Lab Assignment-6

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

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1. **Newton-Raphson method**: Write a Matlab/C program to find a root of the equation $f(x) = 0$ for given initial approximation x_0, where f is a given differentiable function.
   Hint: (You may use the following algorithm)
   INPUT: Initial approximation x_0, tolerance TOL; maximum number of iterations N.
   OUTPUT: Approximate root x or message failure.
   Step-1: Set $k = 1$.
   Step-2: While $k \leq N$ do Steps 3 to 6.

   Step-3: $x = x_0 - f(x_0)/f'(x_0)$, $y = f(x)$.

   Step-4: If $y = 0$, OUTPUT(x). STOP.
   If $|x - x_0| \leq$ TOL or $|f(x)| \leq$ TOL, OUTPUT (x). STOP.

   Step-5: Set $x_0 = x$.

   Step-6: Set $k = k + 1$.

   Step-7: OUTPUT ('Maximum number of iteration reached. The method fail after N iterations.') STOP.

2. **Secant method**: Write a Matlab/C program to find a root of the equation $f(x) = 0$ for given initial approximations x_0, x_1, where f is a given continuous function.
   Hint: (You may use the following algorithm)
   INPUT: Initial approximations x_0, x_1, tolerance TOL; maximum number of iterations N.
   OUTPUT: Approximate root x or message failure.
   Step-1: Set $k = 2$; $y_0 = f(x_0)$ and $y_1 = f(x_1)$.
   Step-2: While $k \leq N$ do Steps 3 to 6.

   Step-3: $x = x_1 - (y_1 \cdot (x_1 - x_0))/(y_1 - y_0)$, $y = f(x)$.

   Step-4: If $y = 0$, OUTPUT(x). STOP.
   If $|x - x_1| \leq$ TOL or $|f(x)| \leq$ TOL, OUTPUT (x). STOP.

   Step-5: Set $x_0 = x_1$, $y_0 = y_1$, $x_1 = x$. and $y_1 = y$.

   Step-6: Set $k = k + 1$.

   Step-7: OUTPUT ('Maximum number of iteration reached. The method fail after N iterations.') STOP.

   STOP.
   You could use other stopping criteria for Newton-Raphson or Secant method.
   Test your coding to find a root of the following equation $f(x) = 0$ where

   $$f(x) = x - \cos(x), \quad x \in \mathbb{R}.$$

   Take $x_0 = \pi/4$ (for Newton-Raphson) and $x_0 = 0.5$, $x_1 = \pi/4$ (for Secant). Compute at least 10 iterations. Print the values of k (number of iterations), $x_k, f(x_k)$ in two tables (one for Newton-Raphson and another for Secant).

3. **Fixed Point method**: Write a Matlab/C program to find a fixed point of g i.e, $x = g(x)$ for given initial approximation x_0, where g is a given continuous function.
Hint: (You may use the following algorithm)
INPUT: Initial approximation x_0, tolerance TOL; maximum number of iterations N.
OUTPUT: Approximate root x or message failure.
Step-1: Set $k = 1$.
Step-2: While $k \leq N$ do Steps 3 to 6.

   Step-3: $x = g(x_0)$.

   Step-4: If $|x - x_0| \leq$ TOL, OUTPUT (x). STOP.

   Step-5: Set $x_0 = x$.

   Step-6: Set $k = k + 1$.

   Step-7: OUTPUT ('Maximum number of iteration reached. The method fail after N iterations.') STOP.
   Test your coding for the following equation:

$$x^3 + 4x^2 - 10 = 0.$$

   There are many ways to write the equation in the fixed point form as $x = g(x)$. Test you coding with following different $g_i's$, $i = 1, 2, 3, 4, 5$.

$$\begin{aligned}
x = g_1(x) &= x - x^3 - 4x^2 + 10, \\
x = g_2(x) &= \left(\frac{10}{x} - 4x\right)^{\frac{1}{2}}, \\
x = g_3(x) &= \frac{1}{2}\left(10 - x^3\right)^{\frac{1}{2}}, \\
x = g_4(x) &= \left(\frac{10}{4+x}\right)^{\frac{1}{2}}, \\
x = g_5(x) &= x - \frac{x^3 + 4x^2 - 10}{3x^2 + 8x}.
\end{aligned}$$

   Choose initial vector as $x^0 = 1.5$. Print your result in a table.

End.