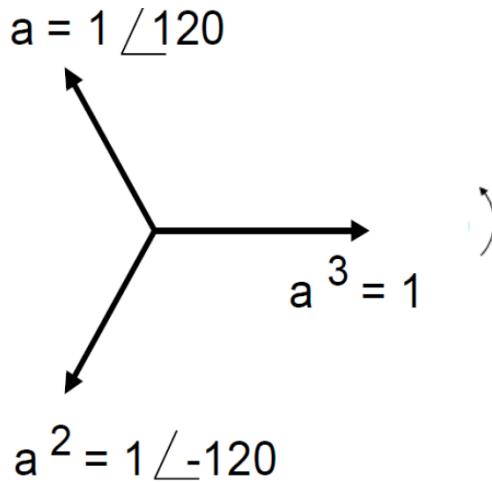


Figure 5.9: 'a' phasors.

The value of the star point changes with fault conditions.

### 5.2.12 Representation using 'a'

Using the 'a' operator then the positive sequence components can be written:

$$V_{a1} = 1 \quad (5.13)$$

$$V_{b1} = (1\angle -120^\circ) = a^2 V_{a1} \quad (5.14)$$

$$V_{c1} = (1\angle 120^\circ) = a V_{a1} \quad (5.15)$$

In other words we have used the 'a' operator to express  $V_{b1}$  and  $V_{c1}$  in terms of  $V_{a1}$ . Similarly for the negative sequence, we have:

$$V_{a2} = 1 \quad (5.16)$$

$$V_{b2} = (1\angle 120^\circ) V_{a2} = a V_{a2} \quad (5.17)$$

$$V_{c2} = (1\angle -120^\circ) V_{a2} = a^2 V_{a2} \quad (5.18)$$

In other words we have used the 'a' operator to express  $V_{b2}$  and  $V_{c2}$  in terms of  $V_{a2}$ . For the zero sequence:

$$V_{a0} = V_{b0} = V_{c0} \quad (5.19)$$

No need for the operator 'a' here as all zero sequence components are in phase!

<i>Function</i>	<i>Polar</i>	<i>Rectangular</i>
$a$	$1/\underline{120^\circ}$	$-0.5 + j0.866$
$a^2$	$1/\underline{240^\circ}$	$-0.5 - j0.866$
$a^3$	$1/\underline{0^\circ}$	$1.0 + j0$
$a^4$	$1/\underline{120^\circ}$	$-0.5 + j0.866$
$1 + a = -a^2$	$1/\underline{60^\circ}$	$0.5 + j0.866$
$1 + a^2 = -a$	$1/\underline{-60^\circ}$	$0.5 - j0.866$
$1 - a$	$\sqrt{3}/\underline{-30^\circ}$	$1.5 - j0.866$
$1 - a^2$	$\sqrt{3}/\underline{30^\circ}$	$1.5 + j0.866$
$a - 1$	$\sqrt{3}/\underline{150^\circ}$	$-1.5 + j0.866$
$a^2 - 1$	$\sqrt{3}/\underline{-150^\circ}$	$-1.5 - j0.866$
$a - a^2$	$\sqrt{3}/\underline{90^\circ}$	$0.0 + j1.732$
$a^2 - a$	$\sqrt{3}/\underline{-90^\circ}$	$0.0 - j1.732$
$a + a^2$	$1/\underline{180^\circ}$	$-1.0 + j0$
$1 + a + a^2$	$0$	$0$

Figure 5.10: List of 'a' phasors.

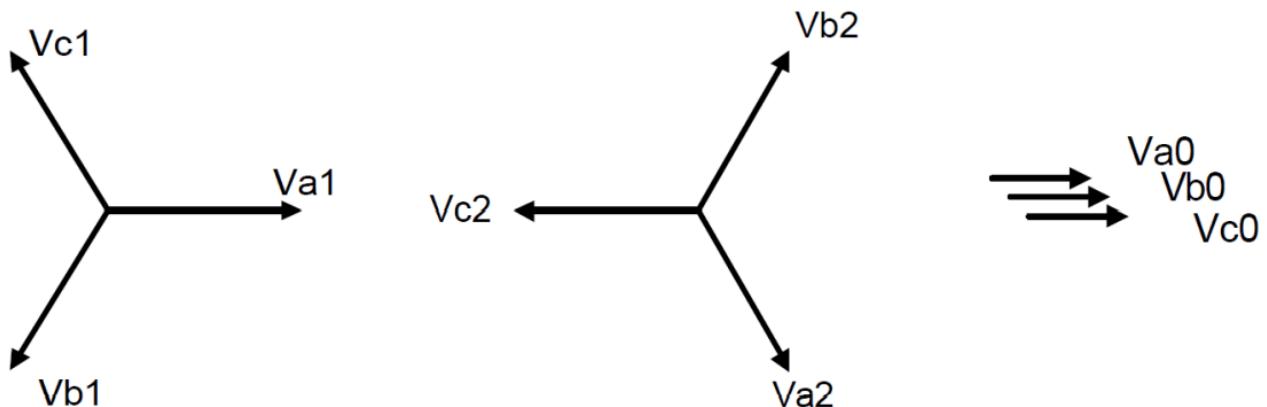
### 5.2.13 Representing all sequence components in terms of $V_a$ sequence components

$$\text{Line} = \sum \text{sequence components} \quad (5.20)$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = V_{a0} + V_{a1} + V_{a2} \quad (5.21)$$

$$V_b = V_{b0} + V_{b1} + V_{b2} = V_{a0} + a^2 V_{a1} + a V_{a2} \quad (5.22)$$

$$V_c = V_{c0} + V_{c1} + V_{c2} = V_{a0} + a V_{a1} + a^2 V_{a2} \quad (5.23)$$

Figure 5.11: Phase voltages expressed in terms of  $V_a$ .

### 5.2.14 ‘a’ matrix

$$\text{Line} = \sum \text{sequence components} \quad (5.24)$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad (5.25)$$

### 5.2.15 Inverse ‘a’ matrix

The sequences may be described by the ‘inverse a matrix’ and phasors:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (5.26)$$

Where:

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (5.27)$$

### 5.2.16 Example

A three-phase star connected load is connected across a three-phase balanced supply system. Obtain a set of equations relating the symmetrical components of a line and its phase voltages. Assuming:

$$V_{ab} = V_a - V_b \quad (5.28)$$

We will do this for one line voltage...

Zero sequence. Since:

$$V_{ab} + V_{bc} + V_{ca} = 0 \quad (5.29)$$

then

$$V_{ab0} + V_{bc0} + V_{ca0} = 0 \quad (5.30)$$

In other words there is no change in the zero sequence relationships. Assume balance

Positive sequence: Choosing  $V_{ab}$  then:

$$V_{ab1} = \frac{1}{3} (V_{ab} + aV_{bc} + a^2V_{ca}) \text{ from inverse ‘a’ matrix} \quad (5.31)$$

$$= \frac{1}{3} [(V_a - V_b) + a(V_b - V_c) + a^2(V_c - V_a)] \quad (5.32)$$

$$\dots \quad (5.33)$$

$$= \frac{1}{3} [(1 - a^2)(V_a + aV_b + a^2V_c)] \quad (5.34)$$

$$= (1 - a^2)V_{a1} \text{ from table} \quad (5.35)$$

$$= \sqrt{3}V_{a1}e^{j30} \text{ using exp form} \quad (5.36)$$

Negative sequence:

$$V_{ab2} = \frac{1}{3} (V_{ab} + a^2 V_{bc} + a V_{ca}) \text{ from inverse 'a' matrix} \quad (5.37)$$

$$= \frac{1}{3} [(V_a - V_b) + a^2 (V_b - V_c) + a (V_c - V_a)] \quad (5.38)$$

$$\dots \quad (5.39)$$

$$= \frac{1}{3} [(1 - a) (V_a + a^2 V_b + a V_c)] \quad (5.40)$$

$$= (1 - a) V_{a2} \text{ from table} \quad (5.41)$$

$$= \sqrt{3} V_{a2} e^{-j30^\circ} \text{ using exp form} \quad (5.42)$$

### 5.2.17 Sequence components and faults

- This lecture started by considering unsymmetrical faults
- The lecture has introduced the method of sequence components and has provided a method analysis of unsymmetrical faults based on Fortescue's theorem
- Manipulation of the voltages and currents using the 'a' matrix is an important step since this provides the analytical means to analyse unsymmetrical faults from sequence, phase and line perspectives
- In the next lecture we will look at unsymmetrical faults by applying this methodology

### 5.2.18 Conclusions

- The analysis shown in this session has explained the system analysis methods for 'unbalanced faults'
- The introduction to the 'a' matrix which will be used for relationships between phase and line values and also introduced sequence components
- Appreciate the need for positive, negative and zero sequence impedances of different components that make up a power system

# Chapter 6

## Full Fault Analysis

### 6.1 Unbalanced impedance

#### 6.1.1 Impedance and sequence components

We have established that in a three-phase unbalanced network there are line and phase voltages and currents that deviate in their relationships from the balanced case. Furthermore, any unbalance can be described as a set of sequence components consisting positive, negative and zero sequence phasors. Now considering impedances in unbalanced networks then we need to ensure that we understand:

- How to change between star and delta arrangements
- Appreciate how sequence impedance is calculated

#### 6.1.2 Unbalanced star and delta equivalence

$Z_{\text{delta}} = 3 \cdot Z_{\text{star}}$  for all three phases when the loads in the three-phase system were balanced. This was helpful when looking at symmetrical faults since we normally convert delta connections into star connections and consider an impedance diagram as representing one phase. When loads are unbalanced then we need to consider each phase independently because they are subjected to different voltages, currents and impedances. Consider the two circuits below. The star and delta equivalence must result in the same line voltages and currents. In other words the impedance between any two impedances must be equivalent.

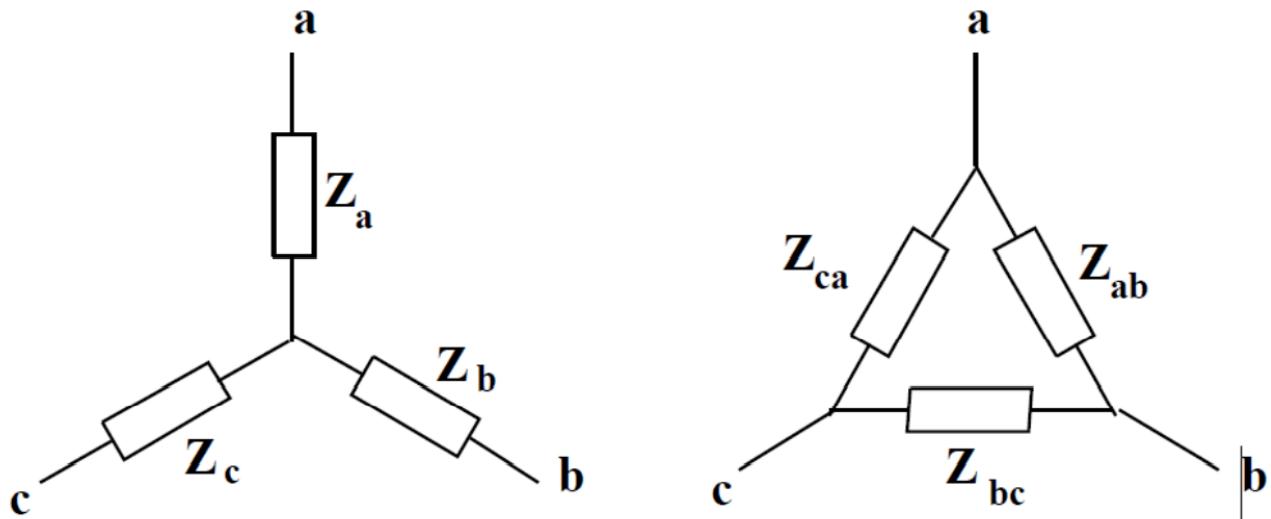


Figure 6.1: Star and delta arrangements.

For example considering phase a and phase b, the impedance equivalence must be:

$$Z_a + Z_b = Z_{ab} // (Z_{ca} + Z_{bc}) \text{ similarly,} \quad (6.1)$$

$$Z_b + Z_c = Z_{bc} // (Z_{ab} + Z_{ca}) \quad (6.2)$$

$$Z_c + Z_a = Z_{ca} // (Z_{bc} + Z_{ab}) \quad (6.3)$$

By manipulation and substitution then it is possible to derive the following relationships:

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c} \quad (6.4)$$

$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a} \quad (6.5)$$

$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b} \quad (6.6)$$

and

$$Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \quad (6.7)$$

$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} \quad (6.8)$$

$$Z_c = \frac{Z_{bc} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \quad (6.9)$$

These relationships are needed when considering impedance in unbalanced loads.

### 6.1.3 Good practice

In many fault calculations it is handy to convert delta impedances into star impedances because:

- It ensures that all balanced arrangements are related to ground or virtual group (floating star point)
- When calculating faults then it is apparent that such calculations are made for one phase and then ‘phase shifted’ to determine impact on other phases. Having everything as a star arrangement (mathematically and circuit wise) assists in ensuring that right values are obtained.

## 6.2 Impedance of sequences

There are positive, negative and zero phase sequence components: In voltage these are represented by  $V_0$ ,  $V_1$  and  $V_2$ . In current these are represented by  $I_0$ ,  $I_1$  and  $I_2$ .

### 6.2.1 Sequence components and impedance

Since  $V = IZ$ , it follows if there are sequence components in both voltage and current then there must be a sequence impedance too:

- $V_1 = I_1 Z_1$  where  $Z_1$  is the positive sequence impedance
- $V_2 = I_2 Z_2$  where  $Z_2$  is the negative sequence impedance
- $V_0 = I_0 Z_0$  where  $Z_0$  is the zero sequence impedance

### 6.2.2 The importance of sequence impedance

The impedance of a network is important for calculating currents for an applied voltage. Remembering earlier work on balanced networks, we established that the impedance limited the fault current i.e. the further from the source you were the greater the impedance. the lower the fault current. Now considering the sequence components, it is apparent that the sequence impedances  $Z_0$ ,  $Z_1$ ,  $Z_2$  will limit sequence currents  $I_0$ ,  $I_1$ ,  $I_2$  for the applied sequence voltages  $V_0$ ,  $V_1$ ,  $V_2$ .

### 6.2.3 Network elements

Different network equipment exhibit different sequence impedances:

- Typically, transmission lines and cables have one impedance value for positive and negative sequence, but an entirely different impedance value for zero sequence
- Typically, rotating machines e.g. generators and motors have different impedance values for all three sequences
- Typically, transformers positive, negative and zero sequence components depend upon connection by positive and negative are often the same value

Appreciating these different impedances is important for accurate calculation of unsymmetrical faults.

### 6.2.4 Transmission lines and distribution cables

Power cables and transmission lines are used to carry power from the source to the load. Typically (over short distances) they can be represented as resistance and inductance. The inductance is comprised of its own self-inductance and mutual inductance between each line or cable.

### 6.2.5 Transmission line analysis

In a three-phase system interconnected between a three-phase generator and three-phase load the lines/cables usually run close to each so there is always mutual inductance and self-inductance of the lines.

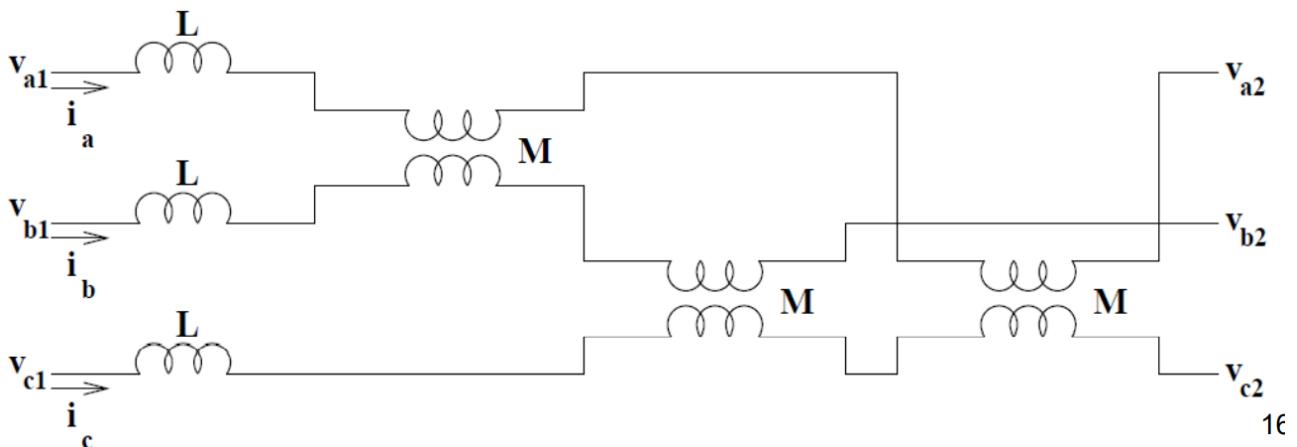


Figure 6.2: Transmission line mutual inductance and self-inductance.

### 6.2.6 Transmission line representation

Hence, we can write the relationship ( $V = XI$ ):

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \quad (6.10)$$

It is reasonable to say that the line and mutual inductances are the same for each transmission line or cable under steady-state balanced conditions. This is not necessarily the case for transient or unbalanced case.

### 6.2.7 Transmission sequence representation

Bringing in the relationship between phase and sequence components we have (ignoring 1/3):

$$I_{sequence} = [A]^{-1} \cdot I_{phase} \quad (6.11)$$

$$V_{sequence} = [A]^{-1} \cdot V_{phase} \quad (6.12)$$

Hence:

$$\begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \quad (6.13)$$

$$[A] [V_{sequence}] = j\omega [LM] \cdot [A] [I_{sequence}] \quad (6.14)$$

### 6.2.8 Transmission line representation

Hence by transformation we obtain:

$$[V_{sequence}] = j\omega [A] \cdot [LM] \cdot [A]^{-1} [I_{sequence}] \quad (6.15)$$

The part  $([A] \cdot [LM] \cdot [A]^{-1})$  provides the inductance sequence relationship for the transmission line or distribution cable. Resolving gives:

$$\begin{pmatrix} L - M & 0 & 0 \\ 0 & L - M & 0 \\ 0 & 0 & L + 2M \end{pmatrix} \quad (6.16)$$

The relationship between sequence components becomes:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L - M & 0 & 0 \\ 0 & L - M & 0 \\ 0 & 0 & L + 2M \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \quad (6.17)$$

The sequence component relationships become:

$$V_1 = j\omega (L - M) I_1 \quad (6.18)$$

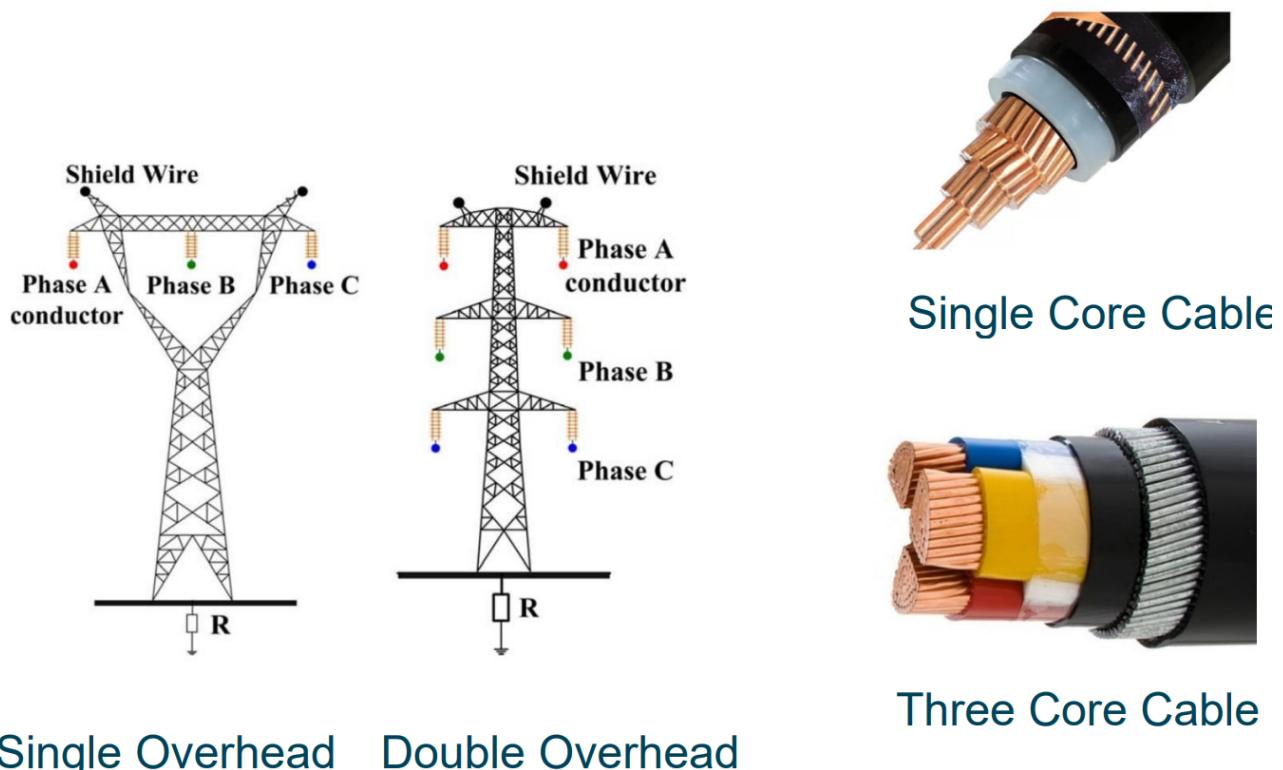
$$V_2 = j\omega (L - M) I_2 \quad (6.19)$$

$$V_0 = j\omega (L - M) I_0 \quad (6.20)$$

The positive, negative and zero sequence reactances of the balanced transmission line are then:

$$Z_1 = Z_2 = j\omega (L - M) \quad (6.21)$$

$$Z_0 = j\omega (L + 2M) \quad (6.22)$$



## Single Overhead    Double Overhead

Figure 6.3: Transmission line and cable arrangements.

### 6.2.9 Lines and cables

The positive and negative sequence impedances are normally balanced i.e.  $Z_1 = Z_2$ . The zero sequence impedance depends upon the nature of the return path through the earth. Typical relative values of  $Z_0$  during faults are

Overhead:

- For a single-circuit arrangement  $(Z_0/Z_1) = 3.5$
- For a double-circuit arrangement  $(Z_0/Z_1) = 5.5$

Cable arrangements:

- For a single-core arrangement  $(Z_0/Z_1) = 1.25$
- For a three-core arrangement  $(Z_0/Z_1) = 4$

### 6.2.10 Synchronous machines (generators)

The positive sequence reactance  $Z_1$  is the value used under balanced operation due to positive sequence currents flowing in the windings of the machine in steady-state and transient. The negative sequence reactance  $Z_2$  is due to negative sequence currents which give rise to fluxes in the air gap of the machine that rotate in the opposite direction during unbalance.  $Z_2$  is different to  $Z_1$  in most designs. The zero sequence reactance  $Z_0$  depends upon the nature of the connection of the star point. Zero sequence currents will not flow when the star point is floating but will flow when there is.

Type of machine	+ve sequence	-ve sequence	zero sequence
440 V 50 Hz 1 MVA	0.16 pu	0.11 pu	0.05 pu
11 kV 50 Hz 75 MVA	0.18 pu	0.14 pu	0.07 pu
16 kV 50 Hz 275 MVA	0.21 pu	0.18 pu	0.08 pu
22 kV 50 Hz 575 MVA	0.28 pu	0.21 pu	0.12 pu

Table 6.1: Table to show typical value of sequence impedances for synchronous generators

### 6.2.11 Neutral connection

The symmetrical components are independent with the voltage-current relationships:

$$V_1 = ZI_1 \quad (6.23)$$

$$V_2 = ZI_2 \quad (6.24)$$

$$V_0 = (Z + 3Z_g) I_0 \quad (6.25)$$

In many generators that are tied to ground at the star point will have additional impedance separately added to reduce the level of zero sequence currents.

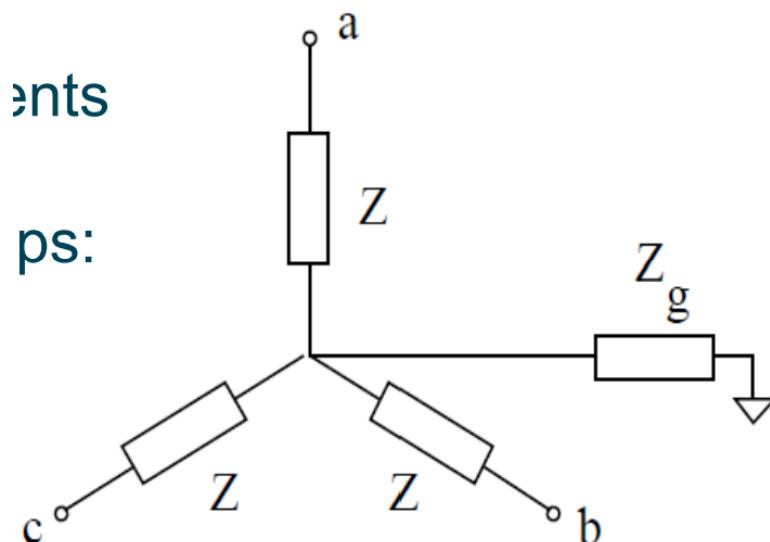


Figure 6.4: Grounded star arrangement.

