

MECH0071 Topic Notes

UCL

HD

October 7, 2022

Contents

1	Introduction	5
1.1	Team	5
1.2	Course Aim	5
1.3	Student learning outcomes	5
1.4	Assessment	6
1.5	Textbooks	6
1.6	Softwares	6
2	The Electrical Line Diagram	7
2.1	Overview of electrical power systems	7
2.1.1	Basic electrical power system	7
2.1.2	What is an electrical power system?	7
2.2	Components of electrical power systems	7
2.2.1	Sources of electrical power include	7
2.2.2	Sources of DC electrical power	8
2.2.3	Generators . . . single and multiphase AC	8
2.2.4	Transmission systems	8
2.2.5	Distribution systems	9
2.2.6	Loads	9
2.3	Representation by the electrical line diagram	10
2.3.1	Electrical system representation	10

2.3.2	Questions for you?	11
2.3.3	The ‘Single Line Diagram’ (SLD)	12
2.3.4	Some common features of SLDs	14
2.3.5	Limitations of the electrical line diagram	14
3	Developing Impedance Diagram	15
3.1	Three Phase Power	15
3.1.1	Three-phase alternating voltages	15
3.1.2	Three-phase emfs (or terminal voltages) can be expressed mathematically	15
3.1.3	Three-phase, six-wire connection	16
3.1.4	Three-phase current	16
3.1.5	Three-phase alternating current	16
3.1.6	Connecting Three-Phases	17
3.1.7	Star and delta connections	17
3.1.8	Phase and line voltages	18
3.1.9	Relationships between star and delta	19
3.1.10	Single-phase impedance triangle	19
3.1.11	Single-phase power triangle	20
3.1.12	Three-phase power	20
3.1.13	Student Activity	20
3.2	Per Unit (PU) System	21
3.2.1	Electrical line diagram to Impedance diagram	21
3.2.2	Simple equivalent impedances	21
3.2.3	How manufacturers of electrical equipment specify ratings	21
3.2.4	The per unit system	22
3.2.5	Values in per unit system	22
3.2.6	Three-Phase system PU conversion	23

3.2.7	Example PU system conversion	25
3.2.8	Reactance diagram	28
3.2.9	Impedance and reactance diagrams	28
3.3	Summary	29

Chapter 1

Introduction

1.1 Team

- Professor Richard Bucknall
- Mr Chris Greenough
- Mr Konrad Yearwood - Helpdesk email: k.yearwood@ucl.ac.uk

1.2 Course Aim

The aim of this course is to provide students with detailed knowledge and understanding of the design, performance and analysis of electrical power systems.

Students will increase their knowledge and understanding through face-to-face/synchronous lectures, asynchronous (including tutorials) tasks and a computer simulation workshop and demonstrate their learning through summative coursework and an examination.

1.3 Student learning outcomes

- Appreciate the components that make up electrical power systems and understand the similarities and differences between large, medium and small scale power systems.
- Develop skills needed to be able to design electrical power systems including analytical and computer based methods.
- Understand the behaviour of steady-state, transient and faulted networks and appreciate how such behaviour influences design.

- Understand the benefits of electrical propulsion for different vehicle types be able to undertake designs.
- Appreciate future developments and applications in electrical power and electrical propulsion systems.

1.4 Assessment

- Coursework - summative assessment exercise based around computer simulations
- Examination - two hour examination in January

1.5 Textbooks

Kirtley, James. *Electric Power Principles: Sources, Conversion, Distribution and Use*. Wiley. 2020. ISBN: 9781119585305.t

1.6 Softwares

- PSCAD

Chapter 2

The Electrical Line Diagram

2.1 Overview of electrical power systems

2.1.1 Basic electrical power system

Most electrical power systems contain:

- Generators to produce electrical energy (often coming from another store of energy e.g. chemical - oil, gas, coal)
- A means to transmit and distribute the electrical energy
- Loads that use the electrical energy for some purpose

2.1.2 What is an electrical power system?

An **electric power system** is a network or grid of electrical components that supply, transfer and use electric energy. Electrical power systems can be a:

- Large grids covering a wide area e.g. a continent
- Medium grid covering a large area e.g. a country
- Small network covering a small area e.g. a ship

2.2 Components of electrical power systems

2.2.1 Sources of electrical power include

Generators (rotating types AC and DC):

- Large AC generators e.g. 25 kV three-phase voltages
- Medium AC generators e.g. 440 V three-phase voltages
- Small AC generators e.g. e.g. single-phase 220 V voltages

Fuel cells:

- DC output voltage (typically 720 V DC)

Batteries (electro-chemical):

- DC output voltage (usually multiples of 12 V)

Photo-voltaic (solar) cells:

- DC output currents (usually mA/cell)

2.2.2 Sources of DC electrical power ...

A fuel cell in a car. Photovoltaics used in a solar farm. Battery energy store. DC systems are increasing in their popularity due to wider use of batteries, solar cells and fuel cells in grids and electrical propulsion.

2.2.3 Generators ... single and multiphase AC

AC generators:

- Large AC generators e.g. 25 kV 3 phase
- Medium AC generators e.g. 11 kV or 440 V 3 phase
- Small generators e.g. 220 V single-phase voltage

2.2.4 Transmission systems

HVAC often three-wire and three-phase e.g. 440 kV, 275 kV and 132 kV.

HVDC often two-wire and bipolar e.g. +/- 330 kV.

2.2.5 Distribution systems

AC distribution:

- 11 kV, 440 V three-phase
- 25 kV single-phase (rail)
- 240 V single-phase

DC distribution:

- 750 V (rail)
- 110 V (emergency lighting)

2.2.6 Loads

Three-phase loads:

- Induction motors to drive pumps, fans and compressors
- Propulsion drives

Single-phase loads:

- Lighting
- Heating
- Appliances e.g. domestic, electronics, small pumps

DC loads:

