



UNIVERSITY COLLEGE LONDON

MENG MECHANICAL ENGINEERING

CEGE0016 FINANCIAL ASPECTS OF PROJECT ENGINEERING

## TOPIC NOTES

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# Contents

<b>List of Figures</b>	<b>vi</b>
<b>List of Tables</b>	<b>viii</b>
<b>I Lecture Notes</b>	<b>1</b>
<b>1 Module Introduction</b>	<b>2</b>
1.1 Teaching Team . . . . .	2
1.2 Assessment . . . . .	2
1.3 Timeline . . . . .	3
<b>2 Introduction to public economics</b>	<b>4</b>
2.1 Introduction and overview of mixed economies . . . . .	4
2.1.1 Aims . . . . .	4
2.1.2 Economies . . . . .	4
2.1.3 Mixed economies . . . . .	4
2.1.4 Private sector . . . . .	5
2.1.5 Public sector . . . . .	5
2.1.6 Classical microeconomics . . . . .	5
2.1.7 Economic models . . . . .	5
2.2 Consumer theory . . . . .	6
2.2.1 Consumer theory: continuous goods . . . . .	6
2.2.2 Utility . . . . .	6
2.2.3 Choice between two continuous goods . . . . .	6
2.2.4 Utility maximisation with budget constraint . . . . .	7
2.2.5 Trade between two agents . . . . .	7
2.2.6 Edgeworth box . . . . .	8
2.2.7 Pareto efficiency . . . . .	8
2.3 Producer theory . . . . .	9
2.3.1 Production of goods and services . . . . .	9
2.3.2 Marginal cost . . . . .	9
2.3.3 Pareto efficient production . . . . .	10
2.3.4 Single market efficiency . . . . .	11
2.4 Fundamental theorems of welfare economics . . . . .	11
2.4.1 Competitive economies . . . . .	11
2.4.2 Efficiency vs equality . . . . .	12
2.4.3 Wealth distribution . . . . .	12
2.4.4 Efficiency and equality? . . . . .	13
2.5 Public sector . . . . .	13
2.5.1 Role of government: the theory . . . . .	13
2.5.2 Role of government: reality . . . . .	13

2.5.3	Market failures . . . . .	13
2.6	Review and recap . . . . .	14
2.6.1	A need for better understanding? . . . . .	14
<b>3</b>	<b>Public Goods and Externalities</b>	<b>15</b>
3.1	Introduction . . . . .	15
3.1.1	Aims . . . . .	15
3.1.2	Market equilibrium . . . . .	15
3.1.3	Public goods and externalities . . . . .	16
3.2	Public goods . . . . .	16
3.2.1	Two dimensions of public good . . . . .	16
3.2.2	Rivalry and capacity . . . . .	16
3.2.3	Continuous scale . . . . .	17
3.2.4	Public goods in free markets . . . . .	17
3.2.5	Public goods and market failure . . . . .	17
3.2.6	Privatisation in the public sector . . . . .	18
3.3	Externalities . . . . .	18
3.3.1	Positive and negative externalities . . . . .	18
3.3.2	Negative production externality . . . . .	18
3.3.3	Negative consumption externality . . . . .	19
3.3.4	Positive production externality . . . . .	19
3.3.5	Positive consumption externality . . . . .	20
3.3.6	Externalities and property rights . . . . .	20
3.3.7	Managing externalities . . . . .	20
3.3.8	Externalities and the environment . . . . .	20
3.3.9	The doughnut economic model . . . . .	21
<b>4</b>	<b>Techniques for Project Evaluation</b>	<b>22</b>
4.1	Introduction to Capital and Interest . . . . .	22
4.1.1	Capital . . . . .	22
4.1.2	Interest . . . . .	22
4.2	Simple interest . . . . .	22
4.3	Compound interest . . . . .	23
4.3.1	Compound vs simple interest . . . . .	23
4.4	What is project evaluation . . . . .	23
4.5	Minimum attractive rate of return (MARR) . . . . .	24
4.6	Project evaluation using Net Present Value . . . . .	24
4.6.1	Net present value (NPV) . . . . .	24
4.6.2	Example . . . . .	25
4.6.3	Advantages and disadvantages of NPV . . . . .	26
4.7	Project evaluation using Internal Rate of Return . . . . .	26
4.7.1	Internal Rate of Return (IRR) . . . . .	26
4.7.2	Example . . . . .	27
4.7.3	Advantages and disadvantages of IRR . . . . .	27
4.8	Project evaluation using Payback Period . . . . .	28
4.8.1	Simple Payback Period Example . . . . .	28
4.8.2	Simple & Discounted Payback Period Example . . . . .	28
4.8.3	Advantages and disadvantages of payback period . . . . .	28
<b>5</b>	<b>Cost-Benefit Analysis</b>	<b>30</b>
5.1	What is cost-benefit analysis? . . . . .	30
5.2	The Benefit-Cost Ratio method . . . . .	30
5.2.1	Conventional BCR method . . . . .	31

5.2.2	Modified BCR method . . . . .	31
5.2.3	Conventional and modified BCRs . . . . .	32
5.2.4	What value of $i$ to use? . . . . .	32
5.2.5	Why do conventional and modified BCRs lead to the same decision? . . . . .	32
5.2.6	Example 1 . . . . .	32
5.2.7	Issues of concern using BCRs . . . . .	33
5.2.8	Treatment of disbenefits . . . . .	34
5.2.9	Example 2 . . . . .	34
5.2.10	Treatment of certain cash flows . . . . .	34
5.2.11	Example 3 . . . . .	34
5.2.12	Treatment of disbenefits/certain cash flows . . . . .	35
5.3	Evaluating independent projects using the BCR method . . . . .	35
5.3.1	What are independent projects? . . . . .	35
5.3.2	Example 4 . . . . .	35
5.4	Evaluating mutually exclusive projects using the BCR method . . . . .	36
5.4.1	What are mutually exclusive projects . . . . .	36
5.4.2	Procedure for evaluating mutually exclusive projects . . . . .	36
5.4.3	Example 5 . . . . .	37
5.4.4	Mutually exclusive projects with unequal lives . . . . .	37
5.4.5	Example 6 . . . . .	38
<b>6</b>	<b>Companies and Financial Accounting</b>	<b>39</b>
6.1	Companies . . . . .	39
6.1.1	What is a company? . . . . .	39
6.1.2	Structure of a Limited Liability Company limited by shares . . . . .	40
6.1.3	What is a shareholder? . . . . .	40
6.1.4	What is the Board of Directors? . . . . .	40
6.1.5	What is a company secretary? . . . . .	41
6.1.6	Limited Liability Company . . . . .	41
6.2	What is accounting? . . . . .	41
6.3	The fundamental accounting equation . . . . .	42
6.3.1	Some important definitions . . . . .	42
6.4	Balance sheet . . . . .	43
6.5	Income statement . . . . .	43
6.5.1	Some important definitions . . . . .	43
6.6	Worked example . . . . .	43
6.7	Cash flow statement . . . . .	45
6.7.1	Cash flow statement . . . . .	46
6.7.2	Financial ratios . . . . .	46
<b>7</b>	<b>Critical Path and Delay Analysis in Construction Projects</b>	<b>50</b>
7.1	Kroll Expert Services . . . . .	50
7.2	Dispute in construction and the role of experts . . . . .	51
7.2.1	What is a dispute? . . . . .	51
7.2.2	Typical sequence to a trial . . . . .	51
7.2.3	Independent expert . . . . .	51
7.3	Critical path and delay analysis . . . . .	52
7.3.1	What is delay analysis? . . . . .	52
7.3.2	What is the critical path? . . . . .	52
7.3.3	Basics of delay . . . . .	52
7.3.4	Information sources . . . . .	53
7.3.5	Methods of delay analysis . . . . .	54
7.3.6	Example 1 . . . . .	54

7.3.7	Why so many methods? . . . . .	63
7.3.8	Example 2 . . . . .	63
7.4	Summary . . . . .	65
<b>8</b>	<b>Financial Project Planning</b>	<b>66</b>
8.1	Budgeting . . . . .	66
8.1.1	What is budgeting? . . . . .	66
8.1.2	What information is used for budgeting? . . . . .	66
8.1.3	Types of budget . . . . .	66
8.1.4	Example 1: Operating budget . . . . .	66
8.2	Cash flow forecast . . . . .	67
8.2.1	Example 2 . . . . .	68
8.2.2	Time value of money . . . . .	69
8.2.3	Example 3 . . . . .	69
8.2.4	Example 4 . . . . .	70
8.2.5	Cash flow forecast: table . . . . .	71
8.2.6	Example 5 . . . . .	71
8.3	S-curves . . . . .	72
8.3.1	What are s-curves used for? . . . . .	72
8.3.2	Why are s-curves useful? . . . . .	72
8.3.3	Example 6 . . . . .	73
8.4	Summary . . . . .	74
<b>9</b>	<b>Project Risk Analysis</b>	<b>76</b>
9.1	What is risk? . . . . .	76
9.1.1	Sources of uncertainty . . . . .	76
9.2	Measuring risk . . . . .	76
9.2.1	Random variables . . . . .	76
9.2.2	Discrete random variables . . . . .	76
9.2.3	Continuous random variables . . . . .	77
9.2.4	Expected value . . . . .	77
9.2.5	Variance . . . . .	78
9.2.6	Multiplying by a constant . . . . .	78
9.2.7	Multiplying by another random variable . . . . .	78
9.2.8	Adding or subtracting random variables . . . . .	78
9.3	Example risk analysis . . . . .	79
9.3.1	Worked example 1 . . . . .	79
9.3.2	Risk analysis with continuous random variables . . . . .	79
9.3.3	Risk analysis with multiple independent random variables . . . . .	80
9.4	Decision trees . . . . .	80
9.4.1	Worked example of a decision tree . . . . .	81
<b>10</b>	<b>Project Management</b>	<b>83</b>
10.1	Environmental liability . . . . .	83
10.1.1	What is environmental liability? . . . . .	83
10.1.2	Environmental impact assessment . . . . .	83
10.2	Conditions of contract . . . . .	84
10.2.1	What is a contract? . . . . .	84
10.2.2	Types of contract . . . . .	84
10.2.3	Why are contracts needed? . . . . .	85
10.2.4	Force majeure . . . . .	85
10.2.5	Conditions of contract . . . . .	85
10.3	Summary . . . . .	85

<b>II Sample Exam Papers</b>	<b>87</b>
<b>11 Sample Exam Paper 1</b>	<b>88</b>
11.1 Question 1 . . . . .	88
11.2 Question 2 . . . . .	88
11.3 Question 3 . . . . .	89
11.4 Question 4 . . . . .	91
<b>12 Sample Exam Paper 2</b>	<b>95</b>
12.1 Question 1 . . . . .	95
12.2 Question 2 . . . . .	95
12.3 Question 3 . . . . .	96
12.4 Question 4 . . . . .	97

# List of Figures

1.1	Module Assessment.	2
1.2	Module Timeline.	3
2.1	Classical microeconomic models.	5
2.2	.	6
2.3	.	7
2.4	.	7
2.5	.	8
2.6	.	9
2.7	.	9
2.8	.	10
2.9	.	11
2.10	.	12
2.11	.	13
3.1	Market equilibrium.	15
3.2	Continuous scale.	17
3.3	Public goods in free markets.	17
3.4	Negative production externality.	18
3.5	Negative consumption externality.	19
3.6	Positive production externality.	19
3.7	Positive consumption externality.	20
3.8	Doughnut economic model.	21
4.1	Blue: simple interest, Orange: compound interest	23
4.2	Effect of interest rate on PV.	25
4.3	Internal Rate of Return.	26
4.4	Simple and discounted payback periods.	28
5.1	Flow chart for evaluating mutually exclusive projects	36
6.1	Structure of a Limited Liability Company limited by shares.	40
7.1	Dispute pathway.	51
7.2	Work programmes can be as simple as a few bar charts.	52
7.3	Or a complex schedule prepared in a specialist software.	53
7.4	Example project with two buildings.	54
7.5	Impact as-planned analysis for example project.	55
7.6	Time impact analysis for example project.	56
7.7	Time slice windows analysis for example project. Window 1	57
7.8	Time slice windows analysis for example project. Window 2.	57
7.9	Time slice windows analysis for example project. Window 3.	58
7.10	Time slice windows analysis for example project. Window 4.	58

7.11	As-planned vs as-built windows analysis for example project. Step 1 & 2 . . . . .	59
7.12	As-planned vs as-built windows analysis for example project. Step 3 . . . . .	59
7.13	As-planned vs as-built windows analysis for example project. Step 4 . . . . .	60
7.14	As-planned vs as-built windows analysis for example project. Step 5 . . . . .	60
7.15	Longest path analysis for example project. Step 1 & 2 . . . . .	61
7.16	Longest path analysis for example project. Step 3 . . . . .	61
7.17	Longest path analysis for example project. Step 4 . . . . .	62
7.18	Longest path analysis for example project. Step 5 . . . . .	62
7.19	Collapsed as-built analysis for example project. . . . .	63
7.20	A simple project programme. . . . .	63
7.21	Actual programme. . . . .	64
7.22	Comparison between planned and actual programmes. . . . .	64
7.23	Critical path analysis. . . . .	64
7.24	But-for delay. . . . .	65
7.25	Reduced exposure to liquidated damages. . . . .	65
8.1	Cash flow forecast. . . . .	68
8.2	Cash flow forecast. . . . .	68
8.3	Cash flow diagram for ABC corporation. . . . .	68
8.4	Time value of money in cash flow forecast. . . . .	69
8.5	Cash flow diagram. Example 3. . . . .	69
8.6	Cash flow diagram. Example 4. . . . .	70
8.7	S-curve. . . . .	72
8.8	S-curve with real-time and projected progress. . . . .	73
8.9	S-curve for example 6. . . . .	74
8.10	S-curve for example 6 with cash flow analysis. . . . .	74
9.1	Decision tree. . . . .	81
9.2	Decision tree for worked example. . . . .	82
11.1	Decision tree for Question 3. The time period is measured in years and represents the end of the corresponding year. . . . .	89
12.1	Decision tree for Question 3. EOY denotes end-of-year and PV denotes the net present value associated with the alternative. . . . .	96

# List of Tables

5.1	Example 1. . . . .	33
5.2	Example 4 information. . . . .	35
5.3	Example 5 information. . . . .	37
5.4	Example 5 required information. . . . .	37
5.5	Example 6 information. . . . .	38
5.6	Example 6 required information. . . . .	38
6.1	Table to show Assets, Liabilities and Owners' Equity. . . . .	42
6.2	Balance sheet. . . . .	43
6.3	Income statement. . . . .	43
6.4	Financial position of John Deere Ltd. . . . .	44
6.5	Deere Consulting Ltd. income statement, month ended 31 May, 2021. . . . .	44
6.6	Cash flow statement. . . . .	46
7.1	Methods of delay analysis. . . . .	54
7.2	Justification for various methods of analysis. . . . .	63
8.1	Deere Consulting Ltd. Income Statement. Year ended 31 December, 2021. . . . .	67
8.2	Deere Consulting Ltd. Income Statement. Year ended 31 December, 2022. . . . .	67
8.3	Alternative A: Rebuild (overhaul) the existing HVAC system. . . . .	71
8.4	Alternative B: Install a new HVAC system that uses existing equipment. . . . .	71
8.5	Example 5 table. . . . .	71
8.6	Example 6 table. . . . .	73
9.1	Worked example 1 table. . . . .	79
9.2	Risk analysis with multiple independent random variables table. . . . .	80
9.3	Risk analysis with multiple independent random variables table with expected and variance values. . . . .	80
9.4	Decision tree example table. . . . .	81
10.1	Table of exemplar contaminants and exposure pathways. . . . .	83
11.1	UCL Engineering plc income statement. . . . .	91
11.2	UCL Engineering plc balance sheet. Note the inventory for the company in 2027 is valued at £2,450 million. . . . .	92
11.3	Financial ratios and their industry average values for Question 4. . . . .	93
12.1	Potential capital costs and benefits of the equipment for Question 2. . . . .	95
12.2	UCL Engineering plc income statement. . . . .	98
12.3	UCL Engineering plc balance sheet. Note the inventory for the company in 2027 is valued at £2,350 million. . . . .	98
12.4	Financial ratios and their industry average values for Question 4. . . . .	99

# **Part I**

# **Lecture Notes**

# Chapter 1

## Module Introduction

### 1.1 Teaching Team

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### 1.2 Assessment

Assessment Type	Description	Weighting
Quiz	Three multiple-choice quizzes <b>over the course of the module</b> , to be turned in within <b>24 hours</b>	15% (3 x 5%)
Course Project	Written report and recorded presentation on a project to be set <b>halfway through the module</b> , to be completed in <b>6 weeks</b>	40%
Individual Exercise	Written set of questions <b>completed in examination conditions</b>	45%

Figure 1.1: Module Assessment.

### 1.3 Timeline

Lecture Number	Date	Lecture Topic	Assessment Set	Assessment Due
1	3 <sup>rd</sup> October 2022	Introduction to Module; Public Economics		
2	10 <sup>th</sup> October 2022	Public Goods and Externalities		
3	17 <sup>th</sup> October 2022	Techniques for Project Evaluation	Quiz 1 set Wednesday 19 <sup>th</sup> October 12pm	Quiz 1 due Thursday 20 <sup>th</sup> October 12pm
4	24 <sup>th</sup> October 2022	Cost-Benefit Analysis	Quiz 2 set Wednesday 26 <sup>th</sup> October 12pm	Quiz 2 due Thursday 27 <sup>th</sup> October 12pm
5	31 <sup>st</sup> October 2022	Guest lectures on HS2 and the Lower Thames Crossing; Introduction to Course Project	Course project set Monday 31 <sup>st</sup> October in class	Course project presentation due Monday 12 <sup>th</sup> December in class; Course project report due Friday 16 <sup>th</sup> December at 5pm
<b>-----Reading Week-----</b>				
6	14 <sup>th</sup> November 2022	Companies and Financial Accounting	Quiz 3 set Wednesday 16 <sup>th</sup> November 12pm	Quiz 3 due Thursday 17 <sup>th</sup> November 12pm
7	21 <sup>st</sup> November 2022	Project Planning Financial Project Planning		
8	28 <sup>th</sup> November 2022	Risk Analysis		
9	5 <sup>th</sup> December 2022	Project Management		
10	12 <sup>th</sup> December 2022	Course Project Presentations	Course project presentation delivered in class	
Lecture Number	Date	Lecture Topic	Assessment Set	Assessment Due
1	3 <sup>rd</sup> October 2022	Introduction to Module; Public Economics		
2	10 <sup>th</sup> October 2022	Public Goods and Externalities		
3	<b>17<sup>th</sup> October 2022</b>	Techniques for Project Evaluation	Quiz 1 set Wednesday 19 <sup>th</sup> October 12pm	Quiz 1 due Thursday 20 <sup>th</sup> October 12pm
4	<b>24<sup>th</sup> October 2022</b>	Cost-Benefit Analysis	Quiz 2 set Wednesday 26 <sup>th</sup> October 12pm	Quiz 2 due Thursday 27 <sup>th</sup> October 12pm
5	<b>31<sup>st</sup> October 2022</b>	Guest lectures on HS2 and the Lower Thames Crossing; Introduction to Course Project	Course project set Monday 31 <sup>st</sup> October in class	Course project presentation due Monday 12 <sup>th</sup> December in class; Course project report due Friday 16 <sup>th</sup> December at 5pm
<b>-----Reading Week-----</b>				
6	<b>14<sup>th</sup> November 2022</b>	Companies and Financial Accounting	Quiz 3 set Wednesday 16 <sup>th</sup> November 12pm	Quiz 3 due Thursday 17 <sup>th</sup> November 12pm
7	21 <sup>st</sup> November 2022	Project Planning Financial Project Planning		
8	28 <sup>th</sup> November 2022	Risk Analysis		
9	5 <sup>th</sup> December 2022	Project Management		
10	<b>12<sup>th</sup> December 2022</b>	Course Project Presentations	Course project presentation delivered in class	

Figure 1.2: Module Timeline.

# Chapter 2

## Introduction to public economics

### 2.1 Introduction and overview of mixed economies

#### 2.1.1 Aims

1. Understand the roles of the private and public sector in mixed economies
2. Become (re)familiar with key microeconomic concepts of consumption and production, including Pareto optimality and market equilibrium
3. Understand the trade-offs between market efficiency and equitable distribution of resources
4. Be aware of the assumptions and limitations of fundamental theorems and associated neoclassical economics

#### 2.1.2 Economies

##### A simplified definition

An area of *production, trade* and *consumption* of goods and services by different *agents*.

##### Agents

- Individuals and households
- Businesses
- Government

#### 2.1.3 Mixed economies

##### Economies today are predominantly *mixed economies*

Private sector:

profit-maximising firms operate in competitive markets

Public sector:

governments/other organisations make interventions in those markets

## 2.1.4 Private sector

### Welfare economics

"he is in this, as in many other cases, led by an invisible hand to promote an end which has no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectively than when he really intends to promote it. (Smith 1776)

## 2.1.5 Public sector

### Public sector aims to balance trade-offs

In particular

efficiency of competitive markets vs. improved equity of distribution of income from regulation

Understanding the role of public sector in mixed economies first requires us to understand operation of free-markets

## 2.1.6 Classical microeconomics

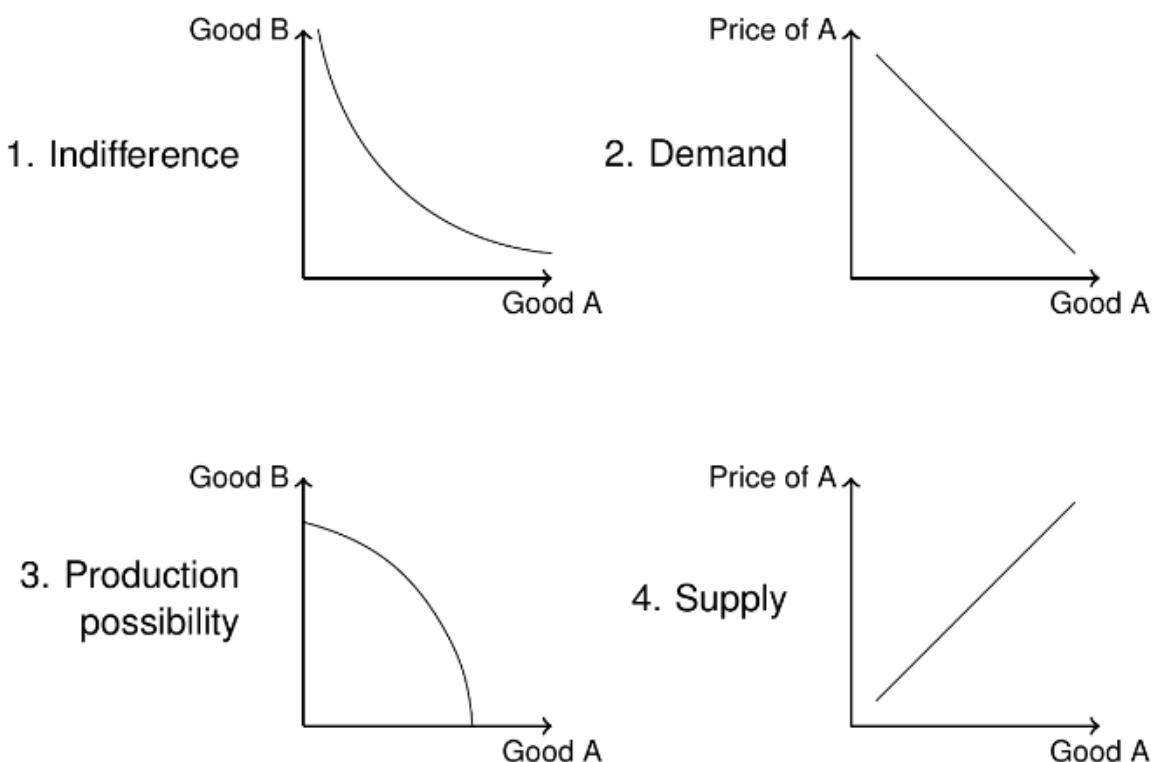


Figure 2.1: Classical microeconomic models.

## 2.1.7 Economic models

"Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful" (Box and Draper 1987)

## 2.2 Consumer theory

### 2.2.1 Consumer theory: continuous goods

- $J$  continuous goods, each good denoted by  $j = 1, \dots, J$
- Each good has associated price per unit  $p_j$ 
  - e.g.,  $j = 1$  corresponds to milk,  $p_j = 90$  pence/litre
  - $j = 2$  corresponds to eggs,  $p_j = 16$  pence/egg
- Consider choice of agent, with total budget  $I$ 
  - Assume agent represents individual
  - Individual chooses quantity  $q_j$  of each good  $j$ , subject to budget constraint

$$\sum_{j=1}^J (p_j q_j) \leq I \quad (2.1)$$

### 2.2.2 Utility

- Individual's choice of goods represented by consumption bundle  $Q$ 
  - i.e. vector of  $q_1, \dots, q_J$
- Individual gets utility  $U(Q)$  from  $Q$ 
  - Utility represents how individuals perceived benefit from consuming/owning  $Q$
  - Assume  $U(Q)$  increases monotonically with increasing  $q_i$

### 2.2.3 Choice between two continuous goods

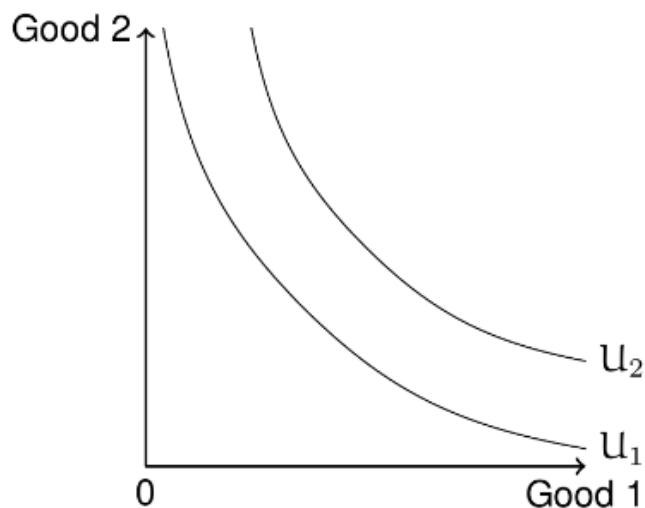


Figure 2.2:

Indifference curves:

Different combinations of each good that yield same level of utility

Marginal Rate of Substitution (MRS):

Gradient of indifference curve

- i.e. how many units of good 2 individual would substitute for 1 unit of good 1
- Assumed to be convex

#### 2.2.4 Utility maximisation with budget constraint

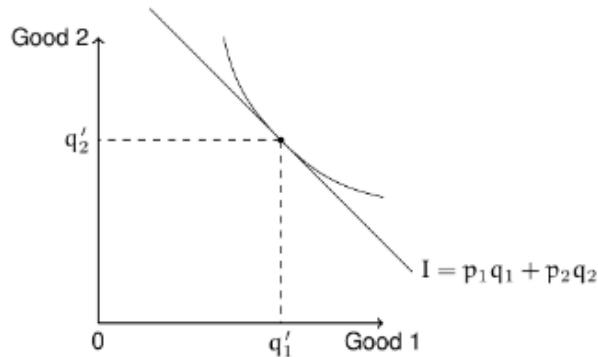


Figure 2.3:

- Assume agents try to maximise utility
- Under maximal utility assumption, optimal solution when indifference curve is tangent to budget line

$$\text{MRS} = \frac{p_1}{p_2} \quad (2.2)$$

#### 2.2.5 Trade between two agents

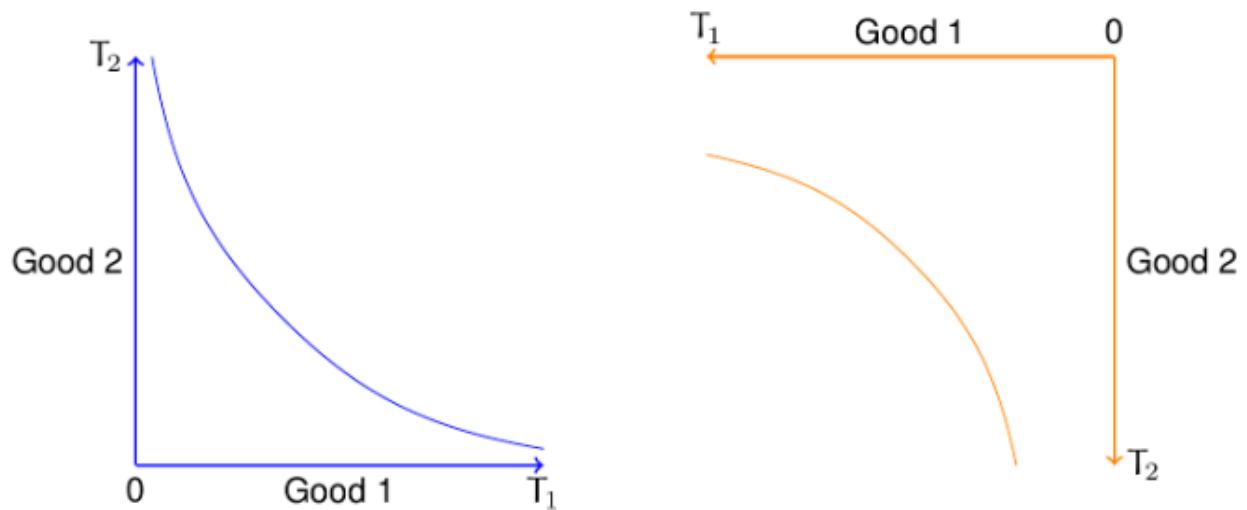


Figure 2.4:

## 2.2.6 Edgeworth box

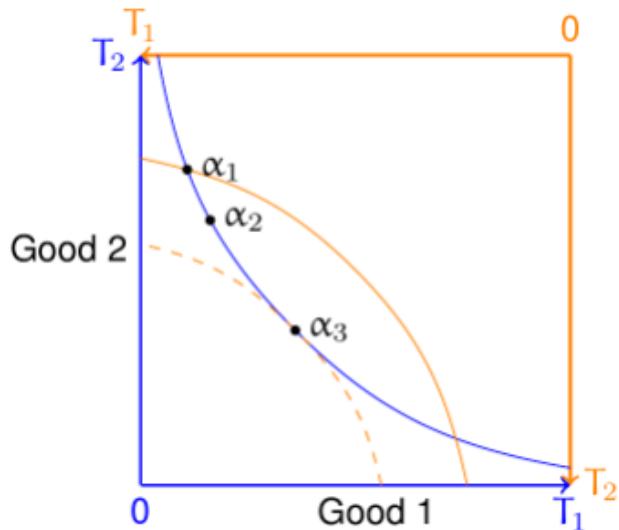


Figure 2.5:

Edgeworth box:

depicts distribution of commodities in closed economy between two agents

Pareto improvement:

A reallocation that improves utility of one individual without reducing anyone else's utility

- $\alpha_2$  is Pareto improvement of  $\alpha_1$
- $\alpha_3$  is Pareto improvement of  $\alpha_2$

Pareto optimal/efficient:

An allocation from which no-one can improve utility without reducing someone else's

