Motion in two Dimension with Constant Acceleration

by

Sheikh Muhammad Zeeshan Iqbal

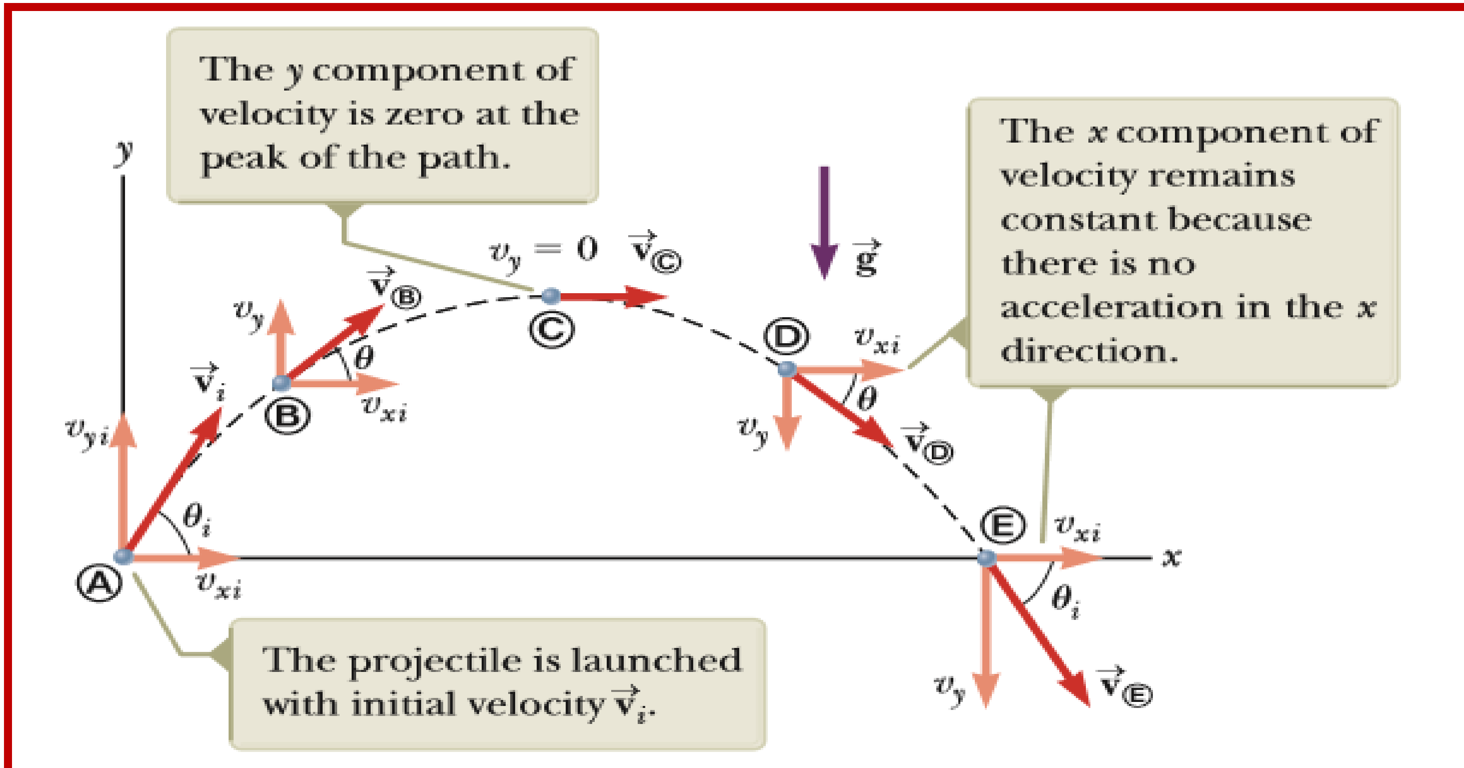
Motion in Two Dimension

- It consists of two independent motion.
- They are perpendicular to each other associated with x and y axes.
- It means any influence in x direction does not affect the motion in y direction, same goes for any influence in y direction.
- The position vector for a particle moving in xy plane.
    - $r = x\hat{i} + y\hat{j}$