- DC motors
- Lighting and heating
- Battery charging

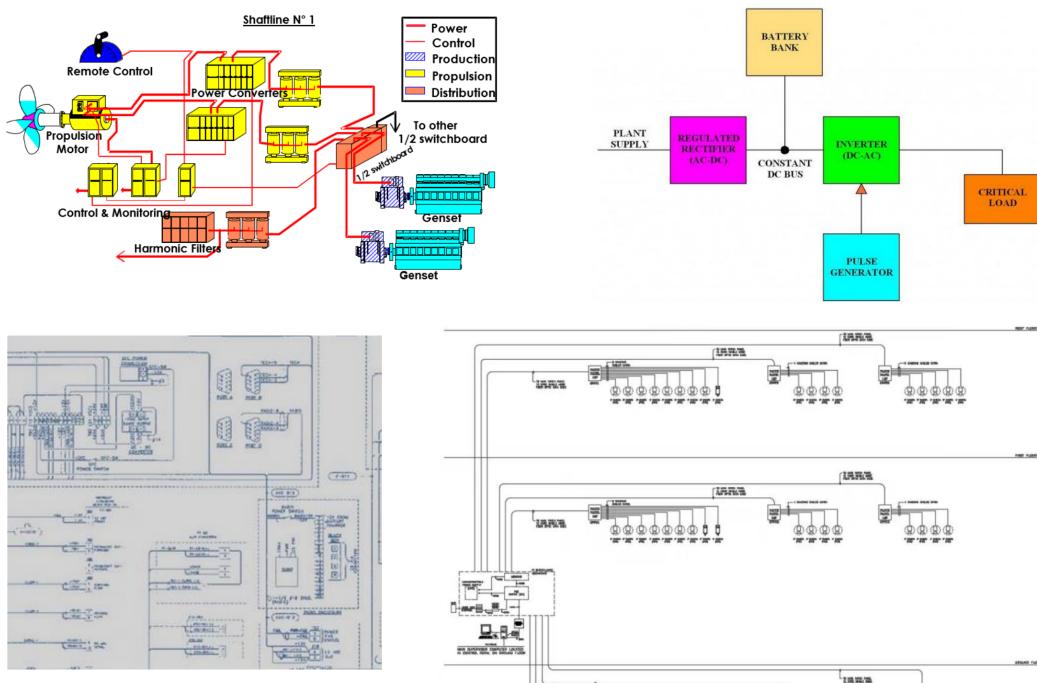
2.3 Representation by the electrical line diagram

2.3.1 Electrical system representation

Electrical systems are commonly represented as one of the following:

- Pictorial diagram
- Block diagram
- Wiring diagram
- Single line diagram
- Riser diagram
- Electrical floor plan
- Layout diagram

Of these the most useful to the *electrical power engineer* is the **Single line diagram**.



18

Figure 2.1: Some types of electrical system representation.

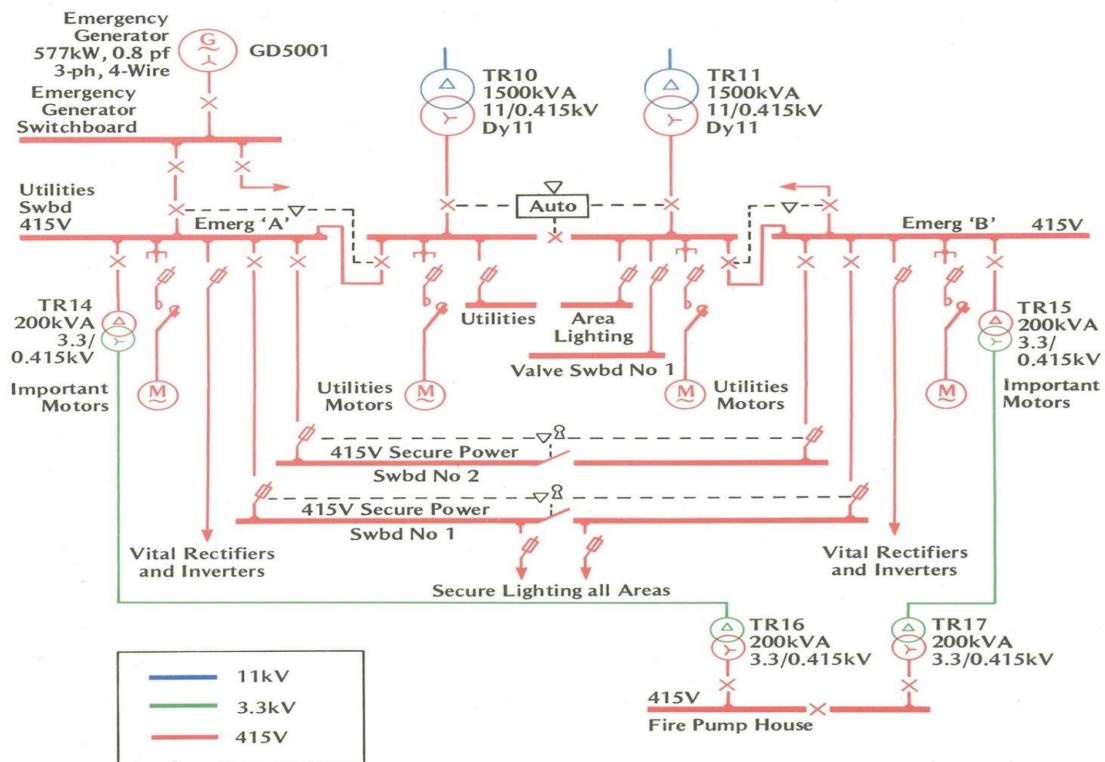


Figure 2.2: Example of a ‘Single Line diagram’.

2.3.2 Questions for you?

1. The number of separate switchboards shown? 14 (each thick line is a separate switchboard)
2. Maximum current that will flow through the supply transformers? $I = \frac{kVA}{kV \times \sqrt{3}}$, (root 3 due to 3-phase)
3. How many different electrical sources supply the fire pump house? All three supplies can be connected to the fire pump house.

Equipment	Single Line Diagram Representation		
AC Machine (Motor and Generator)			
DC Machine (Motor or Generator)			
Transmission Lines and Cables (With circuit breaker)			
Switchboards (with busbar, circuit breakers and feeders)			
Power Conversion (Rectifier AC-DC and Inverter DC-AC)			
Transformer (Two winding transformer, Three winding transformer)			
Star, Delta and Zig-Zag connections.			
Earth			
Passive Components (Resistance, Capacitance and inductance)			

Figure 2.3: Symbols.

2.3.3 The ‘Single Line Diagram’ (SLD)

The ‘Single Line Diagram’ (also known as the ‘One Line Diagram’) represents an electrical power system using single lines regardless of number of cables being used. It can be used to represent:

- Any type of electrical power system: DC, single-phase, three-phase or a mixed voltage electrical system.
- The interconnections between different electrical equipment including generators, switchboards, electrical distribution centres and loads.
- The types of electrical equipment and their main characteristics e.g. ratings of equipment such as voltage, power, power factor, and impedance.

- Emergency features such as reversionary modes, cross-connections and emergency generators. Sometimes these can be represented as single ‘dotted line’ connections rather than the usual solid single line.
- Other details such as ‘earthing arrangements, arrangements of star/delta connections in three-phase systems and any autonomous operating systems such as circuit breakers.

