

# **Measure of Dispersion and Measure of Skewness**

- **Kindly calculate mean , median and mode write now on a piece of paper**

- Batsman A : 50, 63, 45, 36, 68, 33, 55, 72, 47 81

- Batsman B: 0 , 0,0,58,181,33,47,64,96,71

Mean A= 55

Mean B =55

- although **avg** is equal Still it is clear that the two data sets differ.
- There is much more variation in scores of player B than A
- TO describe this **difference quantitatively**, we use measure of dispersion
- Measure of dispersion is used to measure differences in the data
- A measure of dispersion is the degree of closeness of the values from its some measure of central tendency(mean or median or mode, quantiles)

# Descriptive Statistics

- **Measures of Central Tendency/ Measures of Location**
  - Mean, Median, Mode and Quantiles
- **Measures of Dispersion**
- **MOMENTS**
- Measures of Symmetry
- Measures of Peakedness

# Objectives of Current Lecture

## current lecture:

- **Absolute measure of dispersion**
- **Relative Measures of Dispersion**
- **Moments**
  - Moments about Mean (Central Moments)
  - Moments about any arbitrary Origin
  - Moments about Zero
- Relation b/w central moments and moments about origin
- Moment Ratios
- Skewness
- Kurtosis
- Related Excel Demos

# Types: Measures of Dispersion

- Absolute measure of dispersion / Measure of Dispersion
  - Range
  - Quartile deviation
  - Mean Absolute Deviation
  - Mean Squared Deviation OR **Variance**
  - Mean Standard deviation/ **Standard Deviation**
- Relative Measure of Dispersion / Coefficients of Dispersion
  - Coefficient of variation
  - Coefficient of mean deviation
  - Coefficient of range
  - Coefficient of quartile deviation

$$1) \sum (x - \bar{x}) = 0$$

# Introduction to Statistics

## Measure of Dispersion

$$60 - 60 = 0$$

$$60 - 60 = 0$$

$$60 - 60 = 0$$

Measure of central tendency  
mean, median, mode, quantiles

A: 60, 60, 60

mean: 60

dispersion: 0

B: 40, 40, 100

mean: 60

dispersion:

Measure of dispersion

$x_i$	$x - \bar{x}$	$ x - \bar{x} $	$(x - \bar{x})^2$
40	-20	20	400
40	-20	20	400
100	40	40	1600
$\Sigma$	0	80	2400

# RANGE

- $\text{Range} = R = \text{Max} - \text{Min}$  // ungroup data

**OR**

- **$\text{Range} = R = \text{Largest} - \text{Smallest}$**  // ungroup data
- $\text{Range} = \text{U.C.B of Largest class} - \text{L.C.B of smallest class}$  // Group data
- Example question: Find range in the following data:

31,26,15,43,19,27,22,12,36,33,30,24,20,14

Range =  $43 - 12 = 31$

Question#2: Find the range in the following freq distribution

x	10_20	20_30	30_40	40_50	50_60	
f	5	15	20	6	4	

Range =  $60 - 10 = 50$

# Quartile deviation

- FORMULA:  $Q.D = (Q3 - Q1) / 2$

- Find the Q.D for the following data:

10,13,9,6,4,9,18,8,7,5,14

Solution:

For quartiles First of all list data in some order: here we are ascending order:

4,5,6,7,8,9,9,10,13,14,18

$$Q.D = (Q3 - Q1) / 2$$

So we can see total obs= n= 11

Then

$Q1 = [(n+1)/4]$ th value =  $[(11+1)/4]$ th val =  $12/4$ th value = 3<sup>rd</sup> value = 6

$Q3 = [3 * (n+1)/4]$ th value =  $3 * [(11+1)/4]$ th val =  $3 * 3$ th value = 9<sup>th</sup> value = 13

$$Q.D = (13 - 6) / 2 = 3.5$$



$$Q.D = (Q3 - Q1) / 2$$

- Find Q.D for the following distribution:

C.I (C.B)	2__4	4_6	6_8	8_10	10_12
F	10	25	18	12	5

C.B	F	C.F<
2-4	10	10
4-6	25	35
6-8	18	53
8-10	12	65
		70

ution:

$$= (Q3 - Q1) / 2$$

artile formula:

$$= l + h / f [ (\sum f * i) / 4 - c.f < ]$$

$$= l + h / f [ (17.5 - c.f < ) ] = 4 + 2 / 25 [ 17.5 - 10 ] = 4.6$$

$$= l + h / f [ (2 * (70) / 4 - c.f < ) ]$$

10_12	5

# Mean Deviation / Mean Absolute Deviation

- IT is defined as the mean of the absolute values of deviations taken from some average( Mean , median or mode)
- TYPES:

➤ M.D about mean =  $M.D(X) = \frac{\sum |x_i - \bar{x}|}{n}$  // ungroup data ➔  $M.D(X) = \frac{\sum f_i * |x_i - \text{mean}|}{\sum f_i}$

data

➤ M.D about median

➤ M.D about mode

# M.D about median

➤ M.D about mean = M.D(mean) =  $\frac{\sum |x_i - \text{mean}|}{n}$  // ungroup data ➔ M.D(mean) =  $\frac{\sum f_i * |x_i - \text{mean}|}{\sum f_i}$

**Question :** find mean deviation about mean and about median in following data  
2, 7, 9

- Solution:
- i. About Mean:

M.D about mean = M.D(mean) =  $\frac{\sum |x_i - \text{mean}|}{n}$

Mean =  $(2+7+9)/3=6$

= M.D(mean) =  $\frac{\sum |x_i - \text{mean}|}{n} = 8/3=2.67$

x	Xi-mean(6)	xi –mean
2	-4	4
7	1	1
9	3	3
		$\sum  x_i - \text{mean}  = 8$

# Calculate M.D about meadian for

- 2, 7 ,9

x	Xi-median(7)	xi –median
2	-5	5
7	0	0
9	2	2
		$\Sigma   xi- median   =7$

$$\text{M.D}(\text{mean}) = \frac{\Sigma | xi- mean |}{n} = 7/3 = 2.33$$

# Mean Deviation / Mean Absolute Deviation

- GROUP DATA:
- Calculate mean deviation from mean and mean deviation from median in the following data:

C.I	2_4	4_6	6_8	8_10	10_12
f	2	3	6	2	1

- SOLUTION:
- FORMULA:

C.B	x	f	F * x	c.f

# Variance and Standard Deviation

If  $x_1, x_2, x_3, \dots, x_n$  are the  $n$ -values of the variable  $X$  the Variance is defined as :

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2$$

S.D =  $\sqrt{\text{variance}}$ ,

Group data

$$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{\sum f_i x_i^2}{\sum f_i} - \bar{x}^2$$



Measure of dispersion

- 1) Range
- 2) Mean absolute deviation
- 3) Mean squared deviation

Max - Min

$$\frac{\sum |x - \bar{x}|}{3} = 0$$

$$\frac{0}{3} = 0$$

Variance (A)

$$\frac{80}{3}$$

$$\frac{800}{3}$$

20  
20  
40  
400  
400  
1600

Variance (B)



$\begin{matrix} \text{Rs.} \\ X \end{matrix}$	$X - \bar{X}$	$ X - \bar{X} $	$(X - \bar{X})^2$
61	-8 Rs.	8	64 Rs
72	-3 Rs.	3	9
76	1	1	1
76	1	1	1
84	9 Rs.	9	81
375	$\Sigma$	22	156 Rs

which is the same value that we got by using the defining formula.

