

# Assignment#1

1. a die and a coin is tossed together what is the probability that the die shows an even number and the coin shows head?

## ANSWER

The coin is fair, the probabilities of getting a head and a tail are equal to  $1/2$ . The probability of getting an even number on a die is  $3/6=1/2$  because among 6 results there are 3 even numbers.

Finally to calculate the probability of event  $A \cap B$  we have to multiply the calculated probabilities:

$$P(A \cap B) = 1/2 \cdot 1/2 = 1/4$$

3. If one card is drawn at random from a well-shuffled pack of 52 cards, find the chance that the card is:

(i) Spade (ii) Black (iii) Not a diamond (iv) An ace

## ANSWER

(i) There are total 52 cards and there are 4 suits having 13 cards each: spade, diamond, clubs, hearts.

as we want the probability for spade which is 13 in number.

$$13/52 = 1/4 \text{ Probability } \mathbf{ANSWER}$$

(ii) As we should know there are 26 black cards and 26 red cards in a deck so,

$$26/52 = 1/2 \text{ probability } \mathbf{ANSWER}$$

(iii) As we want all cards unit except the diamond so we subtract total card with one unit(diamond),

$$52 - 13 = 39 \text{ cards } \mathbf{ANSWER}$$

so the probability will be,

$$39/52 = 3/4 \text{ probability. } \mathbf{ANSWER}$$

(iv) As there are 4 units and every unit have the same cards so there are 4 Ace in a deck,

$$4/52 = 1/13 \text{ probability } \mathbf{ANSWER}$$

**5. If a permutation of the word "WHITE" is selected at random, Find the probability that the permutation?**

**(a) begins with a consonant? (b) end with a vowel?**

**(c) has the consonant and vowels alternating?**

**ANSWER**

**(a)** "WHITE" have five words and there are two consonants and there will be 5 options for beginning so,

=  $2/5$  probability **ANSWER**

**(b)** As there are two vowel and there will be 5 options for last word because "WHITE" contain 5 words so,

=  $2/5$  probability **ANSWER**

**(c)** Let P be the probability that the permutation has the consonants and vowels alternating. Then,

$$P = (\text{number of permutations with consonant and vowels alternating}) / (\text{total number of permutations})$$

Total number of permutations =  $5! = 120$

Now, for the number of permutations with consonant and vowels alternating:

**There's 3 consonants**

**There's 2 vowels**

That forces us to start the word with consonants since there are more consonants than vowels.

Now let's analyze how many letters can be put for every position in the word.

1st position: 3 possible letters (consonant)

2nd position: 2 possible letters (vowel)

3rd position: 2 possible letters (consonant, we already put one in the 1st position)

4th position: 1 possible letter (vowel, we already put one in the 2nd position)

5th position: 1 possible letter (consonant, we already put one in the 1st and 3rd positions)

Multiplying everything we get that the total number of permutations with consonants and vowels alternating is  $3 \times 2 \times 2 \times 1 \times 1 = 12$ .

That means  $P = 12 / 120 = 1 / 10 = 0.1 = 10\%$  probability

**ANSWER**

**7. A pair of fair dice is rolled, What is the probability of a total of at most 4?**

**ANSWER**

There are only three ways when the total will be 4 (2,2)(1,3)(3,1) so,

$$= 3/36 = 1/12 = 0.08333 \text{ probability } \mathbf{ANSWER}$$

**9. Two numbers are selected from 1,2,3,...10 what is the probability that the sum of the numbers is even ?**

**ANSWER**

As we want the sum to be even number so

2,4,6,8,10,12,14,16,18,20 are the number we can attain so

we will find the propability of every digit seperately then add them to get the propability of the even number,

(2) There are 1 way can get the sum 2 (1,1)

(4) There are 3 way can get the sum 4 (2,2)(1,3)(3,1)

(6)There are 5 way can get the sum 6 (1,5)(2,4)(3,3)(4,2)(5,1)

(8)There are 7 way can get the sum 8

(1,7)(2,6)(3,5)(4,4)(5,3)(6,2)(7,1)

(10)There are 9 way can get the sum 10

(1,9)(2,8)(3,7)(4,6)(5,5)(6,4)(7,3)(8,2)(9,1)

(12)There are 9 way can get the sum

12(2,10)(3,9)(4,8)(5,7)(6,6)(7,5)(8,4)(3,9)(2,10)