- $\alpha_3$  is Pareto optimal

## 2.2.7 Pareto efficiency

- Pareto efficient solutions happen when indifference curves have equal gradient
- i.e. each agent has equal MRS

Pareto frontier:

Set of all possible Pareto efficient allocations

## 2.3 Producer theory

### 2.3.1 Production of goods and services

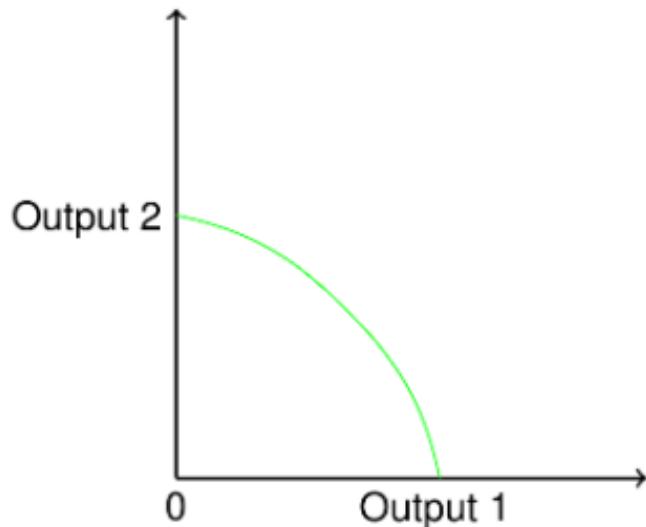


Figure 2.6:

Production Possibilities Frontier (PPF):

possible combinations of outputs (e.g. goods/services) that can be produced by economy with fixed inputs technology

- all points on PPF are production efficient: no more of one output can be produced without sacrificing the other

Marginal Rate of Transformation (MRT):

Gradient of PPF

- Measures amount of Output 2 that must be sacrificed to produce additional unit of Output 1

### 2.3.2 Marginal cost

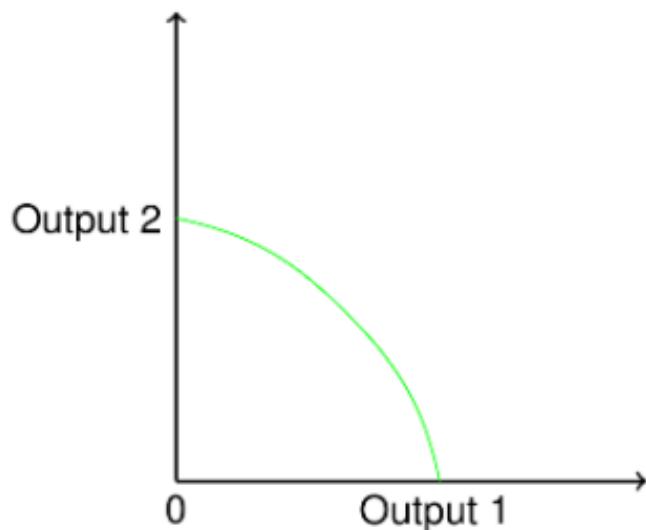


Figure 2.7:

Marginal cost:

Cost of producing one additional unit of output

$$MRT = \frac{MC_{output_1}}{MC_{output_2}} \quad (2.3)$$

- PPF often assumed to be concave under certain conditions (i.e. rewards diversity)
  - Easier to obtain low-hanging fruit

### 2.3.3 Pareto efficient production

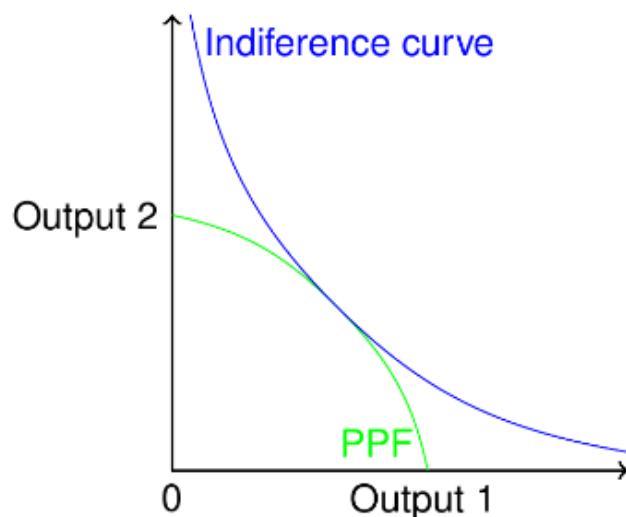


Figure 2.8:

Pareto efficiency only achieved when production of goods matches consumers' willingness to pay

- Gradient of PPF matches combined indifference curve of all consumers
- i.e.  $MRS = MRT$

### 2.3.4 Single market efficiency

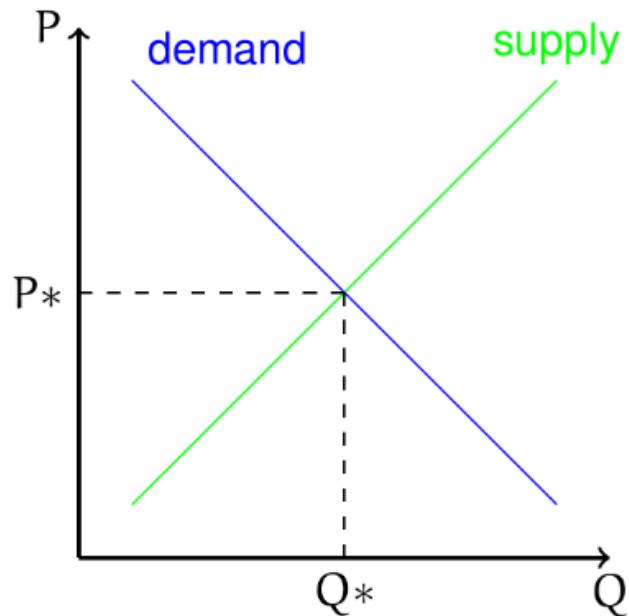


Figure 2.9:

Market equilibrium occurs when supply equals demand

- marginal benefit of consumption is equal to marginal cost of production

## 2.4 Fundamental theorems of welfare economics

### 2.4.1 Competitive economies

#### Fundamental theorems of welfare economics

If the economy is competitive, it is Pareto efficient

### 2.4.2 Efficiency vs equality

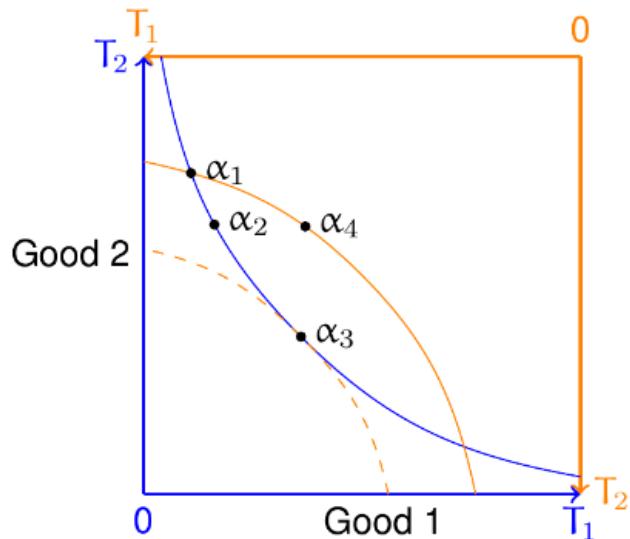


Figure 2.10:

- So far, considered only efficiency of allocations
  - $\alpha_3$  is Pareto efficient
- Social welfare also depends on equitable distribution of goods
- How do we choose between  $\alpha_3$  and  $\alpha_4$ 
  - Do we need to?

### 2.4.3 Wealth distribution

#### Fundamental theorems of welfare economics

- If the economy is competitive, it is Pareto efficient
- Every Pareto efficient resource allocation can be obtained with competitive market process with an appropriate initial redistribution of wealth

#### 2.4.4 Efficiency and equality?

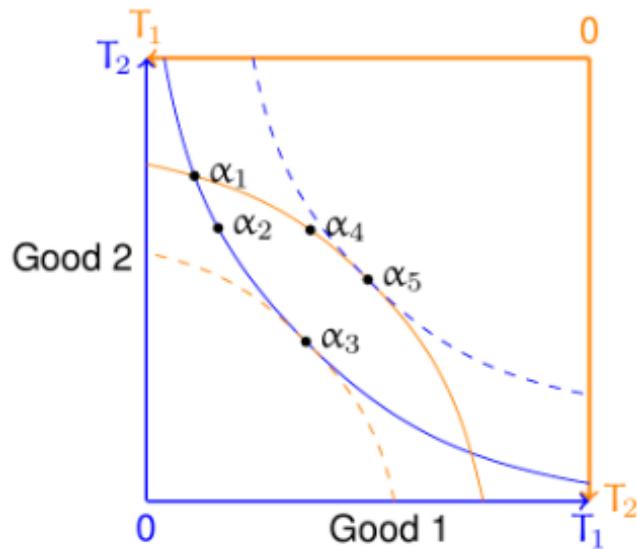


Figure 2.11:

According to second fundamental theorem:

more equitable allocation can be found through suitable assignment of initial endowments and free trade

### 2.5 Public sector

#### 2.5.1 Role of government: the theory

**When and how should governments make interventions in mixed economies?**

According to first fundamental theorem:

government interventions that reduce competition make economies less efficient

Redistribute income and leave markets alone?

#### 2.5.2 Role of government: reality

Note... Governments play an active role in all major economies, including:

- Allocation
- Distribution
- Regulation
- Stabilisation

#### 2.5.3 Market failures

Several situations result in the failure of free markets to achieve optimal solutions. Causes include:

- existence and need for public goods
- existence of externalities

- imperfect competition
- incomplete information and uncertainty

## 2.6 Review and recap

### 2.6.1 A need for better understanding?

- Several strong assumptions
  - Individuals as rational utility maximisers
  - Equivalence of utility, value and price
  - Markets as continuous
  - Statics tastes and preferences
  - Perfect competition
- Fundamental welfare economic theory does not capture
  - unpaid labour
  - social exchange
  - long-term resilience and sustainability

# Chapter 3

## Public Goods and Externalities

### 3.1 Introduction

#### 3.1.1 Aims

1. Recall the two dimensions of public good (rivalry and excludability) and understand how they lead to market failure
2. Identify and describe the occurrence and results of positive and negative externalities
3. Understand the role of the public sector in managing market failures arising from public goods and externalities
4. Be aware of the particular challenges related to climate externalities
5. Become familiar with the dimensions of the environmental ceiling and social foundations of the doughnut economic model

#### 3.1.2 Market equilibrium

Market equilibrium occurs when supply equals demand. Private marginal benefit of consumption is equal to private marginal cost of production.

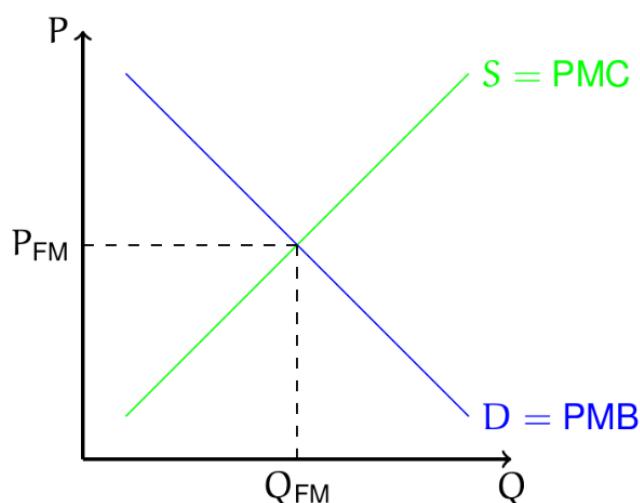


Figure 3.1: Market equilibrium.

### 3.1.3 Public goods and externalities

#### Definitions

Public goods:

Goods which are both non-excludable and have non rivalrous consumption.

Externalities:

Positive or negative effects on third parties arising from the production or consumption of goods, that are not reflected in the price

#### Market failures

Public goods and externalities cause market failures in the allocation of goods/services at the free-market equilibrium.

- i.e.  $Q_{\text{free-market}}$  is not optimal
- addressed through the allocative role of government

## 3.2 Public goods

### 3.2.1 Two dimensions of public good

**Excludability:** the degree to which access to a good, service or resource can be restricted.

- **Excludable:** agents can easily be prevented from using the good/service
- **Non-excludable:** preventing agents from consuming the good/service is impossible (or very expensive)

**Rivalry:** the degree of which consumption by one party affects another party's use of the good.

- **Rivalrous:** consumption by one agent prevents simultaneous consumption by other agents, or reduces the marginal benefit of other agents
- **Non-rivalrous:** once it is provided, the additional resource cost of another person consuming the good is zero (i.e.  $MC = 0$ ) and the marginal benefit does not decrease with number of users
- **(Anti-rivalrous:** marginal benefit increases with the number of users, e.g. social network)

### 3.2.2 Rivalry and capacity

Goods are often non-rivalrous up to a certain capacity, above which they are rivalrous e.g. public transport (bus/train), road bridge, internet bandwidth.

### 3.2.3 Continuous scale

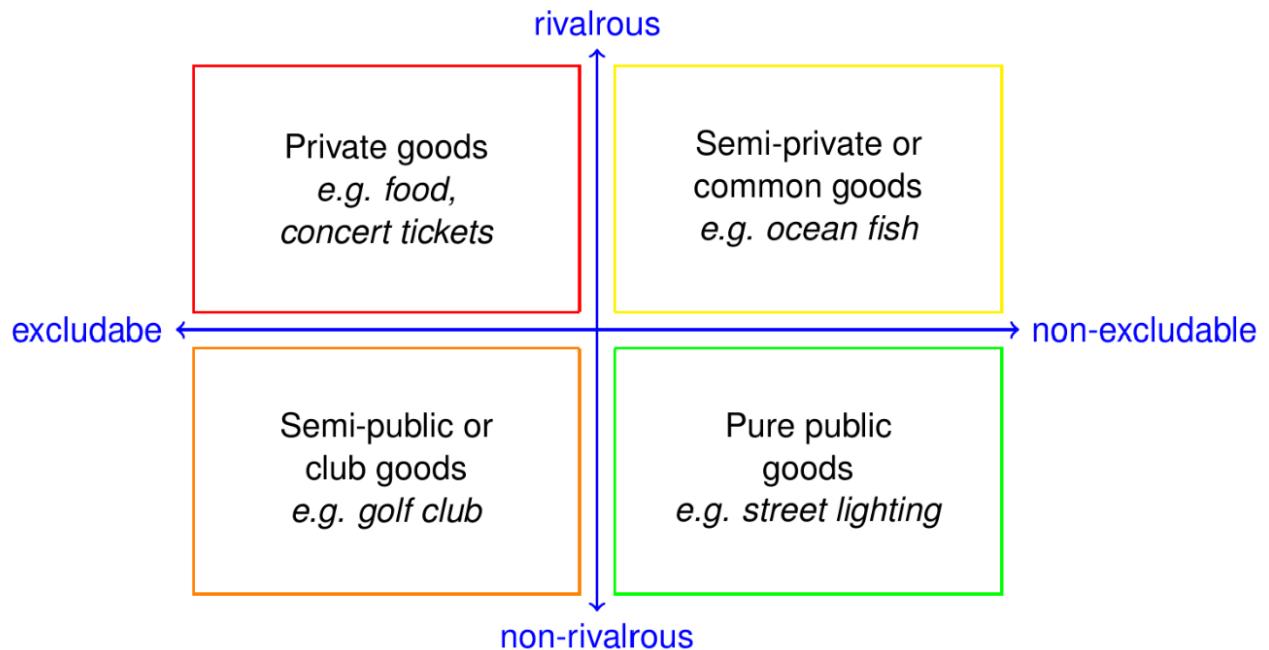


Figure 3.2: Continuous scale.

### 3.2.4 Public goods in free markets

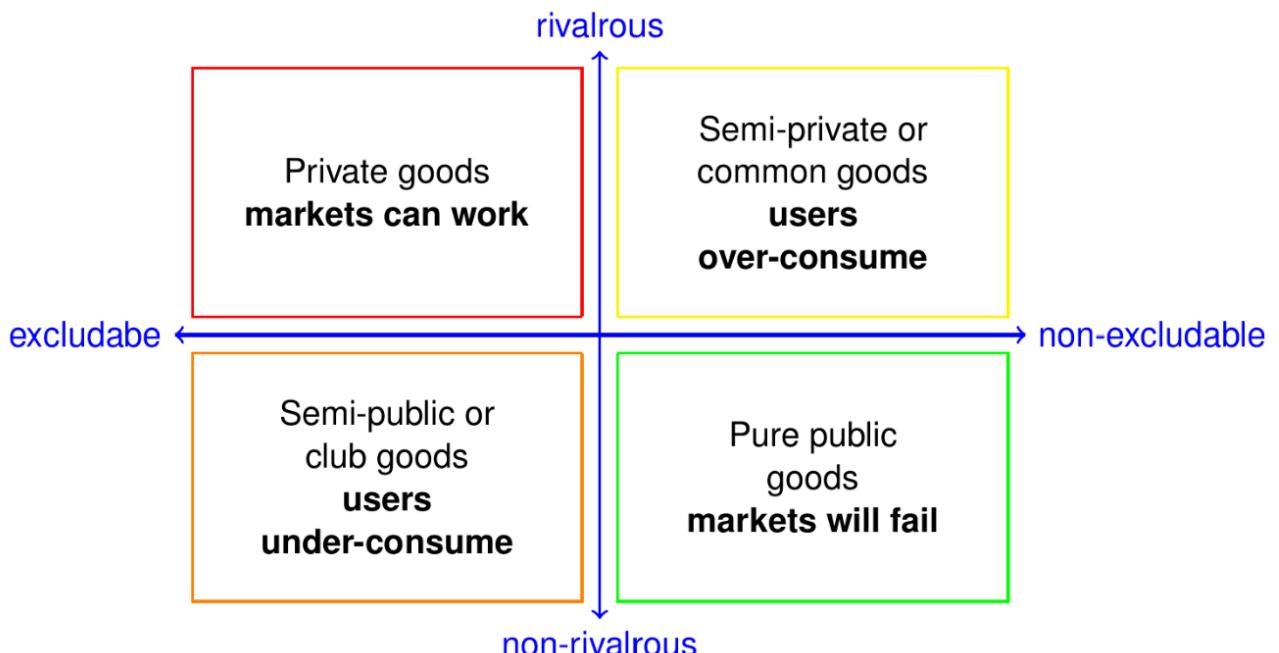


Figure 3.3: Public goods in free markets.

### 3.2.5 Public goods and market failure

Pure public goods are **non-excludable**

- Producers cannot exclude agents from consumption
- Unable to charge and therefore make profit
- Therefore (in theory) would not be produced through market action!

Possibility of funding via private cooperative, but...

### Free rider problem

as size of cooperative increases, possibility of avoiding contributing increases

### Public sector provision

Large group public goods supplied from public sector budget

- Allocative role of government

### 3.2.6 Privatisation in the public sector

Note... Public sector provision  $\neq$  equivalent public sector production.

The creation of markets in public services has been one of the great defining shifts in the way government has been run over the past 30 years (Gash and Roos 2012)

## 3.3 Externalities

### 3.3.1 Positive and negative externalities

Externalities

when the actions of one economic agent directly affect other agent(s) outside the market mechanism (production/consumption)

Externalities can arise from either production or consumption and have a net positive or negative effect.

### 3.3.2 Negative production externality

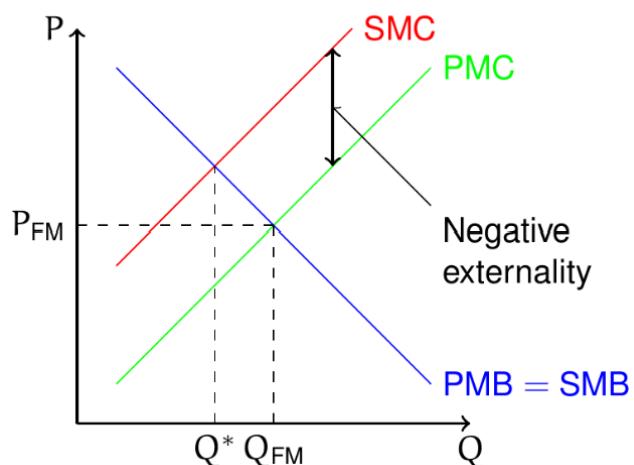


Figure 3.4: Negative production externality.

Production of output reduces well-being of third parties not involved in transaction,

- e.g. oil spills during fuel production pollute oceans and damage wildlife
- leads to overproduction

### 3.3.3 Negative consumption externality

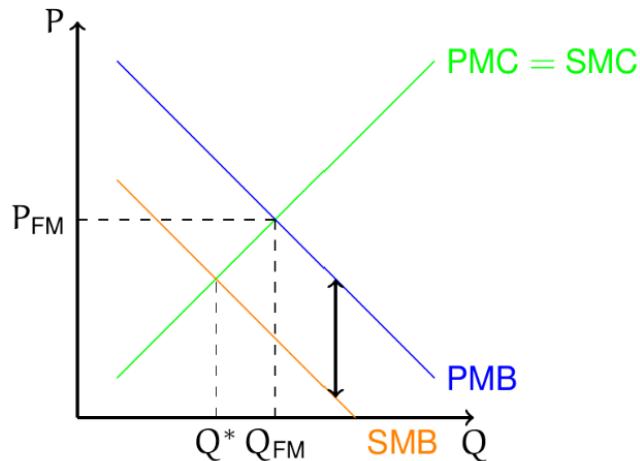


Figure 3.5: Negative consumption externality.

Consumption of output reduces well-being of third parties not involved in transaction,

- e.g. driving cars produces carbon emissions
- leads to overconsumption

### 3.3.4 Positive production externality

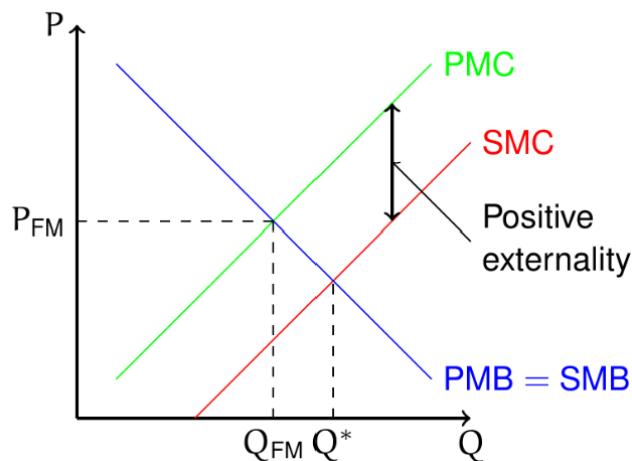


Figure 3.6: Positive production externality.

Production of output reduces well-being of third parties not involved in transaction,

- e.g. creating a new tourist attraction brings increases custom to local shops
- leads to underproduction

### 3.3.5 Positive consumption externality

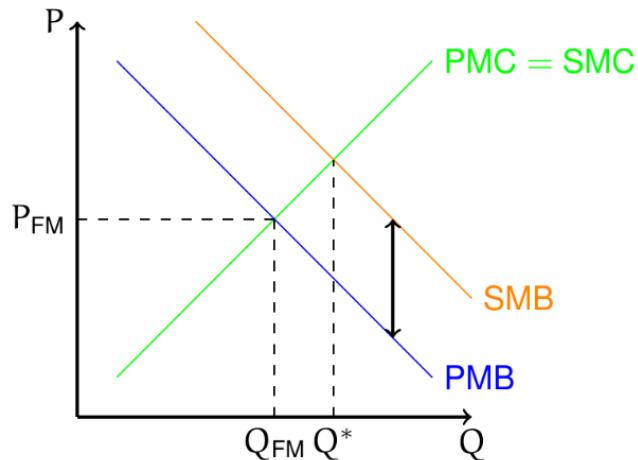


Figure 3.7: Positive consumption externality.

Consumption of output reduces well-being of third parties not involved in transaction,

- e.g. cycling improves peoples general health, reducing pressure on public healthcare
- leads to under-consumption

### 3.3.6 Externalities and property rights

Externalities can be transferred where third party benefit/cost is clear i.e. where property rights are well defined.

### 3.3.7 Managing externalities

Where property rights are not clear, managing externalities relies on allocative role of government

#### Public sector interventions

Negative externalities:

- Corrective taxes
- Quantity restrictions
- Standards

Positive externalities

- Subsidies
- Tax benefits
- Direct production

### 3.3.8 Externalities and the environment

Note... Externalities related to climate change are critical to long term sustainability of the planet.

**COP26**

“Climate change is the single greatest threat facing humanity. While no one is safe from the health impacts of climate change, they are disproportionately felt by the most vulnerable and disadvantaged.” (World Health Organisation 2021)

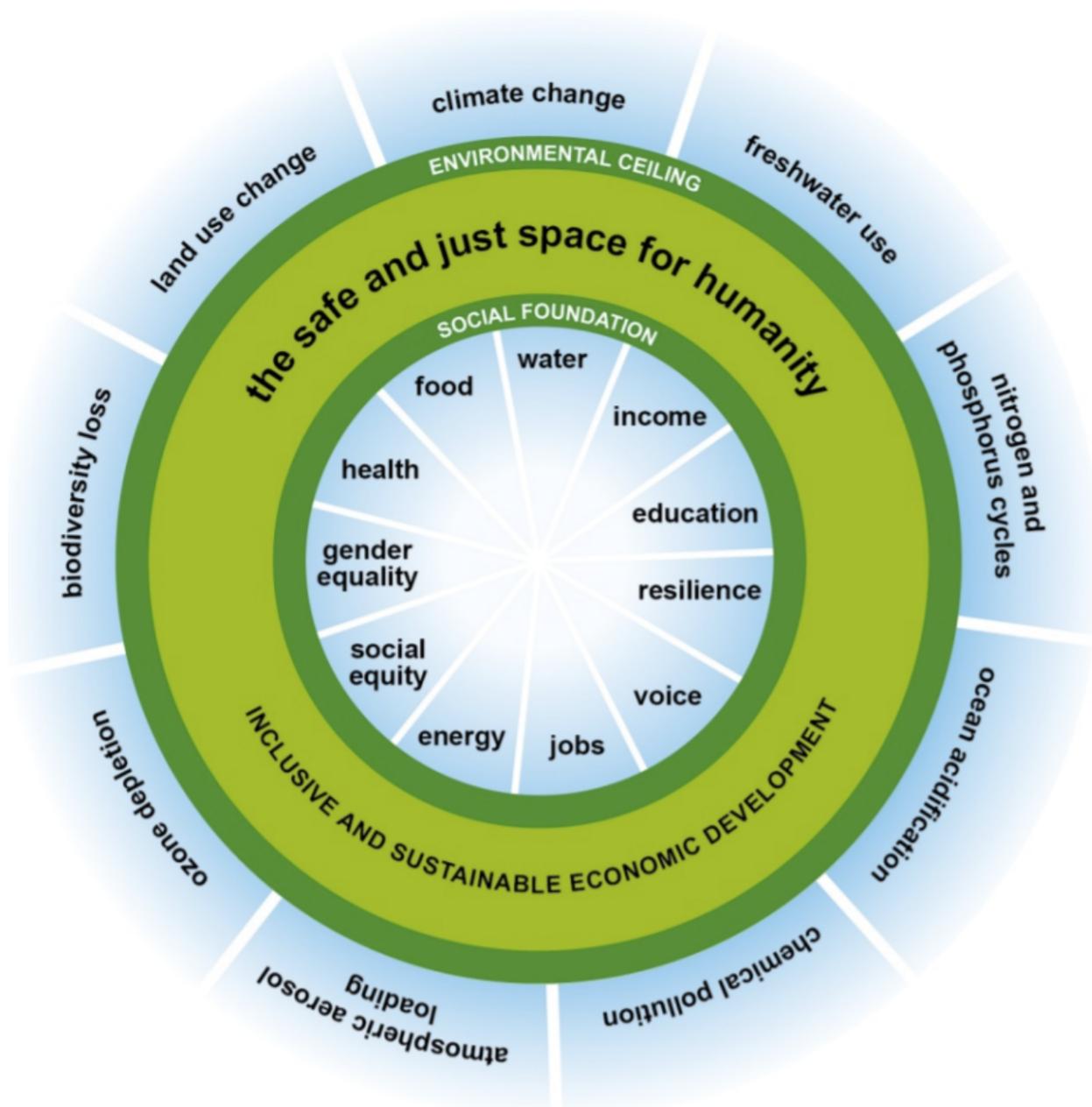
**3.3.9 The doughnut economic model**

Figure 3.8: Doughnut economic model.

# Chapter 4

## Techniques for Project Evaluation

### 4.1 Introduction to Capital and Interest

#### 4.1.1 Capital

- **Capital** is wealth in the form of money or property that can be used to produce more wealth
- A pound is worth more than a pound one or two years from now because of the **interest** it can earn
- Therefore money has a **time value**
- Often the riskiest thing a person can do with money is nothing

#### 4.1.2 Interest

- Interest pays the providers of capital for:
  - Forgoing its use during the time the capital is being used
  - The risk the investor takes in permitting another person or organisation to use their capital
- Investors must decide whether the return on their capital is sufficient to buy into a proposed project or venture
- The interest available from an alternative investment is the opportunity cost of using capital in the proposed undertaking

### 4.2 Simple interest

Interest earned or charged that is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods for which the principal is committed. Simple interest is not used frequently in modern commercial practice.

$$I = PNi \quad (4.1)$$

where:

- $I$  is total simple interest
- $P$  is principal amount lent or borrowed
- $N$  is number of interest periods
- $i$  is interest rate per interest period

The total amount repaid at the end of  $N$  interest periods is  $P + I$ . If £1000 were loaned for three years at a simple interest rate of 10% per year, the interest earned would be £300. The total amount owed at the end of three years would be £1300.

## 4.3 Compound interest

Interest earned or charged that is based on the remaining principal amount plus any accumulated interest charges up to the beginning of that period. Compound interest considers the time value of money, and is much more common than simple interest.

$$I = P(1+i)^N - P \quad (4.2)$$

The total amount repaid at the end of  $N$  interest periods is  $P + I$ . If £1000 were loaned for three years at a compound interest rate of 10% per year, the interest earned would be £331. The total amount owed at the end of three years would be £1331.

### 4.3.1 Compound vs simple interest

Assume that £1000 were loaned for three years at an interest rate of 10% per year.