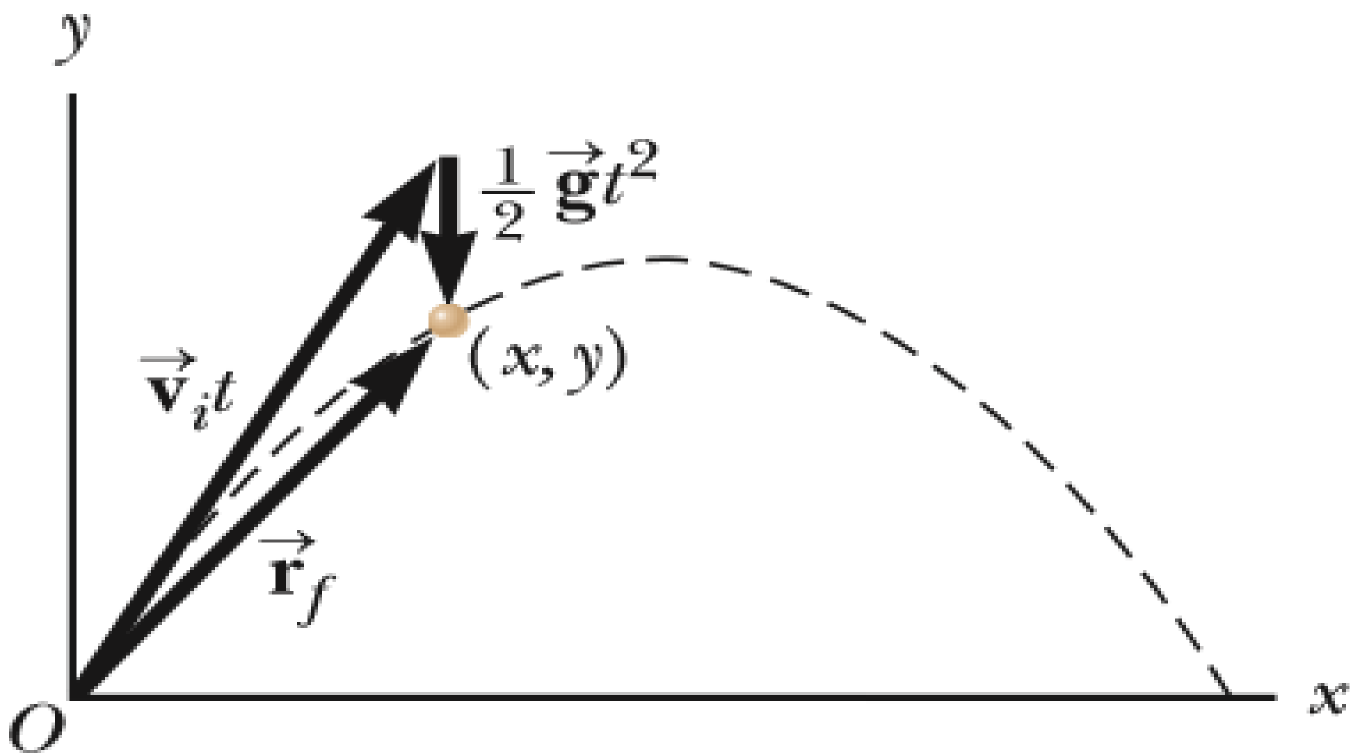
Projectile Motion

- To analyze a projectile motion two assumtion must be made.
  - For the motion in y axis there is a constant acceleration due to gravity directed downward.
  - The effect of air res.istance is negligible