(14)There are 7 way can get the sum

14(4,10)(5,9)(6,8)(7,7)(8,6)(5,9)(4,10)

(16)There are 5 way can get the sum 16

(6,10)(7,9)(8,8)(9,7)(6,10)

(18)There are 3 way can get the sum 18(8,10)(9,9)(10,8)

(20)There are 1 way can get the sum 20(10,10)

so we add all the propability as we select 2 set of number so,

$$n^r = 10^2$$

$$1/100 + 3/100 + 5/100 + 7/100 + 9/100 + 9/100 + 7/100 + 5/100 + 3/100 + 1/100 = 50/100 = 1/2 \text{ Propability. } \mathbf{ANSWER}$$

**11. A urn contains x-red balls and y-green balls. If a ball is taken at random from the urn the propability that it is red is 3/7 write down an equation connecting x and y .If there had been 5 more red balls in the urn, the propability would have been 1/2 find x and y?**

**ANSWER**

An urn contains x red balls and y green balls. if a ball is taken at random from the urn, the probability that it is red is 3/7.

$$x/x + y = 3/7$$

$$7x = 3x + 3y$$

$$4x = 3y$$

$$x = 3y/4 \text{ -----> (1)}$$

If there had been 5 more red balls, the probability would have been 1/2.

$$x + 6/x + y + 6 = 1/2$$

$$2x + 12 = x + y + 6$$

$$x = y - 6$$

Substituting  $x = 3y/4$

$$3y/4 = y - 6$$

$$3y = 4y - 24$$

$$y = 24$$

putting value of y in equation 1

$$x = 3(24)/4$$

$$x = 18$$

The values of X and Y are (18,24) **ANSWER**

**13. There are three candidates for the awards of a scholarship. Candidate C is given 3 times the the chance of either A OR B.**

**(i) What is the probabiltiy that C wins?**

**(ii) What is the Probability that A does not win?**

**ANSWER**

(i) three man are seeking award. candidates A and B are given about the same chance of winning, but candidate C is given thrice the chance of either A or B.

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$$P(a) = x$$

$$P(b) = x$$

$$P(c) = 3x$$

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$$x + x + 3x = 1$$

$$5x = 1$$

$$x = 1/5$$

so the chance for C will be,

$$3x = \mathbf{3/5 \text{ probability ANSWER}}$$

(ii) we will subtract total chances from the chances A had to win

$$= 1 - 1/5 = 4/5 \text{ probability ANSWER}$$

**15. If two dice are thrown simultaneously, what is the probability of obtaining a sum of 7 or a sum of 11?**

**ANSWER**

probability of getting sum 7 or 11

the favourable out come

are (1,6)(2,5)(3,4)(4,3)(5,2)(5,6)(6,1)(6,5)

**Required probability:**

$$\mathbf{8/36 = 2/9 \text{ probability ANSWER}}$$

**17. If A, B and C are mutually exclusive events and  $P(A) = 0.2$ ,  $P(B) = 0.3$  and  $P(C) = 0.2$  find (i)  $P(A \cup B \cup C)$  (ii)  $P[A' \cap (B \cup C)]$  (iii)  $P(B \cup C)$**



## **ANSWER**

$$(i) P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= 0.2 + 0.3 + 0.2$$

$$= 0.7 \text{ ANSWER}$$

(ii) Again since the events are mutually exclusive,  $P[A' \cap (B \cup C)]$  is just  $B \cup C$ ; so  $P[(A') \cap (B \cup C)]$  is  $P(B \cup C) = P(B) + P(C) = 0.3 + 0.2 = 0.5$  **ANSWER**

(iii) Once more, since the events are mutually exclusive,  $B \cup C$ ' is just  $C'$ ; the complement of that set is  $C$ . So  $P(B \cup C)' = P(C) = 0.2$  **ANSWER**

**19. A class consists of 35 girls and 45 boys 9 girls and 17 boys of the class take mathematics as their optional subject. What is the probability that a student selected at random from the class is a boy or take mathematics?**

## **ANSWER**

As there are total 35girls + 45boys = 80students in class  
total students taking mathematics are = 17boys + 9girls  
= 28students

the probability of a boy in class is  $P(A) = 45/80 = 9/16$

the probability of a student taking mathematics is  $P(B)$   
 $= 28/80 = 7/20$

and the probability of student that they are in  
mathematic and are boys  $= 17/80$

so we add A and B probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 9/16 + 7/20 - 17/80 = 7/10 \text{ probability. } \mathbf{ANSWER}$$