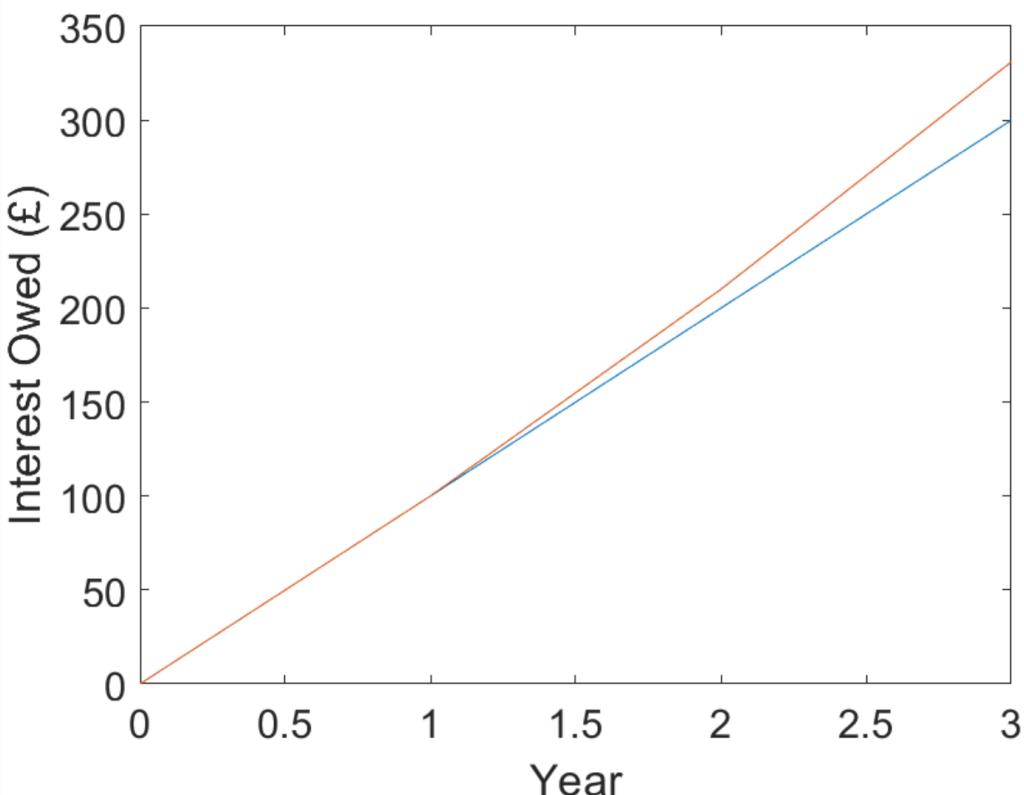


Figure 4.1: Blue: simple interest, Orange: compound interest

## 4.4 What is project evaluation

- Project evaluation considers the return that a given project will or should produce
- Project evaluation involves quantifying project profitability using various methods
- We address whether a proposed capital investment and its associated expenditures can be recovered by revenue (or savings) over a period of time, in addition to a return on the capital that is sufficiently attractive

## 4.5 Minimum attractive rate of return (MARR)

MARR is the **minimum rate of return** on a project that the top management of an organisation is willing to accept before starting a project. MARR depends on numerous factors:

- Amount of money available for investment (as well as the source and costs of funds)
- The number of projects available for investment and their purpose (i.e. whether they are essential or optional)
- The amount of perceived risk and the estimated cost of administering projects over different planning horizons
- The type of organisation involved (government, public utility, private industry)

## 4.6 Project evaluation using Net Present Value

### 4.6.1 Net present value (NPV)

The NPV method examines the equivalent worth of all cash flows relative to some base point in time i.e. the present. The future value ( $FV$ ) of a sum of money has a value today called the present value ( $PV$ ), which depends on the interest rate / that can be obtained (generally the MARR) - note that we are talking about a single sum of money in this case. The  $PV$  of a cashflow in  $n$  years' time as a function of  $i$  is:

$$PV = \frac{FV}{(1+i)^n} \quad (4.3)$$

Note that  $i$  is expressed as a decimal here. A series of uniform (annual) receipts ( $AV$ ) have a value today called the present value ( $PV$ ) which depends on the interest rate / that can be obtained (generally the MARR) - note that we are talking about multiple sums of money in this case. The  $PV$  of a series of cashflows that occur at the end of periods (years) 1 to  $n$  is:

$$PV = AV \frac{(1+i)^n - 1}{i(1+i)^n} = \sum_{k=1}^n \frac{AV}{(1+i)^k} \quad (4.4)$$

$NPV$  then accounts for all cash inflows and outflows:

$$NPV = PV_{\text{cash inflows}} - PV_{\text{cash outflows}} \quad (4.5)$$

To use the NPV method to determine project worthiness, we compute  $NPV$  using the MARR as the interest rate. The higher the interest rate ( $i$ ) and the farther into the future a cash flow occurs, the lower its  $PV$ .

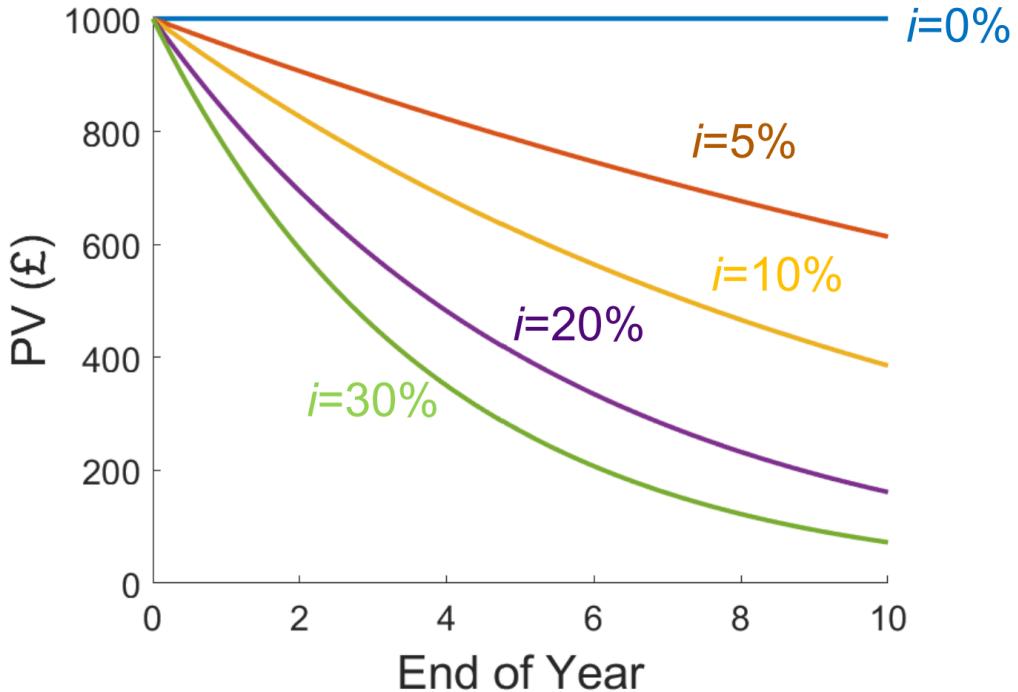


Figure 4.2: Effect of interest rate on PV.

#### 4.6.2 Example

A retrofitted heat-pump system is being considered for a small office building. The system can be installed and purchased for £110,000 and it will save an estimated 300,000 kilowatt-hours of electric power each year over a six-year period. A kilowatt-hour of electricity costs £0.10, and the company uses a MARR of 15% per year in its economic evaluations of refurbished systems. The market value of the system will be £8,000 at the end of six years, and additional annual operating and maintenance expenses are negligible. Use the NPV method to determine whether the system should be installed.

$$NPV = PV \text{ of estimated savings} + PV \text{ of market value} - PV \text{ of cost} \quad (4.6)$$

Estimated value:

$$PV_{ES} = 300000 \times 0.1 = 30000 \quad (4.7)$$

$$PV_{ES,y1} = \frac{30000}{(1 + 0.15)^1} \quad (4.8)$$

$$PV_{ES,y2} = \frac{30000}{(1 + 0.15)^2} \dots \quad (4.9)$$

$$PV_{ES,y6} = \frac{30000}{(1 + 0.15)^6} \quad (4.10)$$

$$\therefore \sum_{k=1}^6 \frac{30000}{(1 + 0.15)^k} \quad (4.11)$$

Market value:

$$PV_{MV} = \frac{8000}{(1 + 0.15)^6} \quad (4.12)$$

Cost:

$$PV_{cost} = 110000 \quad (4.13)$$

Therefore, NPV is:

$$NPV = \sum_{k=1}^6 \frac{30000}{(1 + 0.15)^k} + \frac{8000}{(1 + 0.15)^6} - 110000 \approx 6993 \quad (4.14)$$

### 4.6.3 Advantages and disadvantages of NPV

Advantages

- It accounts for the time value of money
- It accounts for uncertainties about future projections
- It accounts for all cash flows of interest

Disadvantages

- It is highly sensitive to the interest rate used
- It is not useful for comparing projects of different sizes
- It ignores costs that are incurred before the project starts

## 4.7 Project evaluation using Internal Rate of Return

### 4.7.1 Internal Rate of Return (IRR)

The IRR method solves for the interest rate that equates the present value of cash inflows (receipts or savings) to the present value of cash outflows (expenditures, e.g. investment costs). That is, the IRR provides the answer to the question: what interest rate provides an NPV of 0? This method is the most widely using rate-of-return method for performing engineering economic analyses.

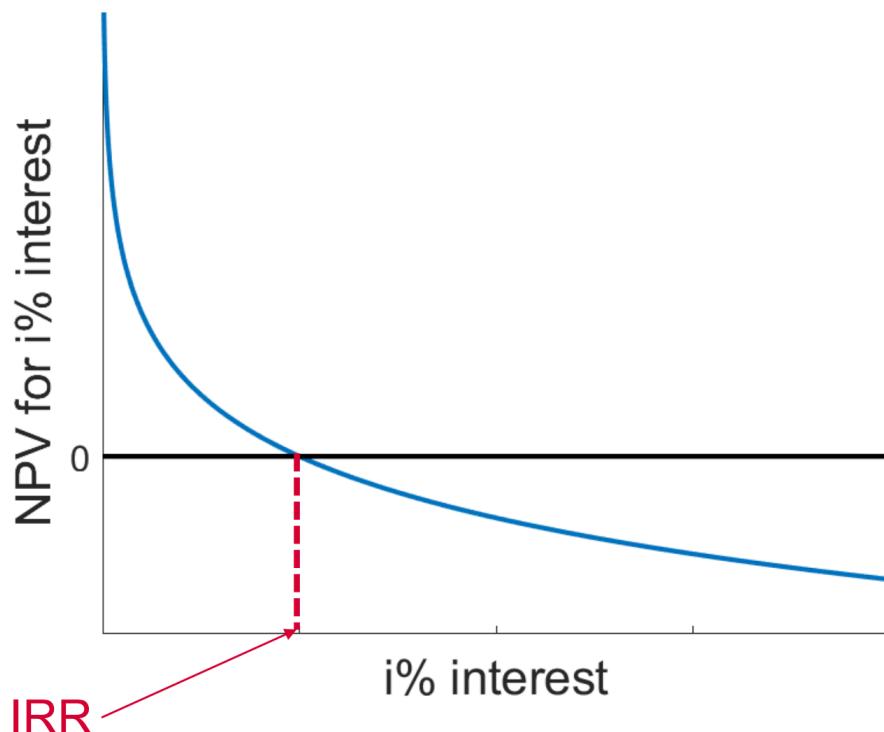


Figure 4.3: Internal Rate of Return.

### 4.7.2 Example

A company is considering the purchase of a digital camera for the maintenance of design specifications by feeding digital pictures directly into an engineering workstation where computer-aided design files can be superimposed over the digital pictures. Differences between the two images can be noted, and corrections as appropriate can then be made by design engineers. The capital investment requirement is £345,000 and the estimated market value of the system after a six-year study period is £115,000. Annual revenues attributable to the new system will be £120,000 and additional annual expenses will be £22,000. You have been asked by management to determine the IRR of this project and to make a recommendation. The corporation's MARR is 20% per year.

Denote IRR as  $i$ . First let's determine an equation for NPV:

$$NPV = PV \text{ of net annual revenue} + PV \text{ of market value} - PV \text{ of cost} \quad (4.15)$$

PV of net annual revenue:

$$PV_{NAR,y1} = \frac{120000 - 22000}{(1+i)^1} \quad (4.16)$$

$$PV_{NAR,y2} = \frac{120000 - 22000}{(1+i)^2} \dots \quad (4.17)$$

$$PV_{NAR,y6} = \frac{120000 - 22000}{(1+i)^6} \quad (4.18)$$

$$\therefore \sum_{k=1}^6 \frac{98000}{(1+i)^k} \quad (4.19)$$

Market Value:

$$PV_{MV} = \frac{115000}{(1+i)^6} \quad (4.20)$$

Cost:

$$PV_{cost} = 345000 \quad (4.21)$$

NPV:

$$NPV = \sum_{k=1}^6 \frac{98000}{(1+i)^k} + \frac{115000}{(1+i)^6} - 345000 \quad (4.22)$$

Lets try  $i = MARR = 20\% = 0.2$ :

$$NPV(i = 0.2) = +19,413 \quad (4.23)$$

However, this is not the IRR... We must calculate  $i$  using a solver to find which value of  $i$  gives and NPV of 0. Using Excel, we find that our IRR is 22%. Interpolation may also be used.

### 4.7.3 Advantages and disadvantages of IRR

Advantages:

- It has widespread acceptance in industry
- It is relatively simple to understand
- It accounts for the time value of money

Disadvantages

- It is difficult to compute
- It ignores the size and scope of projects
- It does not account for the actual reinvestment rate

## 4.8 Project evaluation using Payback Period

The payback period method evaluates the number of years  $\Theta$  it takes for cash inflows to equal cash outflows. Both of the previous evaluation methods focus on profitability. The payback period instead estimates a company's liquidity (i.e. how fast an investment can be recovered). There are two types of payback period methods:

1. Simple payback period - ignores the time value of money
2. Discounted payback period - accounts for the time value of money

### 4.8.1 Simple Payback Period Example

A public school is being renovated for £13.5 million. The building has geothermal heating and cooling, high-efficiency windows, and a solar array that permits the school to sell electricity back to the local electric utility. The annual value of these benefits is estimated to be £2.7 million. In addition, the residual value of the school at the end of its 40-year life is negligible. What is the simple payback period for the renovated school?

The simple payback period is:

$$SPP = \frac{13.5}{2.7} = 5 \text{ years} \quad (4.24)$$

### 4.8.2 Simple & Discounted Payback Period Example

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is £25,000 and the equipment will have a market value of £5,000 at the end of its expected life of 5 years. Increased productivity attributable to the equipment will amount to £8,000 per year after extra operating costs have been subtracted from the value of the additional production. MARR is 20% per year. Calculate the simple and the discounted payback periods.

**Simple Payback Period**

End of Year	Net Cash Flow	Net Present Value (0% interest)
0	-25,000	-25,000
1	8,000	-17,000
2	8,000	-9,000
3	8,000	-1,000
4	8,000	7,000
5	13,000	

**Discounted Payback Period**

End of Year	Net Cash Flow	Net Present Value (20% interest)
0	-25,000	-25,000
1	6,667	-18,333
2	5,556	-12,777
3	4,630	-8,147
4	3,858	-4,289
5	5,223	+934

**This figure includes the market value of the equipment at the end of its expected life**

**Payback period = 4 years**

**How can we compute this number?**

**Payback period = 5 years**

Figure 4.4: Simple and discounted payback periods.

### 4.8.3 Advantages and disadvantages of payback period

Advantages

- It provides a new perspective on performance (by focusing on liquidity)
- It is relatively simple to understand and compute
- It requires relatively few inputs

### Disadvantages

- It does not account for cash flows that occur after the payback period
- It may not consider the time value of money
- It ignores profitability, and should only be used as a secondary evaluation measure.

# Chapter 5

## Cost-Benefit Analysis

### 5.1 What is cost-benefit analysis?

“Assessing costs and benefits across all affected groups or places matters because even a proposal with a relatively low public sector cost such as new regulation, may have significant effects on specific groups in society, places or business”

- A cost-benefit analysis is the process used to measure the benefits of a decision or taking action relative to the associated costs
- If benefits > costs, the decision or action is a good one to take
- If costs > benefits, the proposed action or decision should be reconsidered
- Cost benefit analysis can also be used to compare alternate decisions or actions

Both costs and benefits are required to be expressed in monetary terms, accounting for the time value of money.  
Costs may be categorised as:

- Direct - e.g. labour costs, manufacturing costs, material costs
- Indirect - e.g. utilities, rent
- Intangible - e.g. reduced productivity because of a new process
- Opportunity - lost benefits (opportunities) when pursuing one strategy over another

Benefits may be categorised as:

- Direct - e.g. increased revenue and sales
- Indirect - e.g. increased consumer interest
- Intangible - e.g. improved employee morale
- Competitive - e.g. being an industry leader

### 5.2 The Benefit-Cost Ratio method

The Benefit-Cost Ratio (BCR) is defined as the ratio of the equivalent value of benefits to the equivalent value of costs. The equivalent-value measure can be:

- Annual value (AV)
- Present value (PV)
- Future value (FV)

The BCR method has been the accepted procedure for making decisions and comparing projects in the public sector for many decades.

if  $BCR \geq 1$ , the project is acceptable

Several different formulations of the BCR method have been developed. We examine two formulations of the BCR method that are commonly used by government agencies:

- Conventional BCR method
- Modified BCR method

Both formulations will lead to identical project acceptability decisions (i.e.  $BCR \geq 1$  or  $BCR < 1$ ).

### 5.2.1 Conventional BCR method

Present value (PV) formulation:

$$BCR_{PV} = \frac{PV_{benefits}}{PV_{costs}} \quad (5.1)$$

$$BCR_{PV} = \frac{PV_{benefits}}{I - PV_{MV} + PV_{O\&M}} \quad (5.2)$$

Annual value (AV) formulation:

$$BCR_{AV} = \frac{AV_{benefits}}{AV_{costs}} \quad (5.3)$$

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} \quad (5.4)$$

Both ratios lead to identical numerical results. Where:

- $I$  is initial investment in the proposed project
- $MV$  is market value at the end of useful life
- $O\&M$  is operating and maintenance costs
- $CR$  is capital-recovery amount (i.e. equivalent cost of  $I$ , including an allowance for market or salvage value)

### 5.2.2 Modified BCR method

Present value (PV) formulation:

$$BCR_{PV} = \frac{PV_{benefits}}{PV_{costs}} \quad (5.5)$$

$$BCR_{PV} = \frac{PV_{benefits} - PV_{O\&M}}{I - PV_{MV}} \quad (5.6)$$

Annual value (AV) formulation:

$$BCR_{AV} = \frac{AV_{benefits}}{AV_{costs}} \quad (5.7)$$

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} \quad (5.8)$$

Both ratios lead to identical numerical results.

### 5.2.3 Conventional and modified BCRs

Remember the formula for PV:

$$PV = \frac{FV}{(1+i)^n} \quad (5.9)$$

Present value (PV) of a future value (FV) in  $n$  years, for an interest rate  $i$ . Remember the formula for AV:

$$AV = PV \frac{i(1+i)^n}{(1+i)^n - 1} \quad (5.10)$$

Value of a series of uniform (annual) receipts (AV) that occur at the end of periods (years) 1 to  $n$ , given their present value (PV) and an interest rate  $i$ .

### 5.2.4 What value of $i$ to use?

There are three main considerations when it comes to what interest rate to use for engineering economy studies of public-sector projects:

- the interest rate on borrowed capital
- The opportunity cost of capital to the government agency
- The opportunity cost of capital to the taxpayers

### 5.2.5 Why do conventional and modified BCRs lead to the same decision?

Conventional BCR formulation:

$$BCR_V = \frac{V_{benefits}}{I - V_{MV} + V_{O\&M}} = \frac{B}{C} \quad (5.11)$$

Where subscript  $V$  denotes either PV or AV. Modified BCR formulation:

$$BCR_V = \frac{V_{benefits} - V_{O\&M}}{I - V_{MV} + V_{O\&M} - V_{O\&M}} = \frac{B - X}{C - X} \quad (5.12)$$

Both the numerator and denominator differ by the same constant.

$$\frac{B}{C > 1} \rightarrow B > C \rightarrow B - X > C - X \rightarrow \frac{B - X}{C - X} > 1 \quad (5.13)$$

leading to the same decision.

### 5.2.6 Example 1

The Greater London Authority is considering extending the runways of Stansted Airport so that larger commercial airplanes can use the facility. The land necessary for the runway extension is currently a farmland that can be purchased for £350,000. Construction costs for the runway extension are projected to be £600,000, and the additional annual maintenance costs for the extension are estimated to be £22,500. If the runways are extended, a small terminal will be constructed at a cost of £250,000. The annual operating and maintenance costs for the terminal are estimated at £75,000. Finally, the projected increase in flights will require the addition of two air traffic controllers at an annual cost of £100,000. Annual benefits of the runway extension have been estimated as follows: Apply the BCR method with a study of 20 years and a MARR of 10% per year to determine whether the runways at Stansted airport should be extended.

Description	Annual benefit
Leasing fee receipts from airlines	£325,000
Passenger airport tax receipts	£65,000
Convenience benefit for residents near Stansted	£50,000
Additional tourism money for London	£50,000
<b>Total</b>	<b>£490,000</b>

Table 5.1: Example 1.

Information provided:

$$i = 0.1 \quad (5.14)$$

$$n = 20 \text{ years} \quad (5.15)$$

$$I = £350000 + £600000 + £250000 = £1200000 \quad (5.16)$$

$$AV_{benefits} = £490000 \quad (5.17)$$

$$PV_{MV} = AV_{MV} = £0 \quad (5.18)$$

$$AV_{O\&M} = £22500 + £75000 + £100000 = £197500 \quad (5.19)$$

First, we need to determine PVs and AVs using:

$$PV = AV \frac{(1+i)^n - 1}{i(1+i)^n} \quad (5.20)$$

$$AV = PV \frac{i(1+i)^n}{(1+i)^n - 1} \quad (5.21)$$

$$PV_{benefits} = £4171646 \quad (5.22)$$

$$PV_{O\&M} = £1681429 \quad (5.23)$$

$$AV_I = CR = £140951 \quad (5.24)$$

Conventional BCRs:

$$BCR_{PV} = \frac{PV_{benefits}}{I - PV_{MV} + PV_{O\&M}} = 1.448 \quad (5.25)$$

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.448 \quad (5.26)$$

Modified BCRs:

$$BCR_{PV} = \frac{PV_{benefits} - PV_{O\&M}}{I - PV_{O\&M}} = 2.075 \quad (5.27)$$

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} = 2.075 \quad (5.28)$$

BCR  $\geq 1$  in all cases, so runway should be extended.

### 5.2.7 Issues of concern using BCRs

- The treatment of disbenefits
  - Negative consequences to the public resulting from the implementation of a public sector project
- The treatment of certain cash flows as additional benefits or reduced costs

### 5.2.8 Treatment of disbenefits

Disbenefits can be incorporated in BCR calculations by:

- Reducing benefits accordingly (traditional approach) or
- Increasing costs accordingly

How do these approaches affect the BCR? How do these approaches affect the final decision?

### 5.2.9 Example 2

Refer back to Example 1. Suppose that there are disbenefits associated with the runway extension project. Specifically, the increased noise level from commercial jet traffic will be a serious nuisance to homeowners living along the approach path Stansted Airport. The annual disbenefit to these citizens is estimated to be £100,000.

Reapply the conventional BCR method, with equivalent annual worth, to determine whether this disbenefit affects your recommendation on the desirability of this project.

Disbenefit treated as a reduced benefit:

$$BCR_{AV} = \frac{AV_{benefits} - 100000}{CR + AV_{O\&M}} = 1.152 \quad (5.29)$$

Disbenefit treated as an increased cost:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M} + 100000} = 1.118 \quad (5.30)$$

BCR  $\geq 1$  in both cases, so runway should be extended. The treatment of disbenefits affects the magnitude of the BCR, but not the decision.

### 5.2.10 Treatment of certain cash flows

Certain cash flows can be incorporated in BCR calculations by:

1. Increasing benefits accordingly
2. Reducing costs accordingly

How do these approaches affect the BCR? How do these approaches affect the final decision?

### 5.2.11 Example 3

Transport for London is considering upgrading an ageing bridge across the Thames. The existing two-lane bridge is expensive to maintain and creates a traffic bottleneck because the road is four lanes wide on either side of the bridge. The new bridge can be constructed at a cost of £300,000, and estimated annual maintenance costs are £10,000. The existing bridge has annual maintenance costs of £18,500. The annual benefit of the new four-lane bridge to motorists, due to the removal of the traffic bottleneck, has been estimated to be £25,000.

Conduct a cost-benefit analysis based on equivalent annual worth, using a MARR of 8% and a study period of 25 years, to determine whether the new bridge should be constructed.

Information provided:

$$i = 0.08 \quad (5.31)$$

$$n = 25 \text{ years} \quad (5.32)$$

$$I = \text{£}300000 \quad (5.33)$$

$$AV_{benefits} = \text{£}25000 \quad (5.34)$$

$$PV_{MV} = AV_{MV} = \text{£}0 \quad (5.35)$$

$$AV_{O\&M} = \text{£}10000 - \text{£}18500 = -\text{£}8500 \quad (5.36)$$

Cost is negative because it represents a reduction with respect to the current cost.

Required information:

$$AV_I = CR = £28104 \quad (5.37)$$

Reduced cost treated as a reduced cost (conventional BCR approach):

$$BCR_{AV} = \frac{BCR_{AV}}{CR + AV_{O\&M}} = 1.275 \quad (5.38)$$

Reduced cost treated as an increased benefit (modified BCR approach):

$$BCR_{AV} = \frac{AV_{benefits} - AV_{O\&M}}{CR} = 1.192 \quad (5.39)$$

BCR  $\geq 1$  in both cases, so bridge should be constructed. The classification of the cash-flow items affects the magnitude of the BCR, but not the decision.

### 5.2.12 Treatment of disbenefits/certain cash flows

Arbitrary decisions on the classification of benefits and costs has no bearing on project acceptability because if X is classified as an added benefit:

$$BCR = \frac{B+X}{C} \quad (5.40)$$

and

$$BCR > 1 \rightarrow B + X > C \rightarrow B > C - X \rightarrow \frac{B}{C-X} > 1 \quad (5.41)$$

leading to the same decision.

## 5.3 Evaluating independent projects using the BCR method

### 5.3.1 What are independent projects?

Independent projects are categorised as groupings of projects for which the choice to select any particular project in the group is **independent** of choices regarding all other projects within the group.

It is therefore acceptable to select:

1. None of the projects
2. A combination of the projects
3. All of the projects

Formal comparisons of independent projects is unnecessary. The only criterion for selecting each independent project is  $BCR \geq 1$ .

### 5.3.2 Example 4

Uncontrolled water flow has increased flow conditions along a river. You have independent options to alleviate the problem of building a reservoir and/or improving the channel. Relevant information is as follows:

	Reservoir construction	Channel improvement
$CR + AV_{O\&M}$	£1,642,200	£1,815,100
$AV_{benefits}$	£1,742,200	£2,856,300

Table 5.2: Example 4 information.

Conduct a cost-benefit analysis using the conventional BCR method and equivalent annual worth to determine the best course of action.

Reservoir construction:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.061 \quad (5.42)$$

Channel improvement:

$$BCR_{AV} = \frac{AV_{benefits}}{CR + AV_{O\&M}} = 1.574 \quad (5.43)$$

$BCR \geq 1$  in both cases, so both options should be pursued (the fact that the channel improvement has a higher BCR is irrelevant).

## 5.4 Evaluating mutually exclusive projects using the BCR method

### 5.4.1 What are mutually exclusive projects

Mutually exclusive projects are a group of projects from which, **at most, one project may be selected**. Each mutually exclusive project can be viewed as a feasible design alternative. Because the BCR method provides a ratio of benefits to costs rather than a direct measure of a project's profit potential, **selecting the project that maximises the BCR does not guarantee that the best project is selected**.

### 5.4.2 Procedure for evaluating mutually exclusive projects

1. Calculate equivalent value (PV, AV or FV) of costs for each mutually exclusive project
2. Rank-order mutually exclusive projects by increasing equivalent value of costs (note: the rank-order will be the same for all equivalent value types)
3. Calculate the BCR for the project with the lowest equivalent cost ( $BCR_L$ )
  - If  $BCR_L \geq 1 \rightarrow$  baseline = project with the lowest equivalent cost
  - Else  $\rightarrow$  baseline = “do-nothing”
4. Follow the flow chart in Figure 5.1

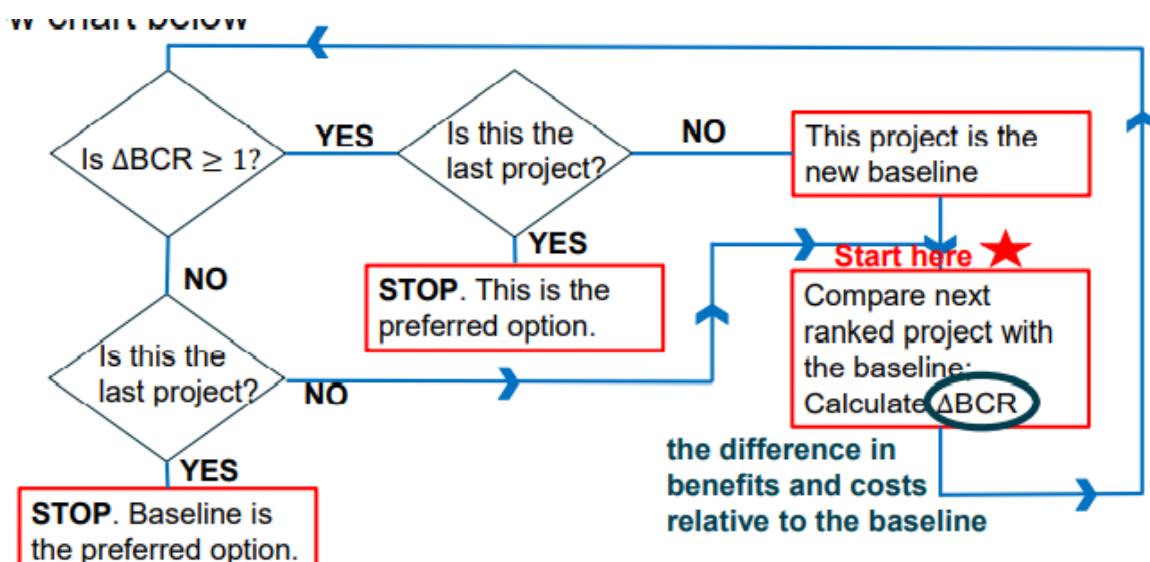


Figure 5.1: Flow chart for evaluating mutually exclusive projects

### 5.4.3 Example 5

Three mutually exclusive public-works projects are currently under consideration. Their respective costs and benefits are included in the table that follows. Each of the projects has a useful life of 50 years, and MARR is 10% per year.

	Project A	Project B	Project C
Capital investment	£8,500,000	£10,000,000	£12,000,000
Annual O&M costs	£750,000	£725,000	£700,000
Market value	£1,250,000	£1,750,000	£2,000,000
Annual benefit	£2,150,000	£2,265,000	£2,500,000

Table 5.3: Example 5 information.

Which, if any of these projects, should be selected?

Information provided and required information:

$$i = 0.1 \quad (5.44)$$

$$n = 50 \text{ years} \quad (5.45)$$

Step 1: convert to PV.

	Project A	Project B	Project C
I	£8,500,000	£10,000,000	£12,000,000
$PV_{O\&M}$	£7,436,111	£7,188,241	£6,940,370
$PV_{MV}$	£10,648	£14,907	£17,037
$PV_{costs}$	£15,925,463	£17,173,333	£18,923,333
$PV_{benefits}$	£21,316,851	£22,457,055	£24,787,036

Table 5.4: Example 5 required information.