### 6.2.12 Typical values of sequence impedances for synchronous generators

Manufacturers will test their machines to obtain the relevant data/ The value of the sequence components may differ from country to country, manufacturer to manufacturer.

### 6.2.13 Transformers

The positive and negative sequence sequence impedances are the normal values obtained from the per-phase equivalent circuit. ( $Z_1 = Z_2$ ). The zero sequence components depend upon the connection of the windings. Zero sequence currents in the windings on one side of the transformer must produce the corresponding ampere-turns in the other. In delta windings the zero-sequence currents circulate through the three-phase windings but do not leave the transformer.

## 6.3 Unbalanced faults

### 6.3.1 Fortescue's symmetrical component process

Symmetrical components are used extensively for fault study calculations. in these calculations the positive, negative and zero-sequence impedance networks are either given by the manufacturer or are calculated by the user using base voltages and base power for the system of interest. Each of the sequence networks are then connected together to calculate fault currents and voltages depending upon the type of fault. Standard circuit arrangements have been derived in this course to keep variation reasonable.

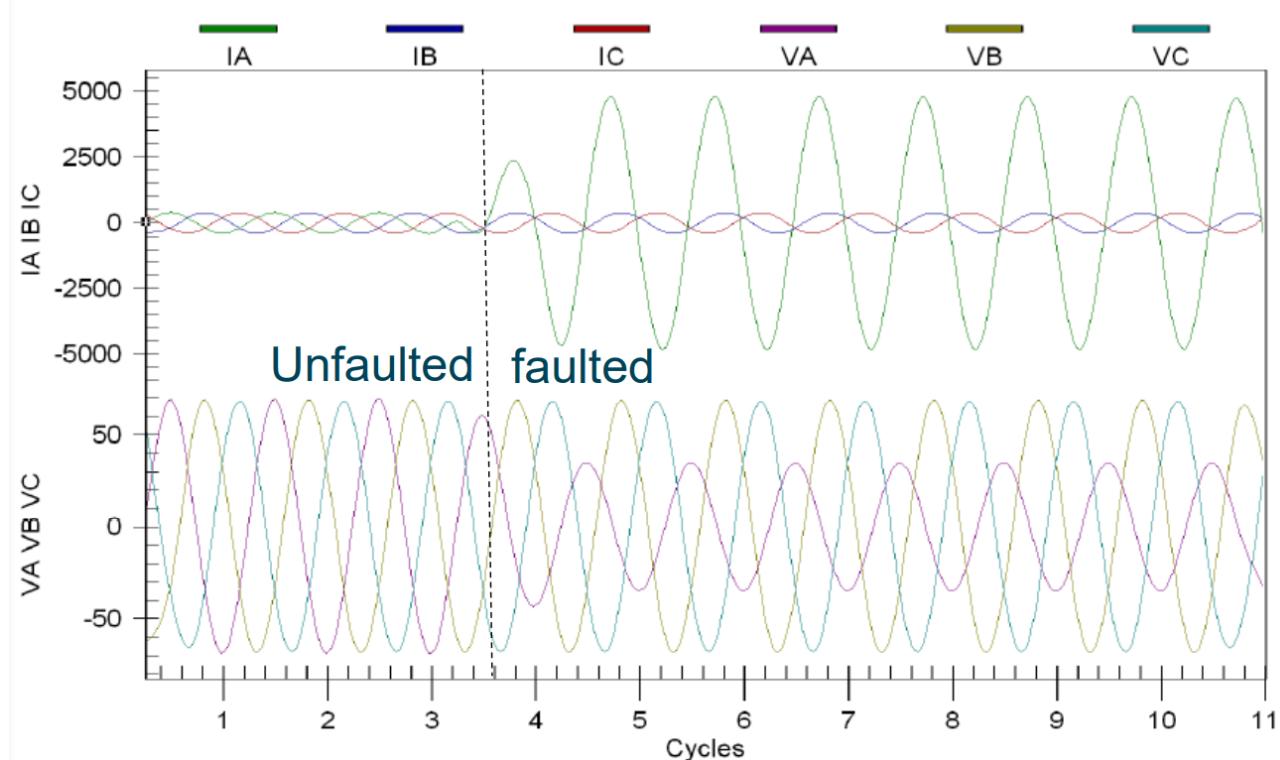


Figure 6.5: Line to ground fault.

### 6.3.2 Standard fault sequence connections - single line to ground

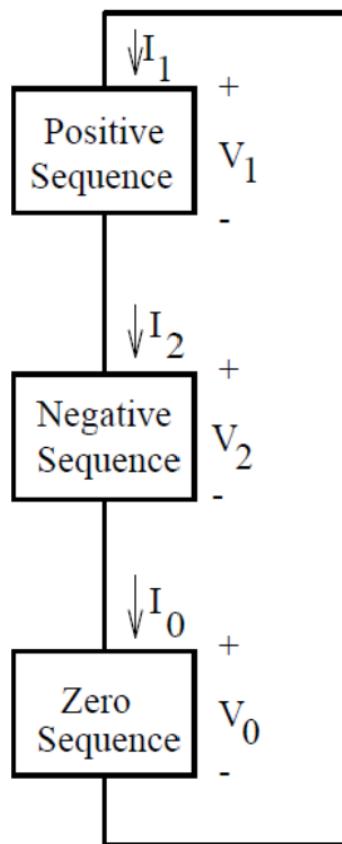


Figure 6.6: Single line to ground connection.

Assumptions:

- $V_a = 0$ ;  $I_a = \text{very large value}$  (faulted line)
- $I_b = 0$  (small in comparison to fault current)
- $I_c = 0$  (small in comparison to fault current)

Hence for phase voltage 'a' we can say:

$$V_0 + V_1 + V_2 = 0 \quad (6.26)$$

And for the current we can say:

$$I_0 + I_1 + I_2 = \frac{1}{3} I_a \quad (6.27)$$

Together, these two expressions describe the sequence network connection.

### 6.3.3 Standard fault sequence connections - line to line

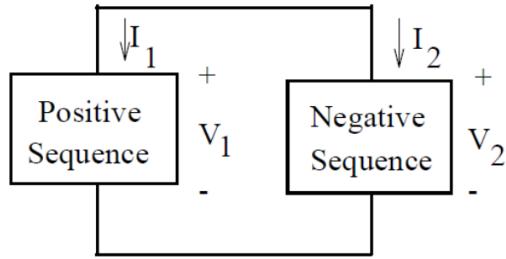


Figure 6.7: Line to line connection.

Assumptions. If the fault occurs between phase b and c then we can say:

- $V_b = V_c$
- $I_b = -I_c$
- $I_a = 0$  (since it is small in comparison with the fault current)

Hence, we can use the phase sequence relationships to say:

$$V_1 = V_2 \text{ and also } I_a = I_1 + I_2 \text{ since } I_0 = 0 \quad (6.28)$$

### 6.3.4 Standard fault sequence connections - double line to ground

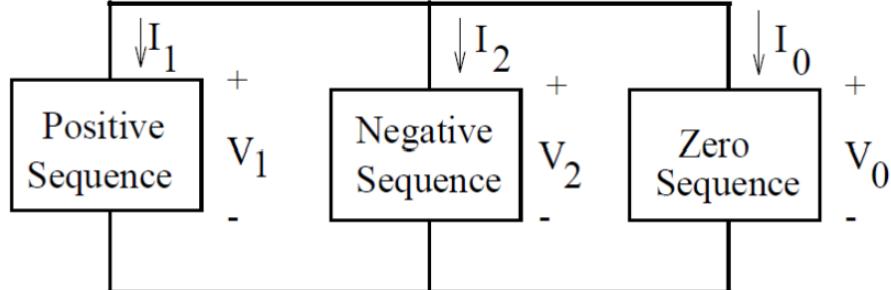


Figure 6.8: Double line to ground connection.

Assumptions. If the fault involves phases b and c to ground then we can say:

- $I_a = 0$  (small in comparison to fault current)
- $V_b = 0$  (faulted line)
- $V_c = 0$  (faulted line)

Hence using phase-sequence relationships we can further say that:

$$V_0 + V_1 + V_2 = 0 \quad (6.29)$$

$$I_a = I_0 + I_1 + I_2 = 0 \quad (6.30)$$

## 6.4 A full fault analysis study

### 6.4.1 Breaker sizing method (most common approach)

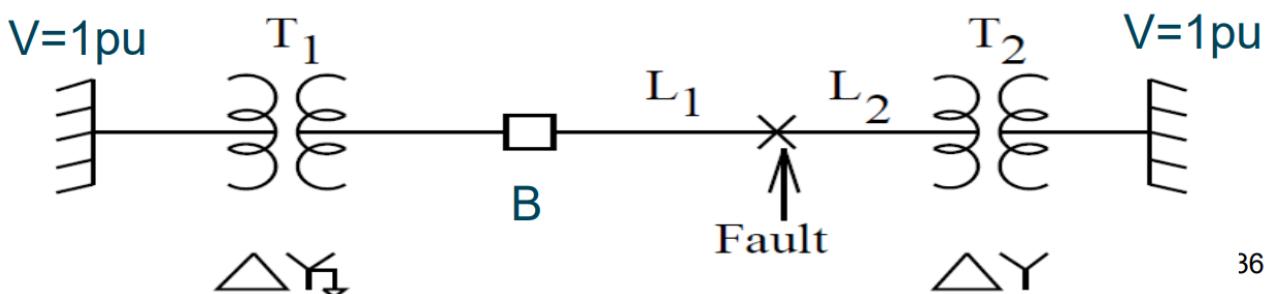
One of the main purposes of circuit breakers is to arrest large currents that flow when there is a fault. Breaker sizing is achieved by understanding currents flowing under both symmetrical and non-symmetrical fault condi-

tions (to be calculated). Calculations are carried out using symmetrical components i.e. positive, negative and zero sequence. Only one phase needs to be considered . . . but all fault types need to be calculated.

#### 6.4.2 Breaker sizing example

Determine the maximum current through the breaker B due to a fault at the location X. Calculate all three types of unbalanced fault and the balanced fault currents.

- System base: voltage 138 kV (1 pu), Power 100 MVA (1 pu)
- Transformer  $T_1$  leakage reactance j0.1 pu
- Transformer  $T_2$  leakage reactance j0.1 pu
- Line L1: positive and negative sequence reactance j0.05 pu, zero sequence reactance j0.1 pu
- Line L2: positive and negative sequence reactance j0.02 pu, zero sequence reactance j0.1 pu



36

Figure 6.9: Breaker sizing example.

#### 6.4.3 Sequence component arrangement

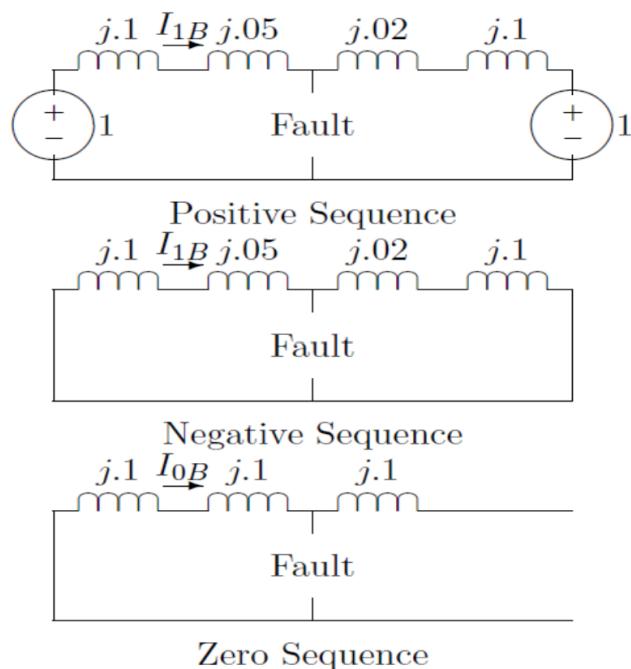


Figure 6.10: Sequence component arrangement.

The sequence networks are exactly like what we would expect to have drawn for equivalent single phase networks. A positive, negative and zero sequence arrangement has been shown for one phase. Only the positive

sequence network has sources, because the infinite bus supplies only positive sequence voltage. The zero sequence network is open at the right hand side because of the delta-wye transformer connection.

#### 6.4.4 Symmetrical fault current

For a symmetrical (three-phase) fault, only the positive sequence network is involved. The fault shorts the network at its position, so that the current is:

$$I_1 = \frac{1}{j0.15} - j6.67 \text{ per unit from LHS} \quad (6.31)$$

$$(I_1 = \frac{1}{j0.12} - j8.33 \text{ per unit from RHS}) \quad (6.32)$$

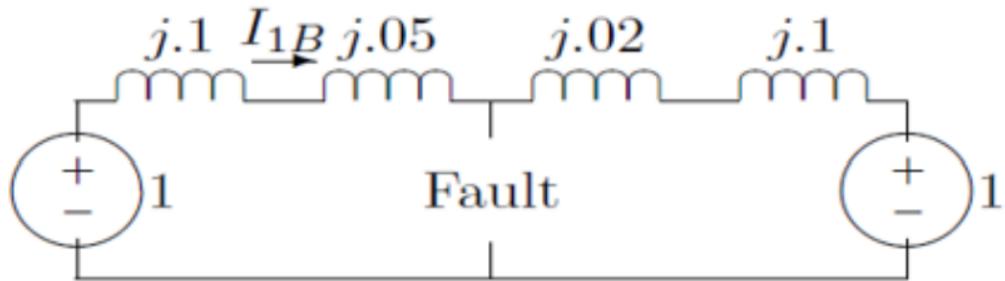


Figure 6.11: Positive sequence impedance in symmetrical fault.

#### 6.4.5 Single line to ground fault

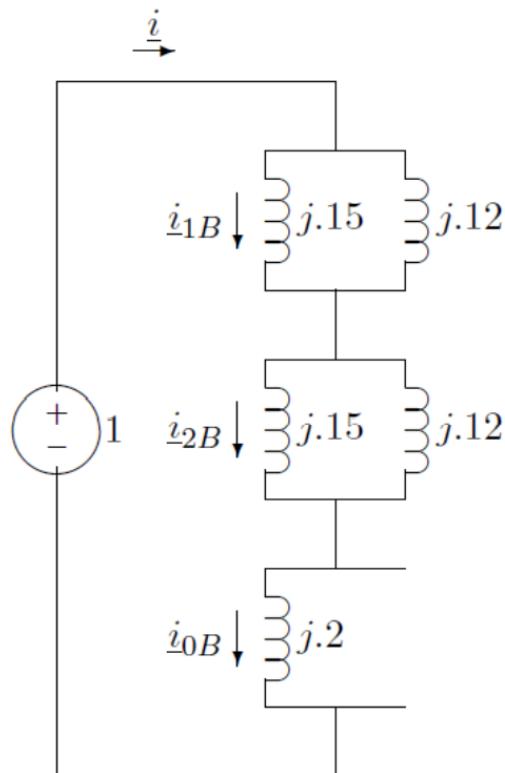


Figure 6.12: Positive sequence impedance in symmetrical fault.

The three networks are in series and the situation is as shown with the total current given by:

$$\underline{i} = \frac{1}{2 \times (j0.15 || j0.12) + j0.2} = -j3.0 \quad (6.33)$$

The sequence currents are:

$$\underline{i}_{1B} = \underline{i}_{2B} \quad (6.34)$$

$$= \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \quad (6.35)$$

$$= -j1.33\underline{i}_{0B} = \underline{i} \quad (6.36)$$

$$= -j3.0 \quad (6.37)$$

#### 6.4.6 Single line to ground fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = \underline{i}_{1B} + \underline{i}_{2B} + \underline{i}_{0B} \quad (6.38)$$

$$\underline{i}_b = \underline{a}^2 \underline{i}_{1B} + \underline{a} \underline{i}_{2B} + \underline{i}_{0B} \quad (6.39)$$

$$\underline{i}_c = \underline{a} \underline{i}_{1B} + \underline{a}^2 \underline{i}_{2B} + \underline{i}_{0B} \quad (6.40)$$

Hence:

$$\underline{i}_a = -j5.66 \text{ pu} \quad (6.41)$$

$$\underline{i}_b = -j1.67 \text{ pu} \quad (6.42)$$

$$\underline{i}_c = -j1.67 \text{ pu} \quad (6.43)$$

#### 6.4.7 Double line to ground fault

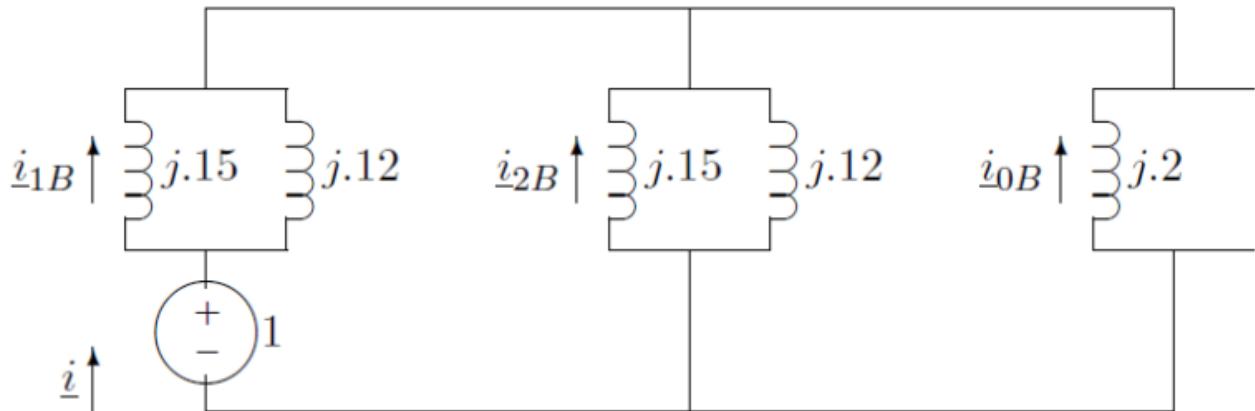


Figure 6.13: Double line-ground fault configuration.

For the double line-ground fault, the networks are in parallel.

$$\underline{i} = \frac{1}{j(0.15||0.12) + j(0.15||0.12||0.2)} \quad (6.44)$$

$$= -j8.57 \quad (6.45)$$

$$i_{1B} = \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \quad (6.46)$$

$$= -j3.81 \quad (6.47)$$

$$i_{2B} = -\underline{i} \times \frac{j0.12||j0.2}{j0.12||j0.2 + j0.15} \quad (6.48)$$

$$= j2.86 \quad (6.49)$$

$$i_{0B} = \underline{i} \times \frac{j0.12||j0.15}{j0.2 + j0.12||j0.15} \quad (6.50)$$

$$= j2.14 \quad (6.51)$$

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = j1.19 \quad (6.52)$$

$$\underline{i}_b = i_{0B} - \frac{1}{2}(i_{1B} + i_{2B}) - \frac{\sqrt{3}}{2}j(i_{1B} - i_{2B}) \quad (6.53)$$

$$= j2.67 - 5.87 \quad (6.54)$$

$$\underline{i}_c = i_{0B} - \frac{1}{2}(i_{1B} + i_{2B}) + \frac{\sqrt{3}}{2}j(i_{1B} - i_{2B}) \quad (6.55)$$

$$= j2.67 + 5.87 \quad (6.56)$$

Hence:

$$|\underline{i}_a| = 1.19 \text{ pu} \quad (6.57)$$

$$|\underline{i}_b| = 6.43 \text{ pu} \quad (6.58)$$

$$|\underline{i}_c| = 6.43 \text{ pu} \quad (6.59)$$

#### 6.4.8 Line to line fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = 0 \quad (6.60)$$

$$\underline{i}_b = -\frac{1}{2}(i_{1B} + i_{2B}) - j\frac{\sqrt{3}}{2}(i_{1B} - i_{2B}) \quad (6.61)$$

Hence:

$$|\underline{i}_b| = 5.77 \text{ pu} \quad (6.62)$$

$$|\underline{i}_c| = 5.77 \text{ pu} \quad (6.63)$$

There are only two networks at play - positive and negative sequence.

	Phase A	Phase B	Phase C
Three-phase fault	2791	2791	2791
Single line-ground, $\phi_a$	2368	699	699
Double line-ground, $\phi_b, \phi_c$	498	2690	2690
Line-line, $\phi_b, \phi_c$	0	2414	2414

Table 6.2: Table to show fault currents.

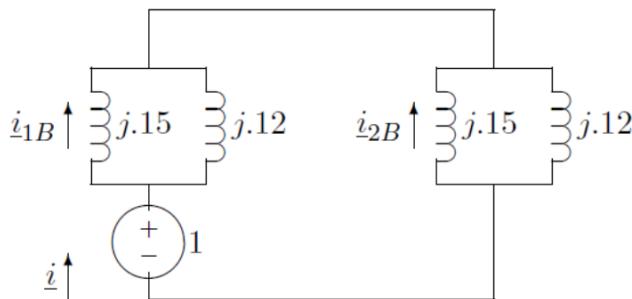


Figure 6.14: Line to line fault configuration.

#### 6.4.9 Conversion to ampere ratings

Having calculated the fault currents then the values in per unit can be expressed as amperes. The value of  $I_B$  is:

$$I_B = \frac{P_B}{\sqrt{3}V_{Bl-l}} = 418.8 \text{ A} \quad (6.64)$$

Hence the fault currents are calculated as being: The worst fault is the balanced three-phase fault.

#### 6.4.10 Practical sizing of breakers

Key information needed for sizing a circuit breaker include:

- Voltage rating
- Normal current rating
- MVA fault level
- Fault current levels
- Withstand voltage levels

There are three main types of circuit breakers: Air, vacuum and SF6.

#### 6.4.11 Conclusions

- Appreciated the need for positive, negative and zero sequence impedances of different components to make up a power system
- Introduced the concept of positive, negative and zero sequence impedance. Examined this at a component level
- A system analysis method has been applied for ‘unbalanced faults’ in a transmission system and fault current table produced

# Chapter 7

# Network Analysis

## 7.1 Electrical networks

Note that the system interconnects in a complex pattern allowing for multiple current paths. How to determine voltage and current flows?

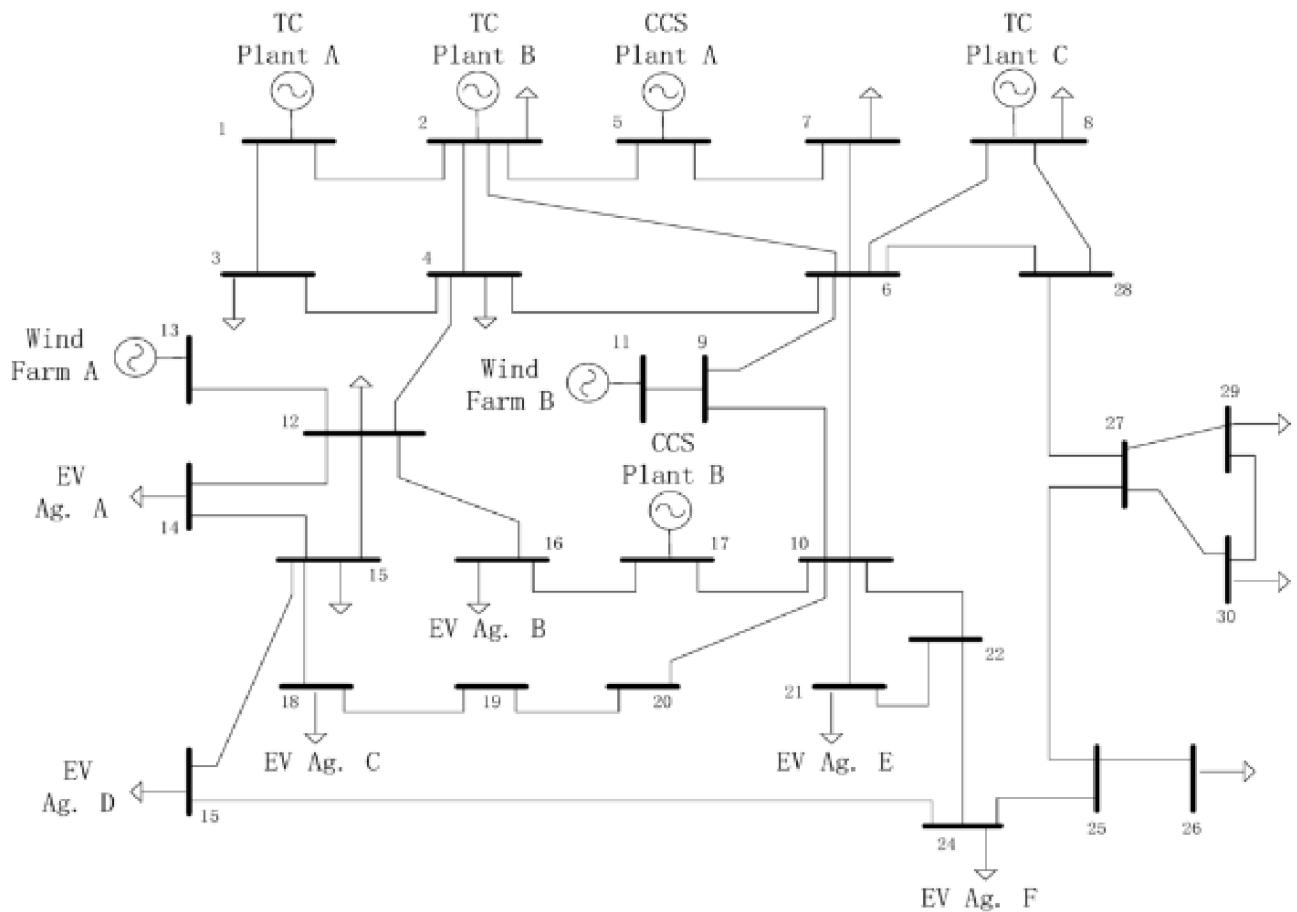


Figure 7.1: Network at a works.

