0.1 Introduction

0.1.1 What is the module about?

Dynamics: the study of forces and the resultant motion.

In this module we will study the oscillatory forces and the resulted motion of bodies, in other words "Mechanical Vibration."

- Harmful vibrations:
  - Vibrations can cause:
    - Resonance
    - Flutter
    - Fatigue
  - It may cause discomfort and even be harmful to the human.
- Good vibrations:
  - Hearing
  - Loudspeakers
  - Musical instruments
  - Electric toothbrush
  - Clocks
  - Material handling
  - Sifting

0.1.2 How to deal?

Analysis:

- Mathematical modelling
- Derivation of governing equations
- Solution of the governing equations
- Interpretation of results

Measurements:

- Appropriate setup
- Interpretation of measurements
- Updating mathematical models

How would this module enable you to analyse dynamical systems?

- Mathematical modelling, week 1 also 2-11.
- Single degree of freedom systems
  - Free vibration
    - Undamped, weeks 1 and 2
    - Damped, week 3
  - Forced vibration of single degree of freedom system
    - Harmonic excitation, weeks 4 and 5
    - Arbitrary excitation, week 7
- Two degree of freedom system, week 8
- Multi degree of freedom systems, week 9
- The use of computer aided engineering to create a refined motor vehicle, invited talk, week 10
- Introduction to continuous systems, week 11
- Vibration measurements
- Vibration control

0.2 Fundamentals

0.2.1 Physical elements of vibrations

Vibration: A vibration or oscillation is a periodic motion, i.e. it repeats itself in all its particular after a certain interval of time.

In general in any mechanical oscillatory system there are:

- A **mass** that can store kinetic energy (accelerates when a load is applied upon)
- A **spring** that can store potential energy (constant displacement due to a constant force)
- A **damper** through which energy dissipates

What are the equivalent electrical elements?

- Force (F) - Voltage (V)
- Mass (M) - Inductance (L)
- Damping (B) - Resistance (R)
- Spring constant (K) - Reciprocal of Capacitance ($\frac{1}{c}$)

- Displacement (x) - Charge (q)
- Velocity (v) - Current (i)

In other words vibration is a result of the interaction of two forces. One a function of displacement (spring):

$$f)k = -kx(t) \tag{1}$$

One a function of acceleration (mass):

$$f_m = m\ddot{x}(t) \tag{2}$$

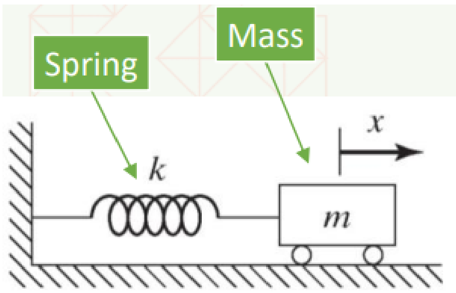

Figure 1: Spring-mass system.

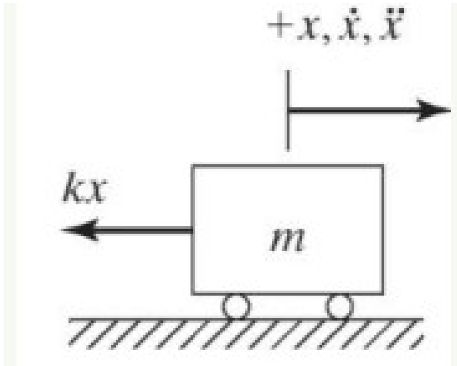

Figure 2: free body diagram.

Equation of motion:

$$\sum F = m\ddot{x}(t) \tag{3}$$
$$m\ddot{x}(t) = -kx(t) \tag{4}$$

Equation of motion:

$$m\ddot{x}(t) + kx(t) = 0 \tag{5}$$

Solution:

$$x(t) = A\sin(\omega_n t + \phi) \tag{6}$$

0.2.2 Degree of freedom

The minimum number of coordinates that are required to define the position of a system is called degree of freedom.