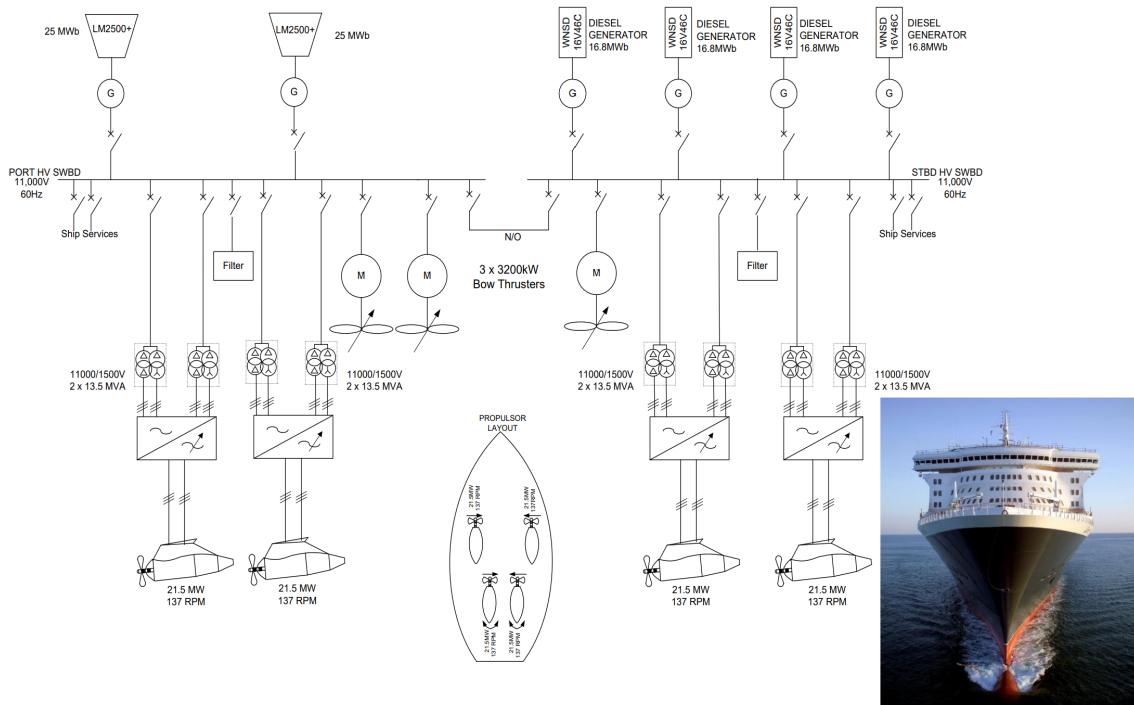
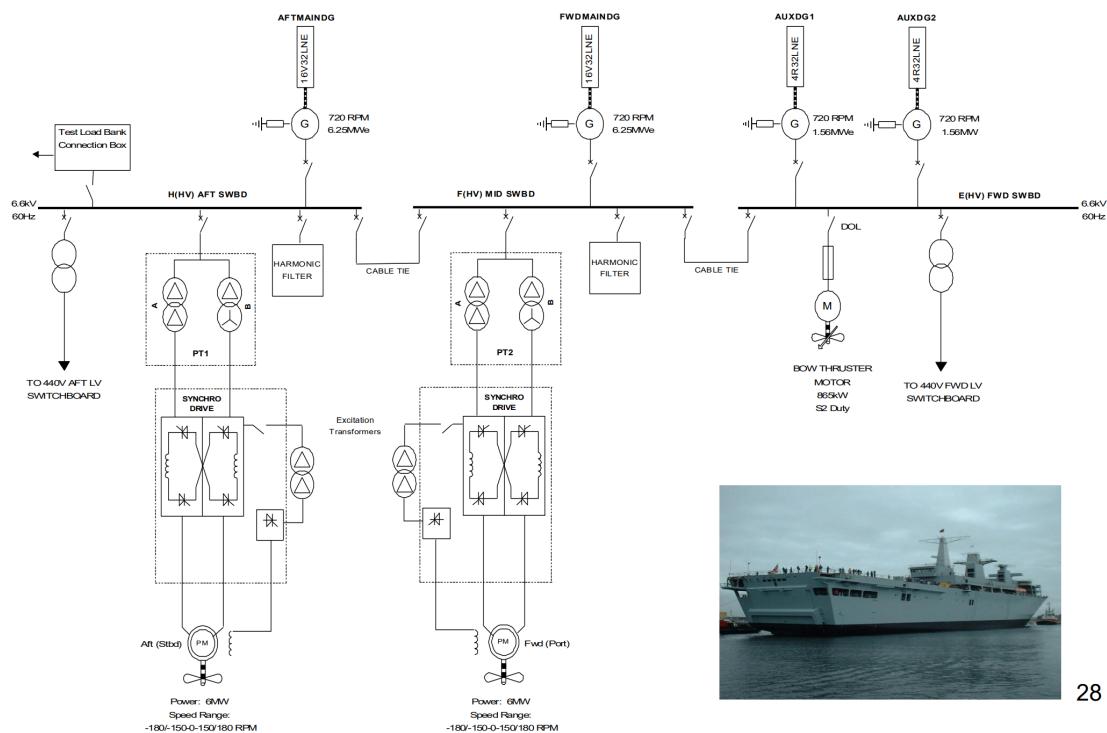


Figure 2.4: Marine SLD.



28

Figure 2.5: Naval SLD.

2.3.4 Some common features of SLDs

- Supplies (shore supplies, generators, incoming supply) are located at the top of the diagram
- The loads (motors, lighting, etc.) are located towards the bottom of the diagram.
- Switchboards are shown as thicker lines with interlocking switchgear being shown using dotted lines.
- Interconnections between equipment is a single-line representation regardless of number of phase (unless there is a good reason not to do so).
- Voltage, Frequency, Power, PF, revolutions, etc. are provided.

2.3.5 Limitations of the electrical line diagram

- The ‘Single Line Electrical Diagram’ is a very useful means of showing how electrical equipment is connected into a system using single lines (representing a three-phase system or some other electrical power system).
- It has very limited use when undertaking analysis. It is not an electrical circuit. To undertake analysis of electrical power systems then it is necessary to change the ‘Single Line Electrical Diagram’ into an ‘Impedance Diagram’.

Chapter 3

Developing Impedance Diagram

3.1 Three Phase Power

3.1.1 Three-phase alternating voltages

A three-phase synchronous generator consists of a rotor and a stator.

- Adjusting excitation current on the rotating field will change the magnitude of the three AC phase emfs generated in the stator.
- Changing the rotational speed changes the frequency of the AC emfs
- The three phases generated are 120° displaced due to special arrangement

3.1.2 Three-phase emfs (or terminal voltages) can be expressed mathematically

$$v_a(t) = V_m \sin(\omega t) \quad (3.1)$$

$$v_b(t) = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \quad (3.2)$$

$$v_c(t) = V_m \sin\left(\omega t - \frac{4\pi}{3}\right) \quad (3.3)$$

V_m is the peak (maximum) voltage, ω is the angular frequency, t is time. The phase displacement between the three-phase waveforms is 120° or $\frac{2\pi}{3}$ radians. v_a , v_b and v_c are the three phase voltages.

