Introduction to Fuzzy Logic Control

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(Corrected by WWK)

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Overview

- **Outline to the left in** green
- **Current topic in** yellow

- **References**
- **Introduction**
- **Crisp Variables**
- **Fuzzy Variables**
- **Fuzzy Logic Operators**
- **Fuzzy Control**
- **Case Study**

References

- Jan Łukasiewicz
- **L. Zadah, "Fuzzy sets as a basis of possibility" Fuzzy Sets Systems, Vol. 1, pp3-28, 1978.**

Introduction

- **Fuzzy logic:**
  - **A way to represent variation or imprecision in logic**
  - **A way to make use of natural language in logic**
  - **Approximate reasoning**
- **Humans say things like "If it is sunny and warm today, I will drive fast"**
- **Linguistic variables:**
  - **Temp: {freezing, cool, warm, hot}**
  - **Cloud Cover: {overcast, partly cloudy, sunny}**
  - **Speed: {slow, fast}**

Crisp (Traditional) Variables

- **Crisp variables represent precise quantities:**
  - **x = 3.1415296 (not so!)**
  - **A ∈{0,1}**
- **A proposition is either True or False**
  - $A \wedge B \Rightarrow C$
- **King(Richard) ∧ Greedy(Richard) ⇒ Evil(Richard)**
- **Richard is either greedy or he isn't:**
  - **Greedy(Richard) ∈{0,1}**

Fuzzy Sets

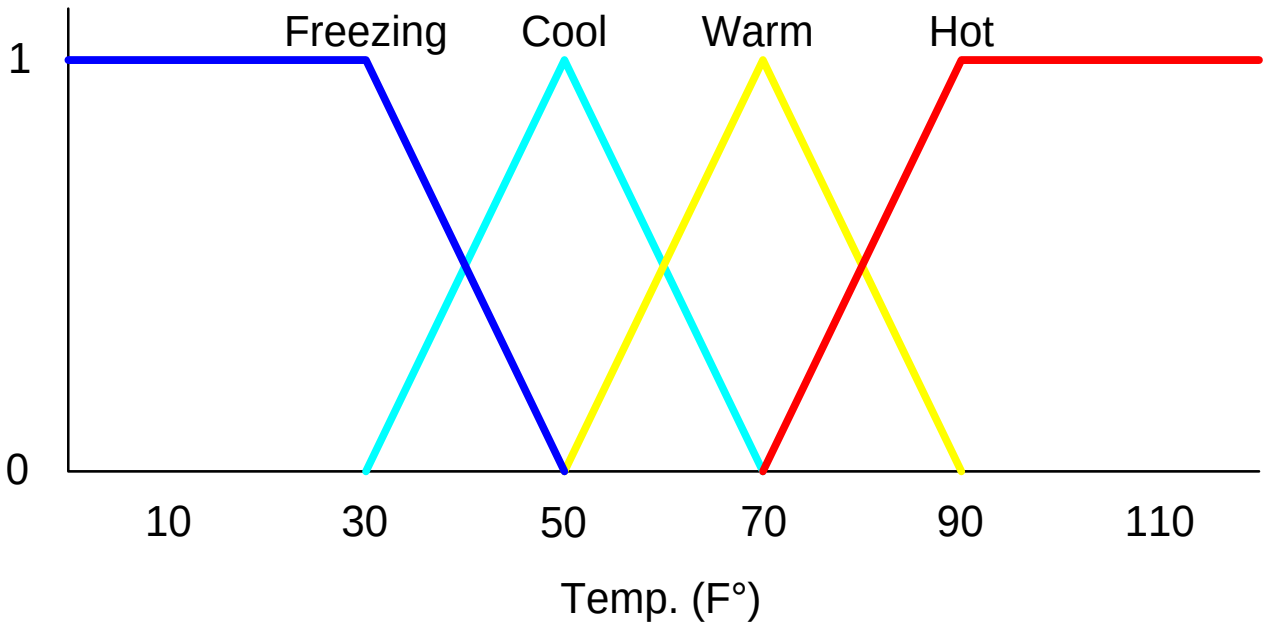
- **What if Richard is only somewhat greedy?**
- **Fuzzy Sets can represent the degree to which a quality is possessed.**
- **Fuzzy Sets (Simple Fuzzy Variables) have values in the range of [0,1]**
- **Greedy(Richard) = 0.7**
- **Question: How evil is Richard?**

Fuzzy Linguistic Variables

- **Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum**
- **Temp:** {Freezing, Cool, Warm, Hot}
- **Membership function**
- **Question: What is the temperature?**
- **Answer: It is warm.**
- **Question: How warm is it?**