- Single degree of freedom (SDOF):
  - Only one coordinate is required to fully define the state of the system

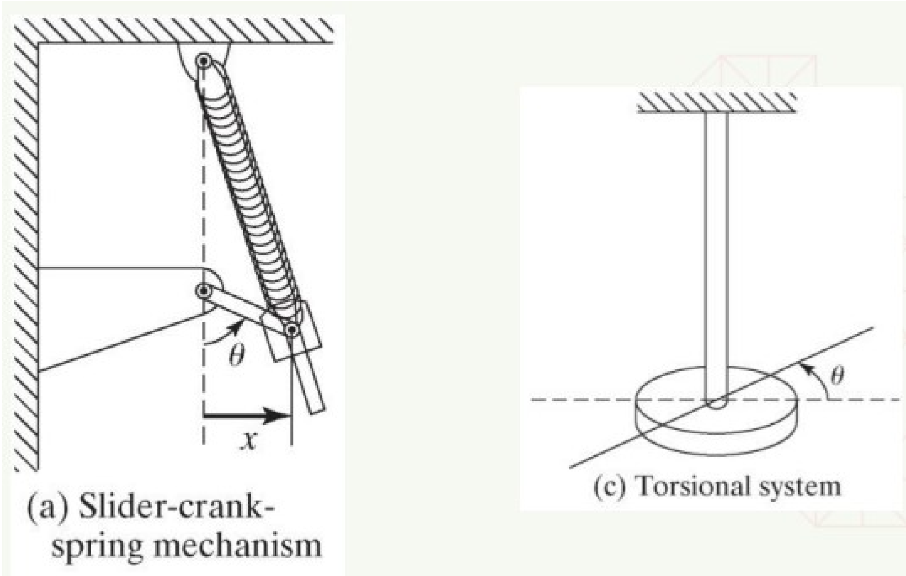

Figure 3: SDOF systems.

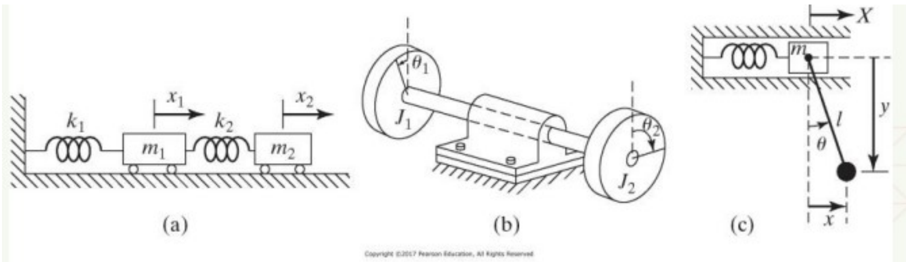

Figure 4: Two degree of freedom systems.

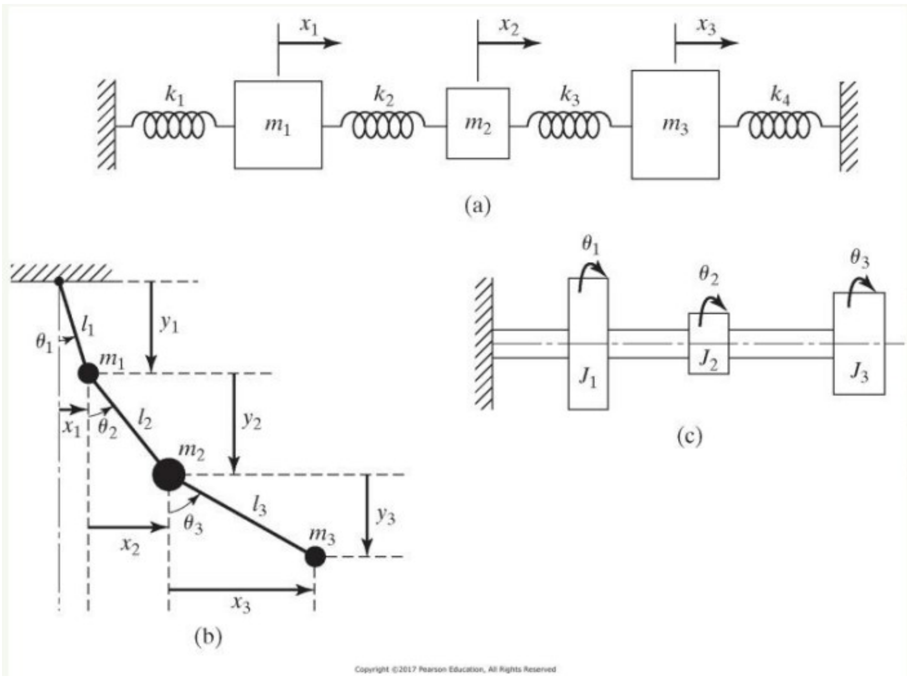

Figure 5: Three degree of freedom systems.

