A Survey of Manoeuvring Target Tracking Methods

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Graham W Pulford
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Graham W. Pulford[†]

[†]Department of Electrical and Electronic Engineering, University of Melbourne, Parkville 3052, Australia and the Cooperative Research Centre for Sensor Signal and Information Processing (CSSIP)

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Abtract

A comprehensive review of the literature on manouevring target tracking for both uncluttered and cluttered measurements is presented. Various discrete-time dynamical models including non-random input, random-input and switching or hybrid system manoeuvre models are presented. The problem of manoeuvre detection is covered. Classical and current filtering methods for manoeuvre tracking are presented including multi-level process noise, input estimation, variable dimension filtering, two-stage Kalman filter, the interacting multiple model algorithm, and generalised pseudo-Bayesian algorithms. Various extensions of these algorithms to the case of cluttered measurements are also described and these include: joint manoeuvre and measurement association, probabilistic data association and multi-hypothesis tracking. Smoothing schemes, including IMM smoothing and batch expectation-maximisation using the Viterbi algorithm, are also described. The use of amplitude information for target measurement discrimination is discussed. It is noted that although many manoeuvre tacking techniques exist, the literature contains surprisingly few performance comparisons to guide the design engineer although a performance benchmark has recently been introduced.

1 Introduction

The problem of tracking a manoeuvring target based on noisy sensor measurements has been approached by a great many researchers in the last few decades. The comprehensive taxonomy of the existing approaches presented in this article should convince the reader that this is a very active research area. Manoeuvres correspond to unknown acceleration terms in the target dynamics, increasing the number of quantities that must be estimated compared with the constant velocity case. The fundamental problems in manoeuvring target tracking are (i) manoeuvre detection: the detection of the onset (and end) of a manoeuvre, and (ii) compensation: the correction of target state estimates to allow for the manoeuvre. A third problem of tracking a manoeuvring target in clutter is discussed subsequently. Classical techniques based on adjustments or slight modifications to the discrete-time Kalman filter include increasing the process noise covariance during a manoeuvre, finite memory (sliding data window) and fading memory (exponential forgetting) filters [69]. All of these approaches increase the effective filter bandwidth so as to respond more rapidly to the measurement data during a manoeuvre; but they are risky in the presence of clutter since they increase the size of the validation gate.

Some of the more recent approaches to manoeuvring target tracking include: adaptive Kalman filters [61, 39], filters using correlated and semi-Markov process noise [120, 119, 60] usually implemented using multiple model (partitioning) filters and filter banks [131, 58, 95, 114, 125, 141]; filters based on Poisson and renewal process models of acceleration [86, 128]; input estimation [35, 36] and input and onset-time estimation [33, 38, 116]; variable dimension filters [13]; track splitting filters with a finite memory constraint [147]; the generalised pseudo-Bayesian (GPB) algorithm [68, 1]; and the interacting multiple model (IMM) algorithm [31, 28, 96]. A second-order extension of the IMM algorithm was developed in [26].

Some researchers have developed approaches to avoid the two-stage nature of manoeuvre detection and compensation implied by methods like variable dimension filtering. These include concurrent input estimation (of acceleration and manoeuvre onset time) and variable-dimension filtering [103]; and an interacting multiple model filter including a bias estimator [144, 25, 3] which treats the acceleration as a bias term to be estimated and used for compensation of the target state estimate. A filter developed in [2] uses a kinematic speed constraint in the measurement equation for tracking constant-speed manoeuvring targets (the acceleration should be orthogonal to the velocity).

A brief overview of some of these techniques is contained in [54], while a detailed treatment of the filter bank approach using a semi-Markov process noise model may be found in [51]. A comparison of input/onset-time estimation with the interacting multiple model algorithm appeared in [13] (with follow-up comments contained in [52] and [65]), demonstrating the comparable performance of the IMM algorithm at lower computational complexity. Several methods based on partitioning (multiple model) filters have been reviewed in the context of bearings-only tracking for non-manoeuvring and manoeuvring targets in [71], and their performance compared with that of a single Kalman filter technique.

A technique for complexity reduction of existing manoeuvring target trackers based on the information form of the Kalman filter was presented in [115, 53]. Manoeuvres were modelled as continuous-time stochastic process with known prior density and a conditional mean estimator derived in [21]; the estimator is implementable when the input process has a finite number of discrete values with known prior probabilities.

A comparison of Kalman and H_∞ filters for tracking constant-speed manoeuvring targets recently appeared in [137]. The conclusion reached was that an "untuned" H_∞ filter does not outperform an appropriately tuned Kalman filter, and that the H_∞ filter tends to be more robust to mistuning. The issue of tuning in H_∞ filtering is important firstly in that it requires the selection of an extra parameter γ, and secondly since the filter is not realisable if γ is made too small.

The literature on manoeuvring target tracking is not restricted to filtering as evidenced by work on fixed-interval smoothing using interacting multiple models [63]; and measurement concatenation (block processing) applied to variable dimension filtering [43, 41, 42]. As remarked in [54], the variable dimension filtering approach is in fact a smoothing technique, since it requires the accelerating target model to be initialised retrospectively, *i.e.*, using measurements prior to the instant of manoeuvre detection.

There are also off-line approaches to the problem of manoeuvring target tracking which require a batch of measurement data for processing. The bearings-only tracking problem for a manoeuvring target has been attempted via a batch method that uses simulated annealing to initialise an optimisation routine that determines maximum likelihood state estimates. The Expectation-Maximisation (EM) algorithm [19, 132, 129] has recently been applied by the author to determine the maximum *a posteriori* sequence of target manoeuvres in a multi-level white noise model [106]. The EM algorithm is a multi-pass technique that requires a complete track estimate for initialisation. A recursive, sub-optimal version of the preceding algorithm has also been developed and has been shown to be closely related to the Viterbi manoeuvre tracker in [8].

Some less conventional approaches to manoeuvring target tracking include fuzzy systems and artificial neural networks. The fuzzy logicians [73, 101] have concentrated on emulating alpha-beta trackers, which are simplified Kalman filters using precomputed steady-state gains. Approaches based on artificial neural networks have considered estimation or modelling of the target acceleration using neural nets [4], or replacing the entire Kalman filter or PDA algorithm with a neural network [130, 153, 152]. The simulations reported in these articles are unconvincing or lack any comparison with accepted techniques. It is also unclear how the important issue of tuning or "learning" should be dealt with, since algorithms based on neural networks often have many adjustable parameters.

The considerably more difficult problem of tracking a manoeuvring target in a cluttered environment has also received much attention. As well as unknown acceleration terms with unknown onset times, the tracker must solve the additional problem of data association, that is, determining which of several possible measurements is to be associated with the target. In the absence of target manoeuvres, tracking in clutter is often achievable using extensions of Kalman filtering such as the probabilistic data association (PDA) filter [12] or other, more computationally intensive techniques [9]. These algorithms must be modified to maintain track during target manoeuvres. It is clear that the objectives of manoeuvre detection and data association are somewhat in conflict, since the appearance of a false measurement can easily lead to an incorrect assumption of a target manoeuvre. The classical approach of nearest-neighbour Kalman filtering often suffers from filter divergence in cluttered environments. Recent PDA-type approaches which have been applied to tracking manoeuvring targets in clutter include: Bayesian adaptive filters with multi-level white or coloured noise [57, 133, 134, 135]; joint measurement and manoeuvre association filters for a single target [82] and for multiple targets [117, 78]; variable dimension Kalman and PDA filters [23, 121]; interactive multiple model PDA [14, 15] and IMM integrated PDA [64]; PDA with "prediction-oriented" multiple models [98]; PDA filters using ranking information [100] or amplitude information [83, 84] to enhance the discrimination of targets from clutter. Limited-memory (N-scan-back) filters featuring multi-level white noise with Markov switching are developed in [72, 140].

A good review of manoeuvring target tracking techniques, together with some comparative studies of algorithm performance is contained in [9, 10]. A MHT tracker using an adaptive re-initialisation procedure is described in [112]; this approach has been applied in target motion analysis of acoustic sources in clutter and performs better than the IMM at low SNR's. A JPDA approach to multiple, manoeuvring target tracking using an innovations-based manoeuvre detector and compensator was presented in [40]; however the simulations assumed an unrealistically low clutter density.

Trackers based on hidden Markov models, using the Viterbi algorithm [55] to perform an efficient search over a discretised state space, have been used to track targets with limited manoeuvrability in the presence of false alarms and other interference [89, 46]. In another paper [47] a HMM-based tracker is presented, but the emphasis is on the ability of the technique to handle non-linear measurement models rather than on manoeuvring in clutter. The discrete state-space HMM trackers developed in [123, 150, 151] for single and multiple frequency line tracking, which work for time-varying tones, are known to be adaptable to the cluttered measurement case by suitable modification of the observation probability density, although this is not explicitly stated therein. Barniv's dynamic programming algorithm [17, 18] for tracking low SNR targets in infra-red image sequences was extended in [6] to allow for cluttered measurements.

The Viterbi algorithm can also be applied to tracking problems *without* discretisation of the state and measurement spaces. A first approach along these lines is contained in [111] in the context of sonar tracking. A subsequent study, reported in [107, 108, 79], extends the previous work to include automatic track maintenance and measurement gating. This technique, called *Viterbi data association* (VDA) has been applied successfully to the data association problem in over-the-horizon radar (OTHR) and more recently extended to allow for manoeuvring targets [80, 81] based on the two-stage Kalman filter. Modifications to the transition cost function of this data association technique to allow for tracking of a manoeuvring target were also briefly described in [111] but have been found to be difficult to tune, at least in OTHR scenarios.

