Uniform Circular Motion

by

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Lecturer # 1

Uniform Circular Motion

- Any thing which move in a circle is a circular motion. For e.g a car moving around a round about.
- If this car moves in a circle with constant speed. We call it a "**uniform circular motion**"
- Keep in mind that I have said constant speed not constant velocity as it continuously changing direction

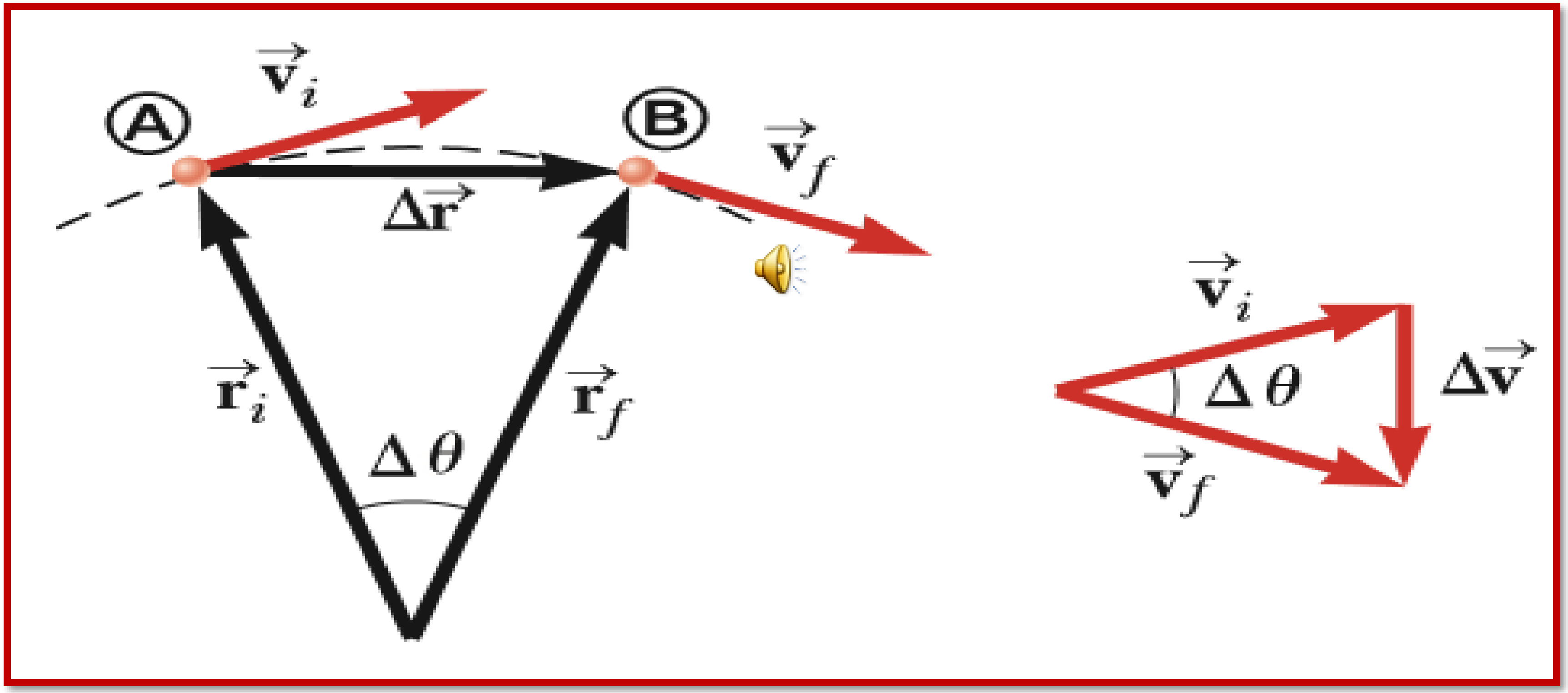
Acceleration in a Uniform Circular Motion

- According to the definition of acceleration, it is actually the change in a velocity.
- As velocity is a vector so it can be changed with respect to its magnitude and with respect to its direction.
- In uniform circular motion as mentioned earlier magnitude (speed) is constant.
- Then acceleration is the result of change in direction

Direction of the Acceleration

- Direction of acceleration in this type of motion in given condition is always perpendicular to the path and directed towards the centre of the circle.
- For the magnitude of the acceleration let us consider two vectors in uniform circular motion.
- One is the position vectors r_i, r_f, change in position Δr and velocity vectors v_i, v_f and change in velocity Δv as shown in the diagram

Centripetal Acceleration



Centripetal Acceleration

- Particle is at 'A' at time interval t_i and at 'B' at time t_f.
- As it is a uniform circular motion then
  - vi = vf =v
- Velocity vector is always perpendicular to the position vector as shown the figure.
- Angle $\Delta\theta$ between the two velocity and two position vectors is same.