Step 2

$$BCR_A = \frac{21316851}{15925463} = 1.339 \geq 1 \rightarrow \text{Project A is the baseline} \quad (5.46)$$

Step 3: establish baseline.

Step 4:

$$\Delta BCR_B = \frac{22457055 - 21316851}{17173333 - 15925463} = 0.914 < 1 \rightarrow \text{Project A is still the baseline} \quad (5.47)$$

Proceed to Project C

$$\Delta BCR_C = \frac{24787036 - 21316851}{18923333 - 15925463} = 1.158 \geq 1 \rightarrow \text{Project C is the preferred option} \quad (5.48)$$

### 5.4.4 Mutually exclusive projects with unequal lives

It is not uncommon for public projects to have different useful lives. How can we conduct BCR analyses in these cases? In these cases, annual values (AVs) should be used to conduct incremental cost-benefit analyses.

### 5.4.5 Example 6

Two mutually exclusive alternative public-works projects are under consideration. Their respective costs and benefits are included in the table that follows. Project A has an anticipated life of 35 years, and the useful life of Project B has been estimated to be 25 years. The effect of inflation is negligible.

	Project A	Project B
Capital investment	£750,000	£625,000
Annual O&M costs	£120,000	£110,000
Annual benefit	£245,000	£230,000

Table 5.5: Example 6 information.

If the MARR is 9% per year, which, if either, of these projects should be selected?

Information provided and required information:

$$i = 0.09 \quad (5.49)$$

$$n = 35 \text{ or } 25 \text{ years} \quad (5.50)$$

Step 1: convert to AV.

	Project A	Project B
$AV_I$	£70,977	£63,629
$AV_{O\&M}$	£120,000	£110,000
$AV_{MV}$	£0	£0
$AV_{costs}$	£190,977	£173,629
$AV_{benefits}$	£245,000	£230,000

Table 5.6: Example 6 required information.

Step 2:

$$BCR_B = \frac{230000}{173629} = 1.325 \geq 1 \rightarrow \text{Project B is the baseline} \quad (5.51)$$

Step 3: establish baseline.

Step 4:

$$\Delta BCR_A = \frac{245000 - 230000}{190977 - 173629} = 0.865 < 1 \rightarrow \text{Project B is the preferred option} \quad (5.52)$$

# Chapter 6

# Companies and Financial Accounting

## 6.1 Companies

### 6.1.1 What is a company?

A company is a legal entity (“personality”) that can issue contracts, enter into agreements or contracts, assume obligations, incur and pay debts, sue and be sued in its own right, and be held responsible for its own actions. There are three main types of companies in the UK:

1. A Sole Trader
2. A Partnership
3. A Limited Liability Company

### Sole Trader

A sole Trader:

- Runs their own business as an individual
- Keeps all of the net profits
- Is personally responsible for any losses their business makes (unlimited liability)

### Partnership

A Partnership:

- Involves two or more partners who share responsibility for the business
- Keeps all of the net profits (jointly)
- Is jointly responsible for any losses their business makes (joint unlimited liability)
- A partner does not have to be an actual person. For example, a limited company counts as a ‘legal person’ and can also be a partner

### Limited Liability Company

A Limited Liability Company that is “limited by shares” or “limited by guarantee.”

Limited by shares:

- Usually businesses that make a profit

- is legally separate from those who run it (limited liability)
- has shares and shareholders
- retains net profits

Limited by guarantee:

- Usually businesses that are “not for profit”
- is legally separate from those who run it (limited liability)
- has guarantors and a “guaranteed amount”
- invests profits it makes back into the company

### 6.1.2 Structure of a Limited Liability Company limited by shares

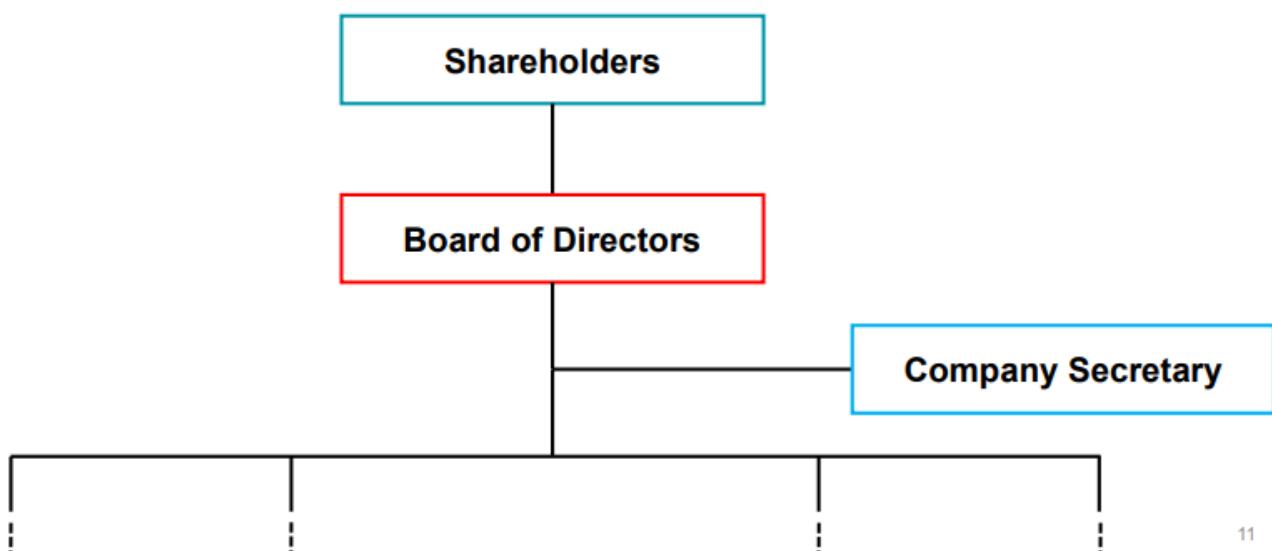


Figure 6.1: Structure of a Limited Liability Company limited by shares.

### 6.1.3 What is a shareholder?

- Shareholders own a Limited Liability Company limited by shares
- They have no day-to-day duties related to the company’s operation
- Each shareholder owns a fraction of the company (and has corresponding voting power) proportional to their shareholding (investment)
- The liability of shareholders is limited to their original investment
- The company’s profits are paid to shareholders as share dividend

### 6.1.4 What is the Board of Directors?

- The Board of Directors is elected by the shareholders at the Annual General Meeting (AGM) to run the company.
  - Directors may also be shareholders (but not necessarily)
  - The shareholders can vote to appoint or sack individual Board of Directors members
- The Board of Directors files the company’s audited accounts (legal requirement)

- The Board of Directors report to the shareholders at the AGM (legal requirement)
- The Board of Directors is controller by the Chairman of the Board
- The Board of Directors can appoint or sack the Chief Executive Officer (CEO - responsible for day-to-day running)

### **6.1.5 What is a company secretary?**

The company secretary ensures the smooth administration of the company. The company secretary is responsible for:

- Making sure the company stays within the law
- Making sure the company maintains proper record books and accounts
- Providing strategic advice to the Board of Directors (sometimes)

The company secretary may or may not be a member of the Board of Directors.

### **6.1.6 Limited Liability Company**

A Limited Liability Company that is “limited by shares” can be public or private.

Private Limited Company (Ltd.):

- Share ownership is controlled
  - is owned privately
  - only one director is required
  - have nine months to file their annual accounts
  - a company secretary is not legally required

Public Limited Company (PLC):

- Share ownership is NOT controlled
  - company shares can be bought and sold publically on the open market (a stock exchange)
  - two directors are required
  - have six months to file their annual accounts
  - a company secretary is legally required

## **6.2 What is accounting?**

Accounting is the collection, analysis and communication of financial information that is used by:

- Those who need to make decisions and plans in an organisation
- Those who need to control an organisation

Accounting helps companies to plan for the future and evaluate past performance. Accounting os often referred to as the language of business.

### 6.3 The fundamental accounting equation

$$\text{Assets} = \text{Liabilities} + \text{Owners' Equity} \quad (6.1)$$

where:

- Assets are resources that a company owns. They have the capacity to provide future services or benefits. Companies use their assets in carrying out activities like production and sales
- Liabilities are claims against assets - existing debts and obligations. They arise from purchasing items on credit or borrowing money from a bank for purchases
- Owners' Equity is what remains of the assets after all liabilities have been paid

“Liabilities + Owners’ Equity” are the rights or claims against the resources of the company.

Assets	Liabilities	Owners' Equity
Cash	Wages due	Owners' capital
Equipment	Bank debt	
Buildings	Accounts payable	
Land		
Inventories		

Table 6.1: Table to show Assets, Liabilities and Owners' Equity.

#### 6.3.1 Some important definitions

##### Assets:

*Economic resources that a company expects to help generate future cash inflows or help reduce future cash outflow.*

##### Current assets:

*A company's cash and other assets that are expected to be converted to cash over the next one year period.*

##### Non-current assets:

*A company's assets that are not expected to be converted to cash over the next one year period.*

##### Inventory:

*Goods (current assets) available for sale or raw materials and components used to produce goods available for sale.*

##### Liabilities:

*Economic obligations of the organisation to outsiders, or claims against its assets by outsiders.*

##### Current liabilities:

*A company's short term obligations, due within a one year period.*

##### Non-current liabilities:

*A company's long-term obligations listed on the balance sheet.*

##### Dividends:

*Money paid regularly to shareholders by the company.*

## 6.4 Balance sheet

The balance sheet is one of the two most common accounting statements. The fundamental accounting equation defines the format of the balance sheet. The balance sheet shows the financial position of the company at one instant in time (e.g. end of the quarter or end of the year).

Assets	Liabilities / Owners' Equity	
Cash	£2,500	Accounts payable
Land	£1,800	Bank note
Accounts receivable	£800	Owners' Equity
Total assets	£5,100	Total liabilities and Owners' Equity

Table 6.2: Balance sheet.

$$\text{Assets} = \text{Liabilities} + \text{Owners' Equity} = £5100 \quad (6.2)$$

## 6.5 Income statement

The income statement is the second of the two most common accounting statements. The income statement summarises the revenue and expense results of operations over a period of time (a moving picture.) It is defined by the following equation:

$$\text{Profit (or Loss)} = \text{Revenues} - \text{Expenses} \quad (6.3)$$

<b>Revenue</b>	
Sales	£3,000
<b>Expenses</b>	
Labour	£1,200
Depreciation	£400
Material	£500
Total Expenses	£2,100
<b>Net income</b>	<b>£900</b>

Table 6.3: Income statement.

### 6.5.1 Some important definitions

#### Turnover:

*The net sales generated by a business.*

Turnover and profit are the beginning and end points of the income statement.

## 6.6 Worked example

John Deere owns and operates a design company called Deere Consulting Ltd. The financial position of his business is:

Cash	£1,720
Accounts receivable	£3,240
Land	£24,100
Accounts payable	£5,400
John Deere, Capital	£23,660

Table 6.4: Financial position of John Deere Ltd.

During May 2021, the following events occurred:

1. Deere received £12,000 as a gift and deposited the cash in the business bank account
2. Deere paid off the beginning balance of the accounts payable
3. Deere performed services for a client and received cash of £1,100
4. Deere collected £750 cash from a customer on account
5. Deere purchased £720 of supplies on account
6. Deere billed a client £5,000 for services rendered
7. Deere invested personal cash of £1,700 in the business
8. Deere recorded £1,860 of business expenses
9. Deere sold supplies to another company for £80 cash (the price of the supplies)
10. Deere withdrew £4,000 cash for personal use

For Deere Consulting Ltd., prepare:

1. The income statement for the month ended 31 May 2021
2. The balance sheet as at 31 May 2021

#### **Income statement, month ended 31 May, 2021**

<b>Revenue</b>	
Services to Client 1	£5,000
Services to Client 2	£1,100
Total revenue	£6,100
<b>Expenses</b>	
Business expenses	£1,860
<b>Net income</b>	£4,240

Table 6.5: Deere Consulting Ltd. income statement, month ended 31 May, 2021.

**Balance sheet, 31 May, 2021**

<b>Assets</b>	<b>Liabilities/Owners' Equity</b>	
Cash	£6,090	Accounts payable
Accounts receivable	£7,490	
Supplies	£640	
Land	£24,100	J. Deere, Capital
<b>Total assets</b>	<b>£38,320</b>	<b>Total liabilities and Owners' Equity</b>
		<b>£38,320</b>

## 6.7 Cash flow statement

The cash flow statement shows how a company generated the cash flows it needed to finance its various opportunities and responsibilities over a period of time (a moving picture). It acts as a bridge between the income statement and the balance sheet by showing how money moved in and out of the business. The cash flow statement has three primary sections:

1. Cash flow from operating activities
  - Cash inflows: generation of funds in normal operations
  - Cash outflows: expenditure of funds in normal operations
2. Cash flow from investing activities
  - Cash inflows: sale of plant and equipment. Liquidation of long-term investment
  - Cash outflows: purchase of plant and equipment. Long-term investments
3. Cash flow from financing activities
  - Cash inflows: sale of bonds, common stock and other securities
  - Csh outflows: repurchase of bonds, common stock and other securities. Payment of cash dividend

The sum of these sections = net cash flow.

### 6.7.1 Cash flow statement

<b>Cash flows from operating activities</b>	
Cash generated from operations	£5,460
Income tax paid	-£1,351
Interest paid	-£40
Net cash flow from operating activities	£4,069
<b>Cash flows from investing activities</b>	
Interest received	£100
Purchases of property, plant and equipment	£5,894
Proceeds on disposal of property	£41
Capital grants received	£1,979
Net cash flow from investing activities	-£3,774
<b>Cash flows from financing activities</b>	
Repayments of borrowings	-£10,991
New loans raised	£10,841
Repayment of liaise liabilities	-£107
Net cash flow from financing activities	-£257
Net increase/(decrease) in cash and cash equivalents	£38
Cash and cash equivalents at beginning of the period	£430
<b>Cash and cash equivalents at end of year</b>	<b>£522</b>

Table 6.6: Cash flow statement.

### 6.7.2 Financial ratios

Financial ratios are mathematical calculations that a company can use to evaluate its performance. They are relative magnitudes of selected numerical values taken from a company's financial statements. Financial ratios may be used by managers, stakeholders or creditors to:

- Determine whether key performance trends are improving or not
- Compare ratios (and therefore company performances) between years
- Define goals for future companies

Financial analysts use financial ratios to compare strengths and weaknesses across various companies. There are five different types of financial ratios:

1. Liquidity ratios - help evaluate a company's ability to pay its bills on a regular week-to-week or month-to-month basis
2. Financial leverage ratios - measure how much of a company's assets belong to the shareholders rather than creditors (lenders)
3. Asset utility ratios - measure how efficient a company is with using its assets to generate revenue
4. Profitability ratios - help evaluate how well the firm generates a profit through its operations
5. Market value ratios - help evaluate the economic status of publicly traded companies and can play a role in identifying stocks that may be overvalued, undervalued or priced fairly

### Liquidity ratios

**Current ratio** - measure the ability to pay short-term debt. If Current ratio  $< 1$ , the company has liquidity problems to cover its short-term liabilities. If Current ratio = 1, the company is able to cover its short-term liabilities. The ideal Current ratio is between 1.2 and 2.

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} \quad (6.4)$$

**Quick ratio** - measure the ability to pay short-term debt. If Quick ratio  $< 1$ , the company finds it hard to fully pay its debt in the short term. If Quick ratio  $> 1$ , the company is able to cover its debt.

$$\text{Quick ratio} = \frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}} \quad (6.5)$$

**Cash ratio** - measure the ability of cash to pay debt. If Cash ratio  $< 1$ , there is insufficient cash to pay off short-term debt. If Cash ratio = 1, there is sufficient cash to pay off short-term debt. If Cash ratio  $> 1$ , there is more than sufficient cash to pay off short-term debt.

$$\text{Cash ratio} = \frac{\text{Cash}}{\text{Current liabilities}} \quad (6.6)$$

### Financial leverage ratios

**Total debt ratio** - measure the degree to which a company has used debt to finance its assets. Total Debt ratio quantifies the proportion of company financing that comes from creditors.

$$\text{Total Debt ratio} = \frac{\text{Total assets} - \text{Total equity}}{\text{Current Assets}} \quad (6.7)$$

**Debt-Equity ratio** - measure the degree to which shareholder equity covers all outstanding debts. Debt-Equity ratio quantifies the proportion of company financing that comes from investors.

$$\text{Debt-Equity ratio} = \frac{\text{Total liabilities}}{\text{Total equity}} \quad (6.8)$$

**Equity Multiplier ratio** - measure the degree to which stakeholder equity covers the company's assets. Equity Multiplier ratio quantifies the proportion of a company's assets that is financed by stakeholder equity.

$$\text{Equity Multiplier ratio} = \frac{\text{Total assets}}{\text{Total equity}} \quad (6.9)$$

**Times Interest Earned ratio** - measure the creditworthiness of a company (the ability of the company to meet its debts). Earning before interest and taxes can be abbreviated as EBIT. Times Interest Earned ratio quantifies the number of times a company could pay the interest on its annual debt.

$$\text{Times Interest Earned ratio} = \frac{\text{Earning before interest and taxes}}{\text{Interest}} \quad (6.10)$$

**Cash Coverage ratio** - measure the ability of a company to service its debt and meet financial obligations (e.g. interest payments and dividend). Cash Coverage ratio quantifies the cash available to a company as a proportion of the interest on its annual debt.

$$\text{Cash Coverage ratio} = \frac{\text{EBIT} + \text{Non-Cash Expenses}}{\text{Interest}} \quad (6.11)$$

### Asset utility ratios

**Inventory turnover** - measure how efficiently a company manages its inventory. Inventory is defined here as average inventory over the course of the period. Inventory turnover quantifies the number of times an inventory is created and sold during the period.

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Inventory}} \quad (6.12)$$

**Day sales in inventory** - measure how long a company's stock of inventory will last. Day sales in inventory quantifies the average time in days that a company takes to turn its inventory into sales.

$$\text{Day sales in inventory} = \frac{365}{\text{Inventory turnover}} \quad (6.13)$$

**Receivables turnover** - measure how quickly a company is collecting its sales that were made on credit. Receivables turnover quantifies the number of times "accounts receivable" have been created through the sale of goods on credit.

$$\text{Receivables turnover} = \frac{\text{Net annual credit sales}}{\text{Accounts receivable}} \quad (6.14)$$

**Day sales in accounts receivable** - measure how quickly a company is collecting cash from its credit sales. Day sales in accounts receivable quantifies the average time in days that a company takes to collect cash from its credit sales.

$$\text{Day sales in accounts receivable} = \frac{365}{\text{Receivable turnover}} \quad (6.15)$$

**Total asset turnover ratio** - measures the value of a company's sales or revenues relative to its assets. Total asset turnover ratio quantifies the size of total sales as a proportion asset investment.

$$\text{Total asset turnover ratio} = \frac{\text{Total annual sales}}{\text{Total assets}} \quad (6.16)$$

**Capital intensity ratio** - measures the amount of assets requires to generate £1 in sales. Capital intensity ratio quantifies the size of asset investment as a proportion of total sales.

$$\text{Capital intensity ratio} = \frac{\text{Total assets}}{\text{Total annual sales}} \quad (6.17)$$

### Profitability ratios

**Profit margin** - measures the degree to which a company makes money. Profit margin quantifies the proportion sales that has turned into profit.

$$\text{Profit margin} = \frac{\text{Net income}}{\text{Net sales}} \quad (6.18)$$

**Return on assets ratio** - measures a company's net income produced from its total assets. Return on assets ratio quantifies company income as a proportion of its total assets.

$$\text{Return on assets ratio} = \frac{\text{Net income}}{\text{Total assets}} \quad (6.19)$$

**Return on equity ratio** - measures the return a company makes on its equity. Return on assets ratio quantifies net income as a proportion of shareholder equity.

$$\text{Return on equity ratio} = \frac{\text{Net income}}{\text{Total equity}} \quad (6.20)$$

## Market value ratios

Note:

- Shares are units of equity ownership in a company
- Stocks are the same as shares
- A stock exchange is a market that matches buyers of company shares with sellers of company shares
- A financial market is a place where financial assets are issued and traded

**Price-earnings ratio** - measures whether a company's stock price is overvalued or undervalued. Price-earnings ratio quantifies current share price as a proportion of profit per share.

$$\text{Price-earnings ratio} = \frac{\text{Price per share}}{\text{Earnings per share}} \quad (6.21)$$

**Market-to-book ratio** - measures whether a company is overvalued or undervalued. Here, Market value per share is the stock price (worth on the market) and Book value per share is the amount of money left if all assets are sold and all liabilities are paid. Market-to-book ratio quantifies a company's current market value relative to its book value.

$$\text{Market-to-book ratio} = \frac{\text{Market value per share}}{\text{Book value per share}} \quad (6.22)$$

## Chapter 7

# Critical Path and Delay Analysis in Construction Projects

### 7.1 Kroll Expert Services

With experience spanning global industries across the world's continents, we are a market leader for expert witnesses, dispute resolution, advisory and investigative services. Our clients include international contractor, government agencies, blue chip multinationals, investors, developers, banks and insurers.

- Delay analysis
  - The assessment of the incidence, extent and causes of delay to the progress and completion of capital investment projects
- Quantum analysis
  - Construction quantum is the assessment of financial entitlement of the parties arising from claims under or out of a construction contract
- Project advisory
  - Our advisory practice draws on our in-depth project delivery, management and industry expert insight to provide pre-dispute advisory services to capital projects, infrastructure programmes and real assets organisations across their delivery lifecycle

## 7.2 Dispute in construction and the role of experts

### 7.2.1 What is a dispute?

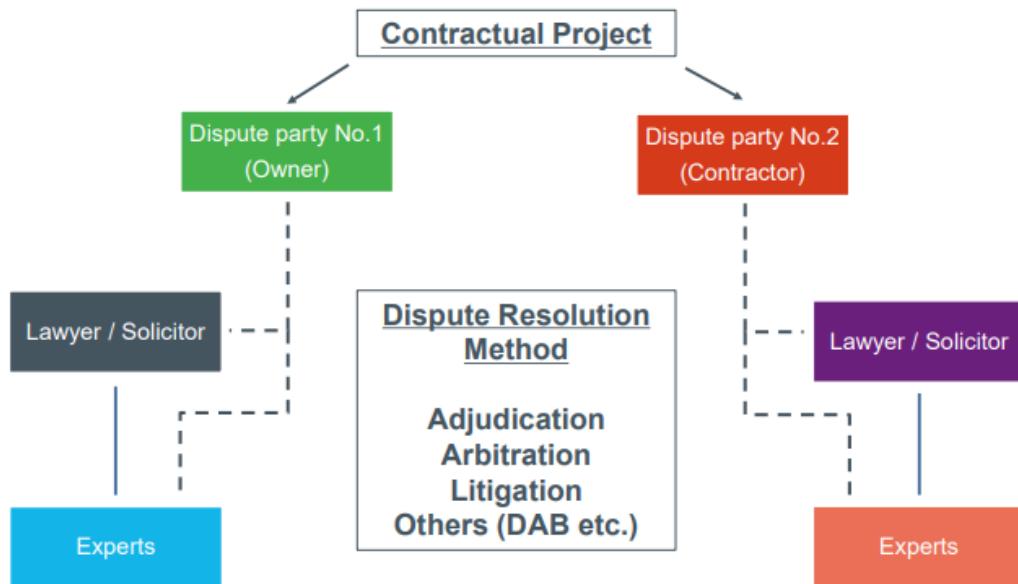


Figure 7.1: Dispute pathway.

### 7.2.2 Typical sequence to a trial

1. Dispute arises between two parties
2. Dispute party hires a lawyer
3. Lawyer engages with an independent expert
4. Dispute party provides the expert with evidence
  - Identify issues
  - Analyse data
  - Prioritise key areas
5. The expert provides independent expert report
  - Report could be used:
    - in dispute resolution
    - for consulting purposes
    - for negotiation purposes
6. Engage with opposing expert
7. Go to trial
  - Oral evidence / cross examination

### 7.2.3 Independent expert

- Independent

- An expert witness is to provide **independent**, impartial and unbiased evidence to the court or tribunal
  - Expert
    - An **expert** is a person engages to give an opinion based on experience, knowledge and expertise
  - Assist
    - The primary duty of an expert witness is to **assist** the court

### 7.3 Critical path and delay analysis

### 7.3.1 What is delay analysis?

**Definition:**

*Delay analysis is the assessment of impact, causes and effects of a construction or engineering project not meeting its original forecast completion date.*

Delay analysis includes:

- Determination of critical path of the project (EFFECT)
  - Assessment of extent of delays (quantification of EFFECT)
  - Assessment of causes of delays (CAUSE)

### 7.3.2 What is the critical path?

*The longest sequence of activities through a project network from start to finish, the sum of whose durations determines the overall project duration.*

### 7.3.3 Basics of delay

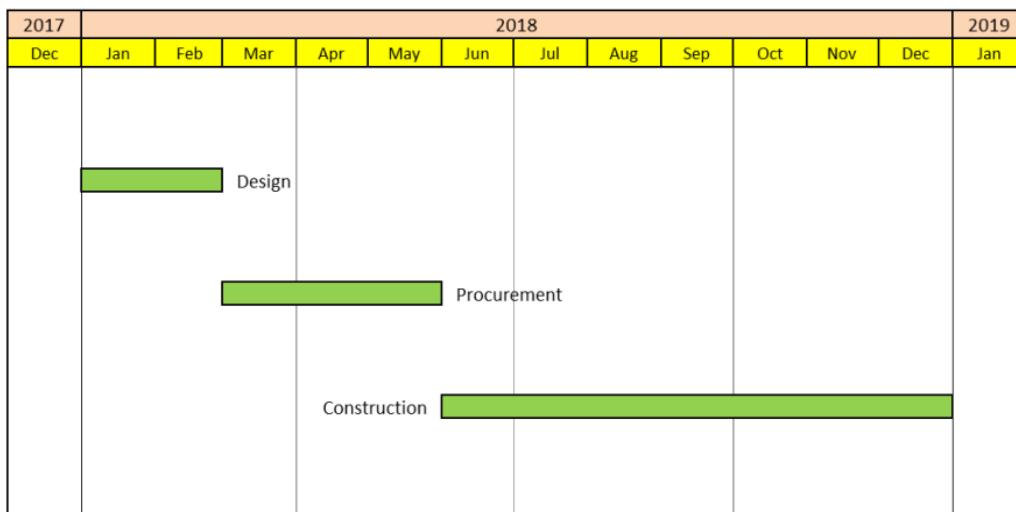


Figure 7.2: Work programmes can be as simple as a few bar charts.

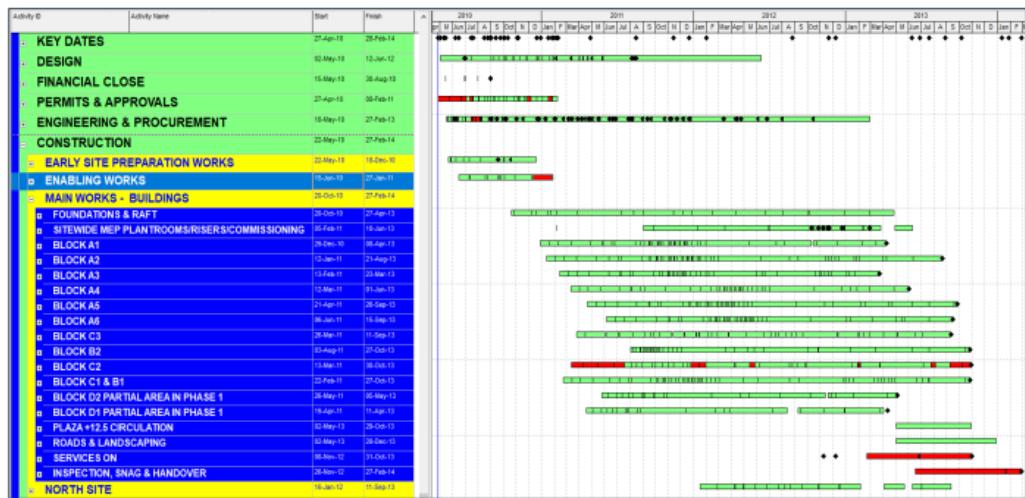


Figure 7.3: Or a complex schedule prepared in a specialist software.

#### 7.3.4 Information sources

Experts do not just rely on the programme. The best intelligently utilise other as-built evidence. A key rule:

*Never rely on assumptions when you know the facts.*

- Primary
    - Photographs
    - Videos
    - Site diaries / daily reports
    - Pour / installation records
    - Inspection sheets
    - Daily allocation sheets L/P/M
    - Detailed timesheets / labour resources timesheets
    - Detailed sign-off sheets
    - Detailed quality records
    - Contemporaneously marked-up progress drawings
    - Quality Assurance documentation
  - Secondary
    - Updated programmes
    - Look-ahead programmes
    - Progress reports
    - Progress meetings' minutes
    - Data spreadsheets
    - Valuations / earned value analysis
    - As-built drawings / records

- Claims
- Tertiary
  - Correspondence
  - General meeting minutes
  - RFIs (Request for Inspections)
  - TQs (Technical Queries)
  - Variations / Changes of Orders
  - Instructions
  - Etcetera

### 7.3.5 Methods of delay analysis

Method of analysis	Analysis type	Critical path determined	Delay impact determined
Impacted as-planned analysis	Cause & Effect	Prospectively	Prospectively
Time impact analysis	Cause & Effect	Contemporaneously	Prospectively
Time slice windows analysis	Effect & Cause	Contemporaneously	Retrospectively
As-planned vs as-built windows analysis	Effect & Cause	Contemporaneously	Retrospectively
Longest path analysis	Effect & Cause	Retrospectively	Retrospectively
Collapsed as-built analysis	Cause & Effect	Retrospectively	Retrospectively

Table 7.1: Methods of delay analysis.

### 7.3.6 Example 1

Consider two buildings:

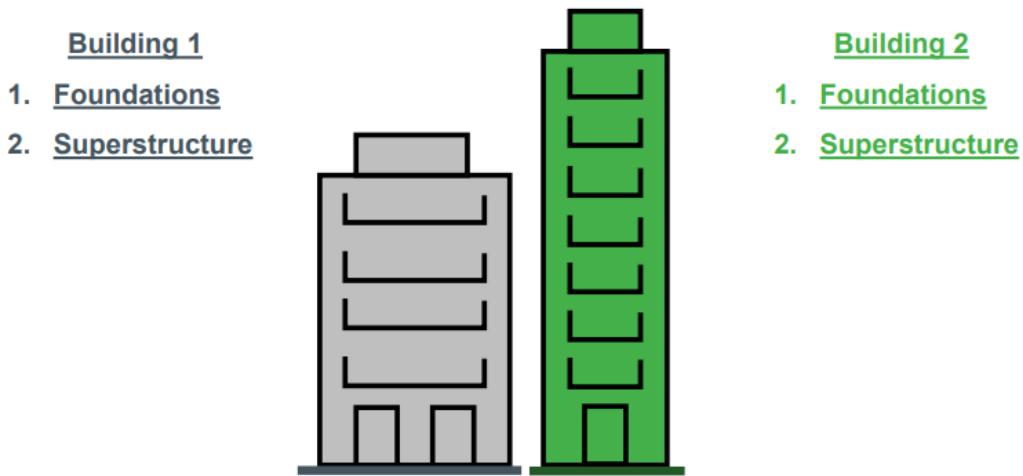


Figure 7.4: Example project with two buildings.

