Uniform Circular Motion:

Flat Curve

and

Banked Curve

by

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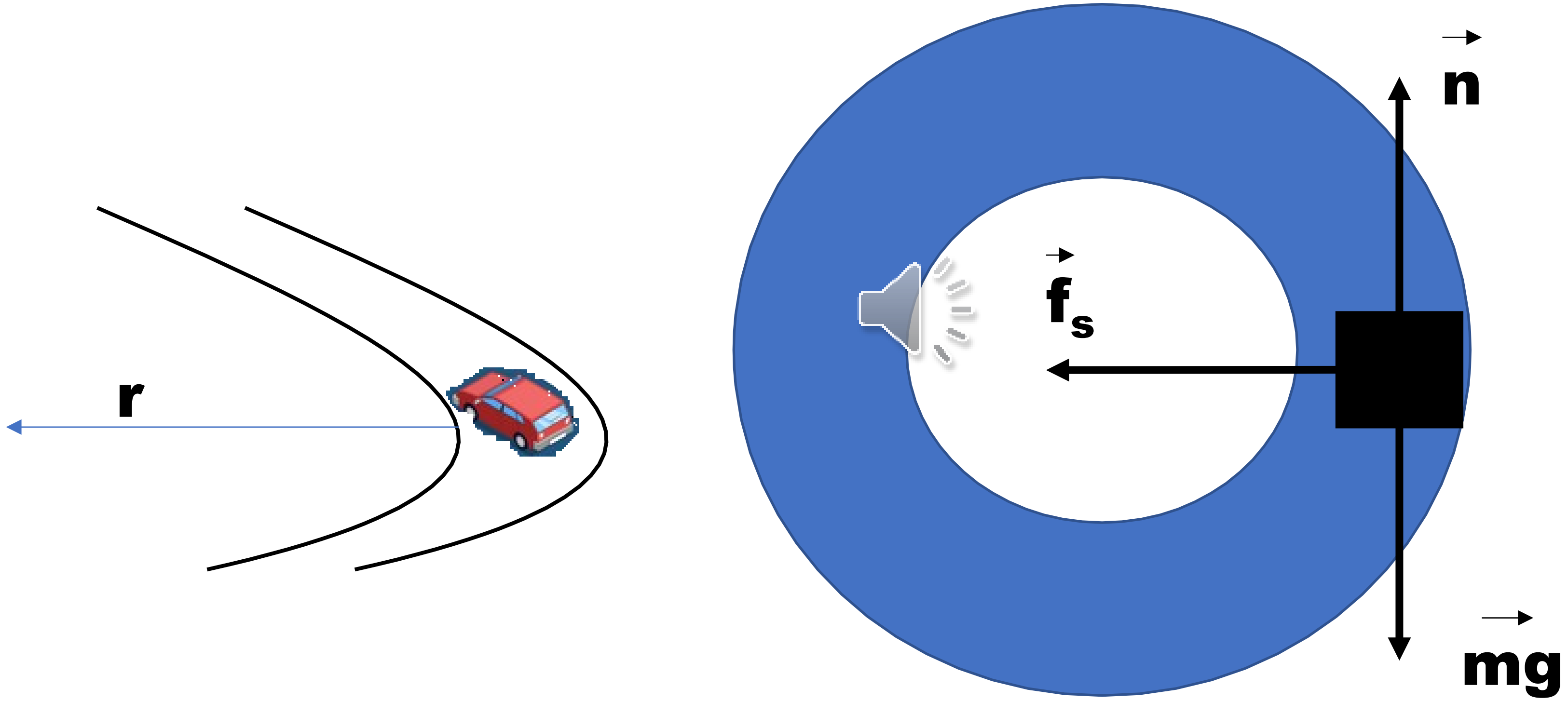
Flat Curve

- A car moving around a flat curve or round about with constant velocity. Find the maximum speed the car can have and still can turn safely.
- First Step
- Imagination and draw an illustration or diagram according to your imagination.
- Fill out with all the possible known and unknown variables
- Then draw a free body diagram.