**21. A bag contains 90 black and 10 white balls. If each ball has an equal chance of being drawn, what is the probability of drawing a black ball also it is given that 50 of the black ball and 3 of the white one are defective. What is the probability of drawing a ball?**

**(i) without defects?**

**(ii) that is black, defective or both?**

**ANSWER**

**(i)** the total balls which are defected are  $= 50\text{black} + 3\text{white} = 53\text{balls}$

and total balls are  $= 90\text{black} + 10\text{white} = 100\text{balls}$

not defected  $= \text{total} - \text{defected}$

$$= 100 - 53$$

$$= 47/100 \text{ ANSWER}$$

(ii) As there are total 90 black balls as we want both so,

$$= 90/100 \text{ ANSWER}$$

**23. The integers 1, 2, 3,...20 are written on slips of paper which are placed in a bowl and thoroughly mixed. A slip is drawn, what is the probability that the number on the slip is either square or divisible by 3?**

### **ANSWER**

To find out that the numbers who are square or divisible by 3 we will write all the digits

Square: 9

Divisible : 3,6,9,12,15,18

So we will count 9 one time because if it is picked it will be both square and divisible.

$$= 6/20 = 3/10 \text{ ANSWER}$$

**25.(a)** Twenty cards have been numbered from 1 to 20. The cards are shuffled and a card is drawn. What is the probability that the number so obtained is

(i) a multiple of 5 or 7?

(ii) a multiple of 3 or 5?

**(b)** A pair of fair die is tossed. Find the probability of getting:

(i) a total of 8, call it  $P(A)$ ?

(ii) Six on one die, call it  $P(B)$ ?

(iii) Compute  $P(A \cup B)$  and  $p(A' \cap B')$ ?

### **ANSWER**

**(a)**

**(i)** For multiple of 5 or 7 we will write down all the digits multiple down,

**(\*)** 5, 7, 10, 14, 15, 20 are the number which are multiple of 5 or 7 so,

No. of favorable outcomes = 6

Total no. of possible outcomes = 20

$$= \frac{6}{20} = \frac{1}{4}$$

**(ii)** we will do the same for multiple of 3 or 5

**(\*)** 3, 5, 6, 9, 10, 12, 15, 18, 20 are the number which are multiple of 3 or 5 so,

No. of favorable outcomes= 9

Total no. of possible outcomes =20

**= 9/20 ANSWER**

**(b)**

**(i)**Number of favourable outcomes =  
(2,6)(3,5)(4,4)(5,3)(6,2) = 5

total no. of possible outcome = 36

$P(A) = \text{Number of favourable outcomes} / \text{total no. of possible outcome} = 5/36 = 0.1388$

**(ii)**to get a SIX on one dice we have 2 possible outcome as a SIX for one dice then for the second one

No. of favorable outcomes= 2

Total no. of possible outcomes =36

$P(B) = \text{Number of favourable outcomes} / \text{total no. of possible outcome} = 2/36 = 1/18 = 0.055$

**(iii)** For  **$P(A \cup B) = P(A) + P(B) - P(A \cap B)$**

$= 0.1388 + 0.055 - 0.007634$  ;  $P(A \cap B) = P(A) * P(B)$

**$P(A \cup B) = 0.186$**

For  **$P(A' \cap B')$**

**$;P(A) + P(A') = 1$**

$$P(A) + P(A') = 1$$

$$P(B) + P(B') = 1$$

$$0.1388 + P(A') = 1$$

$$0.055 + P(B') = 1$$

$$P(A') = 0.8612$$

$$P(B') = 0.945$$

$$P(A' \cap B') = P(A') * P(B')$$

$$P(A' \cap B') = 0.8612 * 0.945$$

$$P(A' \cap B') = \mathbf{0.8138 \text{ ANSWER}}$$

**27.**In a cafeteria students may order any combination of Chips, Sandwiches and Cold drink. The probability that a student chooses a cold drink is 0.45, sandwiches and chips 0.19, cold drink and sandwiches 0.15, cold drink and chips 0.25, cold drink or sandwiches 0.6, cold drink or chips 0.84, cold drink or chips or sandwiches 0.9. Find the probability that a student chooses all the three

### **ANSWER**

In my solution, I will use the letters D, S and C to coding drinks, sandwiches and chips, respectively.

So, we have an Universal set U of all possible combinations, and  $P(U) = 1$ .