Membership Functions

- **Temp: {Freezing, Cool, Warm, Hot}**
- **Degree of Truth or "Membership"**



Membership Functions

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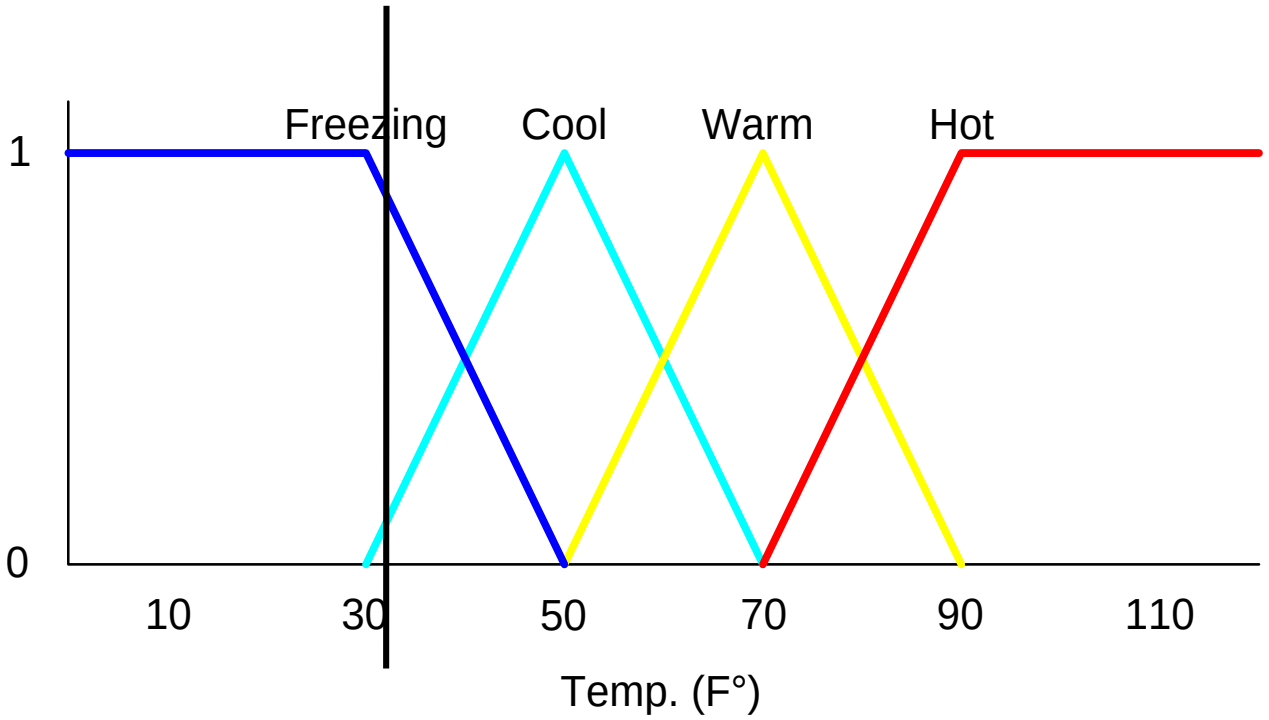
- Crisp Variables
- Fuzzy Sets
- Linguistic Variables
- Membership Functions
- Fuzzy Logic
  - Fuzzy OR
  - Fuzzy AND
  - Example
- Fuzzy Control
  - Variables
  - Rules
  - Fuzzification
  - Defuzzification
- Summary

- **How cool is 36 F° ?**



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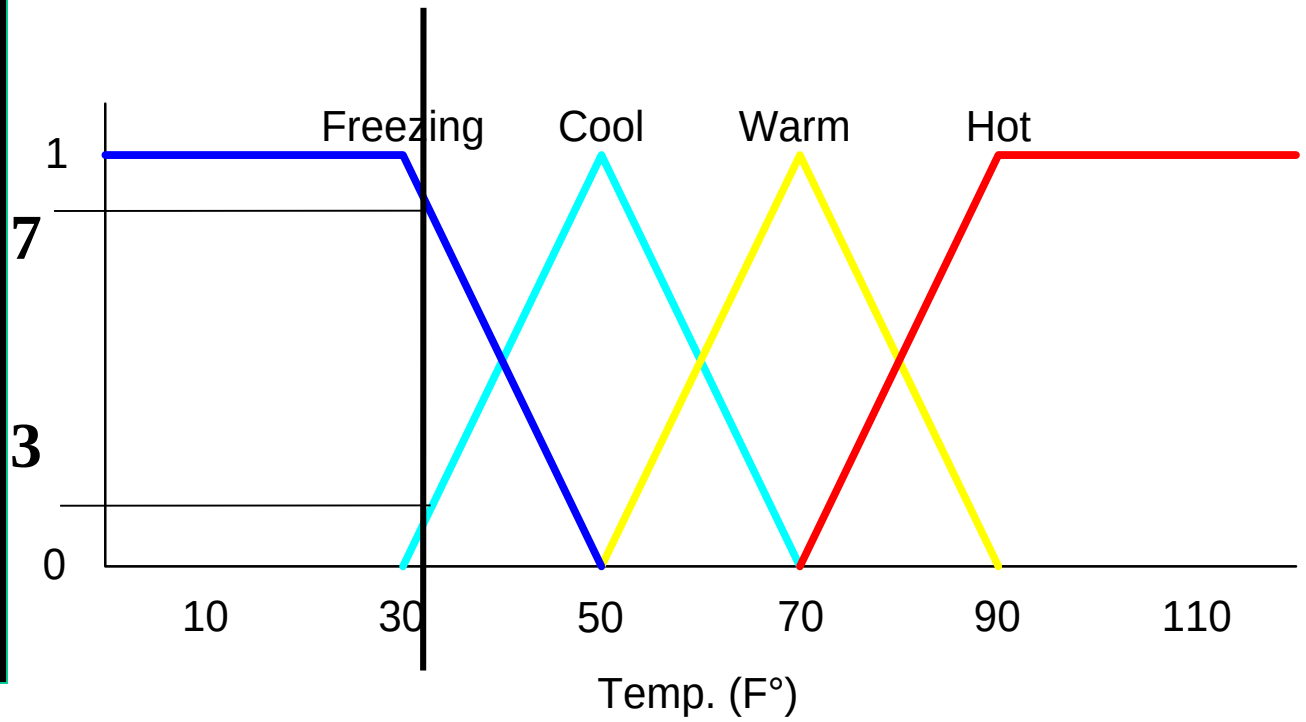
- Summary