The structure of the paper is now described. Section 2 introduces several classes of dynamical model for manoeuvring targets which have been used in radar and sonar. Section 3 deals with the problem of manoeuvre detection, that is, estimation of the onset time of a target manoeuvre based on noisy observations of the target state. In section 4 the main methods of manoeuvring target tracking are summarised. We cover the extension to the case of cluttered measurements in section 5. In section 6 we summarise our observations concerning the manoeuvring target tracking problem based on the surveyed literature.

2 Dynamical Models for Manoeuvring Targets

In the following sections we review the literature on manoeuvring target tracking, focussing first on the case of zero false alarms and later on the cluttered measurement case. The starting point for our survey is a description of dynamical models for manoeuvring targets. These fall into several broad categories depending on how the manoeuvre dynamics $x(k)$ are modelled [9, 10]. We use $y(k)$ to denote the target measurement at time k and Y^k to denote the measurement set to time k. We assume familiarity with the discrete-time Kalman filter and smoother, and with Bayesian estimation theory (refer to, for example, [5, 10]).

1. Systems of the form

$$\begin{aligned} x(k+1) &= F(k)x(k) + G(k)u(k) + v(k) \\ y(k) &= H(k)x(k) + w(k) \end{aligned} \tag{1}$$

   where the unknown input (acceleration) process $u(k)$ is a random process and $v(k)$ is the (small) filter process noise term, which controls the filter bandwidth. In this kind of model the target acceleration $u(k)$ is modelled as an additional process noise term. Usually a finite-state process is assumed to describe the possible types of target manoeuvre as is the case in the Markov and semi-Markov process noise models in which the acceleration switches between several possible levels according to given transition probabilities and with given distributions governing the switching times for each level.

2. Systems with unknown inputs:

$$\begin{aligned} x(k+1) &= F(k)x(k) + G(k)u(k) + v(k) \\ y(k) &= H(k)x(k) + w(k) \end{aligned} \tag{2}$$

   where the input $u(k)$ is deterministic but unknown and must be estimated. Often $u(k)$ is assumed to be piecewise constant, such as in the multi-level noise case

$$\begin{aligned} x(k+1) &= \begin{cases} F(k)x(k) + G(k)u_1(k) + v(k) & \text{for model 1} \\ F(k)x(k) + G(k)u_2(k) + v(k) & \text{for model 2} \\ \vdots & \vdots \\ F(k)x(k) + G(k)u_r(k) + v(k) & \text{for model r} \end{cases} \\ y(k) &= H(k)x(k) + w(k) \end{aligned} \tag{3}$$

   where it is assumed that only one of the r possible target models in effect at any one time.

3. Multiple dynamic models

$$\begin{aligned} x(k+1) &= F(k; M(k))x(k) + v(k; M(k)) \\ y(k) &= H(k; M(k))x(k) + w(k; M(k)) \end{aligned} \tag{4}$$

in which the model type M(k), also called the *mode*, is a finite-state Markov chain taking values in the set $\{M_1, \ldots, M_r\}$. The transition probabilities of the Markov chain are usually assumed to be known and the modes are mutually exclusive.

Systems (3) and (4) are called *hybrid systems* since they contain both continuous-valued states and discrete parameters (the model type or input level). In the control literature, system (3) is referred to as a switching control system, and system (4) a jump-linear system. In tracking, unlike control theory, the input term $u(k)$, whether deterministic or random, is unobservable and must be estimated or allowed for by switching to an appropriate model.

The particular model structures can be chosen to reflect the expected target dynamics. This might include models such as constant velocity, constant acceleration, constant turn-rate and constant speed (co-ordinated turn), *etc.* The filter process noise $v(k)$ is used to represent small variations in acceleration about the nominal target trajectory, while the manoeuvre input $u(k)$, or mode $M(k)$ in the multiple model case, is used to represent significant levels of acceleration. The interested reader is referred to [20, 146, 145, 2, 29] for further examples of target manoeuvre models.

It is well known that optimal (Bayesian or MAP) state estimation for hybrid systems generally requires filters with increasing memory requirements. This has lead to several sub-optimal filtering techniques with bounded complexity such as the interactive multiple-model algorithm [31] and the class of generalised pseudo-Bayesian algorithms [1, 37]. Readable accounts of estimation techniques for switching systems include [104], and [138], which covers partitioning filters, the GPB algorithm and the random sampling algorithm. An IMM algorithm for semi-Markov switching systems was developed in [34]. We return to a discussion of filters for hybrid systems in section 4.

2.1 Correlated Process Noise (Singer) Model

The archetypal work on modelling a manoeuvring target was done in [120]. Target manoeuvres are characterised by large deviations from a constant-velocity trajectory corresponding to a random acceleration $a(t)$. Thus, in one dimension of motion, if the components of the state vector $x(t)$ are $x_1(t)$ - the target displacement and $x_2(t)$ - the velocity, the target dynamics are described by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a(t).$$

Manoeuvres are parametrised by an acceleration variance σ_m^2 and a time constant $1/\alpha$. During a manoeuvre, the target acceleration is *correlated* with autocorrelation function

$$r(\tau) = \mathrm{E}\{a(t)a(t+\tau)\} = \sigma_m^2 \mathrm{e}^{-\alpha|\tau|}.$$

The same autocorrelation results from a linear system

$$\dot{a}(t) = -\alpha a(t) + v(t)$$

driven by white noise $v(t)$ with autocorrelation function $2\alpha\sigma_m^2\delta(t)$ ($\delta(\cdot)$ is the Kronecker delta function). Augmenting the state with the target acceleration $a(t)$, the dynamical model for the manoeuvring target becomes

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\alpha \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v(t) \tag{5}$$

where $v(t)$ is white noise with variance $2\alpha\sigma_m^2$.

The continuous-time model can be discretised in the usual manner [7] by uniform sampling with period T. The resulting discrete-time model is (with a slight abuse of notation)

$$\begin{aligned} x(k) &= \begin{bmatrix} \text{target displacement at time } kT \\ \text{target velocity at time } kT \\ \text{target acceleration at time } kT \end{bmatrix} \\ x(k+1) &= F_\alpha(T)x(k) + u(k) \end{aligned} \tag{6}$$

where the transition matrix $F_\alpha(T)$ may be shown to be

$$F_\alpha(T) = \begin{bmatrix} 1 & T & (\alpha T - 1 + \mathrm{e}^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1-\mathrm{e}^{-\alpha T})/\alpha \\ 0 & 0 & \mathrm{e}^{-\alpha T} \end{bmatrix} \tag{7}$$

and where $u(k)$ is a zero-mean white noise sequence with covariance matrix $Q(k)$ whose elements q_{ij} are given by

$$\begin{aligned} q_{11} &= \sigma_m^2[1 - \mathrm{e}^{-2\alpha T} + 2\alpha T + \frac{2}{3}\alpha^3T^3 - 2\alpha^2T^2 - 4\alpha T\mathrm{e}^{-\alpha T}]/\alpha^4 \\ q_{12} &= \sigma_m^2[\mathrm{e}^{-2\alpha T} + 1 - 2\mathrm{e}^{-\alpha T} + 2\alpha T\mathrm{e}^{-\alpha T} - 2\alpha T + \alpha^2T^2]/\alpha^3 \\ q_{13} &= \sigma_m^2[1 - \mathrm{e}^{-2\alpha T} - 2\alpha T\mathrm{e}^{-\alpha T}]/\alpha^2 \\ q_{22} &= \sigma_m^2[4\mathrm{e}^{-\alpha T} - 3 - \mathrm{e}^{-2\alpha T} + 2\alpha T]/\alpha^2 \\ q_{23} &= \sigma_m^2[\mathrm{e}^{-2\alpha T} + 1 - 2\mathrm{e}^{-\alpha T}]/\alpha \\ q_{33} &= \sigma_m^2[1 - \mathrm{e}^{-2\alpha T}]/\alpha. \end{aligned} \tag{8}$$

The manoeuvre parameter α may be chosen to model various classes of target manoeuvre, *e.g.*, lazy turn, evasive manoeuvre, *etc.* If the sampling time is sufficiently small compared with the manoeuvre parameter, *i.e.*, $\alpha T \ll \frac{1}{2}$, the following asymptotic form of $Q(k)$ may be used:

$$\bar{Q}(k) = 2\alpha\sigma_m^2 \begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix}. \tag{9}$$

If noisy measurements of the target are available, *viz.*

$$y(k) = H(k)x(k) + w(k), \;\; w(k) \sim N\{0, R(k)\},$$

where $N\{m, C\}$ denotes a Gaussian density[1] with mean m and covariance C, then it is straightforward to derive a Kalman filter for minimum mean-square error estimation of the manoeuvring target's state (see section 4).

[1] We will also use $N\{y; m, C\}$ to denote the value at y of the Gaussian density with mean m and covariance C.