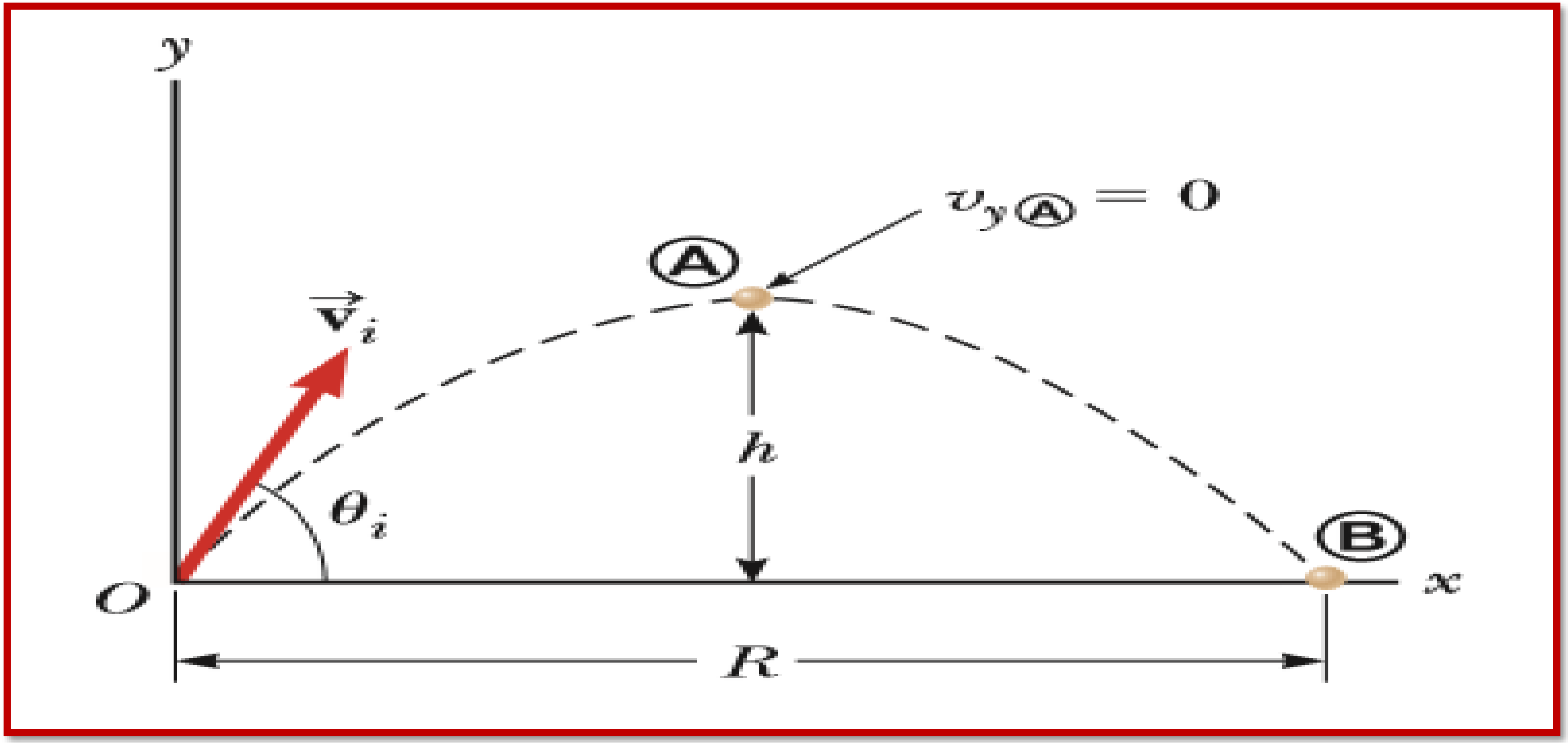
Projectile Motion



Analysis of a Projectile Motion



Analysis of Projectile Motion



Some typical formulae from the past

- From the above figure
- Time to reach maximum height $=\dfrac{v_0 sin\theta}{g}$
- Total Time of flight $= \dfrac{2v_0 sin\theta}{g}$
- Maximum height $= \dfrac{v_0^2 sin^2\theta}{2g}$
- Maximum Range $= \dfrac{v_0^2 sin2\theta}{g}$

Things to Remember

- All these formulae in previous slide must be used when the launching position of the projectile and the landing position od the projectile is at the same height, otherwise we have to get back to the famous three equations of motion we have studied previously.

Problem Solving Strategy for projectile Motion

- Imagine the whole motion and draw the diagram inclusive all the known and unknown variables
- If Initial velocity and launching angle is given first resolved into its components.
- Try to separately solve the horizontal motion using the constant velocity model and vertical motion using the constant acceleration model.
- If it does not work then.