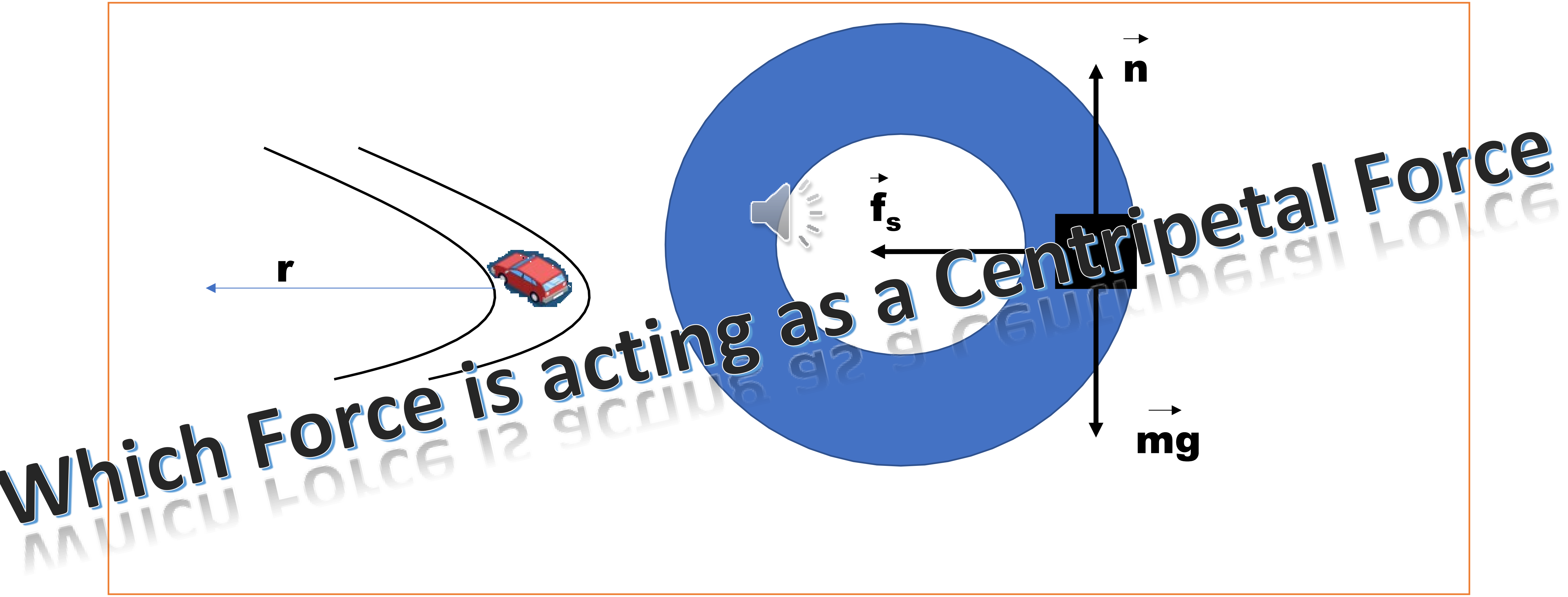
Flat Curve: Free body diagram



Analysis

- First question how many forces?
  - Weight of a car = mg
  - Normal (reaction, third law of motion)= n
  - Force of friction between the tyre and the road = f_s
- Force of friction is trying hard to stick the car on the road
- It is a uniform circular motion means we supposed, or we restrict the driver to move with constant speed.
- Now the million-dollar question? Which force is acting as a centripetal force

Flat Curve: Free body diagram



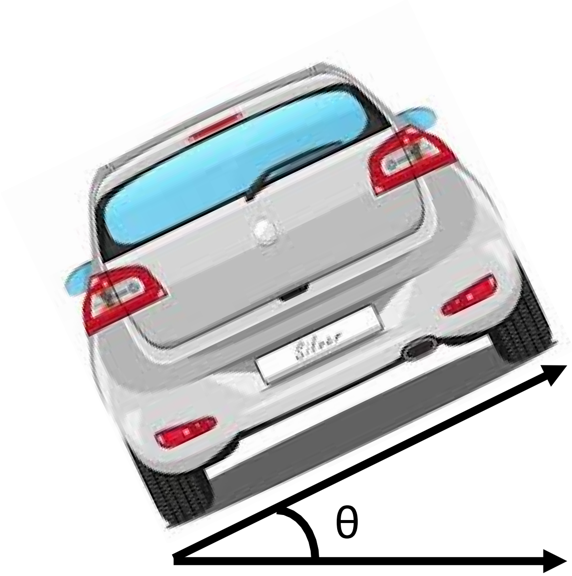
Final expression for the maximum velocity

