The Electrodynamics of Value: Gauge-Theoretic Structure in AI Alignment

A Structural Correspondence Between Field Theory and Invariant Evaluation

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Abstract

For three centuries, ethical formalism has often remained in a "Newtonian" state: modeling value as a scalar magnitude (utility) to be maximized. We argue this scalar picture is often brittle for high-dimensional autonomous systems, particularly when proxy misspecification or representational gaming are concerns [22, 21]. Using gauge theory [8, 9], we show that a broad class of representation-invariant governance formalisms can be modeled using the same geometric ingredients that appear in classical electrodynamics: principal bundles, connections, curvature, and symmetry-derived conservation. We present "Maxwell-like" alignment constraints: a compact set of invariance and consistency conditions that clarify which failures are ruled out by symmetry (representational/semantic gaming) and which remain (grounding adequacy, implementation error, covert channels). The correspondence is structural, not metaphysical: both domains instantiate the same mathematical pattern, but the guarantees are conditional on explicit assumptions we state upfront. This paper is the theoretical companion to the GUASS specification [2], which provides operational protocols for deployment.

1 Formal Spine: Assumptions, Definitions, and Scoped Claims

We use gauge/electrodynamics language as a compact way to talk about invariance, consistency, and exploitable loopholes. The correspondence is conditional: it becomes precise once the objects and assumptions are fixed, and it fails when they are violated.

1.1 The Four Axioms

A1 (Declared Observables). Choose a grounding map $\Psi : \mathcal{X} \to \mathbb{R}^k$ for the deployment domain, where $\mathcal{X}$ is the space of all representations and $\mathbb{R}^k$ is the measurement space. The measurement manifold M is then defined as $M := \Psi(\mathcal{X}) \subseteq \mathbb{R}^k$, which inherits smooth or stratified structure from the measurement space. Specify the measurement pipeline explicitly.

A2 (Measurement Integrity). Assume $\Psi(x)$ is reported within declared tolerances, and that detected tampering or inconsistency triggers fail-closed behavior.

A3 (Re-description Suite). Define a **declared transform suite** $\mathcal{G}_{\text{declared}}$ of Ψ-preserving re-descriptions under which evaluation should be invariant. Formally, each $g \in \mathcal{G}_{\text{declared}}$ is a (possibly partial) map $g : \mathcal{X} \rightharpoonup \mathcal{X}$ satisfying $\Psi(g(x)) = \Psi(x)$ for all $x \in \text{dom}(g)$.

Engineering regime vs. geometric regime: In practical deployments (NLP, vision), transforms in $\mathcal{G}_{\text{declared}}$ may be:

- **Discrete** (not continuous/Lie),
- **Partial** (not defined on all inputs),
- **Non-invertible** (one-way normalizations).

The **engineering regime** uses $\mathcal{G}_{\text{declared}}$ directly for invariance testing without requiring group structure. The **geometric regime** (for principal-bundle constructions, holonomy, curvature) restricts to an invertible subset $G \subseteq \mathcal{G}_{\text{declared}}$ that forms a Lie group acting smoothly on $\mathcal{X}$. The geometric machinery applies only within this subset; the engineering guarantees (BIP) apply to all of $\mathcal{G}_{\text{declared}}$.

Validation of membership: This definition makes invariance hold by construction for declared $\mathcal{G}_{\text{declared}}$. The substantive question is whether the suite is specified correctly. A3 defines an operational equivalence class: the claim is not that $\mathcal{G}_{\text{declared}}$ captures "true semantic equivalence," but that if a deployment standard declares a Ψ-preserving suite and verifies membership, then representational gaming within that declared envelope is structurally removed. In practice, membership can be validated by: (i) provable equivalence under a measurement model, (ii) empirically testable invariance checks on held-out re-descriptions, or (iii) formal verification that the canonicalizer treats $g(x)$ and x identically. Getting $\mathcal{G}_{\text{declared}}$ wrong—either too narrow or too wide—is an explicit failure mode outside the guarantees.