#### Impact as-planned analysis

Potential issue with the result we get as it is all perspective. None of the sequence happened as it should have. Everything in green is a projection i.e. what should have happened. Impacting what should happen

is not consistent with the real world events. For example, in this example, we could add mitigation to the superstructure of B2, moving the project completion to the left (shortening it).

Another issue is that results are quite biased. Using this method, we tend to maximise the impact of delay, so any introduction of a delay event is likely to cause a delay in the completion date, regardless of whether it is critical or not. We have not found a true critical path to impact the delay onto it.

Another issue is that we are only picking certain delay events and not necessarily taking into account other delay events. In this example, we have only taken a delay event for B2. There could be other things happening in B1 that we are not taking into account. This analysis tends to favour the party selecting the events to be modelled; it could be ignoring delay events that are not favourable to the client or the contractor.

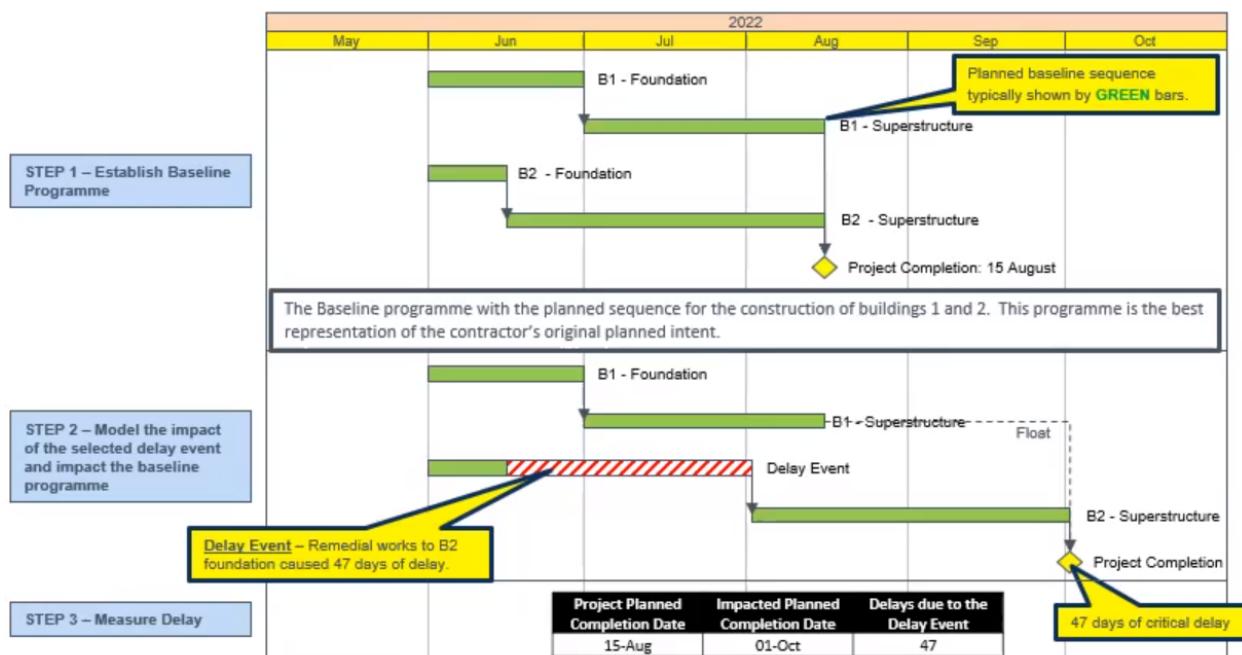


Figure 7.5: Impact as-planned analysis for example project.

### Time impact analysis

This differs from Impact as-planned analysis, as we take into consideration actual progress on the project, represented by the blue bars. Similarly to the previous analysis method, it is still a matter of perspective. We are still projecting the completion date. Again, we can pick certain delay events that are not necessarily taking into account the full impact of other delay events on the other buildings. However, this analysis method is reliable in anticipating the potential delay of an event. This is a method used on live projects that have not been finished yet to predict delay impact.

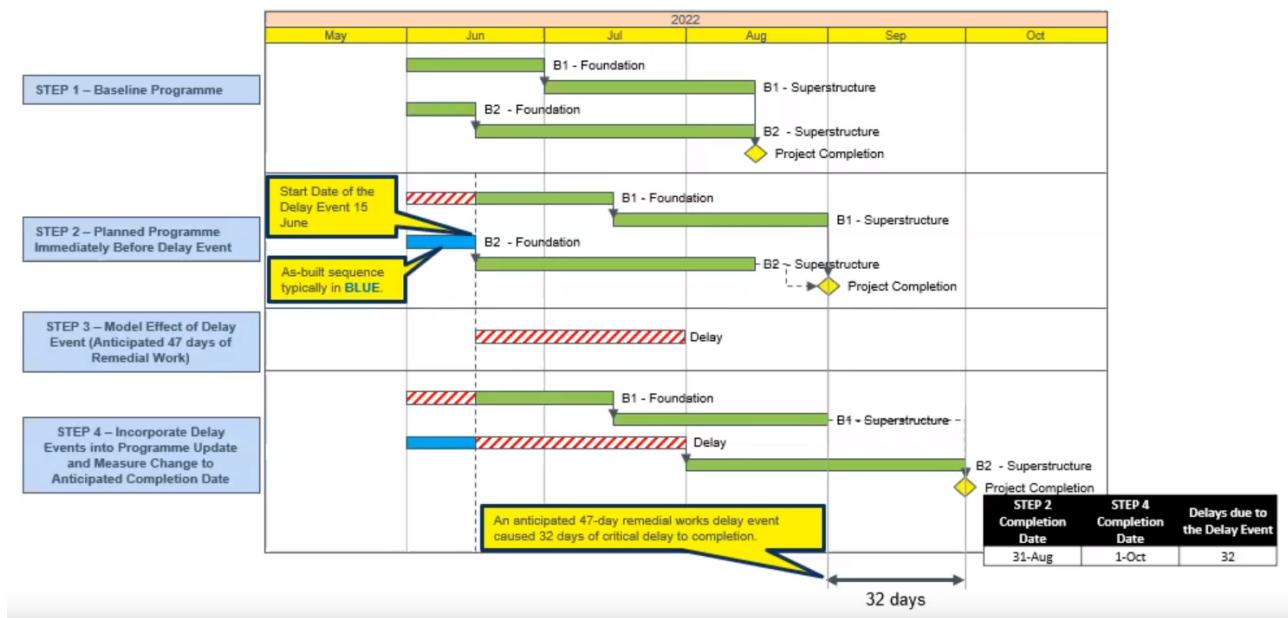


Figure 7.6: Time impact analysis for example project.

### Time slice windows analysis

This method takes into account all that happened during the project. It takes into account all the ongoing parallel issues and makes an assessment of whether it is critical or not. It is quite an accurate method of delay and is used on projects. An issue is that it relies very heavily on programme updates. This could mean it is heavily biased; in the construction programme the contractor may use it to artificially establish an entitlement for an extension of time. This means that the information provided could be skewed to favour the contractor. In ideal circumstances, when we have an ideal programme and we find it is not biased or it has not been used at all by the contractor, we can use this form of analysis, to analyse delay. However, the case is usually that they are not reliable.

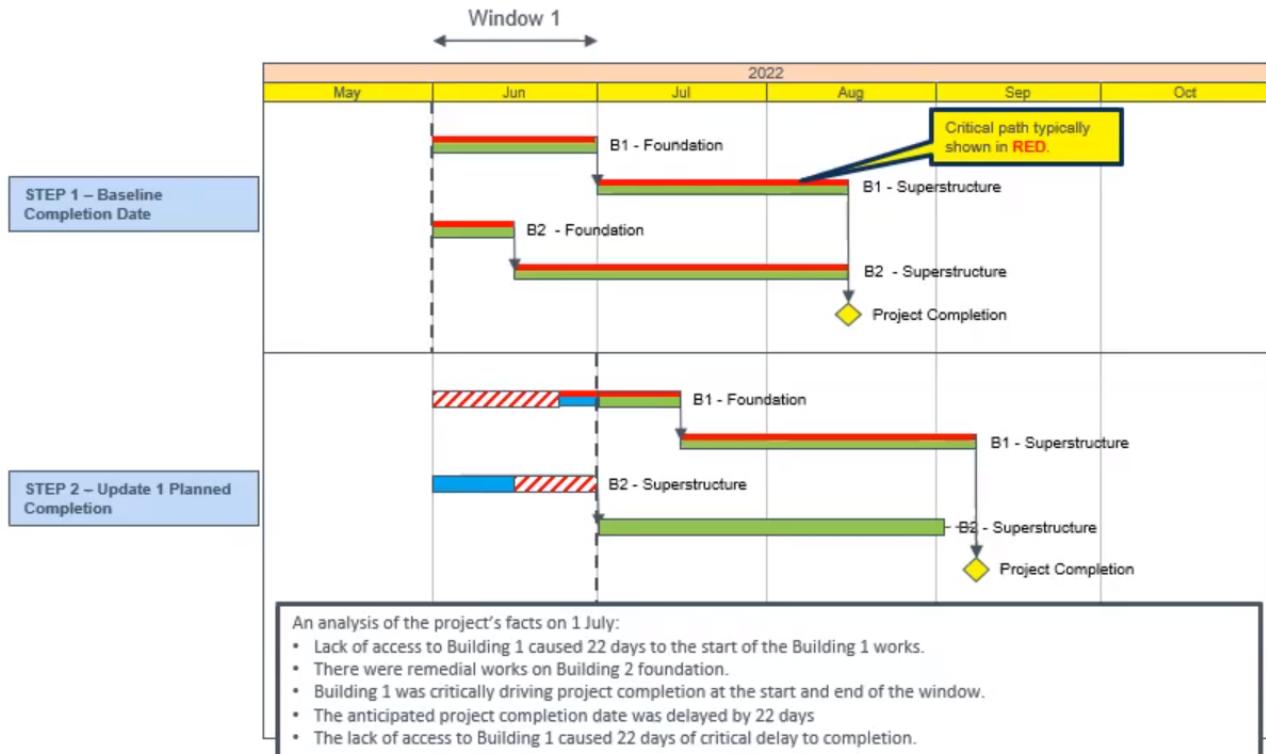


Figure 7.7: Time slice windows analysis for example project. Window 1

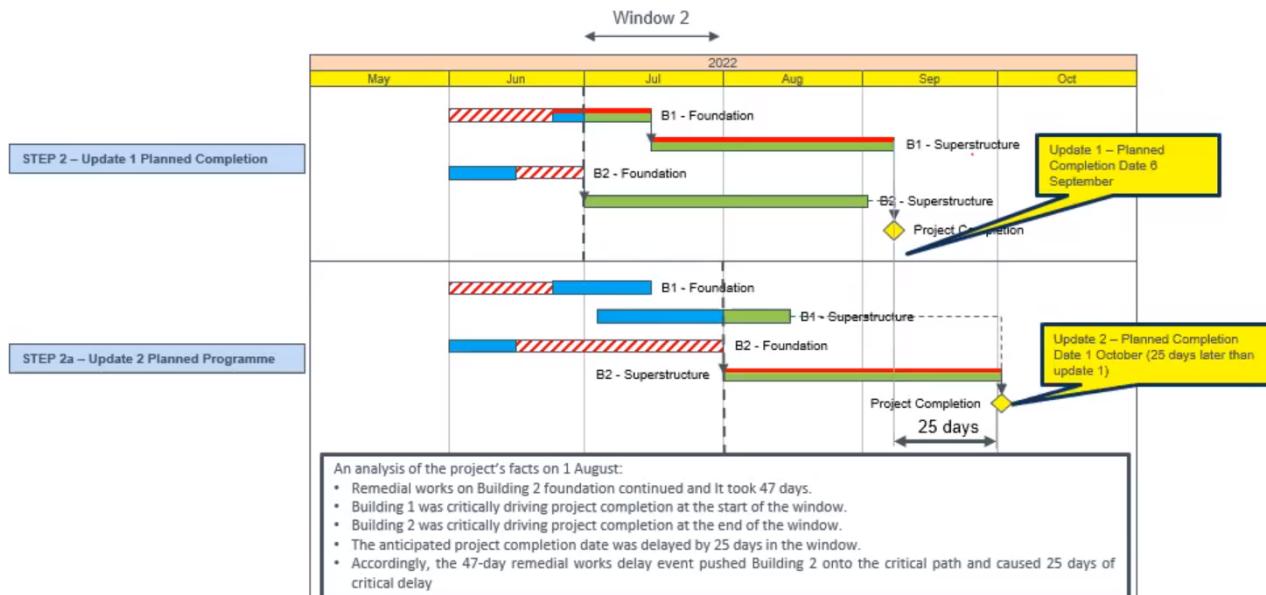


Figure 7.8: Time slice windows analysis for example project. Window 2.

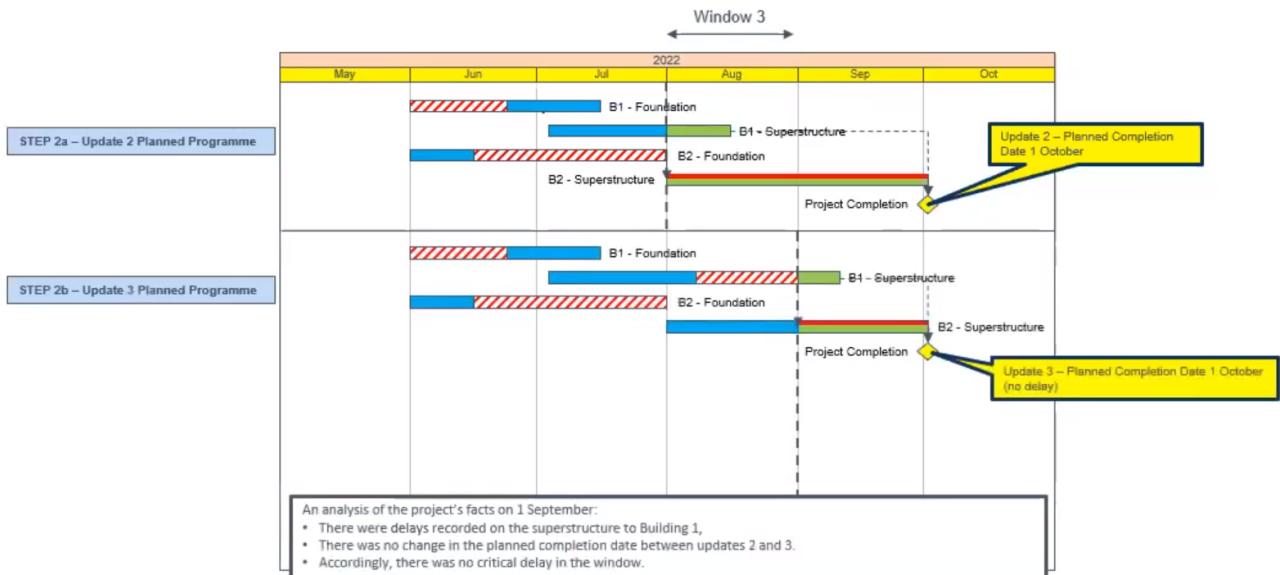


Figure 7.9: Time slice windows analysis for example project. Window 3.

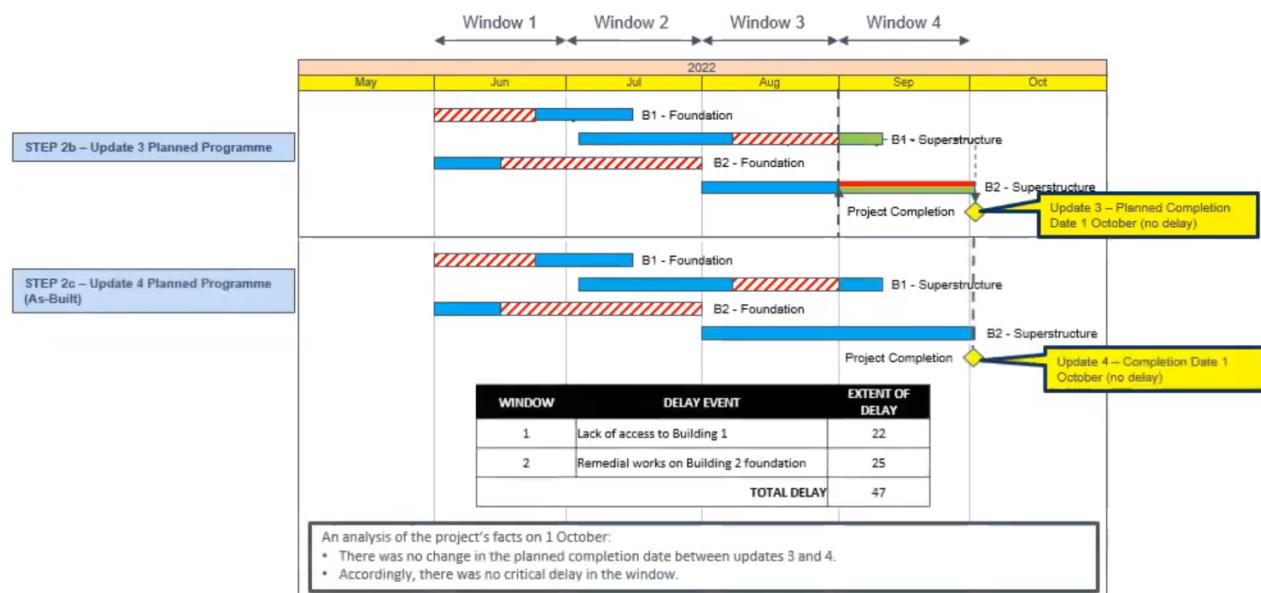


Figure 7.10: Time slice windows analysis for example project. Window 4.

### As-planned vs as-built windows analysis

More effort is required for this analysis. This is usually the best method to use, however a key issue with this method is that all delays are measured against the baseline programme. This can lead to dispute if the baseline programme is quite wrong and unfeasible. A skewed baseline programme would lead to skewed delay results.

This method of analysis defines the critical path through experience and looking through the data, which is something more difficult to defend, because as an expert, you are relying your own subjective opinion to determine the critical path. Lawyers or disputing parties may argue against this. It also requires the project data to be reliable.

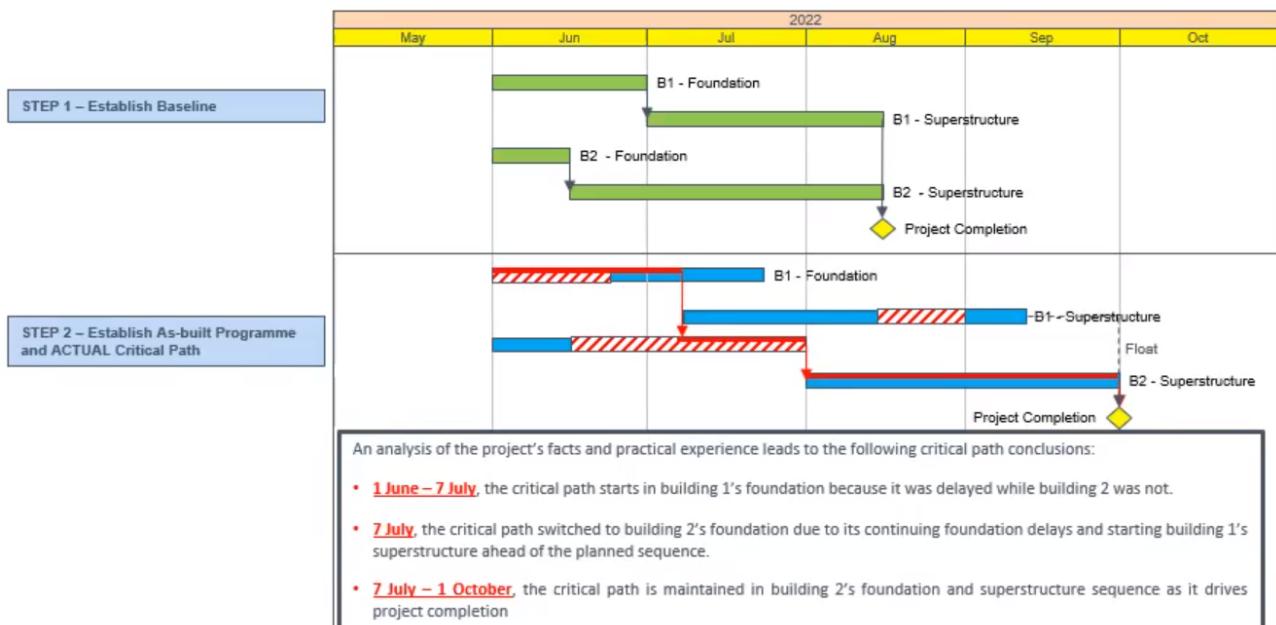


Figure 7.11: As-planned vs as-built windows analysis for example project. Step 1 & 2.

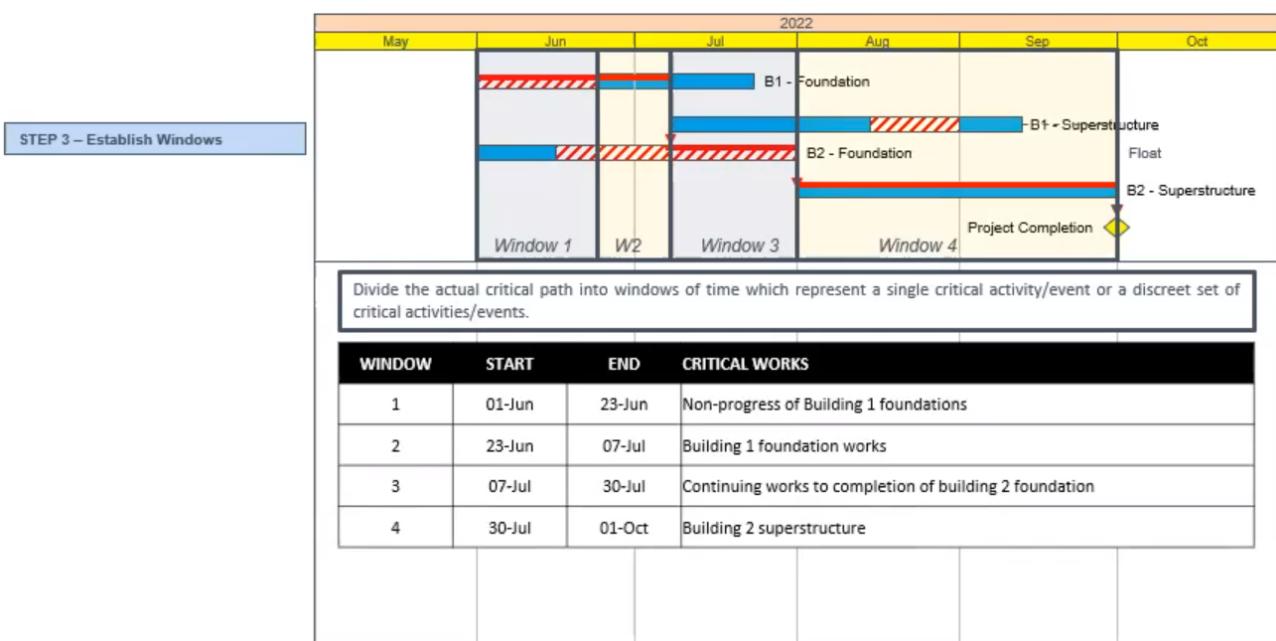


Figure 7.12: As-planned vs as-built windows analysis for example project. Step 3.

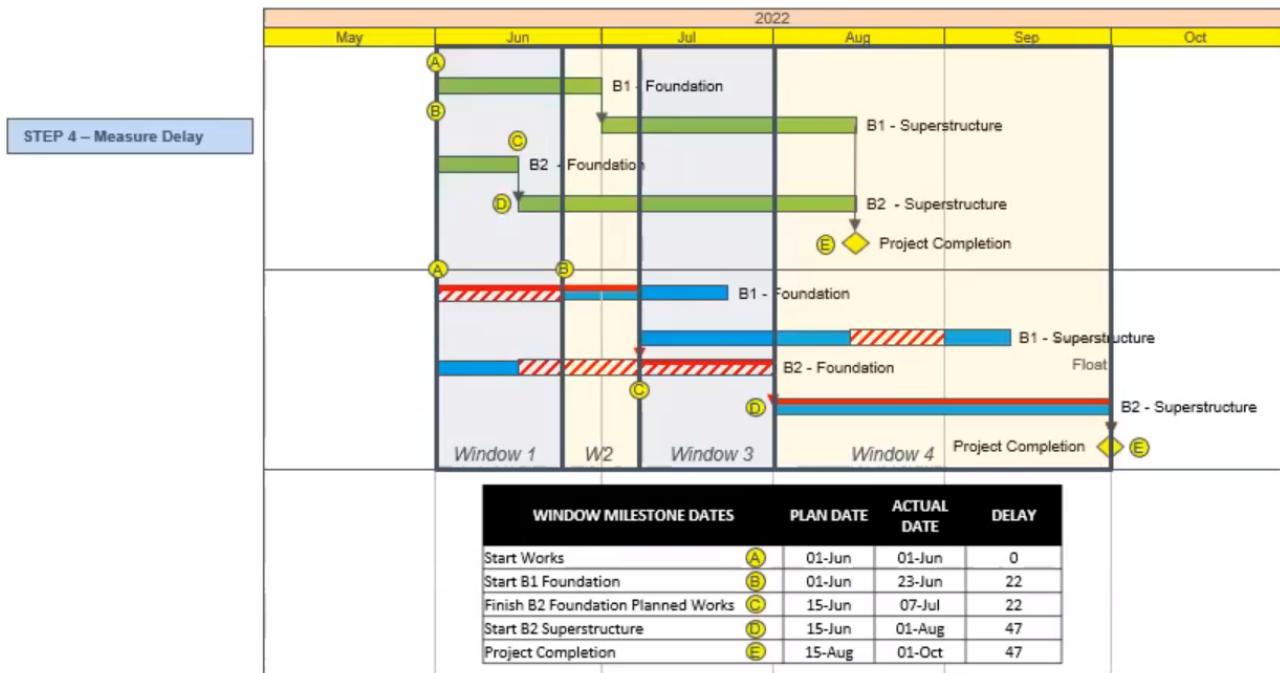


Figure 7.13: As-planned vs as-built windows analysis for example project. Step 4.

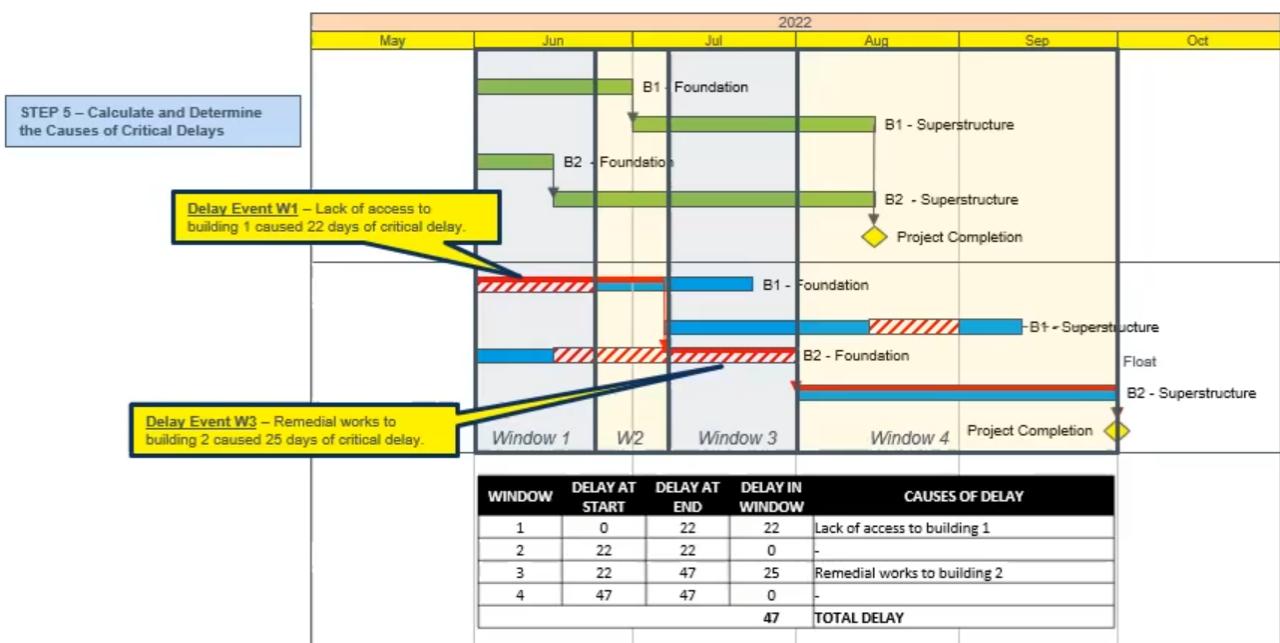


Figure 7.14: As-planned vs as-built windows analysis for example project. Step 5.

### Longest path analysis

Here, the critical path is found retrospectively, which does not really allow for the switching of the critical path between the two buildings. Usually, in large projects, the critical path switches between many different aspects of the project. In a simple project, this type of analysis might work. For larger projects, we do need to analyse the critical path more contemporaneously. This method of analysis is sometimes used when programme updates are too few or unreasonable, which prevents the use of time slice windows.

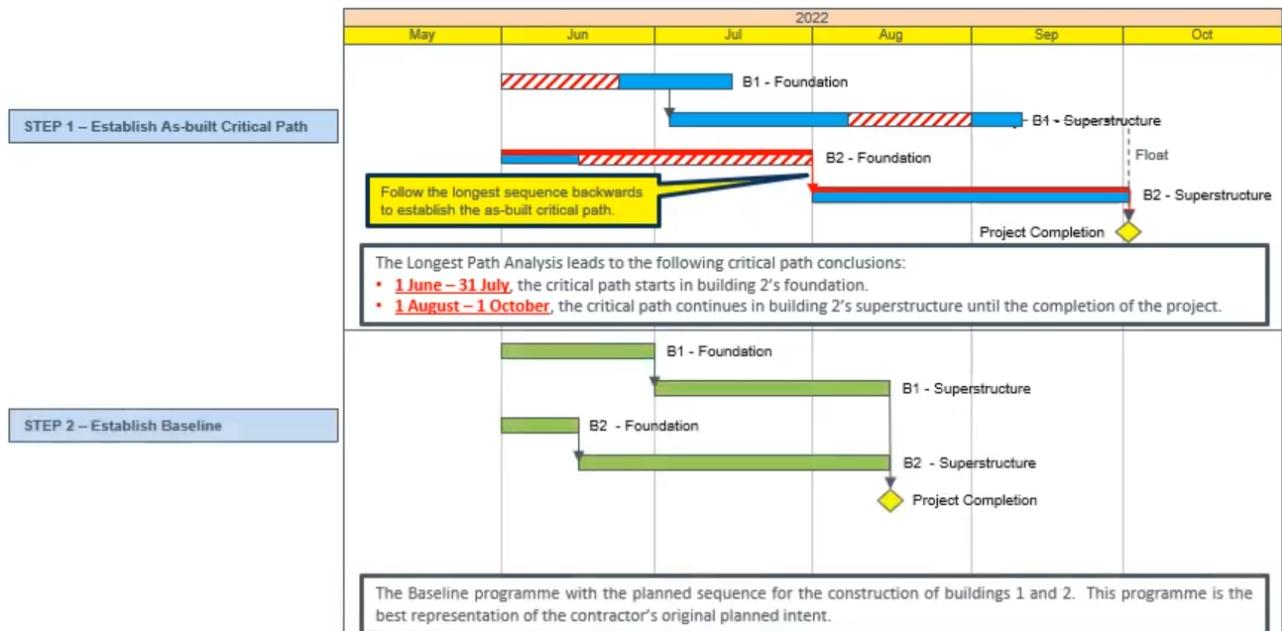


Figure 7.15: Longest path analysis for example project. Step 1 & 2.