- **How cool is 36 F° ?**
- **It is 30% Cool and 70% Freezing**

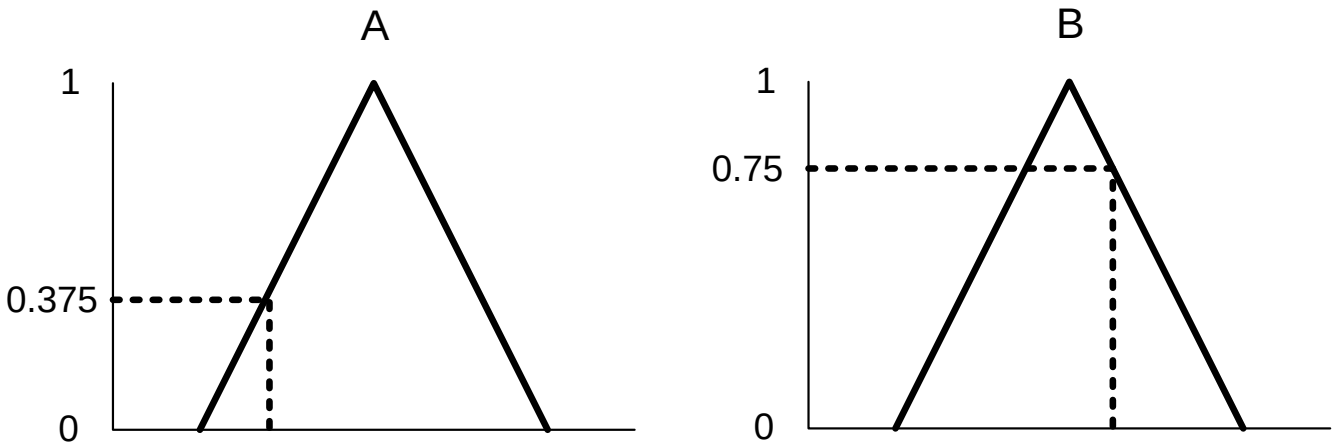


Fuzzy Logic

- **How do we use fuzzy membership functions in predicate logic?**
- **Fuzzy logic Connectives:**
  - **Fuzzy Conjunction, ∧**
  - **Fuzzy Disjunction, ∨**
- **Operate on degrees of membership in fuzzy sets**

Fuzzy Disjunction

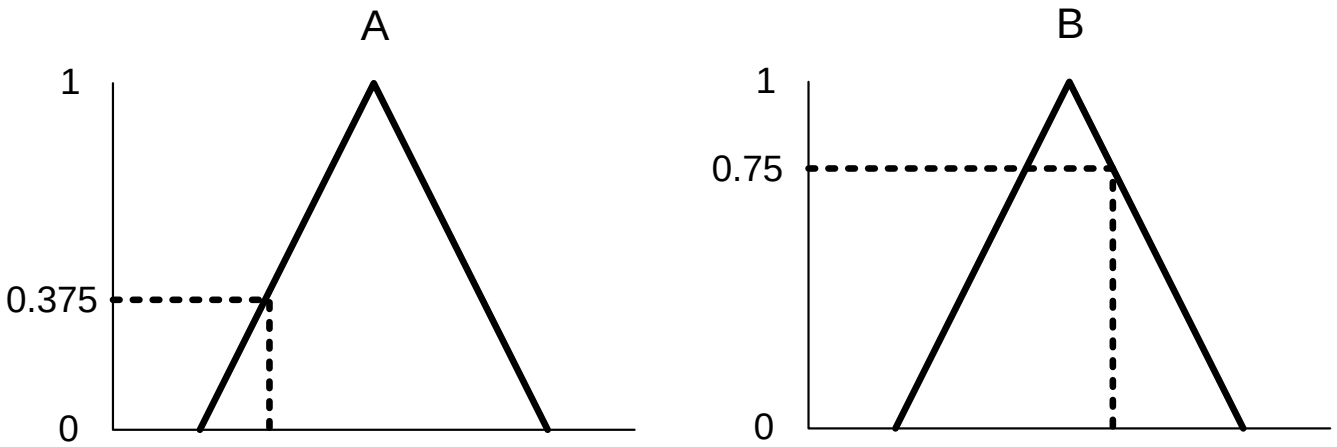
- **$A \vee B \triangleq \max(A, B)$**
- **$A \vee B = C$ "Quality C is the disjunction of Quality A and B"**



- **$(A \vee B = C) \Rightarrow (C = 0.75)$**

Fuzzy Conjunction

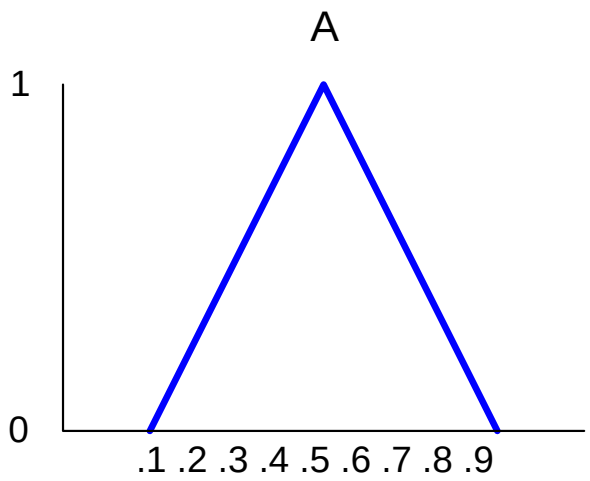
- **$A \wedge B \triangleq \min(A, B)$**
- **$A \wedge B = C$ "Quality C is the conjunction of Quality A and B"**

