Rotational Kinematics

by

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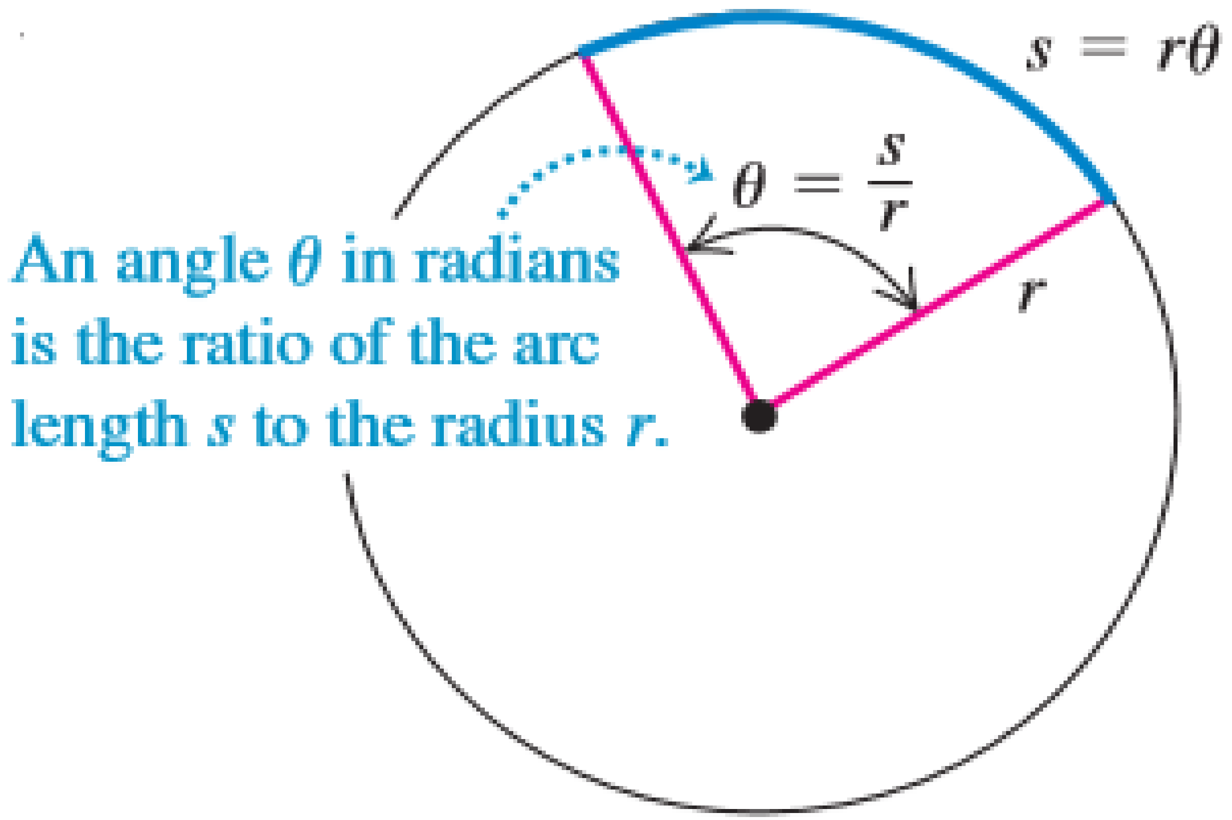
Angular Velocity and Acceleration

- Let's think about a rigid body that rotates about a fixed axis.
- We can write about the relation between the angular displacement with the linear by this expression.
- $\theta = \frac{s}{r}$
- It was shown in the figure an angle is subtended by an arc of length s on a circle of radius r.

Angular Velocity and Acceleration



Angular Velocity

- We can define the angular velocity as follows
- $\omega = \dfrac{\Delta\theta}{\Delta t}$
- Where ω is the angular velocity
- When Δt approaches to zero we can write
- $\omega = \dfrac{d\theta}{dt}$

Angular Acceleration

- Angular acceleration can be given as
- $\alpha = \frac{\Delta\omega}{\Delta t}$
- If Δt approaches to zero, then we can write
- $\alpha = \frac{d\omega}{dt}$ or
- $\alpha = \frac{d^2\theta}{dt^2}$

Relating Linear and Angular Kinematics

- Displacement
- $s = r\theta$
- Velocity
- $v = r\omega$
- Acceleration
- $a_{tan} = r\alpha$
- $a_{radial} = \frac{v^2}{r}$
- $a_{radial} = \omega^2 r$

Energy in Rotational Motion

- When a body rotates its mean a mass in motion, so it is a kinetic energy. We can explain kinetic energy in terms of angular speed and a new entity moment of inertia which we will discuss later.
- Consider a system of particles of masses m_1, m_2,…. At distances r_1,r_2,.... from the axis of rotation moving with velocities v1 , v2,….
- We label the particle with the index i. we can write
- $\frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$, Total Kinetic Energy can be given as
- $K = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2$

