Chapter 1

Full fault analysis

1.1 Unbalanced impedance

1.1.1 Impedance and sequence components

We have established that in a three-phase unbalanced network there are line and phase voltages and currents that deviate in their relationships from the balanced case. Furthermore, any unbalance can be described as a set of sequence components consisting positive, negative and zero sequence phasors. Now considering impedances in unbalanced networks then we need to ensure that we understand:

- How to change between star and delta arrangements
- Appreciate how sequence impedance is calculated

1.1.2 Unbalanced star and delta equivalence

$Z_{delta} = 3 \cdot Z_{star}$ for all three phases when the loads in the three-phase system were balanced. This was helpful when looking at symmetrical faults since we normally convert delta connections into star connections and consider an impedance diagram as representing one phase. When loads are unbalanced then we need to consider each phase independently because they are subjected to different voltages, currents and impedances. Consider the two circuits below. The star and delta equivalence must result in the same line voltages and currents. In other words the impedance between any two impedances must be equivalent.

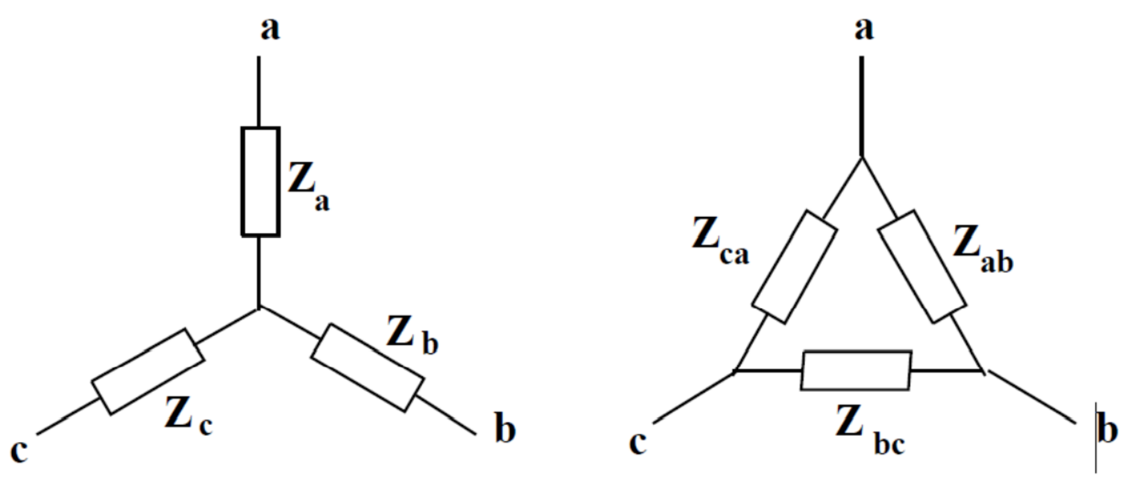

Figure 1.1: Star and delta arrangements.

For example considering phase a and phase b, the impedance equivalence must be:

$$Z_a + Z_b = Z_{ab}//\left(Z_{ca} + Z_{bc}\right) \text{ similarly,} \tag{1.1}$$
$$Z_b + Z_c = Z_{bc}//\left(Z_{ab} + Z_{ca}\right) \tag{1.2}$$
$$Z_c + Z_a = Z_{ca}//\left(Z_{bc} + Z_{ab}\right) \tag{1.3}$$

By manipulation and substitution then it is possible to derive the following relationships:

$$Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c} \tag{1.4}$$
$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a} \tag{1.5}$$
$$Z_{ca} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b} \tag{1.6}$$

and

$$Z_a = \frac{Z_{ab} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{1.7}$$
$$Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{1.8}$$
$$Z_b = \frac{Z_{bc} Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \tag{1.9}$$

These relationships are needed when considering impedance in unbalanced loads.

1.1.3 Good practice

In many fault calculations it is handy to convert delta impedances into star impedances because:

- It ensures that all balanced arrangements are related to ground or virtual group (floating star point)

- When calculating faults then it is apparent that such calculations are made for one phase and then 'phase shifted' to determine impact on other phases. Having everything as a star arrangement (mathematically and circuit wise) assits in ensuring that right values are obtained.

1.2 Impedance of sequences

There are positive, negative and zero phase sequence components: In voltage these are represented by V_0, V_1 and V_2. In current these are represented by I_0, I_1 and I_2.

