0.1 Three Phase Power

0.1.1 Three-phase alternating voltages

A three-phase synchronous generator consists of a rotor and a stator.

- Adjusting excitation current on the rotating field will change the magnitude of the three AC phase emfs generated in the stator.
- Changing the rotational speed changes the frequency of the AC emfs
- The thre phases generated are 120° displaced due to special arrangement

0.1.2 Three-phase emfs (or terminal voltages) can be expressed mathematically

$$v_a(t) = V_m \sin(\omega t) \tag{1}$$

$$v_b(t) = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) \tag{2}$$

$$v_c(t) = V_m \sin\left(\omega t - \frac{4\pi}{3}\right) \tag{3}$$

V_m is the peak (maximum) voltage, ω is the angular frequency, t is time. The phase displacement between the three-phase waveforms is 120° or $\frac{2\pi}{3}$ radians. v_a, v_b and v_c are the three phase voltages.

0.1.3 Three-phase, six-wire connection

The are different arrangements for distributing three-phase electrical power. The three phases can be independent of each other as seen below and treated as three separate circuits. This is known as the *three-phase, six-wire system.*

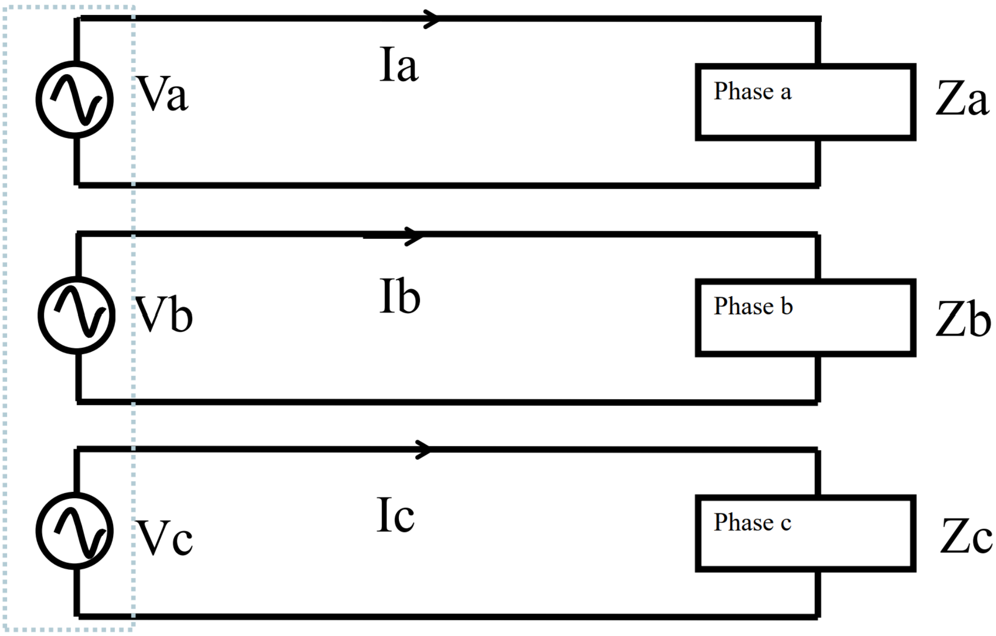

Figure 1: Three-phase, six-wire system.

0.1.4 Three-phase current

The currents flow in a three-phase circuit when there is a three-phase load. We will initially assume that the three-phase load is balanced i.e. the magnitude of voltage, current and the phase-angle is the same for each phase circuit. This is not true for three-phase circuits with unbalanced loads and the mathematical approach is different and more complex so we will examine this later.

0.1.5 Three-phase alternating current

The currents associated with a three-phase system that flow from the supply to the load may be described mathematically by:

$$i_a(t) = I_m \sin(\omega t + \theta) \tag{4}$$

$$i_b(t) = I_m \sin\left(\omega t - \frac{2\pi}{3} + \theta\right) \tag{5}$$

$$i_c(t) = I_m \sin\left(\omega t - \frac{4\pi}{3} + \theta\right) \tag{6}$$

Note: the phase displacement angle (θ) can be positive (leading PF) indicating a capacitive load or negative (lagging PF) indicating an inductive load. A zero phase displacement angle indicates a resistive circuit or a circuit at resonance ($X_L = X_C$).

0.1.6 Connecting Three-Phases

A three-phase six wire system is generally expensive to install and is actually unnecessary due to an inherent balancing characteristic.

In the balanced three-phase system, the algebraic sum of voltage at any point where all three-phase voltages are connected is zero.

The zero voltage point is known as the 'star point' and this may be grounded or left isolated (floating). In most electrical systems the star point is grounded with exceptions being some ship types.

