UCL Mechanical Engineering 2021/2022

MECH0026 Problem Sheet Solutions

HD

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1 Problem Sheet 1

1.1 Q1

1.1.1 a

For plane stress, our simplification is valid when one dimension of an object (e.g. z-direction) is very small compared to others, e.g. a thin sheet, loaded perpendicular to the surface. Stress tensors relating to the z-direction are virtually 0 ($\sigma_z = \tau_{yz} = \tau_{xz} = 0$) and no loads (body or boundary) in z-direction. We can use compliance matrix to find out the components of our stress field.

For plane strain, our simplification is valid when one dimension of an object (e.g. in z-direction) is very large compared to others e.g. a long cylindrical or prismatic body loaded perpendicular to the length. The conditions of plane strain are:

1. Everything is constant in the z-direction $\frac{\partial()}{\partial z} = 0$
2. $w = 0$
3. No loads (body or boundary) in z-direction

Hence, $\epsilon_z = \delta_{yz} = \delta_{xz} = 0$. We can use stiffness matrix to find components of our strain field.

1.1.2 b

1.1.3 c

Compliance matrix $[S]$:

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E} \begin{Bmatrix} 1 & -v & -v & 0 & 0 & 0 \\ -v & 1 & -v & 0 & 0 & 0 \\ -v & -v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+v) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+v) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+v) \end{Bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \tag{1.1}$$

	Plane Strain	Plane strain
Stress Tensor	$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}$
Strain tensor	$\varepsilon = \begin{pmatrix} \varepsilon_x & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}$	$\varepsilon = \begin{pmatrix} \varepsilon_x & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_y & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Table 1: Table to show stress/strain tensors in plane stress/strain.

Stiffness matrix $[C]$:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E\left(1-v\right)}{\left(1+v\right)\left(1-2v\right)} \begin{Bmatrix} 1 & \frac{v}{1+v} & \frac{v}{1+v} & 0 & 0 & 0 \\ \frac{v}{1+v} & 1 & \frac{v}{1+v} & 0 & 0 & 0 \\ \frac{v}{1+v} & \frac{v}{1+v} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} \end{Bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{1.2}$$

1.2 Q2

$$\phi = \frac{p}{20h^3}\left(15h^2x^2y - 5x^2y^3 - 2h^2y^3 + y^5\right) \tag{1.3}$$

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = \frac{p}{20h^3}\left(20y^3 - 30x^2y - 12h^2y\right) \tag{1.4}$$

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = \frac{p}{20h^3}\left(30h^2xy - 10xy^3\right) \tag{1.5}$$

$$\tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = -\frac{p}{20h^3}\left(20h^2x - 30xy^2\right) \tag{1.6}$$

satisfies harmonic relationship. Boundary conditions:

$$\tau_{xy} = 0 \text{ at } y = \pm h \tag{1.7}$$

$$\sigma_{yy} = -p \text{ at } y = -h \tag{1.8}$$

$$\sigma_{yy} = p \text{ at } y = h \tag{1.9}$$

$$\sigma_{xx} = 0 \text{ at } x = 0 \tag{1.10}$$

$$u = v = \frac{\partial v}{\partial u} = 0 \text{ at } x = L \tag{1.11}$$