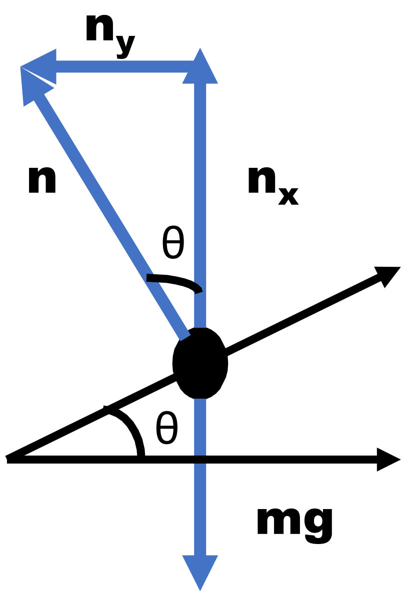
- It's a force of friction
- For y-axis we apply the equilibrium model
- $\sum F_y = 0$
- $n - mg = 0$
- $n = mg$
- $f_s = F_C$
- $\mu_s n = \frac{mv_{max}^2}{r}$
- $v_{max} = \sqrt{\mu_s r g}$

Banked Curve

- An Engineer must design a round about / curved road in such a way that a car does not have to rely on friction between the tyre and the road to prevent itself from skidding. To achieve his goal an Engineer must designed a banked curve.
- First Step
- Imagination and draw an illustration or diagram according to your imagination.
- Fill out with all the possible known and unknown variables
- Then draw a free body diagram.