$$\bar{X} = \frac{\sum X}{n} = 75 \text{ Rs.}$$

$$MAD = \frac{\sum |X - \bar{X}|}{n} = \frac{22}{5} = 4.4 \text{ Rs.}$$

$$MSD = \text{Variance} = \frac{\sum (X - \bar{X})^2}{n}$$

$$= \frac{156}{5} = 31.2 \text{ Rs}^2$$

$$\sigma^2 = \frac{\sum (x - \bar{x})^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\frac{\sum (x - \bar{x})^2}{n} = \frac{\sum (x^2 - 2x\bar{x} + \bar{x}^2)}{n}$$

$$= \left[ \sum x^2 - 2\bar{x} \sum x + \sum \bar{x}^2 \right] \div n$$

$$= \left[ \sum x^2 - 2n\bar{x}^2 + n\bar{x}^2 \right] / n$$

$$= \frac{\sum x^2}{n} - \frac{n\bar{x}^2}{n} = \frac{\sum x^2}{n} - \bar{x}^2$$

$$\begin{aligned} \sum \bar{x} &= \bar{x} + \bar{x} + \dots + \bar{x} \\ &= n\bar{x} \end{aligned}$$

start (5,5)



Spreadsheet

$X$	$X^2$
67	
72	
76	
76	
84	

$$\bar{x} = \frac{375}{5} = 75 \text{ cm}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2 = 56.562 \text{ cm}^2 \otimes$$

$$\sigma = 7.502 \text{ cm} \rightarrow \sigma$$

X	X <sup>2</sup>		
67	4489		
72	5184		
76	5776		
76	5776		
84	7056		
375	28281		
		SD	
mean	75		
population variance	5656.2	75.20771	
Sample variance	7070.25	84.08478	

Measure of dispersion for a group data

$$MAD = \frac{\sum f |x - \bar{x}|}{n}$$

$$\sigma^2 = \frac{\sum f (x - \bar{x})^2}{n}$$

## Population and Sample Data

**Population data:** The values of a variable for the entire population.

**Sample data:** The values of a variable for a sample of the population.

## Population and Sample Distributions; Distribution of a Variable

The distribution of population data is called the **population distribution**, or the **distribution of the variable**.

The distribution of sample data is called a **sample distribution**.

## Sample Standard Deviation

For a variable  $x$ , the standard deviation of the observations for a sample is called a **sample standard deviation**. It is denoted  $s_x$  or, when no confusion will arise, simply  $s$ . We have

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}},$$

where  $n$  is the sample size and  $\bar{x}$  is the sample mean.

# Parameter and Statistic?

## Parameter and Statistic

Parameter: A descriptive measure for a population

Statistic: A descriptive measure for a sample

## Standardized variable?

### Standardized Variable

For a variable  $x$ , the variable

$$z = \frac{x - \mu}{\sigma}$$

is called the **standardized version** of  $x$  or the **standardized variable** corresponding to the variable  $x$ .



# Problem

*Understanding the Basics* Let's consider a simple variable  $x$ —namely, one with possible observations shown in the first row of Table 3.16.

- Determine the standardized version of  $x$ .
- Find the observed value of  $z$  corresponding to an observed value of  $x$  of 5.
- Calculate all possible observations of  $z$ .
- Find the mean and standard deviation of  $z$  using Definitions 3.11 and 3.12. Was it necessary to do these calculations to obtain the mean and standard deviation?
- Show dotplots of the distributions of both  $x$  and  $z$ . Interpret the results.

$x$	-1	3	3	3	5	5
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## Solution

- a. Using Definitions 3.11 and 3.12, we find that the mean and standard deviation of  $x$  are  $\mu = 3$  and  $\sigma = 2$ . Consequently, the standardized version of  $x$  is

$$z = \frac{x - 3}{2}.$$

- b. The observed value of  $z$  corresponding to an observed value of  $x$  of 5 is

$$z = \frac{x - 3}{2} = \frac{5 - 3}{2} = 1.$$

- c. Applying the formula  $z = (x - 3)/2$  to each of the possible observations of the variable  $x$  shown in the first row of Table 3.16, we obtain the possible observations of the standardized variable  $z$  shown in the second row of Table 3.16.

- d. From the second row of Table 3.16,

$$\mu_z = \frac{\sum z_i}{N} = \frac{0}{6} = 0$$

and

$$\sigma_z = \sqrt{\frac{\sum (z_i - \mu_z)^2}{N}} = \sqrt{\frac{6}{6}} = 1.$$

The results of these two computations illustrate that the mean of a standardized variable is always 0 and its standard deviation is always 1. We didn't need to perform these calculations.