0.2.3 Continuous systems

Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom.

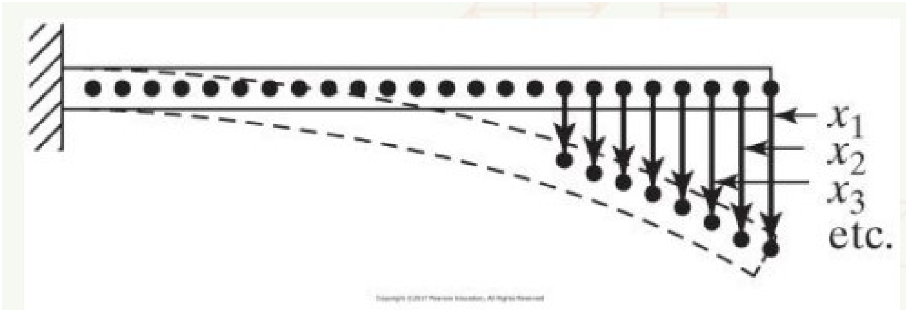

Figure 6: Continuous degree of freedom.

Discrete or lumped parameter systems: A system with finite number of degree of freedom. Continuous or distributed systems: an infinite number of degree of freedom.

0.2.4 Analysis procedure

Mathematical modelling

We need a mathematical model to obtain a solution for the vibrational problem. The model is a compromise between simplicity and accuracy. We make some assumption and our model is valid with the limitation of those assumptions only.

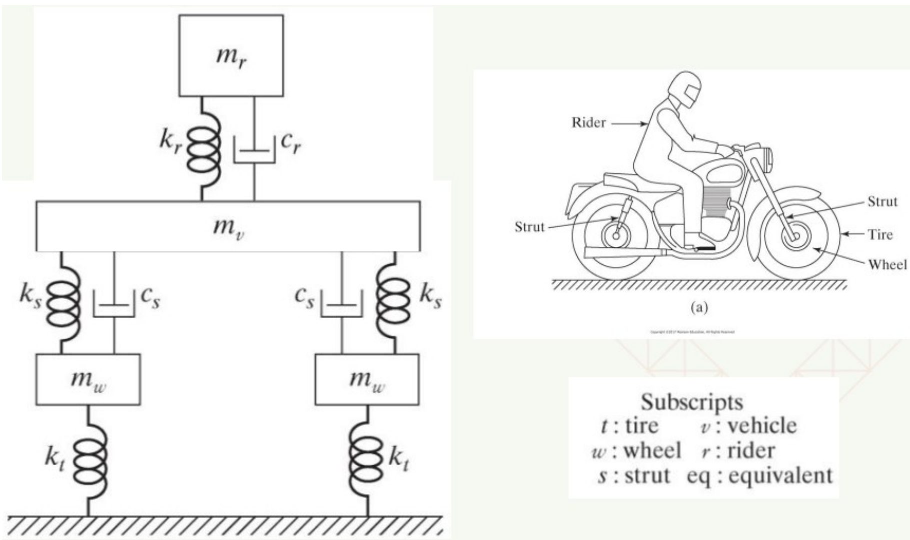

Figure 7: Model of rider-motorbike-wheel system.