The Singer model has been extended in [62] to reflect the tendency of targets to have a constant forward speed with acceleration normal to their velocity. Under this model the turn rate is assumed to be uniformly distributed . A higher order "jerk model" of manoeuvring targets has been developed in [92]. This model is obtained by discretising a system similar to (5) the state of which contains a third-order derivative of the position. It is claimed that this model more accurately reflects the behaviour of agile targets. The filter for this fourth-order model requires three-point initiation with the initial jerk estimate taken as zero.

2.2 Semi-Markov Correlated Process Noise Model

The correlated process noise model of Singer was built on in [95, 94, 58, 114]. By allowing the correlated process noise to have a randomly switching mean value, rather than a zero mean value, a more realistic model of a manoeuvring targets was obtained. This model is also more economical than the white Gaussian process noise with randomly switching mean since it requires fewer mean values for accurate modelling than the latter. The system model for a target with state $x(k)$ and measurement process $y(k)$ is in this case

$$\begin{aligned} x(k+1) &= F(k)x(k) + G(k)(u(k) + v(k)) \\ y(k) &= H(k)x(k) + w(k) \end{aligned} \tag{10}$$

where $u(k)$ is a random process taking values in a finite, discrete set $\{u_1, \ldots, u_N\}$; $v(k)$ is the correlated Gaussian (Singer) process noise; and $w(k)$ is a white Gaussian measurement noise. The Singer process noise covariance and the measurement noise covariance are assumed to be known. Since physical target manoeuvres tend to be sustained for a certain "holding" time, the randomly switching mean is modelled as a *semi-Markov* process [67] (Vol. 2), and this is illustrated in Fig. 1. In addition to the transition probabilities for a (time-invariant) Markov chain

$$p_{ij} = \Pr\{u(k+1) = u_j | u(k) = u_i\},$$

a further set of probabilities, the holding-time distributions, must also be specified:

$$p_i(k) = \Pr(\text{transition from } u_i \,|\, k \text{ successive time instants spent in } u_i).$$

To simplify the characterisation of the semi-Markov process, the holding-time distribution is often assumed to be an monotonically increasing (*e.g.*, exponential) function of time so that a transition from manoeuvre state u_i becomes more likely as the time spent in u_i increases. In practice, to reduce the number of free parameters in the model, it may be assumed that each possible manoeuvre state is held for a random holding-time, with the transition matrix of the Markov chain having diagonal elements near unity and equal off-diagonal elements [95].

Minimum variance (conditional mean) state estimation of the manoeuvring target system (10) requires a filter with exponentially increasing memory [1]. Approximate conditional mean filtering is therefore necessary and this take the form of a truncated, weighted sum of Kalman filter estimates matched to the possible manoeuvre sequences. By forcing the covariance update for each filter in the bank to be identical, it is possible to reduce the filter complexity to that of a Kalman filter with N state updates [51, 94] (see section 4.1).

2.3 Poisson Processes and Renewal Models

Target manoeuvres have been modelled as a finite-state jump process with Markov transitions [86]. The theory for these models is that of discrete-state, continuous-time Markov processes [22] and is beyond the scope of this article. We will however summarise the main points of these alternative approaches to manoeuvre modelling and the different filter structures which arise.

Consider the case of motion in one dimension represented by the discrete-time system

$$\begin{aligned} x(k+1) &= Fx(k) + G(u(k) + v(k)),\ v(k) \sim N\{0, Q\} \\ y(k) &= Hx(k) + w(k),\ w(k) \sim N\{0, R\} \end{aligned}$$

where $u(k)$ is a scalar process corresponding to the jumps from one manoeuvre state to another, and the other symbols have their usual meanings. Let the possible manoeuvre states, shown in Fig. 2, be denoted $\{0, \pm 1, \pm 2, \ldots, \pm n\}$, with state 0 corresponding to a constant-velocity target. Note that transitions are only allowed between adjacent manoeuvre states, and the transition rates λ_{ij} (probability per unit time) between states i and j are assumed to be known constants. If $p_i(k) = \Pr\{u(k) = i\}$ denotes the probability that the target is in manoeuvre state i at time k, then for $i \neq \pm n$,

$$p_i(k + \Delta) = \lambda_{i-1,i}\Delta p_{i-1}(k) + (1 - \lambda_{i,i+1}\Delta - \lambda_{i+1,i}\Delta)p_i(k) + \lambda_{i+1,i}\Delta p_{i+1}(k)$$

where Δ is a small time increment. Similar recursions hold for the two states at the end of the chain $i = \pm n$. A (constrained) linear, minimum variance filter for the state estimates is then derived based on noisy observations. The state and covariance update for the filter are the same as those in the Kalman filter, but the prediction equations for the state and covariance are different. For example, the state prediction is

$$\hat{x}(k+1|k) = F\hat{x}(k|k) + Gb(k+1)$$

where the term $b(k + 1)$ is the predicted acceleration from the Poisson process, and may be computed using the transition rates λ_{ij} and posterior manoeuvre state probabilities $\Pr(u(k) = i|Y^k)$. The probabilities $\Pr(u(k) = i|Y^k)$ presumably satisfy a recursion, although this is not provided in [86]. The covariance prediction is of the form

$$P(k+1|k) = FP(k|k)F' + GQG' + GBG'$$

where B is a diagonal matrix determined from the $\Pr\{u(k) = i\}$ and the $\Pr(u(k) = i|Y^k)$ probabilities.

No comparison with the classical manoeuvre modelling techniques is provided. More importantly, it is unclear how robust the filter is when the transition rates of the manoeuvre process are not precisely known.

Finite-state renewal process models of target manoeuvres have been investigated in [128]. Renewal processes [45] are generated by the differences between consecutive transition times (or sojourn times) of a point process. For example, if $\{t_1, \ldots, t_n, \ldots\}$ is a Poisson process, then $\{\tau_1 = t_1, \tau_2 = t_2 - t_1, \ldots, \tau_n = t_n - t_{n-1}, \ldots\}$ is a renewal process [102]. Renewal processes, which are Markov processes, are characterised up to an initial distribution by a matrix of transition probabilities $\mathbf{P}$ between the manoeuvre states $\{u_1, \ldots, u_N\}$, and a set of sojourn-time distributions $p_i(\tau|u(k) = u_i)$. Thus both the transitions between manoeuvre states and the time spent in each state following a transition are random.

A general characteristic of Markov and semi-Markov manoeuvring target models is that the sojourn times (*i.e.*, the time spent in each manoeuvre state) are exponentially distributed. Thus the sample paths from such processes can contain manoeuvring segments of very short duration, which may not be physically realistic.

A model for an agile target that executes co-ordinated (constant turn rate at constant speed) turns at instants governed by a renewal process in developed in [128]. A dynamical model for constant-turn rate motion in the (x_1, x_2) plane is [10]

$$\ddot{x_1} = -\omega \dot{x_2}, \ \ddot{x_2} = -\omega \dot{x_1}$$

where ω is the (constant) angular velocity. The state-space model for the manoeuvring target is then[2]

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \\ \ddot{x_1}(t) \\ \ddot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\omega(t) \\ 0 & 0 & \omega(t) & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} v(t) \tag{11}$$

where $v(t)$ is a white noise process and $\omega(t)$ a finite-state renewal process modelling the random rate and duration of the turns.

The transition probabilities of the renewal process are assumed known. Each sojourn-time distribution is characterised by Gamma density with known parameters

$$p_\gamma(t) = \begin{cases} \frac{\lambda(\lambda t)^{r-1}}{\Gamma(r)} \exp(-\lambda t) & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $r > 0$ controls the shape of the density and $\lambda > 0$ governs the time scale. Notice that for $r = 1$, the Gamma density reduces to an exponential PDF, but for $r > 0$ the Gamma density function is zero at the origin, so that very short sojourns in the manoeuvre states are uncommon.

The model (11) is not well suited to extended Kalman filtering since it is has non-Gaussian noise multiplying the state components. The dynamical process can be approximated by a Gaussian process with power spectral density (PSD) matched to that of the equilibrium PSD of the non-stationary process generated by (11). Analogously to the Singer case, a state-space system augmented by the manoeuvre $\omega(t)$ process is obtained. A second order matching to the PSD is required for a reasonable approximation to the original system, and depends on the time constant of the matching filter and the turn-rate variance $\mathrm{E}\{\omega^2\}$. The dimension of the equivalent continuous-time Gauss-Markov system is then six and this system is amenable to extended Kalman filtering. Results of simulations performed in [128] indicate that the performance improvement from this matched EKF method is not commensurate with its increased complexity for single-sensor systems. However, the performance gain is significant for image-enhanced systems [127] such as a microwave radar teamed with an infra-red imaging sensor. It should be stressed that the renewal model of manoeuvres was developed for highly agile targets which are not well modelled by the conventional semi-Markov approach.

[2]Strictly speaking, we should write this as a stochastic differential equation since the derivative of the Brownian motion term v is not defined.

3 Manoeuvre Detection

Many manoeuvring target tracking techniques require a method for detecting the manoeuvre before compensation of the state estimate can be accomplished. This is the case in approaches which use a "quiescent" constant-velocity model such as the variable-dimension filter [9, 13] which uses a low order Kalman filter to provide high accuracy tracking of nearly constant velocity targets, and switches to a higher order filter once the manoeuvre is detected. Manoeuvre detection is also used in an adaptive Kalman filter to switch to a higher level of process noise during a manoeuvre. The input estimation technique [35] also requires knowledge of the manoeuvre onset time. The IMM algorithm (section 4.4) and its variants do not require separate manoeuvre detection logic.

