Lab Assignment-8

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

January-April 2024, IIT Kanpur

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1. **Construction of Hermite polynomial**: Write a Matlab/C program to find the coefficient of the Hermite polynomial H for $(n+1)$ data points $(x_i, y_i) = (x_i, f(x_i))$, $i = 0, 1, \ldots, n$ for the function f.
   Hint: (You may use the following algorithm)
   INPUT: Numbers $x_0, x_1, \ldots, x_n$ and values $f(x_0), f(x_1), \ldots f(x_n)$ and $f'(x_0), f'(x_1), \ldots f'(x_n)$.
   OUTPUT: The numbers $Q_{0,0}, Q_{1,1}, \ldots, Q_{2n+1,2n+1}$, where

$$
\begin{aligned}
H(x) = Q_{0,0} &+ Q_{1,1}(x - x_0) + Q_{2,2}(x - x_0)^2 + Q_{3,3}(x - x_0)^2(x - x_1) \\
&+ Q_{4,4}(x - x_0)^2(x - x_1)^2 + \cdots \\
&+ Q_{2n+1,2n+1}(x - x_0)^2(x - x_1)^2 \cdots (x - x_{n-1})^2(x - x_n)
\end{aligned}
$$

   Step-1: For $i = 0, 1, \ldots, n$ do Steps 2 and 3.

   Step-2: Set $z_{2i} = x_i$,
   $z_{2i+1} = x_i$,
   $Q_{2i,0} = f(x_i)$,
   $Q_{2i+1,0} = f(x_i)$,
   $Q_{2i+1,1} = f'(x_i)$,

   Step-3 If $i \neq 0$, then set

$$
Q_{2i,1} = \frac{Q_{2i,0} - Q_{2i-1,0}}{z_{2i} - z_{2i-1}}
$$

   Step-4 For $i = 2, 3, \ldots, 2n+1$
   for $j = 2, 3, \ldots, i$, set

$$
Q_{i,j} = \frac{Q_{i,j-1} - Q_{i-1,j-1}}{z_i - z_{i-j}}
$$

   Step-5: OUTPUT $(Q_{0,0}, Q_{1,1}, \ldots, Q_{2n+1,2n+1})$ STOP.
   Test your coding for the systems given in a separate attached page.

2. **Construction of linear spline**: Write a Matlab/C program to obtain the linear spline S for $(n+1)$ data points $(x_i, y_i) = (x_i, f(x_i))$, $i = 0, 1, \ldots, n$ with $x_0 < x_1 < \cdots < x_n$, for the function f.

   Hint: (You may use the following algorithm)
   INPUT: Numbers $x_0, x_1, \ldots, x_n$ and values $f(x_0), f(x_1), \ldots f(x_n)$.
   OUTPUT: The numbers a_j, b_j, where

$$
S(x) = S_j(x) = a_j + b_j(x - x_j), \quad x \in [x_j, x_{j+1}]
$$

   for $j = 0, 1, \ldots, n-1$.
   Step-1: For $i = 0, 1, 2 \ldots, n-1$ set $a_i = f(x_i)$ and $b_i = (f(x_{i+1}) - f(x_i))/(x_{i+1} - x_i)$.
   Step-2: Output $(a_j, b_j$ for $j = 0, 1, \ldots, n-1)$

3. **Construction of natural cubic spline**: Write a Matlab/C program to obtain the natural cubic spline S for $(n+1)$ data points $(x_i, y_i) = (x_i, f(x_i))$, $i = 0, 1, \dots, n$ with $x_0 < x_1 < \cdots < x_n$, for the function f.