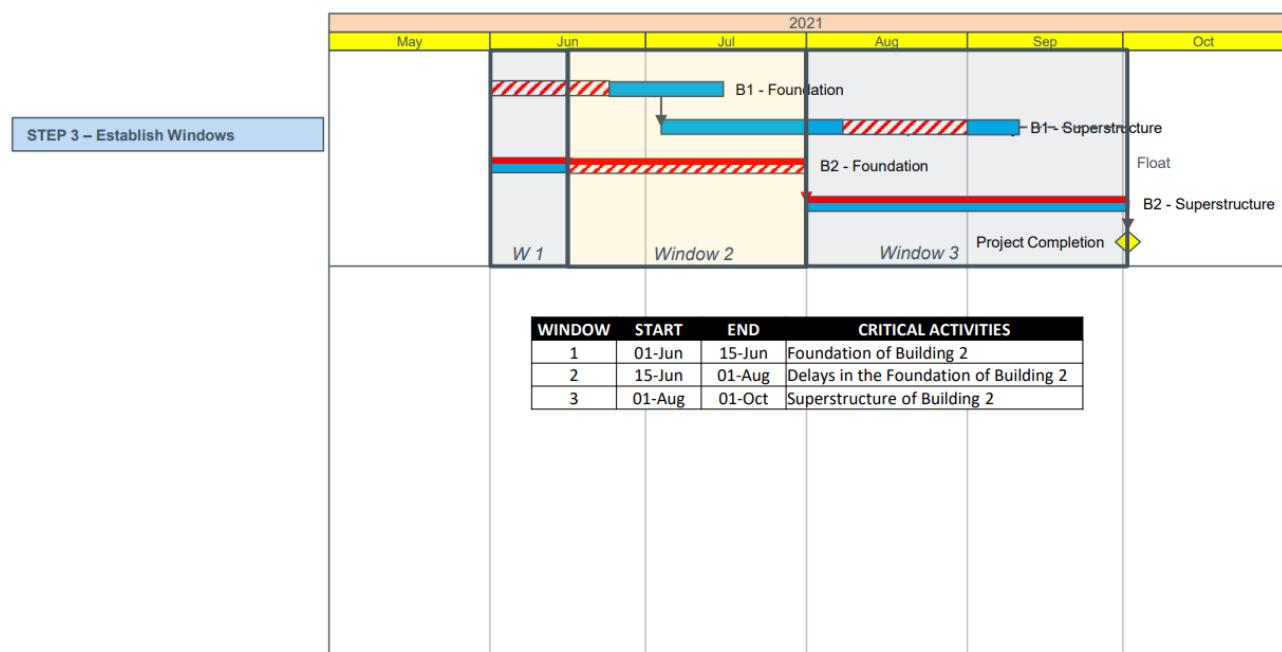


Figure 7.16: Longest path analysis for example project. Step 3.

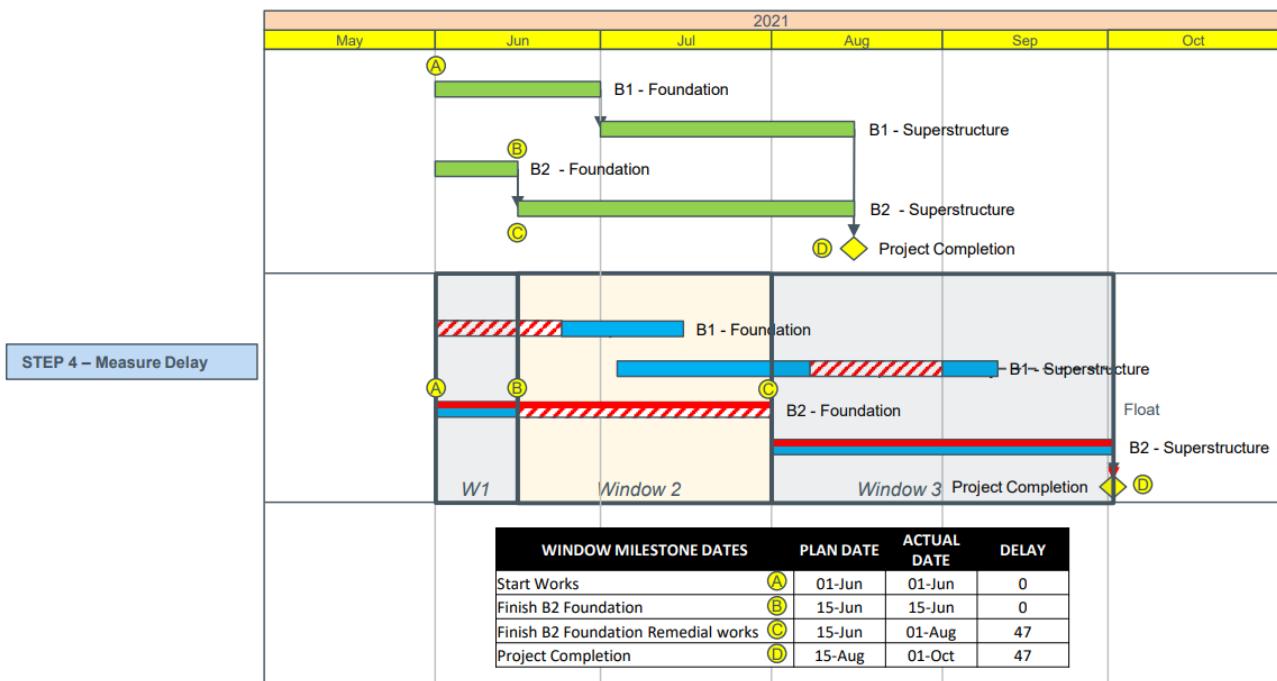


Figure 7.17: Longest path analysis for example project. Step 4.

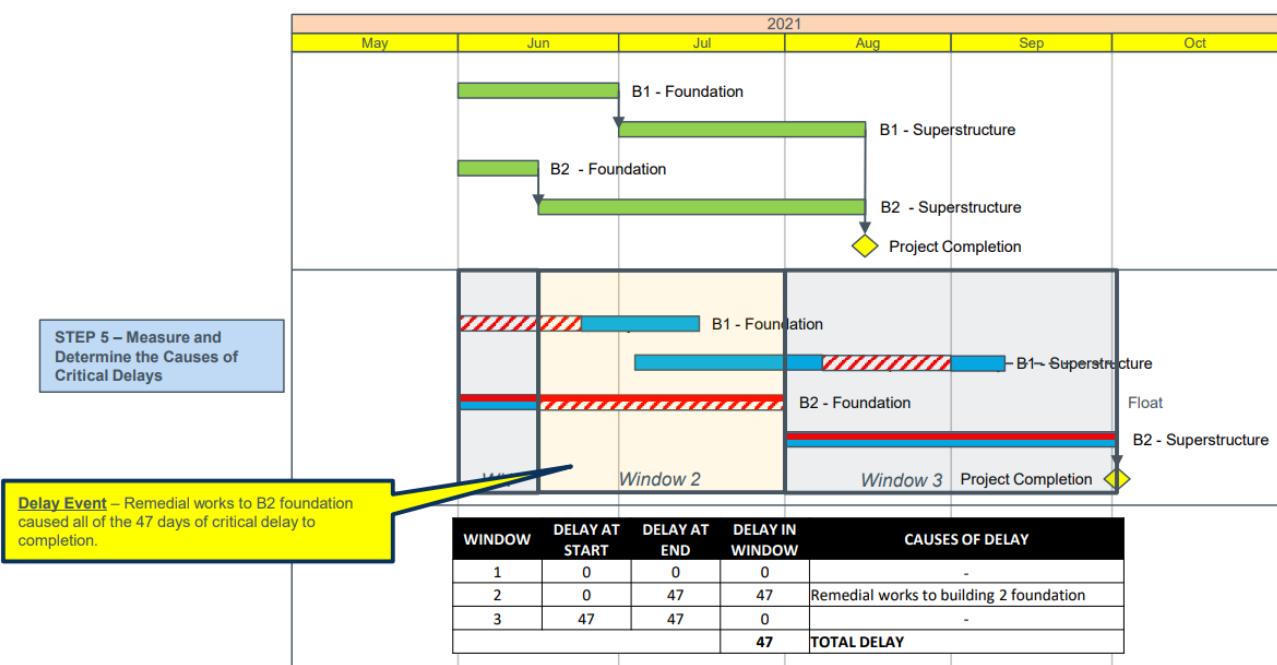


Figure 7.18: Longest path analysis for example project. Step 5.

### Collapsed as-built analysis

This involves building a programme from scratch using the project records (as-built data) and track what happened in the project. Then we can add in our delay events, and calculate the projected completion date if there was no delay.

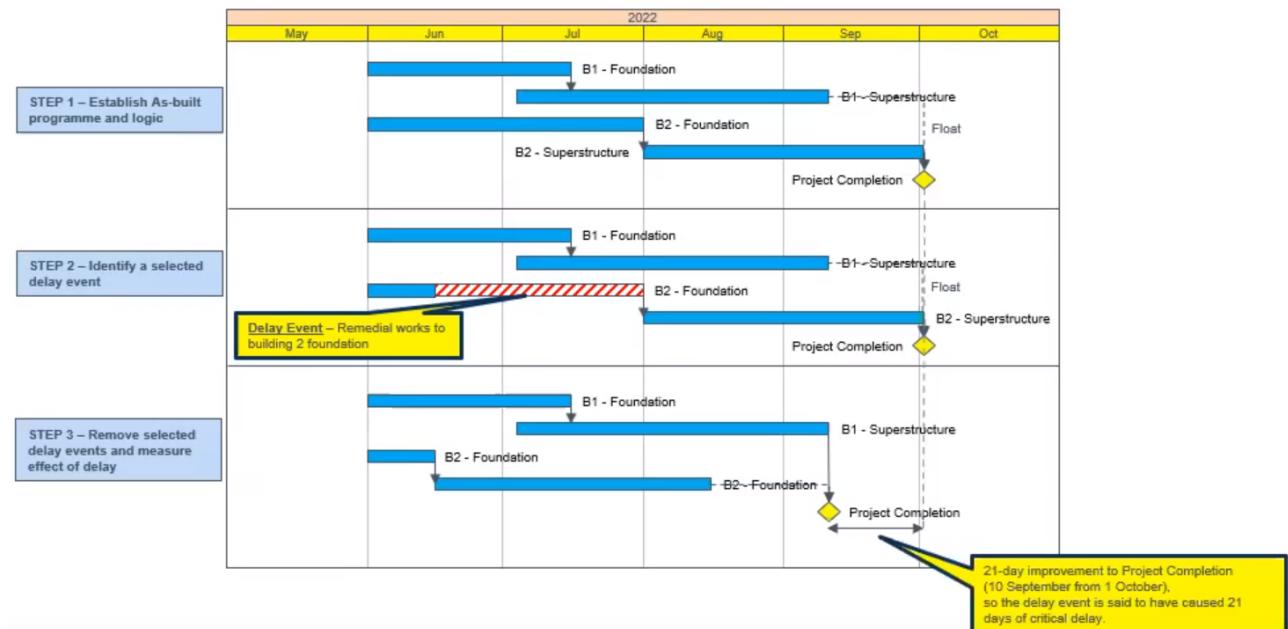


Figure 7.19: Collapsed as-built analysis for example project.

### 7.3.7 Why so many methods?

Blame lawyers and claims consultants.

Method of analysis	The question it answers
Impacted as-planned analysis	What effect would this event(s) have had on the completion date assuming everything else went exactly as planned in the programme?
Time impact analysis	What was the likely effect of this event(s) on the completion date adjudged from the point in time it was instructed or arose?
Time slice windows analysis	What was the contemporaneous or actual critical path to completion throughout the works and what were the causes of delay thereto?
As-planned vs as-built windows analysis	What was the contemporaneous or actual critical path to completion throughout the works and what were the causes of delay?
Longest path analysis	What was the as-built critical path to completion, viewed retrospectively, and what were the causes of delay thereto?
Collapsed as-built analysis	But for the event(s) when would the completion date have been achieved?

Table 7.2: Justification for various methods of analysis.

### 7.3.8 Example 2

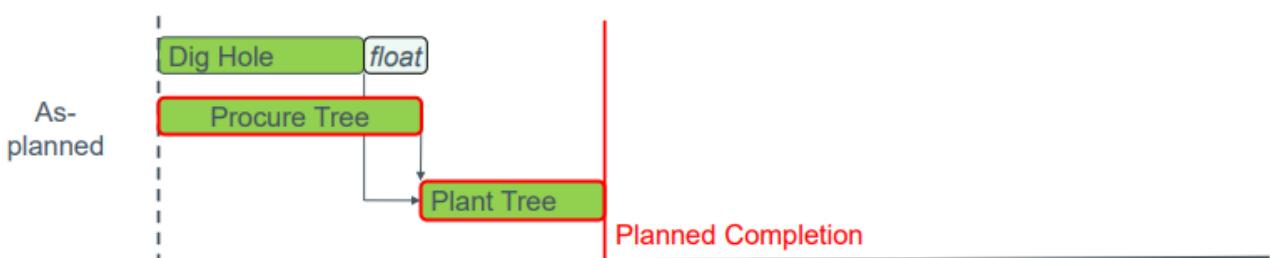


Figure 7.20: A simple project programme.

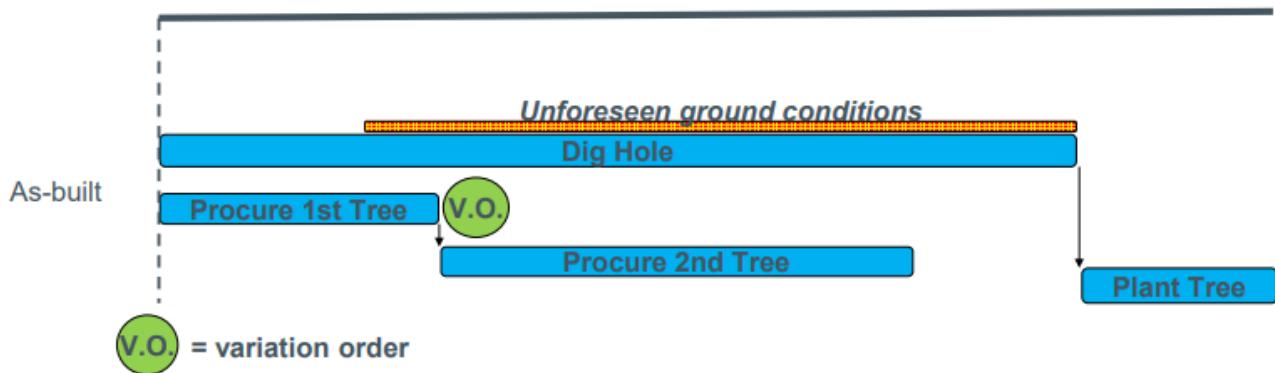


Figure 7.21: Actual programme.

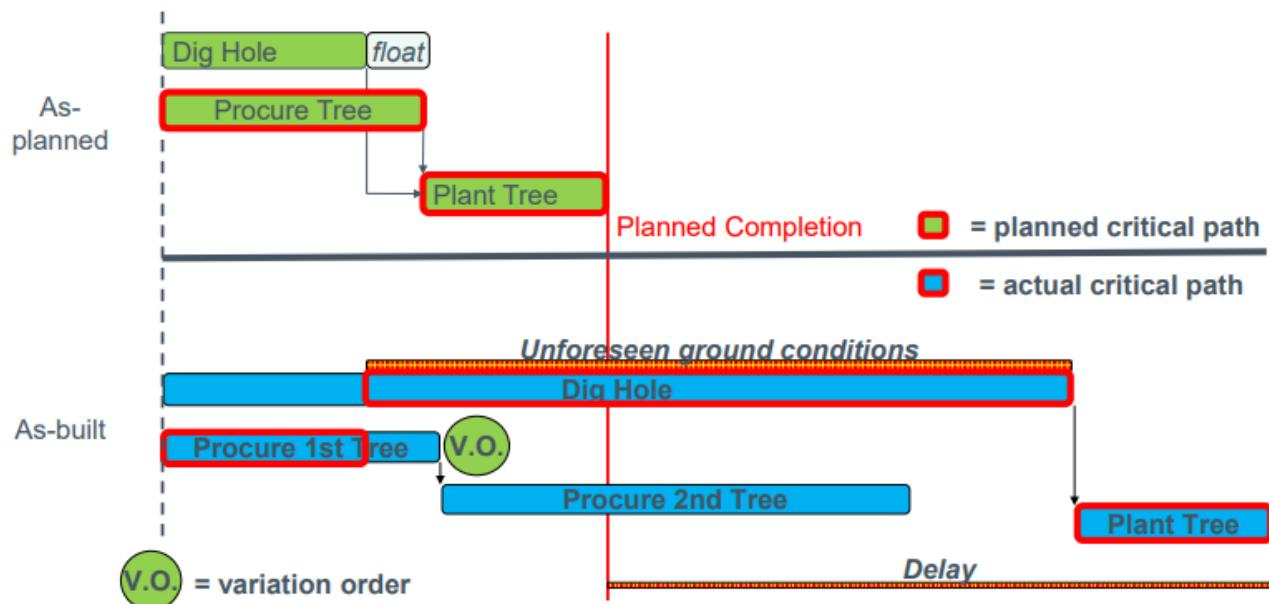


Figure 7.22: Comparison between planned and actual programmes.

Then the unhappy, but clever, instructing lawyer thinks:

*"That's all very well Mr Delay Expert, but what about the classic 'but' for test of causation? Has there been an act of prevention here?"*

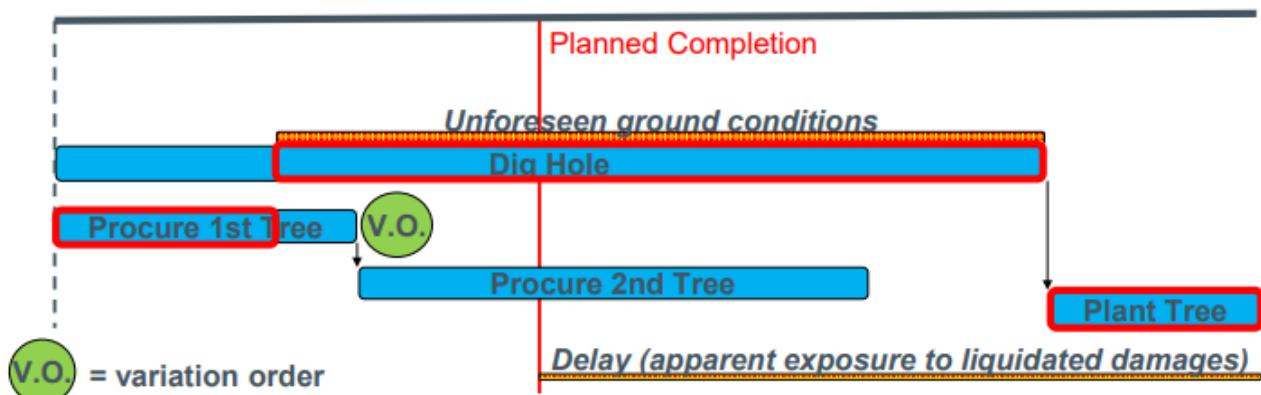


Figure 7.23: Critical path analysis.

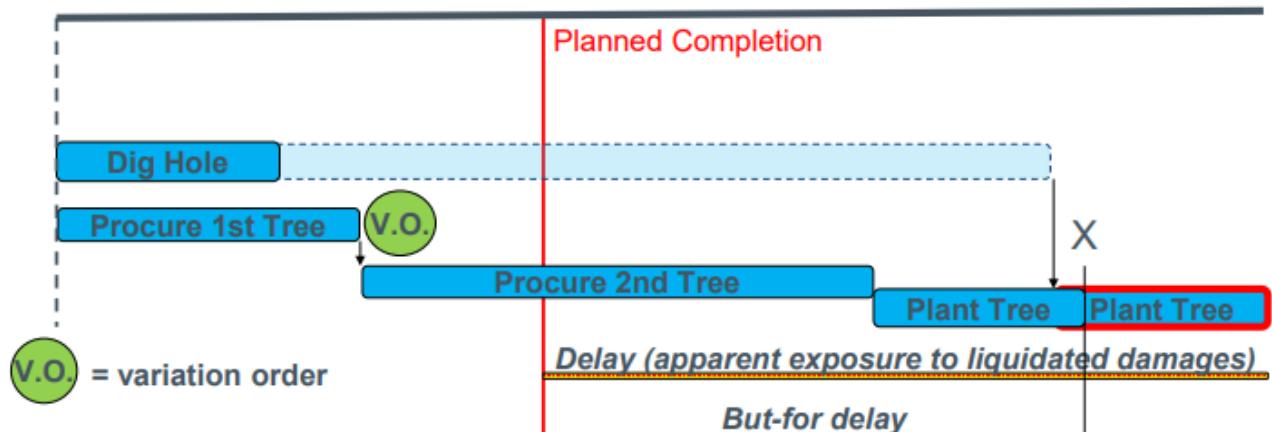


Figure 7.24: But-for delay.

*"Your honour, as a consequence of the instructed VO, there were no circumstances where my client could have completed this project any earlier than X. It would be harsh, indeed immoral, to expose her to Liquidated Damages liability before that date!"*

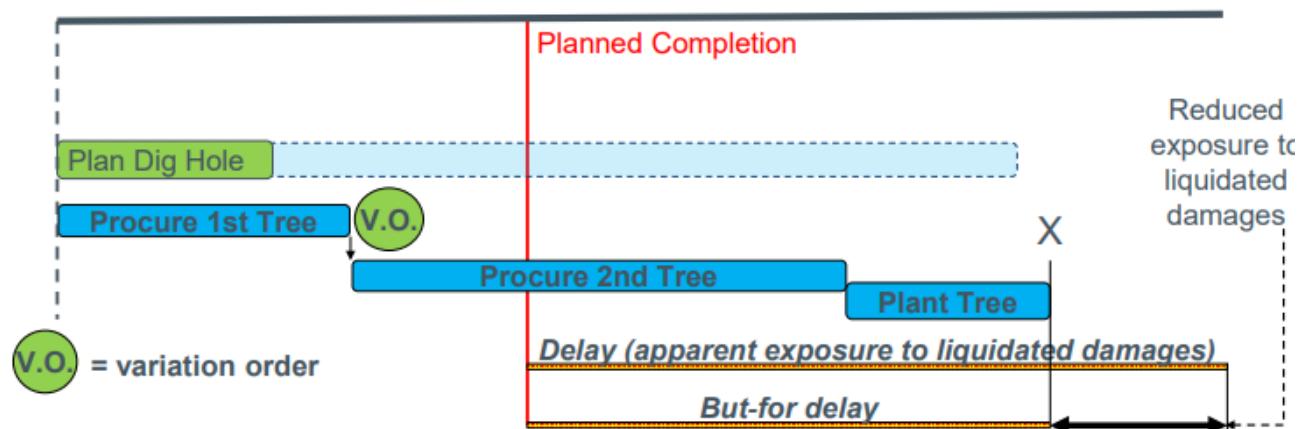


Figure 7.25: Reduced exposure to liquidated damages.

## 7.4 Summary

1. The **delay expert** advises how the project was delayed
2. The **technical expert** advises what technically went wrong
3. The **quantum expert** advise on additional costs
4. The **court** assigns liability and ultimately extension of time and damages

# Chapter 8

## Financial Project Planning

### 8.1 Budgeting

#### 8.1.1 What is budgeting?

Budgeting involves examining:

- Estimated revenues
- Estimated expenditures

over a specific period in the future, to determine a prediction of future company performance.

#### 8.1.2 What information is used for budgeting?

1. Accounting records
2. Other sources within the company e.g. engineering / sales / production teams
3. Sources outside the company e.g. published information, personal contacts
4. Research and development

in decreasing order of importance (approximately).

#### 8.1.3 Types of budget

1. Master budget: aggregation of lower-level budgets
2. Operating budget: “projected income statement”
3. Cash budget: “cash flow forecast”
4. Capital budget: “projected balance sheet”
5. Labour budget: “Staffing forecast”
6. Static budget: fixed revenue and expenses

#### 8.1.4 Example 1: Operating budget

Deere Consulting Ltd. has the following income statement for 2021:

<b>Revenue</b>	<b>£(,000)</b>
Sales	25,323
Cost of goods sold	18,582
Gross profit	6,741
<b>Operating expenses</b>	
Total operating expenses	5,223
Net income	1,280

Table 8.1: Deere Consulting Ltd. Income Statement. Year ended 31 December, 2021.

Deere Consulting Ltd. has provided the following supplementary information to further assist with the budget:

- Operating expenses are expected to increase to £5.62 million and sales are expected to increase proportionally
- The gross profit margin is expected to remain unchanged

Currently:

$$\frac{\text{Sales}}{\text{Op. Ex}} = \frac{25323}{5223} = 4.85 \quad (8.1)$$

$$\text{Gross profit margin} = \frac{\text{Gross profit}}{\text{Sales}} = \frac{6741}{25323} = 26.6\% \quad (8.2)$$

<b>Revenue</b>	<b>£(,000)</b>
Sales	27,257
Cost of goods sold	20,007
Gross profit	7,250
<b>Operating expenses</b>	
Total operating expenses	5,620
Net income	1,360

Table 8.2: Deere Consulting Ltd. Income Statement. Year ended 31 December, 2022.

## 8.2 Cash flow forecast

Cash flow forecasts are useful for clarifying the consequences of flows of money that occur at various times.

$$\text{Net Cash Flow over time } t = \quad (8.3)$$

$$\sum \text{Cash inflows in } t - \sum \text{Cash outflows in } t \quad (8.4)$$

Cash flow forecasts are important because they form the basis of evaluating alternative projects.

Cash flow diagrams can be used to visualise flows of money that occur at various times.



Figure 8.1: Cash flow forecast.

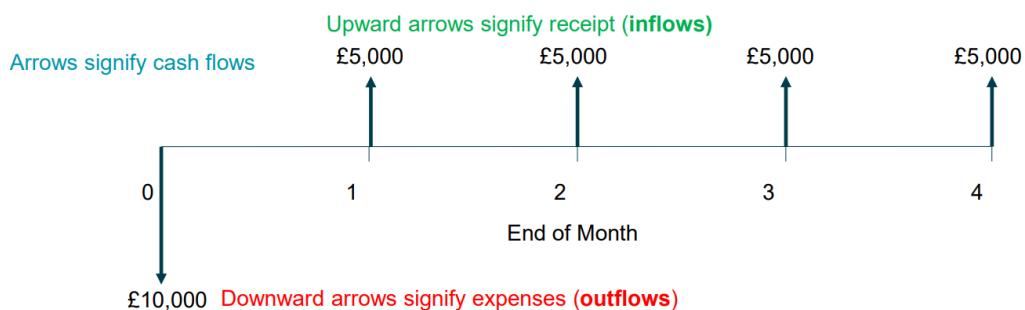


Figure 8.2: Cash flow forecast.

### 8.2.1 Example 2

Before evaluating the economic merits of a proposed investment, the ABC corporation insists that its engineers develop a cash-flow diagram of the proposal. An investment of £10,000 can be made that will produce uniform annual revenue of £5,310 for five years and then have a market (recovery) value of £2,000 at the EOY five. Annual expenses will be £3,000 at the end of each year for operating and maintaining the project.

*Draw a cash-flow diagram for the five-year life of the project. Use the corporation's viewpoint*

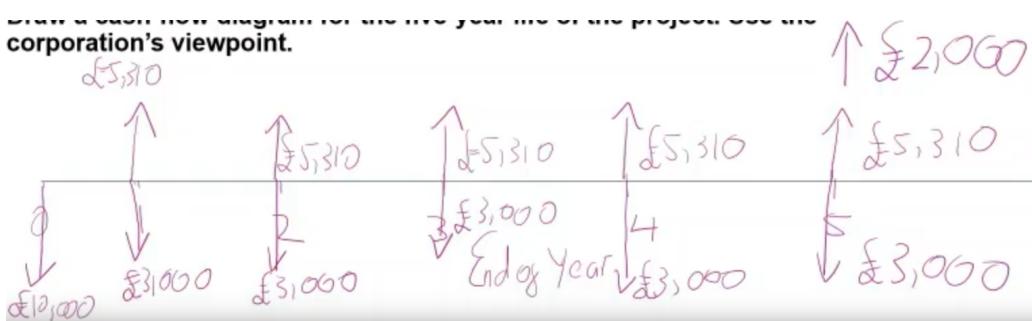


Figure 8.3: Cash flow diagram for ABC corporation.

So far, we have neglected the time value of money.

- So far, we have neglected the **time value of money**

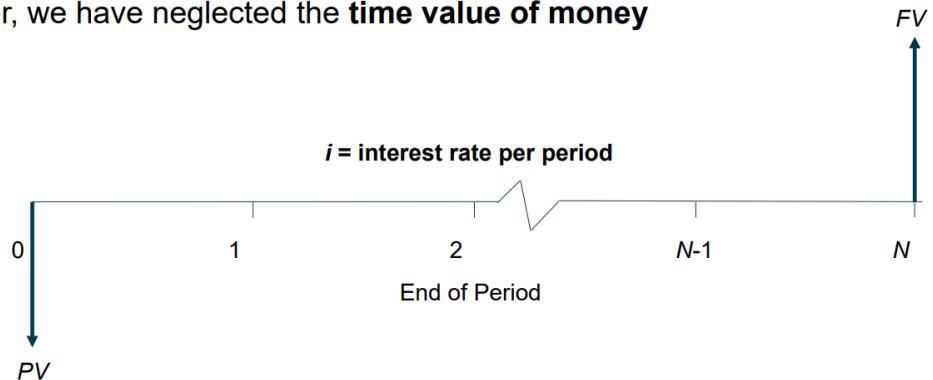


Figure 8.4: Time value of money in cash flow forecast.

### 8.2.2 Time value of money

Remember the formula for PV:

$$PV = \frac{FV}{(1 + i)^n} \quad (8.5)$$

Present value (PV) of future value (FV) in  $n$  periods, for an interest rate  $i$ .

Remember the formula for AV:

$$AV = PV \frac{i (1 + i)^n}{(1 + i)^n - 1} \quad (8.6)$$

Value of a series of uniform (annual) receipts (AV) that occur at the end of periods 1 to  $n$ , given their present value (PV) and an interest rate  $i$ .

### 8.2.3 Example 3

A processing plant is considering the installation of a newly designed solar energy system that will power the majority of its energy operations, and sell £450,000 per year of surplus energy to the grid over its expected lifetime of 10 years. If the interest rate is 12% per year, how much money can the plant afford to invest in the new boiler system?