Manoeuvre detection can be formulated as a hypothesis testing problem and implemented by likelihood ratio test (LRT) based on the Kalman filter innovations. The central idea is that the sequence of Kalman filter innovations are white Gaussian random variables for a correctly modelled system. The probability of the measurement sequence is then expressible solely in terms of the innovations and their covariances. The statistics of the Kalman filter innovations can therefore be checked to see if they are consistent with the hypothesised model of the target dynamics.

Likelihood ratio tests and other forms of hypothesis testing are described in detail in [139]. The likelihood ratio test (LRT) arises from the Neyman-Pearson criterion and uses (i) a test statistic, and (ii) a threshold for the test. If the test statistic, which is a function of the measurement data, exceeds the threshold, a manoeuvre is declared; otherwise no manoeuvre is declared. Assuming that the probability distribution (of the test statistic) under the manoeuvre and non-manoeuvre hypotheses are known, fixing a value for the probability of false alarm P_{MFA} determines the threshold for the test. The probability of manoeuvre detection P_{MD} [131] and can then be calculated for the chosen threshold. There is clearly a compromise since we cannot make P_{MD} arbitrarily large without also raising P_{MFA}.

The following description of an innovations-based manoeuvre detector is from [9]. If $\nu(k)$ denotes the Kalman filter innovation, *i.e.*, the difference between the measurement and its prediction, and $S(k)$ is the corresponding innovations covariance matrix, then the normalised squared innovation is defined as

$$\epsilon(k) = \nu'(k)S^{-1}(k)\nu(k). \tag{12}$$

In a correctly tuned KF with Gaussian inputs, $\epsilon(k)$ is a $\chi^2_{n_y}$ random variable where n_y is the number of measurement components. The modified likelihood function of the Kalman filter is given by

$$\lambda(k) = \sum_{i=1}^{k} \epsilon(k) \tag{13}$$

and forms the test statistic for manoeuvre detection. This function can be made made responsive to recent data by applying an "exponential forgetting factor" [87] yielding the fading-memory likelihood function

$$\rho(k) = \sum_{i=1}^{k} \alpha^{k-i}\epsilon(i) \tag{14}$$

which satisfies the recursion

$$\rho(k) = \alpha\rho(k-1) + \epsilon(k). \tag{15}$$

The forgetting factor α (not the same as in the Singer model) is between 0 and 1, and yields an effective memory length of $1/(1-\alpha)$. Equivalently, the modified likelihood function can be implemented as a sliding window sum. The mean value of $\rho(k)$ can be shown to be $n_y/(1-\alpha)$ [9]. The detection test compares $\rho(k)$ to a threshold to determine whether or not a manoeuvre has occurred. Similarly, a hypothesis test on the significance of the acceleration estimate from the manoeuvring Kalman filter is used to determine when the manoeuvre has terminated.

It is important to detect manoeuvres as soon as they occur, but there is a trade-off between the detection delay and the probability of falsely declaring that a manoeuvre has occurred. In [142] a method was proposed to minimise the delay in manoeuvre detection. Their technique uses a sliding window over which the Kalman filter residuals are summed:

$$D_L(k) = \sum_{i=k-L+1}^{k} \nu(i).$$

The random variable $D_L(k)$ is normally distributed with zero mean for a constant-velocity target but has a non-zero mean if the target is accelerating (the mean can be calculated from the system matrices and the target acceleration over the window). For a given false alarm probability P_{FA} it is possible to compute the average manoeuvre detection delay, which depends on the system matrices, the target acceleration, P_{FA} and window length L. With all parameters except L fixed, it is then possible to determine numerically the optimum window length that minimises the manoeuvre detection delay.

In tracking and other practical scenarios, the probability densities for hypothesis testing may not be precisely known, rendering it difficult to determine a test statistic and threshold for the likelihood ratio test. Moreover, incorrect modelling of the required probability densities decreases the power of the test, *i.e.*, its ability to detect a manoeuvre when one has occurred.

The same remarks apply to generalised likelihood ratio tests (GLRT's) where the probability densities in question depend on an unknown parameter (*e.g.*, the acceleration) or composite hypothesis. In a GLRT, maximum likelihood estimates of the parameter are used in a LRT to make a decision on which hypothesis is true. An example of an application of a GLRT in manoeuvre detection may be found in [91, 77] and is summarised in [51].

If it is desired to estimate both the onset time of the manoeuvre and the target acceleration at this time, the following hypothesis test on the Kalman filter innovation sequence can be set up

$$\begin{aligned} H_1 : &\quad \nu(k) = \nu_0(k) + Gu(\tau) \\ H_0 : &\quad \nu(k) = \nu_0(k) \end{aligned}$$

where the acceleration $u(\tau)$ is zero for $k < \tau$ and constant thereafter, and $\nu_0(k)$ is the residual for a constant-velocity target. A sufficient statistic for the test is

$$L = \frac{\mathrm{p}(\nu(k)|Y^k, H_1)}{\mathrm{p}(\nu(k)|Y^k, H_0)}$$

which is implemented by discretising the possible accelerations and onset times u and τ and passing the filter innovation through a bank of low-pass filters matched to the possible combinations of u and τ. The optimum estimate of u and τ corresponds to the filter with the largest magnitude output. Although this technique estimates both the acceleration and onset time of the manoeuvre, the detection is delayed by the memory of the low-pass filter, and the complexity of the algorithm is substantial.

Several techniques exist in the literature for designing more robust detection tests, these include the marginalised likelihood ratio test (MLRT) [59] and a robust LRT reported in [148]. It seems that these techniques have so far not been applied in manoeuvring target tracking. The MLRT is designed for systems undergoing sudden changes when the onset time and magnitude of the change are unknown. It uses marginalisation to remove the dependence of the probability densities on the parameters and also does not require a threshold. Despite this advantage, the MLRT has a high complexity and also requires data smoothing. The robust LRT in [148] focusses mainly on developing a hypothesis test for nonlinear dynamical systems in low signal-to-noise ratio conditions.

4 Manoeuvring Target Tracking Techniques

In this section we review the existing techniques for tracking a manoeuvring target in the absence of clutter. The literature on this subject is replete with claims of superior performance; however, many simulation studies are unconvincing or fail to take account of existing approaches in their performance comparisons. The general absence of accepted tests for manoeuvring and multi-target tracking has been noted in [27]. Recently a benchmark problem for manoeuvring target tracking with a phased-array microwave radar was established [27]. This benchmark problem requires the tracker to initiate and maintain track on manoeuvring targets within a minimum number of scans (on average), subject to a maximum track loss probability of 4%. Test targets are subject to position and manoeuvrability constraints while fading, missed detections, beam shape and resolution are taken into account.

The Kalman filter is the optimal (MMSE) state estimator for linear observations of a target with known dynamics and white Gaussian process and measurement noise. However, as soon as a target changes its dynamics, *e.g.*, by executing a manoeuvre, the Kalman filter estimates may diverge causing track loss. This divergence is due to incorrect modelling of the target dynamics, rather than finite numerical precision, and is the chief difficulty in manoeuvring target tracking. As summarised in [51], the approaches to manoeuvring target tracking using Kalman filters generally fall into 5 classes:

1. re-initialisation of the Kalman gain (to reduce the effect of past measurements);
2. increasing the process noise covariance;
3. increasing the state estimation covariance;
4. augmenting the filter state with acceleration terms (variable-dimension filter);
5. switching to or applying heavier weighting to another filter in a bank of Kalman filters (multiple-model approaches).

Once a manoeuvre is detected, it becomes necessary to take one of the actions above in order to correct the state estimate and its covariance. Approaches 1–3 above all involve changing the *parameters* of the Kalman filter. For a given application, the adjustment required will depend on the type of manoeuvre, the sampling time, the measurement noise, *etc.* Usually, trial-and-error methods must be used to tune the Kalman filter for manoeuvre tracking. Approaches 4 and 5 involve changing or adapting the *filter structure*, and we describe these later in some detail. The following sections cover the major filtering techniques and structures for manoeuvring target tracking, including: optimal Bayesian filters for multi-level (Markov) process noise and multiple models; single-scan and multiple-scan approximate filters including the interacting multiple-model (IMM) algorithm and the generalised pseudo-Bayesian (GPB) filter; input estimation; the variable dimension filter; the two-stage Kalman filter (which treats the acceleration as a bias term); the IMM smoother; and an expectation-maximisation approach. The multi-hypothesis tracker (MHT) [113] is also of importance in manoeuvring target tracking, and we comment on this in section 5.6.