3.1.3 Three-phase, six-wire connection

There are different arrangements for distributing three-phase electrical power. The three phases can be independent of each other as seen below and treated as three separate circuits. This is known as the *three-phase, six-wire system*.

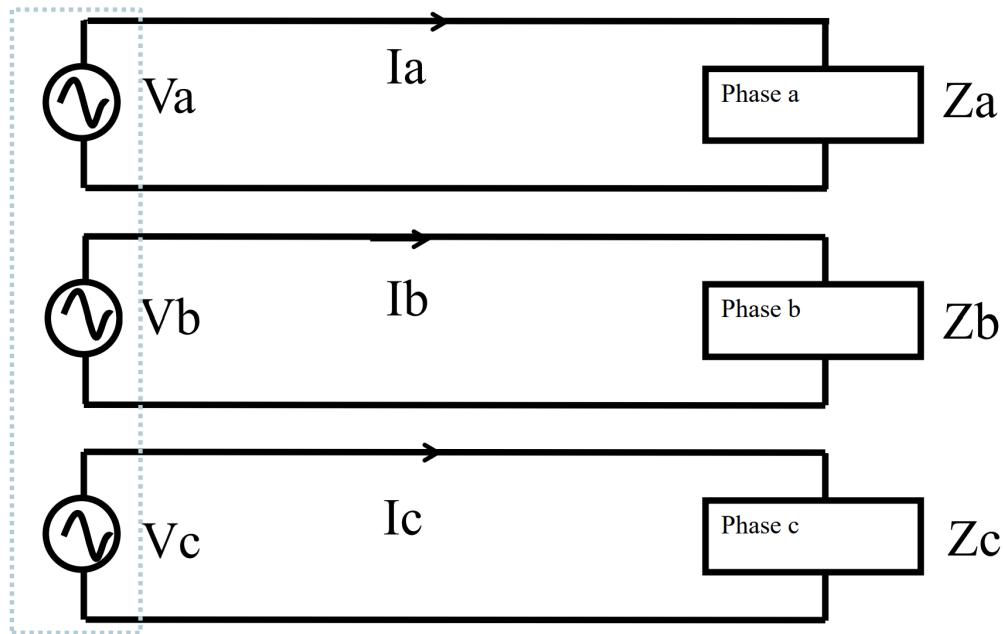


Figure 3.1: Three-phase, six-wire system.

3.1.4 Three-phase current

The currents flow in a three-phase circuit when there is a three-phase load. We will initially assume that the three-phase load is balanced i.e. the magnitude of voltage, current and the phase-angle is the same for each phase circuit. This is not true for three-phase circuits with unbalanced loads and the mathematical approach is different and more complex so we will examine this later.

3.1.5 Three-phase alternating current

The currents associated with a three-phase system that flow from the supply to the load may be described mathematically by:

$$i_a(t) = I_m \sin(\omega t + \theta) \quad (3.4)$$

$$i_b(t) = I_m \sin\left(\omega t - \frac{2\pi}{3} + \theta\right) \quad (3.5)$$

$$i_c(t) = I_m \sin\left(\omega t - \frac{4\pi}{3} + \theta\right) \quad (3.6)$$

Note: the phase displacement angle (θ) can be positive (leading PF) indicating a capacitive load or negative (lagging PF) indicating an inductive load. A zero phase displacement angle indicates a resistive circuit or a circuit at resonance ($X_L = X_C$).

3.1.6 Connecting Three-Phases

A three-phase six wire system is generally expensive to install and is actually unnecessary due to an inherent balancing characteristic.

In the balanced three-phase system, the algebraic sum of voltage at any point where all three-phase voltages are connected is zero.

The zero voltage point is known as the ‘star point’ and this may be grounded or left isolated (floating). In most electrical systems the star point is grounded with exceptions being some ship types.

3.1.7 Star and delta connections

The number of transmission wires can be reduced by connecting the phases in either delta or star configuration.

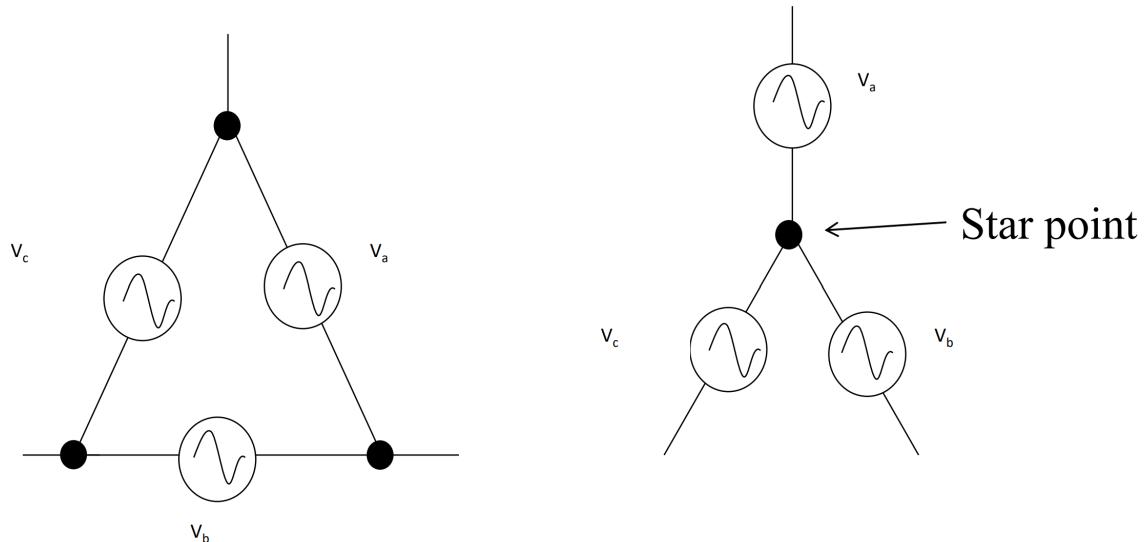


Figure 3.2: Star and delta configurations.