1.2.1 Sequence components and impedance

Since $V = IZ$, it follows if there are sequence components in both voltage and current then there must be a sequence impedance too:

- $V_1 = I_1 Z_1$ where Z_1 is the positive sequence impedance
- $V_2 = I_2 Z_2$ where Z_2 is the negative sequence impedance
- $V_0 = I_0 Z_0$ where Z_0 is the zero sequence impedance

1.2.2 The importance of sequence impedance

The impedance of a network is important for calculating currents for an applied voltage. Remembering earlier work on balanced networks, we established that the impedance limited the fault current i.e. the further from the source you were the greater the impedance. the lower the fault current. Now considering the sequence components, it is apparent that the sequence impedances Z_0, Z_1, Z_2 will limit sequence currents I_0, I_1, I_2 for the applied sequence voltages V_0, V_1, V_2.

1.2.3 Network elements

Different network equipment exhibit different sequence impedances:

- Typically, transmission lines and cables have one impedance value for positive and negative sequence, but an entirely different impedance value for zero sequence
- Typically, rotating machines e.g. generators and mots have different impedance values for all three sequences
- Typically, transformers positive, negative and zero sequence components depend upon connection by positive and negative are often the same value

Appreciating these different impedances is important for accurate calculation of unsymmetrical faults.

1.2.4 Transmission lines and distribution cables

Power cables and transmission linse are used to carry power from the source to the load. Typically (over short distances) they can be represented as resistance and inductance. The inductance is comprised of its own self-inductance and mutual inductance between each line or cable.

1.2.5 Transmission line analysis

In a three-phase system interconnected between a three-phase generator and three-phase load the lines/cables usually run close to each so there is always mutual inductance and self-inductance of the lines.

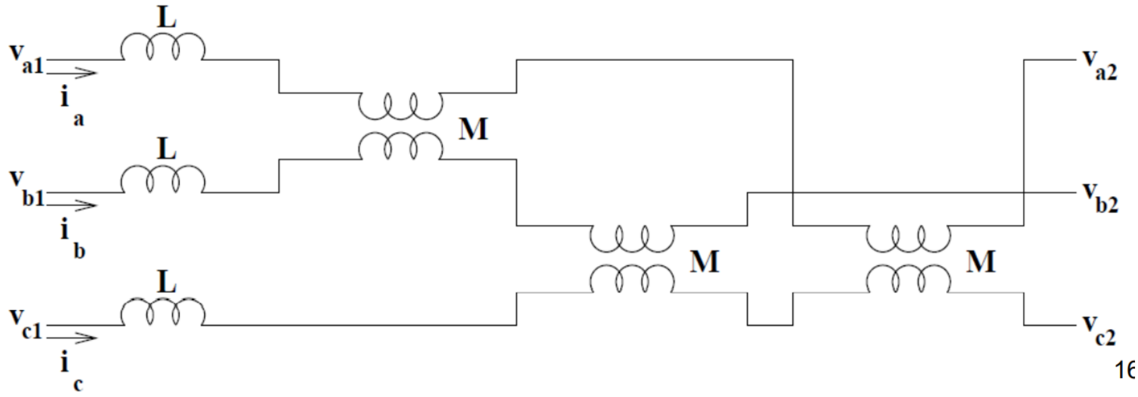

Figure 1.2: Trasmission line mutual inductance and self-inductance.

1.2.6 Transmission line representation

Hence, we can write the relationship ($V = XI$):

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{1.10}$$

It is reasonable to say that the line and mutual inductances are the same for each transmission line or cable under steady-state balanced conditions. This is not necessarily the case for transient or unbalanced case.

1.2.7 Transmission sequence representation

Bringing in the relationship between phase and sequence components we have (ignoring 1/3):

$$I_{sequence} = [A]^{-1} \cdot I_{phase} \tag{1.11}$$

$$V_{sequence} = [A]^{-1} \cdot V_{phase} \tag{1.12}$$

Hence:

$$\begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{1.13}$$

$$[A] \left[V_{sequence}\right] = j\omega \left[LM\right] \cdot [A] \left[I_{sequence}\right] \tag{1.14}$$

1.2.8 Transmission line representation

Hence by transformation we obtain:

$$\left[V_{sequence}\right] = j\omega\,[A]\cdot[LM]\cdot[A]^{-1}\left[I_{sequence}\right] \tag{1.15}$$