We also have given

$$P(D) = 0.45$$

$$P(S \cap C) = 0.19$$

$$P(D \cap S) = 0.15$$

$$P(D \cap C) = 0.25$$

$$P(D \cup S) = 0.6$$

$$P(D \cup C) = 0.84$$

$$P(D \cup C \cup S) = 0.9.$$

There is a formula in elementary probability theory

$$P(D \cup C \cup S) = P(D) + P(C) + P(S) - P(D \cap S) - P(D \cap C) - P(C \cap S) + P(D \cap C \cap S) \quad (1)$$

which is valid for any subsets (sets of events) of the Universal set.

In this formula, many terms are given, but not all. The terms  $P(S)$  and  $P(D)$  are not given.

If I knew these two terms, I would be in position to calculate the value of  $P(D \cap C \cap S)$  from the formula (1) MOMENTARILY.

So, my goal now is to find these terms  $P(S)$  and  $P(D)$ .

To find  $P(S)$ , I will use another basic formula of the elementary probability theory  $P(D \cup S) = P(D) + P(S) - P(D \cap S)$ .

By substituting all given values, I get  $0.6 = 0.45 + P(S) - 0.15$ ; it implies  $P(S) = 0.6 - 0.45 + 0.15 = 0.3$ .

To find  $P(C)$ , I will use similar basic formula  $P(D \cup C) = P(D) + P(C) - P(D \cap C)$ .

By substituting all given values, I get  $0.84 = 0.45 + P(C) - 0.25$ ; it implies  $P(C) = 0.84 - 0.45 + 0.25 = 0.64$ .

Now I substitute all the given and found values into the formula (1)

$$0.9 = 0.45 + 0.3 + 0.64 - 0.19 - 0.15 - 0.2 + P(D \cap C \cap S),$$

which gives me the ANSWER

$$P(D \cap C \cap S) = 0.9 - 0.45 - 0.3 - 0.64 + 0.19 + 0.15 + 0.2 = 0.1. \text{ ANSWER}$$

**29.(a) Assume three events A, B and C are such that  $P(A)=0.5, P(B)=0.6,$**

**$P(C)=0.4, P(A \cap B)=0.3, P(A \cap C)=0.1, P(B \cap C)=0.2$**   
**and  $P(A \cap B \cap C)=0.05$ .**

**Find**

**(i)  $P(A \cap B \cap C')$  (ii)  $P(A \cup B \cup C)$  (iii)  $p(A' \cap B \cap C')$**

**(b) In a survey of 1000 persons it was found that (1) 600 drink, (2) 720 smoke, (3) 560 chew, (4) 380 drink and smoke, (5) 270 drink and chew (6) 350 smoke and chew, (7) 80 drink, smoke and chew. What is the probability that a person is chosen at random from the group**

**(i) do not drink or smoke**

**(ii) drink, smoke, but do not chew**



## **ANSWER**

**(a)**

**(i)** To calculate  $P(A \cap B \cap C')$ , notice that  $(A \cap B \cap C')$  are those elements (events) of  $(A \cap B)$  that do not belong to  $C$ ;

in other words,  $(A \cap B \cap C') = (A \cap B) \setminus (A \cap B \cap C)$ ,  $\rightarrow (1)$

where the sign " $\setminus$ " denotes subtraction of sets (of subsets).

Then equality (1) implies that

$P(A \cap B \cap C') = P(A \cap B) - P(A \cap B \cap C) =$  (substitute the given data)  $= 0.3 - 0.05 = 0.25$ . **ANSWER**

**(ii)** Use the formula

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \rightarrow (2)$

which is valid for any subsets  $A$ ,  $B$  and  $C$  of the universal set  $U$ .

Now, when the formula (2) is proved, simply substitute the given input data in it. You will get

$P(A \cup B \cup C) = 0.5 + 0.6 + 0.4 - 0.3 - 0.1 - 0.2 + 0.05 = 0.95$  **ANSWER**

(iii) To calculate  $P(A' \cap B \cap C')$ , notice that  $(A' \cap B \cap C')$  are those elements (events) of B that do belong NEITHER A NOR C;

in other words,  $(A' \cap B \cap C') = (B \setminus (A \cap B) \setminus (C \cap B)) \rightarrow (3)$

Then equality (2) implies that

$$P(A' \cap B \cap C') = P(B \setminus (A \cap B) \setminus (C \cap B)) = (P(B) - P(A \cap B)) + (P(B) - P(C \cap B)) + P(A \cap B \cap C) \rightarrow (4)$$

Notice that in the right side of the formula (4), I subtracted the probability  $P(A \cap B)$  from  $P(B)$ , then subtracted  $P(C \cap B)$  from  $P(B)$ ,

so I subtracted the probability  $P(A \cap C)$  of the intersection  $(A \cap C)$  twice; therefore, I should add  $P(A \cap B \cap C)$  to restore the equilibrium.

