Lab Assignment-5

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

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1. **Bisection method**: Write a Matlab/C program to find a root of the equation $f(x) = 0$ for a given the continuous function f on the interval $[a, b]$, where $f(a)$ and $f(b)$ have opposite signs.
   Hint: (You may use the following algorithm)
   INPUT: Endpoints a, b; tolerance TOL; maximum number of iterations N.
   OUTPUT: Approximate root x
   Step-1: Set $k = 1$; and $y_0 = f(a)$.
   Step-2: While $k \leq N$ do Steps 3 to 6.

   Step-3: $x = (a + b)/2$; and $y = f(x)$.

   Step-4: If $y = 0$, OUTPUT(x). STOP.
   If $(b - a) \leq$ TOL, OUTPUT $((a + b)/2)$. STOP.

   Step-5: If $y_0 \cdot y > 0$, then set $a = x$ and $y_0 = y$;
   else set $b = x$.

   Step-6: Set $k = k + 1$.

   Step-7: OUTPUT ('Maximum number of iteration reached. Approximate root is x.')
   STOP.

2. **Regula-Falsi method**: Write a Matlab/C program to find a root of the equation $f(x) = 0$ for a given the continuous function f on the interval $[a, b]$, where $f(a)$ and $f(b)$ have opposite signs.
   Hint: (You may use the following algorithm)
   INPUT: Initial approximations a, b; maximum number of iterations N.
   OUTPUT: Approximate root x
   Step-1: Set $k = 1$; $y_0 = f(a)$ and $y_1 = f(b)$.
   Step-2: While $k \leq N$ do Steps 3 to 6.

   Step-3: $x = a - (y_0 \cdot (b - a))/(y_1 - y_0)$ and $y = f(x)$.

   Step-4: If $y = 0$, OUTPUT(x). STOP.

   Step-5: If $y_0.y > 0$, then set $a = x$ and $y_0 = y$;
   else set $b = x$. and $y_1 = y$.

   Step-6: Set $k = k + 1$.

   Step-7: OUTPUT ('Maximum number of iteration reached. Approximate root is x.')
   STOP.

   You could use other stopping criteria for Bisection or Regula-Falsi method.
   Test your coding to find a root of the following equation $f(x) = 0$ where

   $$f(x) = \sqrt{x} - \cos(x), \quad x \in [0, 1].$$

   Compute at least 10 iterations. Print the values of k (number of iterations), $a_k, b_k, x_k, f(x_k)$ in two tables (one for Bisection and another for Regula-Falsi).

3. **Power method**: Write a Matlab/C program on Power method to find the dominant eigen value of a matrix and a corresponding eigen vecgtor.
Hint: (You may use the following algorithm)
INPUT : dimension n, matrix A, initial vector $\mathbf{x}$, tolerance TOL and maximum number of iteration N.
OUTPUT: Approximate eigenvalue μ; approximate eigenvector $\mathbf{x}$ (with $\|\mathbf{x}\|_\infty = 1$) or a message that the maximum number of iterations was exceeded.
Step-1: Set $k = 1$.
Step-2: Find the smallest integer p with $1 \leq p \leq n$ and $|x_p| = \|\mathbf{x}\|_\infty$.
Step-3: Set $\mathbf{x} = \mathbf{x}/x_p$.
Step-4: While $k \leq N$ do Steps 5 to 11.

Step-5: Set $\mathbf{y} = A\mathbf{x}$.

Step-6: $\mu = y_p$.

Step-7: Find the smallest integer p with $1 \leq p \leq n$ and $|y_p| = \|\mathbf{y}\|_\infty$.

Step-8: If $y_p = 0$, then OUTPUT ('Eigenvector', $\mathbf{x}$);
OUTPUT ('A has the eigenvalue 0, select a new vector $\mathbf{x}$ and restart');
STOP.

Step-9: Set ERR= $\|\mathbf{x} - (\mathbf{y}/y_p)\|_\infty$ and $\mathbf{x} = \mathbf{y}/y_p$.

Step-10: If ERR $<$ TOL, then OUTPUT $(\mu, \mathbf{x})$;
STOP.

Step-11: Set $k = k + 1$.
STOP.

Step-12: OUTPUT ('The maximum number of iterations exceeded');
STOP.

Test your coding for the following matrix:

$$\begin{pmatrix} 3 & 0 & 0 \\ -4 & 6 & 2 \\ 16 & -15 & -5 \end{pmatrix}$$

It is known that eigen values of the above matrix are $3, 2, 0$ and corresponding eigen vectors are

$$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 2 \\ -5 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}.$$

Choose initial vector as $x^0 = (1, 0.5, 0.25)^T$. Print your result in a table.

End.