The problem is more difficult in three dimensions. Consider the nonlinear system

$$2x_1 - 3x_2 + x_3 - 4 = 0, \quad 2x_1 + x_2 - x_3 + 4 = 0, \quad x_1^2 + x_2^2 + x_3^2 - 4 = 0.$$

Define three equations using the Maple commands

$eq1 := 2x1-3x2+x3-4 = 0; eq2 := 2x1+x2-x3+4 = 0; eq3 := x1^2+x2^2+x3^2-4 = 0;$

The third equation describes a sphere of radius 2 and center $(0, 0, 0)$, so $x1$, $x2$, and $x3$ are in $[-2, 2]$. The Maple commands to obtain the graph in this case are

$with(plots)$: $implicitplot3d(\{eq1, eq2, eq3\}, x1 = -2..2, x2 = -2..2, x3 = -2..2);$

Various three-dimensional plotting options are available in Maple for isolating a solution to the nonlinear system. For example, we can rotate the graph to better view the sections of the surfaces. Then we can zoom into regions where the intersections lie and alter the display form of the axes for a more accurate view of the intersection's coordinates. For this problem, a reasonable initial approximation is $(x_1, x_2, x_3)^t = (-0.5, -1.5, 1.5)^t$.

EXERCISE SET 10.2

1. Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(2)}$ for each of the following nonlinear systems.

a.
$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 + 8 = 0,$$
$$\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 = 0.$$

b.
$$\sin(4\pi x_1x_2) - 2x_2 - x_1 = 0,$$
$$\left(\frac{4\pi - 1}{4\pi}\right)(e^{2x_1} - e) + 4ex_2^2 - 2ex_1 = 0.$$

c.
$$x_1(1 - x_1) + 4x_2 = 12,$$
$$(x_1 - 2)^2 + (2x_2 - 3)^2 = 25.$$

d.
$$5x_1^2 - x_2^2 = 0,$$
$$x_2 - 0.25(\sin x_1 + \cos x_2) = 0.$$

2. Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(2)}$ for each of the following nonlinear systems.

a.
$$3x_1 - \cos(x_2x_3) - \frac{1}{2} = 0,$$
$$4x_1^2 - 625x_2^2 + 2x_2 - 1 = 0,$$
$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0.$$

b.
$$x_1^2 + x_2 - 37 = 0,$$
$$x_1 - x_2^2 - 5 = 0,$$
$$x_1 + x_2 + x_3 - 3 = 0.$$

c.
$$15x_1 + x_2^2 - 4x_3 = 13,$$
$$x_1^2 + 10x_2 - x_3 = 11,$$
$$x_2^3 - 25x_3 = -22.$$

d.
$$10x_1 - 2x_2^2 + x_2 - 2x_3 - 5 = 0,$$
$$8x_2^2 + 4x_3^2 - 9 = 0,$$
$$8x_2x_3 + 4 = 0.$$

3. Use the graphing facilities of Maple to approximate solutions to the following nonlinear systems.

a.
$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 + 8 = 0,$$
$$\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 = 0.$$

b.
$$\sin(4\pi x_1x_2) - 2x_2 - x_1 = 0,$$
$$\left(\frac{4\pi - 1}{4\pi}\right)(e^{2x_1} - e) + 4ex_2^2 - 2ex_1 = 0.$$

c.
$$x_1(1 - x_1) + 4x_2 = 12,$$
$$(x_1 - 2)^2 + (2x_2 - 3)^2 = 25.$$

d.
$$5x_1^2 - x_2^2 = 0,$$
$$x_2 - 0.25(\sin x_1 + \cos x_2) = 0.$$

4. Use the graphing facilities of Maple to approximate solutions to the following nonlinear systems within the given limits.