UCL Mechanical Engineering 2020/2021

MECH0011 Final Coursework

NCWT3

May 21, 2021

Boat speed V_w (m/s)	Hydrofoil wingspan b (m)	Angle of attack (deg)	Hydrofoil selected	Average chord $\bar{c}$ (m)	x_c/R, y_c/R, R
11	2	0	RG 8 Airfoil		

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1 Question 1

1.1 a

The data was imported into MATLAB and the shape of the hydrofoil, the chord line and the mean camber line were plotted for all four hydrofoils.

```matlab
clc
clear
close all

%define vars
i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS 930 Hydrofoil"]; %index hydrofoil names from sheets for ease
data = zeros(122,2,4); %initialise matrix
counter = 0; %initialise counter
x = linspace(0,1,100); %interpolation range initialisation

%import data
for j = 1:4 %index all data for plots
    counter = counter + 1; %increment counter
    data(:,:,counter) = readmatrix('suppFiles.xlsx','Sheet',i(j),'Range','A3:B124'); %loop through sheets and pull data
end

%camber line calculation
%pull positive and negative coordinate points
%eppler
dataPos1 = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','A3:B37');
dataNeg1 = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','A38:B70');

%naca
dataPos2 = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','A3:B28');
dataNeg2 = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','A29:B54');

%rg
dataPos3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A3:B34');
dataNeg3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A35:B64');

%ys
dataPos4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A3:B65');
dataNeg4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A66:B124');

%interpolate hydrofoil shape with 100 data points from 0 to 1
%eppler
dataIntPos1 = interp1(dataPos1(:,1), dataPos1(:,2), x);
dataIntNeg1 = interp1(dataNeg1(:,1), dataNeg1(:,2), x);

%naca
```

```
dataIntPos2 = interp1(dataPos2(:,1), dataPos2(:,2), x);
dataIntNeg2 = interp1(dataNeg2(:,1), dataNeg2(:,2), x);

%rg
dataIntPos3 = interp1(dataPos3(:,1), dataPos3(:,2), x);
dataIntNeg3 = interp1(dataNeg3(:,1), dataNeg3(:,2), x);

%ys
dataIntPos4 = interp1(dataPos4(:,1), dataPos4(:,2), x);
dataIntNeg4 = interp1(dataNeg4(:,1), dataNeg4(:,2), x);

%calculate camber line
%eppler
camber1 = (dataIntPos1 + dataIntNeg1)./2;

%naca
camber2 = (dataIntPos2 + dataIntNeg2)./2;

%rg
camber3 = (dataIntPos3 + dataIntNeg3)./2;

%ys
camber4 = (dataIntPos4 + dataIntNeg4)./2;

%plot data
subplot(4,1,1)
plot(dataPos1(:,1), dataPos1(:,2),'b',dataNeg1(:,1), dataNeg1(:,2),'b', x
    , camber1,'r')
axis image
grid on
xlabel('Chord')
ylabel('Z(x)')
title('Plot of ' + i(1))
xlim([-0.05 1.05])
ylim([-0.05 0.1])
legend('Hydrofoil profile','Mean camber line')

subplot(4,1,2)
plot(dataPos2(:,1), dataPos2(:,2),'b',dataNeg2(:,1), dataNeg2(:,2),'b', x
    , camber2,'r')
axis image
grid on
xlabel('Chord')
ylabel('Z(x)')
title('Plot of ' + i(2))
xlim([-0.05 1.05])
ylim([-0.05 0.1])
legend('Hydrofoil profile','Mean camber line')

subplot(4,1,3)
plot(dataPos3(:,1), dataPos3(:,2),'b',dataNeg3(:,1), dataNeg3(:,2),'b', x
    , camber3,'r')
axis image
```

```
grid on
xlabel('Chord')
ylabel('Z(x)')
title('Plot of ' + i(3))
xlim([-0.05 1.05])
ylim([-0.05 0.1])
legend('Hydrofoil profile','Mean camber line')

subplot(4,1,4)
plot(dataPos4(:,1), dataPos4(:,2),'b',dataNeg4(:,1), dataNeg4(:,2),'b', x, camber4,'r')
axis image
grid on
xlabel('Chord')
ylabel('Z(x)')
title('Plot of ' + i(4))
xlim([-0.05 1.05])
ylim([-0.05 0.1])
legend('Hydrofoil profile','Mean camber line')
```