Example 1 (Concrete $\mathcal{G}_{\text{declared}}$ for Vision Systems)**.** *Consider an autonomous vehicle's pedestrian detection system where $\mathcal{X}$ = image space and Ψ extracts pedestrian locations and velocities.*

- ***In** $\mathcal{G}_{\text{declared}}$ **(should not change moral assessment):** Lighting changes (brightness, contrast within sensor range), lossy compression artifacts, camera white balance, time-of-day color shifts, sensor noise and weather effects within the validated operating envelope.*
- ***Not in** $\mathcal{G}_{\text{declared}}$ **(should change assessment):** Occlusion (pedestrian hidden), object substitution (pedestrian $\rightarrow$ mannequin), adversarial patches that change classification.*

Membership is validated by: testing that the canonicalizer (e.g., normalization + detection model) produces identical Ψ-outputs for related inputs; flagging cases where related inputs produce different outputs as canonicalizer bugs.

Example 2 (Concrete $\mathcal{G}_{\text{declared}}$ for Text Systems)**.** *Consider a content moderation system where $\mathcal{X}$ = text strings and Ψ extracts semantic intent features.*

- ***In** $\mathcal{G}_{\text{declared}}$**:** Synonym substitution ("car" $\leftrightarrow$ "automobile", "big" $\leftrightarrow$ "large"), trivial paraphrase ("the cat sat on the mat" $\leftrightarrow$ "on the mat sat the cat"), Unicode normalization, whitespace changes, case changes (where semantically irrelevant).*
- ***Not in** $\mathcal{G}_{\text{declared}}$**:** Negation ("I will" $\rightarrow$ "I won't"), target substitution ("harm Alice" $\rightarrow$ "harm Bob"), hypothetical framing ("I will" $\rightarrow$ "What if someone were to").*

*Note that many text transforms are **non-invertible** (e.g., lowercasing) or **partial** (synonym substitution only applies where synonyms exist). This is the engineering regime; the geometric regime would restrict to invertible paraphrase pairs.*

A4 (Verified Canonicalization + External Gate). Implement and verify a canonicalizer $\kappa : \mathcal{X} \rightarrow \mathcal{X}$ and enforce evaluation/actuation through an external monitor so that representational changes cannot bypass checks.

1.2 Core Invariance Property

Given A1–A4, evaluation satisfies the **Bond Invariance Principle (BIP)** [1]:

$$\Sigma(x) = \Sigma(g(x)) \quad \forall g \in \mathcal{G}_{\text{declared}},\ x \in \text{dom}(g)$$

Engineering-regime quotient (canonicalizer-induced): Since $\mathcal{G}_{\text{declared}}$ may include non-invertible transforms, the relation "reachable by transforms" is not symmetric and hence not an equivalence relation. Instead, we define the engineering quotient via the canonicalizer:

$$x \sim_\kappa y \quad \iff \quad \kappa(x) = \kappa(y)$$

This *is* an equivalence relation (reflexive, symmetric, transitive by properties of equality). The quotient map is then $q := \kappa$ (treating canonical forms as equivalence class representatives), and BIP becomes:

$$\Sigma = \tilde{\Sigma} \circ \kappa \quad \text{for some } \tilde{\Sigma} : \text{im}(\kappa) \to V.$$

Note: BIP holds for the full engineering suite $\mathcal{G}_{\text{declared}}$, including partial and non-invertible transforms. The geometric constructions below require restricting to an invertible Lie-group subset G, where the orbit-space quotient $\mathcal{X}^*/G$ is well-defined.

1.3 Geometric Setup and Diagnostic Tools

1.3.1 Bundle Structure (Two-Regime Formulation)

Engineering regime: The invariance guarantee (BIP) holds for all of $\mathcal{G}_{\text{declared}}$ without requiring geometric structure. The canonicalizer κ defines an equivalence relation ($x \sim_\kappa y$ iff $\kappa(x) = \kappa(y)$), and $\Sigma = \tilde{\Sigma} \circ \kappa$ provides operational invariance. In this regime, $\mathcal{X}$ may be a discrete set (e.g., text strings in NLP) with no manifold structure; all that is required is that κ be well-defined and that transforms in $\mathcal{G}_{\text{declared}}$ preserve Ψ-values.

Geometric regime: For principal-bundle constructions [10, 8], restrict to an invertible subset $G \subseteq \mathcal{G}_{\text{declared}}$ that forms a Lie group acting smoothly on $\mathcal{X}$. This regime requires $\mathcal{X}$ (or at least the relevant portion $\mathcal{X}^*$) to carry smooth manifold structure. We work on the **principal stratum** $\mathcal{X}^* \subseteq \mathcal{X}$ where the G-action is free and proper [26]. On $\mathcal{X}^*$, the orbit space

$$B := \mathcal{X}^*/G$$

is a smooth manifold and the projection $\pi : \mathcal{X}^* \to B$ makes $\mathcal{X}^*$ a principal G-bundle over B.