Banked Curve : Free Body Diagram



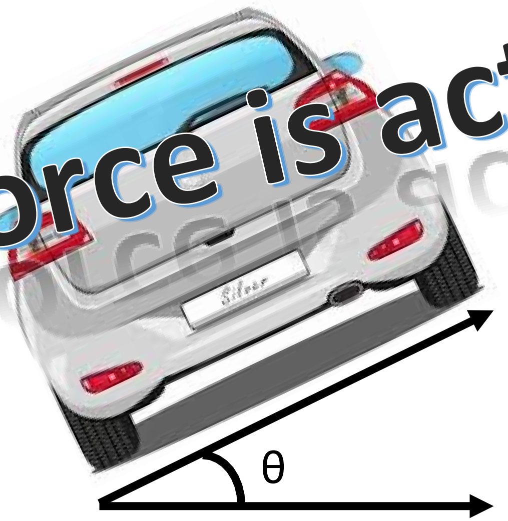


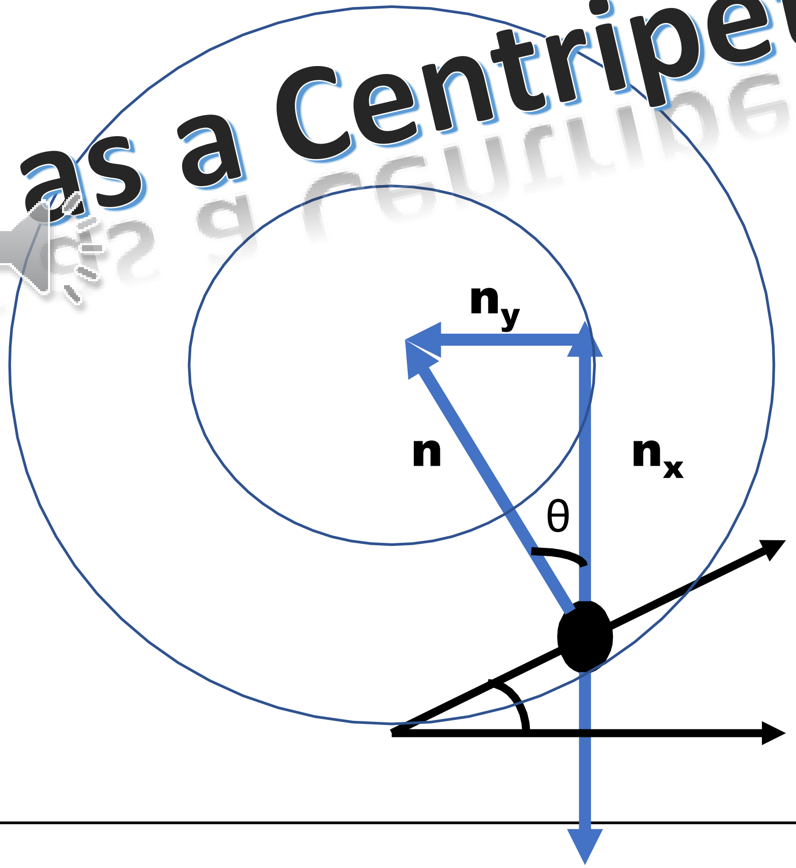
Analysis

- First question how many forces?
  - Weight of a car = ***mg***
  - Normal (reaction, third law of motion)= ***n***
  - Components of Normal = $\boldsymbol{n_x}$ & $\boldsymbol{n_y}$
  - There is no force of friction (Supposed)
- It is a uniform circular motion means we supposed, or we restrict the driver to move with constant speed.
- Now the million-dollar question? Which force is acting as a centripetal force

Banked Curve : Free Body Diagram

Which Force is acting as a Centripetal Force





Final expression for the Banking angle

- It's a vertical component of a normal
- For y-axis we apply the equilibrium model
- $\sum F_y = 0$
- $n cos\theta - mg = 0$
- $n cos\theta = mg$______________________(1)
- $n sin\theta = \frac{mv^2}{r}$ ______________________(2)
- Dividing equation 2 by equation 1
- $tan\theta = \frac{v^2}{rg}$
- $\theta = \tan^{-1}\frac{v^2}{rg}$