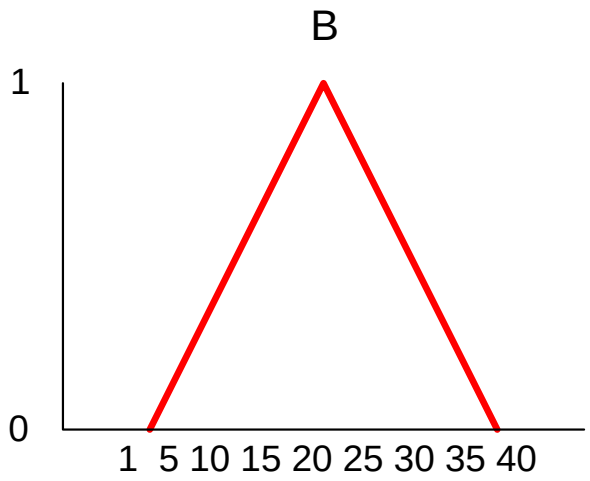


- **$(A \wedge B = C) \Rightarrow (C = 0.375)$**

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20





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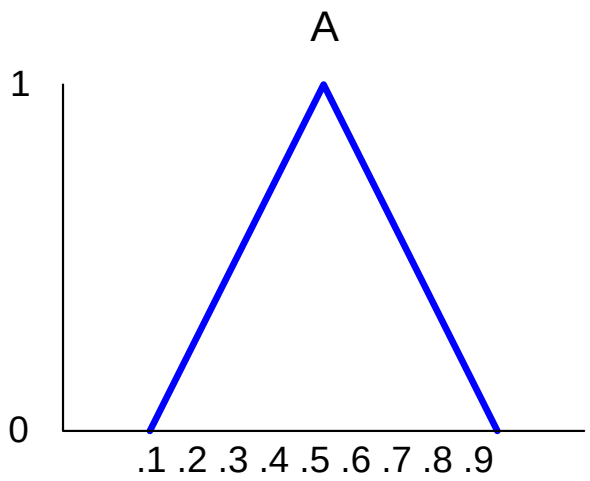

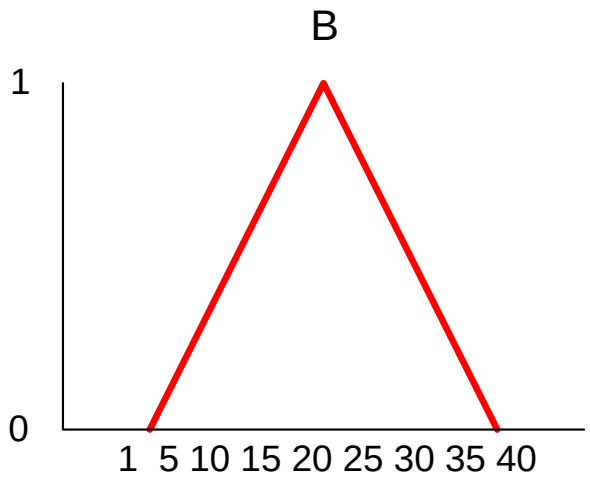

- **Determine degrees of membership:**

Example: Fuzzy Conjunction

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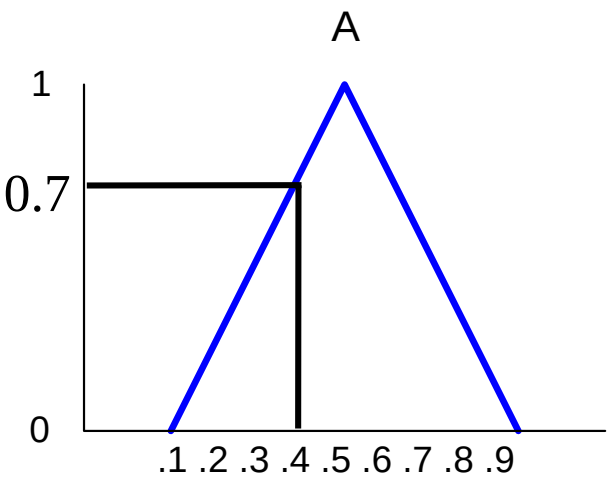