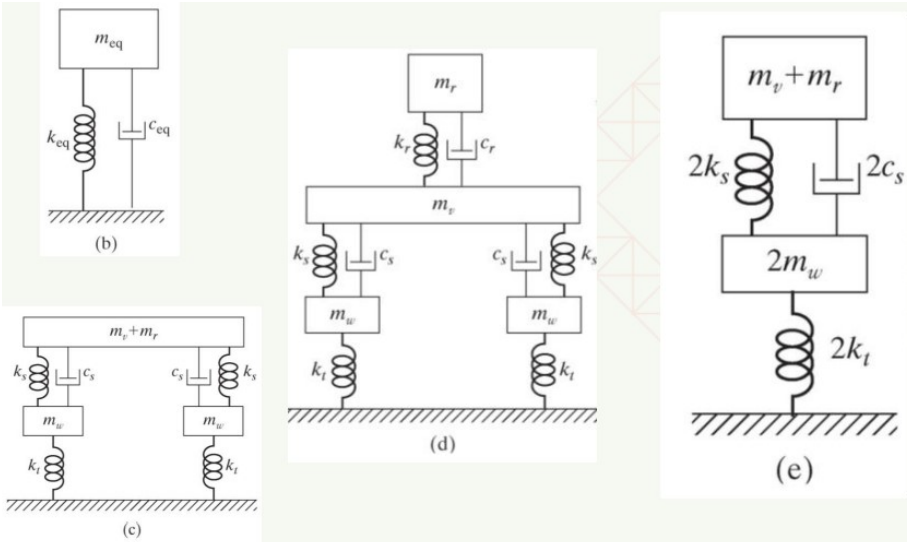

Figure 8: Various models of rider-motorbike-wheel system.

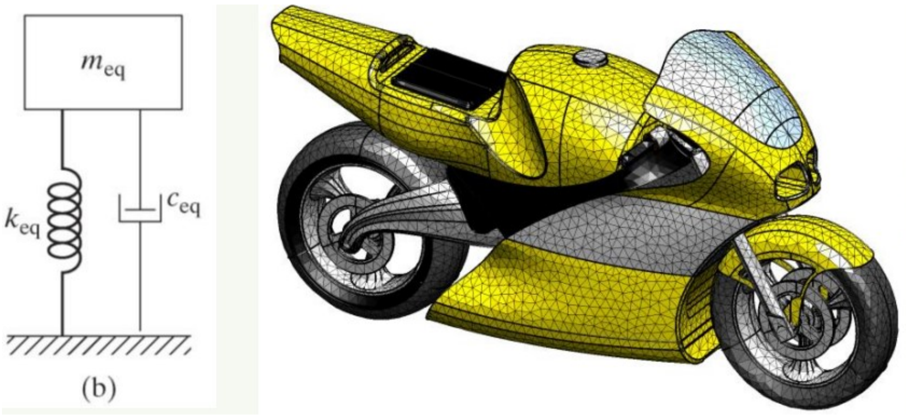

Figure 9: Complex model of rider-motorbike-wheel system.

Derivation of governing equations

Apply principle of dynamics to obtain the equations:

- Newton's second law of motion
- d'Alembert's principle
- The principle of conservation of energy

Usually in the form of a set of ordinary differential equations.

Solution of governing equations

- Standard method of solving differential equations
- Laplace transformation method
- Matrix methods
- Numerical methods

Interpretation of results

- Have a clear view of the purpose of the analysis
- Pay attention to the assumption made to obtain the results.

0.2.5 Classification of vibration

Figure 10: Classification of vibrations.

0.2.6 Example: simple pendulum

Deriving the equation of motion. We have some assumptions:

- The mass of the rod is ignored
- Friction in the hinges is ignored
- The motion remains in a plane

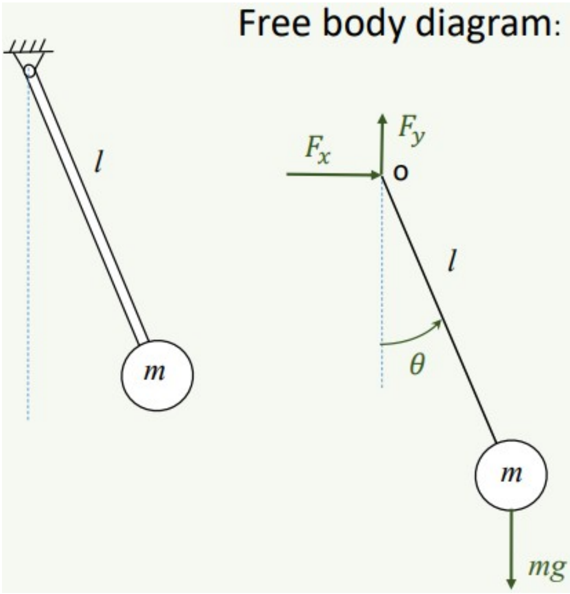

Figure 11: Free body diagram of simple pendulum system.