The part $([A]\cdot[LM]\cdot[A]^{-1})$ provides the inductance sequence relationship for the transmission line or distribution cable. Resolving gives:

$$\begin{pmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{pmatrix} \tag{1.16}$$

The relationship between sequence components becomes:

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = j\omega \begin{pmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L+2M \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \tag{1.17}$$

The sequence component relationships become:

$$V_1 = j\omega\,(L-M)\,I_1 \tag{1.18}$$

$$V_2 = j\omega\,(L-M)\,I_2 \tag{1.19}$$

$$V_0 = j\omega\,(L-M)\,I_0 \tag{1.20}$$

The positive, negative and zero sequence reactances of the balanced transmission line are then:

$$Z_1 = Z_2 = j\omega\,(L-M) \tag{1.21}$$

$$Z_0 = j\omega\,(L+2M) \tag{1.22}$$

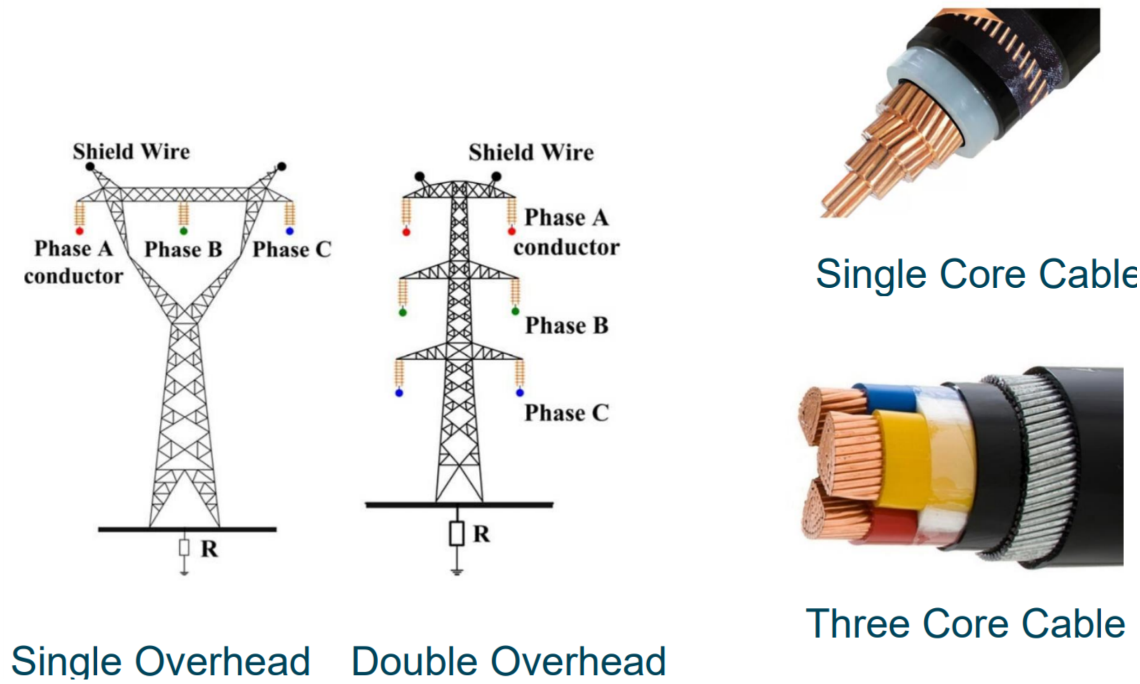

Figure 1.3: Transmission line and cable arrangements.

1.2.9 Lines and cables

The positive and negative sequence impedances are normally balanced i.e. $Z_1 = Z_2$. The zero sequence impedance depends upon the nature of the return path through the earth. Typical relative values of Z_0 during faults are

Overhead:

- For a single-circuit arrangement $(Z_0/Z_1) = 3.5$
- For a double-circuit arrangement $(Z_0/Z_1) = 5.5$

Cable arrangements:

- For a single-core arrangement $(Z_0/Z_1) = 1.25$
- For a three-core arrangement $(Z_0/Z_1) = 4$

1.2.10 Synchronous machines (generators)

The positive sequence reactance Z_1 is the value used under balanced operation due to positive sequence currents flowing in the windings of the machine in steady-state and transient. The negative sequence reactance Z_2 is due to negative sequence currents which give rise to fluxes in the air gap of the machine that rotate in the opposite direction during unbalance. Z_2 is different to Z_1 in most designs. The zero sequence reactance Z_0 depends upon the nature of the connection of the star point. Zero sequence currents will not flow when the star point is floating but will flow when there is.