## 7.2 Split distribution system - high integrity

Non-essential loads (NE) are divided equally between two generators. Essential loads (E) have a cross-over connection capability either manual or automatic. This system ensures the integrity of the electrical system in case of generator failure.

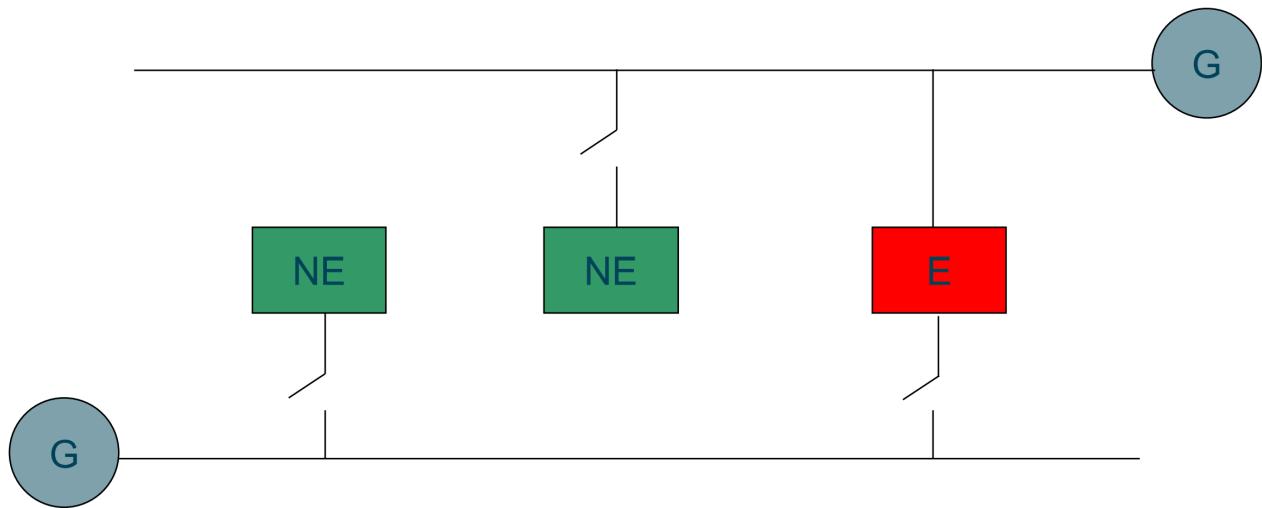


Figure 7.2: Split distribution system - high integrity.

### 7.3 Tree distribution

Electrical power is generated (generators can be used in parallel) by the main generators to supply loads distributed around the vessel. A small emergency generator is used as back-up to supply the emergency switchboard, which is usually supplied by the main board.

(emergency generator)

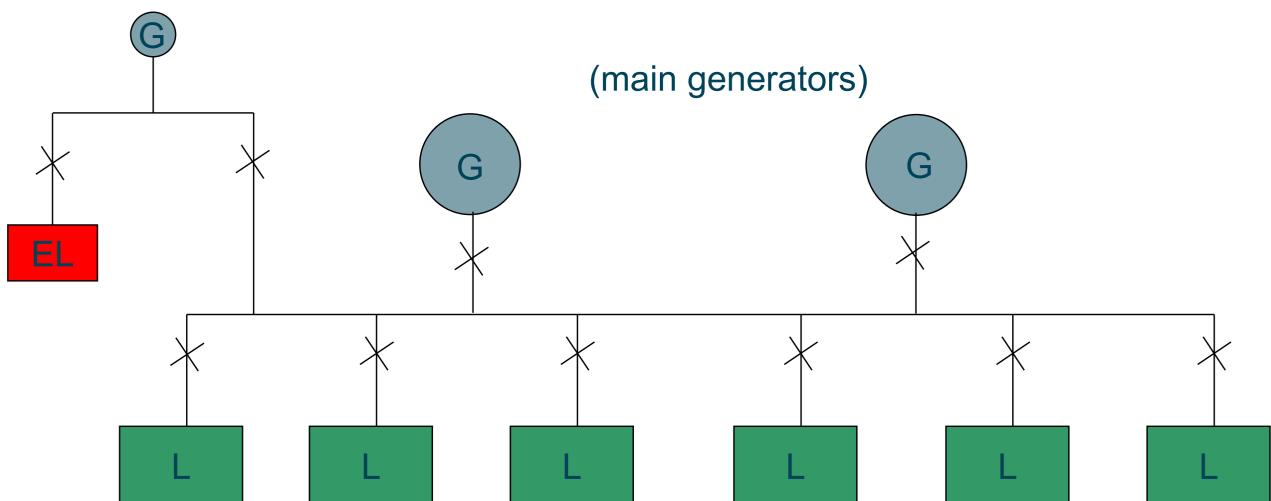


Figure 7.3: Tree distribution.

## 7.4 Ring networks - grids

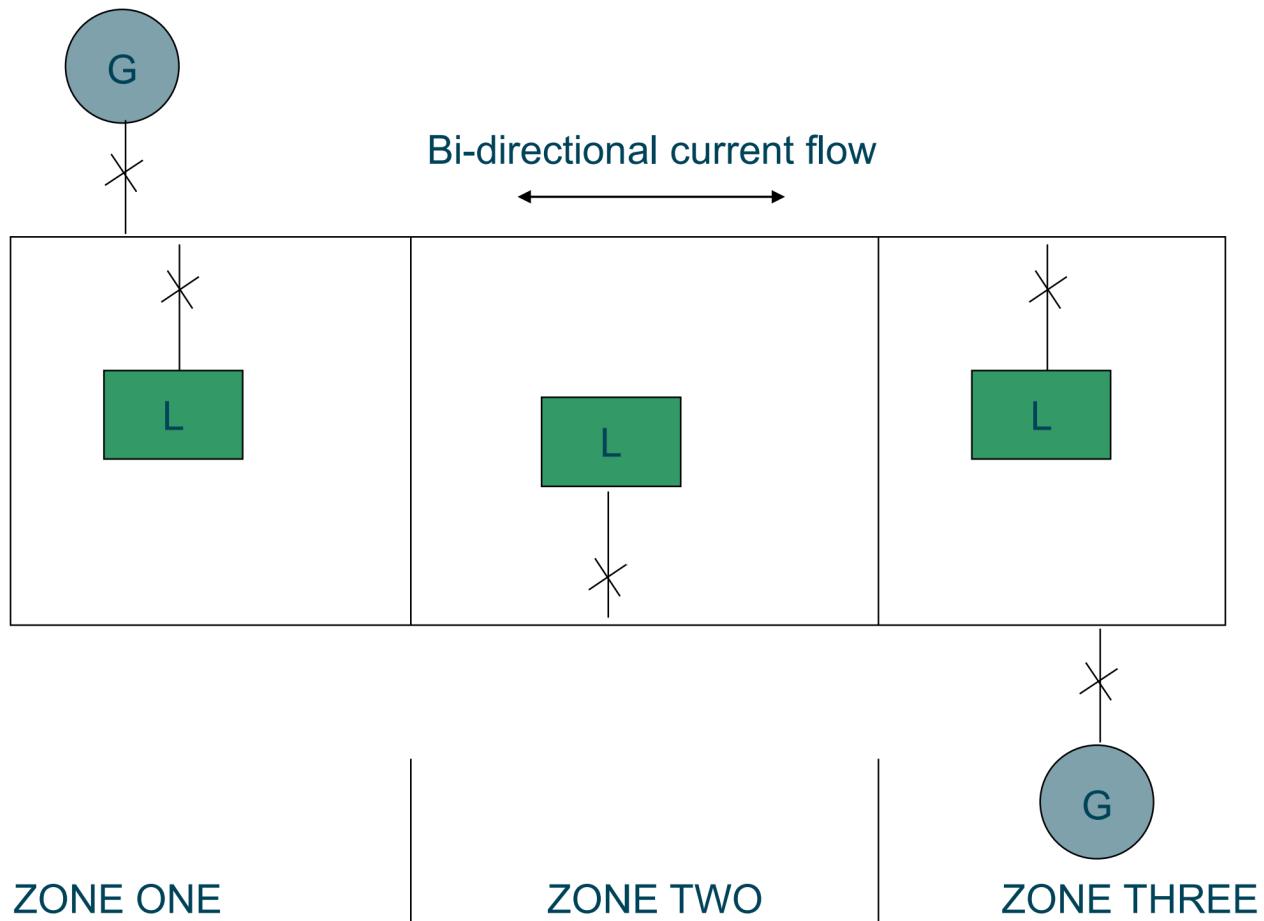


Figure 7.4: Ring networks - grids.

## 7.5 Network analysis

It is essential to be able to analyse the performance of power systems whatever their design both during normal operating conditions and under fault (short-circuit) conditions. The analysis in normal steady-state operation is called a **power-flow study (load-flow study)** and it involves determining the voltages, current and real and reactive power flows in system under given load conditions. Earlier in the course, we examined the impact of an induction motor start. However, that was a single network and relatively straight forward. For more complex system, matrix methods are best used.

Today most power-flow studies are done by computer. The purpose of power flow studies is to plan ahead to be able to account for various hypothetical situations and understand steady-state, transient and faulted conditions. For instance, what if a transmission line within the power system properly supplying loads must be taken offline for maintenance. Can the remaining lines in the system handle the required loads without exceeding their rated parameters? For instance, what happens if a switchboard in a ship becomes faulty and need to be isolated or what happens when new equipment is fitted to an existing network?

## 7.6 Techniques for power-flow studies

A power-flow study (load-flow study) is an analysis of the voltages, currents and power flows in a power system and we will consider steady-state conditions. In such a study, we make an assumption:

1. Either a **voltage** at a bus or the **power** being supplied to a bus for each bus in the power system

2. We then determine the magnitude and phase angles of the bus voltages, line currents, etc. that would result from the assumed combination of voltages and power flows
3. We use iterative methods of analysis to resolve

## 7.7 Power flow calculations

The simplest way to perform power-flow calculations is by iteration.

1. Create a bus admittance matrix  $Y_{bus}$  for the power system.

- 

$$Y = \frac{1}{Z} = (G + jB) \quad (7.1)$$

$G$  is conductance,  $B$  is called susceptance and may be positive or negative. Note that:

$$G = \frac{1}{R}, \quad B \neq \frac{1}{X} \quad (7.2)$$

2. Make an initial estimate for the voltages at each bus in the system (ideally something that is reasonable)
3. Update the voltage estimate for each bus (one at a time), based on the estimates for the voltages and power flows at every other bus and the values of the bus admittance matrix (the voltage at a given bus depends on the voltages at all of the other buses in the system so even the updated voltage will not be correct but it will usually be closer to the answer than the original estimate). - An iterative method
4. Repeat this process to make the voltages at each bus approaching the correct answers closer and closer...

There are three types of defined bus:

- Load bus
- Generator bus
- Slack bus

The process involves defining these buses and then sticking with that definition.

## 7.8 Basic techniques for power-flow studies

The equations used to update the estimates differ for different types of buses. Each bus in a power system can be classified to one of three types.

1. **Load bus** (PQ bus) - a bus at which the real and reactive power are specified, and for which the bus voltage will be calculated. **Real and reactive powers supplied to a power system are defined to be positive, while the powers consumed from the system are defined to be negative. All busses having no generators are load busses**
2. **Generator bus** (PV bus) - a bus at which the magnitude of the voltage is kept constant by adjusting the field current of a synchronous generator on the bus (**remember** - increasing the field current of the generator increases both the reactive power supplied by the generator and the terminal voltage of the system). We assume that the field current is adjusted to maintain a constant voltage  $V_T$ . We also know that increasing the prime mover's governor set points increases the power that the generator supplies to the power system and the frequency. Therefore, we can *specify* the magnitude of the bus voltage and real power supplied
3. **Slack bus** (swing bus) - a special generator bus serving as the reference bus for the power system. Its voltage is assumed to be fixed in both magnitude and phase (for instance,  $1\angle0^\circ$  pu). The real and reactive powers are uncontrolled: the bus supplies whatever real or reactive power is necessary to make the power flows in the system balance.

## 7.9 Approach to analysis

The most common approach to power-flow analysis is based on the bus admittance matrix  $Y_{bus}$ . However, this matrix is slightly different from the system previously since the internal impedances of generators and loads connected to the system are not included in the  $Y_{bus}$ . Instead, they are accounted for as specified real and reactive powers input and output from the buses.

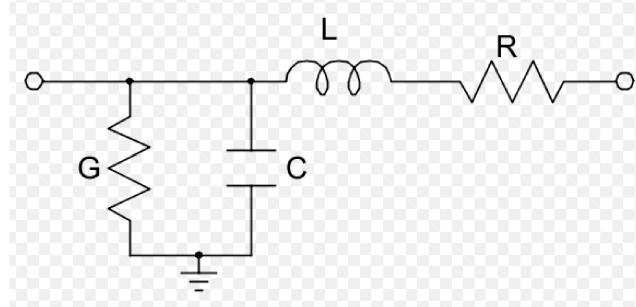


Figure 7.5: Transmission line: only L and R are used in the  $Y_{bus}$ .

## 7.10 Constructing $Y_{bus}$ for power-flow analysis

Example 1: A simple power system has 4 busses, 5 transmission lines, 1 generator and 3 loads.

line no.	Bus to bus	Series Y (pu)
1	1-2	$0.5882 - j2.3529$
2	2-3	$0.3846 - j1.9231$
3	2-4	$0.5882 - j2.3529$
4	3-4	$1.1765 - j4.7059$
5	4-1	$1.1765 - j4.7059$

Table 7.1: Example 1 series per unit admittances.

Table of busses	
Bus 1	Slack bus
Bus 2	Load bus
Bus 2	Load bus
Bus 2	Load bus

Table 7.2: Example 1 table of busses.

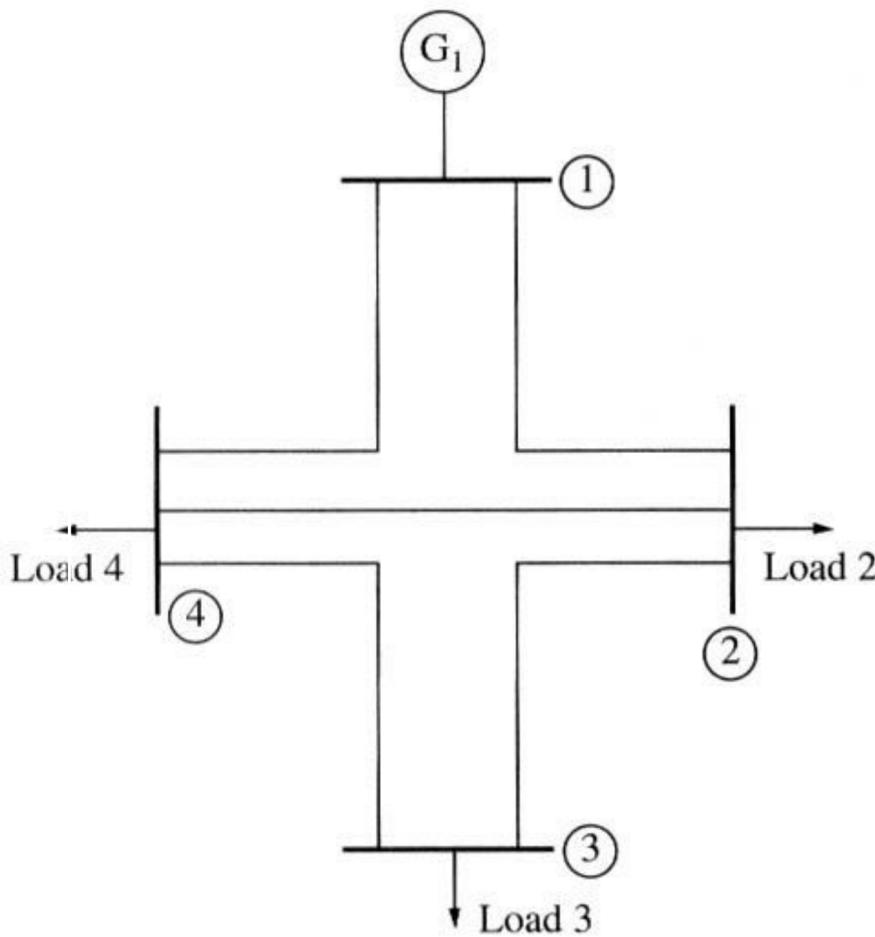


Figure 7.6: Example 1 diagram.

The shunt admittances of the transmission lines are ignored. In this case, the  $Y_{ii}$  terms of the bus admittance matrix can be constructed by summing the admittances of all transmission lines connected to each bus, and the  $Y_{ij}$  ( $i \neq j$ ) terms are just the negative of the line admittances stretching between busses  $i$  and  $j$ . Therefore, for instance, the term  $Y_{11}$  will be the sum of the admittances of all transmission lines connected to bus 1, which are the lines 1 and 5, so:

$$Y_{11} = 1.7647 - j7.0588 \text{ pu} \quad (7.3)$$

Note: if the shunt admittances of the transmission lines are not ignored, the self admittance  $Y_{ii}$  at each bus would also include half of the shunt admittance of each transmission line connected to the bus.

The term  $Y_{12}$  is defined as the negative of all the admittances stretching between bus 1 and bus 2, which will be the negative of the admittance of the transmission line 1, so:

$$Y_{12} = -0.5882 + j2.3529 \quad (7.4)$$

The complete bus admittance matrix can be obtained by repeating these calculations for every term in the matrix.

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.5290 & -0.3746 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix} \quad (7.5)$$

## 7.11 Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system:

$$Y_{bus}V = I \quad (7.6)$$

For the four-bus power system shown above, becomes:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad (7.7)$$

where  $Y_{ij}$  are the elements of the bus admittance matrix,  $V_i$  are the bus voltages, and  $I_i$  are the currents injected at each node. For bus 2 in this system, this equation reduces to:

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2 \quad (7.8)$$

However, real loads are specified in terms of real and reactive powers, not as currents. The relationship between per-unit real and reactive power supplied to the system at a bus and the per-unit current injected into the system at that bus is:

$$S = VI^* = P + jQ \quad (7.9)$$

where  $V$  is the per-unit voltage at the bus,  $I^*$  is the complex conjugate of the per-unit current injected at the bus,  $P$  and  $Q$  are per-unit real and reactive powers. Therefore, for instance, the current injected at bus 2 can be found as:

$$V_2 I_2^* = P_2 + jQ_2 \rightarrow I_2^* = \frac{P_2 + jQ_2}{V_2} \quad (7.10)$$

Now the next steps are

1. To switch  $I_2^*$  and  $V_2$  to use  $I_2$  and  $v_2^*$
2. In doing so we have to change to  $P-Q$  to keep sense
3. Substitute  $I$  for the relationships for  $I = YZ$

Implementing for  $V_2$ , yields...

$$V_2 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^*} - (Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4) \right] \quad (7.11)$$

Similar equations can be created for each load bus in the power system.

### 7.11.1 Gauss-Siedel iterative method

Basic procedure:

1. Calculate the bus admittance matrix  $Y_{bus}$  including the admittances of all transmission lines, transformers, etc., between busses but excludes the admittances of the loads or generators themselves.
2. Select a slack bus: one of the busses in the power system, whose voltage will arbitrarily be assumed as  $1.0\angle0^\circ$ .
3. Select initial estimates for all bus voltages: usually, the voltage at every load bus is assumed as  $1.0\angle0^\circ$  (flat start) lead to good convergence. Write voltage equations for every other bus in the system. The generic form is:

$$V_i = \frac{1}{Y_{ii}} \left( \frac{P_i - jQ_i}{V_i^*} - \sum_{k=1, k \neq i}^N T_{ik} V_k \right) \quad (7.12)$$

4. Calculate an updated estimate of the voltage at each load bus in succession (except for the slack bus).
5. Compare the differences between the old and new voltage estimates: if the differences are less than some specified tolerance for all busses, stop. Otherwise, repeat step 5.
6. Confirm that the resulting solution is reasonable.

## 7.12 Example 2

In a 2-bus power system, a generator attached to bus 1 and loads attached to bus 2. The series impedance of a single transmission line connecting them is  $0.1 + j0.5 \text{ pu}$ . The shunt admittance of the line may be neglected. Assume that bus 1 is the slack bus and that it has a voltage  $V_1 = 1.0\angle0^\circ \text{ pu}$ . Real and reactive powers supplied to the loads from the system at bus 2 are  $P_2 = -0.3 \text{ pu}$ ,  $Q_2 = 0.2 \text{ pu}$ . Determine voltages at each bus for the specified load conditions.

**Table of busses**

Bus 1	Slack bus
Bus 2	Load bus

Table 7.3: Example 2 table of busses.

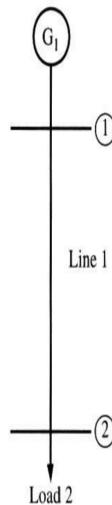


Figure 7.7: Example 2 diagram.

1. We start from calculating the bus admittance matrix  $Y_{bus}$ . The  $Y_{ii}$  terms can be constructed by summing the admittances of all transmission lines connected to each bus, and the  $Y_{ij}$  terms are the negative of the admittances of the line stretching between busses  $i$  and  $j$ . For instance, the term  $Y_{11}$  is the sum of the admittances of all transmission lines connected to bus 1 (a single line in our case). The series admittance of line 1 is:

$$Y_{line1} = \frac{1}{Z_{line1}} = \frac{1}{0.1 + j0.5} = 0.3846 - j1.9231 = Y_{11} \quad (7.13)$$

Applying similar calculations to other terms, we complete the admittance matrix as:

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix} \quad (7.14)$$

2. Next, we select bus 1 as the slack bus since it is the only bus in the system connected to the generator. The voltage at bus 1 will be assumed  $1.0\angle0^\circ$ .

3. We select initial estimates for all bus voltages. Making a flat start, the initial voltage estimates at every bus are  $1.0\angle 0^\circ$ .
4. Next, we write voltage equations for every other bus in the system. For bus 2:

$$V_2 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_{2,old}^*} - Y_{21}V_1 \right] \quad (7.15)$$

Since the real and reactive powers supplied at bus 2 are  $P_2 = -0.3$  pu and  $Q_2 = 0.2$  pu and since  $Y_s$  and  $V_1$  are known, we may reduce the last equation:

$$V_2 = \frac{1}{0.3846 - j1.9231} \left[ \frac{-0.3 - j0.2}{V_{2,old}^*} - ((-0.3846 + j1.9231) V_1) \right] \quad (7.16)$$

$$= \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{V_{2,old}^*} - (1.9612\angle 101.3^\circ) (1\angle 0^\circ) \right] \quad (7.17)$$

5. Next, we calculate an updated estimate of the voltages at each load bus in succession. In this problem we only need to calculate updated voltages for bus 2. since the voltage at the slack bus (bus 1) is assumed constant. We repeat this calculation until the voltage converges to a constant value. The initial estimate for the voltage is  $V_{2,0} = 1\angle 0^\circ$ . The next estimate for the voltage is:

$$V_{2,1} = \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{V_{2,old}^*} - (1.9612\angle 101.3^\circ) (1\angle 0^\circ) \right] \quad (7.18)$$

$$= \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{1\angle 0^\circ} - (1.9612\angle 101.3^\circ) (1\angle 0^\circ) \right] \quad (7.19)$$

$$= 1.0834\angle -9.0275^\circ \quad (7.20)$$

The new estimate for  $V_2$  substituted back to the equation will produce the second estimate:

$$V_{2,2} = \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{1.0834\angle 9.0275^\circ} - 1.9612\angle 101.3^\circ \right] \quad (7.21)$$

$$= 1.0522\angle -9.0275^\circ \quad (7.22)$$

The third iteration will be:

$$V_{2,3} = \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{1.0522\angle 9.0275^\circ} - 1.9612\angle 101.3^\circ \right] \quad (7.23)$$

$$= 1.0542\angle -9.2803^\circ \quad (7.24)$$

The fourth iteration will be:

$$V_{2,4} = \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{1.0542\angle 9.2803^\circ} - 1.9612\angle 101.3^\circ \right] \quad (7.25)$$

$$= 1.0533\angle -9.2803^\circ \quad (7.26)$$

The fifth iteration will be:

$$V_{2,5} = \frac{1}{1.9612\angle -78.8^\circ} \left[ \frac{0.3603\angle -146.3^\circ}{1.0533\angle 9.2803^\circ} - 1.9612\angle 101.3^\circ \right] \quad (7.27)$$

$$= 1.0534\angle -9.2873^\circ \quad (7.28)$$

6. We observe that the magnitude of the voltage is barely changing and may conclude that this value is close to the correct answer and, therefore, stop the iterations. This power system converged to the answer in five iterations. The voltages at each bus in the power system are:

$$V_1 = 1.0\angle 0^\circ \quad (7.29)$$

$$V_2 = 1.0534\angle -9.2873^\circ \quad (7.30)$$

7. Finally, we need to confirm that the resulting solution is reasonable. The results seem reasonable since the phase angles of the voltages in the system differ by only  $10^\circ$ . The current flow from bus 1 to bus 2 is:

$$I_1 = \frac{V_1 - V_2}{Z_{line1}} = \frac{1.0\angle 0^\circ - 1.0534\angle -9.2873^\circ}{0.1 + j0.5} = 0.3389\angle 24.77^\circ \quad (7.31)$$

## 7.13 Conclusions

- Networks can be small, medium or large however flows are important to understand especially for steady-state operation for various load scenarios
- The method of analysis shown here with examples addresses typical yet simple issues. In large networks computer based systems are used but the mathematics behind the code is similar i.e. based on iteration methods
- Understanding voltages and current flows appropriate ratings of cabling and busbars

# Chapter 8

## Marine Electric Propulsion

### 8.1 Introduction

#### 8.1.1 The propulsion requirement

At constant speeds, the thrust produced by the propeller(s) will equal the resistive force experienced by the ship as it moves through the water i.e. balancing of forces at a given speed. When propeller thrust exceeds the resistive force of the ship then it will accelerate. If resistance exceeds thrust then the ship will de-accelerate until the force equilibrium is restored.