Finally
Equation of Trajectory

- If you stuck by following the above rules, then it might need to call the equation of trajectory

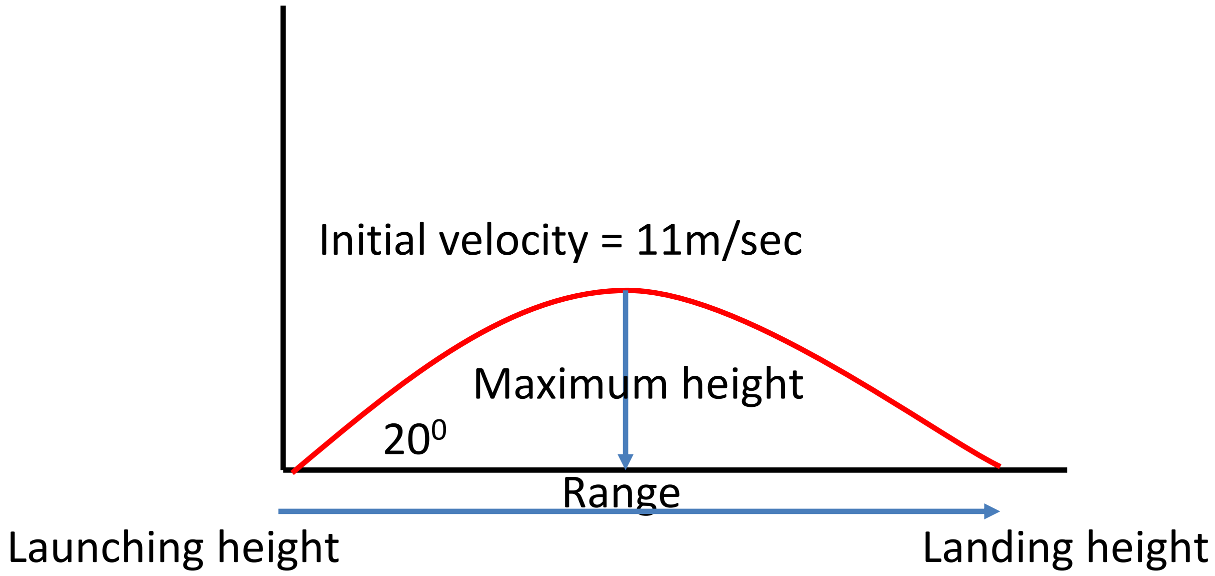
$$y_P = x_P tan\theta - \frac{1}{2} g t^2$$

- Above equation used when you have to consider both horizontal and vertical motion

Some examples when you can use the predefined formulae

- A long jumper leave the ground at an angle of 20^0 above the horizontal and at a speed of 11.0 m/sec.
- How far does he jump in the horizontal direction?
- What maximum height reached?