1.2.11 Neutral connection

The symmetrical components are independent with the voltage-current relationships:

$$V_1 = ZI_1 \tag{1.23}$$

$$V_2 = ZI_2 \tag{1.24}$$

$$V_0 = \left(Z + 3Z_g\right) I_0 \tag{1.25}$$

In many generators that are tied to ground at the star point will have additional impedance separately added to reduce the level of zero sequence currents.

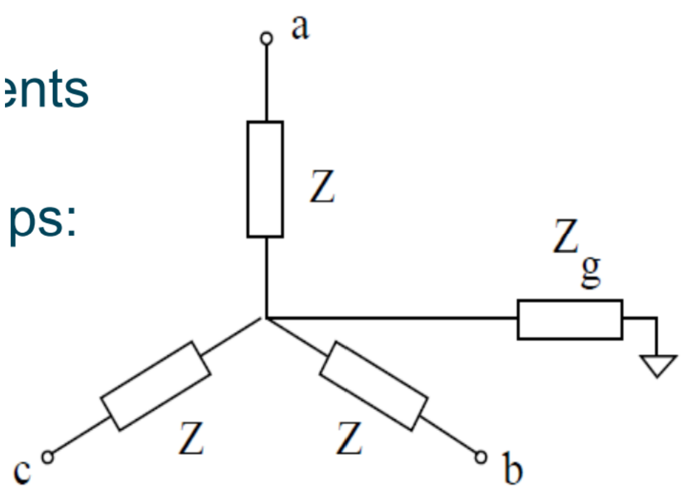

Figure 1.4: Grounded star arrangement.

1.2.12 Typical values of sequence impedances for synchronous generators

Type of machine	+ve sequence	-ve sequence	zero sequence
440 V 50 Hz 1 MVA	0.16 pu	0.11 pu	0.05 pu
11 kV 50 Hz 75 MVA	0.18 pu	0.14 pu	0.07 pu
16 kV 50 Hz 275 MVA	0.21 pu	0.18 pu	0.08 pu
22 kV 50 Hz 575 MVA	0.28 pu	0.21 pu	0.12 pu

Table 1.1: Table to show typical value of sequence impedances for synchronous generators

Manufacturers will test their machines to obtain the relevant data/ The value of the sequence components may differ from country to country, manufacturer to manufacturer.

1.2.13 Transformers

The positive and negative sequence sequence impedances are the normal values obtained from the per-phase equivalent circuit. ($Z_1 = Z_2$). The zero sequence components depend upon the connection of the windings. Zero sequence currents in the windings on one side of the transformer must produce the corresponding ampere-turns in the other. In delta windings the zero-sequence currents circulate through the three-phase windings but do not leave the transformer.

1.3 Unbalanced faults

1.3.1 Fortescue's symmetrical component process

Symmetrical components are used extensively for fault study calculations. in these calculations the positive, negative and zero-sequence impedance networks are either given by the manufacturer or are calculated by the user using base voltages and base power for the system of interest. Each of the sequence networks are then connected together to calculate fault currents and voltages depending upon the type of fault. Standard circuit arrangements have been derived in this course to keep variation reasonable.