Centripetal Acceleration

- Both triangles are same (if one angle is equal and the ratio of the two sides are equal). Then.
- Taking the time

$$\frac{|\Delta\vec{\mathbf{v}}|}{v} = \frac{|\Delta\vec{\mathbf{r}}|}{r}$$

$$|\vec{\mathbf{a}}_{\text{avg}}| = \frac{|\Delta\vec{\mathbf{v}}|}{|\Delta t|} = \frac{v|\Delta\vec{\mathbf{r}}|}{r\Delta t}$$

Finally

- In the end we have the famous equation which everyone has learned by heart

$$a_c = \frac{v^2}{r}$$

Blast by Sir Isaac Newton

Concept of Force

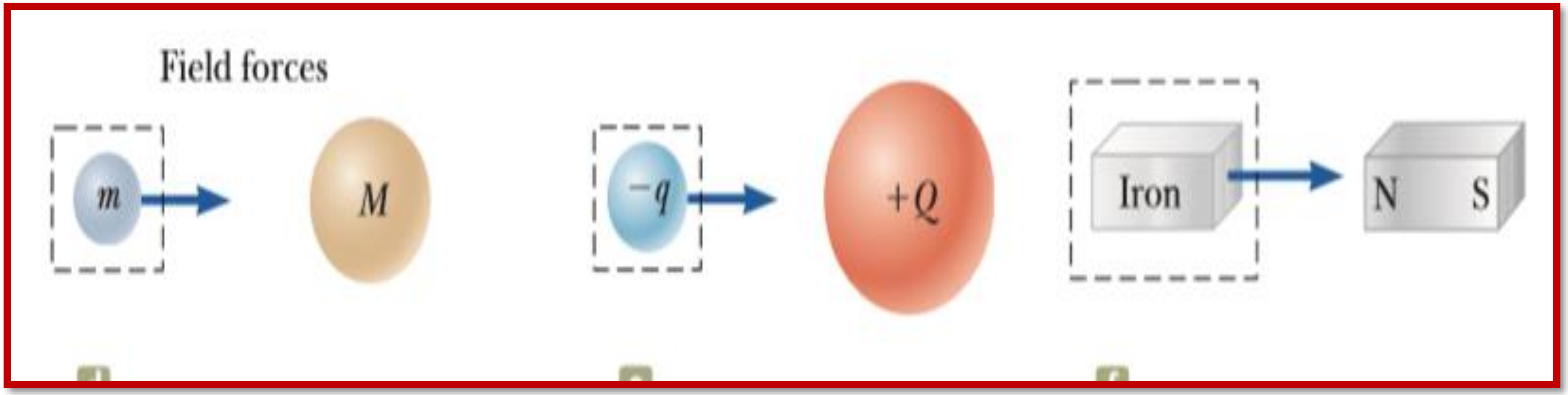
- Definition
  - It is an agent which can cause the motion , which can stop the motion and it can also change the direction of motion.
- Types of forces
  - Contact forces
  - Field forces

Contact Forces

- As it is clear from the name it requires the physical contact.
- Following are some examples of the contact forces

Field Forces

- These kind of forces does not require any contact. They can act through empty spaces.
- Following are some examples of the field forces.



Inertial Frame of Reference

- If the object has zero acceleration, means either zero velocity (at rest) or constant velocity and it does not interact with other objects then it can be identified in an inertial frame of reference. Newton's 1st law of motion is responsible to defines these types of reference frames.

Newton's First Law of Motion

- In the absence of an external force a body which is in rest will remain at rest and if a body is in a state of motion it will remain in the state of motion with constant velocity.
  - Must remember that this explanation is inside an inertial frame of reference, due to this reason 1^{st} law of motion also called law of inertia

Newton's 2nd Law of Motion

- The acceleration is directly proportional to the force applied and inversely proportional to the mass of a body

$$\vec{\mathbf{a}} \propto \frac{\sum \vec{\mathbf{F}}}{m}$$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