- e. Figures 3.14(a) and 3.14(b) show dotplots of the distributions of  $x$  and  $z$ , respectively.

TABLE 3.16

Possible observations of  $x$  and  $z$

$x$	-1	3	3	3	5	5
$z$	-2	0	0	0	1	1

$$\frac{-1 - 3}{2} = -2$$
$$\frac{3 - 3}{2} = 0$$
$$\frac{5 - 3}{2} = 1$$

$$\frac{3 - 3}{2} = 0$$
$$\frac{5 - 3}{2} = 1$$

# Relative Measures of Dispersion

- When one want to compare the extent of variation of two or more distributions whether having differing or identical units of measurements, it is necessary to consider some other measures that reduce the absolute deviation in some relative form. In such cases Relative Measures of Dispersion are used.
- These measures are usually expressed in the form of **ratios** and **percentages** and are **pure numbers**, independent of the unit of measurements.

# Types Relative Measures of Dispersion

- Coefficient of variation
- Coefficient of mean deviation
- Coefficient of range
- Coefficient of quartile deviation

# Coefficient of Variation

- A coefficient of variation is computed as a ratio of the standard deviation of the distribution to the mean of the same distribution.

$$CV = \frac{S_x}{\bar{x}} * 100$$

- It is said to be the most popular and an ideal Relative Measure of Dispersion

# Properties of Coefficient of Variation

- All values are used in the calculation.
- It is only defined for ratio level of data.
- The actual value of the CV is independent of the unit in which the measurement has been taken, so it is a dimensionless number.
  - For comparison between data sets with different units or widely different means, one should use the coefficient of variation instead of the standard deviation.

## Example: Comments on Children in a community

	Height	weight
Mean	40 inch	10 kg
SD	5 inch	2 kg
CV	0.125	0.20

- Since the coefficient of variation for weight is greater than that of height, we would tend to conclude that weight has more variability than height in the population.

## EXAMPLE:

- Which Factory A or B pays out a larger amount as weekly wages
- Which Factory A or B has Greater Variability

<u>Factory</u>	<u>Average</u>	<u>S.D</u>	<u>No of Workers</u>
<b>A</b>	<b>34.5</b>	<b>5</b>	<b>476</b>
<b>B</b>	<b>28.5</b>	<b>4.5</b>	<b>526</b>

Solution: **Complete the solution**

$$CV = \frac{s_x}{\bar{x}} * 100$$

$$C.V \underline{A} = 5 / 34.5 * 100 = 15.79$$



## EXAMPLE:

- Therefore Factory \_\_\_\_\_ has greater Variability

# Coefficient of Mean Deviation

- The Second type of relative measure is the coefficient of mean deviation. As the mean deviation can be computed from mean, median, mode, or from any arbitrary value, a general formula for computing coefficient of mean deviation may be put as follows:

$$\text{Coefficient of mean deviation} = \frac{\text{Mean deviation}}{\text{Mean}} \times 100$$

# Coefficient of Range

- The coefficient of range is a relative measure corresponding to range and is obtained by the following formula:

$$\text{Coefficient of range} = \frac{L - S}{L + S} \times 100$$

- where, “L” and “S” are respectively the largest and the smallest observations in the data set.

# Coefficient of Quartile Deviation

- The coefficient of quartile deviation is computed from the first and the third quartiles using the following formula:

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

# **Lab Activity:**

- Find the following measurement of dispersion from the data set given in the next page:
  - Range
  - Quartile deviation, Mean deviation, Standard deviation
  - Coefficient of variation, Coefficient of mean deviation, Coefficient of range, Coefficient of quartile deviation

## Data for Lab Activity

Example: Rates of return over the past 6 years for two mutual funds are shown below.

Fund A:	8.3	-6.0	18.9	-5.7	23.6	20
Fund B:	12	-4.8	6.4	10.2	25.3	1.4

Problem: Which fund has higher risk?