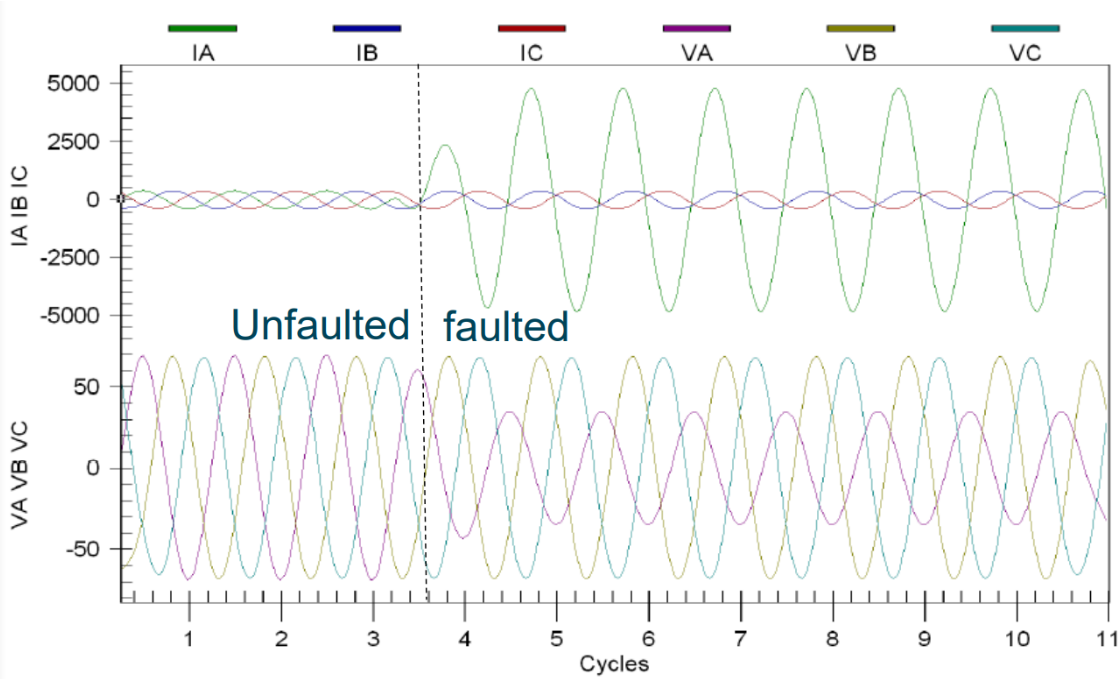

Figure 1.5: Line to ground fault.

1.3.2 Standard fault sequence connections - single line to ground

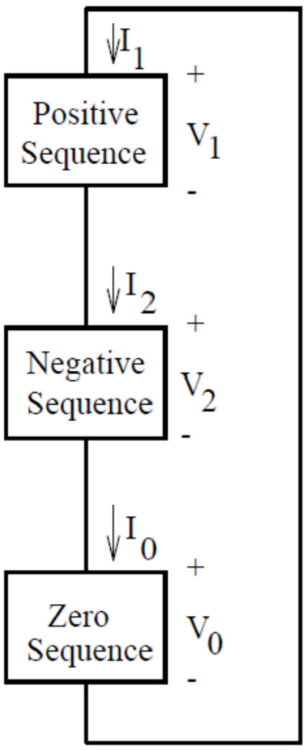

Figure 1.6: Single line to ground connection.

Assumptions:

- $V_a = 0$; $I_a =$ very large value (faulted line)
- $I_b = 0$ (small in comparision to fault current)
- $I_c = 0$ (small in comparision to fault current)

Hence for phae voltage 'a' we can say:

$$V_0 + V_1 + V_2 = 0 \tag{1.26}$$

And for the current we can say:

$$I_0 + I_1 + I_2 = \frac{1}{3} I_a \tag{1.27}$$

Together, these two expressions describe the sequence network connection.

1.3.3 Standard fault sequence connections - line to line

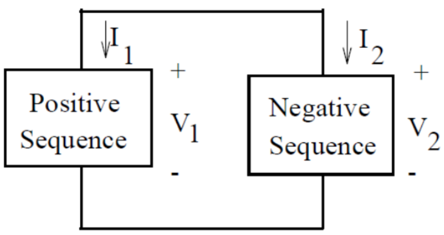

Figure 1.7: Line to line connection.

Assumptions. If the fault occurs between phase b and c then we can say:

- $V_b = V_c$
- $I_b = -I_c$
- $I_a = 0$ (since it is small in comparison with the fault current)

Hence, we can use the phase sequence relationships to say:

$$V_1 = V_2 \text{ and also } I_a = I_1 + I_2 \text{ since } I_0 = 0 \tag{1.28}$$

1.3.4 Standard fault sequence connections - double line to ground

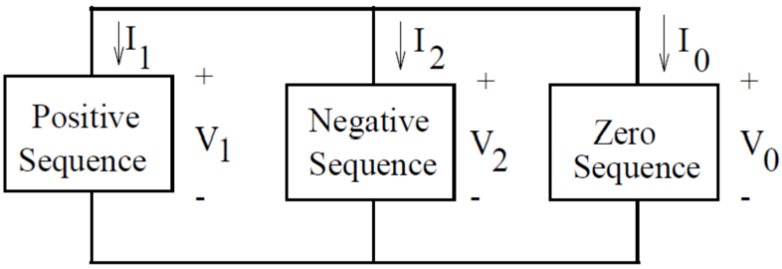

Figure 1.8: Double line to ground connection.