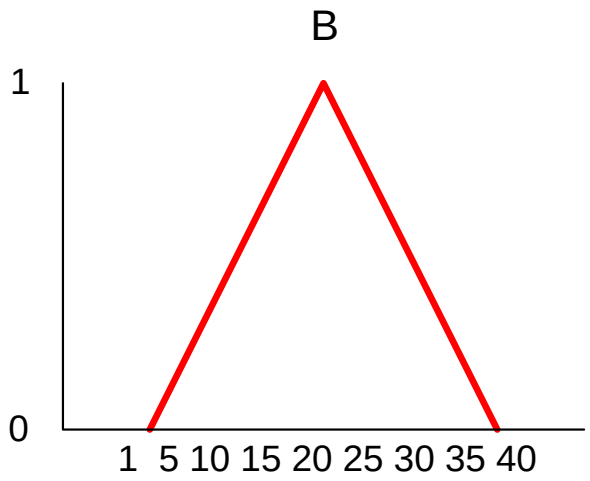

- **Determine degrees of membership:**
  - **A = 0.7**

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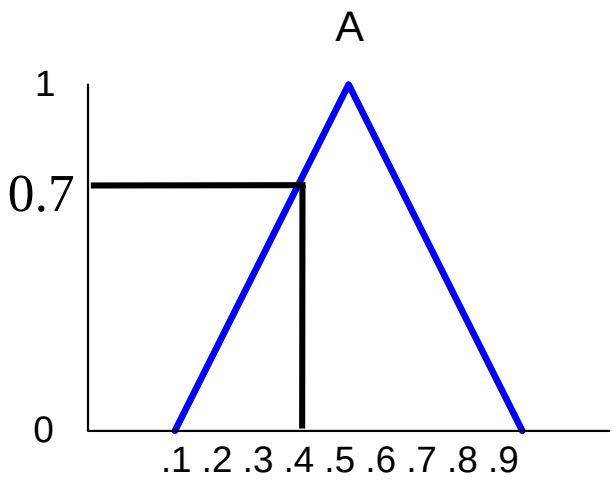
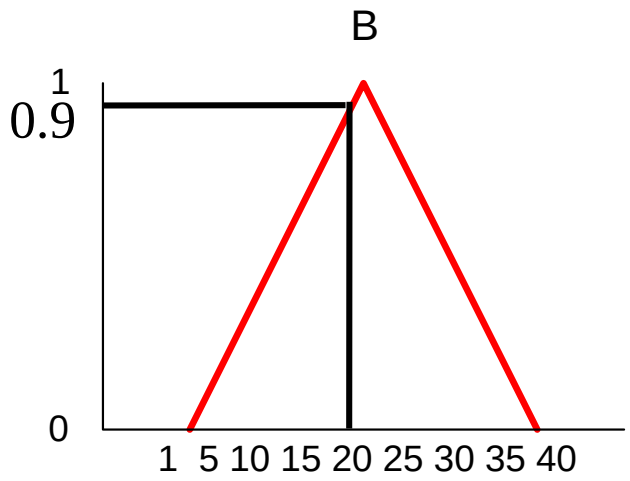
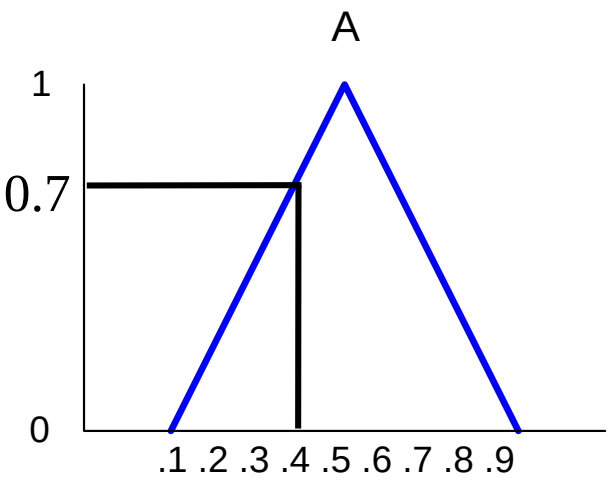


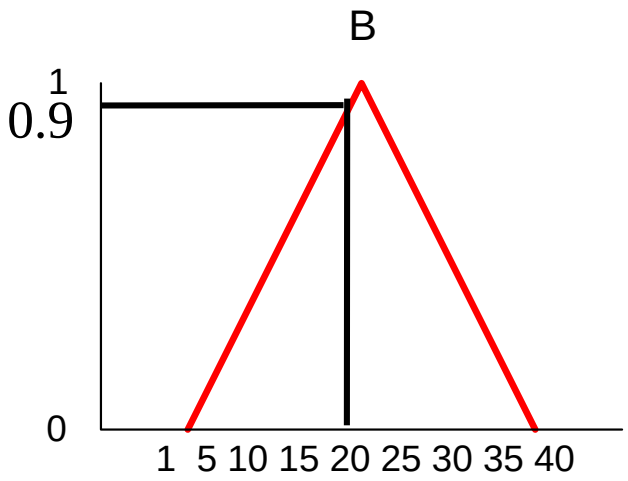


- **Determine degrees of membership:**
  - **A = 0.7 B = 0.9**

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20





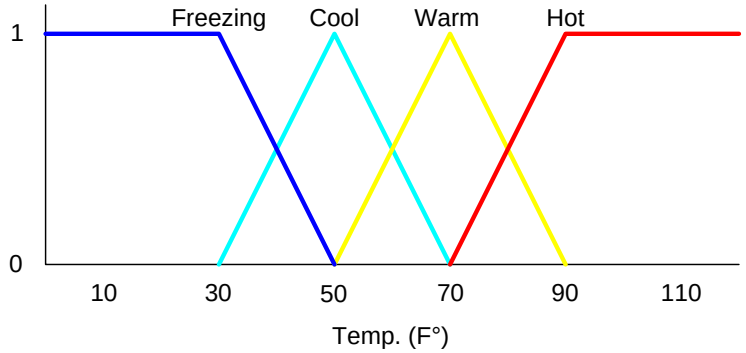
- **Determine degrees of membership:**
  - **A = 0.7 B = 0.9**
- **Apply Fuzzy AND**
  - **A∧B = min(A, B) = 0.7**

Fuzzy control

- **Fuzzy control combines the use of fuzzy linguistic variables with fuzzy logic.**
- **Example: speed control.**
- **How fast am I going to drive today?**
- **It depends on the weather.**
- **Use of isjunctions of conjunctions.**