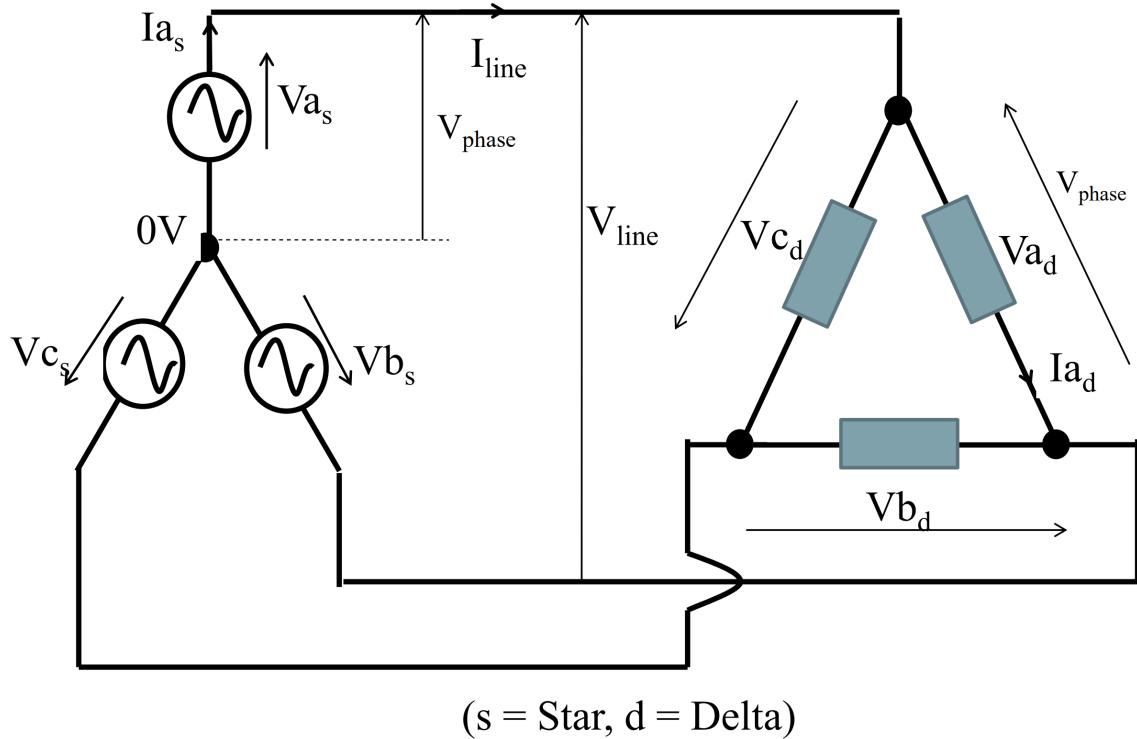


Figure 3.3: Star generator and delta load.

3.1.8 Phase and line voltages

There are therefore two voltage types (either generated as a potential difference) when considering three-phase circuits. These are commonly known as the *phase voltage* and *line voltage*.

The phase voltages in the star-delta circuit are as follows:

- V_{as}, V_{bs}, V_{cs} for the star circuit
- V_{ad}, V_{bd}, V_{cd} for the delta circuit

The line voltages can be measured as follows:

$$V_{ab} = V_{as} - V_{bs} = V_{ad} \quad (3.7)$$

$$V_{bc} = V_{bs} - V_{cs} = V_{bd} \quad (3.8)$$

$$V_{ca} = V_{cs} - V_{as} = V_{cd} \quad (3.9)$$

and if the line voltages measure is reversed:

$$V_{ba} = V_{bs} - V_{as} = -V_{ad} \quad (3.10)$$

$$V_{cb} = V_{cs} - V_{bs} = -V_{bd} \quad (3.11)$$

$$V_{ac} = V_{as} - V_{cs} = -V_{cd} \quad (3.12)$$

Which is why a three-phase system is known as a six-pulse system - (important in power electronic systems).

3.1.9 Relationships between star and delta

For the delta arrangement:

$$V_p = V_l \quad (3.13)$$

$$I_p = \frac{I_l}{\sqrt{3}} \quad (3.14)$$

For the star arrangement:

$$V_p = \frac{V_l}{\sqrt{3}} \quad (3.15)$$

$$I_p = I_l \quad (3.16)$$

Where I_p and V_p are the phase currents and voltages and I_l and V_l are the line currents and voltages respectively. Note: Delta is also known as ‘mesh’; Star is also known as ‘Y’.

3.1.10 Single-phase impedance triangle

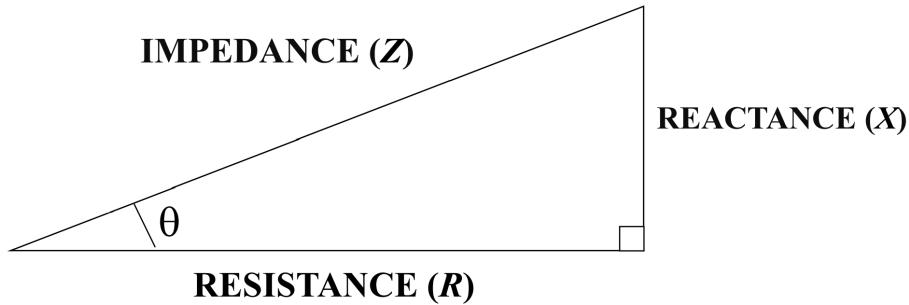


Figure 3.4: Single-phase impedance triangle.

$$Z = R + jX \quad (3.17)$$

$$= R + j(X_L - X_C) \quad (3.18)$$

$$= R + j \left(\omega L - \frac{1}{\omega C} \right) \quad (3.19)$$

Where, Z is impedance, R is resistance, X_L is inductive reactance, X_C is capacitive reactance, ω is angular frequency ($2\pi f$).

3.1.11 Single-phase power triangle

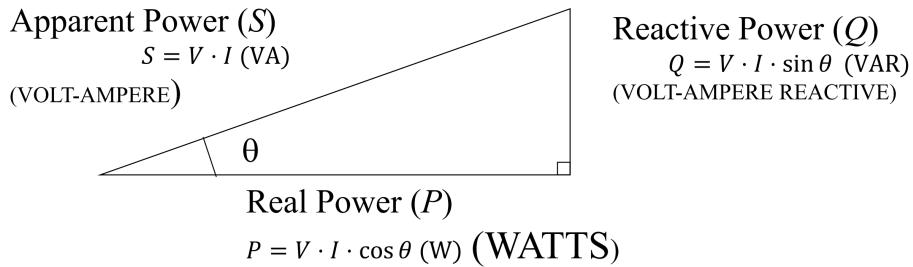


Figure 3.5: Single-phase power triangle.

- Real power (P) is the power that can be put into or taken from the electrical system and is measured in Watts (W).
- Reactive power (Q) is the power that circulates in the electrical system and is measured in Volt-Ampere-Reactive (VAR).
- Apparent power (S) is what is apparent from the product of voltage and current and is measured in Volt-Amperes (VA).

3.1.12 Three-phase power

Since V in the star circuit and I in the delta circuit is subject to change simply by dividing by $\sqrt{3}$, whilst the other variable I and V in star and delta respectively remain unchanged. Hence we get:

$$P = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \cos \theta \quad (3.20)$$

For apparent power (S) and reactive power (Q) we have:

$$S = \sqrt{3} \cdot I_{line} \cdot V_{line} \quad (3.21)$$

$$Q = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \sin \theta \quad (3.22)$$

3.1.13 Student Activity

Three coils each of resistance 5Ω and inductive reactance of 10Ω are connected in (a) star and (b) delta across a 440 VRMS three-phase (line) supply.