4.1 Optimal and Approximate Filters for Markov Noise

The optimal (Bayesian) filter can be derived for tracking a manoeuvring target whose acceleration $u(k)$ is represented by a finite-state Markov chain with state space $\{u_1, \ldots, u_N\}$ and known transition probabilities p_{ij}. The system model is, as in equation (3),

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k) + Gv(k), \;\; v(k) \sim N\{0, Q(k)\} \\ y(k) &= Hx(k) + w(k), \;\; w(k) \sim N\{0, R(k)\} \end{aligned}$$

where the state $x(k)$ contains position, velocity and acceleration terms with corresponding F matrix (7) and $v(k)$ is the zero-mean Singer noise component with covariance Q given by (8). In this derivation we will not use the semi-Markov nature of the manoeuvre process $u(k)$. As in [51], we denote the jth sequence of possible target manoeuvres by

$$\Omega_j^k = \{u_{j_1}, \ldots, u_{j_k}\}.$$

From Bayes' rule, the conditional mean estimate of $x(k)$ based on the measurements contained in Y^k is expressible as

$$\hat{x}(k|k) = \sum_{i=1}^{N^k} \Pr(\Omega_i^k | Y^k) \hat{x}_i(k|k) \tag{16}$$

with conditional estimates defined as $\hat{x}_i(k|k) = \mathrm{E}\{x(k)|\Omega_i^k, Y^k\}$. Alternatively, we could take the MAP estimate, *i.e.*, the $\hat{x}_i(k|k)$ for which $\Pr(\Omega_i^k|Y^k)$ is maximum. In either case, the optimal estimate is seen to require the evaluation of N^k terms which implies that the optimal tracker has exponentially increasing computational requirements, as illustrated in Fig. 3.

A natural way of reducing the complexity of the filter is to approximate the N^{k-1}-term mixture PDF of the state at time $k-1$ by a single Gaussian density, thus obtaining a filter with $O(N)$ complexity [95]:

$$\begin{aligned} \hat{x}(k|k) &\approx \sum_{i=1}^{N} c_i(k)\, \hat{x}_i(k|k) \\ c_i(k) &= \Pr(u(k) = u_i | Y^k) \end{aligned}$$

where $\hat{x}_i(k|k) = \mathrm{E}\{x(k)|u(k) = u_i, Y^k\}$, which is generally not the same as $\mathrm{E}\{x(k)|\Omega_i^k, Y^k\}$. The conditional state estimates are obtained by Kalman filtering in the usual manner:

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + W(k)[y(k) - \hat{y}_i(k|k-1)$$

$$
\begin{aligned}
\hat{x}_i(k|k-1) &= F\hat{x}_i(k-1|k-1) + Gu_i \\
\hat{y}_i(k|k-1) &= H\hat{x}_i(k|k-1) \\
W(k) &= P(k|k-1)H'S^{-1}(k) \\
S(k) &= HP(k|k-1)H' + R(k) \\
P(k|k-1) &= FP(k-1|k-1)F' + GQ(k)G' \\
P_i(k|k) &= (I - W(k)H)P(k|k-1)
\end{aligned}
$$

where $P_i(k|k)$ is the error covariance associated with $\hat{x}_i(k|k)$. The net covariance of the state estimate is obtained using a standard formula for Gaussian mixtures [10]

$$
\begin{aligned}
P(k|k) &= \sum_{i=1}^{N} c_i(k)\{P_i(k|k) + \hat{x}_i(k|k)\hat{x}_i(k|k)'\} - \hat{x}(k|k)\hat{x}(k|k)' \\
&= (I - W(k)H)P(k|k-1) - \sum_{i \neq j} c_i(k)c_j(k)\hat{x}_i(k|k)\hat{x}_j(k|k)'
\end{aligned}
$$

where we have used the fact that the $c_i(k)$ sum to unity, and the assumption that the noise covariances Q and R are identical across all manoeuvre modes. To evaluate the posterior manoeuvre probabilities $c_i(k)$, a recursion can be derived using Bayes' rule:

$$
\begin{aligned}
c_i(k) &= \Pr(u(k) = u_i | y(k), Y^{k-1}) \\
&= \Delta^{-1} \mathrm{p}(y(k)|u(k) = u_i, Y^{k-1}) \Pr(u(k) = u_i | Y^{k-1}),
\end{aligned}
$$

writing the normalising constant as Δ^{-1}. Furthermore the above terms are by assumption

$$
\mathrm{p}(y(k)|u(k) = u_i, Y^{k-1}) = N\{\hat{y}_i(k|k-1), S(k)\}
$$

and

$$
\Pr(u(k) = u_i|Y^{k-1}) = \sum_{j=1}^{N} \Pr(u(k) = u_i|u(k-1) = u_j, Y^{k-1}) \Pr(u(k-1) = u_j|Y^{k-1})
$$

so that the desired recursion is

$$
c_i(k) = \Delta^{-1} N\{\hat{y}_i(k|k-1), S(k)) \sum_{j=1}^{N} p_{ji} c_j(k-1).
$$

Finally the state estimate update can be written

$$
\begin{aligned}
\hat{x}(k|k) &= \sum_{i=1}^{N} c_i(k)\hat{x}_i(k|k) \\
&= \sum_{i=1}^{N} c_i(k)\hat{x}_i(k|k-1) + \sum_{i=1}^{N} c_i(k)Gu_i \\
&= (I - W(k)H)(F\hat{x}(k-1|k-1) + G\hat{u}(k)) + W(k)y(k)
\end{aligned}
$$

where $\hat{u}(k) = \sum_{i=1}^{N} c_i(k)u_i$ is the estimated target acceleration at time k. Notice that the Kalman gain and covariances for the conditional state estimate are identical, which greatly simplifies the filter update.

4.2 Input Estimation

The input estimation technique treats the target acceleration as deterministic bu unknown vector. The ensuing description of the technique is from [9] and is based on [35]. Assume a system model of the form

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k) + v(k), \;\; v(k) \sim N\{0, Q(k)\} \\ y(k) &= Hx(k) + w(k), \;\; w(k) \sim N\{0, R(k)\} \end{aligned} \tag{17}$$

for known matrices F, G, H and noise covariances Q, R. The acceleration sequence $\{u(l)\}$ is assumed to be zero for $l < k$, and non-zero for $k \leq l < k+s$. Denote the state estimates from the constant-velocity $u(k) \equiv 0$ target model as $\hat{x}_0(k|k)$ and those for (17) as $\hat{x}(k|k)$. The prediction of these estimates is accomplished via

$$\hat{x}_0(i+1|i) = F(I - W(i)H)\hat{x}_0(i|i-1) + FW(i)y(i). \tag{18}$$

Denote the estimator transition matrix by $\Phi(i) = F(I - W(i)H)$ and the i-step transition matrix by

$$\Phi(k,i) \triangleq \prod_{j=k}^{i} \Phi(j) = \Phi(i)\Phi(i-1)\cdots\Phi(k)$$

Starting with $\hat{x}_0(k|k-1) = \hat{x}(k|k-1)$, and propagating the difference equation (18) [70] we obtain the constant-velocity predictions for $i = k, \ldots, k+s-1$ as

$$\hat{x}_0(i+1|i) = \Phi(k,i)\hat{x}(k|k-1) + \sum_{j=k}^{i} \Phi(k,j-1)FW(j)y(j), \tag{19}$$

whereas the correct predictions are in fact

$$\hat{x}(i+1|i) = \Phi(k,i)\hat{x}(k|k-1) + \sum_{j=k}^{i} \Phi(k,j-1)(FW(j)y(j) + Gu(j))$$

with innovations $\nu(i+1) = y(i+1) - H\hat{x}(i+1|i)$. The innovations are white, zero-mean and have covariance $S(i+1)$ (the innovations covariance in the Kalman filter for (17)). The innovations sequence of the mistuned (zero-velocity) filter is

$$\nu_0(i+1) = y(i+1) - H\hat{x}_0(i+1|i).$$

By assuming that the accelerations $u(j) = u$ are constant over the interval $[k, k+s-1]$, we have that

$$\nu_0(i+1) = \Psi(i+1)u + \nu(i+1), \;\; \Psi(i) = H\sum_{j=k}^{i} \Phi(k,j-1)G$$

which results in a standard least-squares problem for the unknown input vector u. Now write this as

$$\nu_0 = \Psi u + \nu, \;\; \nu \sim N\{0, S\}$$

where $\nu = [\nu(k+1), \ldots, \nu(k+s)]'$, $S = \text{blockdiag}(S(i))$, *etc.* The least-squares estimate for u is then

$$\hat{u} = (\Psi' S^{-1} \Psi)^{-1} \Psi' S^{-1} \nu_0$$

and has mean-square error $P_u = (\Psi' S^{-1} \Psi)^{-1}$. The same solution can of course be obtained by recursive least-squares. The acceleration estimate $\hat{u}$ can be employed to test the manoeuvring hypothesis, with a manoeuvre declared if

$$d(\hat{u}) = \hat{u}' P_u^{-1} \hat{u} \geq c$$

for some threshold c chosen from a $\chi^2_{n_u}$ table for a given false-alarm probability (n_u is the dimension of the input vector).

When a manoeuvre is declared, the constant-velocity state estimate and its covariance are corrected according to

$$\begin{aligned} \hat{x}(k+s+1|k+s) &= \hat{x}_0(k+s+1|k+s) + M\hat{u} \\ P(k+s+1|k+s) &= P(k+s+1|k+s) + M P_u M' \end{aligned}$$

where $M = \sum_{j=k}^{i} \Phi(k, j-1) G$. The manoeuvre is declared finished when the correction becomes insignificant. The key parameter in the algorithm is the window length s the choice of which depends on the sampling interval.

