Lab Assignment-7

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

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1. **Newton's method for system**: Write a Matlab/C program to find a root of the non-linear system of equations $F(\mathbf{x}) = 0$ for given initial approximation $\mathbf{x}_0$, where F is a given differentiable function.
   Hint: (You may use the following algorithm)
   INPUT: Number of equations n (generally it will be given apriori), Initial approximation x_0, tolerance TOL; maximum number of iterations N.
   OUTPUT: Approximate root $\mathbf{x}$ or maximum number of iterations exceeded.
   Step-1: Set $k = 1$.
   Step-2: While $k \leq N$ do Steps 3 to 7.

   Step-3: Calculate $F(\mathbf{x})$ and the Jacobian matrix $J_F(\mathbf{x})$ where $(F(\mathbf{x}))_i = f_i(\mathbf{x})$ and $(J_F(\mathbf{x}))_{ij} = \frac{\partial f_i(\mathbf{x})}{\partial x_j}$.

   Step-4 Solve the linear system $J(\mathbf{x})\mathbf{y} = F(\mathbf{x})$.

   Step-5 Set $\mathbf{x} = \mathbf{x} + \mathbf{y}$

   Step-6: If $\|\mathbf{y}\| \leq$ TOL and $\|F(\mathbf{x})\| \leq$ TOL, OUTPUT ($\mathbf{x}$). STOP.

   Step-7: Set $k = k + 1$.

   Step-8: OUTPUT ('Maximum number of iteration reached. ') STOP.
   Test your coding for the systems given in a separate attached page. Print your result in a table for at least 10 iterations .

2. **Newton divided difference formula**: Write a Matlab/C program to obtain the divided-difference coefficients of the interpolating polynomial P for $(n+1)$ data points $(x_i, y_i) = (x_i, f(x_i))$, $i = 0, 1, \ldots, n$ for the function f.

   Hint: (You may use the following algorithm)
   INPUT: Numbers $x_0, x_1, \ldots, x_n$ and values $f(x_0), f(x_1), \ldots f(x_n)$ as $F_{0,0}, F_{1,0}, \ldots, F_{n,0}$
   OUTPUT: The numbers $F_{0,0}, F_{1,1}, \ldots, F_{n,1}$.
   Step-1: For $i = 1, 2 \ldots, n$

   For $j = 1, 2 \ldots, i$

   Set $F_{i,j} = \frac{F_{i,j-1} - F_{i-1,j-1}}{x_i - x_i - j}$

   Step-2: Output $(F_{0,0}, F_{1,1}, \ldots, F_{n,1})$
   Test your coding for the exercises given in a separate attached page. Print your result (whole divided difference table) in a table.

End.