Euler's second law:

$$\sum M_o = J\alpha \tag{7}$$

$$J\alpha(t) = -mgl\sin\theta(t) \tag{8}$$

$$ml^2\ddot{\theta}(t) + mgl\sin\theta(t) = 0 \tag{9}$$

Linearisation:

$$\ddot{\theta}(t) + \frac{g}{l}\theta(t) = 0 \tag{10}$$

0.2.7 Simple harmonic motion

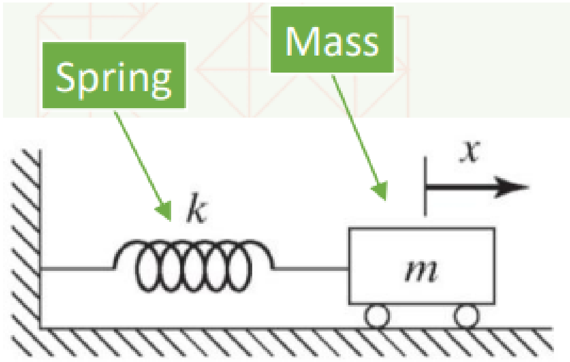

Figure 12: Spring-mass system.

$$x(t) = A\sin(\omega_n t + \phi) \tag{11}$$

$$\dot{x} = A\omega_n\cos(\omega_n t + \phi) \tag{12}$$

$$\ddot{x} = -A\omega_n^2\sin(\omega_n t + \phi) \tag{13}$$

Substitute in equation of motion:

$$-mA\omega_n^2\sin(\omega_n t + \phi) + kA\sin(\omega_n t + \phi) = 0 \tag{14}$$

$$\omega_n^2 = \frac{k}{n} \text{ or } \omega_n = \sqrt{\frac{k}{m}} \tag{15}$$

where 15 is the natural frequency.

0.2.8 Amplitude and phase

$$x(t) = A\sin(\omega_n t + \phi) \tag{16}$$

Initial conditions: initial displacement x_0 and initial velocity v_0 of the mass.

$$x_0 = x(t=0) = A\sin\phi \tag{17}$$

$$v_0 = \dot{x}(t=0) = A\omega_n\cos(\omega_n x_0 + \phi) = A\omega_n\cos\phi \tag{18}$$

$$\frac{v_0}{\omega_n} = A\cos\phi \tag{19}$$

$$x_0^2 + \frac{v_0^2}{\omega_n^2} = A^2\sin^2\phi + A^2\cos^2\phi = A^2 \tag{20}$$

$$\frac{\sin\phi}{\cos\phi} = \frac{x_0}{\frac{v_0}{\omega_n}} \tag{21}$$

Thus:

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} \text{ and } \phi = \arctan \frac{\omega_n x_0}{v_0} \tag{22}$$

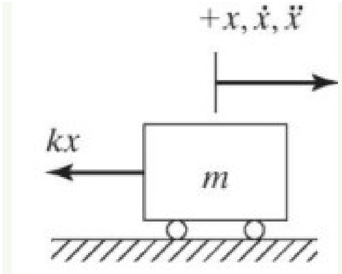

Figure 13: Free body diagram of spring-mass system.

$$m = 2\,\mathrm{kg} \text{ and } k = 200\,\mathrm{N\,m^{-1}} \tag{23}$$

$$1.\ x_0 = -2\,\mathrm{mm} \text{ and } v_0 = 10\,\mathrm{mm\,s^{-1}} \tag{24}$$

$$2.\ x_0 = 2\,\mathrm{mm} \text{ and } v_0 = -10\,\mathrm{mm\,s^{-1}} \tag{25}$$

Solution:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 200\,\mathrm{rad\,s^{-1}} \tag{26}$$

$$A = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} = \frac{\sqrt{10^2 \cdot 2^2 + 10^2}}{10} = 2.2\,\mathrm{mm} \tag{27}$$

$$1.\ \phi = \arctan\left(\frac{\omega_n x_0}{v_0}\right) = \arctan\left(\frac{10 \cdot -2}{10}\right) = -1.107\,\mathrm{rad} \text{ or } -63.4^\circ \tag{28}$$

$$2.\ \phi = \arctan\left(\frac{\omega_n x_0}{v_0}\right) = \arctan\left(\frac{10 \cdot -2}{10}\right) = (-1.107 + \pi)\,\mathrm{rad} \text{ or } 116.6^\circ \tag{29}$$