Figure 8.1: Ship force diagram.

#### 8.1.2 Effective power

**Definition:** the product of the speed of the hullform through the water and its resistance at that speed.

**Equation:**

$$\text{Effective power} = R_T \cdot V_s \text{ kW} \quad (8.1)$$

**Simple example:** at  $3 \text{ m s}^{-1}$  the effective tow rope pull of a naked hull is 50 kN. Find the power of the hull at this speed.

$$P_E = 50 \times 3 = 150 \text{ kW} \quad (8.2)$$

### 8.1.3 The generalised resistance equation

The generalised resistance equation is:

$$R_{total} = R_{frictional} + R_{form} + R_{wave} + R_{air} \quad (8.3)$$

At low speeds  $R_{frictional}$  tends to dominate  $R_{total}$ . At high speeds  $R_{wave}$  tends to dominate  $R_{total}$ . ‘Rule of thumb’ - it is acceptable to assume the resistance of a ship is proportional to the square of the ship speed ( $V_{ship}$ ). For monohulls:

$$R_{total} = C_1 \cdot V_{ship}^2 \quad (8.4)$$

### 8.1.4 Propulsive power requirement

Effective power  $P_E$  is not the same as shaft power. As a first approximation  $P_E$  may be determined from:

$$P_E = R_{total} \cdot V_{ship} \quad (8.5)$$

Combining (8.4) and (8.5), we have:

$$P_E = C_1 \cdot V_{ship}^3 \quad (8.6)$$

where,  $C_1$  is not a constant but contains a factor  $C_0$  that is speed dependent and a multiplying factor  $y$ , which depends upon ship operational characteristics and in particular degradation of performance.

### 8.1.5 Relationship between speed and power

This means that if the ship’s speed is doubled then the power required to achieve that speed is increased eight fold. This also means fuel consumption could also increase by a similar factor.

$$C_1 = y \cdot C_0 V_{ship} \quad (8.7)$$

$$y = \text{function/fouling, displacement, sea-state, water-depth} \quad (8.8)$$

Losses incurred by the propulsors mean a higher shaft power is required from the engines. Typically, FPP (fixed pitch propeller) efficiency is 70-75% but values are different at different speeds and actual values depend upon propulsor design characteristics such as pitch, diameter, rotational speeds and also upon operational conditions such as depth of propeller in the water and wake characteristics.  $\eta_{propeller}$  is therefore speed dependent.

### 8.1.6 Shaft power and effective power

The required shaft power,  $P_s$ , is calculated from:

$$P_s = P_E + \text{propulsor lost power} = \underline{P}_e(\eta_{propeller}) \quad (8.9)$$

The shaft power,  $P_s$ , is supplied by the propulsion machinery to the propulsion shaft and is calculated from:

$$P_s = \omega \cdot T_s \quad (8.10)$$

$$\omega = 2\pi N_s \quad (8.11)$$

where  $T_s$  is shaft torque and  $N_s$  is shaft revolutions per second. There are limitations on the maximum rotational speed of the propulsor hence torques can be large!

### 8.1.7 Ship power/speed curves

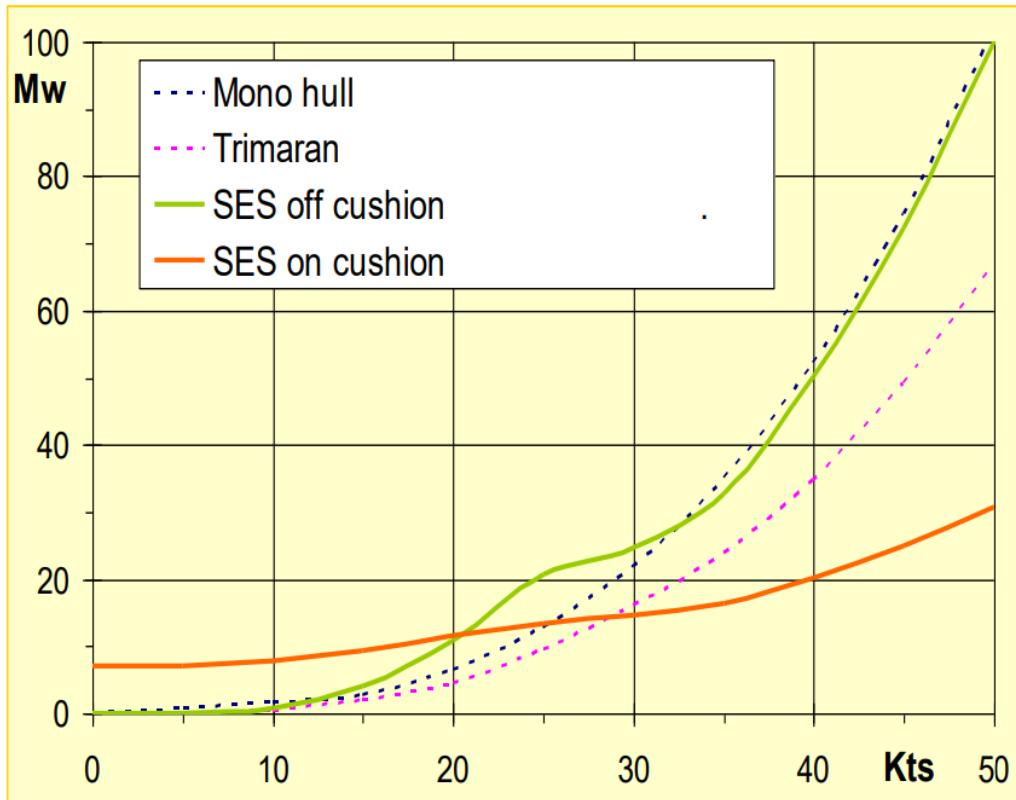


Figure 8.2: Ship power/speed curve.

This relationship is acceptable for relatively low speeds but at high speeds the resistance will tend to increase at increasing rates with increases in ship's speed. Note: the difference between the curves for monohull and multi-hull vessels for this fast corvette example.

### 8.1.8 Power speed/curve - two shafts

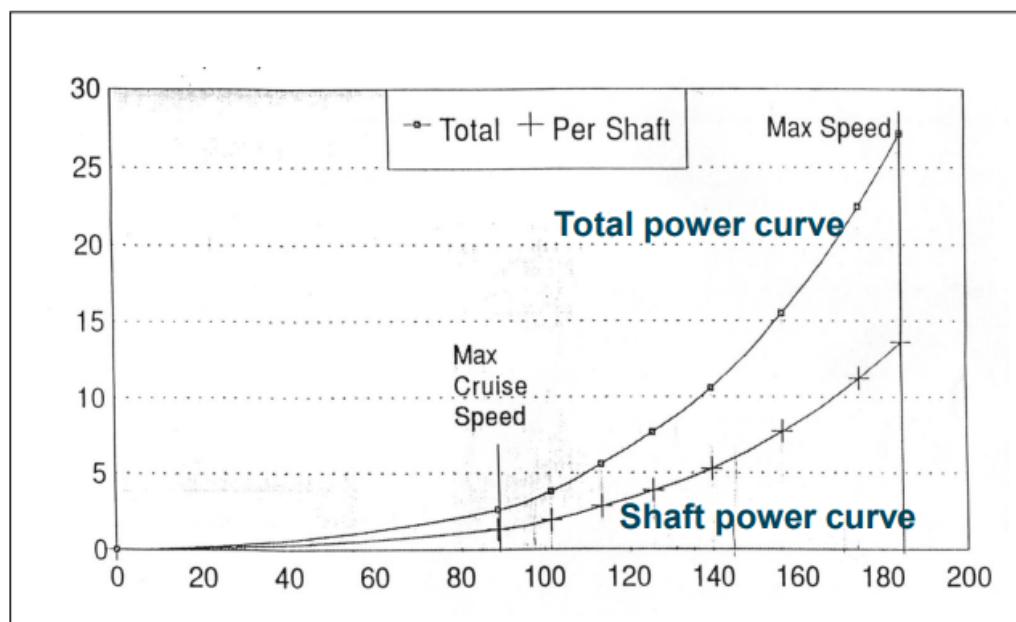


Figure 8.3: Power speed/curve - two shafts.

Twin shaft naval frigate with maximum speed of 29 knots (185 rpm) and cruising speed of 14 knots (90 rpm).

### 8.1.9 Main components of a marine propulsions system

The propulsion system is one of the key ‘systems’ in any ship or submarine. The function of any propulsion system is to generate thrust to move the ship at some desired speed in some direction. The main components of a propulsion system are:

- The Prime-mover(s)
- The Transmission system(s)
- The Propulsor(s)

### 8.1.10 Efficiency of electrical propulsion

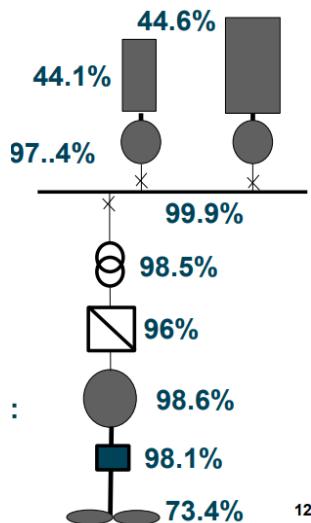


Figure 8.4: Ship SLD efficiency.

Item	Efficiency
Diesel Engine 1	44.6%
Diesel Engine 2	44.1%
Generator	97.4%
Transmission	99.9%
Transformer	98.5%
Converter	96%
Motor	98.6%
Gearbox	98.1%
Propeller	73.4%

Table 8.1: Example efficiencies of components in a marine propulsion system.

‘Overall propulsion’ efficiency can be defined as:

$$\eta = \frac{\text{Energy available for useful thrust}}{\text{Calorific energy available in fuel}} \quad (8.12)$$

Efficiency of system defined in Table 8.1: 28.97% (at normal speed).

## 8.2 Marine electric propulsion

### 8.2.1 The early days

- The pioneers
- Battery powered propulsion
- Turbo-electric (AC) propulsion
- Diesel-electric (DC) propulsion
- Reasons for the decline of conventional electric propulsion systems

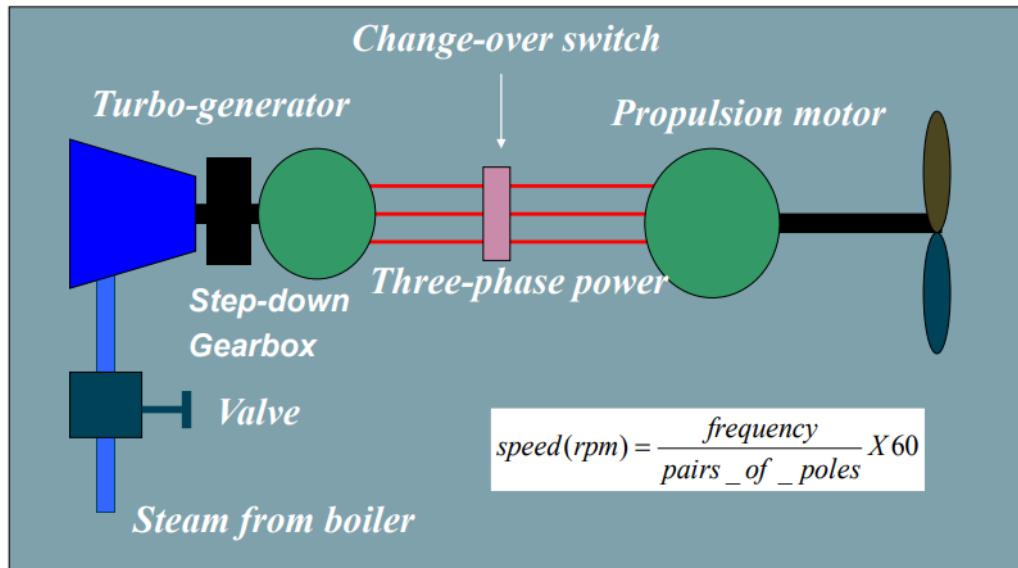


Figure 8.5: Turbo-electric propulsion (Emmet) system.

An ‘electrical speed reduction gearbox’ simply facilitated by the step-up ratio of generator to motor poles.

#### Features of the turbo-electric propulsion:

- Avoided the need for a complex gearbox to reduce revolutions between a high speed turbine and a low speed propeller shaft
- Avoided the need for stringent alignment of the propulsion system within the ship thus allowing greater flexibility in layout especially in large ships
- Enabled simple reversing by use of change-over switch rather than a separate reversing turbine
- It is perceived that more conversion stages meant more equipment in the shaft line hence greater losses (especially at high ship speeds) therefore greater through life cost

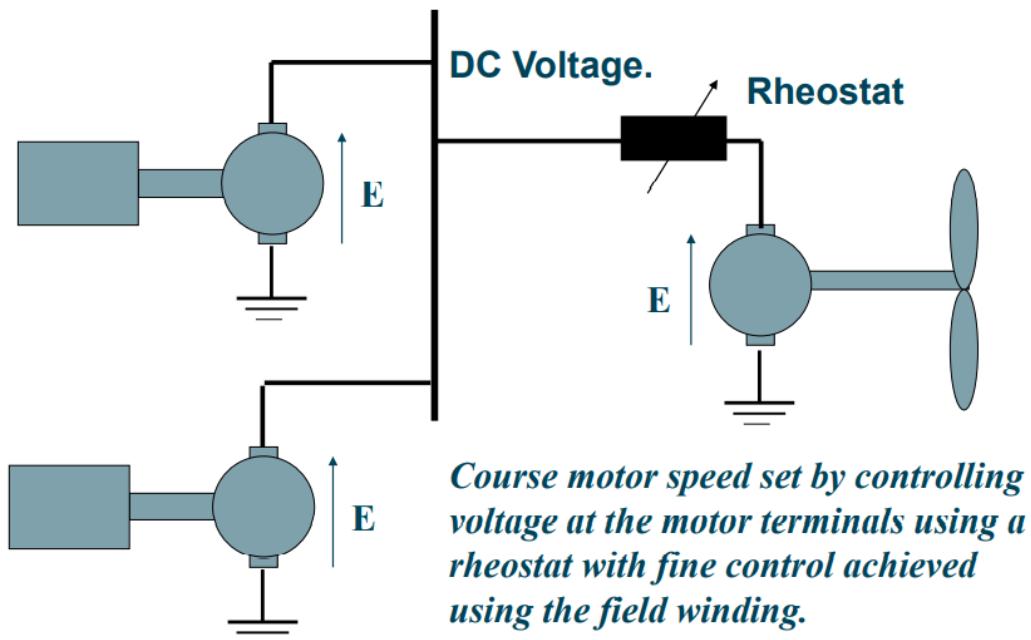


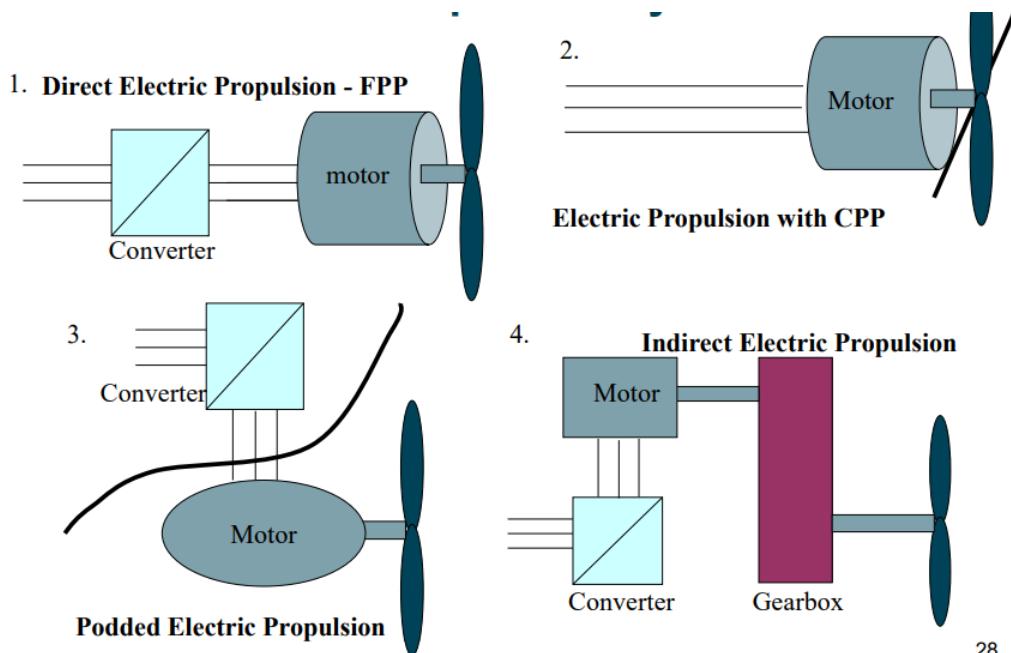
Figure 8.6: Diesel-electric (DC) propulsion system.

#### Decline of the conventional propulsion systems:

- Cost of oil increased leading to the demand for more efficient propulsion systems (e.g. 1970's oil crisis)
- Growth of offshore exploration for oil and gas led to the demand for greater controllability of propulsion power including dynamic positioning control (e.g. North Sea and Gulf of Mexico)
- The demand for AC distribution systems for ship's services and to integrate with propulsion power (i.e. DC was considered old fashioned)
- The invention of power electronic devices (especially the thyristor) and the introduction of power electronic converters

#### 8.2.2 Modern ship designs

- The modern diesel-electric DC propulsion system
- The constant speed propulsion motor system
- The re-engineering of the QEII



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Figure 8.7: Modern electric propulsion systems.

FPP - fixed pitch propeller, CPP - controllable pitch

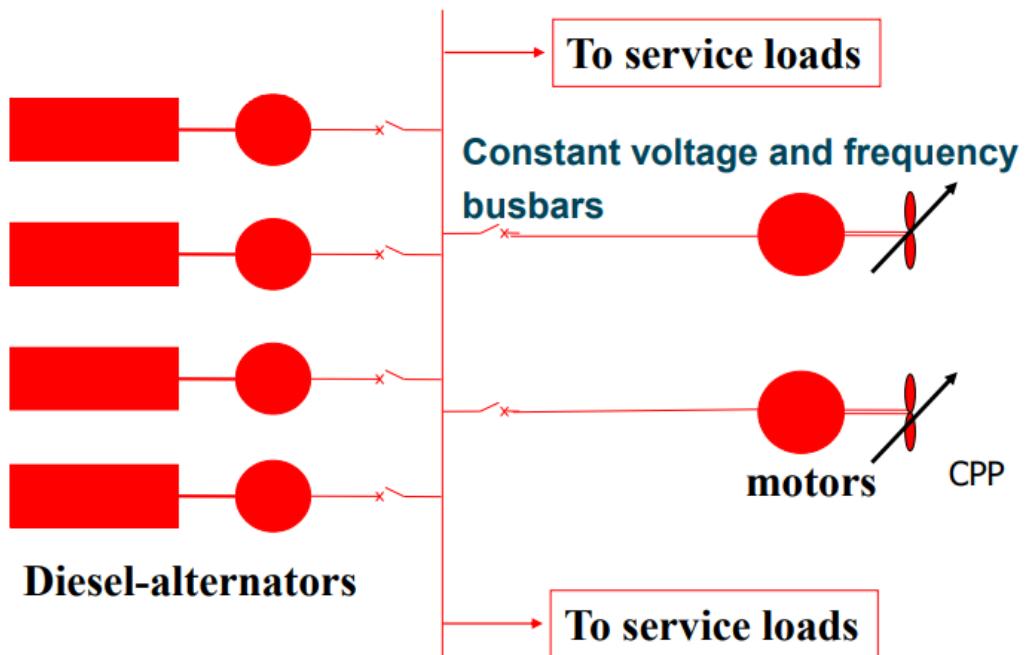


Figure 8.8: Electrical propulsion with CPPs.

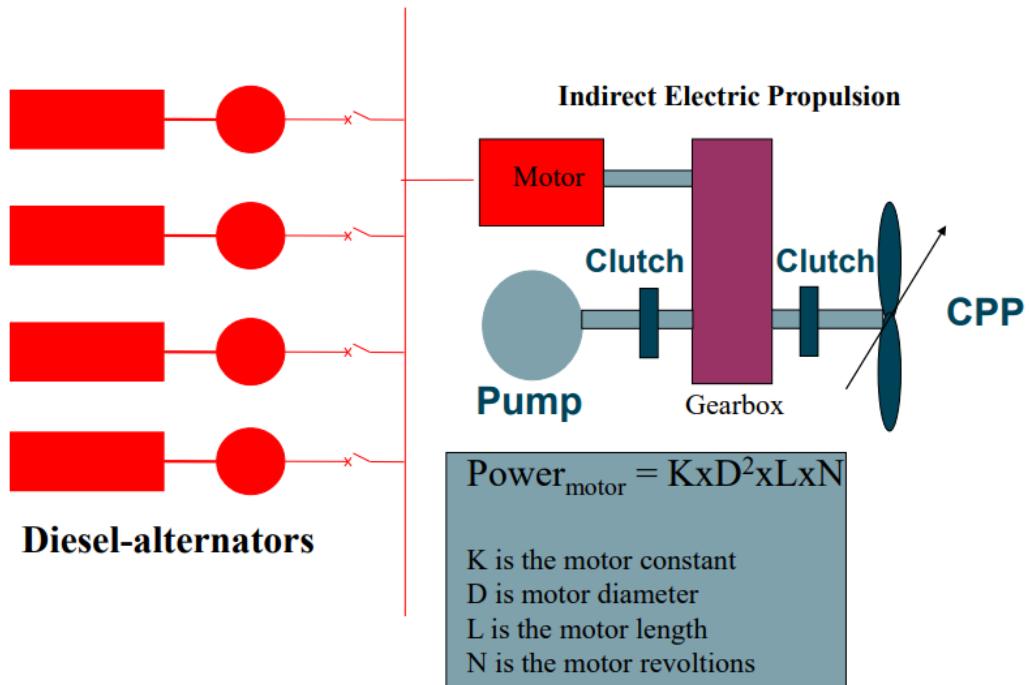


Figure 8.9: Electrical propulsion with gearboxes.

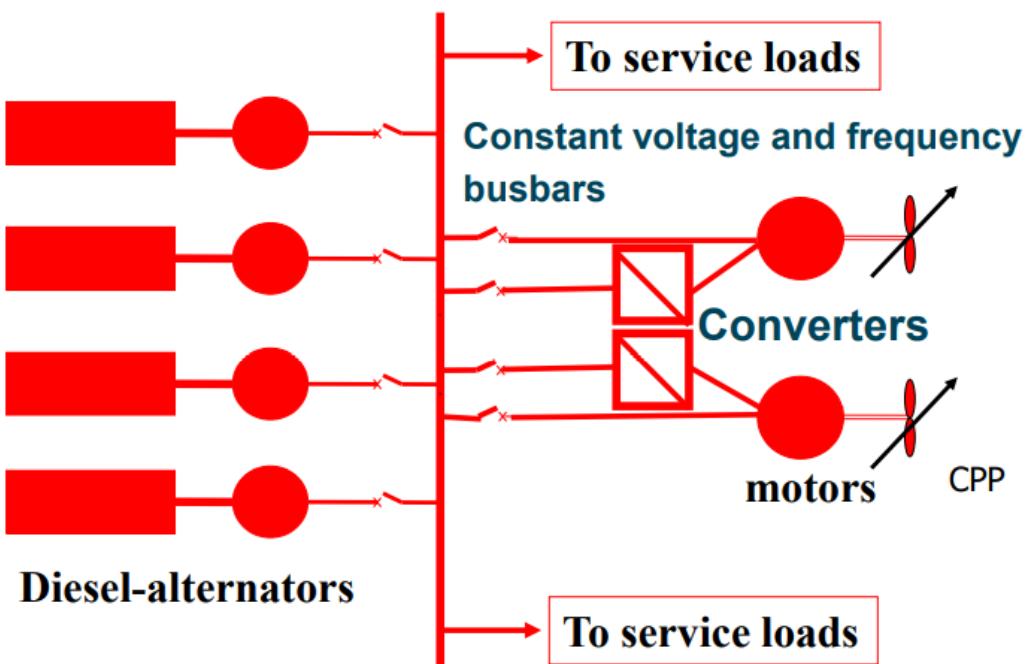


Figure 8.10: Electrical propulsion with converters and CPPs.