Because G is Ψ-preserving (A3), Ψ descends to the quotient: there exists a unique map $\bar{\Psi} : B \to \mathbb{R}^k$ such that

$$\Psi = \bar{\Psi} \circ \pi.$$

The paper's "measurement manifold" is then the image $M := \Psi(\mathcal{X}) \subseteq \mathbb{R}^k$.

Remark 1 (Quotient Regularity)**.** *Outside the principal stratum $\mathcal{X}^*$, the quotient $\mathcal{X}/G$ may be an **orbifold** or **stratified space** rather than a smooth manifold (e.g., when the action has fixed points or varying stabilizer dimensions). We restrict to $\mathcal{X}^*$ for smoothness; the engineering-regime guarantees (BIP, gauge-fixing consistency) still apply outside $\mathcal{X}^*$, but the differential-geometric constructions (connection, curvature, holonomy) require the smooth structure of $B = \mathcal{X}^*/G$.*

Remark 2 (When M Can Serve as "The Base")**.** *The measurement manifold M is **not** automatically the correct base for the principal bundle structure. The correct base is $B = \mathcal{X}^*/G$. However, M can be treated as the base when:*

1. ***Injectivity condition:*** *$\bar{\Psi} : B \to M$ is injective on the region of interest (distinct orbits map to distinct measurements).*
2. ***Submersion condition:*** *$\bar{\Psi}$ is a submersion (or at least an immersion), so M inherits smooth structure from B.*

*When these conditions hold, we may identify $B \cong \bar{\Psi}(B) \subseteq M$ and work directly with M as the base. When these conditions **fail**, M is a coarser space than B: multiple orbits may map to the same measurement, and the bundle structure should be understood over B, with $\bar{\Psi} : B \to M$ as an additional map.*

1.3.2 Canonicalizers as Gauge Choices

A canonicalizer $\kappa : \mathcal{X}^* \to \mathcal{X}^*$ is a **gauge-fixing rule** that picks a representative per orbit. Formally, on an open set $U \subseteq B$, a gauge choice is a local section

$$\sigma : U \to \mathcal{X}^* \quad \text{with} \quad \pi \circ \sigma = \mathrm{id}_U.$$

A global section exists only if the bundle is trivial; in general, σ (and hence κ) should be understood as local or defined only on a restricted domain.

Important: A section does not, by itself, induce a connection. A connection is additional structure that must be specified explicitly.

1.3.3 Connection (Explicit Construction)

A connection is an equivariant choice of horizontal subspaces $H_x \subset T_x\mathcal{X}^*$ complementary to the vertical/orbit directions $V_x := \ker(d\pi_x)$ [10, 12].

Mechanical connection construction: If $\mathcal{X}^*$ carries a G-invariant Riemannian metric $\langle \cdot, \cdot \rangle$, define horizontals by orthogonality:

$$H_x := V_x^{\perp}.$$

This yields a principal connection with associated connection 1-form $\omega \in \Omega^1(\mathcal{X}^*, \mathfrak{g})$ characterized by [9]:

- $\omega(\xi_{\mathcal{X}^*}) = \xi$ for each fundamental vertical vector field
- $\ker \omega = H$
- $R_g^* \omega = \mathrm{Ad}(g^{-1})\omega$

The **curvature** is the $\mathfrak{g}$-valued 2-form [8]:

$$\Omega := d\omega + \frac{1}{2}[\omega, \omega].$$

Remark 3 (Existence of G-Invariant Metrics)**.** *A G-invariant metric exists when G is **compact** (by averaging any metric over the Haar measure). When G is non-compact, a G-invariant metric may not exist, and alternative connection constructions are needed (e.g., specifying horizontal subspaces directly, or using a non-invariant metric with appropriate corrections). In the **engineering regime** (discrete/partial transforms), where no Lie-group structure is assumed, the "alignment transport" approximation below serves as a practical substitute without requiring a geometric connection.*

1.3.4 Two Distinct Diagnostics

We distinguish two complementary tests that serve different purposes:

Diagnostic A: Gauge-Fixing Consistency Test (Engineering Regime). *Purpose:* Detect canonicalizer bugs, non-determinism, or implementation errors.