Various modifications of the input estimation technique have appeared [53, 33, 103]. The technique does not require retrospective corrections to the estimate, and reportedly yields reasonable performance [54], but at a relatively high complexity. Input estimation was compared with the IMM filter in [16, 52, 65]. The assumption of known manoeuvre onset time is addressed in [33] where a bank of filters was used to estimate the onset time of the manoeuvre— increasing the complexity of the technique by at least an order of magnitude. It is claimed [53] that enhanced performance can be obtained at significantly reduced complexity using the information form of the Kalman filter [5]. In [33] it is suggested that the technique is well-suited to implementation in the framework of a multi-hypothesis tracker (see section 5).

4.3 Variable-Dimension Filter

The idea behind variable-dimension filtering was already mentioned in section 3. A constant-velocity model for the target is adopted during quiescent (non-manoeuvring) tracking periods. Manoeuvre detection is accomplished by monitoring a fading-memory average of the normalised residuals from the Kalman filter. Once a manoeuvre is detected, the constant-velocity model is abandoned and the algorithm "switches" to an augmented model such as the one-dimensional, mean-jerk acceleration (Wiener process noise) model:

$$x(k+1) = Fx(k) + Gv(k), \ v(k) \sim N\{0, Q(k)\}$$

$$F = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \ Q = \begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix} q \tag{20}$$

where the quantity $\sqrt(qT)$ models the change in acceleration over the sampling interval [9]. Alternatively the Singer model in section 2.1 could be used. For a manoeuvre detector with (effective) window length s declaring a manoeuvre at time k, the augmented model must be initialised at time $k - s$, that is, *retrospectively.* In [54] it is pointed out that, strictly speaking, the variable-dimension filter is not a filter due to the latter requirement. A Monte Carlo comparison of the variable-dimension and input estimation approaches appears in [9]: the variable-dimension filter performed better on average than input estimation, despite the latter being considerably more complex; a two-level white noise model, the simplest algorithm of the three, performed better than the input estimation technique and only slightly worse than the variable-dimension filter.

4.4 Optimal and Approximate Filters for Jump-Linear Models

In contrast to the multiple model approaches based on multi-level noise or Markov inputs, the focus of this section is on variable structure systems of the type (4), repeated here:

$$\begin{aligned} x(k+1) &= F(k;M(k))x(k) + v(k;M(k)),\;\; v(k) \sim N\{0, Q(M(k))\} \\ y(k) &= H(k;M(k))x(k) + w(k;M(k)),\;\; w(k) \sim N\{0, R(M(k))\}. \end{aligned} \tag{21}$$

The "mode process" $M(k)$ is a finite-state Markov chain taking values in $\{M_1, \ldots, M_r\}$ with *known* transition probabilities p_{ij}. The notation $M_i(k)$ signifies that model $M(k) = M_i$ is assumed to be in effect during the time interval $[k-1, k)$. We also denote by M_l^k the lth mode history, in other words

$$M_l^k = \{M_{l_1}(1), \ldots, M_{l_k}(k)\} = \{M_s^{k-1}, M_l(k)\}.$$

There are a total of r^k possible mode histories at time k. We can derive the optimal Bayesian (MMSE) state estimator for (21) by decomposing the posterior PDF of the state as follows:

$$\mathrm{p}(x(k)|Y^k) = \sum_{l=1}^{r^k} \mu_l(k)\, \mathrm{p}(x(k)|M_l^k, Y^k) \tag{22}$$

where $\mu_l(k) = \Pr(M_l^k|Y^k)$ are the *mode probabilities.* A recursion on the mode probabilities is obtained in a similar manner to section 4.1 as

$$\begin{aligned} \mu_l(k) &= \delta^{-1}\mathrm{p}(y(k)|M_l^k, Y^{k-1})\, \Pr(M_l^k|Y^{k-1}) \\ &= \delta^{-1}\mathrm{p}(y(k)|M_l^k, Y^{k-1})\, \Pr(M_l(k)|M_s^{k-1}, Y^{k-1})\, \Pr(M_s^{k-1}|Y^{k-1}) \\ &= \delta^{-1}\mathrm{p}(y(k)|M_l^k, Y^{k-1})\, p_{ij}\, \mu_s(k-1). \end{aligned}$$

Note that each term $\mathrm{p}(y(k)|M_l^k, Y^{k-1})$ involves running a Kalman filter on the entire mode history up to time k.

Computation of (22) is clearly infeasible since it involves a sum with an exponentially growing number of terms. It is therefore necessary to consider sub-optimal "N-scan-back" approaches that "combine" mode histories prior to time $k - N$ [10]. This amounts to approximating the mixture PDF of the state (or measurement) by a single Gaussian, *i.e.*

$$\mathrm{p}(x(k-N)|Y^{k-N}) \sim N\{\hat{x}, P\}.$$

The resulting algorithm is called a generalised pseudo-Bayesian filter of order N, GPB(N), and requires only r^N parallel Kalman filter computations at each time, rather than r^k. In particular, for $N = 1$ we obtain the GPB(1) filter which summarises the mixture PDF (22) to time $k-1$ by a single Gaussian density and calculates r KF estimates at time k, collapsing this r-term mixture to a single Gaussian PDF. The probabilistic data association filter is analogous to the GPB(1) filter with the replacement of mode hypotheses by target/clutter associations.

The GPB(2) filter would consider the r^2 possible Kalman filter estimates based on the last two manoeuvre modes $\{M(k-1), M(k)\}$ starting with a prior Gaussian density at time $k-2$. It then recondenses the resulting mixture of r^2 terms to a single Gaussian density at time k. In other words, the GPB(2) filter carries the state estimate and covariance at time $k-2$ and the r mode-conditioned state estimates and their covariances at time $k-1$, running each of the r conditional estimates through r mode-matched Kalman filters at time k. Obviously this procedure quickly becomes unworkable, although it is generally accepted that near-optimal performance is obtained, at least in good SNR conditions, with GPB(2).

Instead of combining the r^2 mode-conditioned estimates at the end of the filter cycle, as in GPB(2), it is possible to run only r Kalman filters in parallel, each with an appropriately weighted combination of state estimates as a "mixed initial condition. This reduces the amount of processing almost to that of the GPB(1) approach while obtaining performance near that of the GPB(2) approach. This idea is the basis of the interacting multiple-model (IMM) algorithm [31]. Returning to the mixture density (22), we have

$$\mathrm{p}(x(k)|Y^k) = \sum_{j=1}^{r} \mu_j(k)\, \mathrm{p}(x(k)|M_j(k), y(k), Y^{k-1}).$$

Now, by definition

$$\mathrm{p}(x(k)|M_j(k), y(k), Y^{k-1}) = \frac{\mathrm{p}(y(k)|M_j(k), x(k))}{\mathrm{p}(y(k)|M_j(k), Y^{k-1})}\, \mathrm{p}(x(k)|M_j(k), Y^{k-1}).$$

Conditioning on $M_i(k-1)$, it follows that

$$\begin{aligned} \mathrm{p}(x(k)|M_j(k), Y^{k-1}) &= \sum_{i=1}^{r} \Pr(M_i(k-1)|M_j(k), Y^{k-1})\, \mathrm{p}(x(k)|M_j(k), M_i(k-1), Y^{k-1}) \qquad (23) \\ &\approx \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\, \mathrm{p}(x(k)|M_j(k), M_i(k-1), \hat{x}_i(k-1|k-1), P_i(k-1|k-1)) \end{aligned}$$

where $\mu_{i|j}(k-1|k-1) \stackrel{\triangle}{=} \Pr(M_i(k-1)|M_j(k), Y^{k-1})$ and the approximation rests on taking the mode-conditioned Kalman filter estimates $\{\hat{x}_i(k-1|k-1), P_i(k-1|k-1)\}$ $l = 1, \ldots, r$ as sufficient statistics for the data Y^{k-1}. The mixture PDF in (23) is then approximated by a Gaussian mixture

$$\mathrm{p}(x(k)|M_j(k), Y^{k-1}) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\, N\{x(k); \hat{x}_i^j(k-1|k-1), P_i^j(k-1|k-1)\} \quad (24)$$

where

$$\begin{aligned} \hat{x}_i^j(k-1|k-1) &= \mathrm{E}\{x(k)|M_j(k), \hat{x}_i(k-1|k-1), P_i(k-1|k-1)\} \\ P_i^j(k-1|k-1) &= \mathrm{Cov}\{x(k)|M_j(k), \hat{x}_i(k-1|k-1), P_i(k-1|k-1)\} \end{aligned}$$

are the r^2 Kalman filter estimates calculated in the GPB(2) approach (we will wee that these are not required in the IMM algorithm). The mean of the Gaussian mixture in (24) is

$$\begin{aligned} \hat{x}_j^0(k-1|k-1) &= \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\, \mathrm{E}\{x(k)|M_j(k), \hat{x}_i(k-1|k-1), P_i(k-1|k-1)\} \\ &= \mathrm{E}\{x(k)|M_j(k), \sum_{i=1}^{r} \mu_{i|j}\hat{x}_i,\ \mathrm{cov}\{\sum_{i=1}^{r} \mu_{i|j}\hat{x}_i\}\} \\ &= \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\, \hat{x}_i(k-1|k-1) \end{aligned}$$

by linearity of the Kalman filter (*i.e.*, summing many KF estimates is the same as applying a single KF to a sum of estimates). The covariance of (24) is

$$\begin{aligned}
P_j^0(k-1|k-1) &= \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\,\{P_i(k-1|k-1) + \hat{x}_i(k-1|k-1)\,\hat{x}_i(k-1|k-1)'\} \\
&\quad - \hat{x}_j^0(k-1|k-1)\hat{x}_j^0(k-1|k-1)'
\end{aligned}$$

Only the first two moments $\hat{x}_j^0(k-1|k-1)$ and $P_j^0(k-1|k-1)$, $j = 1, \ldots, r$, of the mixture (24) are retained in the IMM.