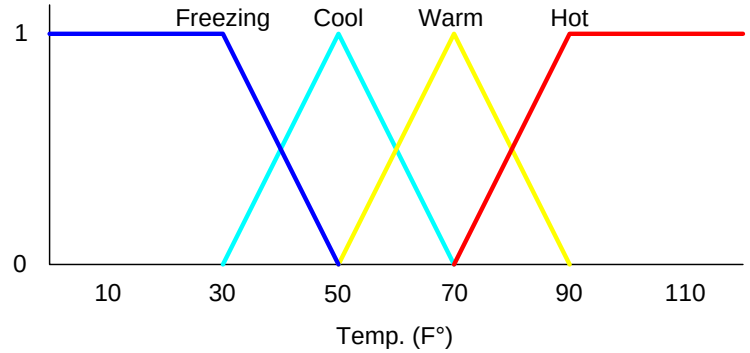
Inputs: Temperature

- **Temp: {Freezing, Cool, Warm, Hot}**

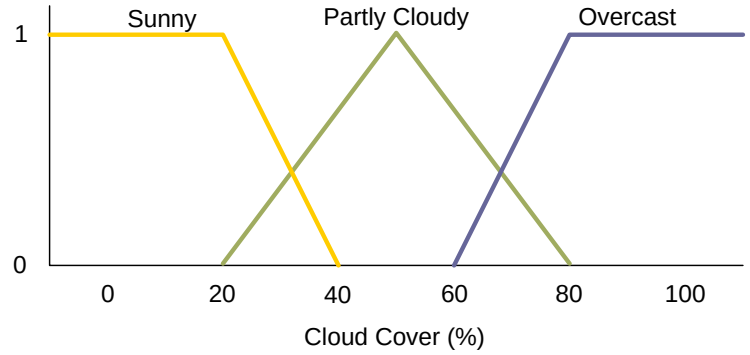


Inputs: Temperature, Cloud Cover

- **Temp: {Freezing, Cool, Warm, Hot}**

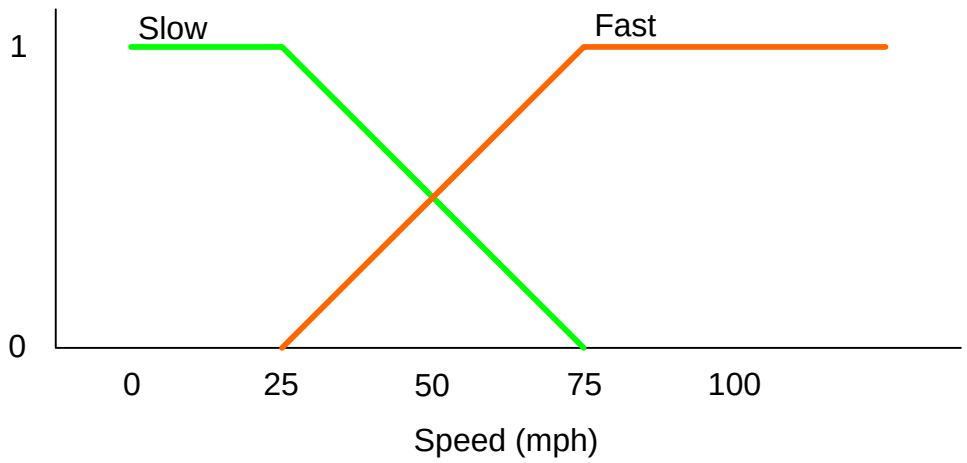


- **Cover: {Sunny, Partly, Overcast}**



Output: Speed

- **Speed: {Slow, Fast}**



Rules

- **If it's Sunny and Warm, drive Fast**

  $Sunny(Cover) \wedge Warm(Temp) \Rightarrow Fast(Speed)$

- **If it's Cloudy and Cool, drive Slow**

  $Cloudy(Cover) \wedge Cool(Temp) \Rightarrow Slow(Speed)$

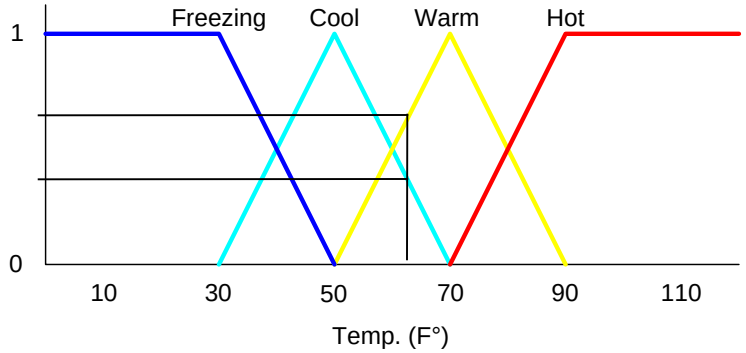
- **Driving Speed is the combination of output of these rules...**

Example Speed Calculation

- **How fast will I go if it is**
  - **65 F°**
  - **25 % Cloud Cover ?**

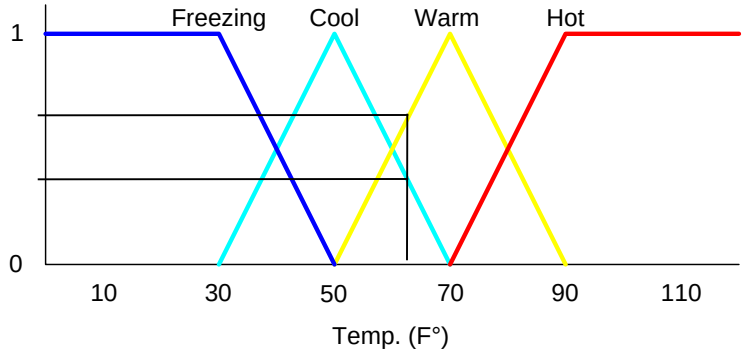
Fuzzification: Calculate Input Membership Levels

- **65 F° ⇒ Cool = 0.4, Warm= 0.7**

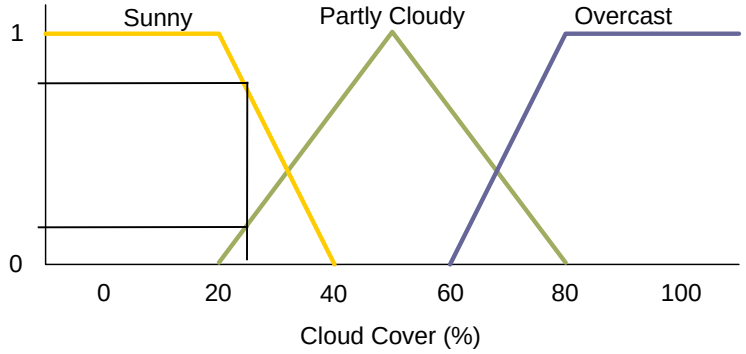


Fuzzification: Calculate Input Membership Levels

- **65 F° ⇒ Cool = 0.4, Warm= 0.7**



- **25% Cover ⇒Sunny = 0.8, Cloudy = 0.2**



...Calculating...