Assumptions. If the fault involves phases b and c to ground then we can say:

- $I_a = 0$ (small in comparison to fault current)
- $V_b = 0$ (faulted line)
- $V_c = 0$ (faulted line)

Hence using phase-sequence relationships we can further say that:

$$V_0 + V_1 + V_2 = 0 \tag{1.29}$$
$$I_a = I_0 + I_1 + I_2 = 0 \tag{1.30}$$

1.4 A full fault analysis study

1.4.1 Breaker sizing method (most common approach)

One of the main purposes of circuit breakers is to arrest large currents that flow when there is a fault. Breaker sizing is achieved by understanding currents flowing under both symmetrical and non-symmetrical fault conditions (to be calculated). Calculations are carried out using symmetrical components i.e. positive, negative and zero sequence. Only one phae needs to be considered ... but all fault types need to be calculated.

1.4.2 Breaker sizing example

Determine the maximum current through the breaker B due to a fault at the location X. Calculate all three types of unbalanced fult and the balanced fault currents.

- System base: voltage 138 kV (1 pu), Power 100 MVA (1 pu)
- Transformer T_1 leakage reactance j0.1 pu
- Transformer T_2 leakage reactance j0.1 pu
- Line L1: positive and negative sequence reactance j0.05 pu, zero sequence reactance j0.1 pu
- Line L2: positive and negative sequence reactance j0.02 pu, zero sequence reactance j0.1 pu

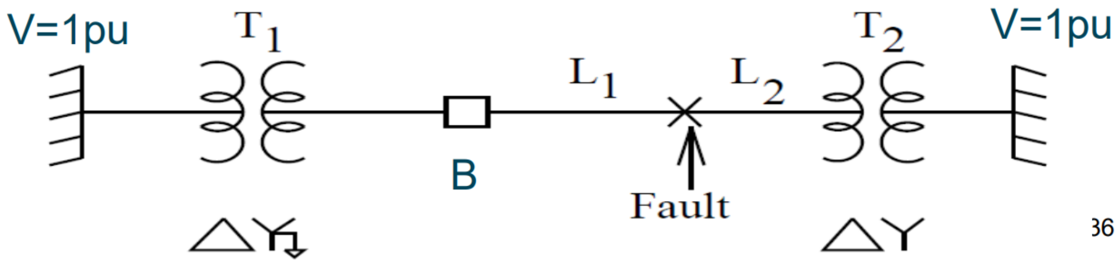

Figure 1.9: Breaker sizing example.

1.4.3 Sequence component arrangement

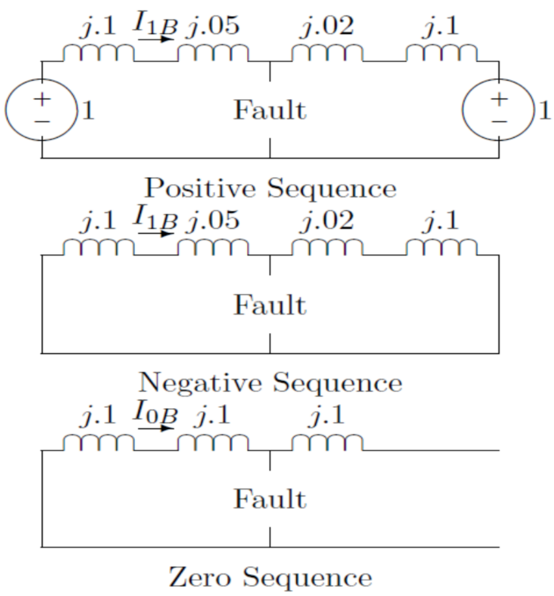

Figure 1.10: Sequence component arrangement.

The sequence networks are exactly like what we would expect to have drawn for equivalent single phase networks. A positive, negative and zero sequence arrangmenet has been shown for one phase. Only the positive sequence network has sources, because the infinite bus supplies only positive sequence voltage. The zero sequence network is open at the right hand side because of the delta-wye transformer connection.