# Moments

A moment is a quantitative measure of the shape of a set of points.

The first moment is called the mean which describes the center of the distribution.

The second moment is the variance which describes the spread of the observations around the center.

Other moments describe other aspects of a distribution such as how the distribution is skewed from its mean or peaked.

A moment designates the power to which deviations are raised before averaging them.

# Central (or Mean) Moments

In mean moments, the deviations are taken from the mean.

**For Ungrouped Data:**

$$\text{First Population Moment about Mean} = \mu_1 = \frac{\sum(x_i - \mu)}{N}$$

$$\text{Second Population Moment about Mean} = \mu_2 = \frac{\sum(x_i - \mu)^2}{N}$$

$$\text{First Sample Moment about Mean} = m_1 = \frac{\sum(x_i - \bar{x})}{n}$$

$$\text{Second Sample Moment about Mean} = m_2 = \frac{\sum(x_i - \bar{x})^2}{n}$$

In General,

$$r^{th} \text{ Population Moment about Mean} = \mu_r = \frac{\sum(x_i - \mu)^r}{N}$$

$$r^{th} \text{ Sample Moment about Mean} = m_r = \frac{\sum(x_i - \bar{x})^r}{n}$$



# Central (or Mean) Moments

## Formula for Grouped Data:

$$r^{th} \text{ Population Moment about Mean} = \mu_r = \frac{\sum f (x_i - \mu)^r}{\sum f}$$

$$r^{th} \text{ Sample Moment about Mean} = m_r = \frac{\sum f (x_i - \bar{x})^r}{\sum f}$$

# Central (or Mean) Moments

**Example:** Calculate first four moments about the mean for the following set of examination marks:

X
45
32
37
46
39
36
41
48
36

**Solution:** For solution, move to MS-Excel.

# Central (or Mean) Moments

**Example:** Calculate: first four moments about mean for the following frequency distribution:

Weights (grams)	Frequency (f)
65-84	9
85-104	10
105-124	17
125-144	10
145-164	5
165-184	4
185-204	5
Total	60

**Solution:** For solution, move to MS-Excel.

# Moments about (arbitrary) Origin

If the deviations are taken from some arbitrary number ('a' called origin), then moments are called moments about arbitrary origin 'a'.

## For Ungrouped Data:

$$r^{th} \text{ Population Moment about Origin 'a'} = \mu'_r = \frac{\sum (x_i - a)^r}{N}$$

$$r^{th} \text{ Sample Moment about Origin 'a'} = m'_r = \frac{\sum (x_i - a)^r}{n}$$

## For Grouped Data:

$$r^{th} \text{ Population Moment about Origin 'a'} = \mu'_r = \frac{\sum f(x_i - a)^r}{\sum f}$$

$$r^{th} \text{ Sample Moment about Origin 'a'} = m'_r = \frac{\sum f(x_i - a)^r}{\sum f}$$

# Moments about zero

If origin is taken as zero. i.e.  $a=0$ , moments are called moments about zero.

## For Ungrouped Data:

$$r^{th} \text{ Population Moment about Zero} = \mu'_r = \frac{\sum (x_i - 0)^r}{N} = \frac{\sum (x_i)^r}{N}$$

$$r^{th} \text{ Sample Moment about Zero} = m'_r = \frac{\sum (x_i - 0)^r}{n} = \frac{\sum (x_i)^r}{n}$$

## For Grouped Data:

$$r^{th} \text{ Population Moment about Zero} = \mu'_r = \frac{\sum f(x_i - 0)^r}{\sum f} = \frac{\sum f(x_i)^r}{\sum f}$$

$$r^{th} \text{ Sample Moment about Zero} = m'_r = \frac{\sum f(x_i - 0)^r}{\sum f} = \frac{\sum f(x_i)^r}{\sum f}$$

# Moments about zero

**Example:** Calculate first four moments about zero (origin) for the following set of examination marks:

X
45
32
37
46
39
36
41
48
36

**Solution:** For solution, move to MS-Excel.

# Moments about zero

**Example:** Calculate: first four moments about zero (origin) for the following frequency distribution:

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65-84	9
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185-204	5
Total	60