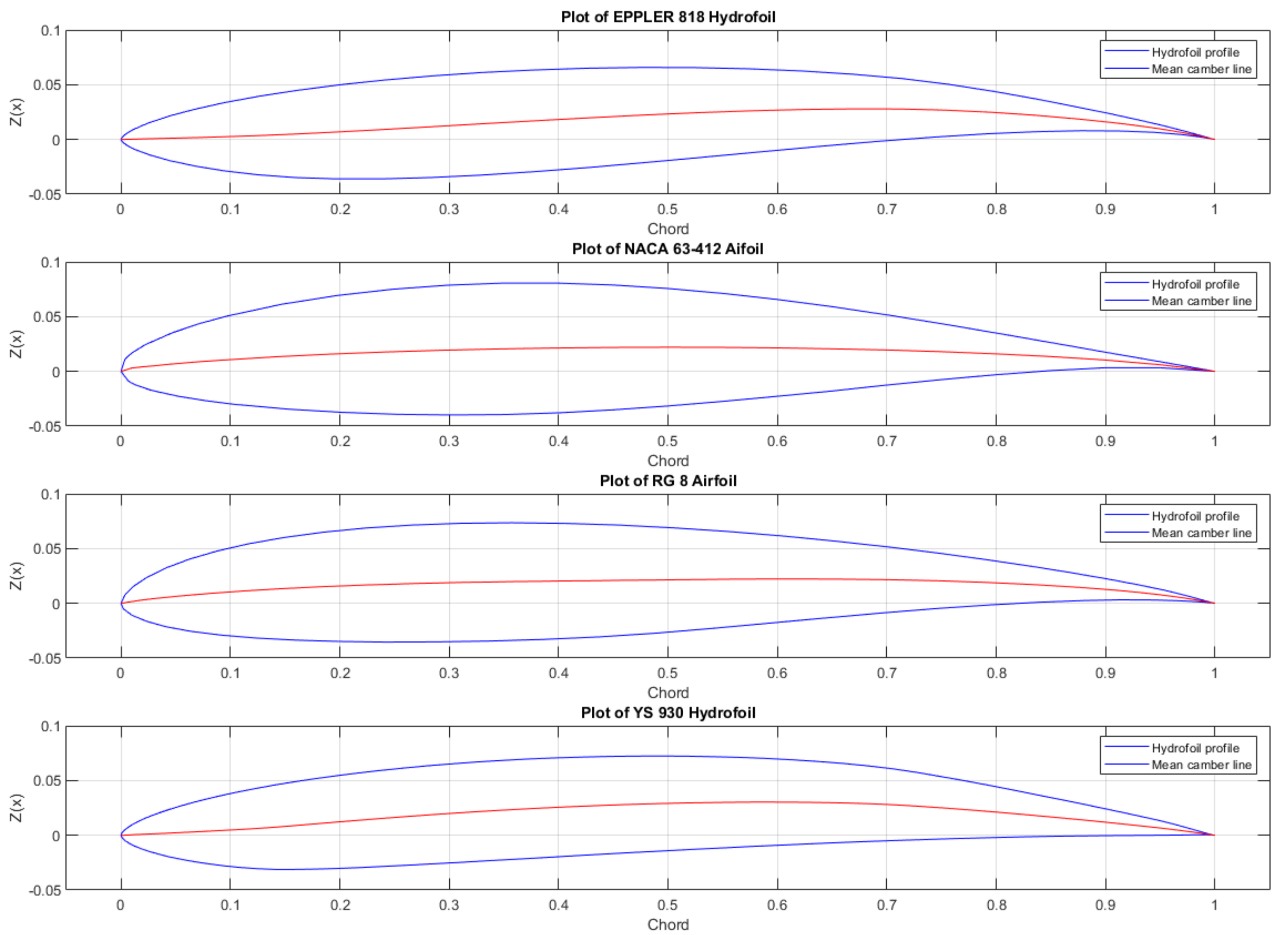

Figure 1: Graphs to show hydrofoil shape, chord line and mean camber line for four different hydrofoils.

1.2 b

MATLAB was used to calculate the lift-to-drag ratio for each hydrofoil.

```matlab
clc
clear
close all

%define vars
i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS 930 Hydrofoil"]; %index hydrofoil names from sheets for ease

%pull cl and cd from data
%eppler
epplerCL = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','E2:E79');
epplerCD = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','F2:F79');

%naca
nacaCL = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','E2:E107');
nacaCD = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','F2:F107');

%rg
rgCL = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','E2:E101');
rgCD = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','F2:F101');

%ys
ysCL = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','E2:E78');
ysCD = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','F2:F78');

%plot data
plot(epplerCL, epplerCD,'r',nacaCL,nacaCD,'g',rgCL,rgCD,'b',ysCL,ysCD,'magenta')
legend(i(1), i(2), i(3), i(4))
axis square
grid on
xlabel('C_L')
ylabel('C_D')
title('Plot of lift-to-drag ratio for ' + i(4))
```

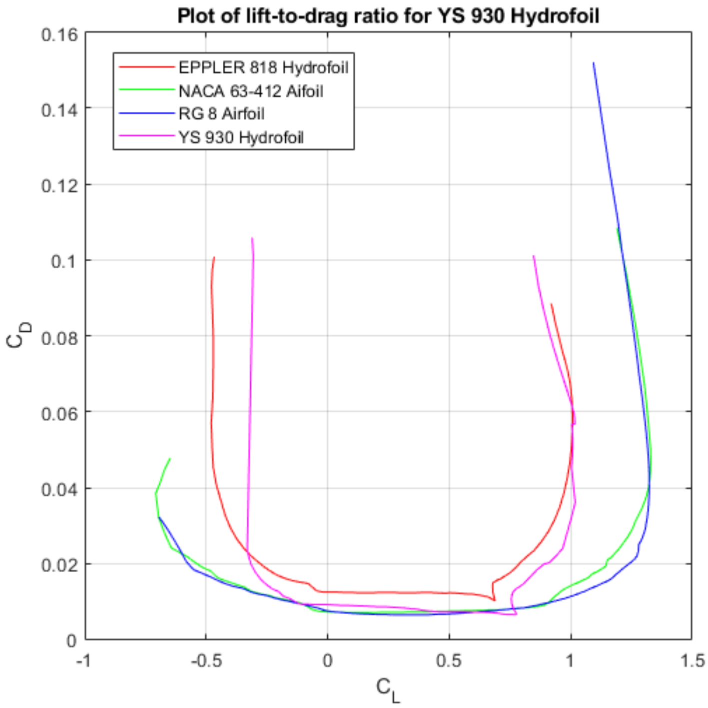

Figure 2: Graph to show lift-to-drag ratio for four different hydrofoils.

1.3 c

At a low angle of attack, the flow is attached to the hydrofoil and separates close to the trailing edge, leaving a small wake. The streamlined shape of the hydrofoil exerts a large shear force on the flow passing over it, leading to a large skin drag but low form drag. As we increase the angle of attack, the flow separation moves along the top of the hydrofoil. This decreases the shear force acting on the hydrofoil and the skin drag starts to reduce, with form drag increasing. At the stall angle, the pressure distribution along the top of the hydrofoil dramatically changes due to flow separation, suction pressure is mostly/all lost and a large wake with turbulent effects is present. The primary drag effect here is now form drag, as the flow is not attached along the length of the hydrofoil surface.

1.4 d

MATLAB was used to calculate each of the variables for each hydrofoil.

```
clc
clear
close all

%define vars
i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS
    930 Hydrofoil"]; %index hydrofoil names from sheets for ease
x = linspace(0,1,100); %interpolation range initialisation

%max percentage camber line calculation
%camber line calculation
%pull positive and negative coordinate points
```

```matlab
%eppler
dataPos1 = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','A3:B37');
dataNeg1 = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','A38:B70');

%naca
dataPos2 = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','A3:B28');
dataNeg2 = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','A29:B54');

%rg
dataPos3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A3:B34');
dataNeg3 = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','A35:B64');

%ys
dataPos4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A3:B65');
dataNeg4 = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','A66:B124');

%interpolate hydrofoil shape with 100 data points from 0 to 1
%eppler
dataIntPos1 = interp1(dataPos1(:,1), dataPos1(:,2), x);
dataIntNeg1 = interp1(dataNeg1(:,1), dataNeg1(:,2), x);

%naca
dataIntPos2 = interp1(dataPos2(:,1), dataPos2(:,2), x);
dataIntNeg2 = interp1(dataNeg2(:,1), dataNeg2(:,2), x);

%rg
dataIntPos3 = interp1(dataPos3(:,1), dataPos3(:,2), x);
dataIntNeg3 = interp1(dataNeg3(:,1), dataNeg3(:,2), x);

%ys
dataIntPos4 = interp1(dataPos4(:,1), dataPos4(:,2), x);
dataIntNeg4 = interp1(dataNeg4(:,1), dataNeg4(:,2), x);

%calculate camber line
%eppler
camber1 = (dataIntPos1 + dataIntNeg1)./2;

%naca
camber2 = (dataIntPos2 + dataIntNeg2)./2;

%rg
camber3 = (dataIntPos3 + dataIntNeg3)./2;

%ys
camber4 = (dataIntPos4 + dataIntNeg4)./2;

%maximum camber per hydrofoil
%eppler
percCamber1 = max(camber1);

%naca
percCamber2 = max(camber2);