- **If it's Sunny and Warm, drive Fast**

Sunny(Cover)∧Warm(Temp)⇒Fast(Speed)

0.8 ∧ 0.7 = 0.7

⇒ Fast = 0.7

- **If it's Cloudy and Cool, drive Slow**

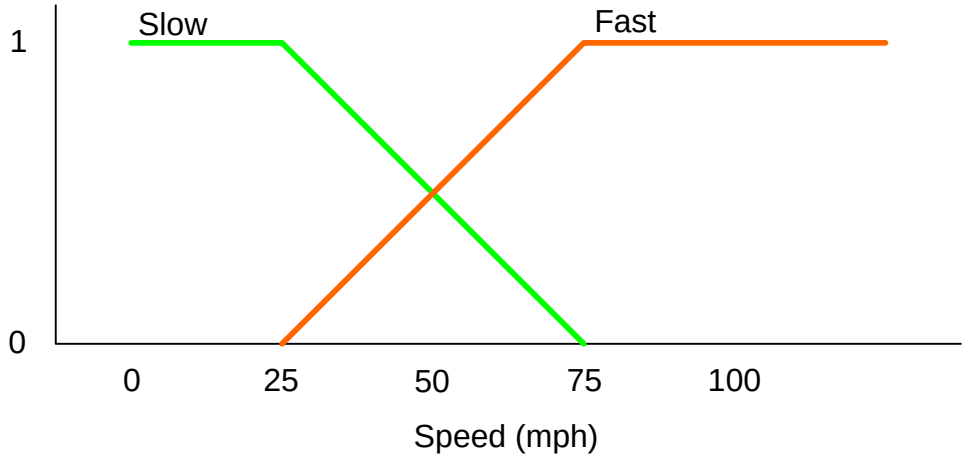
Cloudy(Cover)∧Cool(Temp)⇒Slow(Speed)

0.2 ∧ 0.4 = 0.2

⇒ Slow = 0.2

Defuzzification: Constructing the Output

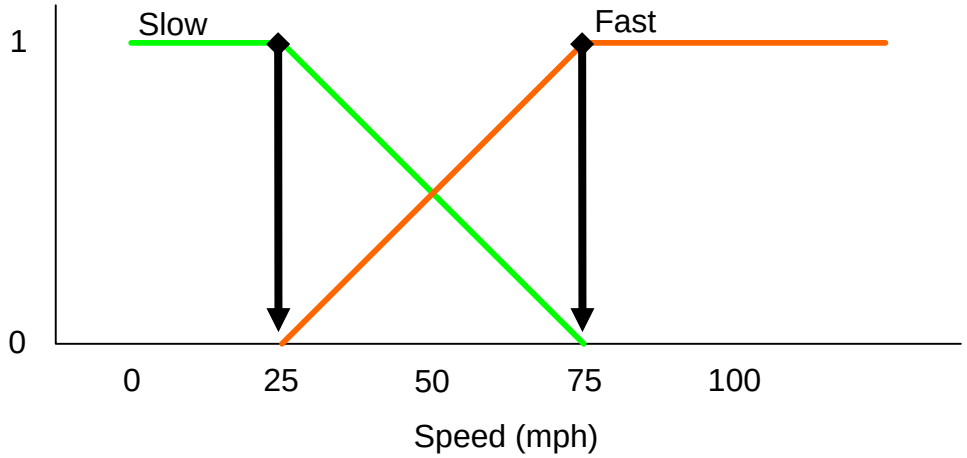
- **Speed is 20% Slow and 70% Fast**



- **Find centroids: Location where membership is 100%**

Defuzzification: Constructing the Output

- **Speed is 20% Slow and 70% Fast**



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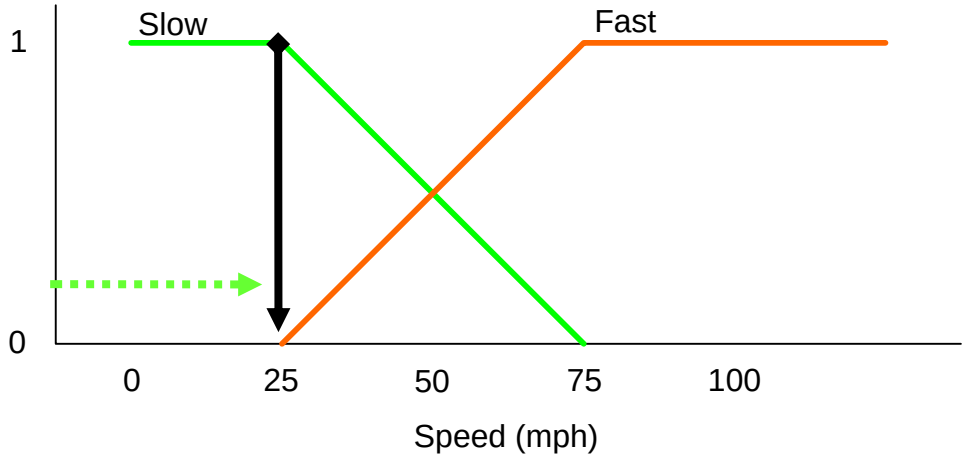

- **Speed = weighted mean**

  = (2*25+...