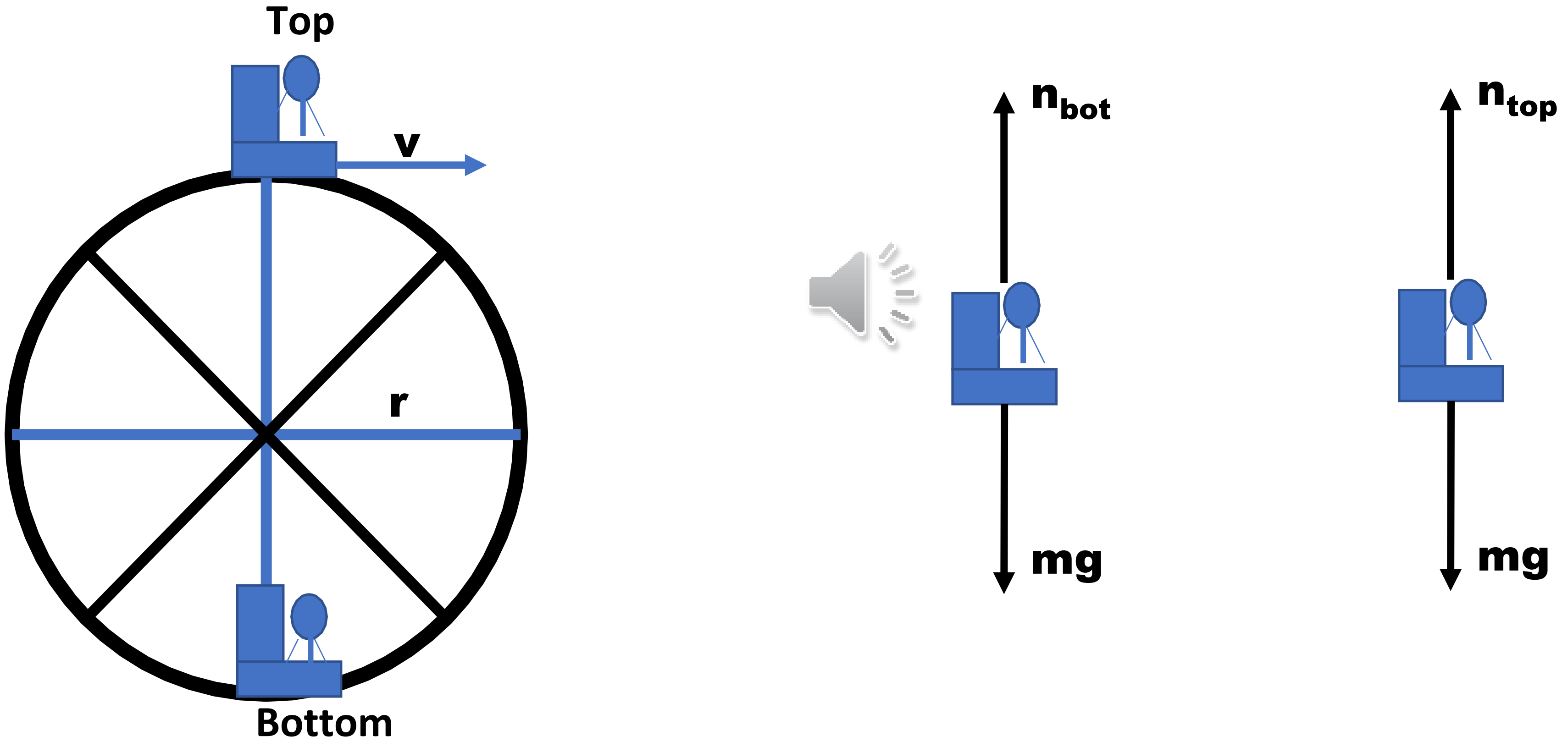
Assignment

- Consider three cars moving with velocities of 35 m/s, 45m/s and 85 m/s moved around a round-about with radius R= 200m. The coefficient of static friction between the road and tyre is 0.85.
- (a)find out which of these cars can turn safely.
- (b) If we must increase the maximum velocity to 5 m/s without increasing a radius of the round-about what we will have to do?

Ferris Wheel

- A child of mass m rides on a Ferris wheel. The child moves in a vertical circle of radius r moving with constant velocity v.
- Determine the force exerted by the seat on the child (a) at the top of the wheel (b) at the bottom of the wheel.
- First Step
- Imagination and draw an illustration or diagram according to your imagination.
- Fill out with all the possible known and unknown variables
- Then draw a free body diagram.