```

```
%rg
percCamber3 = max(camber3);

%ys
percCamber4 = max(camber4);

%clean-up output
percCamber = 100.*[percCamber1 percCamber2 percCamber3 percCamber4];

%maximum percentage thickness calculation
%eppler
thickness1 = (abs(dataIntPos1) + abs(dataIntNeg1));

%naca
thickness2 = (abs(dataIntPos2) + abs(dataIntNeg2));

%rg
thickness3 = (abs(dataIntPos3) + abs(dataIntNeg3));

%ys
thickness4 = (abs(dataIntPos4) + abs(dataIntNeg4));

%max thickness per hydrofoil
%eppler
maxThick1 = max(thickness1);

%naca
maxThick2 = max(thickness2);

%rg
maxThick3 = max(thickness3);

%ys
maxThick4 = max(thickness4);

%clean-up output
maxThick = 100.*[maxThick1 maxThick2 maxThick3 maxThick4];

%maximum lift coefficient
%pull angle of attack, cl and cd from data
%eppler
epplerData = readmatrix('suppFiles.xlsx','Sheet',i(1),'Range','D2:F79');

%naca
nacaData = readmatrix('suppFiles.xlsx','Sheet',i(2),'Range','D2:F107');

%rg
rgData = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','D2:F101');

%ys
ysData = readmatrix('suppFiles.xlsx','Sheet',i(4),'Range','D2:F78');

%find max cl
```

```
%eppler
maxEpplerCL = max(epplerData(:,2));

%naca
maxNacaCL = max(nacaData(:,2));

%rg
maxRgCL = max(rgData(:,2));

%ys
maxYsCL = max(ysData(:,2));

%clean-up output
maxCL = [maxEpplerCL maxNacaCL maxRgCL maxYsCL];

%find angle of attack at max cl
%eppler
critEppler = epplerData(epplerData(:,2) == maxEpplerCL,1);

%naca
critNaca = nacaData(nacaData(:,2) == maxNacaCL,1);

%rg
critRg = rgData(rgData(:,2) == maxRgCL,1);

%ys SPECIAL CASE VALUE ASCERTAINED EMPIRICALLY
critYs = 9;

%clean-up output
crit = [critEppler critNaca critRg critYs];

%lift coefficient for alpha = 0
%eppler
liftAlpha0Eppler = epplerData(epplerData(:,1) == 0,2);

%naca
liftAlpha0Naca = nacaData(nacaData(:,1) == 0,2);

%rg
liftAlpha0Rg = rgData(rgData(:,1) == 0,2);

%ys
liftAlpha0Ys = ysData(ysData(:,1) == 0,2);

%clean-up output
liftAlpha0 = [liftAlpha0Eppler liftAlpha0Naca liftAlpha0Rg liftAlpha0Ys];

%angle of attack corresponding to cl = 0
%find min cl
%eppler
minEpplerCL = min(abs(epplerData(:,2)));

%naca
```

```
minNacaCL = min(abs(nacaData(:,2)));

%rg
minRgCL = min(abs(rgData(:,2)));

%ys
minYsCL = min(abs(ysData(:,2)));

%find angle of attack at min cl
%eppler
AOAMinEppler = epplerData(epplerData(:,2) == minEpplerCL,1);

%naca
AOAMinNaca = nacaData(nacaData(:,2) == -minNacaCL,1);

%rg
AOAMinRg = rgData(rgData(:,2) == -minRgCL,1);

%ys
AOAMinYs = ysData(ysData(:,2) == -minYsCL,1);

%clean-up output
AOAMin = [AOAMinEppler AOAMinNaca AOAMinRg AOAMinYs];

%generate table
T = table(i', percCamber', maxThick', maxCL', crit', liftAlpha0', AOAMin
    ');
```

	Maximum		
Hydrofoil	% camber	% thickness	lift coefficient
EPPLER 818 Hydrofoil	2.792	9.362	1.008
NACA 63-412 Aifoil	2.204	11.992	1.330
RG 8 Airfoil	2.226	10.795	1.323
YS 930 Hydrofoil	3.028	9.088	1.018

Table 1: Table to show maximum percentage camber and thickness and the maximum lift coefficient for four hydrofoils.

Hydrofoil	Stall angle	Lift coefficient for $\alpha = 0^\circ$	Angle of attack α_0 corresponding to $C_L = 0$
EPPLER 818 Hydrofoil	7.75°	0.361	-3°
NACA 63-412 Aifoil	13°	0.338	-3°
RG 8 Airfoil	12.75°	0.382	-3°
YS 930 Hydrofoil	9°	0.391	-3.75°

Table 2: Table to show the stall angle, lift coefficient for $\alpha = 0^\circ$ and the angle of attack α_0 corresponding to $C_L = 0$ for four hydrofoils.

The hydrofoils both have a higher percentage camber than the airfoils. They also have a lower percentage thickness than the airfoils. We can see that this leads to a reduction in the stall angle and subsequently their maximum lift coefficients are lower. However, we can see that they perform better at $\alpha = 0°$. This can be attributed to the fact that at small α, larger camber generates more lift, at the expense of a smaller stall angle, as the boundary layer is more prone to separation at higher α. However, increasing either percentage camber or thickness will increase C_D.

1.5 e

The RG 8 Airfoil was selected as it has a high stall angle, with a subsequently large maximum lift coefficient, the lift coefficient at $\alpha = 0°$ is also the second highest. We can calculate the lift on the hydrofoil using 1.1:

$$L = \frac{1}{2}\rho C_L V_w^2 A \tag{1.1}$$

where ρ is the density of seawater, C_L is the lift coefficient for a given angle of attack, V_w is the velocity of the fluid relative to the wing and A is the projected surface area of the wing. The surface area A can be found by integration:

$$A = \int_{x_0}^{b+x_0} (-0.07x + c_0)\,\mathrm{d}x \tag{1.2}$$

$$A = \left[-0.035x^2 + c_0 x\right]_{x_o}^{b+x_0} \tag{1.3}$$

$x_0 = 0.1$ as given in the material. I have also selected $b = 2\,\mathrm{m}$, hence:

$$A = -0.035(2.1)^2 + c_0(2.1) + 0.035(0.1)^2 - c_0(0.1) \tag{1.4}$$

$$A = 2c_0 - 0.154 \tag{1.5}$$

Substituting 1.5 into 1.1:

$$L = \frac{1}{2}\rho C_L V_w^2 (2c_0 - 0.154) \tag{1.6}$$

Rearranging for c_0:

$$(2c_0 - 0.154) = \frac{L}{\frac{1}{2}\rho C_L V_w^2} \tag{1.7}$$

$$c_0 = \frac{L}{\rho C_L V_w^2} + 0.077 \tag{1.8}$$

The following values were used: $\rho = 1036\,\mathrm{kg\,m^{-3}}$ [1], $V_w = 11\,\mathrm{m\,s^{-1}}$ ($39.6\,\mathrm{km\,h^{-1}}$), $L = 2000 \times 9.81 \approx 20\,000\,\mathrm{N}$. MATLAB was used to plot the chord length against the angle of attack. As V_w represents the upper speed limit of the boat, we want to select a low angle of attack, to reduce the amount of drag on the foil. Hence, the value for C_L at $\alpha = 0$ will be used.

```
clc
clear
close all

%define vars
i = ["EPPLER 818 Hydrofoil", "NACA 63-412 Aifoil", "RG 8 Airfoil", "YS 930 Hydrofoil"]; %index hydrofoil names from sheets for ease
Vw = 11; %39.6km/h
VwLower = 6; %21.6km/h

%pull angle of attack, cl and cd from data
```

```
%rg
rgData = readmatrix('suppFiles.xlsx','Sheet',i(3),'Range','D2:F101');

%chord length equation
RGChord = ((20000)./(1036.*(Vw^2).*(rgData(rgData(:,1) == 0,2)))) + 0.077;%finds CL at alpha = 0 and inputs into equation and solves
L = 0.5.*1036.*(Vw^2).*(rgData(rgData(:,1) == 0,2)).*((2.*RGChord)-0.154); %check
LLower = 0.5.*1036.*(VwLower^2).*(rgData(rgData(:,1) == 11,2)).*((2.*RGChord)-0.154);%alpha = 11 degrees
```

Our chord length is calculated to be $c_0 = 0.4948\,\mathrm{m}$. We must now check whether this chord length is sufficient to lift the boat at lower speeds. In order to do this we require a high angle of attack, to generate more lift. The value for C_L at 11° was used to test the lift at $6\,\mathrm{m\,s^{-1}}$ ($21.6\,\mathrm{km\,h^{-1}}$) and MATLAB gives a lift value of 20 215 N, allowing the boat to be out of the water at lower speeds as well.

2 Question 2

2.1 a

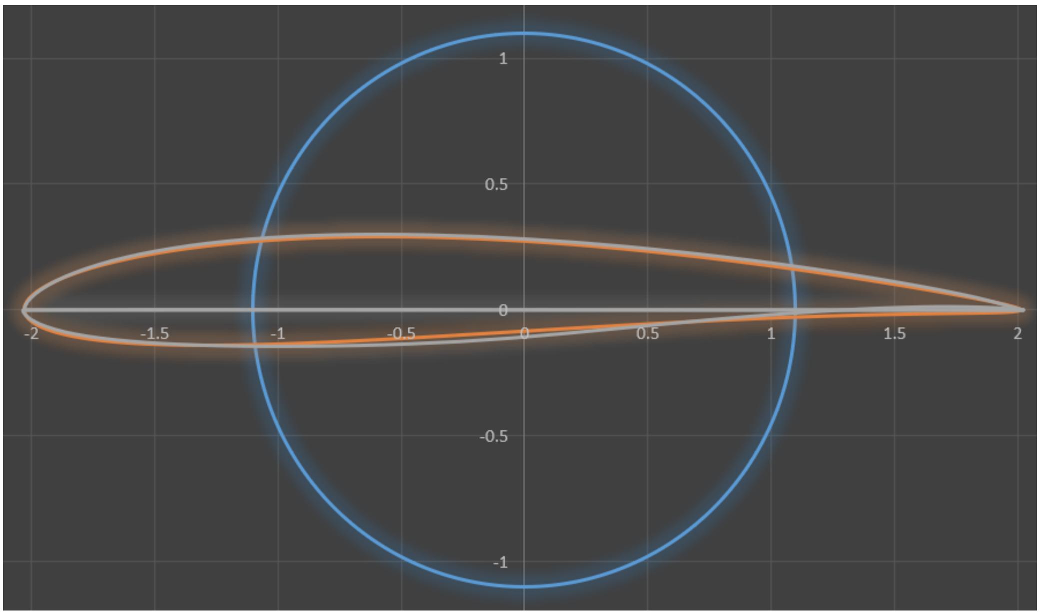

Figure 3: Graph to show conformal map onto RG 8 airfoil.

The Excel sheet from the Fluids lab was used to find values for R, x_c and y_c. They were found to be:

$$R = 1.1,\ x_c = -0.065,\ y_c = 0.051 \tag{2.1}$$

2.2 b

2.3 c

3 Question 3

3.1 a

MATLAB was used to plot the boundary layer velocity profile.

```
clc
clear
close all

%import data
data = readmatrix('suppFiles.xlsx','Sheet','Boundary Layer','Range','A2:B102');

%plot data
plot(data(:,1), data(:,2))
axis image
grid on
xlim([0 1])
ylim([0 1])
ylabel('y/\delta')
xlabel('u/U')
title('Graph to show the boundary layer velocity profile')
```

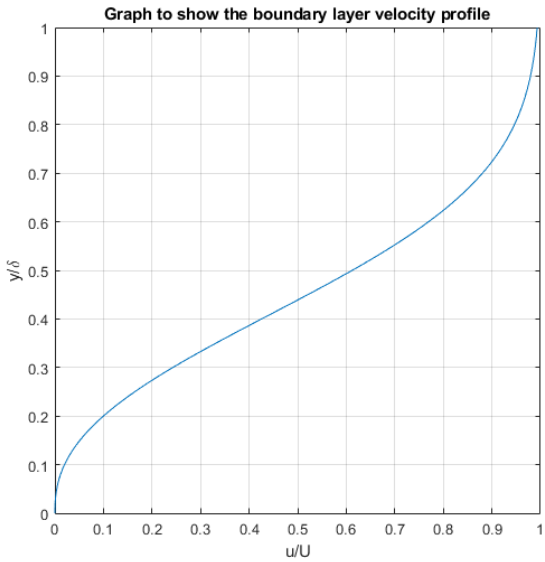

Figure 4: Graph to show boundary layer velocity profile.

3.1.1 i

One can expect to see this boundary layer velocity profile in the area near the streamline separation point on the airfoil surface. We cannot see any negative velocity (indicative of backflow) nor do we see the viscous effects confined to a thing layer. For $\alpha = 0$, this would occur at the trailing edge of the airfoil.

3.1.2 ii

Under stall conditions, our angle of attack is high and the airflow around the airfoil is prone to boundary layer separation early. Hence, we can expect to see this velocity profile close to the leading edge of the airfoil. The flow along the rest of the airflow is highly chaotic, hence we would only see this velocity profile near to the boundary layer separation point.