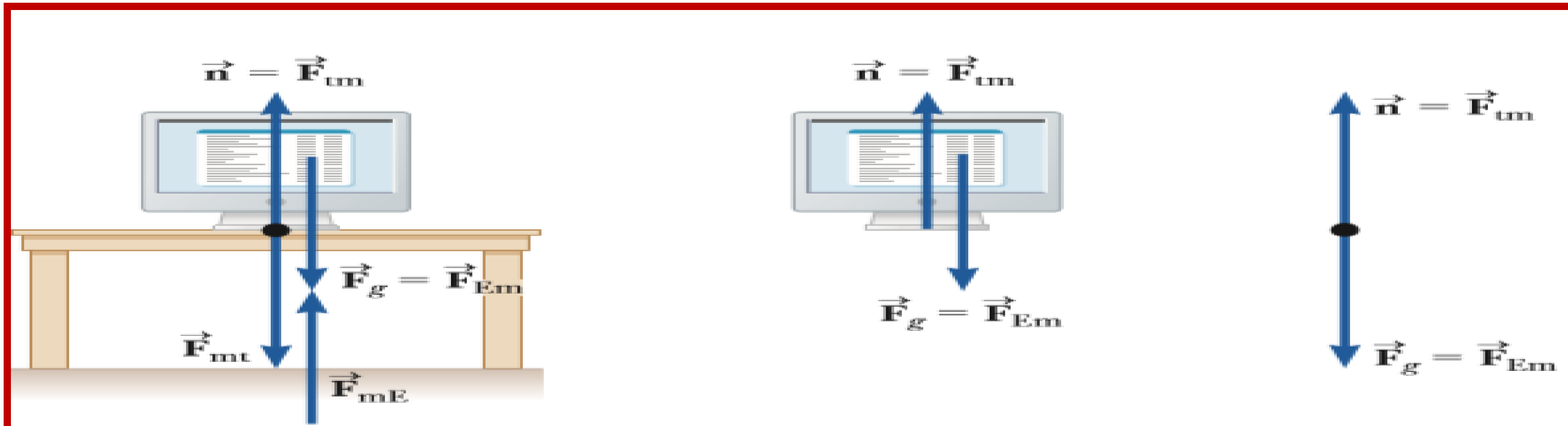
3rd Law of Motion

- Every Action ha equal and opposite reaction.
- If F_{12} is a force exerted by an object 1 on object 2 then in result object 2 will exert an opposite and equal force F_{21} on object 1
  - $F_{12} = - F_{21}$

Free Body Diagram

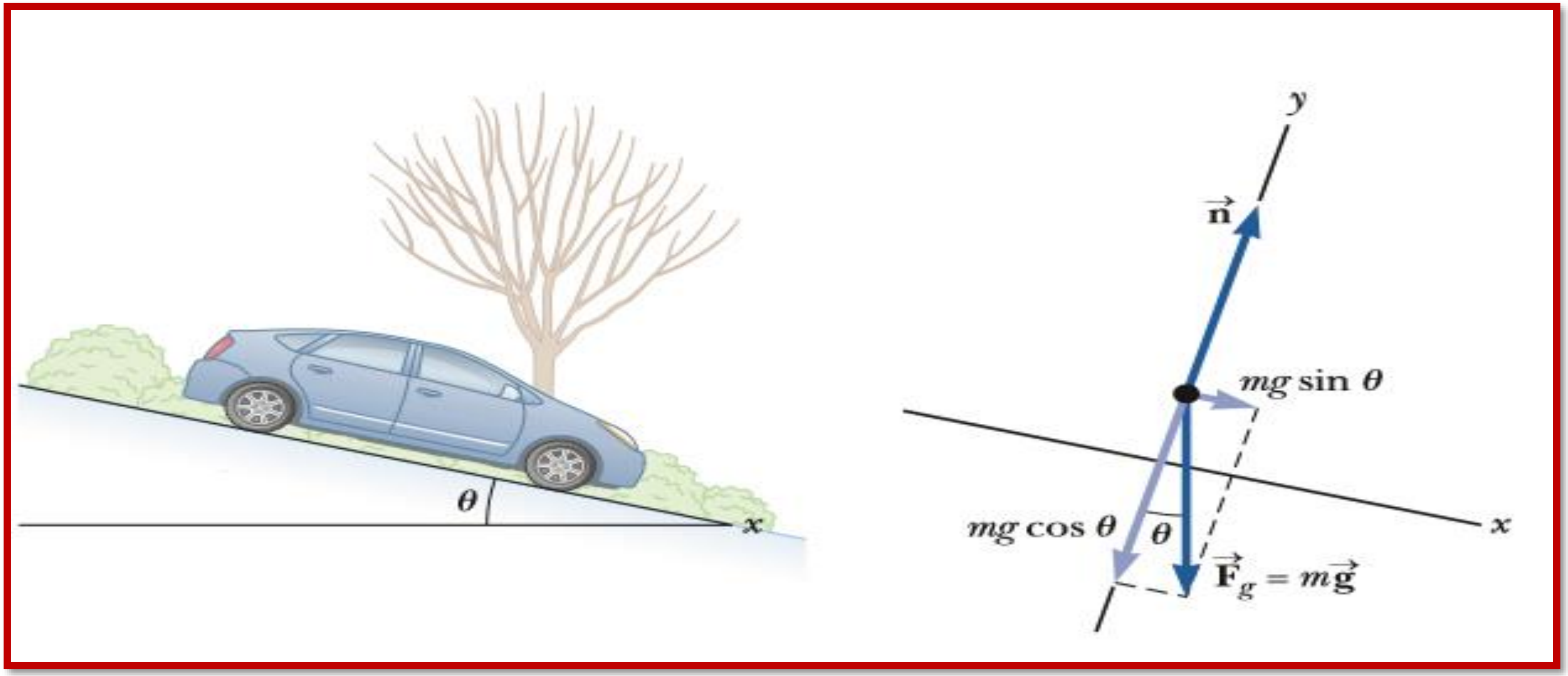
- To solve the problem related to the forces it is necessary to understand and able to draw a free body diagram.
- When drawing a free body diagram
- All the forces acted on an isolated object must be drawn .
- Include all the field forces as well such as gravitational force.