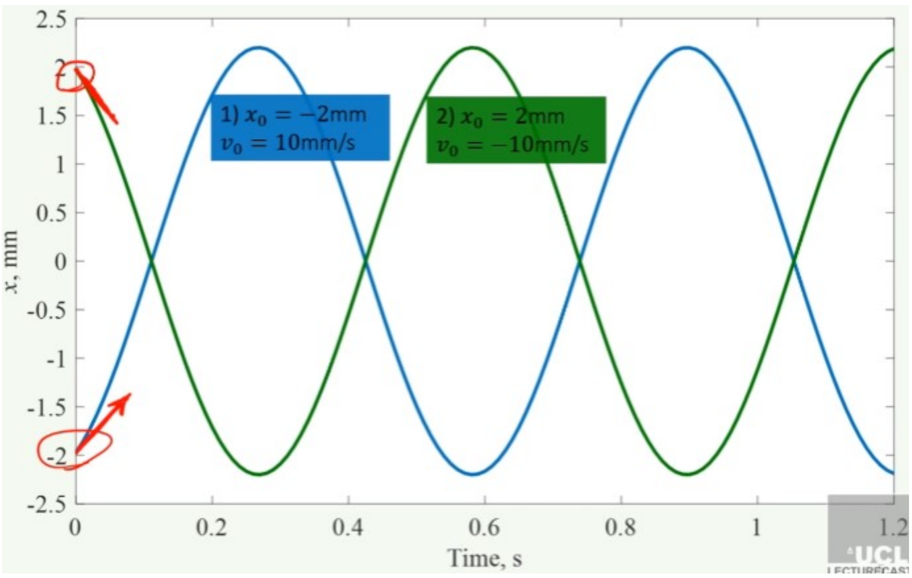

Figure 14: Plots of SHM.

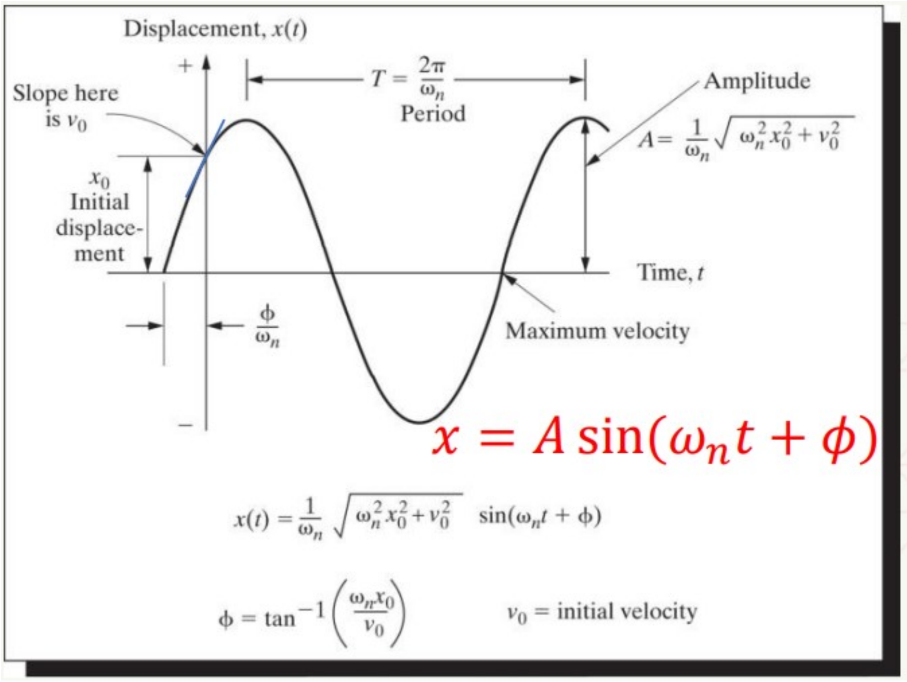

Figure 15: Analysis of SHM plots.

0.2.9 Springs

Hooke's law (linear spring):

$$F_k = kx \tag{30}$$

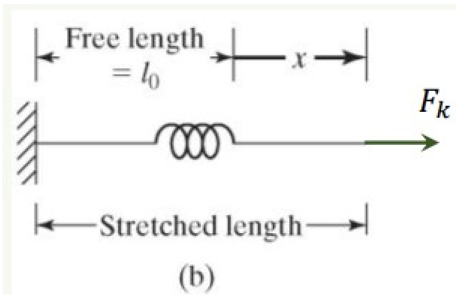

Figure 16: Spring system.