   Hint: (You may use the following algorithm)
   INPUT: Numbers $x_0, x_1, \dots, x_n$ and values $f(x_0), f(x_1), \dots f(x_n)$.
   OUTPUT: The numbers a_j, b_j, c_j, d_j, where

   $$S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3, \quad x \in [x_j, x_{j+1}]$$

   for $j = 0, 1, \dots, n-1$.
   Step-1: For $i = 0, 1, 2 \dots, n$ set $a_i = f(x_i)$.
   Step-2: For $i = 0, 1, 2 \dots, n-1$ set $h_i = x_{i+1} - x_i$.
   Recall: Solving a tri-diagonal system
   Step-3: For $i = 1, 2 \dots, n-1$ set

   $$v_i = \tfrac{3}{h_i}(a_{i+1} - a_i) - \tfrac{3}{h_{i-1}}(a_i - a_{i-1})$$

   Step-4: Set $l_0 = 1$, $u_0 = 0$, $z_0 = 0$.
   Step-5: For $i = 1, 2 \dots, n-1$

   $$\begin{aligned} \text{set } l_i &= 2(h_{i-1} + h_i) - h_{i-1}u_{i-1}, \\ u_i &= h_i / l_i, \\ z_i &= (v_i - h_{i-1}z_{i-1})/l_i. \end{aligned}$$

   Step-6: Set $l_n = 1$, $z_n = 0$, $c_n = 0$.
   Step-7: For $j = n-1, n-2, \dots, 1, 0$

   $$\begin{aligned} \text{set } c_j &= z_j - u_j c_{j+1}, \\ b_j &= (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3 \\ d_j &= (c_{j+1} - c_j)/(3h_j). \end{aligned}$$

   Step-8: Output $(a_j, b_j, c_j, d_j$ for $j = 0, 1, \dots, n-1)$

4. **Construction of clamped cubic spline**: Write a Matlab/C program to obtain the clamped cubic spline S for $(n+1)$ data points $(x_i, y_i) = (x_i, f(x_i))$, $i = 0, 1, \dots, n$ with $x_0 < x_1 < \cdots < x_n$, satisfying $S'(x_0) = f'(x_0)$ and $S'(x_n) = f'(x_n)$, for the function f.

   Hint: (You may use the following algorithm)
   INPUT: Numbers $x_0, x_1, \dots, x_n$ and values $f(x_0), f(x_1), \dots f(x_n)$ and $f'(x_0)$ and $f'(x_n)$.
   OUTPUT: The numbers a_j, b_j, c_j, d_j, where

   $$S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3, \quad x \in [x_j, x_{j+1}]$$

   for $j = 0, 1, \dots, n-1$.
   Step-1: For $i = 0, 1, 2 \dots, n$ set $a_i = f(x_i)$ and $L = f'(x_0)$ and $R = f'(x_n)$.
   Step-2: For $i = 0, 1, 2 \dots, n-1$ set $h_i = x_{i+1} - x_i$
   Step-3: Set $v_0 = 3(a_1 - a_0)/h_0 - 3L$.

   $$v_n = 3R - 3(a_n - a_{n-1})/h_{n-1},$$

   Step-4: For $i = 1, 2 \dots, n-1$ set

   $$v_i = \tfrac{3}{h_i}(a_{i+1} - a_i) - \tfrac{3}{h_{i-1}}(a_i - a_{i-1})$$

   Solving a tri-diagonal system
   Step-5: Set $l_0 = 2h_0$, $u_0 = 0.5$, $z_0 = v_0/l_0$.
   Step-6: For $i = 1, 2 \dots, n-1$

   $$\text{set } l_i = 2(h_{i-1} + h_i) - h_{i-1}u_{i-1},$$

$$u_i = h_i/l_i,$$
$$z_i = (v_i - h_{i-1}z_{i-1})/l_i.$$

Step-7: Set $l_n = h_{n-1}(2 - u_{n-1})$,

$$z_n = (v_n - h_{n-1}z_{n-1})/l_n,$$
$$c_n = z_n.$$

Step-8: For $j = n-1, n-2, \ldots, 1, 0$

set $c_j = z_j - u_j c_{j+1}$,

$$b_j = (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3$$
$$d_j = (c_{j+1} - c_j)/(3h_j).$$

Step-9: Output $(a_j, b_j, c_j, d_j$ for $j = 0, 1, \ldots, n-1)$

Test your coding for the exercises given in a separate attached page. Try to plot graph of Spline functions with set increasing number of points and compare the differences.

End.