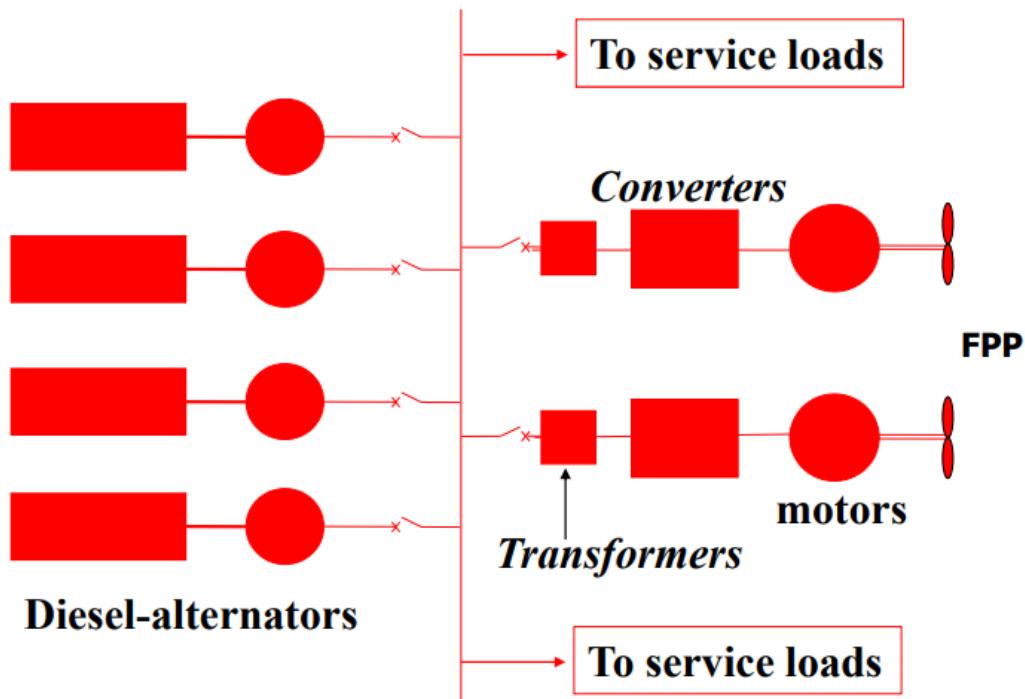


Figure 8.11: Electrical propulsion with converters.

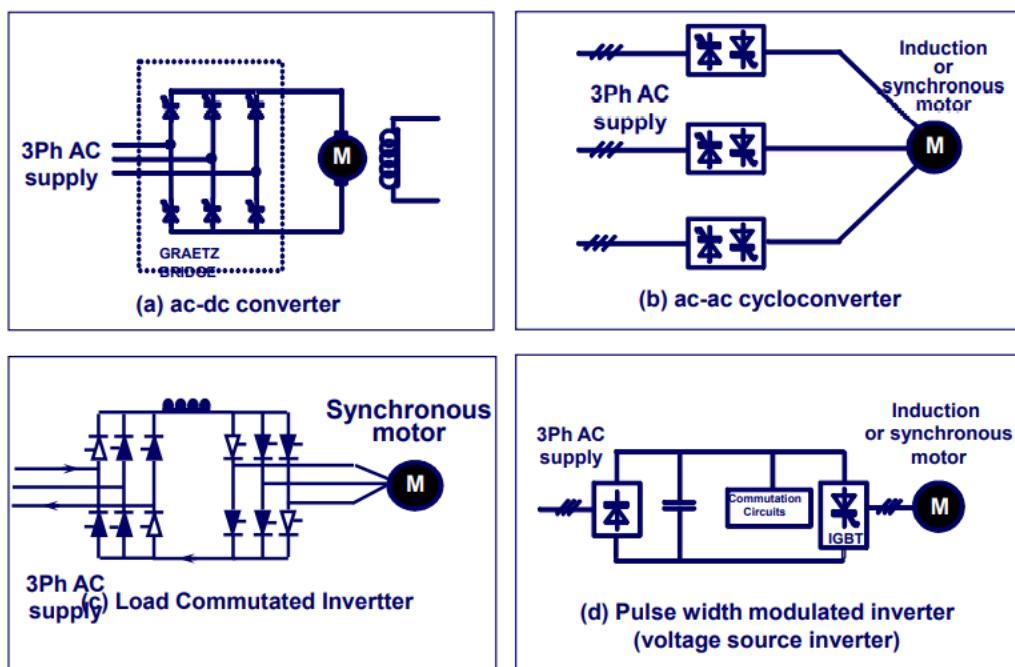


Figure 8.12: Main types of converters.

**Modern power converters:**

- DC rectifier with DC motor
  - Limited to 10 MW at 200 rpm
  - Old technology now replaced with AC drives
- Cycloconverter drive with AC motor
  - Unlimited power

- Synchronous or Induction motors
- Transformers required
- Large size
- Load commutator inverter with AC motor
  - Unlimited power
  - Synchronous motors only
  - Transformers required
  - Compact size
  - Waveform distortion
- Pulse width modulated drive with AC motor
  - Power limited to 24 MW approximately
  - Synchronous or Induction motors
  - Good waveform quality
  - Developing technology

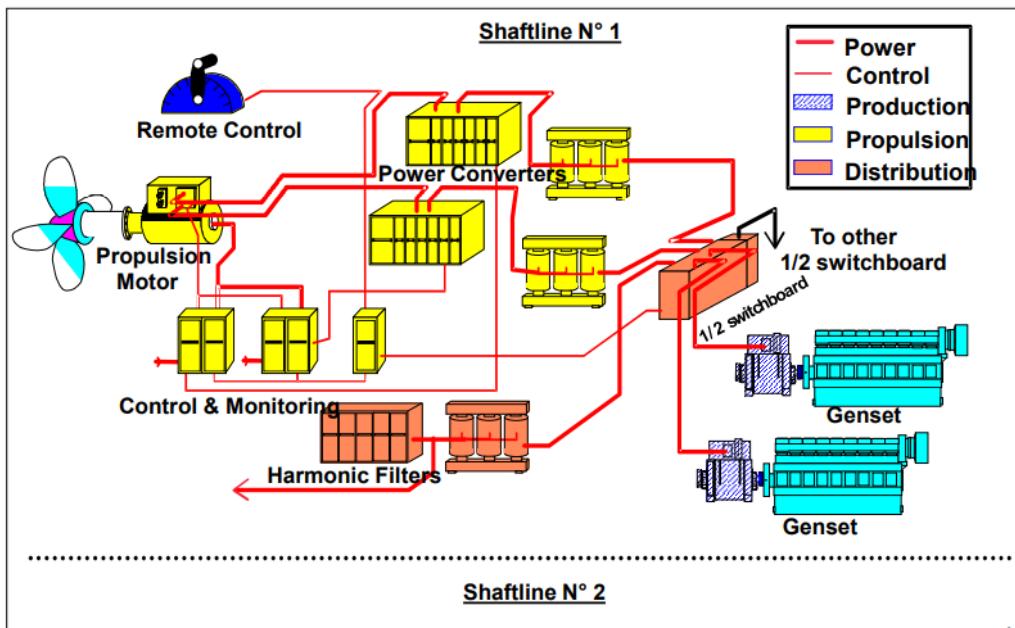


Figure 8.13: Electrical propulsion system arrangement.

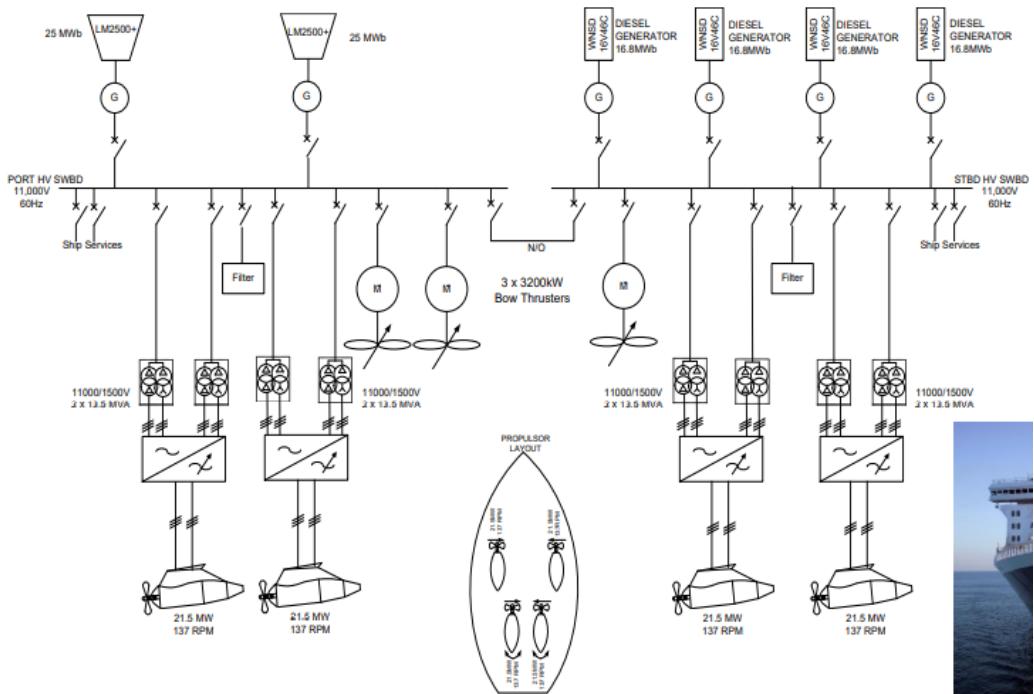


Figure 8.14: Queen Elizabeth 2 electrical propulsion system arrangement.

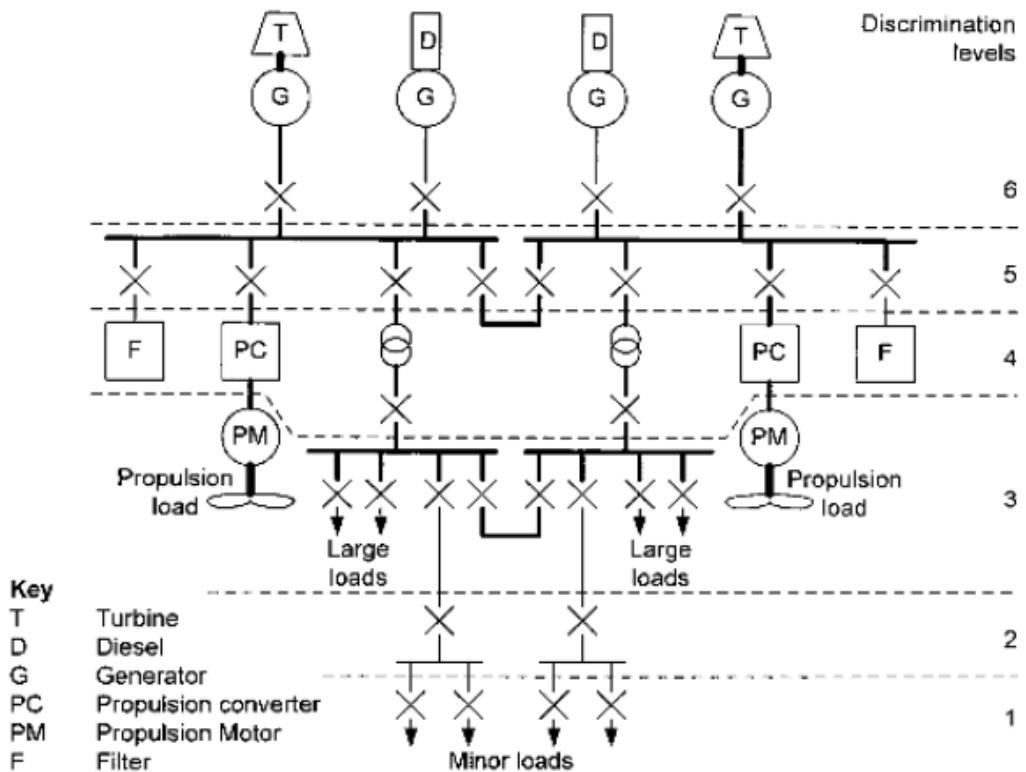


Figure 8.15: T45 Frigate electrical line diagram (not exact).

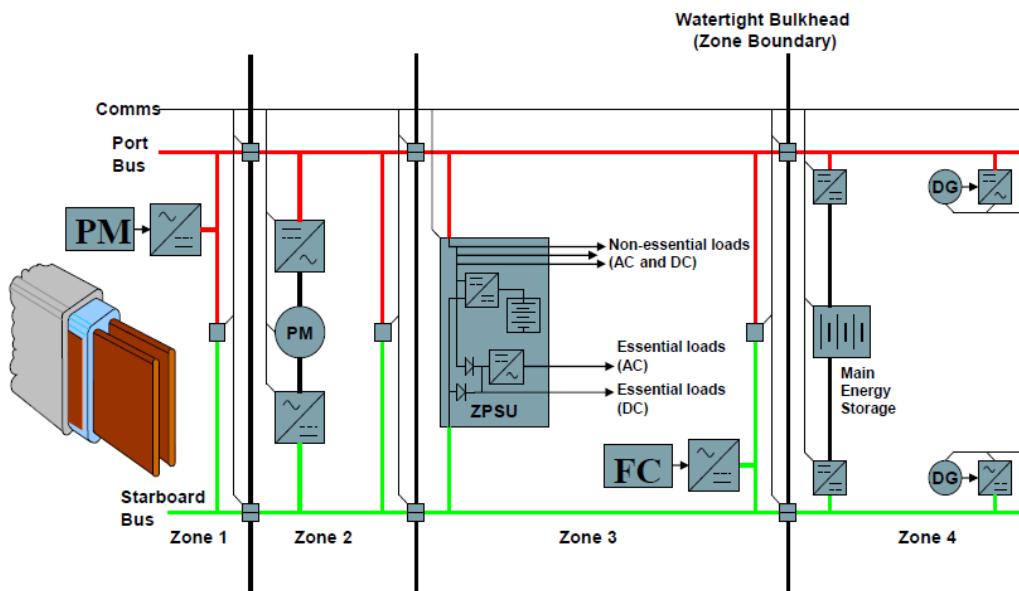


Figure 8.16: Zonal power example architecture.

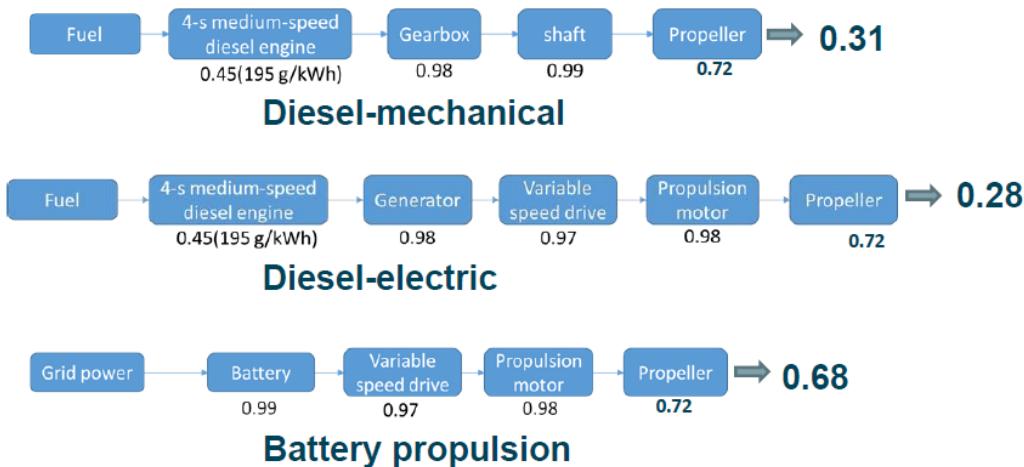


Figure 8.17: System configuration efficiencies.

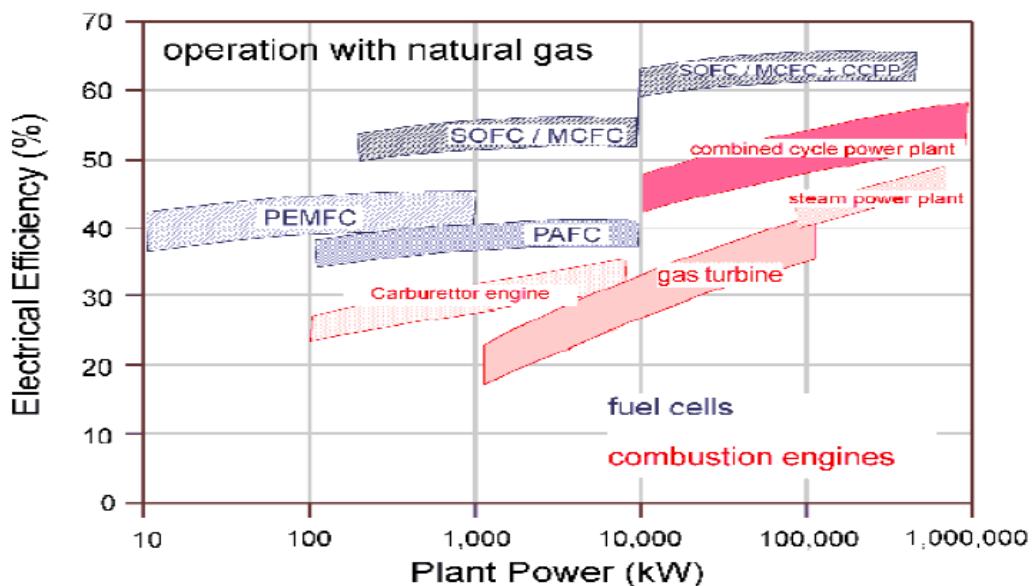


Figure 8.18: Potential of fuel cell technology.

Power system	Efficiency	Weight power density [kg kW <sup>-1</sup> ]	Volume power density [m <sup>3</sup> kW <sup>-1</sup> ]
PEFC	39-42%	2.7-5.4	0.005-0.009
SOFC	45-60%	9.1-13.6	0.017-0.034
MCFC	40-55%	18.1-27.2	0.028-0.060
PAFC	38-42%	13.6-20.9	0.026-0.043
Diesel generator	31%	14.2	0.024
Gas turbine generator	26%	12.2	0.026

Table 8.2: Current fuel cell technology.

**Perceived advantages of modern electric propulsion:**

- Can be more fuel efficient
- Can reduce emissions
- Lower maintenance saving
- Flexibility of operation
- Flexibility of design
- Greater redundancy
- Lower noise
- Easily reversible and good manoeuvrability

**Perceived disadvantages of modern electric propulsion:**

- Greater initial cost of machinery
- Greater machinery volume taken up in hull
- Greater weight of machinery
- Poor efficiency at full speed

### 8.2.3 Summary

- Electrical propulsion has been used in ships for over a century. It was first established in small boats and submarines
- Modern electrical propulsion systems are extensive in design but are largely based upon the use of power conversion methods
- The purpose of power conversion is to convert a fixed voltage/fixed frequency supply to a variable voltage and variable frequency for the control of the propulsion motor speed
- Electrical propulsion is firmly established in UK naval ships and is being seriously considered for use in future US naval vessels and other naval ships across the world. It is already used extensively in merchant ships of all kind
- Electric propulsion technology continues to develop with new equipment and systems designs

# **Chapter 9**

## **Seminar on Marine Propulsion**

### **9.0.1 Propulsion exercise**

It is usually necessary to consider in the design of a propulsion the estimated fuel consumption and associated emissions. This is usually achieved for a specific set of conditions such as the ‘Millbrook Circuit’ as used for many road vehicles. The fuel consumption is associated with prime-movers. For simple arrangements e.g. diesel drives it is simple, for hybrid drives it is more interesting.

#### **Propulsion example**

We will consider a propulsion system for a marine application (hybrid drive). However, the methodology can be applied to different transport modes (with obvious modifications).

### **9.1 Task 1**

- Sketch a CODOG arrangement
- CODOG - combined diesel or gas turbine
- Explain how cruise speed is achieved
- Explain how full speed is achieved
- What design issues are there for
  - The gas turbine
  - The diesel engine
  - The gearbox
  - The propeller

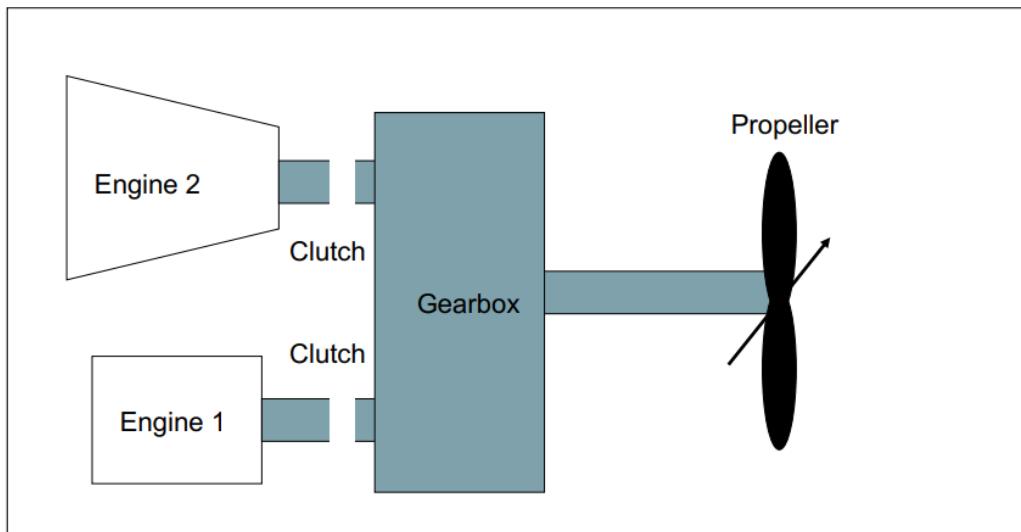


Figure 9.1: CODOG arrangement with CPP.

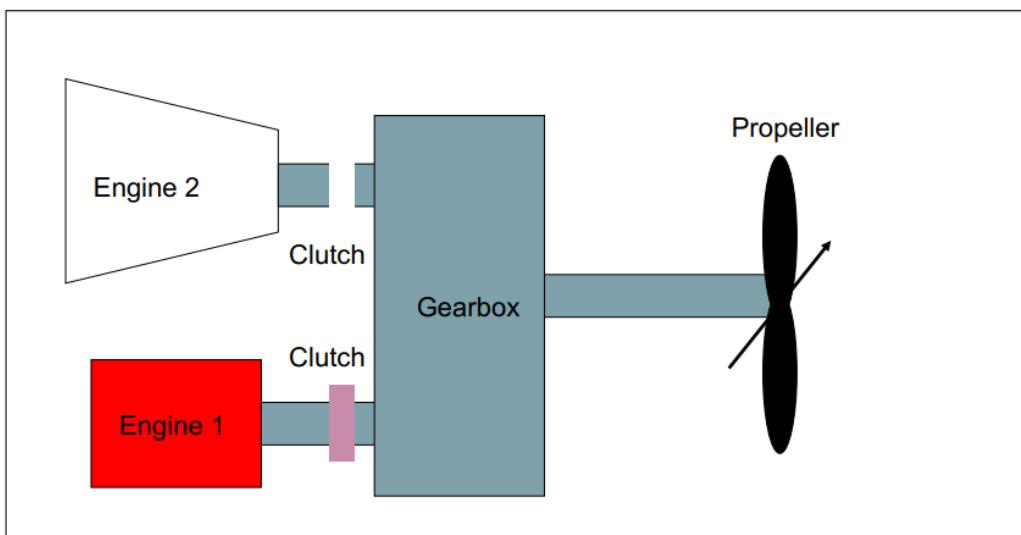


Figure 9.2: Engine 1 (low) power available for cruise speed.

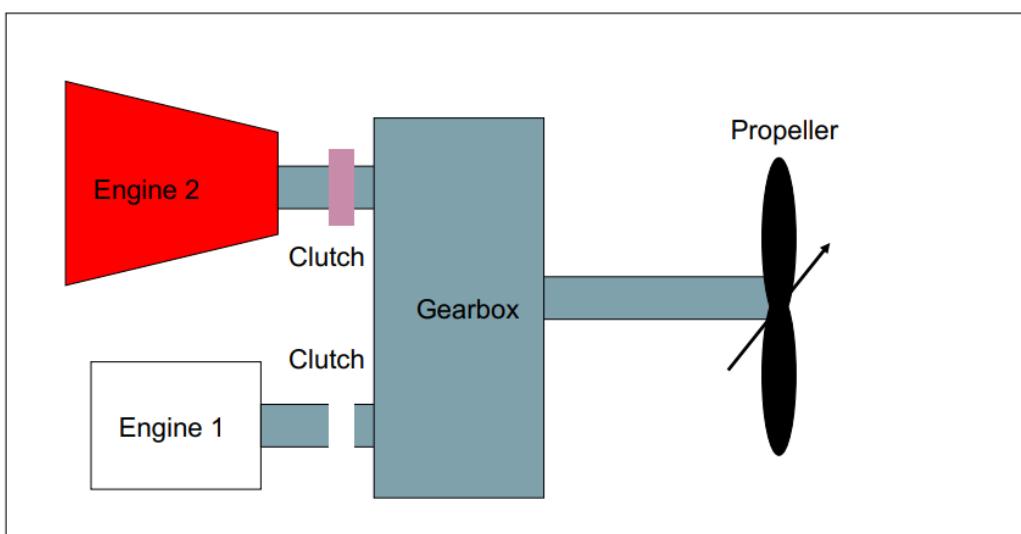


Figure 9.3: Engine 2 (large) power available for top speed.