Substitute the given data into the formula (4), you get the ANSWER

$$\begin{aligned} P(A' \cap B \cap C') &= (0.6 - 0.3 - 0.2) + 0.05 \\ &= 0.15 \text{ ANSWER} \end{aligned}$$

(b)

$$D = 600$$

$$S = 720$$

$$C = 560$$

$$DS = 380$$

$$DC = 270$$

$$SC = 350$$

$$DSC = 80$$

Finding out the value of a person who either smoke ,drink and chew

$$P(DUSUC) = P(D) + P(S) + P(C) - P(DS) - P(DC) - P(SC) + P(DSC)$$

$$= 600 + 720 + 560 - 380 - 270 - 350 + 80$$

$$= 960$$

A person who neither smoke,drink or chew  $1000 - 960 = 40$

**(i)**and a person who dont smoke or drink is

$$P(DUSUC) - P(D) - P(S) + P(DS) + 40$$

$$= 960 - 600 - 720 + 380 + 40$$

$$= 60$$

$$= 60/1000$$

$$= 3/50 \text{ **ANSWER**}$$

**(ii)**probability of a person who smoke drink but do not chew

$$P(DS) - P(DSC)$$

$$= 380 - 80$$

$$=340$$

$$=340/1000$$

$$=17/50 \text{ ANSWER}$$

**31. Mr. A can solve 75% of the problems in a book. Mr. B can solve 70% and Mr. C can solve 60 problems of the same book. If a problem is selected at random find the chance that it is solved?**

### **Answer**

Let A be the event solving 75% of the problems

$$P(A) = 75/100 = 3/4$$

Let B be the event solving 70% of the problems

$$P(B) = 70/100 = 7/10$$

A and B are mutually independent events;

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$3/4 + 7/10 - (3/4)(7/10) = 37/40 \text{ Answers}$$

**33. One bag contains 4 white and 2 black beads and another contains 3 of each color a bead is drawn from each bag. What is the probability**

that one is white and one is black? And what is the probability that both are of same color?

**Answer**

$P(\text{one white ball and one black ball}) = P(\text{black ball from bag 1 and white$

ball from bag 2 or white ball from bag 1 and black ball from bag 2)

$$= \frac{2}{6} * \frac{3}{8} + \frac{4}{6} * \frac{5}{8}$$

$$= \frac{26}{48} = \frac{13}{24} \text{ Answers}$$

**35. A bag contains 2 white, 5 red and 3 blue balls. One ball is drawn, its color noted and put back in the bag. Then a second ball is drawn Find the probability that**

**i) The second ball drawn is either white or blue?**

**ii) Two balls are of same color.**

**Answer**

i) The first ball is selected and then put in the bag. This means that

the second selection has the same condition as the first selection.

Nothing has changed. So the two selections are independent.

We have  $2 + 5 + 3 = 10$  balls total ,of whic h2+ 3 are white or blue.

The probability of selecting either a white or blue ball is  $5/10 = 1/2$

$$(ii) P(\text{white}) = 2_{\text{white}}/10_{\text{total}} = 1/5$$

$$A = P(2_{\text{white}})$$

$$A = (1/5) * (1/5)$$

$$A = 1/25 \text{ **Answers**}$$

Note that  $P(\text{white})$  stays the same due to the events being independent(only because we put the first ball black)

$$P(\text{red}) = (5_{\text{red}})/(10_{\text{total}}) = 1/2$$

$$B = P(2 \text{ red})$$

$$B = P(\text{red}) \times P(\text{red})$$

$$B = (1/2) \times (1/2)$$

$$A + B + C = 1/25 + 1/4 + 9/100$$

$$A + B + C = 4/1000 + 25/100 + 9/100$$

$$A + B + C = 38/100$$

$$A + B + C = (19 * 2)/(50 * 2)$$

$$A + B + C = 19/50$$

The probability of picking two balls of same color is  $19/50$ .