Free Body Diagram



- 1st figure shows the forces acting on the monitor, on the table and on the earth.
- 2nd figure shows the forces acting only on a monitor.
- In 3rd figure object is shown by a dot and the forces acting on a dot. This figure is free body diagram

Another Example



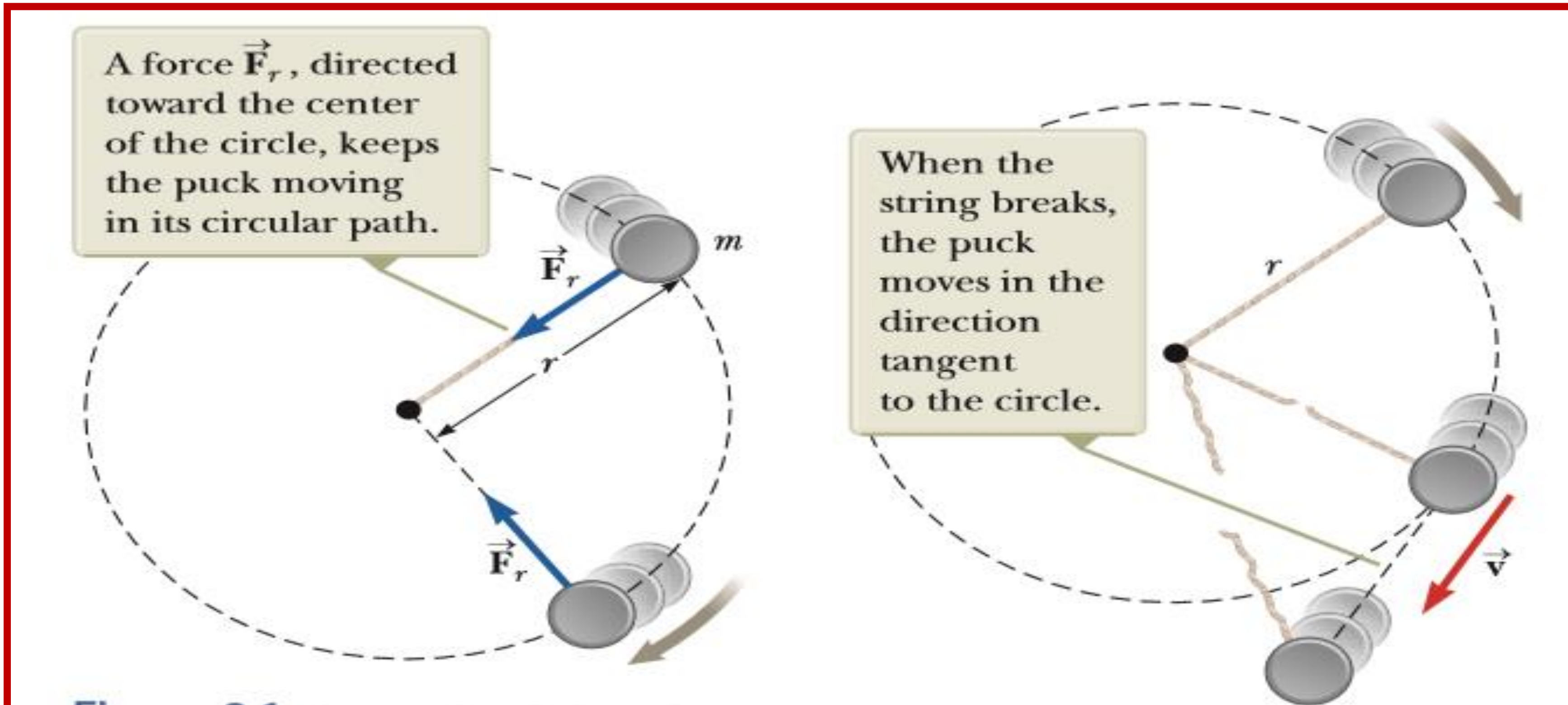
Conceptual Quiz

- A large man and a boy facing each other on a frictionless surface. They put their hands together and push against each other so they move apart.
- Who moves away with the higher speed?
- Who moves farther while their hands are in contact?

Quiz

- According to Newton's 3rd law it's an action and reaction pair of forces.
- Both experience the same force but boy has a less mass so it experience the greater acceleration and it has a larger average speed.
- In the same way the boy moves farther as compared to the man

Centripetal Acceleration



Centripetal Acceleration

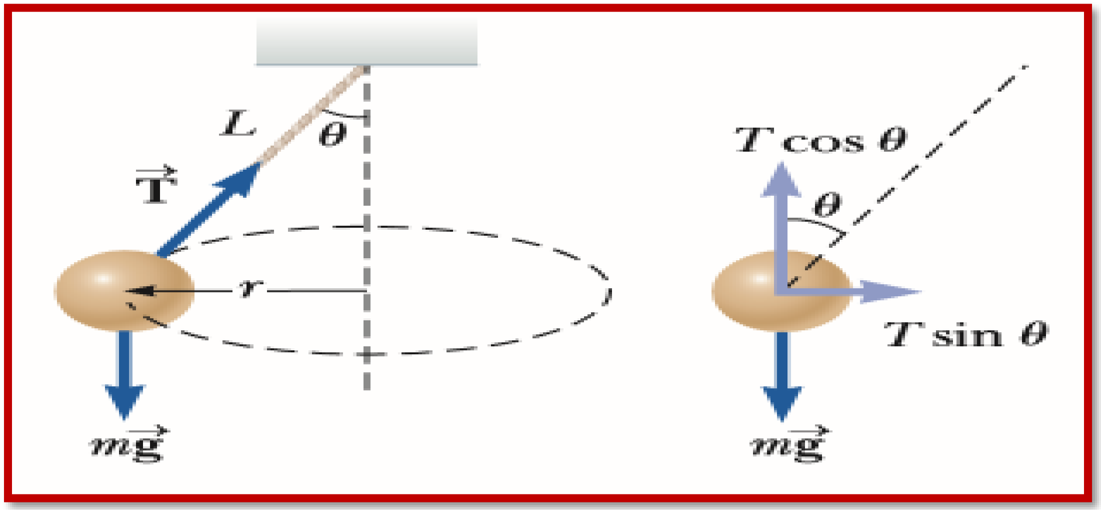
- The force which is associated with the centripetal acceleration and it is responsible for making a particle to strictly moves in a circle by attracting it towards the center of the cirle is known as the Centripetal Acceleration

$$F_c = ma_{c=}\frac{mv^2}{r}$$

- If this force is vanished the particle will stop moving in a circle and start moving in a straight line as mentioned in above figure

Conical Pendulum

- A small ball of mass $\boldsymbol{m}$ is suspended from a string of Length $\boldsymbol{L}$. The ball revolves in a horizontal circle of radius $\boldsymbol{r}$ with constant speed. Find the expression of $\boldsymbol{v}$



Steps to solve

- First of all the free body diagram as shown in the above figure.
- Categorize
  - 2^{nd} it does noes accelerate vertically mean particle is in equilibrium in vertical direction.
  - Then we have to find the force which is acting as a centripetal acceleration responsible for uniform circular motion.

Steps to solve

- Analyze
  - Let θ be the angle between the string and a vertical
  - There will be a tension T in the string and as it is a vector it will resolved in to two components.
  - T cosθ is balanced by a gravitational force means weight.
  - T sinθ is acting as a centripetal force attracting the mass towards the center

Derivation

$$\sum F_y = T\cos\theta - mg = 0$$

$$(1) \quad T\cos\theta = mg$$

$$(2) \quad \sum F_x = T\sin\theta = ma_c = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{Lg\sin\theta\ \tan\theta}$$

Assignment

- You have to find out the Time Period.