**Solution:** For solution, move to MS-Excel.

# Conversion from Moments about Mean to Moments about Origin

**Sample Moments about Mean in terms of Moments about Origin.**

$$m_1 = m'_1 - m'_1 = 0$$

$$m_2 = m'_2 - (m'_1)^2$$

$$m_3 = m'_3 - 3m'_2m'_1 + 2(m'_1)^3$$

$$m_4 = m'_4 - 4m'_3m'_1 + 6m'_2(m'_1)^2 - 3(m'_1)^2$$

**Population Moments about Mean in terms of Moments about Origin.**

$$\mu_1 = \mu'_1 - \mu'_1 = 0$$

$$\mu_2 = \mu'_2 - (\mu'_1)^2$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^2$$



# Moment Ratios

Ratios involving moments are called moment-ratios.

Most common moment ratios are defined as:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \beta_2 = \frac{\mu_4}{\mu_2^2}$$

Since these are ratios and hence have no unit.

For symmetric distributions,  $\beta_1$  is equal to zero. So it is used as a measure of skewness.

$\beta_2$  is used to explain the shape of the curve and it is a measure of peakedness.

For normal distribution (Bell-Shaped Curve),  $\beta_2 = 3$ .

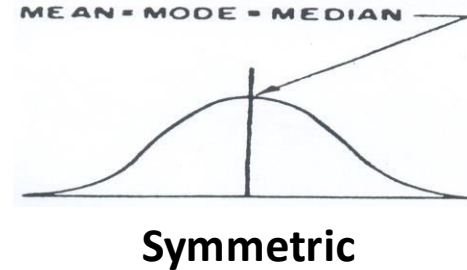
For sample data, moment ratios can be similarly defined as:

$$b_1 = \frac{m_3^2}{m_2^3}, b_2 = \frac{m_4}{m_2^2}$$

# Skewness

A distribution in which the values equidistant from the mean have equal frequencies and is called Symmetric Distribution.

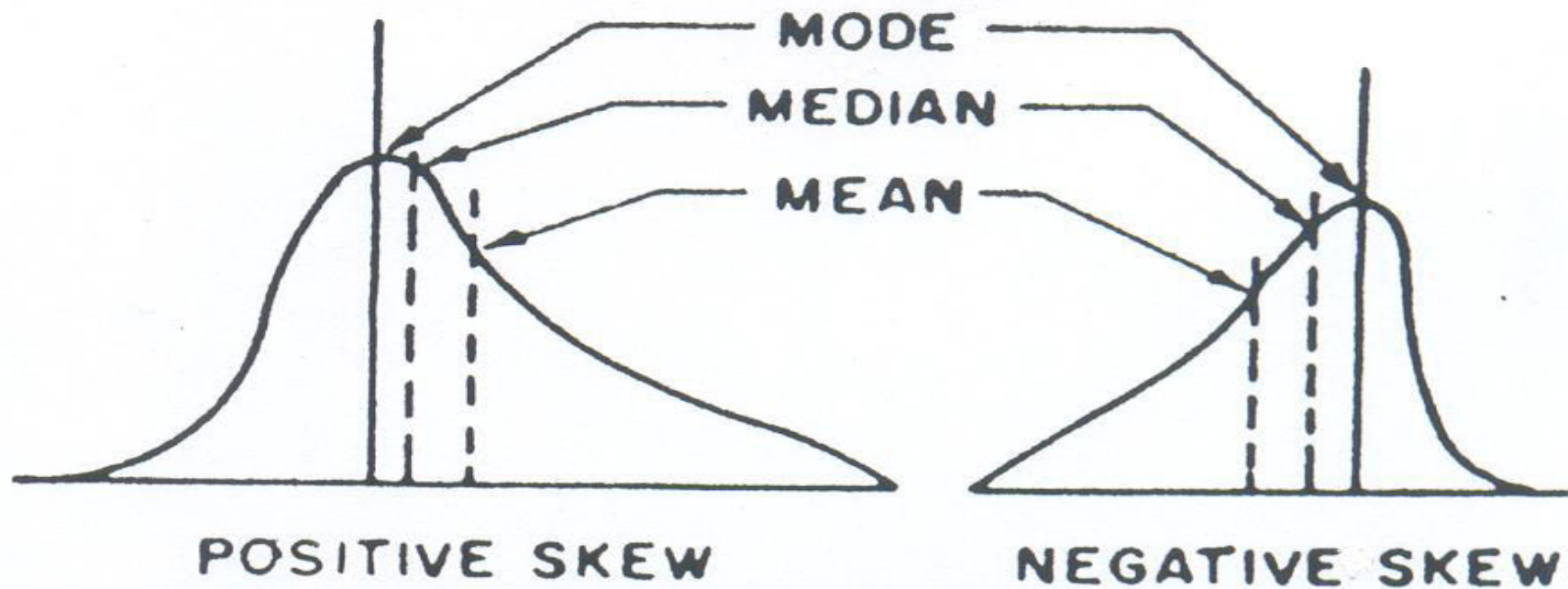
Any departure from symmetry is called **skewness**.