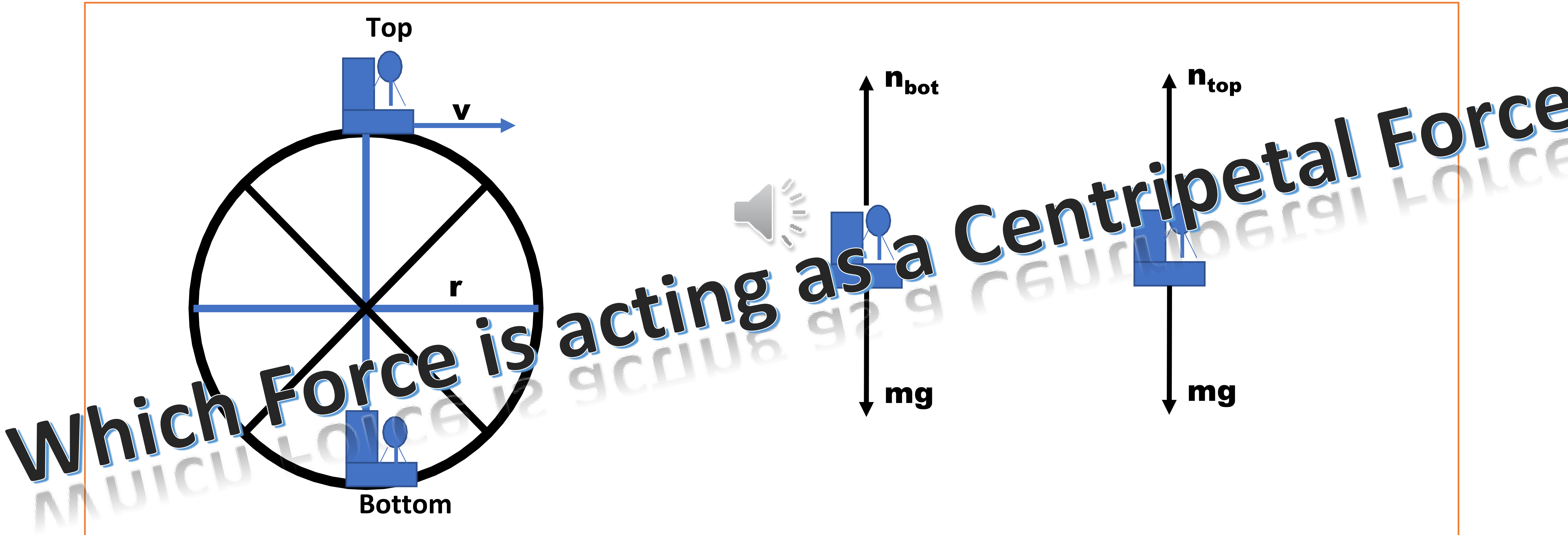
Ferris Wheel: Free Body Diagram



Analysis

- First question how many forces?
  - Weight of a child = $\boldsymbol{mg}$
  - Normal (reaction, third law of motion)= $\boldsymbol{n_{top}}$ **&** $\boldsymbol{n_{bot}}$
- It is a uniform circular motion means we supposed, or we restrict the operator to move Ferris wheel with constant speed.
- Now again the million-dollar question? Which force is acting as a centripetal force.

Ferris Wheel: Free Body Diagram



Analysis

- This problem is different from previous problems. To understand it clearly, we must recall our previous concept that for the continuous circular motion there must be a centripetal force which us acting towards the center of the circle.
- At the top there is a centripetal force, but it is actually the difference of two forces weight and the normal where weight is greater and at the bottom same case but this time normal is greater.

Final expression

- For the force exerted by seat on child at the top of the Ferris wheel
- As mentioned above for the boy at the top of the Ferris wheel we can write
- $\sum F = mg - n_{top} = \frac{mv^2}{r}$
- $n_{top} = mg - \frac{mv^2}{r}$
- $n_{top} = mg\left(1 - \frac{v^2}{rg}\right)$

Final expression…

- For the force exerted by seat on child at the bottom of the Ferris wheel
- As mentioned above for the boy at the bottom of the Ferris wheel we can write
- $\sum F = n_{bot} - mg = \frac{mv^2}{r}$
- $n_{top} = \frac{mv^2}{r} + mg$
- $n_{top} = mg\left(1 + \frac{v^2}{rg}\right)$

Conclusion

- At the top of the Ferris Wheel the force exerted by the seat on the child is less than true weight of the boy i.e. child feels lighter.
- At the bottom of the Ferris wheel the force exerted by the seat on the child is greater than the true weight of the boy i.e. child feels heavier

Assignment

- Consider a conical pendulum with a bob of mass $\boldsymbol{m}$ = 80.0 kg on a string of length $\boldsymbol{L}$ = 10.0 m that makes an angle of $\boldsymbol{\theta} = 5.00^0$ with the vertical. Determine (a) the horizontal and vertical components of the force exerted by the string on the pendulum and (b) the radial acceleration of the bob.

Assignment

- Why is the following situation impossible? The object of mass m=4.00 kg in Figure is attached to a vertical rod by two strings of length ℓ= 2.00 m. The strings are attached to the rod at points a distance d=3.00 m apart. The object rotates in a horizontal circle at a constant speed of v=3.00 m/s, and the strings remain taut. The rod rotates along with the object so that the strings do not wrap onto the rod. What If? Could this situation be possible on another planet?