Procedure:

1. Sample transforms $g_1, g_2 \in \mathcal{G}_{\text{declared}}$ and input $x \in \mathcal{X}$ where both compositions are defined.
2. Compute $\kappa(g_1(g_2(x)))$ and $\kappa(g_2(g_1(x)))$.
3. Measure $\Delta = d(\kappa(g_1(g_2(x))), \kappa(g_2(g_1(x))))$.
4. If $\Delta > \tau$ (threshold), flag as canonicalizer inconsistency.

What it detects: Failure of κ to yield consistent canonical representatives; cases where different transform sequences that should produce $\sim_\kappa$-equivalent results instead yield different canonical forms.

What it does NOT measure: Curvature in the gauge-theoretic sense. Applying transforms from $\mathcal{G}_{\text{declared}}$ does not move you in the base B (geometric regime) or change the $\sim_\kappa$ equivalence class (engineering regime)—you remain in the same "fiber" over a fixed base point or canonical representative.

Applicability: This test applies in both engineering and geometric regimes, and works with partial/non-invertible transforms.

Diagnostic B: Holonomy Loop Test (Geometric Regime). *Purpose:* Detect genuine path dependence of parallel transport—the operational signature of nonzero curvature $\Omega \neq 0$.

Applicability: This test requires the **geometric regime**: an invertible Lie-group subset $G \subseteq \mathcal{G}_{\text{declared}}$, smooth structure on $\mathcal{X}^*$, and either an explicit connection or the alignment-transport approximation.

Key distinction: The loop is formed by **scenario/context perturbations** that move you in the base B, **not** by applying re-description transforms $g \in G$ (which keep you in the same fiber over a fixed base point).

Prerequisites:

- Four nearby **base points** $b_{00}, b_{10}, b_{11}, b_{01} \in B$ forming a small "rectangle." These correspond to different **scenarios/contexts** (e.g., different pedestrian configurations, different semantic situations), not to re-descriptions of the same scenario.
- Representatives $x_{ij} \in \mathcal{X}^*$ with $\pi(x_{ij}) = b_{ij}$
- Either an explicit connection or the practical alignment rule below

Alignment rule (practical stand-in for parallel transport): Given two nearby representatives $x \in \pi^{-1}(b)$ and $x' \in \pi^{-1}(b')$ over **different base points**, compute an approximate transport element:

$$g^*(x, x') := \text{ApproxArgMin}_{g \in G}\, d(x, x' \cdot g)$$

where ApproxArgMin denotes a bounded optimization procedure with:

- **Bounded search:** Terminate after fixed iterations or when improvement falls below threshold
- **Deterministic tie-breaking:** If multiple near-optimal g exist, select lexicographically or by predefined ordering
- **Failure handling:** If no g achieves $d(x, x' \cdot g) < d_{\max}$, return $\perp$ (undefined) and flag the edge as "transport failed"

This approximation is practical for engineering purposes; it does not require G to be compact or the infimum to be attained.

Procedure:

1. Pick a start representative x_{00}.
2. Compute edge transports (any $\perp$ result aborts with "transport failure" flag):

$$g_{00\to 10} = g^*(x_{00}, x_{10}), \quad g_{10\to 11} = g^*(x_{10}, x_{11}),$$
$$g_{11\to 01} = g^*(x_{11}, x_{01}), \quad g_{01\to 00} = g^*(x_{01}, x_{00}).$$

3. Form the loop product (holonomy estimate):

$$h := g_{01\to 00}\, g_{11\to 01}\, g_{10\to 11}\, g_{00\to 10}.$$

4. Measure deviation from identity: $D_G(h, e)$ (e.g., $\|\log(h)\|$ for matrix Lie groups).

Interpretation:

- $h \approx e$ on small loops suggests **flat** behavior (no path dependence under the chosen connection/transport rule).
- Persistent $h \neq e$ indicates **curvature-driven path dependence**—the correct mathematical analog of "loop exploits" in gauge terms (money-pumping, specification gaming via sequences of **scenario changes**).
- Transport failure ($\perp$) on any edge indicates the alignment rule is inadequate for that region; treat as out-of-distribution and escalate.

Noether Diagnostic (Optional, Conditional). If a suitable action functional S is invariant under a continuous symmetry group, Noether's theorem [13] yields a conserved current J. We propose "alignment current" as a monitorable signal under these assumptions.