In a perfectly **symmetric distribution**, **Mean=Median=Mode** and the two tails of the distribution are equal in length from the mean. These values are pulled apart when the distribution departs from symmetry and consequently one tail become longer than the other.

If right tail is longer than the left tail then the distribution is said to have **positive skewness**. In this case, **Mean>Median>Mode**

If left tail is longer than the right tail then the distribution is said to have **negative skewness**. In this case, **Mean<Median<Mode**



$\text{Mean} > \text{Median} > \text{Mode}$

$\text{Mean} < \text{Median} < \text{Mode}$

# Skewness

**When the distribution is symmetric, the value of skewness should be zero.  
Karl Pearson defined coefficient of Skewness as:**

$$Sk = \frac{Mean - Mode}{SD}$$

**Since in some cases, Mode doesn't exist, so using empirical relation,**

$$Mode = 3Median - 2Mean$$

**We can write,**

$$Sk = \frac{3(Median - Mean)}{SD}$$

**(it ranges b/w -3 to +3)**

# Skewness

According to Bowley (a British Statistician):

Bowley's coefficient of skewness (also called Quartile skewness coefficient)

$$sk = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1} = \frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1} = \frac{Q_1 - 2Median + Q_3}{Q_3 - Q_1}$$

Another measure of skewness mostly used is by using moment ratio (denoted by  $\sqrt{\beta_1}$ ):

$$sk = \frac{\mu_3}{\sigma^3}, \quad \text{for population data}$$

$$sk = \frac{m_3}{s^3}, \quad \text{for sample data}$$

For symmetric distributions, it is zero and has positive value for positively skewed distribution and take negative value for negatively skewed distributions.

# Kurtosis

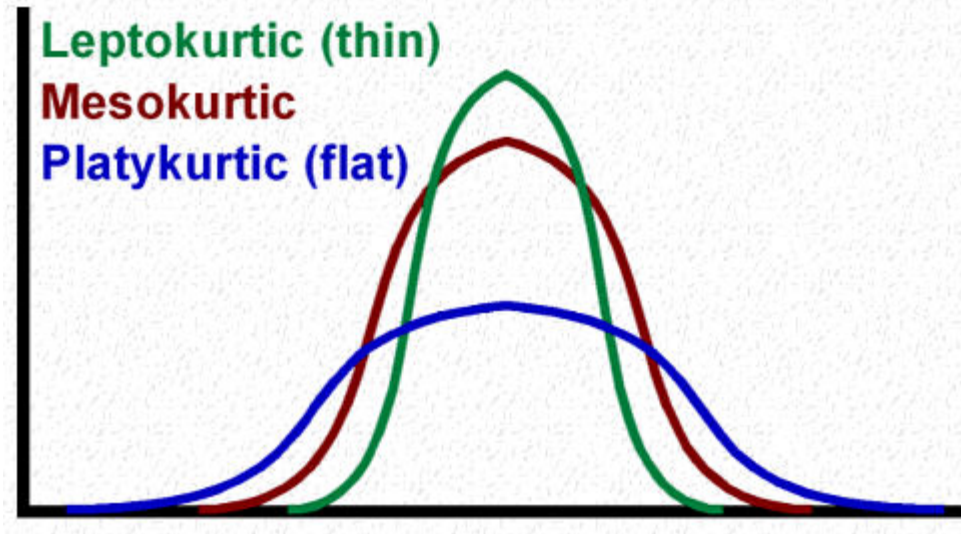
Karl Pearson introduced the term Kurtosis (literally the amount of hump) for the degree of peakedness or flatness of a unimodal frequency curve.