3.2 b

We can use the following formula to calculate the shape factor:

$$H = \frac{\int_0^\delta \left(1 - \frac{u}{U}\right) \mathrm{d}y}{\int_0^\delta \left(\frac{u}{U}\left(1 - \frac{u}{U}\right)\right) \mathrm{d}y} \tag{3.1}$$

Replacing our integrals with the trapezium method of integration:

$$H = \frac{\sum_{i=1}^{100} \left[\frac{\left(1 - \frac{u}{U}\right)_i + \left(1 - \frac{u}{U}\right)_{i+1}}{2} \cdot \left(\frac{y_{i+1}}{\delta} - \frac{y_i}{\delta}\right) \right]}{\sum_{i=1}^{100} \left[\frac{\left(\frac{u}{U}\left(1 - \frac{u}{U}\right)\right)_i + \left(\frac{u}{U}\left(1 - \frac{u}{U}\right)\right)_{i+1}}{2} \cdot \left(\frac{y_{i+1}}{\delta} - \frac{y_i}{\delta}\right) \right]} \tag{3.2}$$

This was implemented in MATLAB:

```
clc
clear
close all

%import data
data = readmatrix('suppFiles.xlsx','Sheet','Boundary Layer','Range','A2:
    B102');

%setup data and vars
var1 = 1 - data(:,1);
var2 = data(:,1).*(1-data(:,1));
dispThick = zeros(1,100);
momThick = zeros(1,100);
```

```matlab

%displacement thickness integral
for i = 1:100 %hundred strips
    dispThick(i) = ((var1(i,1)) + (var1(i+1,1)))*(1/100)/2; %calc area of trapezium and store
end
momThickInt = sum(dispThick); %sum values to find integral

%momentum thickness integral
for i = 1:100
    momThick(i) = ((var2(i,1)) + (var2(i+1,1)))*(1/100)/2;
end
dispThickInt = sum(momThick);

%calc shape factor
H = momThickInt/dispThickInt;
```

The code yielded a shape factor $H = 4$. The shape factor of a BLVP upstream from the one in the supplementary material would be lower than the one given. This is because there will be a velocity at the boundary and shear stresses will be present, increasing momentum thickness. We would also see a reduction in the size of the thickness of the boundary layer, further reducing the shape factor. For a downstream BVLP, we would see a large separation of the flow from the boundary layer, hence a large displacement thickness. As there is no longer a flow of fluid at the boundary of the airfoil, there will be a large deficit of momentum; combined these factors would increase the shape factor.

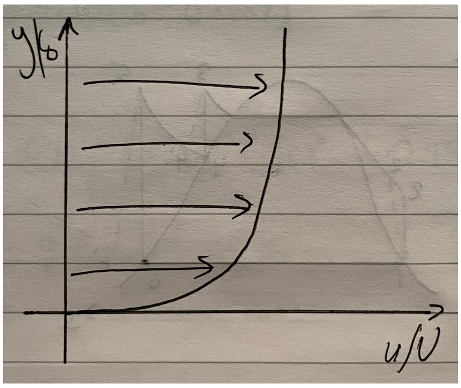

Figure 5: Graph to show boundary layer velocity profile upstream.

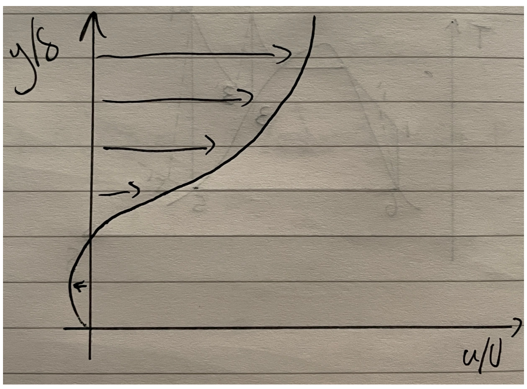

Figure 6: Graph to show boundary layer velocity profile downstream.

4 Question 4

4.1 a

The balanced reaction equation takes the general form:

$$C_xH_y + a\,(O_2 + 3.76\,N_2) \longrightarrow b\ CO_2 + c\ H_2O + 3.76d\ N_2 + e\ O_2 \tag{4.1}$$
$$\tag{4.2}$$

Since this is the reaction of methane with 110% stoichiometric air, we can put in our values and balance our equation

$$CH_4 + 2.2\,(O_2 + 3.76\,N_2) \longrightarrow CO_2 + 2\,H_2O + 8.272\,N_2 + 0.2\,O_2 \tag{4.3}$$

4.2 b

The air-fuel ratio on a mass basis can be found using 4.4

$$AF = \bar{AF} \times \frac{M_{air}}{M_{fuel}} \tag{4.4}$$

Where and $\bar{AF}$ is volumetric air ratio and M is the molar mass. Assuming that the air only consists of oxygen and nitrogen, we can input molar masses from the property tables into the 4.4:

$$AF = \frac{2.2\,(1+3.76)}{1} \times \frac{\left(\frac{32+3.76\,(28.01)}{4.76}\right)}{16.04} \tag{4.5}$$
$$AF = 18.8\,\mathrm{kg_{air}\,kg_{fuel}^{-1}} \tag{4.6}$$

4.3 c

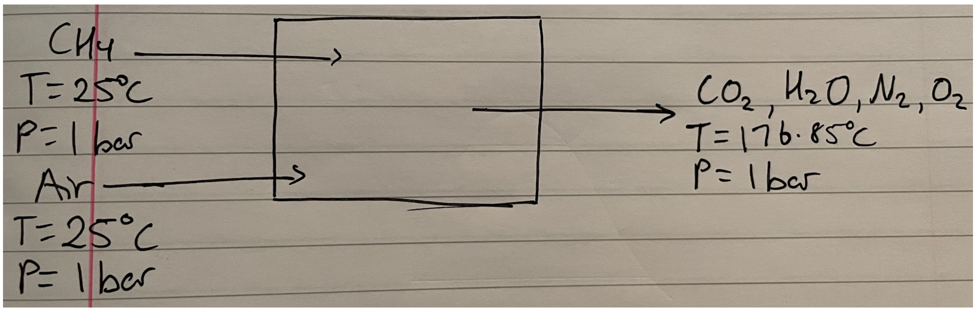

Figure 7: Diagram to show combustion.

