UCL Mechanical Engineering 2021/2022

MECH0026 Problem Sheet 1 Solutions

HD

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1.1

For plane stress, our simplification is valid when one dimension of an object (e.g. z-direction) is very small compared to others, e.g. a thin sheet, loaded perpendicular to the surface. Stress tensors relating to the z-direction are virtually 0 ($\sigma_z = \tau_{yz} = \tau_{xz} = 0$) and no loads (body or boundary) in z-direction. We can use compliance matrix to find out the components of our stress field.

For plane strain, our simplification is valid when one dimension of an object (e.g. in z-direction) is very large compared to others e.g. a long cylindrical or prismatic body loaded perpendicular to the length. The conditions of plane strain are:

1. Everything is constant in the z-direction $\frac{\partial()}{\partial z} = 0$
2. $w = 0$
3. No loads (body or boundary) in z-direction

Hence, $\epsilon_z = \delta_{yz} = \delta_{xz} = 0$. We can use stiffness matrix to find components of our strain field

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$$\phi = \frac{P}{20h^3}\left(15h^2x^2y - 5x^2y^3 - 2h^2y^3 + y^5\right) \tag{2.1}$$

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} = \frac{P}{20h^3}\left(20y^3 - 30x^2y - 12h^2y\right) \tag{2.2}$$

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = \frac{P}{20h^3}\left(30h^2xy - 10xy^3\right) \tag{2.3}$$

$$\tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} = -\frac{P}{20h^3}\left(20h^2x - 30xy^2\right) \tag{2.4}$$

satisfies harmonic relationship