### 9.1.1 CODOG design issues

- Gas turbine supplied the power for maximum speed
- Diesel supplies the power for cruise speed (80% MCR)
- The prime-movers are connected to the gearbox via clutches
- The engines work separately and not together
- The gearbox steps down the speeds of the prime-movers. (GT typically 3600-16000 rpm, propulsion diesels typically 400-1000 rpm)
- The gearbox may also provide a reversing capability
- The propeller can be a fixed pitch or controllable pitch type (maximum shaft revolutions typically 150-200 rpm)

### 9.1.2 Alternative propulsion arrangements

- CODAG - combined diesel and gas turbine
- CODOD - combined diesel or diesel
- COGAG - combined gas turbine and gas turbine
- COGOG - combined gas turbine or gas turbine
- CODLAG - combined diesel electric and gas turbine
- COFCAG - combined fuel cell and gas turbine
- IFEP - integrated full electric propulsion

Note: first engine is cruise engine and second is the sprint engine. AND / OR arrangements should be noted.

## 9.2 Task 2

A CODOG frigate has 3500 tonnes displacement. The specific delivered power coefficient ( $C_D$ ) is 0.03 and may be assumed independent of ship's speed and power. The relationship can be expressed as:

$$P_B = 1.04 \cdot C_D \cdot \rho^{0.33} \cdot \Delta^{0.67} \cdot V_S^3 \text{ (W)} \quad (9.1)$$

- $\rho$  is the seawater density  $1025 \text{ kg m}^{-3}$
- $\Delta$  is the displacement in kg
- $V$  is the speed of the advance in  $\text{m s}^{-1}$  (1 knot =  $0.514 \text{ m s}^{-1}$ )

The vessel has two shafts and a maximum speed of 30 knots and a cruising speed of 20 knots.

#### Calculate the power rating of the engines.

$$P_B = 1.04 \times 0.03 \times 1025^{0.33} \times \left(3500 \times 10^3\right)^{0.67} \cdot V_S^3 \quad (9.2)$$

$$P_B = \left(7.5 \times 10^3\right) \cdot V_S^3 \quad (9.3)$$

#### Notes:

1. The relationship here is between engine break power and vessel speed of advance
2. The relationship can be expressed as using effective power

For the gas turbines here then  $V_S = 30 \text{ knots} = 15.43 \text{ m s}^{-1}$ .

- $P_{B,GTS} = 7.5 \times 10^3 \times 15.43^3 = 27\,500 \text{ kW}$
- Each gas turbine would be rated at 13 750 kW

Gas turbines operating at flat out gives maximum power. For the diesel engines then  $V_S = 20 \text{ knots} = 10.29 \text{ m s}^{-1}$ .

- $P_{B,DE} = 7.5 \times 10^3 \times 10.29^3 = 8200 \text{ kW}$
- Each diesel engine would be rated at 5125 kW assuming that they are rated at  $0.8 \times \text{MCR}$ .

#### Notes:

1. MCR - Maximum continuous rating
2. 1 knot =  $0.514 \text{ m s}^{-1}$
3. We are assuming engines without NOx suppression

### 9.3 Task 3

The frigate has an electrical service demand of 0.3 kW/(Tonne  $\Delta$ ). Auxiliary power is to be supplied by two diesel generators such that in the normal condition they run at 80% power. Generator efficiency is 95%.

#### Determine the size of the diesel generator sets.

Calculating the electrical load:

$$0.3 \times 3500 = 1050 \text{ kW} \quad (9.4)$$

Allowing for the generator efficiency of 95%, then diesel engine output power must be 1105 kW. Two diesel engines have to operate at this mean load with 80% power. Installed sets are therefore 1380 kW. Each diesel generator set will therefore be rated at 690 kW and operate at 80% of MCR. Maximum electrical power available is:

$$\frac{1105}{0.8} = 1.381 \text{ MW} \quad (9.5)$$

### 9.4 Task 4

A journey is planned where at distance of 528 nautical miles must be covered in 24 hours. The captain is considering two alternatives to accomplish the mission:

1. One speed of advance for the whole journey
2. Fast sailing at 28 knots for 8 hours followed by the remaining time at a lower speed of advance

For each option, you - the engineer, are to provide the captain with fuel consumption NOx emissions.

% Power	Gas turbine		Main diesel		Diesel generator	
	SFC g kW <sup>-1</sup> h <sup>-1</sup>	NOXER g kg <sup>-1</sup>	SFC g kW <sup>-1</sup> h <sup>-1</sup>	NOXER g kg <sup>-1</sup>	SFC g kW <sup>-1</sup> h <sup>-1</sup>	NOXER g kg <sup>-1</sup>
25-34	350	5	250	74	245	74
35-44	300	8.5	240	70	240	70
45-54	280	9.5	230	66	235	65
55-64	262	11	220	62	230	60
65-74	258	12	208	58	225	51
75-84	255	13	195	55	220	47
85+	256	14	195	55	220	47

Table 9.1: Data on fuel consumption NOx emissions - Task 4.

- SFC - specific fuel consumption

Speed of advance scenario 1:

$$V_S = \frac{D}{T} = \frac{528}{24} = 22 \text{ knots} = 11.31 \text{ m s}^{-1} \quad (9.6)$$

Speed of advance scenario 2. Phase 1:

$$V_S = 28 \text{ knots for 8 hours} \quad (9.7)$$

$$D = 8 \times 28 = 224 \text{ nm} \quad (9.8)$$

Phase 2. Distance to cover in phase 2 is 304 nm. Time to complete this distance is 16 h.

$$V_S = \frac{D}{T} = 19 \text{ knots} = 9.77 \text{ m s}^{-1} \quad (9.9)$$

#### 9.4.1 Calculations - Scenario 1

Calculate part load power for propulsion at 22 knots ( $11.31 \text{ m s}^{-1}$ ).

$$P = 7.5 \times 10^3 \times 11.31^3 = 10850 \text{ kW} \quad (9.10)$$

Calculate this as a percentage of maximum output power. (note:  $P_{B,GTS}$  is used here)

$$\% \text{ Power} = \frac{10850}{27500} = 39\% \quad (9.11)$$

From Table 9.1:

- SFC:  $300 \text{ g kW}^{-1} \text{ h}^{-1}$
- NOXER:  $8.5 \text{ g kg}^{-1}$

Propulsion - calculate specific NOx emissions (SNE) ( $\text{g kW}^{-1} \text{ h}^{-1}$ ):

$$\text{SNE} = \text{NOXER} \times \text{SFC} \quad (9.12)$$

$$\text{SNE} = 8.5 \times \frac{300}{1000} = 2.55 \text{ g kW}^{-1} \text{ h}^{-1} \quad (9.13)$$

Propulsion - calculate fuel consumption:

$$\text{mass/hour} = \text{SFC} \times P = 300 \times 10850 = 3.26 \text{ tonnes/h} \quad (9.14)$$

$$\text{Fuel consumed} = \text{mass/hour} \times T = 3.26 \times 24 = 78.2 \text{ tonnes} \quad (9.15)$$

Propulsion - calculate NOx emissions:

$$\text{Mass of NOx/hour} = \text{SNE} \times P = 2.55 \times 10850 = 27.7 \text{ kg h}^{-1} \quad (9.16)$$

$$\text{NOx produced} = 27.7 \times 24 = 665 \text{ kg} \quad (9.17)$$

Calculate fuel consumption and NOx emissions for diesel generator sets.

- Diesel generator are working at 80% loading
- SFC =  $220 \text{ g kW}^{-1} \text{ h}^{-1}$  (Table 9.1)
- NOXER =  $47 \text{ g kg}^{-1}$  (Table 9.1)

Generation - calculate fuel consumption:

$$\text{mass of fuel} = 220 \times 1105 \times 24 = 5.8 \text{ tonnes} \quad (9.18)$$

Generation - calculate NOx emission:

$$\text{mass of NOx} = 47 \times \frac{220}{1000} \times 1105 \times 24 = 275 \text{ kg} \quad (9.19)$$

Total fuel consumed (propulsion + generation):

$$78.2 + 5.8 = 84 \text{ tonnes} \quad (9.20)$$

Total NOx emitted (propulsion + generation):

$$665 + 275 = 940 \text{ kg} \quad (9.21)$$

#### 9.4.2 Calculations - Scenario 2

First 8 hours at 28 knots ( $14.4 \text{ m s}^{-1}$ ):

$$P = 7.5 \times 10^3 \times 14.4^3 = 22400 \text{ kW (81\%)} \quad (9.22)$$

$$\text{SFC} = 255 \text{ g kW}^{-1} \text{ h}^{-1} \quad (9.23)$$

$$\text{NOXER} = 13 \text{ g kg}^{-1} \quad (9.24)$$

$$\text{SNE} = \text{NOXER} \times \text{SFC} = 13 \times \frac{255}{1000} = 3.22 \text{ g kW}^{-1} \text{ h}^{-1} \quad (9.25)$$

$$\text{mass of fuel used} = 255 \times 22400 \times 8 = 45.7 \text{ tonnes} \quad (9.26)$$

$$\text{NOx} = 3.22 \times 22400 \times 8 = 595 \text{ kg} \quad (9.27)$$

Last 16 hours at 19 knots ( $9.8 \text{ m s}^{-1}$ ):

$$P = 7.5 \times 10^3 \times 9.8^3 = 7 \text{ MW (85\%)} \quad (9.28)$$

$$\text{SFC} = 195 \text{ g kW}^{-1} \text{ h}^{-1} \quad (9.29)$$

$$\text{NOXER} = 55 \text{ g kg}^{-1} \quad (9.30)$$

$$\text{SNE} = \text{NOXER} \times \text{SFC} = 55 \times \frac{195}{1000} = 10.7 \text{ g kW}^{-1} \text{ h} \quad (9.31)$$

$$\text{mass of fuel used} = 195 \times 7000 \times 16 = 21.9 \text{ tonnes} \quad (9.32)$$

$$\text{NOx} = 10.7 \times 7000 \times 16 = 1198 \text{ kg} \quad (9.33)$$

Total fuel consumed (propulsion + generation):

$$45.7 + 21.9 + 5.8 = 73.4 \text{ tonnes} \quad (9.34)$$

Total NOx emitted (propulsion + generation):

$$595 + 1198 + 275 = 2068 \text{ kg} \quad (9.35)$$

#### 9.4.3 Observations of study

What is the difference in fuel consumption between 1 and 2?

10.6 tonnes *i.e.* 2 is 87% of 1.

What vessel speed would give the worst fuel consumption?

*GTS light load - 22 knots approximately*

Which scenario has the best NOx performance and by how much?

1128 kg less *in favour of scenario 1*

## A word on emissions!

Carbon dioxide is dependent solely on fuel burnt i.e. the carbon factor of the fuel and the fuel consumption. For diesel the fuel consumption is approximately 3.2 tonnes of carbon dioxide for 1 tonne of fuel burnt. NOx is dependent on the combustion processes and in particular temperature and pressure. The higher the combustion pressure the more efficient the engine but more NOx produced. Sulphur emissions depends upon the amount of sulphur in the fuel. Particulates is dependent upon fuel quality and combustion processes.

### Carbon dioxide emissions

Carbon dioxide emissions are directly related to fuel burnt regardless of how this is done whether in an internal combustion engine, gas turbine or boiler. 1 tonne of distilled marine diesel fuel will emit 3.2 or more tonnes of CO<sub>2</sub> depending upon fuel quality and lube oil consumption. We will use 3.2 for simplicity.

- Scenario 1 produces:  $84 \times 3.2 = 269$  tonnes CO<sub>2</sub>
- Scenario 2 produces:  $73.4 \times 3.2 = 234$  tonnes CO<sub>2</sub>

Note the conflict: Burn less fuel and pollute with NOx more OR burn more fuel and pollute with NOx less... but CO<sub>2</sub> more! In part this is why many diesel engines are being fitted with exhaust after treatment to eliminate (as much as possible, NOx).

## 9.5 Task 5 (formative)

Escorting the frigate is a sister ship (same displacement) which has an Integrated Full Electric Propulsion (IFEP) arrangement rather than the CODOG arrangement. The electrical propulsion system consists of generators, power converters and propulsion motors as seen on the next slide. The ship designer has selected the same prime-mover ratings for this design. Each gas turbine is rated at 13 750 kW and each diesel rated at 5815 kW thereby providing engine compatibility across the fleet. The IFEP feeds both propulsion and service power loads. Because the diesels and gas turbines are operated along the generator line rather than the propulsion curve the data provided in the following slides should be used for NOXER values. Efficiency values of electrical machines are also provided to understand where losses occur.

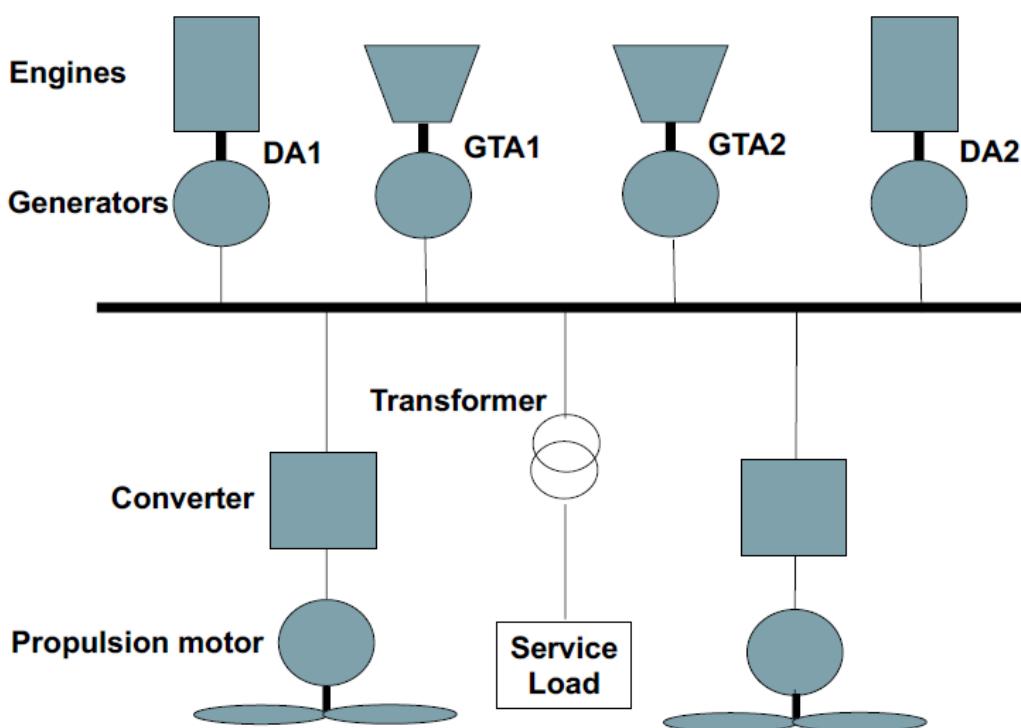


Figure 9.4: Task 5 Line diagram (use CODOG prime-movers).

<b>Power input</b>	<b>Generator (constant N)</b>	<b>Motor (Variable N)</b>
0%	0%	0%
20%	65%	60%
40%	85%	80%
60%	90%	90%
80%	95%	96%
100%	95%	96%

Table 9.2: Efficiency of generators and motors

Power converters (AC:DC:AC): can be considered to be 95% across the full power range. Transformer efficiency: 95%.

<b>% Power</b>	<b>Gas turbine</b>		<b>Diesel generator</b>	
	<b>SFC</b> g kW <sup>-1</sup> h <sup>-1</sup>	<b>NOXER</b> g kg <sup>-1</sup>	<b>SFC</b> g kW <sup>-1</sup> h <sup>-1</sup>	<b>NOXER</b> g kg <sup>-1</sup>
25-34	330	5	245	74
35-44	290	8.5	240	70
45-54	275	9.5	235	65
55-64	259	11	230	60
65-74	257	12	225	51
75-84	255	13	220	47
85+	255	14	220	47

Table 9.3: Data on fuel consumption NOx emissions - Task 5.

### 9.5.1 Task A

1. Suggest the required electrical equipment ratings i.e. kW ratings of generator, converter and motor based on given installed prime-mover powers for the electric frigate
2. Determine for your design the fuel consumptions and exhaust gas emissions for scenario 1 and scenario 2 for the electric frigate
3. Compare and contrast between CODOG and IFEP arrangements (use tables and graphs)
  - Compare maximum possible speed of the two vessels
  - Ideal cruise speeds of the two vessels
  - Running hours of the engines for the two scenarios
  - The likely cooling requirements for each vessel's propulsion system

### 9.5.2 Task B

Using library resources and web resources, summarise an investigation into commercial shipping use of electrical propulsion today. Discuss at least three key technologies that are under development.

# Chapter 10

## Generators

### Generator types used in power systems

- DC generator - usually separately excited machines in dedicated DC systems e.g. typically 5-600 V<sub>DC</sub> output (still in service, rarely built)
- AC synchronous generator - by far the most common, can be single-phase or three-phase. Usually 50 Hz or 60 Hz; 240 V-30 kV; kW to GW
- AC asynchronous generator - most common in renewable energy applications e.g. some wind-turbines. Usually three-phase, 50 Hz or 60 Hz; 440 V-11 kV; between 2-10 MW

### 10.1 Synchronous machine

Synchronous machines are the primary source of electrical energy generation (or conversion). They are used to convert the mechanical power output of steam-turbines, gas-turbines, reciprocating engines (prime movers), hydro turbines into electrical power. Synchronous machines can be extremely large with power ratings up to 2 GW or very small at a few Watts. Known as synchronous machines because they operate at synchronous speeds (speed of rotor always determines supply frequency).

#### 10.1.1 Petrol/diesel generators

- Commonly used at low, medium and high powers (a few kW to 10's MW)
- Often direct connection between diesel and synchronous generator
- Efficiency typically 35% at full load without waste heat recovery
- Efficiency typically 55% at full load with waste heat recovery
- Commonly used at low powers: single-phase back-up power units
- Common use at medium powers: traction, ships, standby generators
- Common use at high powers in generating stations

#### 10.1.2 Gas turbine generators

- Commonly used at medium and high powers (1 MW to 10's MW)
- Connected via gearbox at low powers and directly at high powers
- Efficiency typically 25% at full load for simple cycle types
- Efficiency typically 55% at full load in combined heat and power (CHP systems)

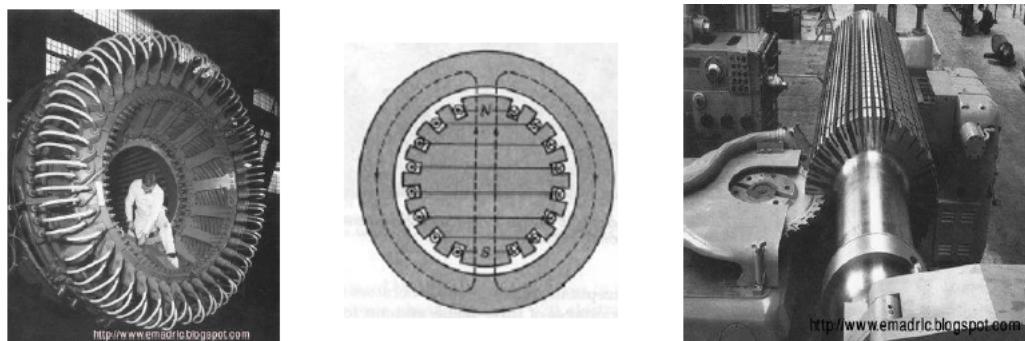
- Common use at medium powers: naval ships and standby generators
- Common use at high powers as CHP or CC in generating stations

### 10.1.3 Steam turbine generators

Steam turbine generators usually driven from coal-fired boilers or nuclear power. An example 1.2 GW steam plant uses a hydrogen cooled generator.

- Commonly used at medium and high powers (1 MW to several GW)
- Connected via gearbox at low powers and directly at high powers
- Efficiency typically 60% with sophisticated steam energy management system
- Common use at medium powers: ships using waste heat recovery
- Common use at high powers in the majority of power stations including nuclear

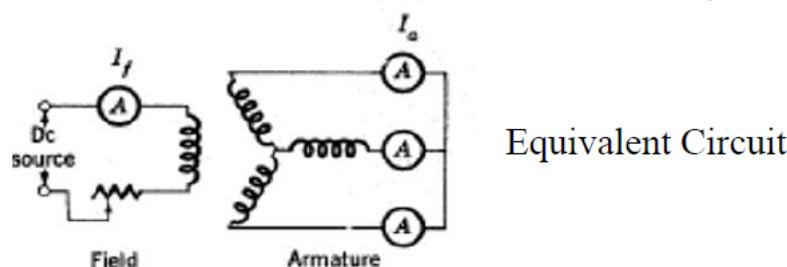
### 10.1.4 Synchronous machine basics



Stator (Armature)

Schematic Arrangement

Rotor (Field)



10

Figure 10.1: Synchronous machine basics.

### 10.1.5 Concept of back emf and internal resistance

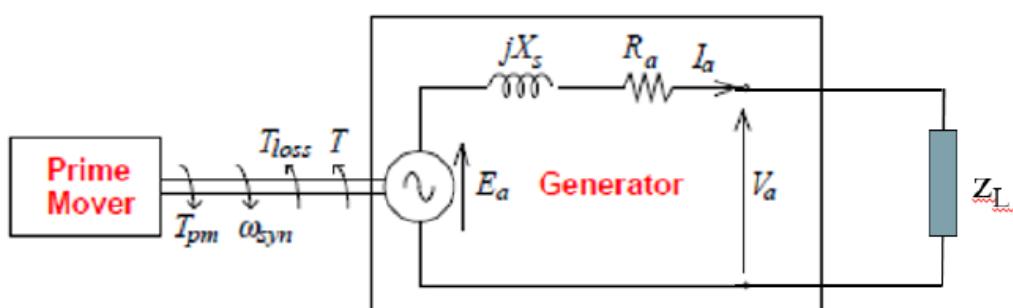


Figure 10.2: Generator diagram.

Back electro-motive force (emf =  $E_a$ ) is always present in a power source as power supplies have internal resistance ( $R_a$ ) and reactances  $X_s$  which cause voltage drops internally and which increases as current flow increases ( $I_a$ ).

### 10.1.6 Phasor diagram representation

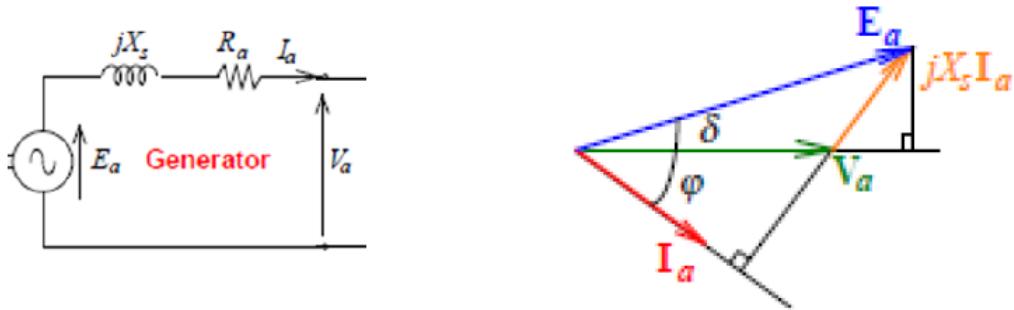


Figure 10.3: Phasor diagram.

The voltage drops can be represented in a phasor diagram. There are two important angles in this diagram known as  $\phi$  and  $\delta$ .

- $\phi$  is the phase angle (cosine of this angle is the power factor)
- $\delta$  is the load angle

Which conditions must exist for  $E_a = V_a$ ?

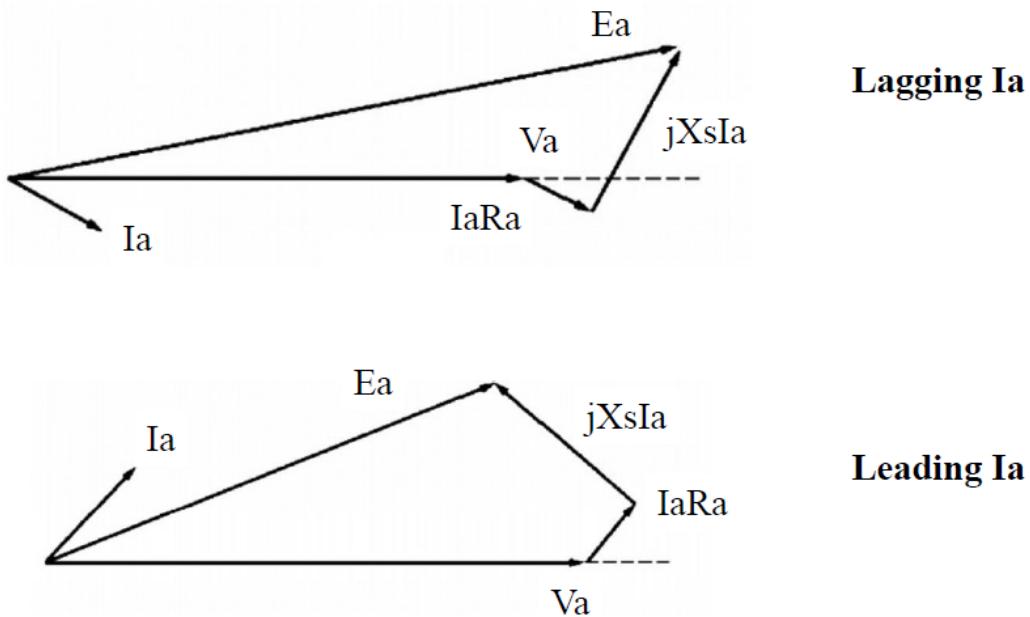


Figure 10.4: Current magnitude and phase effects on  $V_a$ .