Scope & Limitations: On Discrete Systems: Standard Noether's theorem requires continuous time and smooth Lagrangian dynamics. Most RL agents operate in discrete time (MDPs) with discontinuous policies (argmax). For discrete systems, the relevant analog is the discrete Noether theorem for symplectic/variational integrators [16], which yields approximate conservation laws with bounded drift. Alternatively, one can use Noether's theorem for difference equations [14, 15], which provides exact discrete conservation laws when the discrete action admits the symmetry. If neither applies, the "alignment current" becomes a monitored quantity rather than a conserved quantity—drift in J signals symmetry-breaking or model mismatch, even if exact conservation fails.

1.4 The Scoped Claim

What the framework provides (given A1–A4):

1. Purely representational changes (within declared $\mathcal{G}_{\text{declared}}$) cannot change compliance outcomes. [Engineering regime]
2. Gauge-fixing consistency tests detect canonicalizer bugs and implementation errors. [Both regimes]
3. Holonomy/curvature diagnostics detect path-dependent exploits arising from loops in the base. [Geometric regime only]
4. (Conditional) Conservation-style audit signals when Noether applies; monitored drift signals when it doesn't.

What the framework does NOT provide:

1. That Ψ is complete (captures all morally relevant features).
2. That $\mathcal{G}_{\text{declared}}$ is correctly specified (too narrow or too wide).

3. Prevention of physical compromise (sensor spoofing, hardware attacks).
4. Solution to value choice (which Ψ to use is a governance problem).
5. Implementation correctness (bugs can violate guarantees).
6. Exact Noether conservation for discrete-time or dissipative systems.
7. Geometric constructions (holonomy, curvature) for non-Lie-group transform suites.

The framework localizes where remaining risk lives; it does not eliminate all risk.

1.5 Contributions

The core invariance property ($\Sigma = \tilde{\Sigma} \circ \kappa$) is mathematically standard. The contributions of this paper are:

- **Two-regime framework:** Distinguishing the engineering regime ($\mathcal{G}_{\text{declared}}$, partial/non-invertible, $\mathcal{X}$ possibly discrete) from the geometric regime (G Lie group, smooth structure on $\mathcal{X}^*$).
- **Canonicalizer-induced quotient:** Defining the engineering-regime equivalence via $x \sim_\kappa y \Leftrightarrow \kappa(x) = \kappa(y)$, avoiding the ill-defined "orbit space" when $\mathcal{G}_{\text{declared}}$ is not a group.
- **Two-part diagnostic framework:** Distinguishing gauge-fixing consistency (canonicalizer bugs, both regimes) from holonomy-based curvature detection (path-dependent exploits, geometric regime).
- **Correct bundle geometry:** Using $B = \mathcal{X}^*/G$ as the base with explicit conditions for when M can serve as proxy, and noting orbifold/stratified structure outside $\mathcal{X}^*$.
- **Well-posed alignment transport:** Replacing exact argmin with approximate optimization including tie-breaking and failure handling.
- **Maxwell-like constraint checklist:** Organizing invariance conditions as source, consistency, and propagation constraints with explicit failure-mode mappings, domain (M vs B), and time-parameter semantics.
- **Stratified barrier encoding:** Formalizing hard vetoes as infinite-cost strata with implementable barrier functions.
- **Discrete Noether framing:** Recasting conservation as "monitored drift" for discrete-time systems where exact Noether fails.
- **Explicit scoping:** The A1–A4 axiom structure that makes guarantees conditional and localizes residual risk.

1.6 Threat Model: Attack → Axiom Violated

Attack Vector	Axiom Violated / Status
Sensor spoofing / tampering	Violates A2 (Measurement Integrity)
Side-channels bypassing monitor	Violates A4 (External Gate)
Out-of-distribution inputs breaking Ψ	Violates A1/A3 (validated envelope)
Re-descriptions outside declared $\mathcal{G}_{\text{declared}}$	Outside suite $\Rightarrow$ no invariance claim
Stealth harms (Ψ fixed, world harmed)	Violates Ψ-completeness (outside scope)
Exploiting discrete-time gaps	Noether degrades to monitored drift
Learned policy finds novel loophole	Holonomy diagnostic may detect; else suite was too narrow
Canonicalizer implementation bugs	Gauge-fixing consistency test detects
Alignment transport failure	Escalate as OOD; indicates inadequate coverage

This mapping makes explicit that the framework provides guarantees within the declared envelope; attacks that violate the axioms are outside scope by design, not by oversight.

2 The Maxwellian Shift

2.1 The Scalar Error

In the history of physics, "interaction" was once viewed as action-at-a-distance between fixed points. Then came Maxwell: the interaction isn't just a number connecting two particles; it's a field with geometric structure.