When the peak of a curve becomes relatively high then that curve is called **Leptokurtic**.

When the curve is flat-topped, then it is called **Platykurtic**.

Since normal curve is neither very peaked nor very flat topped, so it is taken as a basis for comparison.

The normal curve is called Mesokurtic.



# Kurtosis

Another measure of Kurtosis, known as Percentile coefficient of kurtosis is:

$$\text{Kurt} = \frac{Q.D}{P_{90} - P_{10}}$$

Where,

Q.D is semi-interquartile range= $Q.D = (Q_3 - Q_1)/2$

$P_{90}$ =90<sup>th</sup> percentile

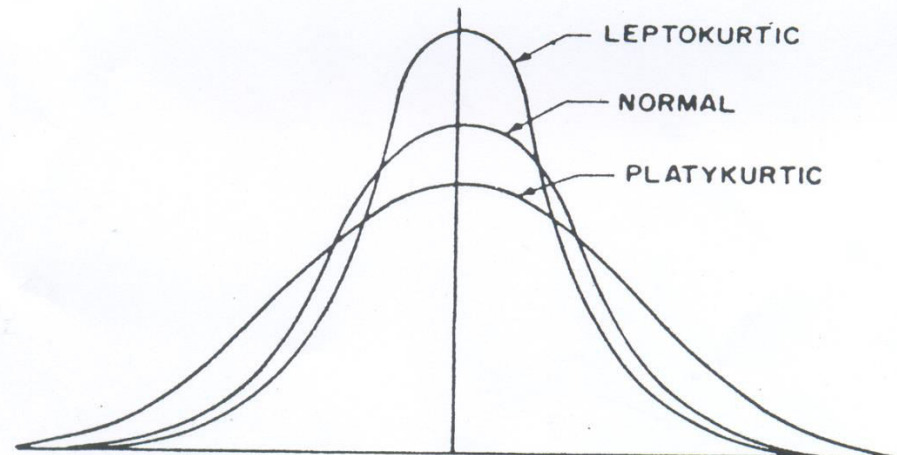
$P_{10}$ =10<sup>th</sup> percentile

# Measures of Kurtosis

- If  $\beta_2 - 3 > 0$ , the distribution is leptokurtic.
- If  $\beta_2 - 3 < 0$  the distribution is platykurtic.
- If  $\beta_2 - 3 = 0$  the distribution is mesokurtic.

The most important measure of kurtosis based on the second and fourth moments is

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$





# Co-efficient of Skewness and Kurtosis using Moments

- Co-efficient of Skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

- Kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

# Exercise-1: Find coefficient of Skewness using Pearson's, Kelly's and Bowley's formula

Income of daily Labour	Frequency (f)	(x)
40-50	3	45
50-60	5	55
60-70	10	65
70-80	8	75
80-90	4	85
90-100	4	95
100-110	1	105
Sum	N=35	

# Exercise-2: Find coefficient of skewness and Kurtosis using moments

Class	Frequency (f)	x	fx	$f(x-\mu)$	$f(x-\mu)^2$	$f(x-\mu)^3$	$f(x-\mu)^4$
25-30	2						
30-35	8						
35-40	18						
40-45	27						
45-50	25						
50-55	16						
55-60	7						
60-65	2						
Sum	N=35						