Temperature of reactants is given as 298 K. Temperature of products was selected to be 450 K (176.85 °C). Assumptions:

- Control volume is in a steady state
- N_2 is an inert gas
- Isobaric process

- Neglect kinetic and potential energies of the particles
- No mechanical work

We also know that there is 3.76 moles of nitrogen for every mole of oxygen in the air. Standard enthalpy of reaction is given as:

$$\Delta\bar{h}_r = \sum_{\text{products}} n_i\bar{h}_i(T) - \sum_{\text{reactants}} n_i\bar{h}_i(T) \tag{4.7}$$

Reaction equation:

$$\mathrm{CH_4} + 2.2\,(\mathrm{O_2} + 3.76\,\mathrm{N_2}) \longrightarrow \mathrm{CO_2} + 2\,\mathrm{H_2O} + 8.272\,\mathrm{N_2} + 0.2\,\mathrm{O_2} \tag{4.8}$$

Calculating the sum of the products using the formula $\bar{h} = \bar{h}^o_{f,i} + \Delta\bar{h}_i(T)$:

$$\sum_{\text{reactants}} n_i\bar{h}_i(T) = \left[\bar{h}^o_f + \bar{h}(450) - \bar{h}(289)\right]_{\mathrm{CO_2}} + 2\left[\bar{h}^o_f + \bar{h}(450) - \bar{h}(289)\right]_{\mathrm{H_2O}} + 8.272\left[\bar{h}(450) - \bar{h}(289)\right]_{\mathrm{N_2}} + 0.2\left[\bar{h}(450) - \bar{h}(289)\right]_{\mathrm{O_2}} \tag{4.9}$$

Using table values:

$$\sum_{\text{reactants}} n_i\bar{h}_i(T) = [-393520 + 15483 - 9369] + 2\,[-241820 + 15080 - 9904] + 8.272\,[13105 - 8669] + 0.2\,[13228 - 8682] \tag{4.10}$$

Calculating the sum of the reactants using standard enthalpies of reaction:

$$\sum_{\text{products}} n_i\bar{h}_i(T) = \left[\bar{h}^o_f\right]_{\mathrm{CH_4}} + 2.2\left[\bar{h}^o_f\right]_{\mathrm{O_2}} + 8.272\left[\bar{h}^o_f\right]_{\mathrm{N_2}} \tag{4.11}$$

Using table values:

$$\sum_{\text{products}} n_i\bar{h}_i(T) = [-74870] + 2.2\,[0] + 8.272\,[0] \tag{4.12}$$

Hence, the final enthalpy of reaction is:

$$\Delta\bar{h}_r = -823090.208 - (-74870) = -748\,220.208\,\mathrm{kJ\,kmol^{-1}} \tag{4.13}$$

4.4 d

We know that all the water obtained in the combustion is a vapour, hence we can deduce that the enthalpy of reaction value obtained is the lower heating value.

5 Question 5

5.1 a

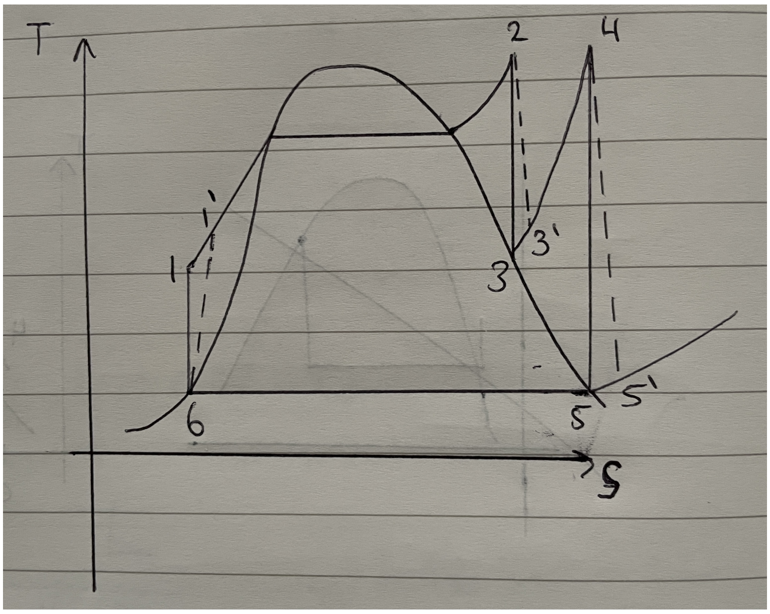

Figure 8: Graph to show Rankine cycle on T-s diagram.

In an ideal Rankine reheat cycle, we would see that the expansion and compression processes are reversible and isentropic. These are shown as straight vertical lines from 6 - 1 (pump compression), 2 - 3 and 4 - 5 (turbine expansion). However, in an actual Rankine reheat cycle, we will have irreversible losses present. For a pump, work must be done to overcome frictional forces. For a turbine, heat transfer from the turbine to the surroundings represents a loss. This process of transfer of energy as work results in entropy being produced within the system, hence we would see the entropy increase for these processes. These have been represented as the curved lines 6 - 1', 2 - 3' and 4 - 5'.

5.2 b

All components in the Rankine reheat cycle are steady flow devices. Neglect kinetic and potential energy changes as they are usually small relative to the work and heat transfer terms. Starting with SFEE formula:

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m}_1 h_1 = \dot{Q}_{out} + \dot{W}_{out} + \dot{m}_2 h_2 \tag{5.1}$$

$\dot{m}_1 = \dot{m}_2 = \dot{m}$. Dividing by $\dot{m}$:

$$q_{in} + w_{in} + h_1 = q_{out} + w_{out} + h_2 \tag{5.2}$$

The thermal efficiency is given by:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{turbines} - w_{pump}}{q_{in}} \tag{5.3}$$

We know that the total heat input will come from the primary heating stage and the reheating stage, hence:

$$q_{in} = q_{\text{primary}} + q_{\text{reheat}} = (h_2 - h_1) + (h_4 - h_3) \tag{5.4}$$

Total power output from the turbines can be calculated as:

$$w_{turbines} = w_{turb1} + w_{turb2} = (h_2 - h_3) + (h_4 - h_5) \tag{5.5}$$

Pump power can be calculated as:

$$w_{pump} = (h_1 - h_6) \tag{5.6}$$

Therefore, the thermal efficiency is:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} \tag{5.7}$$

Let us look at the principal states at each stage on our T-s diagram. State 2 (enthalpy and entropy derived from table values):

$$P_2 = 70\,\text{bar} \tag{5.8}$$
$$T_2 = 500\,°\text{C} \tag{5.9}$$
$$h_2 = 3410\,\text{kJ}\,\text{kg}^{-1} \tag{5.10}$$
$$s_2 = 6.796\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1} \tag{5.11}$$

At state 3, our entropy is the same as that at state 2, hence using table values:

$$P_3 = 5\,\text{bar} \tag{5.12}$$
$$s_3 = 6.796\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1} \tag{5.13}$$

Finding the quality:

$$6.796 = 1.860 + 4.962x \tag{5.14}$$
$$x = 0.99476 \tag{5.15}$$

Therefore:

$$h_3 = 640 + 2109\,(0.99476) \tag{5.16}$$
$$h_3 = 2737.95\,\text{kJ}\,\text{kg}^{-1} \tag{5.17}$$

State 4 (table values):

$$P_4 = 5\,\text{bar} \tag{5.18}$$
$$T_4 = 500\,°\text{C} \tag{5.19}$$
$$h_4 = 3484\,\text{kJ}\,\text{kg}^{-1} \tag{5.20}$$
$$s_4 = 8.087\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1} \tag{5.21}$$

