BOOLEAN ALGEBRA

INTRODUCTION:

George Boole, a nineteenth-century English Mathematician, developed a system of logical algebra by which reasoning can be expressed mathematically. In 1854, Boole published a classic book, “An Investigation of the Laws of thought” on which he founded the Mathematical theories of Logic and Probabilities, Boole’s system of logical algebra, now called Boolean algebra, was investigated as a tool for analyzing and designing relay switching circuits by Claude E. Shannon at the Massachusetts institute of Technology in 1938. Shannon, a research assistant in the Electrical Engineering Department, wrote a thesis entitled “A” symbolic Analysis of Relay and Switching Circuits. As a result of his work, Boolean algebra is now, used extensively in the analysis and design of logical circuits. Today Boolean algebra is the backbone of computer circuit analysis.

Fundamental Concepts of Boolean Algebra:

Boolean algebra is a logical algebra in which symbols are used to represent logic levels. Any symbol can be used, however, letters of the alphabet is generally used. Since the logic levels are generally associated with the symbols 1 and 0, whatever letters are used as variables that can take the values of 1 or 0. Boolean algebra has only two mathematical operations, addition and multiplication. These operations are associated with the OR gate and the AND gate, respectively.

Logical Addition:

When the + (the logical addition) symbol is placed between two variables, say X and Y, since both X and Y can take only the role 0 and 1, we can define the + Symbol by listing, all possible combinations for X and Y and the resulting value of X + Y.
The possible input and output combinations may arranged as follows:

$$0 + 0 = 0$$
$$0 + 1 = 1$$
$$1 + 0 = 1$$
$$1 + 1 = 1$$

This table represents a standard binary addition, except for the last entry. When both' X and Y represents 1‟s, the value of X + Y is 1. The symbol + therefore does not has the "Normal" meaning, but is a Logical addition symbol. The plus symbol (+) read as "OR", therefore X +Y is read as X or Y.

This concept may be extended to any number of variables for example A + B + C +D = E Even if A, B, C and D all had the values 1, the sum of the values i.e. is 1.

Logical Multiplication:

We can define the "." (logical multiplication) symbol or AND operator by listing all possible combinations for (input) variables X and Y and the resulting (output) value of X. Y as,

$$0 . 0 = 0$$
$$0 . 1 = 0$$
$$1 . 0 = 0$$
$$1 . 1 = 1$$

Note: Three of the basic laws of Boolean algebra are the same as in ordinary algebra; the commutative law, the associative law and the distributive law.\

The commutative law for addition and multiplication of two variables is written as,

$$A + B = B + A$$
$$\text{And } A . B = B . A$$

The associative law for addition and multiplication of three variables is written as,

$$(A + B) + C = A + (B + C)$$
$$\text{And } (A . B) . C = A. (B. C)$$

The distributive law for three variables involves, both addition and multiplication and is written as,

$$A (B+ C) = A B + AC$$

Note that while either '+' and „." s can be used freely. The two cannot be mixed without ambiguity in the absence of further rules.

For example does A . B + C means (A . B) + C or A . (B+ C)? These two form different values for A = O, B = 1 and C = 1, because we have
(A . B) + C = (0.1) + 1 = 1
and A . (B + C) = 0 . (1 + 1) = 0
which are different. The rule which is used is that „." is always performed before '+'. Thus X . Y + Z is (X.Y) + Z.

Basic Duality in Boolean Algebra:

We state the duality theorem without proof. Starting with a Boolean relation, we can derive another Boolean relation by:

1. Changing each OR (+) sign to an AND (.) sign
2. Changing each AND (.) sign to an OR (+) sign.
3. Complementary each 0 and 1

For instance

$$A + 0 = A$$

The dual relation is

$$A . 1 = A$$

Also since A (B + C) = AB + AC by distributive law. Its dual relation is
A + B C = (A +B) (A + C)

Simplification of Boolean functions:

Using the theorems of Boolean Algebra, the algebraic forms of functions can often be simplified, which leads to simpler (and cheaper) implementations.

Boolean Variables

Boolean algebra allows the concise description and manipulation of binary variables; although it by no means restricted to base 2 systems. Variables in Boolean algebra have a unique characteristic; they may assume only one of two possible values.

Value (Bit)
Alternate Names
0 {F, False, No, OFF, LOW}
1 {T, True, Yes, ON, HIGH}