In AI alignment, we often remain pre-Maxwell: treating "Human Value" as a scalar reward signal R to be maximized [19, 20]. This paper proposes the **Maxwellian Shift for Ethics**:

1. **Value is not only a scalar:** It can be represented as a valuation potential that varies over configuration space. (Scalar utility can be adequate in well-specified, low-dimensional settings; the shift is motivated by high-dimensional systems where proxy gaming and representational degrees of freedom create failure modes.)
2. **Objectivity as invariance:** In the BIP sense, evaluation should not change under semantics-preserving re-descriptions.
3. **Safety via conserved diagnostics:** When a suitable action functional is invariant under continuous symmetry, Noether yields a conserved quantity that can be monitored.

3 The Structural Correspondence

This is more than metaphor: under the Formal Spine definitions, the governance objects form a gauge-theoretic structure formally analogous to classical electrodynamics [24, 25]. We use this correspondence to derive invariance constraints and diagnostics; we do not claim physical identity.

3.1 The Correspondence Table

Electrodynamics	**Alignment Analog**	**Status**
Principal bundle P	Principal stratum $\mathcal{X}^*$ (where G-action is free/proper)	Geometric regime; requires Lie-group G
Base manifold	Orbit space $B = \mathcal{X}^*/G$	Natural base; $\bar{\Psi} : B \to M$ descends
Projection $\pi : P \to M$	Quotient map $\pi : \mathcal{X}^* \to B$	Standard bundle projection
Gauge group $U(1)$	Re-description group G	Invertible subset of $\mathcal{G}_{\text{declared}}$
Connection 1-form A	Connection ω on $\mathcal{X}^* \to B$	Must be specified explicitly
Curvature $F = dA$	Curvature $\Omega = d\omega + \frac{1}{2}[\omega, \omega]$	Detected via holonomy loop test
Gauge transform	Re-description $x \mapsto g \cdot x$	Action of G on $\mathcal{X}^*$
Gauge-invariant $F_{\mu\nu}$	Invariant evaluation $\tilde{\Sigma} \circ q$	Core BIP property
Parallel transport	Horizontal lift along paths in B	Defined by connection
Holonomy around loop	Loop product h	Measures path dependence
Charge density ρ	Moral status density ρ_Ψ	Sources constraint field; $\rho_\Psi > 0$
Magnetic field B	Contextual twist	Heuristic (see Remark 4)
Current J^μ	Alignment current J	Conditional / monitored

Remark 4 (The Magnetic Field Analog—Heuristic Status). *In electrodynamics, $\nabla \cdot B = 0$ is a hard geometric constraint: magnetic field lines form closed loops because there are no magnetic monopoles. In the alignment analog, we interpret B as contextual twist—the component of moral structure that makes evaluation path-dependent or history-sensitive.*

Honest status: We do not have a rigorous proof that contextual twist must be divergence-free in ethical models. The constraint $\nabla \cdot B = 0$ is included for heuristic completeness of the Maxwell analogy, not because the ethical domain demands it. An "open line" of contextual twist would correspond to a situation where path-dependence accumulates without bound in one direction—a kind of "moral ratchet." Whether such configurations are possible or pathological in ethical models is an open question. We flag this as the weakest element of the correspondence.

Remark 5 (Sign Convention for the Obligation Field). *We model ethical constraints as repulsive fields, analogous to electrostatic repulsion between like charges. Moral status is positively charged: a region with $\rho_\Psi > 0$ (e.g., a human) sources field lines pointing outward, exerting "pressure" on the agent's trajectory to prevent collision (harm). The force $F = qE$ points away from the moral patient. This is a constraint model: the field prevents harmful configurations rather than attracting toward beneficial ones.*

Remark 6 (Conservation of Moral Status). *In electrodynamics, charge is locally conserved: $\partial_t \rho + \nabla \cdot J = 0$. Is moral status conserved?*

Cases where ρ_Ψ changes:

- *A human walks into/out of the sensor field $\to$ ρ_Ψ changes smoothly via flux through the boundary.*
- *A human dies $\to$ ρ_Ψ drops discontinuously (no conservation).*
- *An entity gains moral status (e.g., AI sentience recognized) $\to$ ρ_Ψ increases discontinuously.*

Implication: Moral status is not generally conserved. The continuity equation $\partial_t \rho_\Psi + \nabla \cdot J_\Psi = 0$ holds only when status changes occur via spatial flow (movement), not via creation/destruction. When ρ_Ψ can "pop" into existence, the Source Equation ($\nabla \cdot E = \rho_\Psi/\varepsilon_0$) still holds instantaneously, but the dynamical coupling to the Ampère-Maxwell analog requires modification: the "displacement current" term must account for $\partial_t \rho_\Psi$ even when $\nabla \cdot J_\Psi \neq -\partial_t \rho_\Psi$.