At state 5, our entropy is the same as that at state 4, hence using table values:

$$P_5 = 0.08\,\text{bar} \tag{5.22}$$
$$s_5 = 8.087\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1} \tag{5.23}$$

Finding the quality:

$$8.087 = 0.593 + 7.634x \tag{5.24}$$
$$x = 0.98166 \tag{5.25}$$

Therefore:

$$h_5 = 174 + 2404(0.98166) \tag{5.26}$$
$$h_5 = 2531.949\,699\,\text{kJ}\,\text{kg}^{-1} \tag{5.27}$$

State 6:

$$P_6 = 0.08\,\text{bar} \tag{5.28}$$
$$s_6 = 0.593\,\text{kJ}\,\text{kg}^{-1}\,\text{K}^{-1} \tag{5.29}$$
$$h_6 = 174\,\text{kJ}\,\text{kg}^{-1} \tag{5.30}$$

State 1:

$$P_1 = 70\,\text{bar} \tag{5.31}$$
$$h_1 \approx h_6 + v_6\left(P_1 - P_6\right) \tag{5.32}$$
$$h_1 \approx 174 + \left(1.0084 \times 10^{-3}\right)\left(70 - 0.08\right)\left(\frac{10^5}{10^3}\right) \tag{5.33}$$
$$h_1 = 181.0507\,\text{kJ}\,\text{kg}^{-1} \tag{5.34}$$

Therefore, our thermal efficiency is:

$$\eta_{th} = \frac{3410 - 2737.95 + 3484 - 2531.949 - 181.0507 + 174}{3410 - 181.0507 + 3484 - 2737.95} \tag{5.35}$$
$$\eta_{th} = 0.4068 = 40.68\% \tag{5.36}$$

5.3 c

If the HPT and LPT do not expand isentropically, our thermal efficiency will become:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{(h_2 - h_{3'}) + (h_4 - h_{5'}) - (h_{1'} - h_6)}{(h_2 - h_{1'}) + (h_4 - h_{3'})} \tag{5.37}$$

As the enthalpies at the actual points is more than those at the ideal points, we see a net decrease in the thermal efficiency.

5.4 d

Population of my local area, Redbridge, is given as 300 thousand [2]. Therefore, the total power that needs to be generated is 300 MW. We know that 80% of combustion heat is used to generate vapour for the steam turbine, hence:

$$0.8 \times 748220.208 = 598\,576.1664\,\text{kJ}\,\text{kmol}^{-1} \tag{5.38}$$

Next, to find the mass flow rate of the steam, we can use the following formula:

$$\dot{W}_{net} = \dot{W}_{turbines} - \dot{W}_{pump} \tag{5.39}$$
$$\dot{W}_{net} = \dot{m}\left(h_2 - h_3\right) + \left(h_4 - h_5\right) - \left(h_1 - h_6\right) \tag{5.40}$$
$$\dot{m} = \frac{\dot{W}_{net}}{\left(h_2 - h_3 + h_4 - h_5 - h_1 + h_6\right)} \tag{5.41}$$
$$\dot{m} = \frac{3 \times 10^5}{3410 - 2737.95 + 3484 - 2531.949 - 181.0507 + 174} \tag{5.42}$$
$$\dot{m} = 185.523\,\text{kg}\,\text{s}^{-1} \tag{5.43}$$

The energy into our boiler from initial and reheat stages is:

$$\frac{\dot{Q}_{in}}{\dot{m}} = \left(h_2 - h_1\right) + \left(h_4 - h_3\right) \tag{5.44}$$
$$\dot{Q}_{in} = \left(185.523\right)\left(3410 - 181.0507 + 3484 - 2737.95\right) \tag{5.45}$$
$$\dot{Q}_{in} = 737\,453.7391\,\text{kJ}\,\text{s}^{-1} \tag{5.46}$$

Now we can calculate the mass flow rate of fuel required to supply the boiler with the above required rate of energy.

$$\dot{m}_{\mathrm{CH_4}} = \frac{\dot{Q}_{in}}{Q_{vapour}} \tag{5.47}$$

$$\dot{m}_{\mathrm{CH_4}} = \frac{737453.7391}{598576.1664} \tag{5.48}$$

$$\dot{m}_{\mathrm{CH_4}} = 1.232\,\mathrm{kmol\,s^{-1}} \tag{5.49}$$

6 Question 6

6.1 a

A heat exchange efficiency of 90% is assumed. Heat transfer out of the condenser is given by:

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_5 - h_6 \tag{6.1}$$

Mass flow rate of the steam in the condenser was already calculated as $185.523\,\mathrm{kg\,s^{-1}}$ in 5.43 and since our efficiency is only 90%, $\dot{Q}_{out} = 0.9 \times \dot{Q}$. Therefore:

$$\frac{0.9 \times \dot{Q}}{185.523} = 2531.95 - 174 \tag{6.2}$$

$$\dot{Q} = 486\,059.9532\,\mathrm{kJ\,s^{-1}} \tag{6.3}$$

Let us now look at the water entering the condenser:

$$\frac{\dot{Q}}{\dot{m}_{cw}} = h_7 - h_9 \tag{6.4}$$

$$\dot{m}_{cw} = \frac{\dot{Q}}{h_7 - h_9} \tag{6.5}$$

State 7 (from tables):

$$T_7 = 40\,^{\circ}\mathrm{C} \tag{6.6}$$

$$h_7 = 167.5\,\mathrm{kJ\,s^{-1}} \tag{6.7}$$

State 9:

$$T_9 = 25\,^{\circ}\mathrm{C} \tag{6.8}$$

$$h_9 = 104.8\,\mathrm{kJ\,s^{-1}} \tag{6.9}$$

Therefore:

$$\dot{m}_{cw} = \frac{\dot{Q}}{h_7 - h_9} \tag{6.10}$$

$$\dot{m}_{cw} = \frac{486059.9532}{167.5 - 104.8} \tag{6.11}$$

$$\dot{m}_{cw} = 7752.152\,\mathrm{kg\,s^{-1}} \tag{6.12}$$

6.2 b

Mass flow rate of air can be calculated using the following assumptions:

- Neglect kinetic and potential energies
- Isobaric process
- The control volume is in a steady state

Mass balance of dry air in the cooling tower:

$$\dot{m}_{10a} = \dot{m}_{11a} = \dot{m}_a \tag{6.13}$$

Mass balance of water in the condenser:

$$\dot{m}_7 = \dot{m}_8 + \dot{m}_9 \tag{6.14}$$

$$\dot{m}_8 = \dot{m}_7 - \dot{m}_9 \tag{6.15}$$

Mass balance of the cooling tower:

$$\dot{m}_7 + \dot{m}_8 + \dot{m}_{10} = \dot{m}_9 + \dot{m}_{11} \tag{6.16}$$

$$\dot{m}_7 - \dot{m}_9 + \dot{m}_8 = \dot{m}_1 1 - \dot{m}_{10} \tag{6.17}$$

$$2\dot{m}_8 = \dot{m}_{11} - \dot{m}_{10} \tag{6.18}$$

$$\therefore \dot{m}_8 = \frac{\dot{m}_a\left(\omega_{11} - \omega_{10}\right)}{2} \tag{6.19}$$