**Answers**

**37. The probability that a candidate passes a certain professional examination is 0.7. Find the probability that a candidate would pass the examination before the fourth attempt.**

**Answer**

$$\text{Probability of passing} = P(\text{pass}) = 0.7$$

$$\begin{aligned}\text{Probability of not passing(fail)} &= P(\text{fail}) = 1 - 0.7 \\ &= P(\text{fail}) = 0.3\end{aligned}$$

He failed two times and before the fourth means he will pass the test in the third attempt.

$$P(\text{passing in third attempt}) = 0.3 \times 0.3 \times 0.7 = 0.063$$

**Answers**

**39. A can hit a target 3 times in 5 shots. B can 2 times in 5 shots and C can 3 times in 4 shots, they fire a volley. What is the probability of their being hitting two shots?**

**Answer**

$$P(A) = 3/5 \quad P(A') = 2/5 \quad (\text{probability of } A \text{ hitting and not hitting})$$

$$P(B) = 2/5 \quad P(B') = 3/5$$

$$P(C) = 3/4 \quad P(C') = 1/4$$

*Exactly two of them it*

$$= P(A \& B \text{ hits} \& C \text{ misses}) + P(A \& C \text{ hits} \& B \text{ misses})$$

$$+ P(B \& C \text{ hits} \& A \text{ misses})$$

$$= [P(A) \times P(B) \times P(C')] + [P(A) \times P(C) \times P(B')] + [P(B) \times P(C) \times P(A')]$$

$$= 6/100 + 27/100 + 12/100$$

$$= 45/100 \text{ Answers}$$

**41. Three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. Find the chance that the three selected comprise 1 girl and 2 boys.**

**Answer**

Group X: 3 girls and 1 boy

Group Y: 2 girls and 2 boys

Group Z: 1 girl and 3 boys

The combination of 1 girl and 2 boys can be selected in;

i)  $X - \text{girl}, Y - \text{boy}, Z - \text{boy}: 3/4 * 2/4 * 3/4 = 9/32$

ii)  $X - \text{boy}, Y - \text{girl}, Z - \text{boy}: 1/4 * 2/4 * 1/4 = 1/32$

Hence Required probability



$$= 9/32 + 3/32 + 2/64 = 13/32 \text{ Answers}$$

**43. Two cards are drawn in succession from a deck of cards without replacement. What is the probability that all cards are greater than 2 and less than 10?**

**Answer**

The cards would be 3, 4, 5, 6, 7, 8, 9, 10

7 cards in 4 suits;

The probability for 2nd draw =  $28/52 = 7/13$

Since the card is not replaced,

The probability for 2nd draw =  $27/51 = 9/17$

The probability of both is;

$$P(\text{BOTH} > 2 \ \& \ < 10) = 7/13 * 9/17 = 63/221 \approx 0.285$$

**Answers**

**45. Three cards are drawn in succession from a deck of cards without replacement. What is the probability that all cards are greater than 2 and less than 10?**

**Answer**

The cards would be 3, 4, 5, 6, 7, 8, 9, 10

7 cards in 4 suits;

The probability for 1st draw =  $28/52 = 7/13$

since the cards are not replaced

The probability for 2nd draw =  $27/51 = 9/17$

The probability for 3rd draw =  $26/50 = 13/25$

The probability of all is

$$\begin{aligned} P(\text{all} > 2 \text{ and} < 10) &= 7/13 * 9/17 * 13/25 \\ &= 63/425 \approx 0.148 \text{ Answers} \end{aligned}$$

**47. A bag contains three red, four white and three black, identical balls. Three balls are drawn at random without replacement from the bag. What is the probability that**

**i) All the balls are of the same colors?**

**ii) All the balls are of different colors?**

**iii) At least a ball is white?**

**Answer**

i) As the balls are drawn without replacement,

probability of drawing all red balls =  $3/10 * 2/9 * 1/8$

$$= 1/120$$

probability of drawing all white balls =  $4/10 * 3/9 * 2/8 = 1/30$

probability of drawing all black balls =  $\frac{3}{10} * \frac{2}{9} * \frac{1}{8} = \frac{1}{120}$

Hence the required probability is;

$$= \frac{1}{20} + \frac{1}{20} + \frac{1}{120} = \frac{1}{20}$$

ii) The probability for all of different colors would be

$$3! * (\frac{3}{10} * \frac{4}{9} * \frac{3}{8}) = \frac{3}{10} \text{ **Answers**}$$