This is a dis-analogy with electrodynamics. We retain the Source Equation as a static constraint but flag that the full dynamical system differs when moral status is non-conserved.

3.2 Where the Correspondence is Structural (Not Literal)

- **Dynamics:** The mapping is primarily kinematic unless you specify a concrete Lagrangian.
- **Group structure:** EM uses abelian $U(1)$; alignment groups may be large or non-abelian; engineering suites may not be groups at all.
- **Geometry:** Spacetime is Lorentzian; ethical spaces may be Riemannian or stratified.
- **Monopoles:** $\nabla \cdot B = 0$ is heuristic in ethics (Remark 4).
- **Charge conservation:** ρ_Ψ is not generally conserved.
- **Discrete time:** Noether requires continuous dynamics; discrete systems need separate treatment.
- **Quantization:** No "quantum ethics" is claimed.

4 Maxwell-Like Constraints: What They Detect

Remark 7 (Notation Convention)**.** *We write vector-calculus forms ($\nabla\cdot$, $\nabla\times$) for intuition on the Euclidean portion of $M \subseteq \mathbb{R}^k$. Interpret E and B as components of curvature/connection-derived objects under a chosen decomposition; the vector-calculus notation is mnemonic, not a claim about literal electric and magnetic fields. The coordinate-free formulation uses differential forms. These constraints are best read as a checklist of consistency conditions for any system claiming the Formal Spine, not as a claim that ethics literally instantiates electromagnetism.*

Domain clarification: *These constraints are written on $M \subseteq \mathbb{R}^k$ (the measurement manifold) for notational convenience. Strictly, when $M \neq B$, they should be pulled back via $\bar{\Psi} : B \to M$. The constraints remain meaningful on M when the injectivity and submersion conditions (Remark 1.2) hold.*

Time parameter: *The variable t represents a* ***decision-step index or physical time****, depending on context:*

- *In discrete decision systems: $t \in \mathbb{Z}$ indexes decision steps; ∂_t becomes a finite difference Δ_t.*
- *In continuous-time control: $t \in \mathbb{R}$ is physical time; ∂_t is the standard time derivative.*

The static-regime constraints ($\partial_t B = 0$, $\partial_t E = 0$) apply when context is unchanging between decisions.

4.1 Constraint I: Source Equation (Gauss's Law Analog)

Form: $\nabla \cdot E = \rho_\Psi / \varepsilon_0$

Here $\rho_\Psi : M \to \mathbb{R}_{\geq 0}$ is a scalar moral-status density (positively charged per Remark 3.2).

Generating assumption	Moral patients ($\rho_\Psi > 0$) source the constraint field.
Failure mode detected	Phantom obligations (constraints without patients); invisible harms (patients undetected).
Does not guarantee	Completeness of Ψ; conservation of ρ_Ψ.

4.2 Constraint II: Consistency Equation (Faraday's Law Analog)

Form: $\nabla \times E = -\partial_t B$

When context is static ($\partial_t B = 0$), the obligation field is curl-free. When context changes, curl is induced—order of actions matters. (In simply connected regions of M, curl-free implies a potential structure; globally, holonomy and nontrivial topology can reintroduce path effects even when local curl vanishes.)

Generating assumption	Evaluation is conservative when context is static.
Failure mode detected	Money-pumping; spurious path dependence.
Does not guarantee	Applies only to static regime ($\partial_t B = 0$).

4.3 Optional Heuristic: No Monopoles (Gauss B Analog)

Form: $\nabla \cdot B = 0$

Generating assumption	Contextual twist forms closed loops (no isolated sources).
Failure mode detected	Unbounded directional accumulation of path-dependence.
Does not guarantee	This constraint is heuristic; we lack proof it holds in ethical models.

4.4 Constraint IV: Dynamic Consistency (Ampère-Maxwell Analog)

Form: $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \partial_t E$

Generating assumption	Changes in constraint and context fields propagate consistently.
Failure mode detected	Inconsistent updates leading to global incoherence.
Does not guarantee	Correct propagation law; conservation of ρ_Ψ (coupling may differ).