Defuzzification: Constructing the Output

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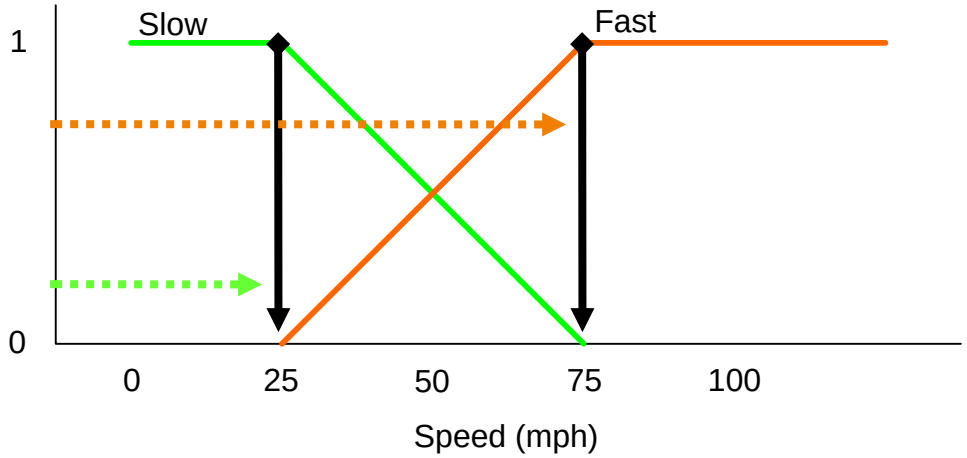


- **Speed = weighted mean**

  = (.2*25+.7*75)/(.9)

  = 63.8 mph

Comments

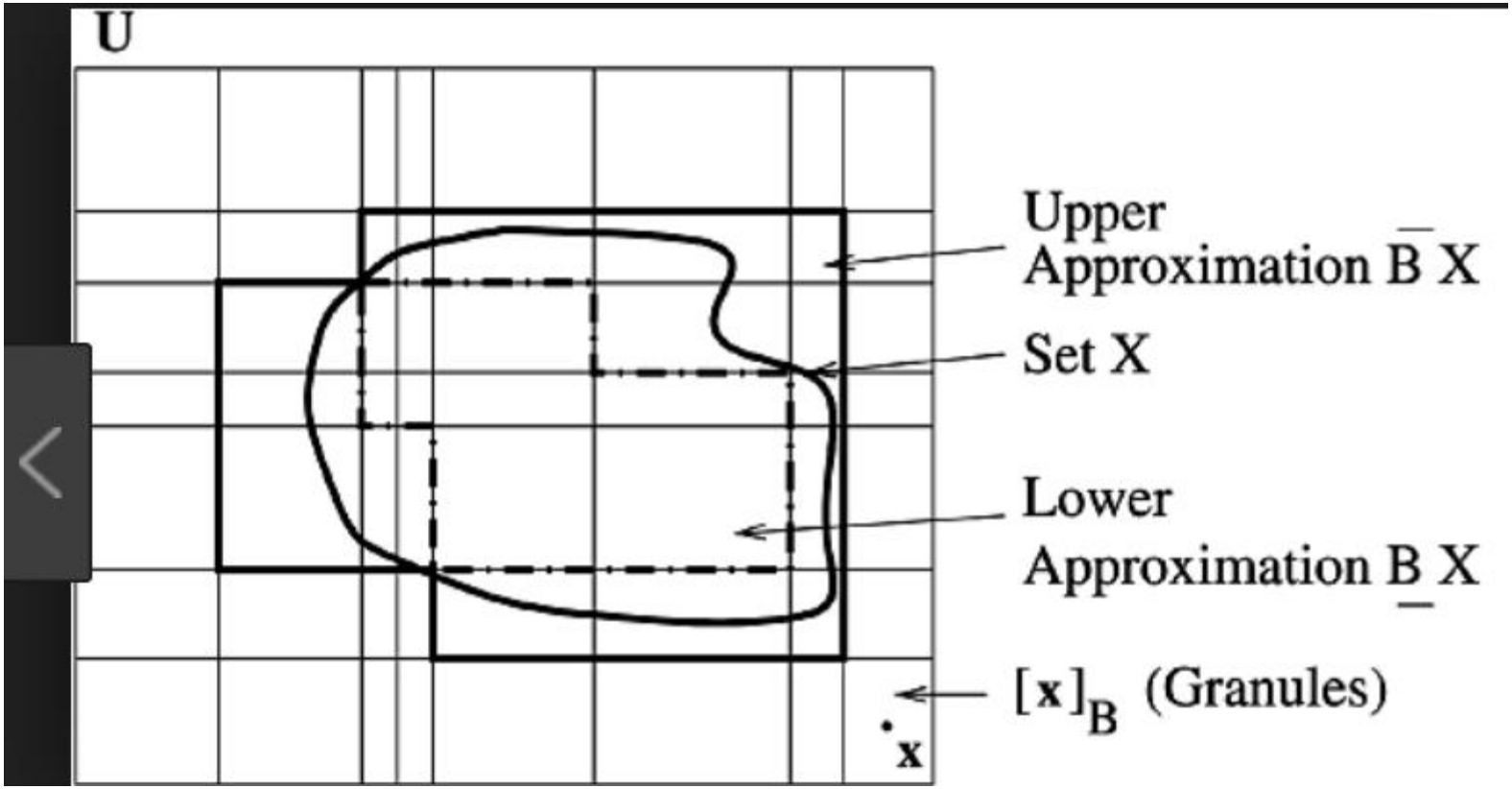
- **The smooth interpolation between variable centroids with relatively few rules is done by fuzzy logic controller.**
- **This does not work with crisp (traditional) logic.**
- **Helps to encode some types of human expertise in a computer program.**

Drawbacks of uzzy logic

- **Solution depends on the membership function definition.**
- **Deals with imprecision, and vagueness, but not uncertainty**
- **FL control may not scale well to large or complex problems.**

Summary

- **Fuzzy logic helps with imprecision and vagueness**
- **Fuzzy logic can be used to represent some kinds of human expertise.**
- **Fuzzy membership function.**
- **Fuzzy linguistic variables**
- **Fuzzy AND and OR**
- **Fuzzy control**
- **FL has its advantages/disadvantages.**


U
Upper
Approximation $\overline{B}$ X
Set X
Lower
Approximation $\underline{B}$ X
$[x]_B$ (Granules)
x