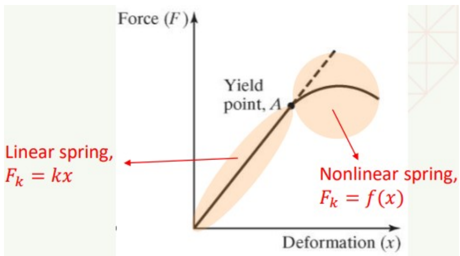

Figure 17: F vs x of spring.

0.2.10 Modelling springs

Springs in parallel:

$$W = k_1\delta_{st} + k_2\delta_{st} \tag{31}$$
$$= (k_1 + k_2)\,\delta_{st} \tag{32}$$
$$W = k_{eq}\delta_{st} \tag{33}$$
$$k_{eq} = (k_1 + k_2) \tag{34}$$

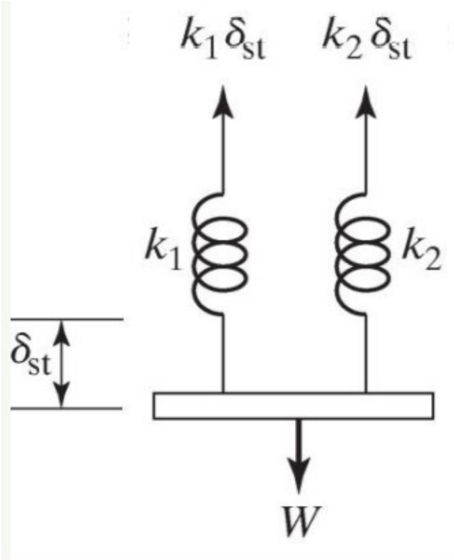

Figure 18: Parallel springs.

Springs in series:

$$\delta_{st} = \delta_1 + \delta_2 \tag{35}$$
$$w = k_1\delta_1 \tag{36}$$
$$w = k_2\delta_2 \tag{37}$$

or

$$\delta_{st} = \frac{w}{k_1} + \frac{w}{k_2} \tag{38}$$
$$\delta_{st} = \frac{w}{k_{eq}} \tag{39}$$
$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \tag{40}$$

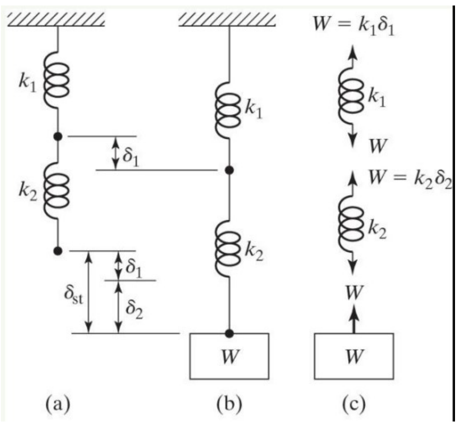

Figure 19: Series springs.

A cantilever beam with a mass at the free end:

$$\delta_{st} = \frac{Wl^3}{3EI} \tag{41}$$

$$k_{eq} = \frac{W}{\delta_{st}} \tag{42}$$

$$k_{eq} = \frac{3EI}{l^3} \tag{43}$$

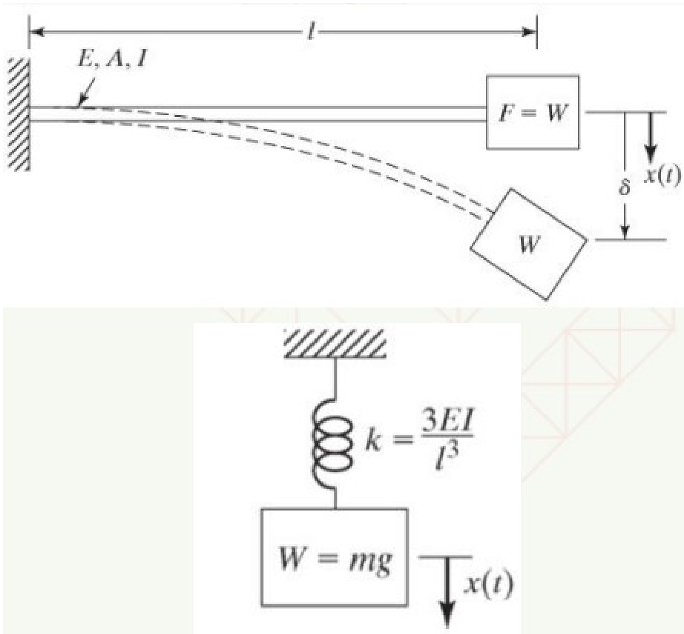

Figure 20: Cantilever beam with mass at free end.