(iii) first we will find out the probability of non white

$$= \frac{7}{10} * \frac{6}{9} * \frac{5}{8}$$

$$= 0.29$$

$$P(\text{Whiteball}) = 1 - P(\text{Nonwhiteball})$$

$$= 1 - 0.29$$

$$= 0.71 \text{ **Answer**}$$

**49. Find  $P(A \text{ and } B)$  if;**

**i)  $P(A) = \frac{1}{2}$   $P(B) = \frac{2}{3}$   $P(A \text{ or } B) = \frac{3}{4}$**

**ii)  $P(A) = \frac{1}{2}$   $P(B) = \frac{2}{3}$   $P(A|B) = \frac{2}{5}$**

**(b) The probability that an automobile being filled with gasoline will also need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.18. If the oil had to be changed what is the probability that a new oil filter is needed?**

## Answer

$$(i) P(A \& B) = P(A) \cdot P(B) = 1/2 * 2/3 = 1/3$$

$$(ii) \therefore P(A|B) = P(A \& B) / P(B)$$

$$= P(A \& B) = P(A|B) \cdot P(B)$$

$$= P(A \& B) = 2/5 * 2/3 = 4/15 \text{ Answers}$$

**(b)** From the given information we have

$P(\text{oil changed}) = 0.25$ ,  $P(\text{need new filter}) = 0.40$ ,  $P(\text{oil changed and need new filter}) = 0.18$

(a)  $P(\text{need new filter} \mid \text{oil changed}) = P(\text{oil changed and need new filter}) / P(\text{oil changed}) = 0.18 / 0.25$

$= 0.72$  **ANSWER**

**51. In a certain town 40% of the people have brown hairs, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random from this town**

**(i) If he has brown hair, what is the probability that he has also brown eyes?**

**(ii) If he has brown eyes, what is the probability that he does not have brown hairs?**

**(iii) What is the probability that he has neither brown hairs nor brown eyes?**

## Answer

i) Let BE = Brown eyes, BH = Brown hair.

According to the question,

$$\therefore P(BH) = 40/100 = 0.4$$

$$P(BE) = 25/100 = 0.25$$

$$P(BH \& BE) = 15/100 = 0.15$$

$$P(BHBE) = P(BH \& BE)P(BE) = 0.15/0.4 = 15/40 = 3/8$$

ii) We want to know  $P(BH/BE)$

$$P(BH/BE) = P(BH \& BE)/P(BE) = 0.15/0.4 = 3/8$$

iii) Since we are given with;

Percentage of people having brown hair = 40%

Percentage of people having brown eyes = 25%

Percentage of people having brown hair and brown eyes = 15%

$$= P(BH) + P(BE) - P(BH \cap BE)$$

$$= 40 + 25 - 15$$

$$= 50\%$$

So the percentage of people who has neither brown hair nor brown eyes

is given by;

$$100\% - 50\% = 50\% \text{ **Answers**}$$

**53.** In a high school class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history and neither mathematics nor psychology, 10 studied all three subjects and 8 did not take any of the three. If a student is selected at random, find the probability that

(i) he takes history and psychology but not mathematics?

(ii) a person enrolled in history takes all three subjects?

(iii) he takes mathematics only?

Let M denotes that event that student studies Mathematic

Let P denotes that event that student studies Psychology

Let H denotes that event that student studies History

There are  $n = 100$  students in a class

It is given that:

42 student studied mathematics,i.e

$$n(M) = 42$$

68 student studied psychology,i.e

$$n(P) = 68$$

54 student studied History,i.e

$$n(H) = 54$$

22 student studied both Maths and history,i.e

$$n(M \cap H) = 22$$

25 student studied both Maths and psychology,i.e

$$n(M \cap P) = 25$$

7 student studied history but neither mathematics nor psychology

$$n(H \cap M' \cap P') = 7$$

10 studied all three subject i.e;

$$n(M \cap P \cap H) = 10$$

8 didnot take any of these three i.e

$$n(M' \cap P' \cap H') = 8$$

(i) We need to find the probability that a person which doesnot take maths but takes both history and psychology

$$P(MnH/P')$$

According to definition of the conditional probability we have,

$$P(MnH/M') = P(M'nHnP)/P(M') \text{ ----(4)}$$

We have:

$$n(M'nHnP) = n(PnH) - n(MnHnP) = 25 - 10 = 15$$

Therefore,

$$P(M'nHnP) = n(M'nHnP)/n = 15/100 = 0.15$$

Also, we have

$$n(M') = n - n(M) = 100 - 42 = 58$$

Therefore,

$$P(M') = n(P')/n = 58/100 = 0.58 \text{ --- (6)}$$

Using equation (4),(5) and(6) we finally get,:

$$P(HnP/M') = P(MnPnH)/P(M) = 0.15/0.58$$

$$= \mathbf{0.2586 \text{ ANSWER}}$$

(ii) We need to find the the probability that a person enrolled in HISTORY takes all three subnjects i.e we need to find

$$P(MnP/H)$$



According to definition of the conditional probability, we have

$$P(MnP/H) = P(MnHnP)/P(H) \text{ -----(1)}$$

Lets now find  $P(MnHnP)$  and  $P(H)$  We have:

$$P(H) = n(H)/n = 54/100 = 0.54 \text{ ----- (2)}$$

and

$$P(MnHnP) = n(MnHnP)/n = 10/100 = 0.1 \text{ ---(3)}$$

Using equation

$$\begin{aligned} P(MnHnP) &= P(MnHnP)/P(H) = 0.1/0.54 \\ &= 0.185 \text{ **ANSWER**} \end{aligned}$$

So, the probability that a person enrolled in history take all three subjects is 0.185

(iii) 25 students are those who take psychology and mahs and 10 are those who are maths and history students

$$= 15 - 10 = 5 \text{ **ANSWER**}$$

**55. In a survey of a group of people the following results are obtained:**

40% own a tape recorder, but not a color T.V.

10% own a color T.V. but not a Tape Recorder

20% own neither color T.V. nor a Tape Recorder

30% own both color T.V. and a Tape Recorder

(i) What is the percentage of people who own either a color T.V. or a Tape Recorder or both?

(ii) What is the percentage of people who own a Tape Recorder among the color T.V. owners?

## **ANSWER**

(i)

$$n(\text{TV}) = 40\%$$

$$n(\text{tape}) = 10\%$$

$$n(\text{TV} \cup \text{Tape})$$

$= 40\% + 10\% + \text{no. of person who own two of the items - who owns none of these}$

$$= 40\% + 10\% + 30\% - 20\% \text{ **ANSWER**}$$

$$= 80\% \text{ owns any of these}$$

(ii) 40% have tv out of which 30% T.R for 100% multiply both by 0.25

$$40\% * 0.25 * 30\% * 0.25 = 75\% \text{ **ANSWER**}$$

**57.**One bag contains three balls of red color and four balls of black color and second bag contains two balls of red color and three of black color. If one bag is chosen at random and a ball is drawn, what is the probability that it is red?

**ANSWER**

7 balls in bag 1, 4 of which are red ( $\frac{4}{7}$ )

5 balls in bag 2, 2 of which are red ( $\frac{2}{5}$ )

Add these (rather than multiply) because one doesn't depend on the other. So

$$(\frac{4}{7}) + (\frac{2}{5}) = \frac{34}{35}.$$

Multiply that result by  $\frac{1}{2}$  because there's a 50% chance of which bag is chosen.

$$\frac{34}{35} * \frac{1}{2} = \frac{17}{35} \text{ **ANSWER**}$$

**59.**A bag contains 6 red and 4 black balls. Another contains 3 red and 8 black balls. A ball is drawn from the first bag and is placed in the second. A ball is then drawn from the second bag. What is the probability that it is red?

**ANSWER**

so in order to find the probability of red ball we will first find the probability of red or black drawn from Bag1

### **Case1:**

First case is that a red ball is placed in Bag2 so,

= 6/10 chances red ball goes to Bag2

then there will be 4Red and 8Black balls in Bag2

4/12 chances of red ball in Bag2

so the probability of red ball from Bag2 will be,

$$(6/10) * (4/12)$$

$$= 1/5 \text{ **ANSWER**}$$

### **Case2:**

Second case is that a black ball is placed in Bag2 so,

= 4/10 chances black ball goes to Bag2

then there will be 3Red and 9Black balls in Bag2

3/12 chances of red ball in Bag2

so the probability of red ball from Bag2 will be,

$$(4/10) * (3/12)$$

$$\text{Total probability} = 6/10 * 4/12 + 4/10 * 3/12$$

$$= 1/5 + 1/10$$

$$= 3/10 \text{ **ANSWER**}$$