First the Diagram



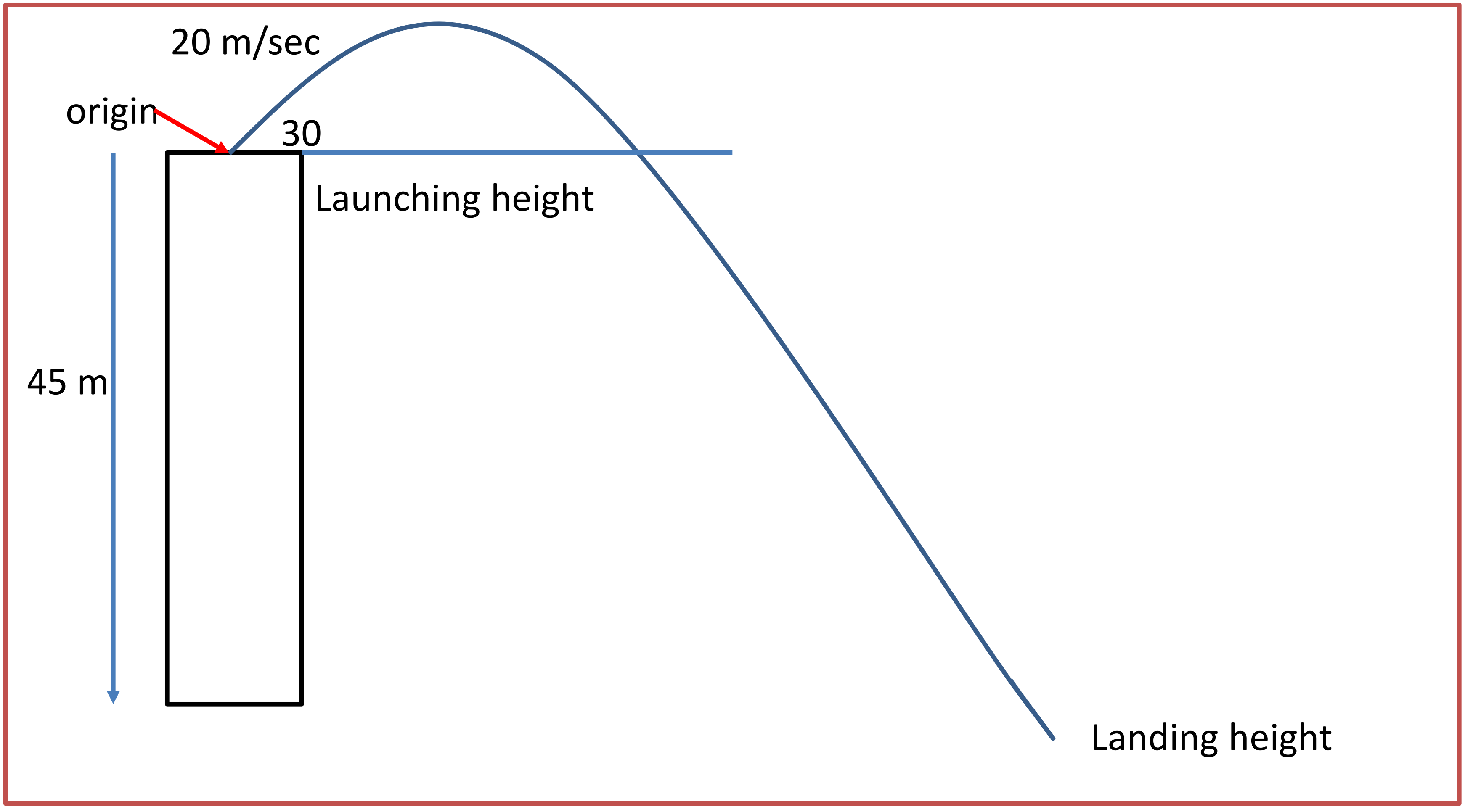
Solution

- 2nd step resolve the velocity into its components but as I mentioned earlier this is a simple problem in which launching, and landing height are same.
- So we can use the the predefined formulae
- Maximum Range $= \frac{v_0^2 sin2\theta}{g} = 7.94$ m
- Maximum height $= \frac{v_0^2 sin^2\theta}{2g} = 0.722$ m

Another Example

- A stone is thrown from the top of the building upward to an angle of 30^0 to the horizontal with an initial speed of 20.0 m/sec. The height from which the stone is thrown is 45 m above the ground.
- How long does it take the stone to reach the ground?
- What is the stone just before it strikes the ground?

First Step “Imagination”



Analysis

- It is clear from the figure that the launching height and landing height is different. So there is thick chance that our predefined formulae does not work there.
- 2nd resolve the velocities into two components.
- $v_x = v_0 cos\theta = 17.3\,\frac{m}{sec}$
- $v_y = \ v_0 sin\theta = 10\,\frac{m}{sec}$

Analysis

- We must decide the origin because we will define every position from there. So if the origin is where the stone was thrown then the height of building will be taken as a depth and used with –ve sign.
- We can divide the whole situation in to two separate and independent parts horizontal and vertical.

Solution…

- Data:
- $v_x = 17.3 \frac{m}{sec}$
- $v_y = \frac{10m}{sec}$
- $y = -45\, m$
- Required:
- Time required to reach the ground=t=?
- Final velocity=? (remember its not vertical not a horizontal it's a resultant velocity)

Solution…

- For the time we can not use the horizontal velocity (constant velocity model) as we do not know the range yet, so we stuck with vertical velocity (constant acceleration model)
- We will use the 2nd equation of motion
- $y = v_{0i}t - \frac{1}{2}gt^2$
- It's a quadratic equation

Solution…

- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- $-45 = 10t \ - \frac{1}{2} 9.8t^2$
- t=4.22 m
- Then 1st equation of motion
- $v_{yf} = v_{yi} \ - gt$
- $v_{yf} = -31.3 \frac{m}{sec}$