### 10.1.7 The speed of rotation of synchronous generators

The electrical frequency is synchronised to the rotor speed. Recall that the magnetic field created by a 3-phase 4-pole machines moves  $180^\circ$  while the stator currents vary  $360^\circ$ .

$$f_e(\text{Hz}) = \frac{n_m (\text{r/min}) P \#}{120} \quad (10.1)$$

Therefore, a 2-pole generator must turn at 3600 r/min to produce a 60 Hz voltage while a 4-pole must turn at 1500 r/min to produce a 50 Hz power.

### 10.1.8 Frequency and voltage control

Governor control: frequency is controlled by the speed of rotation and the number of poles/ As the latter is fixed (a construction constraint) it is the speed of rotation that is important.

Automatic voltage regular control: voltage is controlled by the magnetic flux in the air gap in a fixed speed machine. This is essentially controlled by the field current within magnetic saturation limits.

### 10.1.9 Control of generators

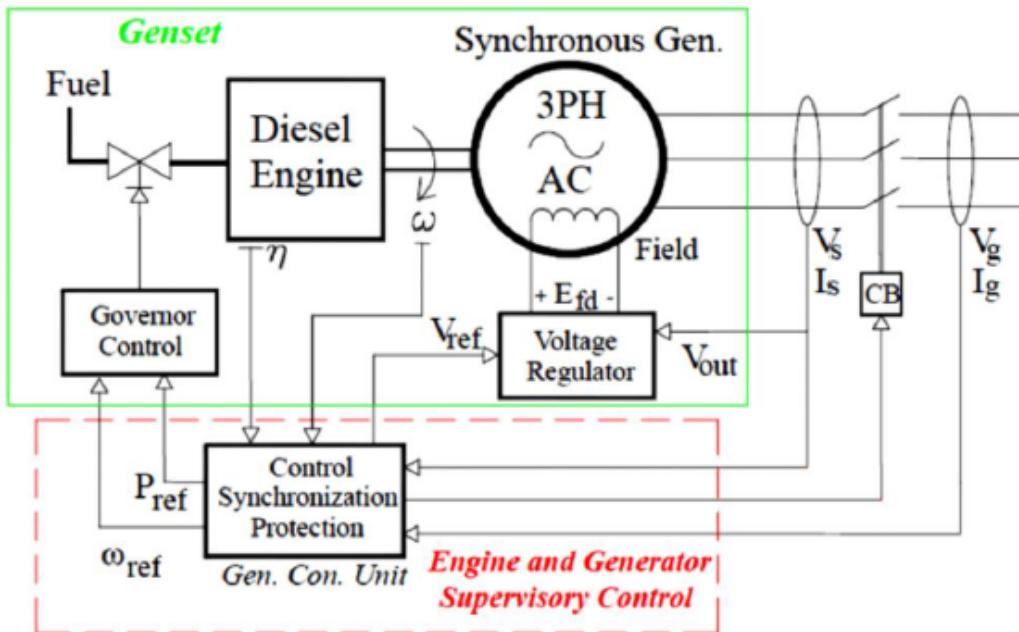


Figure 10.5: Control of generators.

The diesel-generator converts fuel into electricity. The electricity is three-phase, constant voltage and constant frequency. The frequency is controlled by the governor and voltage regulator controls field current. Control synchronisation protection controls the CB.

### 10.1.10 Single-generator operation - real power

In systems where there is a single generator operation then all **real power** (kW) and **reactive power** (kVAR) comes from that single generator. When more **real** power is demanded from the generator the prime-mover begins to decelerate (stall) and speed (and therefore frequency) drops. This is countered by the governor which increases the fuel supplied to the prime-mover thereby providing more power to the generator in an attempt to recover and maintain speed and frequency. The governor cannot act instantaneously and by this means avoid a frequency transient.

When more **reactive** power is demanded from the generator the voltage at the terminals begins to fall due to internal voltage loss. This is countered by an AVR which increases the field current supplied to the generator thereby providing more reactive power whilst maintaining terminal voltage (this can differ for a leading load). When load is shed from the power system the AVR compensates by reducing field current. The AVR cannot act instantaneously to avoid a voltage transient.

### 10.1.11 Automatic voltage regulator

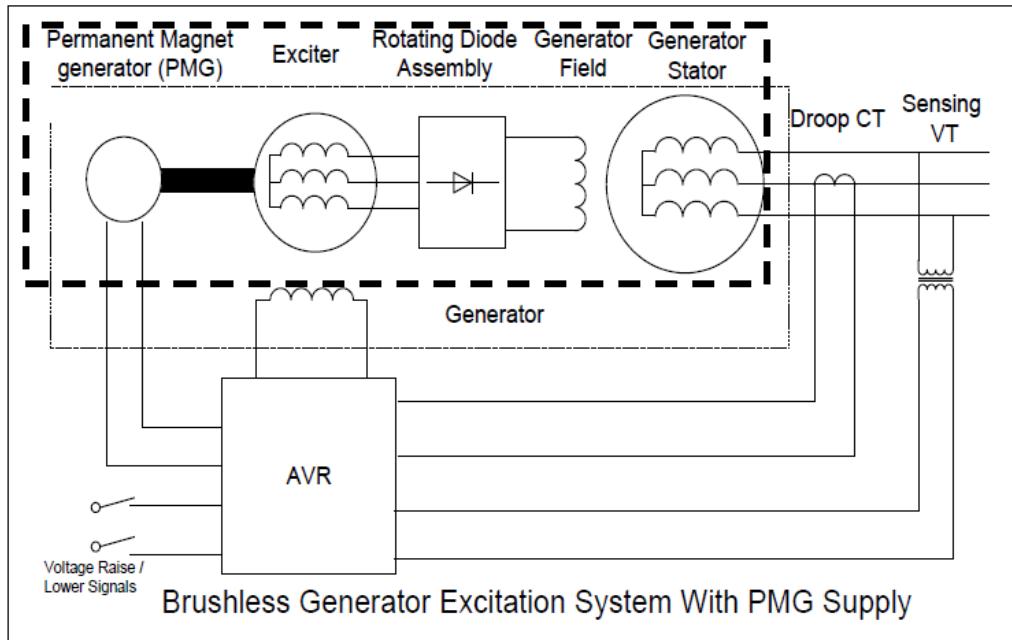


Figure 10.6: Brushless generator excitation system with PMG supply.

### 10.1.12 Circuit breaker and protector initiation

- Reverse power (current)
- Overvoltage
- Significant imbalance (voltage and current)
- Reverse rotation
- Loss of excitation
- Negative sequence overcurrent
- Zero sequence overcurrent
- Over frequency and under frequency
- Thermal overloads

## 10.2 Multi-synchronous generator operation

### 10.2.1 Multi-generator operation

When generators operate in parallel to supply a power system then power may be shared between them (i.e. both kW and kVAR). The governor and AVRs of each machine are designed to allow parallel operation. Two methods are employed:

- Isochronous control: this is a modern control system which enables all generators to be controlled by computer to optimise efficiency. A computer sets the governor and AVR set points as needed by the grid.
- Droop control: conventional method where droop is introduced to enable sharing of kW and kVAR

### 10.2.2 Connection requirements

Synchronise incoming generator with an existing system:

- Same phase rotation
- Same frequency
- Same voltage level
- Same phase sequence

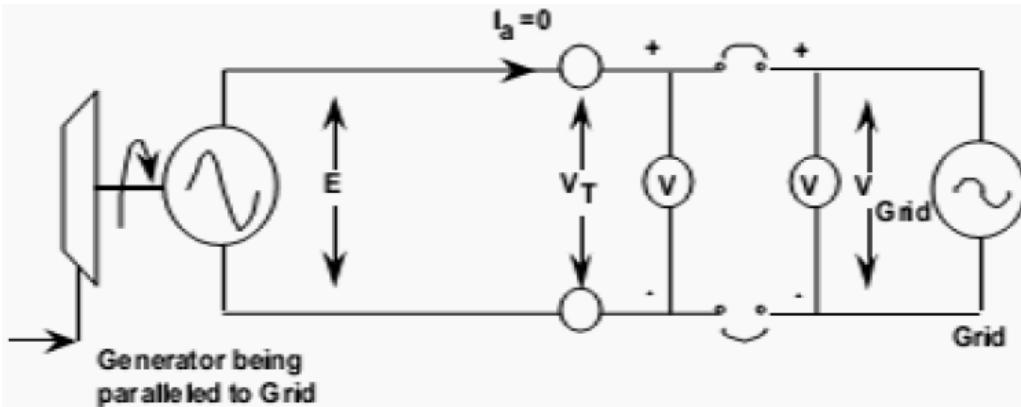


Figure 10.7: Synchronisation of generator to grid.

### 10.2.3 Engine speed control

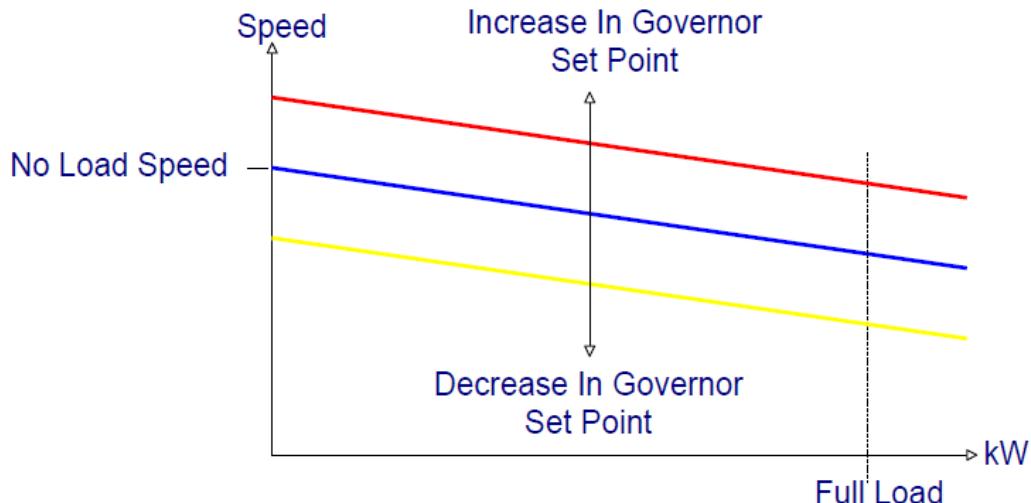


Figure 10.8: Single machine - governor droop characteristics (exaggerated).

- Governor controls the fuel supply to the prime mover (e.g. diesel engine or gas turbine) and forms part of a closed loop control system
- An increase in the governor set point gives a corresponding increase in generator speed and vice-versa
- Speed control system normally configured for droop control i.e. generator speed will fall as load increases
- 

$$\text{Droop (\%)} = \frac{\text{No load speed} - \text{Full load speed}}{\text{No load speed}} \times 100 \quad (10.2)$$

- Typical droop values 3-5%
- Droop characteristic required for stable parallel operation of generators

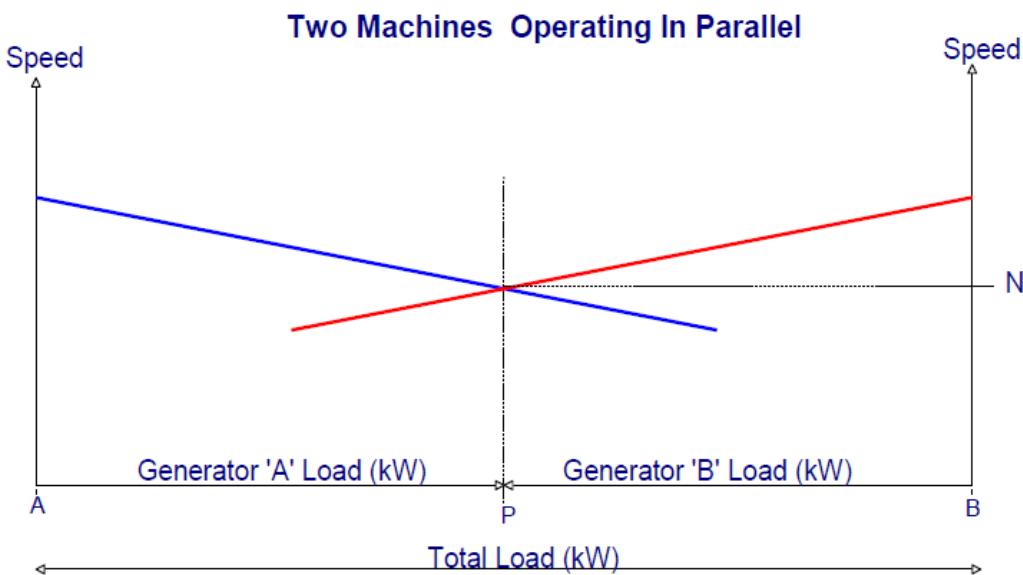


Figure 10.9: Two machines operating in parallel.

- Fuel supply to engine determines active power supplied by the prime mover
- Two identical generators with the same governor droop setting will share the load equally ( $PA = PB = \frac{AB}{2}$ )
- Both machines are locked in synchronism and therefore their speeds are identical
- The common speed or system frequency is at the point where the two lines intersect ( $N$ )

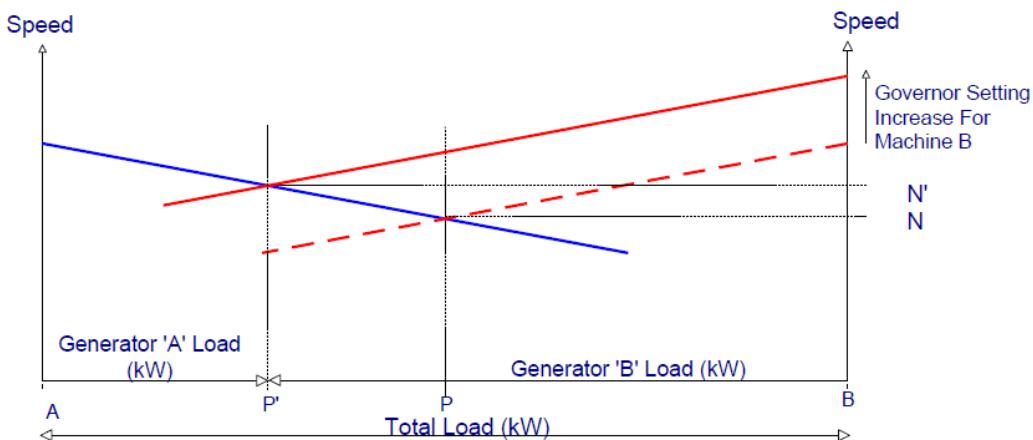


Figure 10.10: Two machines operating in parallel - effects due to governor adjustment.

- An increase in governor setting for machine B will cause the following:
  - System frequency to increase to  $N'$
  - Machine B taking a greater share of the load  $BP'$
  - Machine A taking a smaller share of the load  $AP'$
- System frequency may be restored back to  $N$  by simultaneous reduction in both machine governor settings

#### 10.2.4 Voltage and reactive power control

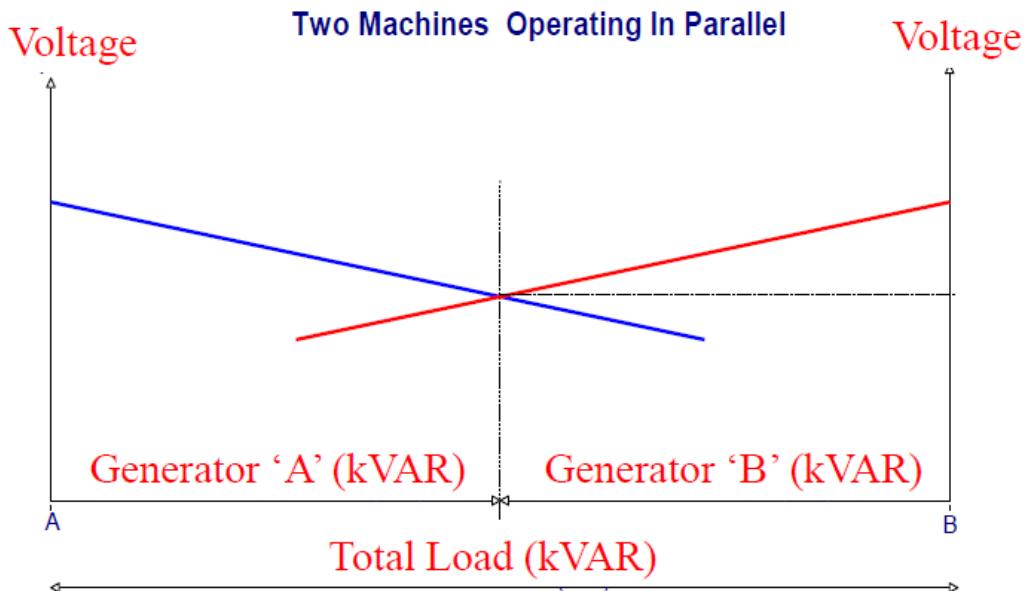


Figure 10.11: Two machines operating in parallel.

The AVR controls both voltage and reactive power flow. In this arrangement droop is needed (typically 1% over the power range to ensure the two AVRs do not fight!)

#### 10.2.5 Steady state performance

For power stations feeding a large national grid system then the voltage and frequency are stiff - this means the governor and AVR are used for controlling real and reactive power flow only. In small systems e.g. ship electrical propulsion systems, the magnitude of system load can be subject to frequency load variations and voltages and frequency is easily disturbed. Therefore, system voltage and frequency will also vary due to AVR % governor droop respectively. Usually a Power Management Systems (PMS) is employed to supervise power system operation. PMS functionality may include the simultaneous trimming of AVR % governor set points to maintain power system voltage and frequency to nominal, pre-set values.

### 10.3 Generator transient performance

#### 10.3.1 Transient performance

Transient performance depends on the ‘strength of a system.’ A weak system is subject to greater transient phenomena. A load transient, whether a predicted disturbance such as a motor start, or an unpredicted disturbance such as a fault at the generator busbars, will influence the power system in different ways:

- Transient voltage excursions
- Transient frequency excursions
- Generator / power system instability

*Various studies are performed at the design stage to determine the limits of power system stability taking into account both safety and operational scenarios.*

### 10.3.2 Transient load response of a generator

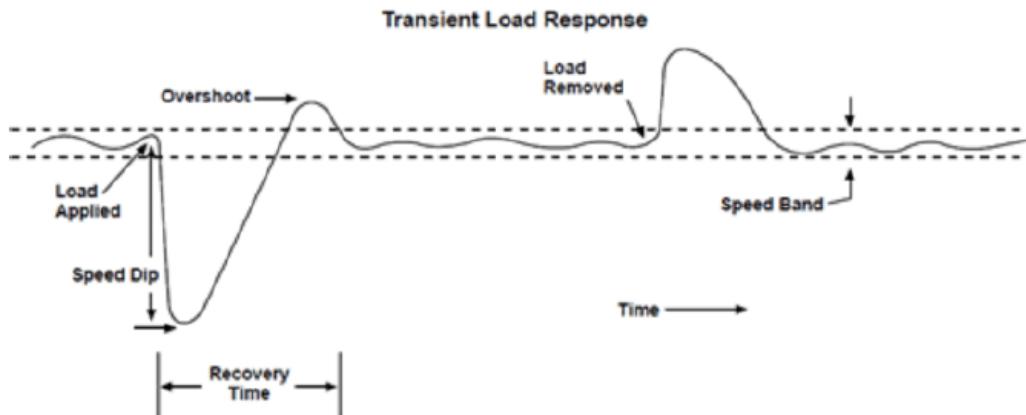


Figure 10.12: Transient load response of a generator.

The response to a frequency (speed) transients depends upon the governor time constants and machine characteristics.

#### Frequency transients

The application of an active (kW) load will result in an increased torque and hence fuel (energy) supply requirements from the generator's prime mover. This control function is performed by the prime mover governor. Different types of prime movers react in different ways to a step load application. A lightly loaded turbocharged diesel engine will have poor transient performance when compared to its normally aspirated equivalent.

*The inherent voltage dip experienced on load application has the second order effect of reducing the electrical kW load placed on the generator. This can result in improved prime mover load pick-up performance.*

#### Voltage transients

Careful setting of the generator AVR V/Hz characteristic can further improve prime mover load pick-up. If the AVR can respond by reducing generator excitation and hence generator terminal voltage when a pre set frequency level is breached, the step load change as seen by the prime mover will be reduced. The gradient of the AVR V/Hz slope will also affect performance. The larger the slope, the better the load pick-up performance. However, the overall voltage dip experienced on the system will be increased.

### 10.3.3 AVR arrangement for generator

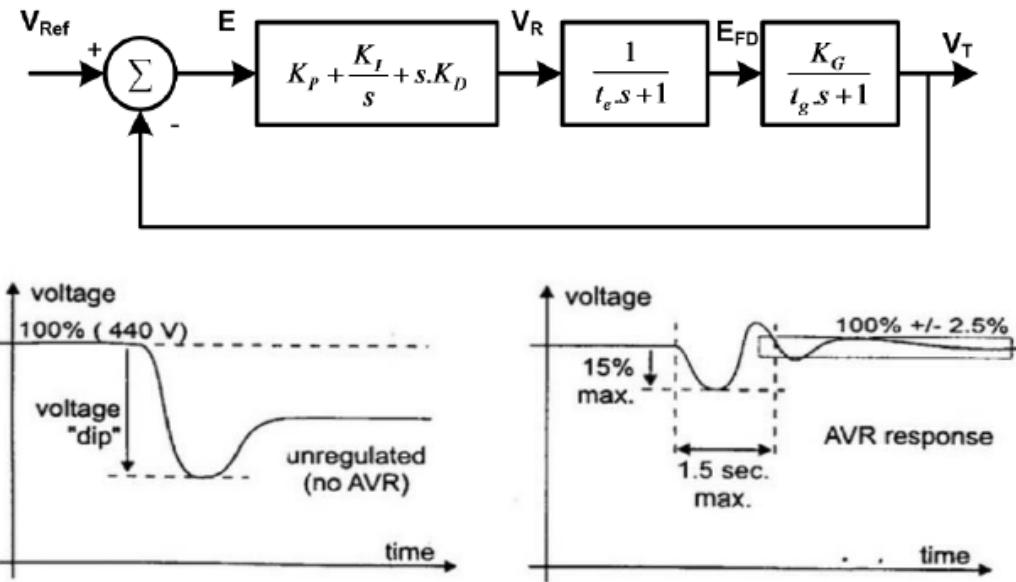


Figure 10.13: Typical AVR controller showing time constants for PID for exciter, regulator and field. Response must be within certain limits by regulation.

#### Voltage transients

A load transient will influence system voltage. The transient voltage response of the system will be dependent on the size of the load applied in relation to the generation capacity and inertia. As the generator circuit is mainly reactive, the effect on generator terminal voltage will be dependent on the power factor of the load.

*Excessive transient voltage dips may cause connected equipment to trip on under voltage, causing a supply outage to essential pieces of equipment.*

Three quantities that predominantly affect the transient performance of the generator:

- Direct axis sub transient reactance  $X_d''$
- Direct axis transient reactance  $X_d'$
- Direct axis synchronous reactance  $X_d$

For a given load application, the initial voltage dip is a function of  $X_d''$  and cannot be affected by an external control system such as the generator AVR. For a constant level of excitation, generator voltage would fall to a value governed by  $X_d'$  after 1 or 2 cycles. For the same level of excitation, generator voltage would eventually fall to a value consistent with  $X_d$ . The effects of  $X_d'$  and  $X_d$  can be minimised by the generator AVR supplying levels of excitation in excess to full load value (field forcing).

*There are numerous methods and tools available in the market place to assist the engineer in determining the stability of a given power system.*

## 10.4 Generator faulted performance

### 10.4.1 Synchronous machine - three-phase short circuit

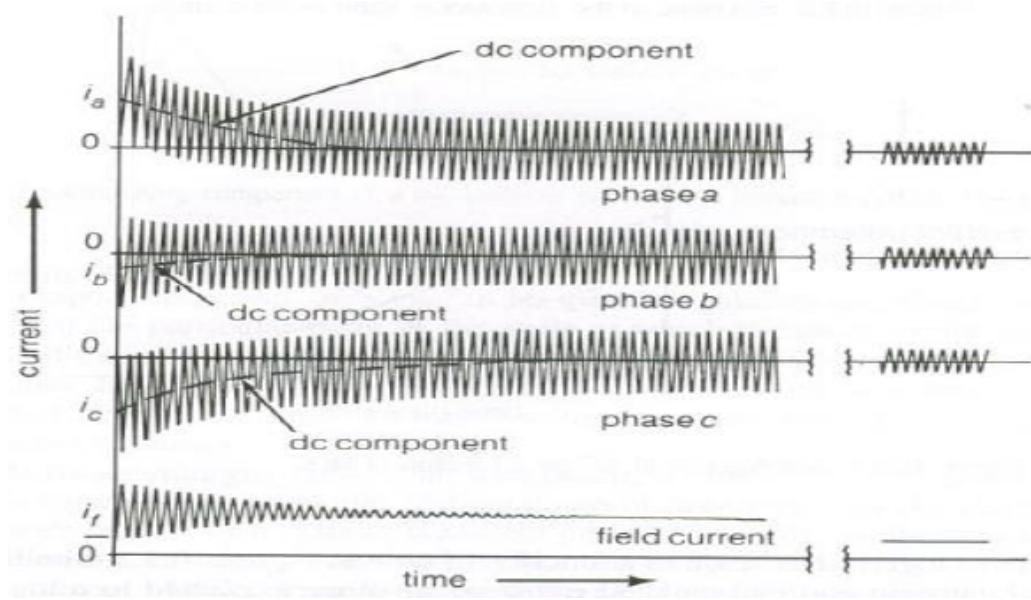


Figure 10.14: Typical response to a sudden three-phase short circuit at the terminals of a generator. Note the asymmetrical arrangement of the waveforms.