0.1.7 Star and delta connections

The number of transmission wires can be reduced by connecting the phases in either delta or star configuration.

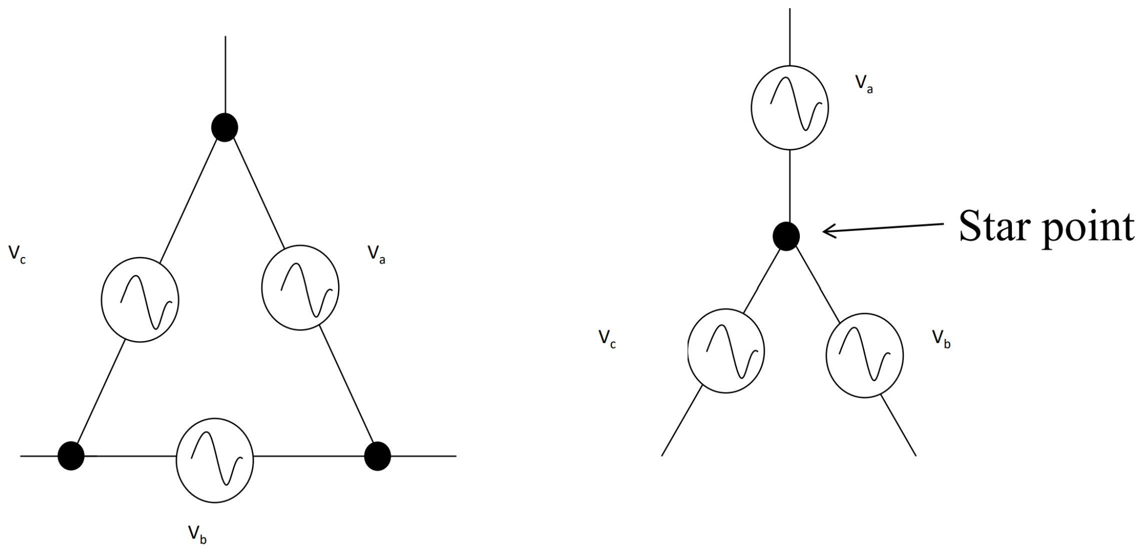

Figure 2: Star and delta configurations.

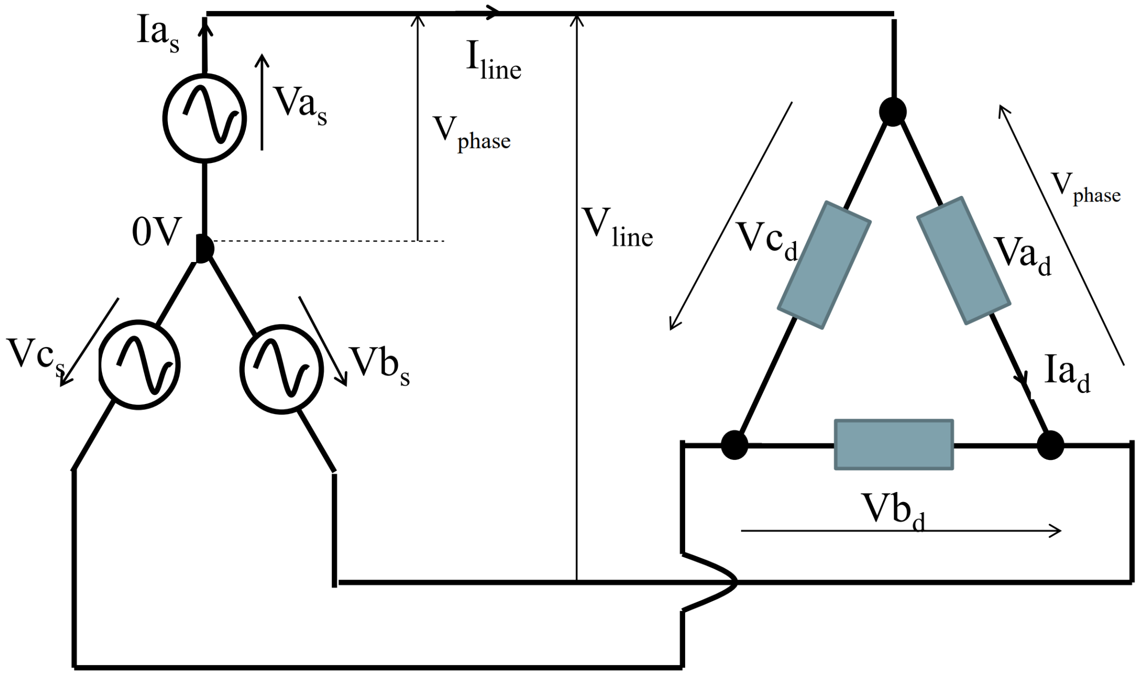

Figure 3: Star generator and delta load.