If each coil has a capacitor connected in parallel having capacitive reactance of 20Ω then calculate the line and phase currents and the total power absorbed.

3.2 Per Unit (PU) System

3.2.1 Electrical line diagram to Impedance diagram

- The ‘*electrical line diagram*’ - a schematic which allows an understanding of equipment and system arrangements.
- The ‘*impedance diagram*’ - a schematic which allows an understanding of the equipment and system impedances.
- The layout of both the ‘electrical line diagram’ and ‘impedance diagram’ should be similar but in the ‘impedance diagram’ all equipment and lines are replaced with impedances.
- All impedances will need to be calculated to a *common base* - hence use of a per unit system.

3.2.2 Simple equivalent impedances

For the purposes of steady-state analysis the Electrical Line Diagram is converted to an ‘Impedance Line Diagram’ where the equipment is represented as an ‘Equivalent Impedance’. Typical *simple* impedances representing equipment are: (note: not all R , L and C values may be given).

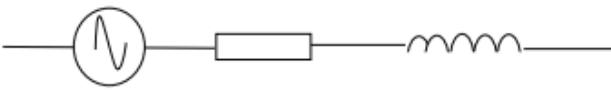
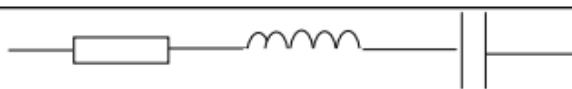
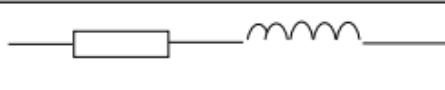
Equipment	Equivalence Impedance Representation
AC Generator or Motor	
DC Machine (Motor or Generator)	
Transmission Lines and Cables	
Transformer	

Figure 3.6: Equivalent Impedance Representations.

3.2.3 How manufacturers of electrical equipment specify ratings

Manufacturers of electrical equipment would usually specify electrical equipment as follows:

e.g. A synchronous generator

- $S = 10 \text{ MVA}$ (value of apparent power)
- $V = 3.3 \text{ kV}$ (line voltage rating of the equipment)
- Phase = 3 (number of phases)
- $\text{PF} = 0.8$ (usual value of power factor of equipment)
- $N = 1500 \text{ rpm}$ (design speed of rotation)
- $F = 50 \text{ Hz}$ (frequency of the alternating current & voltage)
- $X = 0.14$ (Reactance given as a pu value or as a %)
- Connection = star (stator windings)

3.2.4 The per unit system

In Electrical Power System Analysis the per unit system is the preferred method for analysing circuit behaviour rather than the standard SI system of units (Watts, Volts, Amperes, etc.)

The advantages of the per unit system are:

- Computations for power systems have several voltage levels because of connected transformers is very cumbersome when using the SI system because values need to be referred across the transformer turns ratio. The per unit system (overcomes or simplifies) this problem.
- All powers, voltage, currents and impedances are expressed as per unit values of specified base values. This means they are easily compared with one another which is very helpful for equipment specification and selection and in power system design and its analysis.

3.2.5 Values in per unit system

In the per unit system five base values are needed. These are **power**, **current**, **voltage**, **impedance** and **power factor**. It is necessary to choose two base values and to calculate two base values.

Usually the base values defined are:

- the Apparent Power (Base_VA)
- Voltage (Base_V)

Power Factor is already expressed in per unit form. Once the base values are calculated then ‘actual values’ in the circuit can be expressed in per unit form.

3.2.6 Three-Phase system PU conversion

Step one

The per unit relationships for Base_VA and Base_V are defined and Base_I and Base_Z are calculated:

$$\text{Base_VA} = \text{Defined by Engineer} \quad (3.23)$$

$$\text{Base_V} = \text{Defined by Engineer} \quad (3.24)$$

$$\text{Base_I} = \frac{\text{Base_VA}}{\sqrt{3} \cdot \text{Base_V}} \quad (3.25)$$

$$\text{Base_Z} = \frac{\text{Base_V}}{\text{Base_I}} \quad (3.26)$$

Step two

Having calculated the Base Values, these are then defined as being 1 per unit values:

- $\text{Base_V} = 1$ per unit Voltage
- $\text{Base_VA} = 1$ per unit Apparent Power
- $\text{Base_I} = 1$ per unit Current
- $\text{Base_Z} = 1$ per unit Impedance

Step three

In the circuit all apparent powers, voltage, currents and impedances are expressed as per unit values:

$$\text{Per_Unit_S} = \frac{\text{Actual_Value_S}}{\text{Base_S}} \quad (3.27)$$

$$\text{Per_Unit_V} = \frac{\text{Actual_Value_V}}{\text{Base_V}} \quad (3.28)$$

$$\text{Per_Unit_I} = \frac{\text{Actual_Value_I}}{\text{Base_I}} \quad (3.29)$$

$$\text{Per_Unit_Z} = \frac{\text{Actual_Value_Z}}{\text{Base_Z}} \quad (3.30)$$

Step four

Sometimes parameters e.g. Z are already expressed in per unit form rather than as SI units but have been calculated to a different base (S and V). These can be converted as follows:

$$(Per_Unit_Z)_{new_base} = \frac{(Base_VA)_{new_base}}{(Base_VA)_{old_base}} \cdot \frac{(Base_V)_{old_base}^2}{(Base_V)_{new_base}^2} \cdot (Per_Unit_Z)_{old_base} \quad (3.31)$$

Some manufacturers and engineers prefer to work with the percentage system rather than the per unit system which of course is a simple matter of multiplying by 100/ Equipment manufacturers use a machine's own S and V to determine base values from which Z pu is then calculated.