### 10.4.2 Synchronous machines short-circuit envelope

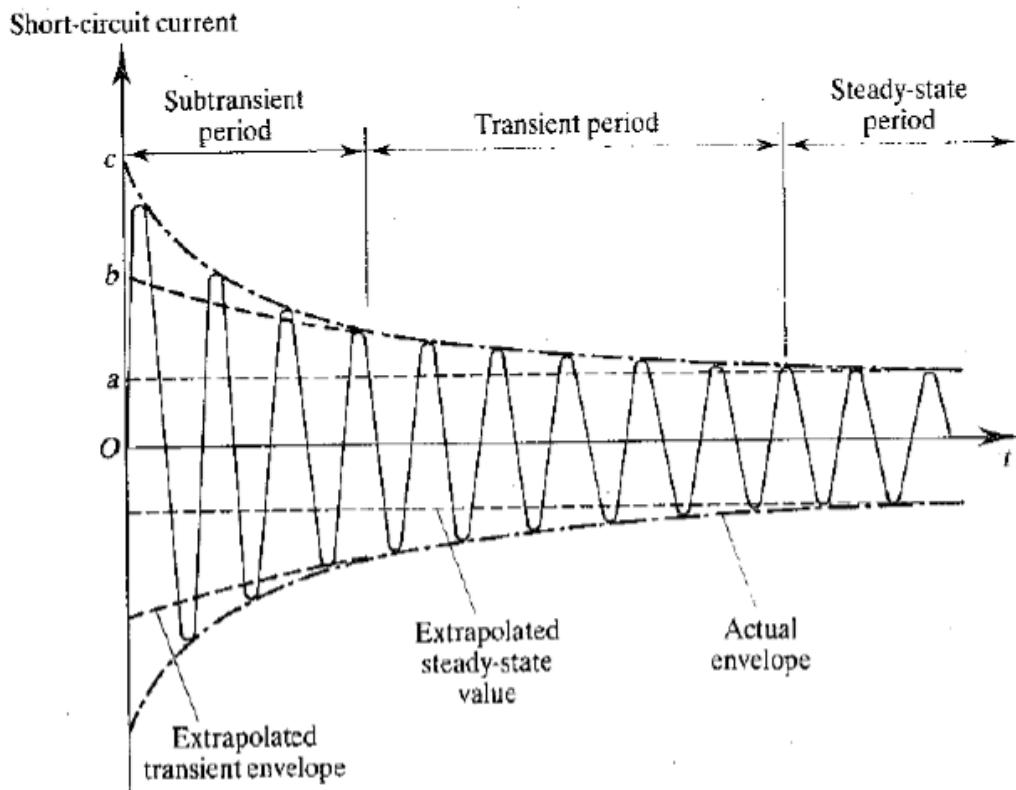


Figure 10.15: Synchronous machine short-circuit envelope.

### 10.4.3 Synchronous machine short-circuit

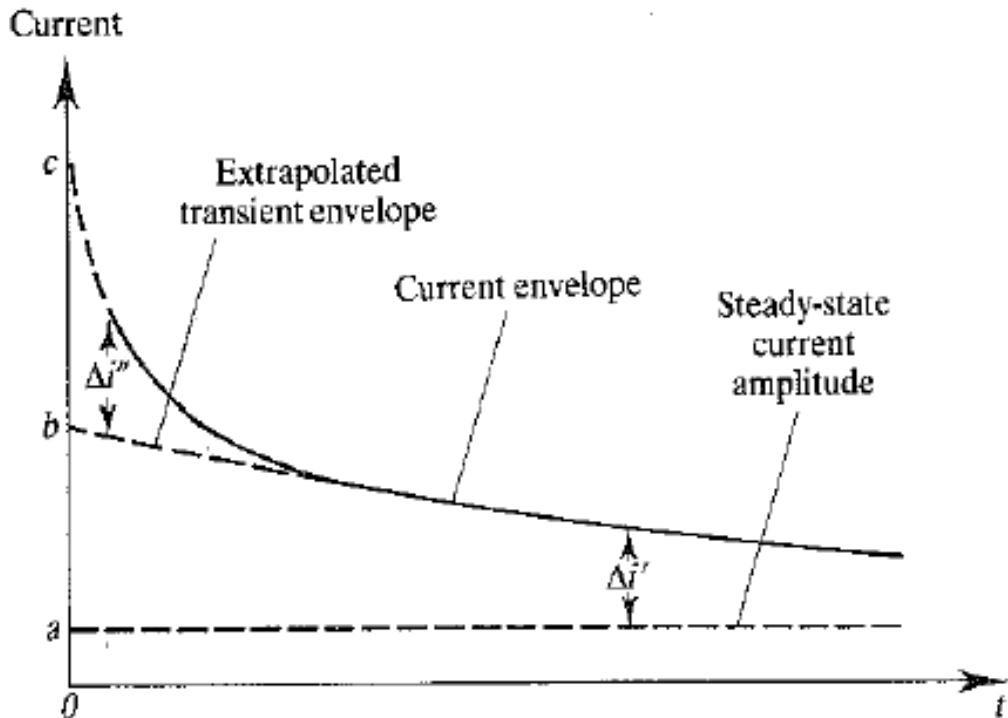


Figure 10.16: Synchronous machine short-circuit.

$$|I| = \frac{Oa}{\sqrt{2}} = \frac{|E_g|}{X_d} \quad (10.3)$$

$$|i'| = \frac{Ob}{\sqrt{2}} = \frac{|E_g|}{X'_d} \quad (10.4)$$

$$|i''| = \frac{Oc}{\sqrt{2}} = \frac{|E_g|}{X''_d} \quad (10.5)$$

$i''$ ,  $X''$  and  $T''$  are known as the subtransient current, subtransient reactance and subtransient time constant respectively.  $i'$ ,  $X'$  and  $T'$  are known as the transient current, transient reactance and transient time constant respectively.

#### 10.4.4 Balanced three-phase component of the short-circuit current

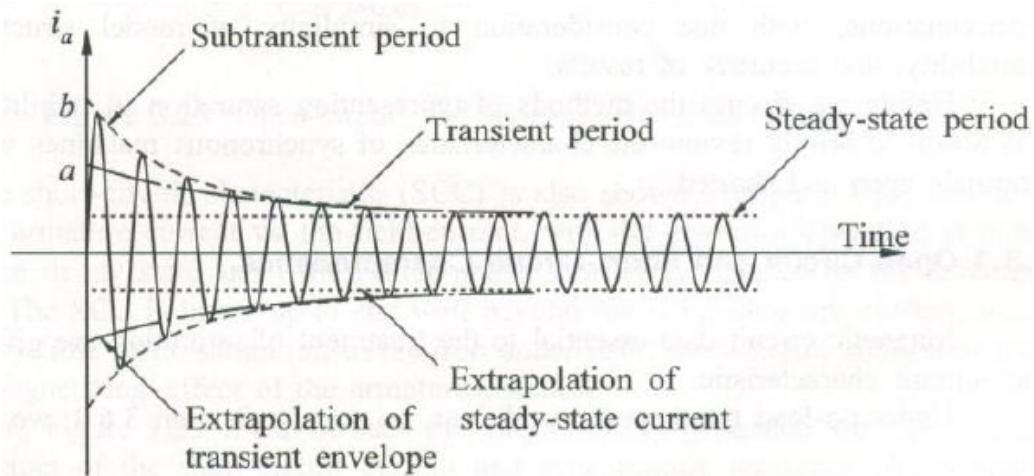


Figure 10.17: Balanced three-phase component of the short-circuit current.

$$I = \left( \frac{E}{X_d''} - \frac{E}{X_d'} \right) e^{-\frac{t}{T_d''}} + \left( \frac{E}{X_d'} - \frac{E}{X_d} \right) e^{-\frac{t}{T_d'}} + \frac{E}{X_d} \quad (10.6)$$

- \$I\$: rms value of the balanced AC component
- \$E\$: rms value of the phase voltage prior to the short-circuit
- \$t\$: time after the instant of the short-circuit
- \$X\_d'':\$ d-axis subtransient reactance
- \$X\_d'\$: d-axis transient reactance
- \$X\_d\$: d-axis synchronous reactance
- \$T\_d'':\$ d-axis short-circuit subtransient time constant
- \$T\_d'\$: d-axis short-circuit transient time constant

#### 10.4.5 Class example 1

An 11.8 kV busbar is fed from three synchronous generators having the following ratings and reactances:

- 20 MVA and \$X' = 0.08\$ pu
- 60 MVA and \$X' = 0.1\$ pu
- 20 MVA and \$X' = 0.09\$ pu

Calculate the fault current and MVA if a three-phase symmetrical fault occurs on the busbars using a 60 MVA base. Explain why the transient reactance is being used instead of the subtransient for this calculation.

The transient reactance of the 20 MVA machine has a base of \$60/20 \times 0.08 = 0.24\$ and \$60/20 \times 0.09 = 0.27\$. Hence as they are all in parallel (to a 60 MVA base):

$$X_{eq} = \frac{1}{\frac{1}{0.24} + \frac{1}{0.27} + \frac{1}{0.1}} = 0.056 \text{ pu} \quad (10.7)$$

Therefore fault MVA:

$$\frac{60}{0.056} = 1071 \text{ MVA} \quad (10.8)$$

Fault current:

$$\frac{1071 \times 106}{3^{0.5} \times 11800} = 52402 \text{ A} \quad (10.9)$$

#### 10.4.6 Worked example

The per-unit reactances of a synchronous generator are  $X_d = 1$ ,  $X'_d = 0.35$  and  $X''_d = 0.25$ . The generator supplier a 1 per-unit load at 0.8 power factor lagging. Calculate the voltages behind the synchronous, transient and subtransient reactances. Use  $V_t = 1 + j0$  as base.

Since: (ignore  $R_a$  as it is usually small)

$$E_g = V_t + jI_L X_d \quad (10.10)$$

Then similarly:

$$E'_g = V_t + jI_L X'_d \quad (10.11)$$

$$E''_g = V_t + jI_L X''_d \quad (10.12)$$

Therefore:

$$E_g = (1 + j0) + j1.0(0.8 - j0.6) = 1.79 \text{ pu} \quad (10.13)$$

$$E'_g = (1 + j0) + j0.35(0.8 - j0.6) = 1.24 \text{ pu} \quad (10.14)$$

$$E''_g = (1 + j0) + j0.25(0.8 - j0.6) = 1.17 \text{ pu} \quad (10.15)$$

#### 10.4.7 Class example 2

The system shown below is initially on no-load. Calculate the subtransient fault current that results when a three-phase fault occurs given the transformer voltage on the high side is 66 kV. Use base of 69 kV and 75 MVA. (Transformer = 0.1 pu at these values).

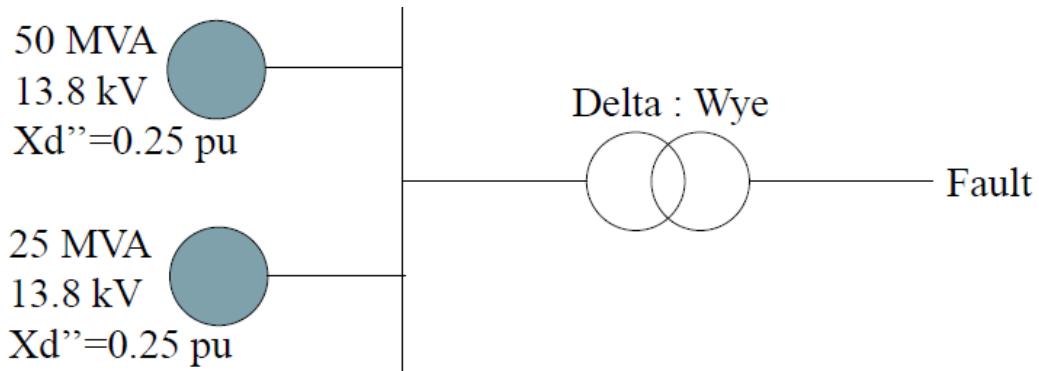


Figure 10.18: Class example 2 SLD.

Let the base voltage (on high side) be 69 kV, 75 MVA. For G1:

$$X''_d = 0.25 \times \frac{75000}{50000} = 0.375 \text{ pu} \quad (10.16)$$

$$E_{g1} = \frac{66}{69} = 0.957 \text{ pu} \quad (10.17)$$

For G2:

$$X''_d = 0.25 \times \frac{75000}{25000} = 0.750 \text{ pu} \quad (10.18)$$

$$E_{g2} = 0.957 \text{ pu} \quad (10.19)$$

$$X'' = \frac{0.375 \times 0.75}{0.375 + 0.75} = 0.25 \quad (10.20)$$

$$I'' = \frac{0.957}{j0.25 + j0.1} = -j2.735 \text{ pu} \quad (10.21)$$

## 10.5 Summary

- The synchronous generator is commonly used in power systems as a stand-alone and parallel operation
- The synchronous machine is controlled by a generator and AVR. This permits frequency and voltage to be controlled
- Protection of the generator is important for significant damage can occur if operated incorrectly.
- Transient behaviour is more significant in small networks. Fault behaviour gives rise to high fault levels and currents

# **Chapter 11**

## **Electric / Hybrid RV Propulsion**

### **11.1 Introduction**

#### **11.1.1 Reasons forcing change**

- Global warming e.g. CO<sub>2</sub> emissions
- Health e.g. NO<sub>x</sub> emissions
- Efficiency e.g. low efficiency of vehicles presently
- Technology advances e.g. batteries
- Increasing demand e.g. in developing countries
- Custom demand e.g. environmental concerns

### 11.1.2 Global CO<sub>2</sub> emissions

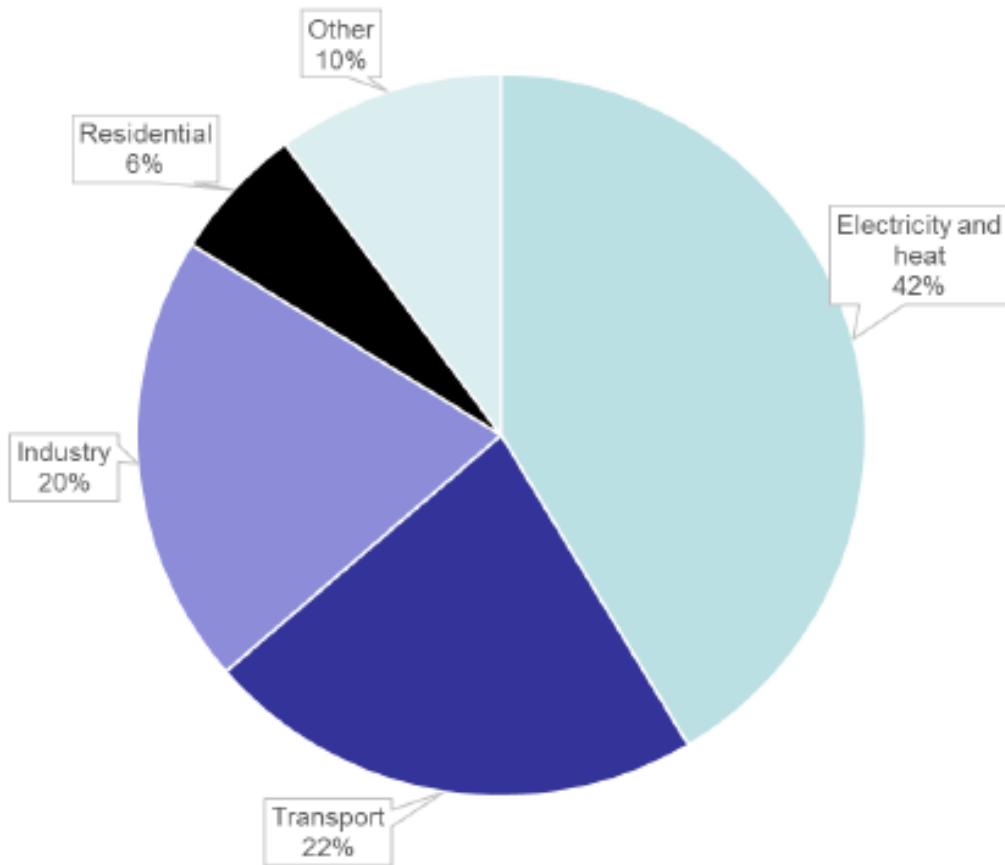


Figure 11.1: Global CO<sub>2</sub> emissions.

Transport is a significant contribution to CO<sub>2</sub> global emissions. It is a bigger emitter than electricity generation in many countries.

### 11.1.3 Typical driving energy losses (city use)

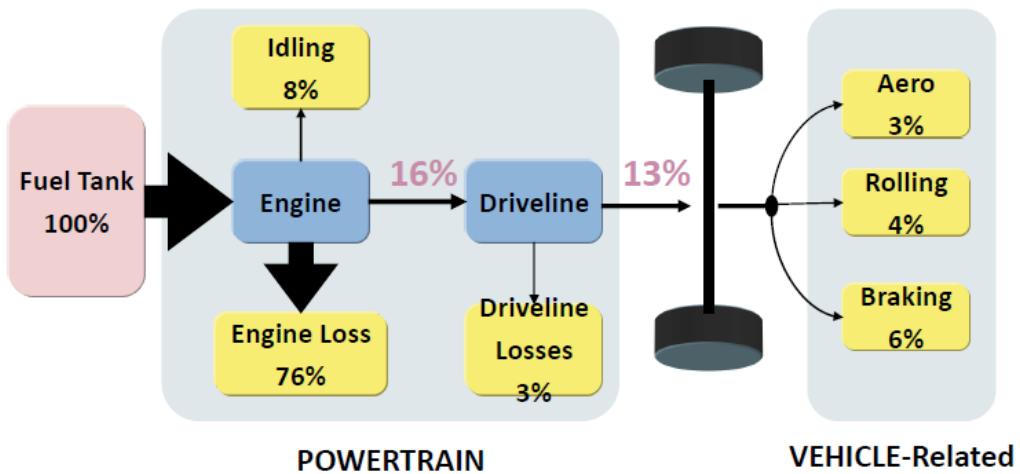
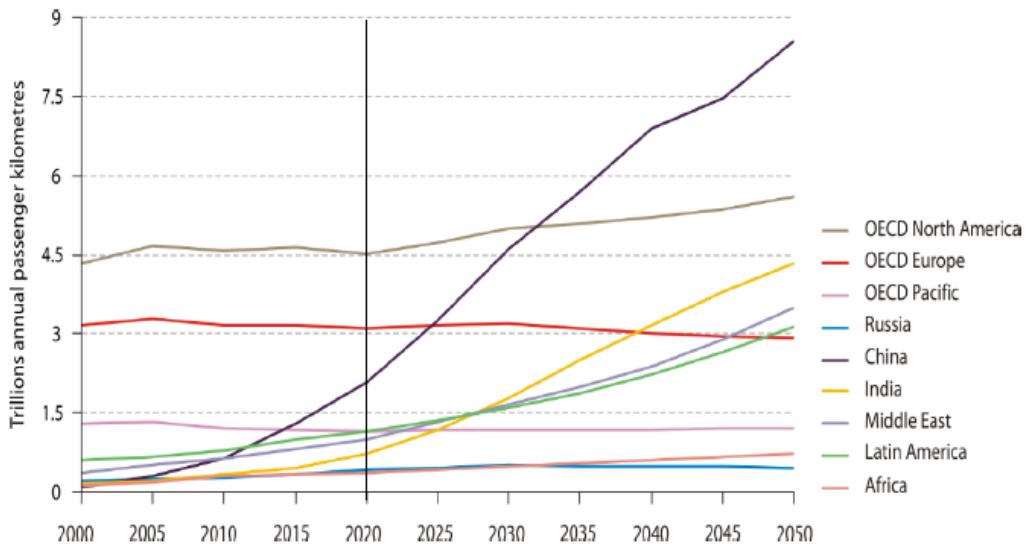


Figure 11.2: Typical driving energy losses (city use).

The efficiency of ICE engines in road vehicles is low.

### 11.1.4 Transport sector growth prediction



Source: <http://www.iea.org/publications/freepublications/publication/>

Figure 11.3: Transport sector growth prediction.

Transport growth is mainly in developing countries whereas developed countries are expected to remain unchanged.

### 11.1.5 Gasoline: the (almost) perfect fuel

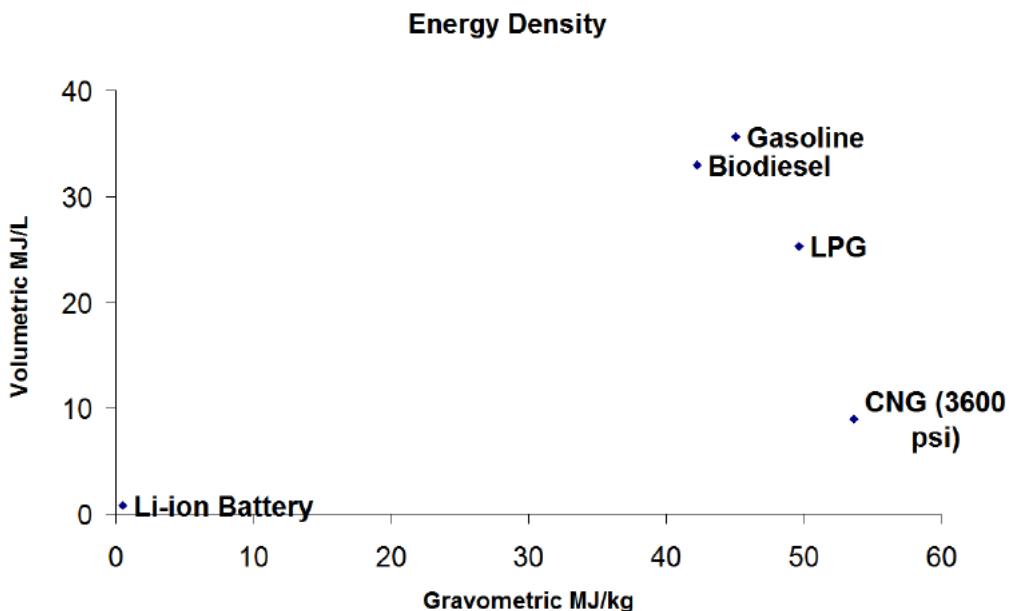


Figure 11.4: Transport sector growth prediction.

### 11.1.6 Towards zero emissions

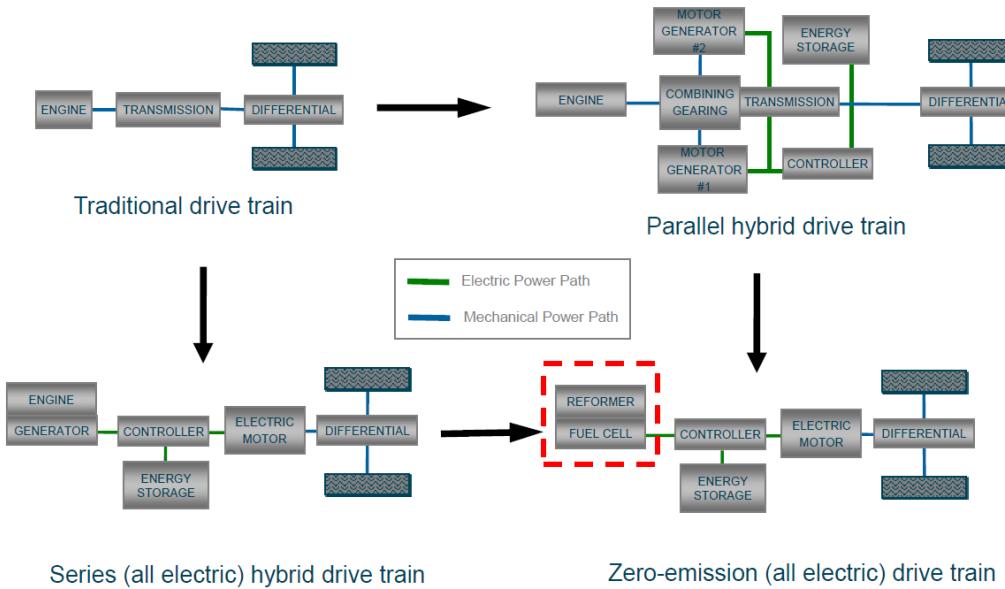


Figure 11.5: Drive trains for various vehicle types.