0.1.8 Phase and line voltages

There are therefore two voltage types (either generated as a potential difference) when considering three-phase circuits. These are commonly known as the *phase voltage* and *line voltage*.

The phase voltages in the star-delta circuit are as follows:

- Vas, Vbs, Vcs for the star circuit
- Vad, Vbd, Vcd for the delta circuit

The line voltages can be measured as follows:

$$Vab = Vas - Vbs = Vad \tag{7}$$
$$Vbc = Vbs - Vcs = Vbd \tag{8}$$
$$Vca = Vcs - Vas = Vcd \tag{9}$$

and if the line voltages measure is reversed:

$$Vba = Vbs - Vas = -Vad \tag{10}$$
$$Vcb = Vcs - Vbs = -Vbd \tag{11}$$
$$Vac = Vas - Vcs = -Vcd \tag{12}$$

Which is why a three-phase system is known as a six-pulse system - (important in power electronic systems).

0.1.9 Relationships between star and delta

For the delta arrangement:

$$V_p = V_l \tag{13}$$
$$I_p = \frac{I_l}{\sqrt{3}} \tag{14}$$

For the star arrangement:

$$V_p = \frac{V_l}{\sqrt{3}} \tag{15}$$
$$I_p = I_l \tag{16}$$

Where I_p and V_p are the phase currents and voltages and I_l and V_l are the line currents and voltages respectively. Note: Delta is also known as 'mesh'; Star is also known as 'Y'.

0.1.10 Single-phase impedance triangle

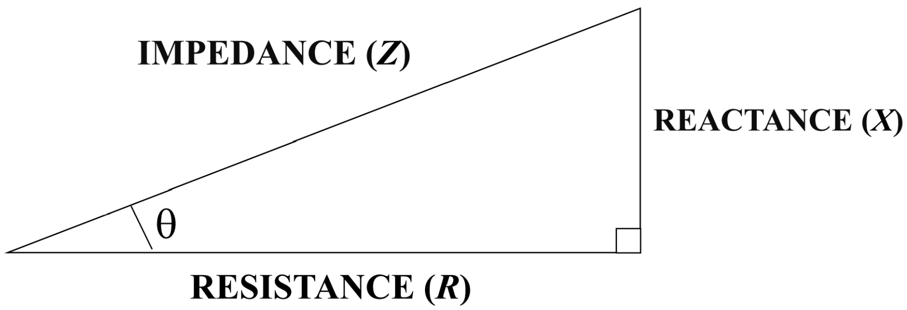

Figure 4: Single-phase impedance triangle.

$$\begin{aligned} Z &= R + jX && (17)\\ &= R + j\left(X_L - X_C\right) && (18)\\ &= R + j\left(\omega L - \frac{1}{\omega C}\right) && (19) \end{aligned}$$

Where, Z is impedance, R is resistance, X_L is inductive reactance, X_C is capacitive reactance, ω is angular frequency ($2\pi f$).

0.1.11 Single-phase power triangle

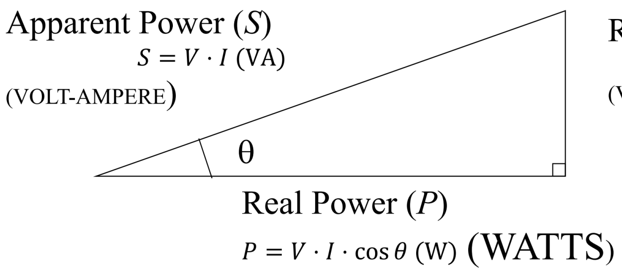

Figure 5: Single-phase power triangle.

- Real power (P) is the power that can be put into or taken from the electrical system and is measured in Watts (W).
- Reactive power (Q) is the power that circulates in the electrical system and is measured in Volt-Ampere-Reactive (VAR).
- Apparent power (S) is what is apparetn from the product of voltage and current and is measured in Volt-Amperes (VA).

0.1.12 Thre-phase power

Since V in the star circuit and I in the delta circuit is subject to change simply by dividing by $\sqrt{3}$, whilst the other variable I and V in star and delta respectively remain unchanged. Hence we get:

$$P = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \cos\theta \tag{20}$$

For apparent power (S) and reactive power (Q) we have:

$$S = \sqrt{3} \cdot I_{line} \cdot V_{line} \tag{21}$$

$$Q = \sqrt{3} \cdot I_{line} \cdot V_{line} \cdot \sin\theta \tag{22}$$