We also know that:

$$\dot{m}_{10} = \dot{m}_a\left(1 + \omega_{10}\right) \tag{6.20}$$

Let us now look at the steady state energy equation:

$$0 = \dot{Q}_{net} - \dot{W}_{net} + \dot{m}_7 h_7 + \dot{m}_8 h_8 - \dot{m}_9 h_9 + \left[\dot{m}_{10a} h_{10a} + \dot{m}_{10} h_{10}\right] - \left[\dot{m}_{11a} h_{11a} + \dot{m}_{11} h_{11}\right] \tag{6.21}$$

$$0 = \dot{m}_{cw}\left(h_7 - h_9\right) + \frac{h_8 \dot{m}_a\left(\omega_{11} - \omega_{10}\right)}{2} + \dot{m}_a\left[h_{10a} + \omega_{10} h_{10}\right] - \dot{m}_a\left[h_{11a} + \omega_{11} h_{11}\right] \tag{6.22}$$

$$0 = \dot{m}_{cw}\left(h_7 - h_9\right) + \dot{m}_a\left[\frac{h_8\left(\omega_{11} - \omega_{10}\right)}{2} + h_{10a} + \omega_{10} h_{10} - h_{11a} - \omega_{11} h_{11}\right] \tag{6.23}$$

$$\dot{m}_a = \frac{\dot{m}_{cw}\left(h_9 - h_7\right)}{\frac{h_8\left(\omega_{11} - \omega_{10}\right)}{2} + h_{10a} + \omega_{10} h_{10} - h_{11a} - \omega_{11} h_{11}} \tag{6.24}$$

$$labeleq : q6b1 \tag{6.25}$$

State 7 (fusion, values taken from tables, all pressures at 1 bar):

$$T_7 = 40\,^{\circ}\mathrm{C} \tag{6.26}$$

$$h_7 = 167.5\,\mathrm{kJ\,kg^{-1}} \tag{6.27}$$

State 8 and 9:

$$T_{8,9} = 25\,^{\circ}\mathrm{C} \tag{6.28}$$

$$h_{8,9} = 104.8\,\mathrm{kJ\,kg^{-1}} \tag{6.29}$$

State 10:

$$T_{10} = 25\,^{\circ}\mathrm{C} \tag{6.30}$$
$$h_{10} = 2547.2\,\mathrm{kJ\,kg^{-1}} \tag{6.31}$$

State 10 (air):

$$T_{10a} = 25\,^{\circ}\mathrm{C} \tag{6.32}$$
$$h_{10a} = 298.3326\,\mathrm{kJ\,kg^{-1}} \tag{6.33}$$
$$p_v = 0.3 \times p_{g,25\,^{\circ}\mathrm{C}} \tag{6.34}$$
$$p_v = 0.3 \times 0.03169 \tag{6.35}$$
$$p_v = 0.009\,507\,\mathrm{bar} \tag{6.36}$$
$$\omega_{10} = 0.622 \cdot \frac{0.009507}{1 - 0.009507} \tag{6.37}$$
$$\omega_{10} = 5.9701 \times 10^{-3} \tag{6.38}$$

State 11:

$$T_{11} = 36\,^{\circ}\mathrm{C} \tag{6.39}$$
$$h_{11} = 2567.1\,\mathrm{kJ\,kg^{-1}} \tag{6.40}$$

State 11 (air):

$$T_{11a} = 36\,^{\circ}\mathrm{C} \tag{6.41}$$
$$h_{11a} = 309.3866\,\mathrm{kJ\,kg^{-1}} \tag{6.42}$$
$$p_v = 0.9 \times p_{g,36\,^{\circ}\mathrm{C}} \tag{6.43}$$
$$p_v = 0.3 \times 0.05947 \tag{6.44}$$
$$p_v = 0.053\,523\,\mathrm{bar} \tag{6.45}$$
$$\omega_{11} = 0.622 \cdot \frac{0.053523}{1 - 0.053523} \tag{6.46}$$
$$\omega_{11} = 0.0352 \tag{6.47}$$

Substituting all values into 6.12. $\dot{m}_{cw}$ has been calculated in 6.12:

$$\dot{m}_a = \frac{\dot{m}_{cw}\,(h_9 - h_7)}{\dfrac{h_8\,(\omega_{11} - \omega_{10})}{2} + h_{10a} + \omega_{10} h_{10} - h_{11a} - \omega_{11} h_{11}} \tag{6.48}$$

$$\dot{m}_a = \frac{7752.152\,(104.8 - 167.5)}{\dfrac{104.8\left(0.0352 - 5.9701 \times 10^{-3}\right)}{2} + 298.3326 + \left(5.9701 \times 10^{-3} \times 2547.2\right) - 309.3866 - (0.0352 \times 2567.1)} \tag{6.49}$$

$$\dot{m}_a = 5729.291\,\mathrm{kg\,s^{-1}} \tag{6.50}$$

Hence, mass flow rate of air from atmosphere can be calculated from 6.20:

$$\dot{m}_{10} = 5729.291\,(1 + 0.0059701) \tag{6.51}$$
$$\dot{m}_{10} = 5762.46\,\mathrm{kg\,s^{-1}} \tag{6.52}$$

6.3 c

A fan blowing the air out of the cooling tower would increase the exit mass flow rate. This would cause the water in the air to condense at a faster rate, leading to a reduction in the moistness of the air leaving the tower. As the flow of air is being accelerated, the air would also be cooler. This has the net effect that a larger air mass flow rate is required and the energy to be dissipated from the system would also increase as there will be a transfer from energy from the fan to the surroundings.

7 Question 7

The Rankine reheat cycle is an effective energy generation method. It seeks to increase the efficiency of the Rankine cycle, which it does so successfully. Our steam is superheated to near the limit for turbines (620 °C), leading to high thermal efficiency, net work and heat input, however, this may require higher quality components and increased maintenance cycles to avoid failure. A high turbine temperature mitigates the risk of low quality at turbine exit, preventing damage to the turbine blades. Reheating the steam after the first turbine stage and passing it through a second turbine improves our cycle efficiency by around 4-5 percent. This is an effective and widely used method in steam power plants. 1-2 stages of reheat are a good compromise as more reheat cycles provide diminishing returns (more complex to reheat and costly for every reheat cycle). One more way to improve our thermal efficiency further is by using a regenerative cycle. An open feedwater heater can be added which is simple and inexpensive. Adding multiple feedwater heaters can become costly and complex, since a pump is required for each cycle.

References

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