1.4.4 Symmetrical fault current

For a symmetrical (three-phase) fault, only the positive sequence network is involved. The fault shorts the network at its position, so that the current is:

$$I_1 = \frac{1}{j0.15} - j6.67 \text{ per unit from LHS} \tag{1.31}$$

$$(I_1 = \frac{1}{j0.12} - j8.33 \text{ per unit from RHS}) \tag{1.32}$$

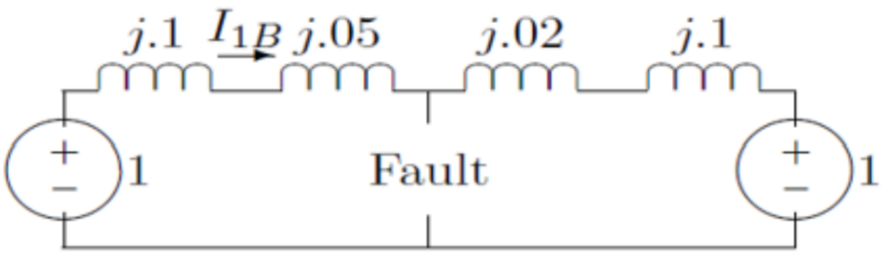

Figure 1.11: Positive sequence impedance in symmetrical fault.

1.4.5 Single line to ground fault

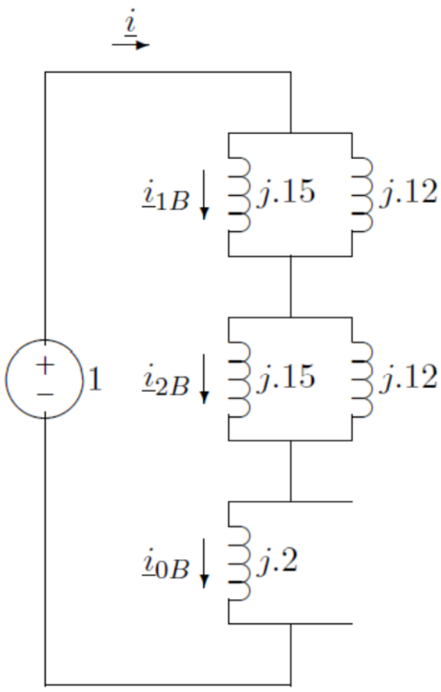

Figure 1.12: Positive sequence impedance in symmetrical fault.

The three networks are in series and the situation is as shown with the total current given by:

$$\underline{i} = \frac{1}{2 \times \left(j0.15 || j0.12\right) + j0.2} = -j3.0 \tag{1.33}$$

The sequence currents are:

$$\underline{i}_{1B} = \underline{i}_{2B} \tag{1.34}$$

$$= \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \tag{1.35}$$

$$= -j1.33\underline{i}_{0B} \qquad = \underline{i} \tag{1.36}$$

$$= -j3.0 \tag{1.37}$$

1.4.6 Singe line to ground fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = \underline{i}_{1B} + \underline{i}_{2B} + \underline{i}_{0B} \tag{1.38}$$

$$\underline{i}_b = \underline{a}^2\underline{i}_{1B} + \underline{a}\underline{i}_{2B} + \underline{i}_{0B} \tag{1.39}$$

$$\underline{i}_c = \underline{a}\underline{i}_{1B} + \underline{a}^2\underline{i}_{2B} + \underline{i}_{0B} \tag{1.40}$$

Hence:

$$\underline{i}_a = -j5.66\,\text{pu} \tag{1.41}$$

$$\underline{i}_b = -j1.67\,\text{pu} \tag{1.42}$$

$$\underline{i}_c = -j1.67\,\text{pu} \tag{1.43}$$

1.4.7 Double line to ground fault

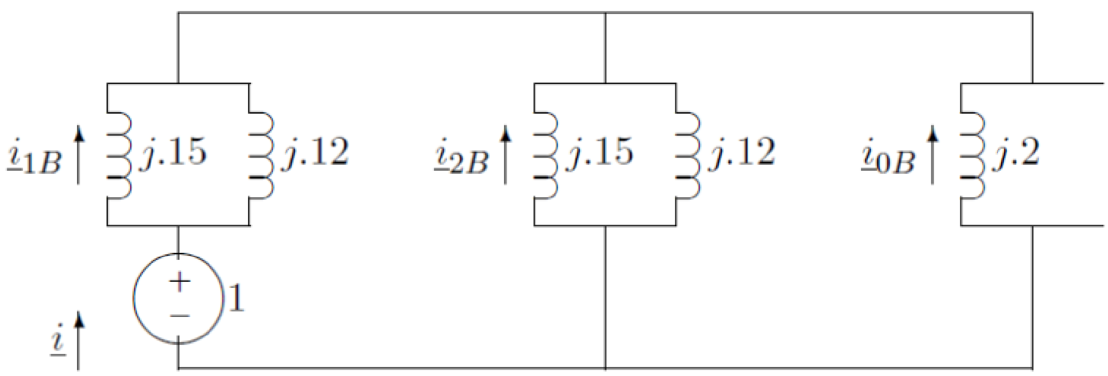

Figure 1.13: Double line-ground fault configuration.