The mean and covariance of the density in (22) can then be computed by r Kalman filters each tuned to a different *mixed* initial condition. Each KF in the bank of r produces its estimates via

$$\{\hat{x}_j^0(k-1|k-1),\ P_j^0(k-1|k-1)\} \rightarrow \boxed{\text{Kalman Filter}} \rightarrow \{\hat{x}_j(k|k),\ P_j(k|k)\}.$$

The conditional state estimates are combined in the usual way to give the overall state estimate and covariance

$$\begin{aligned}
\hat{x}(k|k) &= \sum_{i=1}^{r} \mu_j(k)\hat{x}_j(k|k) \\
P(k|k) &= \sum_{i=1}^{r} \mu_j(k)\{P_j(k|k) + \hat{x}_j(k|k)\hat{x}_j(k|k)'\} - \hat{x}(k|k)\hat{x}(k|k)'.
\end{aligned}$$

Note that the r estimates and covariances $\hat{x}_j(k|k)$ and $P_j(k|k)$ must be stored as they are needed in the next iteration of the algorithm. To complete the description of the IMM algorithm, we need to develop a recursion for the mixing probabilities and evaluate the mode probabilities $\mu_j(k)$. The mixing probabilities are computed with an application of Bayes' rule as

$$\mu_{i|j}(k-1|k-1) = c_j^{-1} p_{ij}\, \mu_i(k-1),\ i, j = 1, \ldots, r$$

where c_j is a normalisation constant given by

$$c_j = \sum_{i=1}^{r} p_{ij}\, \mu_i(k-1).$$

The mode probabilities are computed as in section 4.1

$$\mu_j(k) = c^{-1}\Lambda_j(k) \sum_{i=1}^{r} p_{ij}\, \mu_i(k-1),\ j = 1, \ldots, r,$$

in which the likelihood function for mode j is evaluated as

$$\Lambda_j(k) = \mathrm{p}(y(k)|M_j(k), Y^{k-1}) = N\{y(k); \hat{y}^j(k|k-1), S^j(k)\}$$

where the predictions for mode j are based on mixed initial conditions, *i.e.*,

$$\begin{aligned}
\hat{y}^j(k|k-1) &= HF\hat{x}_j^0(k-1|k-1) \\
S^j(k) &= HP^j(k|k-1)H' + R \\
P^j(k|k-1) &= FP_j^0(k-1|k-1)F' + Q.
\end{aligned}$$

The mode probabilities $\mu_j(k)$ must also be stored during the processing. An attractive property of the IMM algorithm for manoeuvring target tracking is that it does not require an explicit manoeuvre detector since a range of possible manoeuvres are built into the model and these are selected by a probabilistic weighting.

In [16, 10] a two-model and a three-model IMM filter are compared with the standard input estimation method and the variable-dimension filter for tracking a target executing a 90 degree turn. The models were taken to be: M_1 constant-velocity, M_2 Wiener process acceleration model (20) with acceleration increment $q = 0.001$, M_3 constant acceleration model with zero process noise. A Monte Carlo simulation showed a substantial reduction in position error for both IMM filters during the turn, with a factor of two decrease in RMS position error during constant-velocity periods. The complexity of the input estimation filter was around 10 times that of the three-model IMM filter. It is claimed that the results are not very sensitive to the choice of Markov transition probabilities p_{ij} as long as the matrix remains diagonally dominant. Adjustment of the (diagonal) transition probabilities merely trades off the peak error during the manoeuvre with the RMS error for constant-velocity tracking. However in [50] it is shown that performance of the IMM algorithm may depend strongly on the choice of transition probabilities for discrimination of the correct target manoeuvring regime. It is also indicated that careful selection of the different manoeuvre models is required.

It is clear that the choice of models in the IMM is an important implementational consideration. In particular the "dynamic range" of the set of models should be sufficient to give an adequate description of the anticipated target dynamics. In [85], a method for implementing IMM filters with variable structure was proposed. The algorithm adapts the models used in an IMM filter over a discrete set. This avoids over-parametrising the dynamics (which can be as bad as having too few models) while retaining sufficient coverage for good tracking. The case of a continuum of models was treated in [90, 118]. In [57] the need for selecting the best model set was also mentioned, although the solution presented there was iterative rather than adaptive.

A comparison of the IMM algorithm and a Viterbi algorithm (VA) (applied to solve the manoeuvre association problem) for a system with multi-level white process noise was covered in [8]. Their conclusions were: 1) the performance of both the IMM and the VA depends monotonically on the (maximal) difference between the discrete accelerations u_i and on the sampling time, and inversely on the measurement noise; 2) the VA is generally outperforms the IMM algorithm for high sampling rates, low measurement noise and a large number of discrete accelerations; but 3) the IMM provides better state estimation immediately following the onset of the manoeuvre.

4.5 Two-Stage Kalman Filter

A well-known application of Kalman filtering is the simultaneous estimation of a dynamic process with a constant, or slowly-varying unknown bias that is correlated with the state. The dynamical state is augmented by the constant and the estimation is performed in the usual way. This technique has been applied to estimate the acceleration of a manoeuvring target by treating the target acceleration as a bias term on a constant-velocity model [3]. We now summarise the approach.

Consider the following system description

$$x(k+1) \quad = \quad Fx(k) + Gu(k) + v(k) \tag{25}$$

$$
\begin{aligned}
u(k+1) &= u(k)+n(k) &(26)\\
y(k) &= Hx(k)+w(k) &(27)
\end{aligned}
$$

where $v(k)$, $n(k)$, $w(k)$ are zero-mean, white Gaussian noise sequences with covariance Q_v, Q_n and R respectively, $v(k)$ and $w(k)$ are uncorrelated, and $v(k)$ and $n(k)$ are jointly Gaussian with covariance $\mathrm{E}\{v(k)n'(k)\} = Q_{vn}$. Another way of writing (25) is

$$
\begin{aligned}
\begin{bmatrix} x(k+1) \\ u(k+1) \end{bmatrix} &= \begin{bmatrix} F & G \\ 0 & I \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} v(k) \\ n(k) \end{bmatrix} \qquad (28)\\
y(k) &= \begin{bmatrix} H & 0 \end{bmatrix}\begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + w(k)
\end{aligned}
$$

Under certain observability/controllability conditions on the system matrices [5], the system (28) is stable in the sense that the covariance of the Kalman filter converges to a unique positive-definite solution. From a practical standpoint, it may be undesirable to increase the dimension of the Kalman filter. However, it is also possible to write the recursions implied in (28) as a two-stage system: the first stage is a Kalman filter for the system (25) assuming zero bias ($u(k) \equiv 0$), the second stage is a filter based only on the bias term $u(k)$. The outputs of the stages are combined to yield the same estimate sequence as that which would be obtained by an augmented filter for (28).

The bias-free filter produces $\hat{x}_0(k|k)$ according to the recursions below:

$$
\begin{aligned}
\hat{x}_0(k|k-1) &= F\hat{x}_0(k-1|k-1)\\
\hat{x}_0(k|k) &= \hat{x}_0(k|k-1) + W_0(k)\nu_0(k)\\
\nu_0(k) &= y(k) - H\hat{x}_0(k|k-1)\\
P_0(k|k-1) &= FP_0(k-1|k-1)F' + Q_0\\
P_0(k|k) &= (I - W_0(k)H)P_0(k|k-1)\\
W_0(k) &= P_0(k|k-1)H'S_0^{-1}(k)\\
S_0(k) &= HP_0(k|k-1)H' + R,
\end{aligned}
$$

for some covariance Q_0 yet to be defined. The bias filter computes its estimate via:

$$
\begin{aligned}
\hat{u}(k|k-1) &= \hat{u}(k-1|k-1)\\
\hat{u}(k|k) &= \hat{u}(k|k-1) + W_u(k)\nu_0(k) - C(k)\hat{u}(k|k-1)\\
P_u(k|k-1) &= P_u(k-1|k-1) + Q_u\\
P_u(k|k) &= (I - W_u(k)C(k))P_u(k|k-1)\\
W_u(k) &= P_u(k|k-1)C(k)'S_u^{-1}(k)\\
S_u(k) &= C(k)P_u(k|k-1)C(k)' + HP_0(k|k-1)H' + R,
\end{aligned}
$$

where the bias gain is computed using

$$
\begin{aligned}
C(k) &= HU(k)\\
U(k) &= FV(k-1) + G = F(I - W_0(k-1)H)U(k-1) + G\\
V(k) &= (I - W_0(k)H)U(k).
\end{aligned}
$$

The two-stage Kalman filter estimate is formed using the bias-free state and bias estimates

$$
\hat{x}(k|k) = \hat{x}_0(k|k) + V(k)\hat{u}(k|k)
$$

$$
\begin{aligned}
\hat{x}(k|k-1) &= \hat{x}_0(k|k-1) + U(k)\hat{u}(k|k-1) \\
P_{11}(k|k) &= P_0(k|k) + V(k)P_u(k|k)V(k)' \\
P_{12}(k|k) &= V(k)P_u(k|k) \\
P_{12}(k|k-1) &= U(k)P_u(k|k-1) \\
P_{22}(k|k) &= P_u(k|k) \\
P_{22}(k|k-1) &= P_u(k|k-1)
\end{aligned}
$$

where P_{ij} are the blocks of the covariance matrix for the augmented state $(x', u')'$. A theorem in [3] shows that the two-stage Kalman filter is equivalent to the augmented Kalman filter when

$$
\begin{aligned}
Q_{vn} &= U(k+1)Q_u \\
Q_0 &= Q_v - U(k+1)Q_nU'(k+1) \geq 0.
\end{aligned}
\tag{29}
$$

The positive semi-definiteness of Q_0 is required for stability of the filter.