0.2.11 Longitudinal motion of a bar

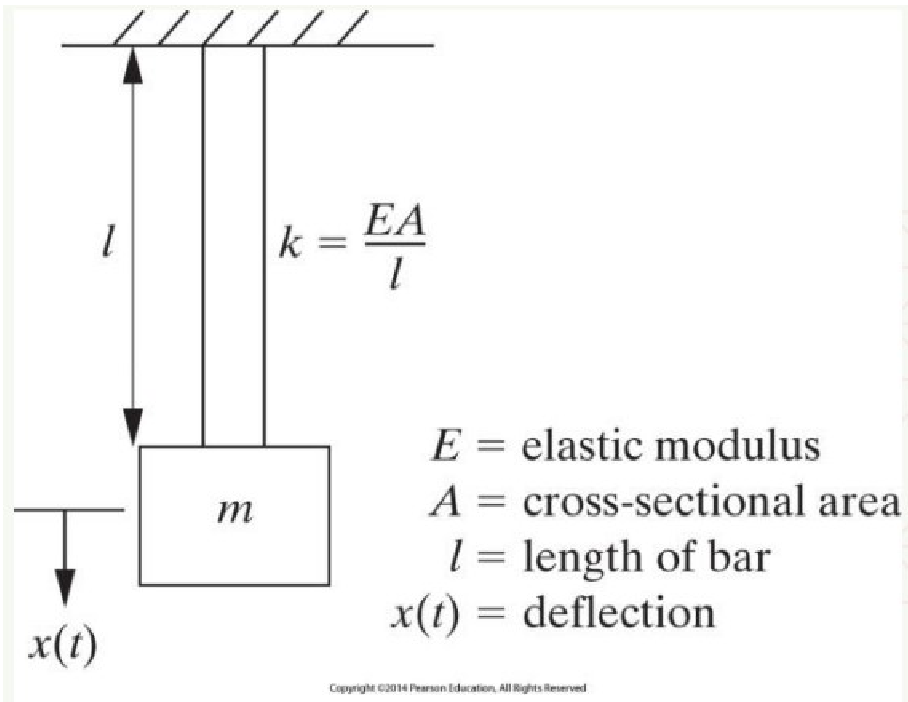

Figure 21: Longitudinal motion of a bar.

0.2.12 Torsional rotation of a shaft

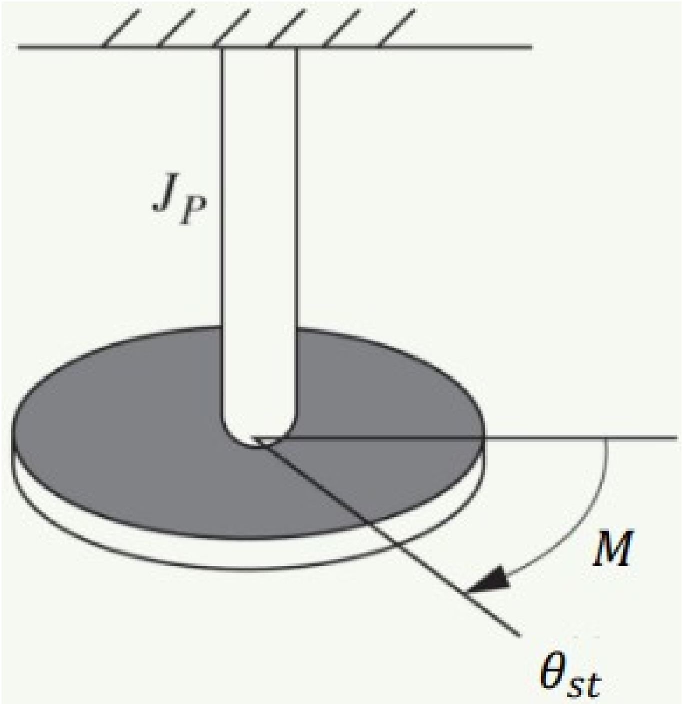

Figure 22: Torsional disc system.

$$M = k_{eq}\theta_{st} \tag{44}$$

$$k_{eq} = \frac{GJ_p}{l} \tag{45}$$

- G: Shear modulus of rigidity
- J_p: Polar second moment of area
- l: Length of the shaft