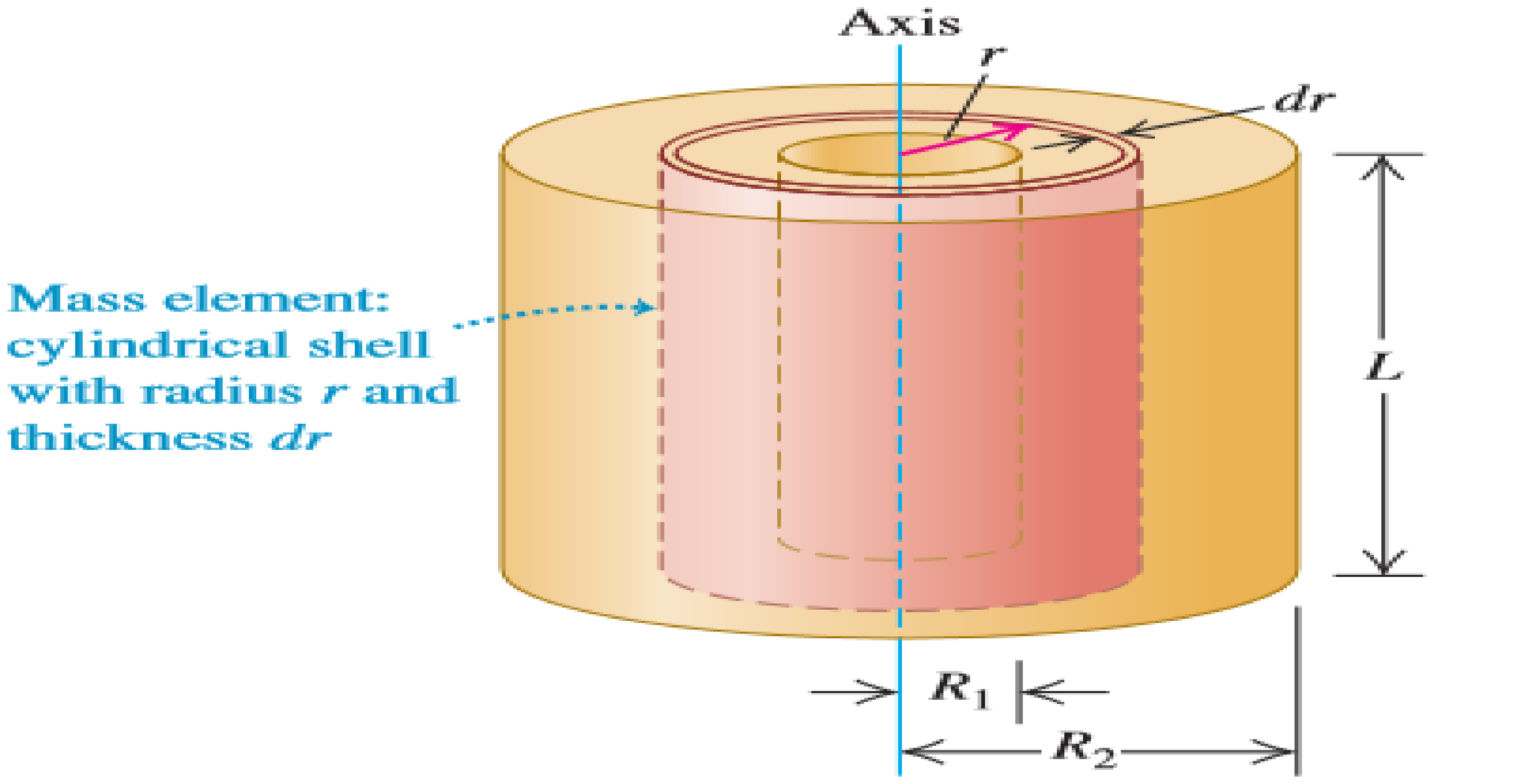
Moment of Inertia

- Moment of inertia depends upon the mass of a body and how the mass is distributed from the axis of rotation. It is denoted by I
- Finally the kinetic Energy can be given as
- $K = I\omega^2$
- Rotational Kinetic energy of the body

Calculation of the Moment of Inertia

- We can us calculus for the calculation of moment of inertia
- $I = \int r^2 dm$
- Where mass can be written in terms density and volume
- $I = \int r^2 \rho dV$
- where ρ is the density and dV is the differential volume
- In the ened
- $I = \rho \int r^2 dV$

Moment of Inertia of a hollow or Solid Cylinder rotating about axis of symmetry



Calculation

- $I = \int r^2 dm$
- $dm = \rho dV$
- $V = \pi r^2 L$
- $dV = 2\pi r L dr$
- $dm = \rho(2\pi r L dr)$
- $I = \int r^2 \rho \, (2\pi r L)$dr
- $I = \int_{R1}^{R2} r^2 \rho(2\pi r L) \, dr$

Calculation

- $I = \int_{R1}^{R2} r^2 \rho (2\pi r L)$dr
- $I = 2\pi L\rho \int_{R1}^{R2} r^3 dr$
- $I = 2\pi L\rho \frac{1}{4}(R_2^4 - R_1^4)$
- $I = 2\pi L\rho \frac{1}{4}(R_2^2 - R_1^2)\,(R_2^2 + R_1^2)$
- Volume of the cylinder=$\pi L(R_2^2 - R_1^2)$
- Then Mass M=$\rho\ \pi L(R_2^2 - R_1^2)$
- $I = \frac{1}{2} M\ (R_2^2 + R_1^2)$

- $I = \frac{1}{2} M \left(R_2^2 + R_1^2\right)$
- $R_1 = 0$
- $I = \frac{1}{2} M \left(R_2^2\right)$(Solid cylinder)
- $R_1 \approx R_2 = R$
- $I = MR^2 (for\ a\ very\ thin\ walled\ cylinder)$