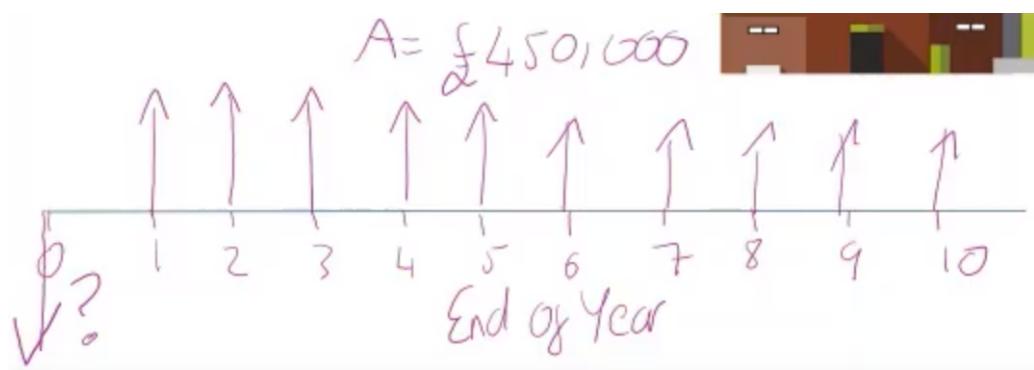


Figure 8.5: Cash flow diagram. Example 3.

$$i = 0.12 \quad n = 10 \quad (8.7)$$

$$PV = AV \frac{(1+i)^n - 1}{i(1+i)^n} \quad (8.8)$$

$$PV = 450000 \times \frac{(1+0.12)^{10} - 1}{0.12(1+0.12)^{10}} \quad (8.9)$$

$$PV = £2,542,600 \quad (8.10)$$

which is the upper limit on what the plant can afford to spend on the steam system.

### 8.2.4 Example 4

Automotive Engineering Ltd. is required to pay money into a compensatory fund, to cover damages caused by a company-related accident. The company will pay in £3 billion at the end of the third quarter of 2022 and another £2 billion in the fourth quarter of 2022. Twelve additional payments of £1.25 billion each quarter thereafter will result in a total of £20 billion having been paid into the fund. The interest rate is 3% per quarter.

*What will be the value of this payment stream at the beginning of the third quarter of 2022?*

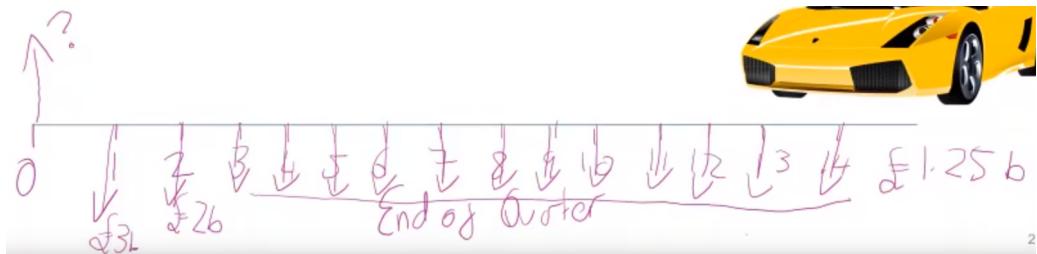


Figure 8.6: Cash flow diagram. Example 4.

PV of first end-of-quarter payment (£3 billion):

$$\frac{3}{(1+0.03)^1} = £2.91b \quad (8.11)$$

PV of second end-of-quarter payment (£2 billion):

$$\frac{2}{(1+0.03)^2} = £1.89b \quad (8.12)$$

FV of subsequent end-of-quarter payment (£1.25 billion):

$$1.25 \times \frac{(1+0.03)^{12} - 1}{0.03(1+0.03)^{12}} = £12.44b \quad (8.13)$$

PV of subsequent end-of-quarter payment (£1.25 billion):

$$\frac{12.44}{(1+0.03)^2} = £11.73b \quad (8.14)$$

PV of payment stream:

$$2.91 + 1.89 + 11.73 = £16.53b \quad (8.15)$$

### 8.2.5 Cash flow forecast: table

Cash flow tables are useful when the complexity of a situation makes it difficult to show all cash-flow amounts on a diagram. Cash flow tables can facilitate the analysis of different plans and designs. Cash flow tables clarify:

1. The timing of cash flows
2. The assumptions being made
3. The amount of data available

### 8.2.6 Example 5

CEGE Ltd. is renovating its office building and have identified two feasible alternatives for upgrading the heating, ventilation and air conditioning (HVAC) system. Either alternative A or alternative B must be implemented. The costs are as follows.

Equipment, labour and materials to rebuild	£18,000
Annual cost of electricity	£32,000
Annual maintenance expenses	£2,400
Estimated market value (after eight years)	£2,000

Table 8.3: Alternative A: Rebuild (overhaul) the existing HVAC system.

Equipment, labour and materials to rebuild	£60,000
Annual cost of electricity	£9,000
Annual maintenance expenses	£16,000
Replacement of a major component in four years	£9,400
Estimated market value (after eight years)	£8,000

Table 8.4: Alternative B: Install a new HVAC system that uses existing equipment.

Assume that both alternatives will provide equivalent services over an eight-year period.

*Use a cash flow table to tabulate the net cash flows for both alternatives.*

*Determine the annual net cash flow difference between the alternatives (B-A).*

End of year	Alternative A Net cash flow (£)	Alternative B Net cash flow (£)	Difference B-A (£)	Cumulative Difference (£)
0	-18,000	-60,000	-42,000	-42,000
1	-34,400	-25000	9,400	-32,600
2	-34,400	-25000	9,400	-23,200
3	-34,400	-25000	9,400	-13,800
4	-34,400	-34400	0	-13,800
5	-34,400	-25000	9,400	-4,400
6	-34,400	-25000	9,400	5,000
7	-34,400	-25000	9,400	14,400
8	-32,400	-17000	15,400	29,800
Total	-291,200	-261,400		29,800

Table 8.5: Example 5 table.

Some important notes:

- Timing is everything
  - When is money being received?
  - When is money being paid?
- Cash flows deal with expected actual receipts and payments
  - Debtors and creditors (from the balance sheet) do not appear
- Cash flow forecasts can act as an early warning system
  - Can help identify the need for a loan or overdraft far in advance

### 8.3 S-curves

Display cumulative data (costs, hours, quantities etc.) for a project.

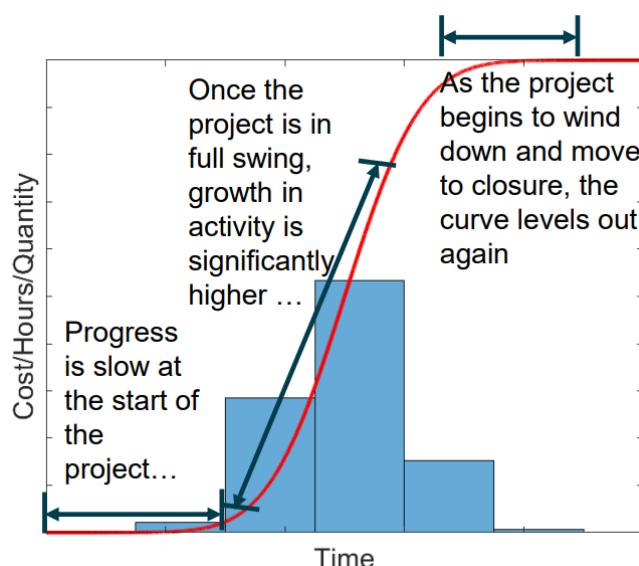


Figure 8.7: S-curve.

#### 8.3.1 What are s-curves used for?

Resource allocation planning:

- S-curves of expected person-hours versus time shows how the project team will need to flex over the project lifecycle
- This can aid the recruitment process

Project expenditure:

- S-curves can show planned expenditure over the project time period
- If this is overlaid with information on when project funds are expected, cash flow problems can be identified

#### 8.3.2 Why are s-curves useful?

S-curves can be used to compare real-time and projected progress.

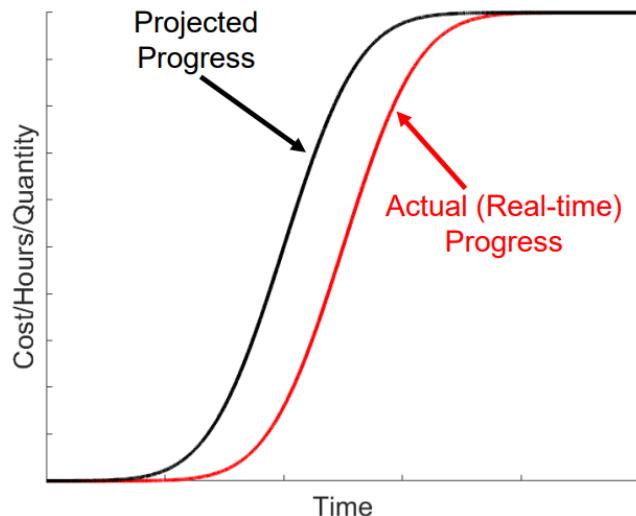


Figure 8.8: S-curve with real-time and projected progress.

- S-curves help you to predict when resources will be heavily utilised
- S-curves help you manage stakeholder expectations
- S-curves help you plan for different schedule scenarios

### 8.3.3 Example 6

The ABC Corporation is planning a construction project with monthly projected receipts and expenditures as presented in the table below.

<b>ABC Corporation Construction Project</b>													
Month	1	2	3	4	5	6	7	8	9	10	11	12	13
Projected total receipts (£)	0	186,861	0	0	373,723	0	0	0	0	934,307	0	186,861	186,861
Projected total expenditures (£)	30,000	114,338	50,000	40,062	110,396	200,000	200,000	441,535	55,215	55,675	40,000	40,000	0

Table 8.6: Example 6 table.

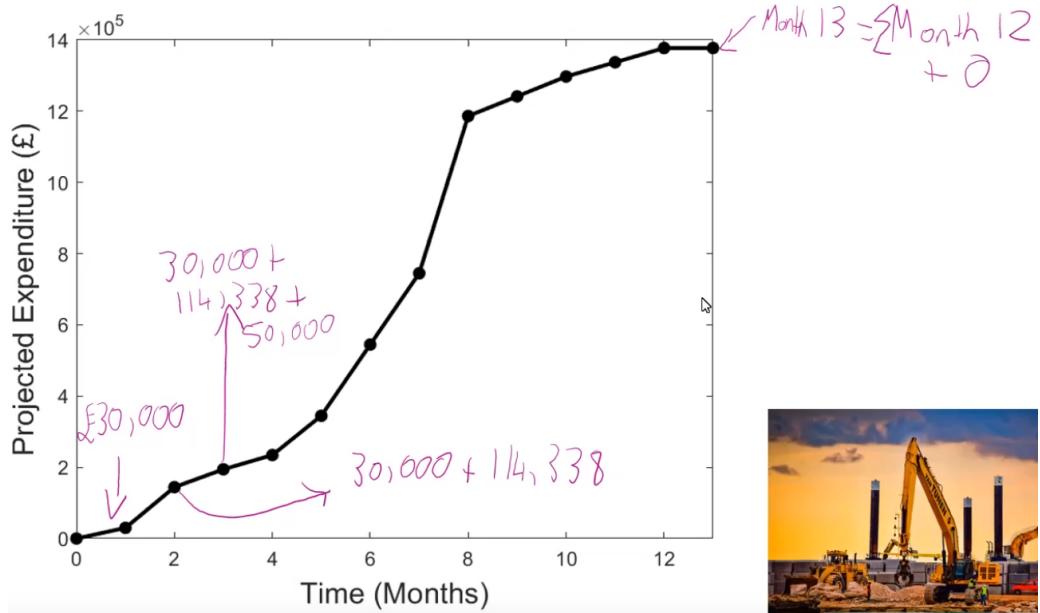


Figure 8.9: S-curve for example 6.

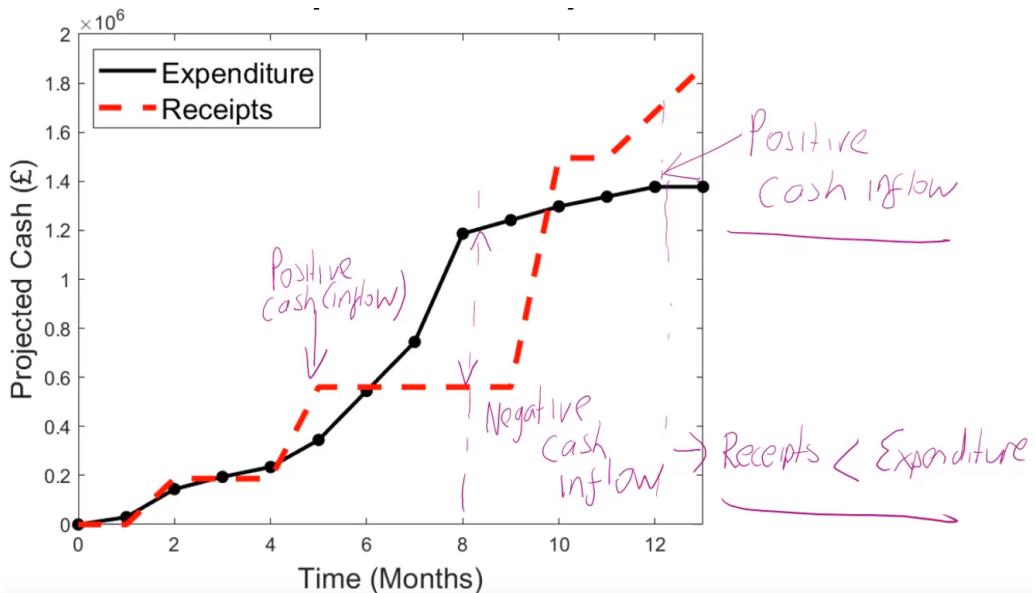


Figure 8.10: S-curve for example 6 with cash flow analysis.

## 8.4 Summary

- Budgeting
  - Definition of budgeting
  - Description of budget types
  - Demonstration of an operating budget
- Cash flow forecasts
  - Description of cash flow diagrams
  - Description of cash flow tables

- Demonstrations of cash flow diagrams and tables
- S-curves
  - Description of s-curves
  - Uses of s-curves
  - Demonstration of constructing an s-curve for expenditure

# Chapter 9

## Project Risk Analysis

### 9.1 What is risk?

Risk is a condition where there is a possibility of adverse deviation from a desired and expected outcome. Risk is caused by a lack of precise knowledge regarding future business conditions, technological development, synergies among projects etc. Decisions under risk are decisions in which the analyst models the decision problem in terms of assumed possible future outcomes, or scenarios, whose probabilities of occurrence can be estimated.

#### 9.1.1 Sources of uncertainty

Factors that affect uncertainty in modelling future economic consequences of an engineering project are:

- Possible inaccuracy of the cash-flow estimates
- The type of business involved in relation to the future health of the economy
- The type of physical plant and equipment involved
- Length of the study period used

### 9.2 Measuring risk

#### 9.2.1 Random variables

Factors that have probabilistic outcomes (e.g., costs, revenues, useful life) are called random variables. Capital letters  $X, Y, Z$ , are usually used to represent random variables and lowercase letters  $x, y, z$  are used to denote the particular values these variables take on in the *sample space*<sup>1</sup>. Information about random variables that is particularly helpful for risk analysis is their expected value and variances. Random variables can be defined as either discrete or continuous.

#### 9.2.2 Discrete random variables

A random variable  $X$  is said to be discrete if it can take on at most a countable (finite) number of values ( $x_1, x_2, \dots, x_L$ ). The probability that a discrete random variable  $X$  takes on the value  $x_i$  is given by:

$$\Pr\{X = x_i\} = P(x_i) \text{ for } i = 1, 2, \dots, L \quad (9.1)$$

where  $i$  is the sequential index of the discrete values and  $p(x_i) \geq 0$  and  $\sum_i p(x_i) = 1$ .

---

<sup>1</sup>Sample space: the set of all possible outcomes for each variable.

The probability that the value of  $X$  is contained in the closed interval  $[a, b]$  is given by the probability mass function:

$$\Pr \{a \leq X \leq b\} = \sum_{i:a \leq X_i \leq b} p(x_i) \quad (9.2)$$

The probability that the value of  $X$  is less than or equal to  $h$  is given by the cumulative distribution function:

$$\Pr \{X \leq h\} = p(h) = \sum_{i:X_i \leq h} p(x_i) \quad (9.3)$$

In most practical cases, discrete random variables represent countable data such as:

- The number of defective columns in a building
- The number of maintenance jobs per week
- The number of employees

### 9.2.3 Continuous random variables

A random variable  $X$  is said to be continuous if it can take on any value within a set of real numbers  $[c, d]$ . The probability that a continuous random variable  $X$  takes on a value within the set  $[c, d]$  is given by:

$$\Pr \{c \leq X \leq d\} = \int_c^d f(x) dx \quad (9.4)$$

where  $\int_{-\infty}^{\infty} f(x) dx = 1$ .

The probability that the value of  $X$  assumes exactly any one of its values is 0. The probability that the value of  $X$  is less than or equal to  $h$  is given by the cumulative distribution function:

$$\Pr \{X \leq h\} = F(h) = \int_{-\infty}^h f(x) dx \quad (9.5)$$

$$\Pr \{c \leq X \leq d\} = \int_c^d f(x) dx = F(d) - F(c) \quad (9.6)$$

In most practical cases, continuous random variables represent measured data such as:

- Time
- Cost
- Revenue

### 9.2.4 Expected value

The expected value of a random variable  $X$  is denoted as  $E(X)$ .  $E(X)$  is a weighted average of the distribution values of  $x$  that it takes on and is a measure of the central location of the distribution.  $E(X)$  is called the mean (or central / first moment) of the distribution.

For a discrete random variable:

$$E(X) = \sum_i x_i p(x_i) \quad (9.7)$$

For a continuous random variable:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx \quad (9.8)$$

### 9.2.5 Variance

The variance of a random variable  $X$  is denoted as  $V(X)$ .  $V(X)$  is a measure of the dispersion of the values  $X$  takes on around  $E(X)$  (note that the square root of  $V(X)$  is equal to the standard deviation  $SD(X)$ ).  $V(X)$  is called the second moment of the distribution.

For a discrete random variable:

$$V(X) = \sum_i [x_i - E(X)]^2 p(x_i) \quad (9.9)$$

For a continuous random variable:

$$V(X) = \int_{-\infty}^{\infty} [x - E(X)]^2 f(x) dx \quad (9.10)$$

### 9.2.6 Multiplying by a constant

Random variables are commonly multiplied by a constant value. e.g.:

- The estimated maintenance labour expense for a time period  $Y = cX$ , where the number of labour hours per period is a random variable  $X$  and the cost per labour hour  $c$  is constant.

For a discrete random variable:

$$E(cX) = cE(X) = c \sum_i x_i p(x_i) \quad (9.11)$$

For a continuous random variable:

$$E(cX) = cE(X) = c \int_{-\infty}^{\infty} x f(x) dx \quad (9.12)$$

For both random variable types:

$$V(cX) = c^2 V(X) \quad (9.13)$$

### 9.2.7 Multiplying by another random variable

Sometimes a random variable  $Z$  results from the product of two other independent random variable ( $XY$ ), e.g.:

- The estimated annual expenses  $Z$  for a repair part repetitively procured during the year on a competitive basis, when the unit price  $X$  and the number of units used per year  $Y$  are independent random variables.

The expected value of  $Z$  is:

$$E(Z) = E(X) E(Y) \quad (9.14)$$

The variance of  $Z$  is:

$$V(Z) = E[XY - E(XY)]^2 = E(X^2) E(Y^2) - [E(X) E(Y)]^2 \quad (9.15)$$

### 9.2.8 Adding or subtracting random variables

Sometimes a random variable ( $Z$ ) results from adding or subtracting two independent variables ( $X + Y$  or  $X - Y$ ) e.g.:

- Alternative A has a cost of  $X$  and alternative B has a cost of  $Y$ . The probability that alternative A is less costly than alternative B is equal to:

$$\Pr\{X < Y\} = \Pr\{X - Y < 0\} = \Pr\{Z < 0\} \quad (9.16)$$

The expected value of  $Z$  is:

$$E(Z) = E(X) \pm E(Y) \quad (9.17)$$

The variance of  $Z$  is:

$$V(Z) = V(X) + V(Y) \quad (9.18)$$

## 9.3 Example risk analysis

### 9.3.1 Worked example 1

Suppose that the estimated probability of obtaining various capacity utilisations in a premixed-concrete plant project are as follows:

Capacity (%)	Probability	Annual Revenue	AV (15%)
50	0.10	£405,000	-£25,093
65	0.30	£526,500	£22,136
75	0.50	£607,500	£53,622
90	0.10	£729,000	£100,850

Table 9.1: Worked example 1 table.

*Estimate E (AV) and V (AV)*

$$E(X) = \sum_i x_i p(x_i) \quad (9.19)$$

$$E(AV) = 0.1 \times -25903 + 0.3 \times 22136 + 0.5 \times 53622 + 0.1 \times 100850 = £41,028 \quad (9.20)$$

$$V(X) = \sum_i [x_i - E(X)]^2 p(x_i) \quad (9.21)$$

$$\begin{aligned} V(AV) &= 0.1 \times (-25903 - 41028)^2 + 0.3 \times (22136 - 41028)^2 + \\ &0.5 \times (53622 - 41028)^2 + 0.1 \times (100850 - 41028)^2 = £9.9 \times 10^8 \end{aligned} \quad (9.22)$$

$$E(AV) = £41,028 \quad (9.23)$$

$$V(AV) = £9.9 \times 10^8 \quad (9.24)$$

$$\sqrt{V(AV)} = £31,327 \quad (9.25)$$

By evaluating both  $E(AV)$  and  $V(AV)$  for the concrete plant, indications of the venture's average profitability and riskiness are obtained. The mean value is above one standard deviation from £0. This leads to the conclusion that there is low risk, and that this is a good investment.

### 9.3.2 Risk analysis with continuous random variables

Supposed that the NPV of a project is £153 and the corresponding variance is £484,416. If the NPV is a normally distributed<sup>2</sup> random variable, what is the probability of having a negative NPV?

From Excel, use `=norm.dist(0, E(NPV), sqrt(V(NPV)), 1)`. The first value is 0, because we want to find the probability of it being less than the mean. The last 1 gives us the cumulative distribution. This results in a 41.3% of having a NPV less than 0.

$$\Pr\{NPV \leq 0\} = F(0) = \int_{-\infty}^0 f(npv) dnpv = 0.413 \quad (9.26)$$

<sup>2</sup>Commonly used distribution that is easy to calculate using a computer and can be entirely described by the mean and variance parameters.

### 9.3.3 Risk analysis with multiple independent random variables

Suppose that the estimated cash flow data for a project are show in the following table for a five-year study period. Each annual net cash-flow amount,  $F_k$ , is a linear combination of two statistically independent random variables,  $X_k$  and  $Y_k$ , where  $X_k$  is a revenue factor and  $Y_k$  is an expense factor. The  $X_k$  cash-flow amounts are statistically independent of each other, and the same is true of the  $Y_k$  amounts. Both  $X_k$  and  $Y_k$  are continuous random variables, but the form of their probability distributions is not known. Interest rate is 20% per year.

*Estimate  $E(NPV)$ ,  $V(NPV)$  and  $SD(NPV)$  of the project's cash flows.*

End of year $k$	Net cash flow	$E[X_k]$	$E[Y_k]$	$SD[X_k]$	$SD[Y_k]$
0	$F_0 = X_0 + Y_0$	£0	-£100,000	£0	£10,000
1	$F_1 = X_1 + Y_1$	£60,000	-£20,000	£4,500	£2,000
2	$F_2 = X_2 + 2Y_2$	£65,000	-£15,000	£8,000	£1,200
3	$F_3 = 2X_3 + 3Y_3$	£40,000	-£9,000	£3,000	£1,000
4	$F_4 = X_4 + 2Y_4$	£70,000	-£20,000	£4,000	£2,000
5	$F_5 = 2X_5 + 2Y_5$	£55,000	-£18,000	£4,000	£2,300

Table 9.2: Risk analysis with multiple independent random variables table.

$$E(F_k) = E(a_k X_k + b_k Y_k) = E(a_k X_k) + E(b_k Y_k) = a_k E(X_k) + b_k E(Y_k) \quad (9.27)$$

$$V(F_k) = V(a_k X_k + b_k Y_k) = V(a_k X_k) + V(b_k Y_k) = a_k^2 V(X_k) + b_k^2 V(Y_k) \quad (9.28)$$

End of year $k$	Net cash flow	$E[X_k]$	$E[Y_k]$	$SD[X_k]$	$SD[Y_k]$	$E[F_k]$	$V[V_k]$
0	$F_0 = X_0 + Y_0$	£0	-£100,000	£0	£10,000	-£100,000	$100 \times 10^6$
1	$F_1 = X_1 + Y_1$	£60,000	-£20,000	£4,500	£2,000	£40,000	$24.2 \times 10^6$
2	$F_2 = X_2 + 2Y_2$	£65,000	-£15,000	£8,000	£1,200	£35,000	$69.8 \times 10^6$
3	$F_3 = 2X_3 + 3Y_3$	£40,000	-£9,000	£3,000	£1,000	£53,000	$45 \times 10^6$
4	$F_4 = X_4 + 2Y_4$	£70,000	-£20,000	£4,000	£2,000	£30,000	$32 \times 10^6$
5	$F_5 = 2X_5 + 2Y_5$	£55,000	-£18,000	£4,000	£2,300	£74,000	$85.2 \times 10^6$

Table 9.3: Risk analysis with multiple independent random variables table with expected and variance values.

$$E(NPV) = \sum_{k=0}^5 \frac{E(F_k)}{(1+i)^n} = £32,517 \quad (9.29)$$

$$V(NPV) = \sum_{k=0}^5 \frac{V(F_k)}{(1+i)^{2n}} = £186.75 \times 10^6 \quad (9.30)$$

$$SD(NPV) = \sqrt{V(NPV)} = £13,666 \quad (9.31)$$

## 9.4 Decision trees

Decision trees are powerful means of depicting and facilitating the analysis of important problems. Decision trees break down a large, complicated problem into a series of smaller, simple problems. They enable objective analysis and decision making that includes explicit consideration of the risk and effect of the future.

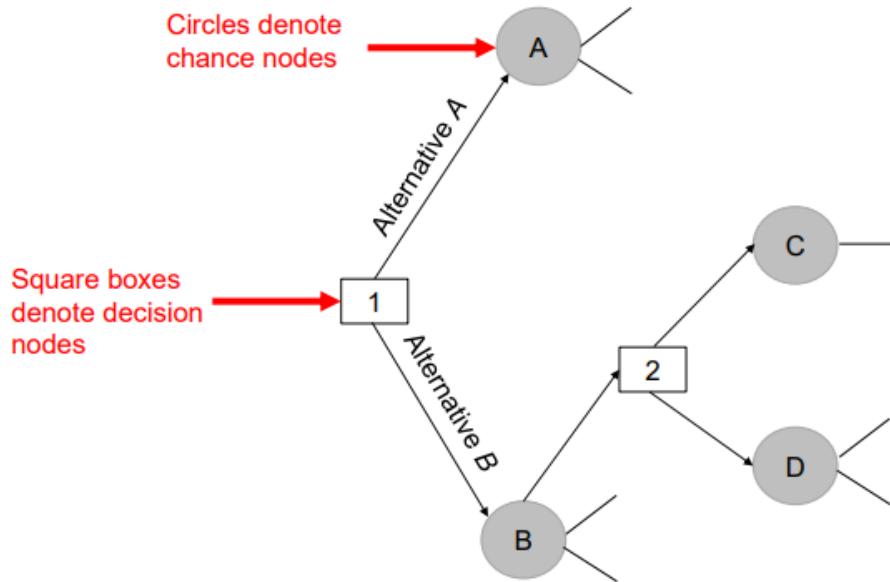


Figure 9.1: Decision tree.

#### 9.4.1 Worked example of a decision tree

CEGE Corporation manufactures compressors for commercial air-condition systems. A new compressor design is being evaluated as potential replacement for the most frequently used unit. The new design involves major changes that have the expected advantage of better operating efficiency. From the perspective of a typical user, the new compressor (as an assembled component in an air-conditioning system) would have an increased investment of £8,600 relative to the present unit and an annual expense saving dependent on the extent to which the design goal is met in actual operation. Estimates by the multidisciplinary design team of the new compressor achieving four levels (percentages) of the efficiency design goal and the probability and annual expense saving at each level are as follows.

Level of design goal met (%)	Probability $p(L)$	Annual expense saving
90	0.25	£3,470
70	0.40	£2,920
50	0.25	£2,310
30	0.10	£1,560

Table 9.4: Decision tree example table.

Use  $MARR = 18\%$ , an analysis period of 6 years, and  $E(NPV)$  as the decision criterion, is the new compressor design economically preferable to the current unit?

Convert to present value figures using the formula for AV.

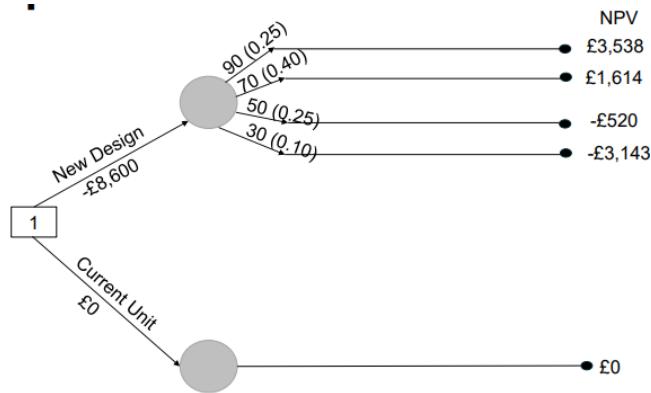


Figure 9.2: Decision tree for worked example.

$$E(X) = \sum_i x_i p(x) \quad (9.32)$$

$$E(NPV) = 0.25 \times 3538 + 0.40 \times 1614 + 0.25 \times (-520) + 0.10 \times (-3413) = 1086 \quad (9.33)$$

There is a 35% chance of a negative NPV value, hence depending on the risk appetite of the company, we find that this may not be a good investment. However, we do have a positive  $E(NPV)$  value.

# Chapter 10

## Project Management

### 10.1 Environmental liability

#### 10.1.1 What is environmental liability?

The Environmental Damage (Prevention and Remediation) (England) Regulations 2015 impose obligations on operators of economic activities, requiring them to:

- Prevent
- Limit
- Remediate

environmental damage caused by their operations. Construction activities have the potential for significant water, soil and air pollution. The risk of this pollution is influenced by:

- Contamination conditions
- Exposure pathways

Construction activities may also result in significant damage to:

- Natural habitats
- Protected species
- Other biodiversity

Contaminants	Exposure pathways
Metals	Storm water / sediment run-off
Inorganic compounds (e.g., sulphuric acid)	Noise and vibration from heavy machinery
Oils and tars	Groundwater disturbance from borehole drilling
Pesticides	Disturbance of unknown pre-existing contamination
Flammable / combustible substances (e.g., petrol and diesel)	Interference with the site's natural resources
Fibres (e.g., synthetic mineral fibres)	

Table 10.1: Table of exemplar contaminants and exposure pathways.