4.5 Summary Table

Constraint	Detects	Regime	Status
I. Source (Gauss E)	Phantom obligations	All	Strong analog
II. Consistency (Faraday)	Money-pumping	Static	Strong analog
(Optional) No monopoles	Unbounded twist	All	Heuristic only
III. Propagation (Ampère)	Inconsistent updates	Dynamic	Modified if ρ_Ψ non-conserved

5 From Smooth Fields to Hard Vetoes

Standard gauge theory assumes smooth manifolds. Real ethical constraints include hard vetoes ("never do X").

5.1 The Stratified Extension

Definition 1 (Hard Veto as Cost Barrier)**.** *A hard veto is a region $M_i \subset M$ modeled by a barrier cost: $c(x, v) \to +\infty$ as $x \to M_i$.*

Lemma 1 (Barrier Impassability—Conditional)**.** *If a forbidden region M_i has $c(x, v) = +\infty$ for $x \in M_i$, then any finite-cost trajectory cannot enter M_i.*

Remark 8 (Computational Implementation of Barriers)**.** *The mathematical statement "$c = +\infty$" is clean but computationally hazardous. In gradient-based learning:*

- ***Problem:*** *Infinite cost $\Rightarrow$ undefined or exploding gradients.*
- ***Solution 1 (Log barriers):*** *Use $c(x) = -\mu \log(d(x, M_i))$ where d is distance to forbidden region. As $x \to M_i$, $c \to +\infty$, but gradients remain finite for $x \notin M_i$. This is standard in interior-point optimization [17, 18].*
- ***Solution 2 (Projection):*** *After each gradient step, project back to the admissible set. The "infinite barrier" is implemented as a hard constraint in the optimizer, not in the loss.*
- ***Solution 3 (Reflex gating):*** *The learner never sees the barrier directly. An external monitor (DEME-style [4]) intercepts trajectories approaching M_i and overrides actions. The learner operates in a "padded" space where the true boundary is never reached.*

The mathematical guarantee (finite-cost trajectories cannot enter) holds; the implementation requires one of these mechanisms to avoid numerical collapse.

Scope & Limitations: The stratified extension assumes the cost formulation extends to stratified settings. Implementation requires barrier functions, projection methods, or external gating—not literal $+\infty$ in the loss.

6 Conclusion

6.1 What This Formalization Provides

We are not relying solely on behavioral exhortations or learned preferences. We are building systems where certain classes of misalignment-by-representation are as constrained as violating an invariance law—within a declared measurement and verification envelope.

The Conservative Claim:

Given Axioms A1–A4, the gauge-theoretic framework makes semantic and representational evasion structurally unavailable. The guarantees are:

- **Unconditional given A1–A4:** Invariance under declared $\mathcal{G}_{\text{declared}}$ [Engineering regime]
- **Conditional on Lie-group structure:** Holonomy-based curvature diagnostics for path-dependent exploits [Geometric regime]
- **Conditional on continuous dynamics:** Noether conservation (or monitored drift for discrete systems)
- **Conditional on barrier implementation:** Hard veto impassability

6.2 What This Does NOT Provide

- **Choosing Ψ:** Grounding adequacy remains a governance problem.
- **Specifying $\mathcal{G}_{\text{declared}}$ correctly:** Verifying semantic equivalence in high-dimensional spaces (LLMs, vision) remains hard.
- **Implementation correctness:** Bugs can violate guarantees.
- **Physical security:** Sensor spoofing requires separate engineering.
- **Conservation of moral status:** ρ_Ψ can be created/destroyed, breaking some dynamical analogs.
- **Monopole constraint:** $\nabla \cdot B = 0$ is heuristic, not proven for ethical models.
- **Exact Noether for discrete systems:** Discrete analogs provide approximate or modified conservation.
- **Literal $+\infty$ costs:** Implementation requires barrier functions or projection, not infinite loss values.
- **Connection specification:** Curvature diagnostics require explicitly constructing a connection (e.g., via G-invariant metric), not automatic from canonicalizer choice.
- **Geometric regime for all transforms:** Principal-bundle constructions require a Lie-group subset G; the full engineering suite $\mathcal{G}_{\text{declared}}$ may include partial/non-invertible transforms outside geometric scope.

The framework localizes where risk lives; it does not eliminate all risk.

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