For the double line-ground fault, the networks are in parallel.

$$\underline{i} = \frac{1}{j\left(0.15||0.12\right) + j\left(0.15||0.12||0.2\right)} \tag{1.44}$$
$$= -j8.57 \tag{1.45}$$
$$\underline{i}_{1B} = \underline{i} \times \frac{j0.12}{j0.12 + j0.15} \tag{1.46}$$
$$= -j3.81 \tag{1.47}$$
$$\underline{i}_{2B} = -\underline{i} \times \frac{j0.12||j0.2}{j0.12||j0.2 + j0.15} \tag{1.48}$$
$$= j2.86 \tag{1.49}$$
$$\underline{i}_{0B} = \underline{i} \times \frac{j0.12||j0.15}{j0.2 + j0.12||j0.15} \tag{1.50}$$
$$= j2.14 \tag{1.51}$$

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = j1.19 \tag{1.52}$$
$$\underline{i}_b = \underline{i}_{0B} - \frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) - \frac{\sqrt{3}}{2}j\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{1.53}$$
$$= j2.67 - 5.87 \tag{1.54}$$
$$\underline{i}_c = \underline{i}_{0B} - \frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) + \frac{\sqrt{3}}{2}j\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{1.55}$$
$$= j2.67 + 5.87 \tag{1.56}$$

Hence:

$$|\underline{i}_a| = 1.19\,\text{pu} \tag{1.57}$$
$$|\underline{i}_b| = 6.43\,\text{pu} \tag{1.58}$$
$$|\underline{i}_c| = 6.43\,\text{pu} \tag{1.59}$$

1.4.8 Line to line fault

Having calculated the sequence currents, the phase currents can be reconstructed:

$$\underline{i}_a = 0 \tag{1.60}$$
$$\underline{i}_b = -\frac{1}{2}\left(\underline{i}_{1B} + \underline{i}_{2B}\right) - j\frac{\sqrt{3}}{2}\left(\underline{i}_{1B} - \underline{i}_{2B}\right) \tag{1.61}$$

Hence:

$$|\underline{i}_b| = 5.77\,\text{pu} \tag{1.62}$$
$$|\underline{i}_c| = 5.77\,\text{pu} \tag{1.63}$$

There are only two networks at play - positive and negative sequence.

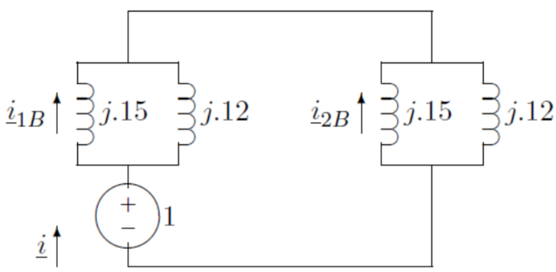

Figure 1.14: Line to line fault configuration.

1.4.9 Conversion to ampere ratings

Having calculated the fault currents then the values in per unit can be expressed as amperes. The value of I_B is:

$$I_B = \frac{P_B}{\sqrt{3}V_{Bl-l}} = 418.8\,\mathrm{A} \tag{1.64}$$

Hence the fault currents are calculated as being:

	Phase A	Phase B	Phase C
Three-phase fault	2791	2791	2791
Single line-ground, ϕ_a	2368	699	699
Double line-ground, ϕ_b, ϕ_c	498	2690	2690
Line-line, ϕ_b, ϕ_c	0	2414	2414

Table 1.2: Table to show fault currents.

The worst fault is the balanced three-phase fault.

1.4.10 Practical sizing of breakers

Key information needed for sizing a circuit breaker include:

- Voltage rating
- Normal current rating
- MVA fault level
- Fault current levels
- Withstand voltage levels

There are three main types of circuit breakers: Air, vacuum and SF6.

1.4.11 Conclusions

- Appreciated the need for positive, negative and zero sequence impedances of different components to make up a power system
- Introduced the concept of positive, negative and zero sequence impedance. Examined this at a component level

- A system analysis method has been applied for 'unbalanced faults' in a transmission system and fault current table produced