#### 10.1.2 Environmental impact assessment

Some construction projects require an environmental impact assessment e.g.:

- Nuclear power stations

- Industrial plants
- Large wastewater treatment plants
- Motorways

Environmental impact assessments are governed by the Town and Country Planning (Environmental Impact Assessment) Regulations 2017. The aim of environmental impact assessment is to protect the environment. They assist local planning authorities in making planning decision on projects that are likely to have significant environmental effects.

There are four stages of an environmental impact assessment:

1. Screening and scoping
  - Is an environmental impact assessment required and to what scale?
2. Preparing an environmental statement
  - What data will be collected to determine environmental risk?
  - What measures will be followed to minimise environmental impacts?
  - Must include:
    - (a) A description of the proposed development
    - (b) A description of the likely significant environmental effects of the proposed development
    - (c) A description of measures to mitigate adverse environmental effects
    - (d) A description of the reasonable alternatives, and why the chosen option was selected, accounting for environmental effects
3. Making a planning application
  - The environmental statement is publicised electronically and by public notice
4. Decision making
  - Local planning authority / Secretary of State will make the final decision on the permit issuance

## 10.2 Conditions of contract

### 10.2.1 What is a contract?

*A contract is a promise or set of promises which the law will enforce.*

It is a legal document. Contracts form a part of everyday life, whenever you buy or sell something, a contract is made. Contract documents also serve as the foundation of an engineering project.

### 10.2.2 Types of contract

There are two main types of contract:

- Unilateral
  - A promise is met with a requested action
  - Only one party to the contract has a legally binding liability
- Bilateral
  - An action is met with another action

- Both actions are obligations and will be legally enforceable

There are two types of bilateral contract:

- Formal
  - Made under seal
  - The contractor's liability under the contract lasts for 12 years
- Informal
  - Signed without a seal
  - The contractor's liability under the contract lasts for 6 years

### 10.2.3 Why are contracts needed?

Contracts define and cover all activities during a construction project. They also apportion risk between the purchaser and the contractor. Examples of risk covered by contracts are:

- Unanticipated cash flow problems
- Unanticipated delays
- Change in the cost of raw materials
- Poor project management
- Force majeure

### 10.2.4 Force majeure

A force majeure clause in a contract is intended to protect the contracting parties if part of the contract cannot be performed because of some exceptional event. It usually allows a contractor more time to perform the contract (but does not allow additional costs for rising raw material prices).

### 10.2.5 Conditions of contract

Terms and conditions refer to the contractual rights and obligations of a party to any contract. The construction industry has developed a range of Standard sets of terms and conditions of contract, over a length of time. It is now common practice to specify one of these Standard documents.

#### Example conditions of contract

There are many examples of Standard terms and conditions of contract. The following Engineering Institutions publish their own examples:

- The Institute of Chemical Engineers (IChemE)
- The Institute of Civil Engineers (ICE)
- The Institution of Mechanical Engineers (IMechE)
- The International Federation of Consulting Engineers (FIDIC)

## 10.3 Summary

Environmental liabilities

- Definition of environmental liabilities

- Description of environmental impact assessments

Conditions of contract

- Definition of a contract
- Distinction between different contract types
- Explanations of why contracts exist
- Explanation and examples of conditions of contract

## **Part II**

# **Sample Exam Papers**

# Chapter 11

## Sample Exam Paper 1

### 11.1 Question 1

A manufacturer makes 1,000,000 chips per year, using a 1.5 Watt machine. Each chip takes 10 minutes to make, and energy is currently priced at £0.60 per Watt-hour. A new process would reduce the unit production time by 1 minute and energy consumption by 50%. If all other costs remain the same, what is the maximum amount of money you would pay for this new process? Assume that the new process must pay for itself by the end of the first year.

**5 marks**

Old process:

$$10 \times 10^6 \text{ min} \times \frac{1}{60} = 166\,666.67 \text{ h} \quad (11.1)$$

$$166\,666.67 \text{ h} \times 1.5 \text{ W} = 250\,000 \text{ Wh} \quad (11.2)$$

$$250\,000 \text{ Wh} \times 0.6 \text{ £} = 150\,000 \text{ £} \quad (11.3)$$

New process:

$$9 \times 10^6 \text{ min} \times \frac{1}{60} = 150\,000 \text{ h} \quad (11.4)$$

$$150\,000 \text{ h} \times 0.75 \text{ W} = 112\,500 \text{ Wh} \quad (11.5)$$

$$112\,500 \text{ Wh} \times 0.6 \text{ £} = 67\,500 \text{ £} \quad (11.6)$$

Maximum amount that should be paid:

$$150\,000 \text{ £} - 67\,500 \text{ £} = 82\,500 \text{ £} \quad (11.7)$$

### 11.2 Question 2

Two mutually exclusive investment alternatives are being considered by an automotive engineering manufacturer, and one of them must be selected. Alternative A requires an initial investment of £13,000 in equipment. Annual operating and maintenance costs are anticipated to be normally distributed, with a mean of £5,000 and a standard deviation of £500. The terminal salvage value at the end of the eight-year planning horizon is anticipated to be normally distributed, with a mean of £2,000 and a standard deviation of £800. Alternative B requires end-of-year annual expenditures over the eight-year planning horizon, with the annual expenditure being normally distributed with a mean of £7,500 and a standard deviation of £750. Using a MARR of 15% per year, what is the probability that Alternative A is the most economic alternative (i.e., the least costly)?

Hint: The addition or subtraction of two normally distributed random variables is a normally distributed random variable.

**25 marks**

Alternative A		Alternative B		
Initial costs	£13000	Annual O&M	£7500 mean	£750 STD
Annual O&M	£5000 mean	£500 STD		
Salvage value	£2000 mean	£800 STD		

Mean NPV Alternative A:

$$-13000 - 5000 \times \frac{(1.15)^8 - 1}{0.15(1.15)^8} + 2000 \times \frac{1}{1.15^8} = -34782.80 \mathcal{L} \quad (11.8)$$

Variance of NPV Alternative A:

$$500^2 \sum_{k=1}^8 \frac{1}{1.15^{2k}} + 800^2 \times \frac{1}{1.15^{16}} = 760746 \mathcal{L}^2 \quad (11.9)$$

Mean NPV Alternative B

$$-7500 \times \frac{(1.15)^8 - 1}{0.15(1.15)^8} = -33655 \mathcal{L} \quad (11.10)$$

Variance of NPV Alternative B

$$750^2 \sum_{k=1}^8 \frac{1}{1.15^{2k}} = 1557794 \mathcal{L}^2 \quad (11.11)$$

### 11.3 Question 3

The tree diagram in Figure 1 describes the uncertain cash flows for an engineering project. The analysis period is two years (as shown in Figure 11.1) and MARR = 15% per year.

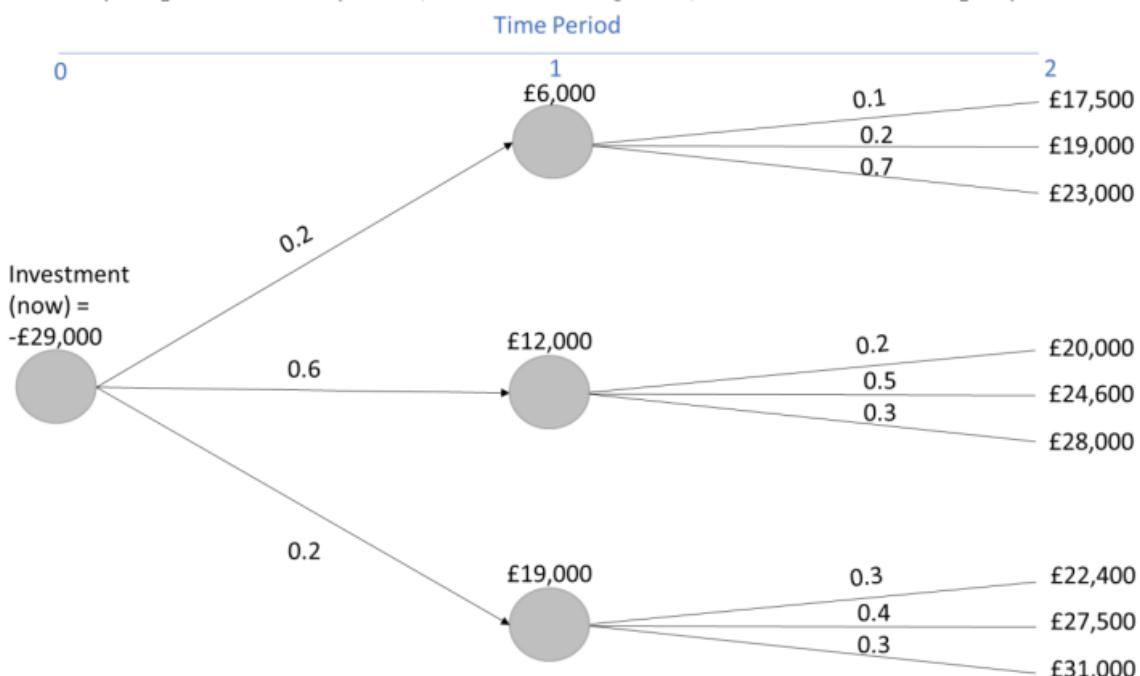


Figure 11.1: Decision tree for Question 3. The time period is measured in years and represents the end of the corresponding year.

*What is the average, variance and standard deviation of NPV?*

### 25 marks

From the decision tree, it can be observed that there are nine possible values of NPV, calculated as follows:

$$(1) \quad -29000 + \frac{6000}{1.15} + \frac{17500}{1.15^2} = -10551 \quad p(1) = 0.02 \quad (11.12)$$

$$(2) \quad -29000 + \frac{6000}{1.15} + \frac{19000}{1.15^2} = -9417 \quad p(2) = 0.04 \quad (11.13)$$

$$(3) \quad -29000 + \frac{6000}{1.15} + \frac{23000}{1.15^2} = -6392 \quad p(3) = 0.14 \quad (11.14)$$

$$(4) \quad -29000 + \frac{12000}{1.15} + \frac{20000}{1.15^2} = -3443 \quad p(4) = 0.12 \quad (11.15)$$

$$(5) \quad -29000 + \frac{12000}{1.15} + \frac{24600}{1.15^2} = 35 \quad p(5) = 0.30 \quad (11.16)$$

$$(6) \quad -29000 + \frac{12000}{1.15} + \frac{28000}{1.15^2} = 2606 \quad p(6) = 0.18 \quad (11.17)$$

$$(7) \quad -29000 + \frac{19000}{1.15} + \frac{22400}{1.15^2} = 4459 \quad p(7) = 0.06 \quad (11.18)$$

$$(8) \quad -29000 + \frac{19000}{1.15} + \frac{27500}{1.15^2} = 8315 \quad p(8) = 0.08 \quad (11.19)$$

$$(9) \quad -29000 + \frac{19000}{1.15} + \frac{31000}{1.15^2} = 10962 \quad p(9) = 0.06 \quad (11.20)$$

We know that:

$$E(X) = \sum x_i p(x_i) \quad (11.21)$$

$$V(X) = \sum [x_i - E(x)]^2 p(x_i) \quad (11.22)$$

Thus average is:

$$E(NPV) = (-10551 \times 0.02) + (-9417 \times 0.04) + (-6392 \times 0.14) + (-3443 \times 0.12) + (35 \times 0.30) + (2606 \times 0.18) + (4459 \times 0.06) + (8315 \times 0.08) + (10962 \times 0.06) \quad (11.23)$$

$$E(NPV) = 175 \text{ £} \quad (11.24)$$

Variance:

$$\begin{aligned} V(NPV) = & \left( (-10551 - 175)^2 \times 0.02 \right) + \left( (-9417 - 175)^2 \times 0.04 \right) + \left( (-6392 - 175)^2 \times 0.14 \right) + \\ & \left( (-3443 - 175)^2 \times 0.12 \right) + \left( (35 - 175)^2 \times 0.30 \right) + \left( (2606 - 175)^2 \times 0.18 \right) + \\ & \left( (4459 - 175)^2 \times 0.06 \right) + \left( (8315 - 175)^2 \times 0.08 \right) + \left( (10962 - 175)^2 \times 0.06 \right) \end{aligned} \quad (11.25)$$

$$V(NPV) = 29.04 \times 10^6 \text{ £}^2 \quad (11.26)$$

Standard deviation:

$$SD(NPV) = \sqrt{V(NPV)} = \sqrt{29.04 \times 10^6} = 5295 \text{ £} \quad (11.27)$$

*What is the probability that NPV < 0?*

### 5 marks

The probability that NPV < 0 is the sum of the probabilities associated with negative values of NPV:

$$P(NPV < 0) = 0.02 + 0.04 + 0.14 + 0.12 = 0.32 \quad (11.28)$$

## 11.4 Question 4

UCL Engineering plc is a multinational engineering equipment trader and distribution company that has been established for many years. Its income and balance sheet for two years are provided in Table 11.1 and Table 11.2 respectively.

<b>UCL Engineering Income Statement</b>		
	2028	2029
	£m	£m
<b>Revenue</b>	27,049	28,436
Cost of sales	(4,653)	(6,229)
<b>Gross profit</b>	22,396	22,207
Selling, general and administrative expenses	-10,729	-13,159
Research and development expenses	-6,013	-5,641
Other income	316	362
<b>Profit from operations</b>	5,970	3,769
Finance costs	-1,088	-1,048
<b>Profit before tax</b>	4,882	2,721
Tax	-227	-307
<b>Net Income</b>	4,655	2,414

Table 11.1: UCL Engineering plc income statement.

<b>UCL Engineering Balance Sheet</b>		
	2028	2029
	£m	£m
<b>Assets</b>		
<b>Non-current assets</b>		
Land and buildings	6,611	6,510
Plant and machinery	12,572	12,121
Intangible assets	13,010	11,322
<b>Total non-current assets</b>	<b>32,193</b>	<b>29,953</b>
<b>Current assets</b>		
Inventories	2,229	1,979
Trade receivables	6,953	7,718
Cash and cash equivalents	6,584	6,984
<b>Total current assets</b>	<b>15,766</b>	<b>16,681</b>
<b>Total assets</b>	<b>47,959</b>	<b>46,634</b>
<b>Equity and Liabilities</b>		
<b>Equity</b>		
Ordinary share capital (£1 shares)	11,891	12,346
Share premium	4,544	4,384
Retained earnings	13,004	14,745
<b>Total equity</b>	<b>29,439</b>	<b>31,475</b>
<b>Non-current liabilities</b>		
Bank loans	12,836	9,205
<b>Total non-current liabilities</b>	<b>12,836</b>	<b>9,205</b>
<b>Current liabilities</b>		
Trade payables	3,569	4,053
Tax liabilities	1,517	2,583
<b>Total current liabilities</b>	<b>5,086</b>	<b>6,636</b>
<b>Total liabilities</b>	<b>17,922</b>	<b>15,841</b>
<b>Total equity and liabilities</b>	<b>47,361</b>	<b>47,316</b>

Table 11.2: UCL Engineering plc balance sheet. Note the inventory for the company in 2027 is valued at £2,450 million.

*Explain the terms “gross profit”, “non-current assets” and “current liabilities” that feature in either the Income Statement or the Balance Sheet.*

### 10 marks

- Gross profit is the income generated from sales, minus the costs of selling the goods. It also accounts for expenses like general expenses and taxes that need to be subtracted to find net income.
- Non-current assets are a company’s long-term (economic) obligations listed on the balance sheet. Long-term refers to assets that are not expected to be converted to cash over the next one-year period.
- Current liabilities are a company’s short term (economic) obligations, due within a one year period.

*Calculate the financial ratios shown in Table 11.3 and use them to provide a one-paragraph interpretation of the company’s general performance.*

### 30 marks

	Formula	2028	2029	Industry average
Current ratio	$\frac{\text{Current assets}}{\text{Current liabilities}}$	$\frac{15766}{5086} = 3.09$	$\frac{16681}{6636} = 2.51$	2.2
Quick ratio	$\frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}}$	$\frac{15766 - 2229}{5086} = 2.66$	$\frac{16681 - 1979}{6636} = 2.66$	1.6
Debt-Equity ratio	$\frac{\text{Total liabilities}}{\text{Total equity}}$	$\frac{17922}{29439} = 0.61$	$\frac{15841}{31475} = 0.50$	37.38%
Inventory turnover	$\frac{\text{Cost of sales}}{\text{Average inventory}}$	$\frac{4653}{0.5(2229 + 2450)} = 1.99$	$\frac{6229}{0.5(2229 + 1979)} = 2.96$	2.99
Day sales in inventory	$\frac{365}{\text{Inventory turnover}}$	$\frac{365}{1.99} = 183.87$	$\frac{365}{2.96} = 123.29$	1.3
Total asset turnover ratio	$\frac{\text{Total annual sales}}{\text{Total assets}}$	$\frac{27049}{47959} = 0.56$	$\frac{28436}{46634} = 0.61$	0.81
Profit margin	$\frac{\text{Net income}}{\text{Net sales}}$	$\frac{4655}{27049} = 0.17$	$\frac{2414}{28436} = 0.08$	8.3%
Return on assets ratio	$\frac{\text{Net income}}{\text{Total assets}}$	$\frac{4655}{47959} = 0.10$	$\frac{2414}{46634} = 0.05$	5.2%
Return on equity ratio	$\frac{\text{Net income}}{\text{Total equity}}$	$\frac{4655}{29439} = 0.16$	$\frac{2414}{31475} = 0.08$	2.1%

Table 11.3: Financial ratios and their industry average values for Question 4.

In comparison to industry averages, the company’s financial performance exhibits both strengths and weaknesses. In terms of liquidity, the company surpasses the industry average with a current ratio of 3.09 in 2028 and 2.51 in 2029, above the standard of 2.2, indicating a healthier ability to meet short-term obligations. However, the company poses a higher risk for shareholders than the industry standard, as evidenced by a debt-equity ratio surpassing the 50% mark in both years, significantly higher than the average of 37.38%.

In 2028, the company’s inventory turnover nearly matches the industry average, although its day sales in inventory and total asset turnover ratio, at 183.87 and 0.56 respectively, suggest less efficiency in converting inventory and assets into sales compared to the average industry values of 1.3 and 0.81.

Profitability, however, presents a mixed picture. The company showed superior profitability in 2027, but by 2028, its profit margin regressed to the industry average of 8.3%. While the return on assets ratio approximated the industry average, the return on equity ratio was notably higher, indicating a better-than-average efficiency at converting equity into profits.

Comparing the company’s performance between the years 2028 and 2029, a general decline in liquidity is

observed, indicated by the decrease in both the current and quick ratios. Conversely, there is a reduction in shareholder risk, as shown by the lowered debt-equity ratio in 2029. Additionally, the company has shown improvement in asset efficiency, as suggested by the increased inventory turnover and total asset turnover ratio. However, profitability shows a general decline, with profit margin and return on both assets and equity witnessing a drop in 2029.

# Chapter 12

## Sample Exam Paper 2

### 12.1 Question 1

A car factory makes 850,000 cars per year. Each car takes four 8-hour days of direct labour at the rate of £21 per hour. Overhead costs are estimated at £11 per direct labour hour. A new production process would reduce the direct labour per car by 25%, but increase overhead costs by 20% per direct labour hour. If all other costs remain the same, what is the maximum amount of money you would pay for this new process? Assume that the new process must pay for itself by the end of the first year. .

**5 marks**

Old process:

$$850000 \times 32 \times (21 + 11) = 870\,400\,000 \text{ £} \quad (12.1)$$

New process:

$$850000 \times 24 \times (21 + 13.2) = 697\,680\,000 \text{ £} \quad (12.2)$$

Maximum amount that should be paid:

$$870\,400\,000 \text{ £} - 697\,680\,000 \text{ £} = 172\,720\,000 \text{ £} \quad (12.3)$$

### 12.2 Question 2

A manufacturer is deciding whether or not to purchase a new piece of equipment. Uncertain market conditions mean that the capital costs and annual benefits associated with this piece of equipment are uncertain (see Table 12.1). The useful life of the equipment is three years. Assume that MARR=10% and the market value at the end of its useful life is zero.

**25 marks**

Capital costs (£)	Annual benefits (£)	Probability of outcome
300,000	120,000	0.1
500,000	200,000	0.3
400,000	150,000	0.6

Table 12.1: Potential capital costs and benefits of the equipment for Question 2.

What is the average NPV?

**25 marks**

$$-300000 + 120000 \times \frac{1.1^3 - 1}{0.1 \times 1.1^3} = -1577.76 \text{ £} \quad (12.4)$$

$$-500000 + 200000 \times \frac{1.1^3 - 1}{0.1 \times 1.1^3} = -2629.60 \text{ £} \quad (12.5)$$

$$-500000 + 150000 \times \frac{1.1^3 - 1}{0.1 \times 1.1^3} = -26972.20 \text{ £} \quad (12.6)$$

$$E(NPV) = (-1577.76 \times 0.1) + (-2629.60 \times 0.3) + (-26972.20 \times 0.6) = -17129.98 \text{ £} \quad (12.7)$$

What is the probability that the decision to buy the piece of equipment is a good one?

**5 marks**

$$P(NPV > 0) = 0 \quad (12.8)$$

No outcomes give a positive NPV value, therefore buying the equipment is not a good choice in any case.

### 12.3 Question 3

A firm must decide between constructing a new facility or renting office space to accommodate the expected growth of this company over the next 10 years (see Figure 12.1).

If the company opts to construct a new facility, additional space may be required in five years (the probability of this occurring can be worked out from other probabilities provided in the tree), depending on its size. If additional space is required, there will be an option to build an office addition or rent space for the additional space requirements. If additional space is not required, operating and maintenance costs will be larger than £200,000 per year in the first five years.

MARR is 10% per year. An NPV analysis is to be conducted on the alternatives. Which course of action should be recommended?

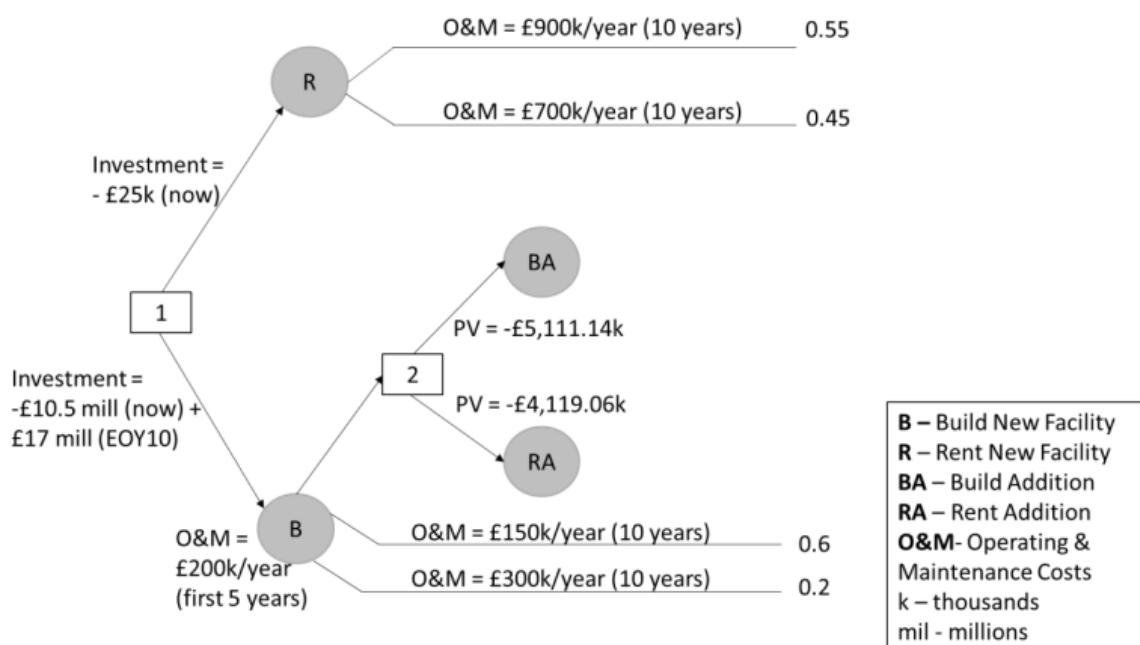
**30 marks**

Figure 12.1: Decision tree for Question 3. EOY denotes end-of-year and PV denotes the net present value associated with the alternative.

Consider the renting option:

$$(1) - 25000 - 900000 \times \frac{1.1^{10} - 1}{0.1 \times 1.1^{10}} = -5555110.40 \text{ £} \quad p(1) = 0.55 \quad (12.9)$$

$$(2) - 25000 - 700000 \times \frac{1.1^{10} - 1}{0.1 \times 1.1^{10}} = -4326196.97 \text{ £} \quad p(2) = 0.45 \quad (12.10)$$

Therefore average NPV for renting is:

$$E(NPV_{\text{renting}}) = (0.55 \times -5555110.40) + (0.45 \times -4326196.97) = -5002099.36 \text{ £} \quad (12.11)$$

Consider the building option and additional space is required (we choose rent addition as build addition is more expensive and eliminated from consideration):

$$(1) - 10500000 - 200000 \times \frac{1.1^5 - 1}{0.1 \times 1.1^5} - 4119060 + \frac{17000000}{1.1^{10}} = -8822981.43 \text{ £} \quad p(1) = 0.2 \quad (12.12)$$

Consider the buying option and no additional space is required:

$$(2) - 10500000 - 200000 \times \frac{1.1^5 - 1}{0.1 \times 1.1^5} - 150000 \times \frac{1.1^{10} - 1}{0.1 \times 1.1^{10}} + \frac{17000000}{1.1^{10}} = -5625606.50 \text{ £} \quad p(2) = 0.6 \quad (12.13)$$

$$(3) - 10500000 - 200000 \times \frac{1.1^5 - 1}{0.1 \times 1.1^5} - 300000 \times \frac{1.1^{10} - 1}{0.1 \times 1.1^{10}} + \frac{17000000}{1.1^{10}} = -6547291.57 \text{ £} \quad p(3) = 0.2 \quad (12.14)$$

Therefore average NPV for building is:

$$E(NPV_{\text{building}}) = (0.2 \times -8822981.43) + (0.6 \times -5625606.50) + (0.2 \times -6547291.57) = -6449418.5 \text{ £} \quad (12.15)$$

Average NPV for renting is lower than the average NPV for building, hence it should be recommended to rent office space to accommodate growth.

## 12.4 Question 4

UCL Engineering plc is a multinational engineering equipment trader and distribution company that has been established for many years. Its income and balance sheet for two years are provided in Table 12.2 and Table 12.3 respectively.

<b>UCL Engineering Income Statement</b>		
	2028	2029
	£m	£m
<b>Revenue</b>	25,832	28,759
Cost of sales	(4,865)	(6,038)
<b>Gross profit</b>	20,967	22,721
Selling, general and administrative expenses	(10,969)	(13,345)
Research and development expenses	(5,771)	(5,734)
Other income	323	365
<b>Profit from operations</b>	4,550	4,007
Finance costs	(1,176)	(1,038)
<b>Profit before tax</b>	3,374	2,969
Tax	(209)	(323)
<b>Net Income</b>	3,165	2,646

Table 12.2: UCL Engineering plc income statement.

<b>UCL Engineering Balance Sheet</b>		
	2028	2029
	£m	£m
<b>Assets</b>		
<b>Non-current assets</b>		
Land and buildings	7,013	6,036
Plant and machinery	12,372	12,449
Intangible assets	12,820	11,733
<b>Total non-current assets</b>	32,205	30,218
<b>Current assets</b>		
Inventories	2,235	1,973
Trade receivables	6,985	7,589
Cash and cash equivalents	6,722	6,514
<b>Total current assets</b>	15,942	16,076
<b>Total assets</b>	48,147	46,294
<b>Equity and Liabilities</b>		
<b>Equity</b>		
Ordinary share capital (£1 shares)	12,215	11,971
Share premium	4,395	4,492
Retained earnings	13,265	14,402
<b>Total equity</b>	29,875	30,865
<b>Non-current liabilities</b>		
Bank loans	13,323	8,985
<b>Total non-current liabilities</b>	13,323	8,985
<b>Current liabilities</b>		
Trade payables	3,610	4,121
Tax liabilities	1,526	2,408
<b>Total current liabilities</b>	5,136	6,529
<b>Total liabilities</b>	18,459	15,514
<b>Total equity and liabilities</b>	48,334	46,379

Table 12.3: UCL Engineering plc balance sheet. Note the inventory for the company in 2027 is valued at £2,350 million.

*Explain the terms “net income”, “intangible assets” and “equity” that feature in either the Income Statement or the Balance Sheet.*

### 10 marks

- Net income is the amount of profit left after all expenses have been paid.
- Intangible assets are assets that are not physical in nature. They are a type of non-current asset meaning that they are not expected to be converted to cash over the next one-year period.
- Equity is what the company owes to shareholders and is what remains of the assets after all liabilities have been paid.

*Calculate the financial ratios shown in Table 12.4 and use them to provide a one-paragraph interpretation of the company’s general performance.*

### 30 marks

	<b>Formula</b>	<b>2028</b>	<b>2029</b>	<b>Industry average</b>
Current ratio	$\frac{\text{Current assets}}{\text{Current liabilities}}$	$\frac{15942}{5136} = 3.10$	$\frac{16076}{6529} = 2.46$	2.4
Debt-Equity ratio	$\frac{\text{Total liabilities}}{\text{Total equity}}$	$\frac{18459}{29875} = 0.62$	$\frac{15514}{30865} = 0.50$	38.38%
Total asset turnover ratio	$\frac{\text{Total annual sales}}{\text{Total assets}}$	$\frac{25832}{48147} = 0.54$	$\frac{28759}{46294} = 0.62$	0.85
Profit margin	$\frac{\text{Net income}}{\text{Net sales}}$	$\frac{3165}{25832} = 0.12$	$\frac{2646}{22721} = 0.09$	8.5%

Table 12.4: Financial ratios and their industry average values for Question 4.

Compared to industry averages, the company demonstrates a variety of strengths and shortcomings in its financial performance. The company’s liquidity is robust, outpacing the high industry standard with a current ratio of 3.10 in 2028 and 2.46 in 2029, exceeding the industry average of 2.4. Conversely, the company embodies a higher risk to shareholders than the industry standard, with a debt-equity ratio of 0.62 in 2028 and 0.50 in 2029, surpassing the average of 38.38%.

In terms of profitability, the company markedly surpassed the industry average in 2028. Although the profit margin of 9% in 2029 is only slightly above the industry average of 8.5%, it suggests a continuation of the prior year’s performance. However, in both 2028 and 2029, the company underperformed the industry average in utilising assets to generate sales, as indicated by the total asset turnover ratio of 0.54 and 0.62, respectively, falling short of the industry average of 0.85.

Inter-year comparison of these ratios presents a discernible trend. The company’s liquidity is generally declining, as shown by the decreasing current ratio from 2028 to 2029. Despite this, the company is becoming less risky for shareholders, evidenced by the reduced debt-equity ratio. Furthermore, there is an indication of improving efficiency in asset utilisation, given the increase in the total asset turnover ratio. Nonetheless, there is a general downturn in profitability, with the profit margin declining from 2028 to 2029.