3.2.7 Example PU system conversion

Single Line Diagram

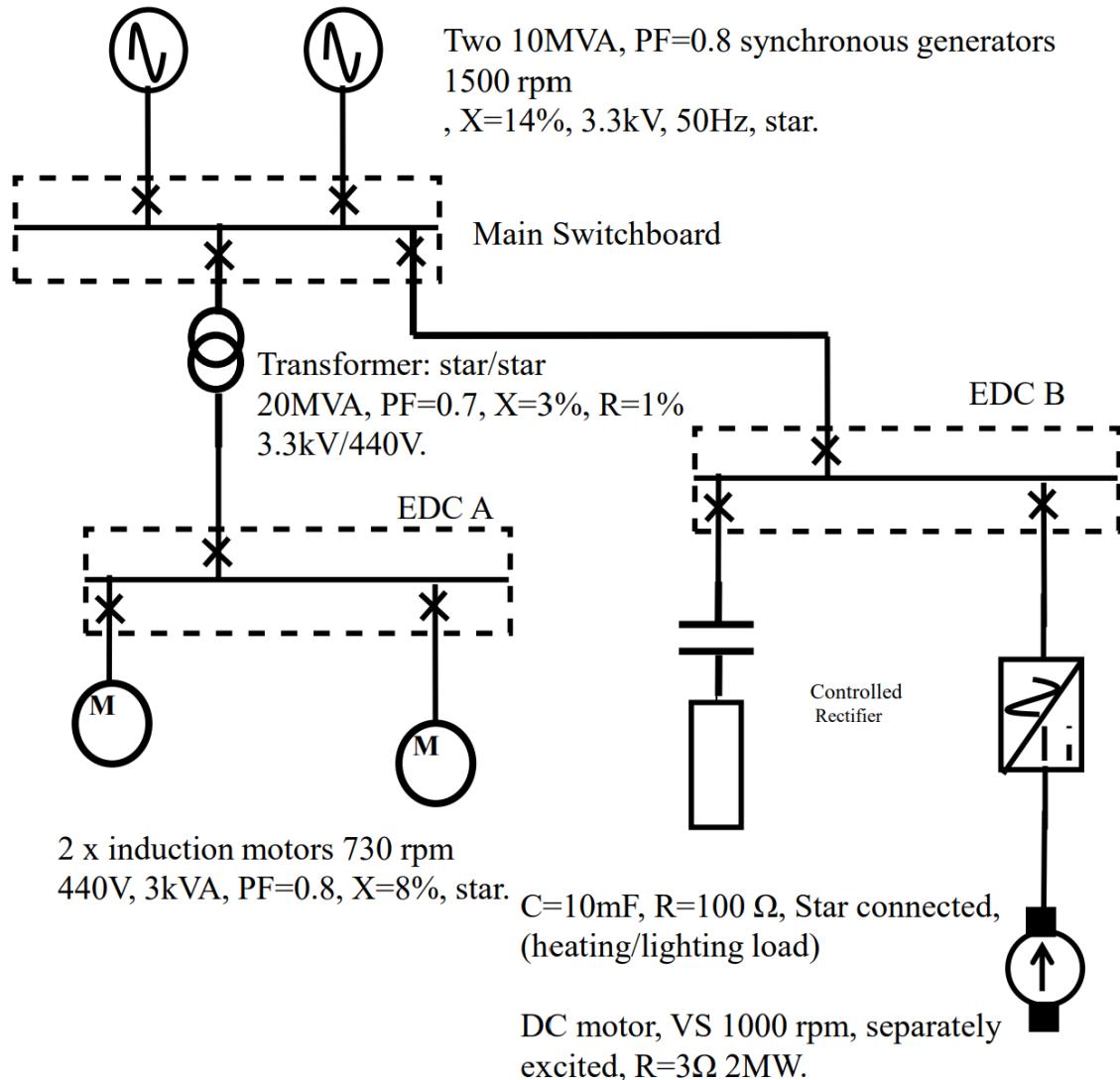


Figure 3.7: Single Line Diagram.

Step one - calculating the base current and base impedance

Selecting 10 MVA as Base_S and 3.3 kV as Base_V (because it seems sensible considering the generators) then we have:

$$\text{Base}_I = \frac{10^6}{\sqrt{3} \times 3.3 \times 10^6} = 1749.5 \text{ A} \quad (3.32)$$

$$\text{Base}_Z = \frac{3.3 \times 10^3}{1749.5} = 1.886 \Omega \quad (3.33)$$

Step two - defininng 1 p.u. values

- $3.3 \times 10^3 \text{ V} = 1 \text{ per unit Voltage} = 1 \text{ pu V}$
- $10 \times 10^6 \text{ VA} = 1 \text{ per unit Apparent Power} = 1 \text{ pu S}$
- $1749.5 \text{ A} = 1 \text{ per unit Current} = 1 \text{ pu A}$
- $1.886 \Omega = 1 \text{ per unit Impedance} = 1 \text{ pu Z}$

Sometimes % values are preferred by some engineers i.e. 1 pu = 100%

Step three - converting impedances expressed in SI units to per unit form

The only ‘actual values’ i.e. expressed in SI units, are the heating load and the DC machine:

For the lighting/heating load:

$$-jXC = -j \left(\frac{1}{2\pi \cdot 50 \cdot 10 \times 10^{-3}} \right) = -j0.318 \quad (3.34)$$

$$-jXC = \frac{-j0.318}{1.886} = -j0.168 \text{ pu} \quad (3.35)$$

$$R = \frac{100}{1.886} = 53.022 \text{ pu} \quad (3.36)$$

For DC motor:

$$R = \frac{3}{1.886} = 1.591 \text{ pu} \quad (3.37)$$

$$S = P + \frac{2}{10} = 0.2 \text{ pu} \quad (3.38)$$

Step four - converting impedances expressed in per unit form to another base

For the synchronous generators:

$$S = \frac{10}{10} = 1 \text{ pu} \quad (3.39)$$

$$V = 3.3 \text{ kV} = 1 \text{ pu} \quad (3.40)$$

$$X = \frac{14}{100} = 0.14 \text{ pu} \quad (3.41)$$

$$PF = 0.8 \text{ pu} \quad (3.42)$$

For the transformer:

$$S = \frac{20}{10} = 2 \text{ pu} \quad (3.43)$$

$$X = \frac{3}{100} \times \frac{10}{20} = 0.015 \text{ pu} \quad (3.44)$$

$$R = \frac{1}{100} \times \frac{10}{20} = 0.005 \text{ pu} \quad (3.45)$$

$$PF = 0.7 \text{ pu} \quad (3.46)$$

For the induction motors:

$$S = \frac{3}{10000} = 0.0003 \text{ pu} \quad (3.47)$$

$$X = \frac{8}{100} \times \frac{10000}{3} = 266.667 \text{ pu} \quad (3.48)$$

$$PF = 0.8 \quad (3.49)$$

Step five - drawing the impedance diagram

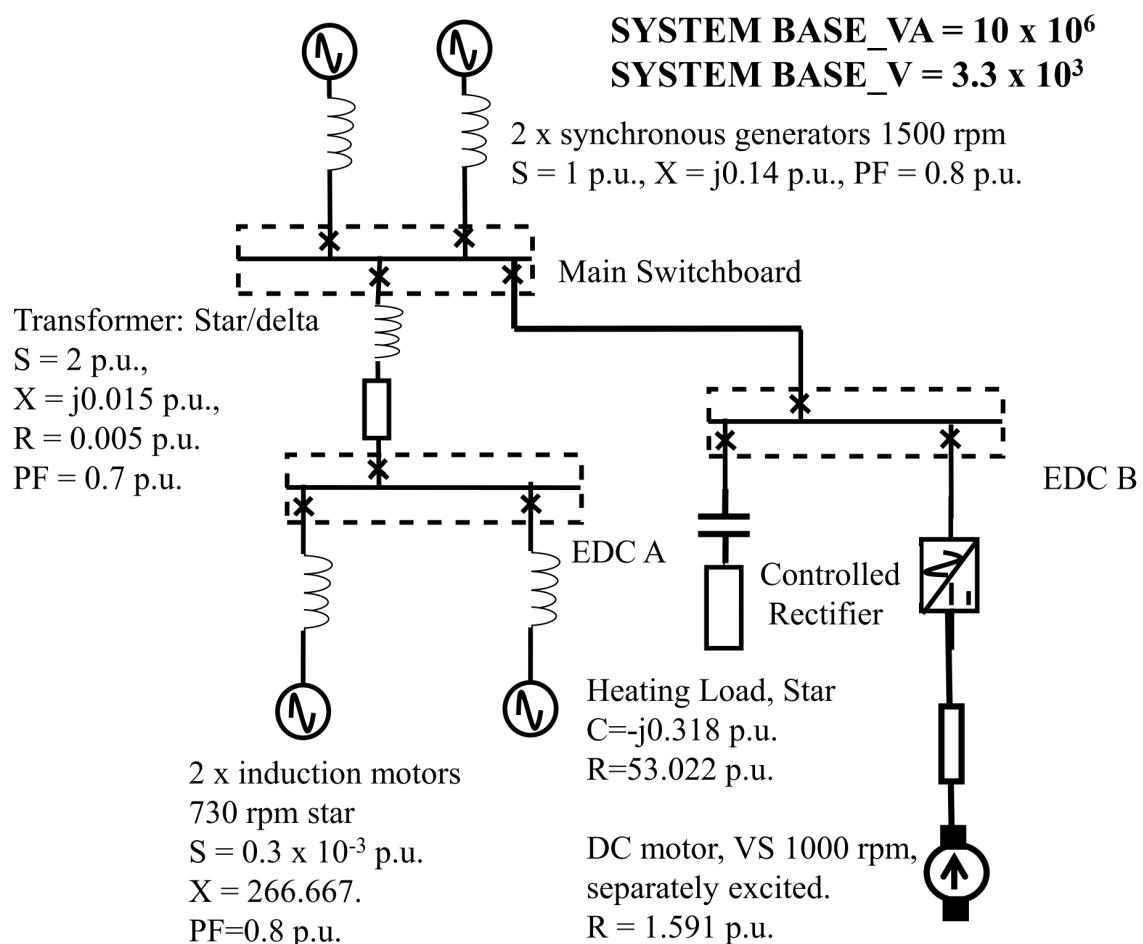


Figure 3.8: Impedance Diagram.

3.2.8 Reactance diagram

The reactance diagram is a modification to the impedance diagram where only per unit reactances are shown. In a reactance diagram all resistances are ignored. The reactance diagram is useful because it allows ‘first pass’ calculations to be made in a power system without too much mathematical complexity due to having $(R \pm jX)$.

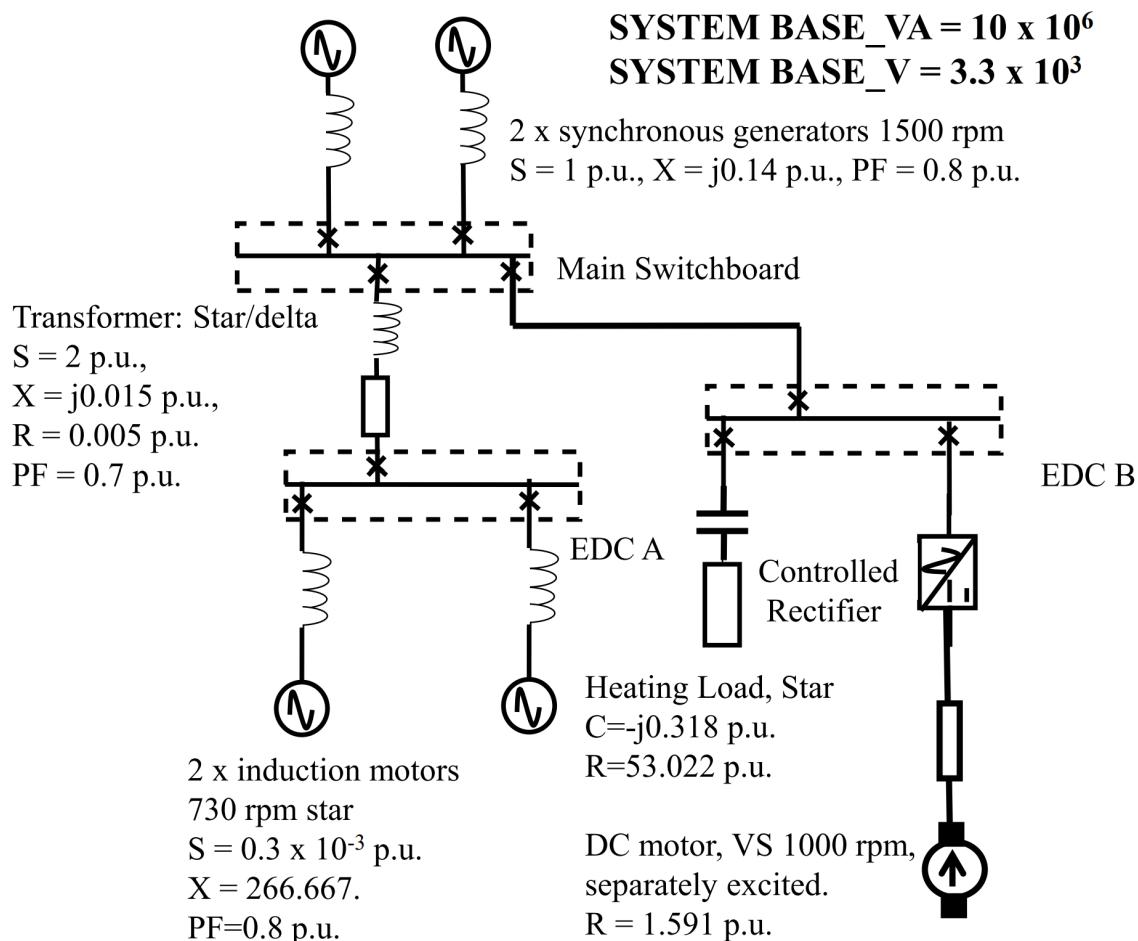


Figure 3.9: Reactance Diagram.

3.2.9 Impedance and reactance diagrams

Converting the electrical line diagram to an impedance diagram or reactance diagram is essential for:

- Potential difference (voltage drop) calculations
- Current flows in cables/lines
- Calculations of losses
- Power flows around an electrical system
- Understand transient effects

- Calculate fault level and fault currents
- Waveform distortion and its penetration
- Impacts when adding new equipment to the network

3.3 Summary

The per unit system allows powers, voltages, currents and impedances to be expressed relative to each other. This allows the designer to understand the relationships between different parts of the circuit.

Using the per-unit transformer model eliminates the need to scale quantities by the transformer turns ratio, thus eliminating a common source for error in electrical calculations.