The overall complexity of the bias-free and bias filters is less than that of the augmented Kalman filter in most cases of interest. The operation of the two-stage filter is akin to a variable-dimension filter with a higher-order (acceleration) estimator that can be switched off in quiescent mode. A standard exponentially-weighted innovations manoeuvre detector (section 3) is used to detect the onset of the manoeuvre. If the effective window length is Δ, and a manoeuvre is detected at time k, the acceleration (bias) filter, but not the bias-free filter, must be initialised (retrospectively) at time $k - \Delta$ to correct the constant-velocity filter estimates. The covariance Q_0 is also increased during the manoeuvre. Termination of the manoeuvre is sensed by monitoring the innovations of the bias-free filter. In a simulation in [3] the two-stage Kalman filter was compared with the variable-dimension filter. The results indicated that the estimation errors were similar before and during a manoeuvre, but the two-stage filter detected the end of the manoeuvre more rapidly.

Regarding initialisation of the two-stage Kalman filter, the initial bias estimate is taken to be uncorrelated with the initial bias-free state estimate

$$
P_{12}(0|0) = 0 \;\Rightarrow\; V(0) = 0,\; U(0) = 0.
$$

The bias-free process noise Q_0 (29) sets the desired response of the constant-velocity filter; the bias filter process noise Q_u determines the manoeuvre response.

The two-stage Kalman filter has been applied within the IMM framework by [97, 96] with the objective of making the assumed target acceleration a variable parameter in the IMM. In this way, only two filters per co-ordinate, rather than several fixed-acceleration models, are required to cover the possible range of target motions. The results of this study indicate similar performance at lower complexity compared with the standard IMM approach. A difficulty is the delay required by the two-stage Kalman filter in estimating the manoeuvre acceleration, although this is somewhat alleviated by the subsequent matching of the IMM model to the target acceleration.

In [25] an interacting multiple model algorithm with bias estimation was proposed, again based on a two-stage Kalman filter. By using only bias filters in the IMM as opposed to augmented-state Kalman filters, a complexity reduction is achieved. A simulation was provided for an IMM with 3 bias models corresponding to: zero bias (constant velocity), constant non-zero bias (constant acceleration), and constant bias with a constant-speed constraint. The performance obtained was similar to a 3-model IMM filter while requiring roughly 50% of the computation.

Lastly an interacting acceleration compensation (IAC) technique was put forward in [144] as a means of obviating the requirement for manoeuvre detection of the two-stage Kalman filter. The IAC technique, illustrated in Fig. 4 (from [28]), assumes a two-stage estimator with a bias-free (constant-velocity) filter and two acceleration (bias) models: one for a constant-velocity target and one for constant acceleration. Since the IMM does not employ a conventional manoeuvre detector, it can provide an acceleration estimate in real time with which to compensate the bias-free filter. The IAC algorithm operates at roughly half the computational cost of an IMM filter with two models, but provides similar performance.

4.6 IMM Algorithm for Fixed Interval Smoothing

A fixed-interval smoothing approach to manoeuvring target tracking has been derived in [63], following earlier work by [56, 32]. The forward-running and backward-running Kalman filters in the usual fixed-interval smoother are replaced by interacting multiple model filters, together with a technique for fusing the conditional estimates from the two filters. The backward-running filters are obtained by time-reversal of the corresponding forward-running filters. Although the proposed algorithm is a batch algorithm with high complexity, its high performance makes it useful as an estimator for generating benchmark tracks in ground truthing or for sensor alignment in multi-sensor tracking systems.

In the notation of section 4.4, assuming r models, suppose that at time k model $M_i(k)$ is in effect for the forward filter with the backward filter assuming model $M_j(k+1)$. The conditional state "predictions" and their covariances are computed as $\hat{x}_i^F(k|k-1)$, $P_i^F(k|k-1)$, $\hat{x}_i^B(k|k-1)$, $P_i^B(k|k-1)$. Assuming a fixed interval of size N, the predictions are used to obtain r^2 fused initial estimates according to

$$\begin{aligned}\hat{x}_{ij}^0(k|N) &= P_{ij}^0(k|N)[(P_i^F(k|k-1))^{-1}\hat{x}_i^F(k|k-1) + (P_j^B(k|k-1))^{-1}\hat{x}_j^B(k|k-1)] \\ P_{ij}^0(k|N) &= (P_i^F(k|k-1))^{-1} + (P_j^B(k|k-1))^{-1}.\end{aligned}$$

These quantities are then mixed to yield prior state and covariances for the Kalman smoother matched to model $M_j(k)$

$$\begin{aligned}x_i^0(k|N) &= \sum_{j=1}^{r} \mu_{j|i}(k+1|N)\,\hat{x}_{ji}^0(k|N) \\ P_i^0(k|N) &= \sum_{j=1}^{r} \mu_{j|i}(k+1|N)\,[P_{ji}^0(k|N) + \hat{x}_{ij}^0(k|N)\hat{x}_{ij}^0(k|N)'] - x_i^0(k|N)x_i^0(k|N)'.\end{aligned}$$

The mode probabilities $\mu_j(k|N)$ are updated via the smoothed measurements $\hat{y}_j(k|N)$ from smoother $M_j(k)$. Finally the conditional estimates are combined to give

$$\hat{x}(k|N) = \sum_{j=1}^{r} \mu_j(k|N)\,\hat{x}_j(k|N)$$

The IMM smoother was compared under simulation in [63] with a single model (constant-velocity) smoother and yielded similar performance, although the former method seemed to produce larger spikes at the manoeuvre onset times.

4.7 Expectation-Maximisation Algorithm

This technique, from [106], utilises multi-level process noise of the form (3) to model the unknown target accelerations. A zero acceleration level is included for a constant-velocity target. We give only a brief description of the technique here.

A length K batch of uncluttered target measurements is assumed available for processing. The expectation maximisation (EM) algorithm [48, 19, 93] is applied to the problem of estimating the MAP sequence of target accelerations U^K:

$$\arg\max_{U^K} \; \mathrm{p}(X^K, Y^K, U^K) \tag{30}$$

where X^K denotes the (unknown) state sequence of length K and Y^K the measurement sequence. This results in a multi-pass, batch estimator of U^K. At each pass an objective function is evaluated (the E step) and then maximised (the M step). The expectation step involves computation of state estimates from a bank of Kalman smoothers tuned to the possible manoeuvre sequences. The maximisation step is efficiently implemented using forward dynamic programming.

An on-line estimator is also derived using a modified EM-type cost function. To obtain a dynamic programming recursion, the target state is assumed to satisfy a Markov property with respect to the manoeuvre sequence. This results in a recursive but sub-optimal estimator implementable on a Viterbi trellis. The transition costs of the latter algorithm, which depend on filtered estimates of the state, are compared with the costs arising in a Viterbi-based manoeuvre estimator developed in [8]. It is shown that the two criteria differ only in the weighting matrix of the quadratic part of the cost function.

Testing of the batch EM manoeuvre tracking algorithm (reported in [109]) has demonstrated superior transient error erformance during manoeuvres compared with the recursive version, which has errors of similar magnitude to the approach in [8]. The false manoeuvre alarm performance of the latter algorithm was the best of the three techniques tested in [109].

5 Manoeuvring Target Tracking in Clutter

Tracking a constant-velocity target in the presence of false measurements or clutter involves solving the *data association* problem. This is the problem of deciding which, if any, of a number of candidate detections is more likely to have arisen from the target of interest. The data association problem involves prediction of the measurement to the next scan followed by a process that correlates measurements with the prediction. In heavily cluttered environments it is well to apply *gating* to reduce the number of plausible target measurements to a reasonable number for processing. The process of gating or measurement validation is covered in [51, 24, 9] and illustrated in Fig. 5.

Since the optimal (Bayesian) solution [120] to the data association problem has combinatorially increasing computational requirements, there are many sub-optimal approaches of which the nearest neighbour Kalman filter, probabilistic data association (PDA) [12], the track-splitting filter [122], and the multiple-hypothesis (MHT) filter [113] are among the more common. A block diagram of the PDA filter is given in Fig. 6, although the same structure applies to any single-scan Bayesian algorithm for state estimation.

As we mentioned in the Introduction, the problem of tracking a manoeuvring target is fundamentally at odds with the data association problem. This is because any method that employs distance from a predicted measurement (*i.e.*, the filter innovation) to determine the onset of a manoeuvre may be falsely triggered in the presence of nearby clutter detections. In this section we review the optimal Bayesian approach to tracking a manoeuvring target in clutter and its natural sub-optimal realisations. We also outline some adaptive PDA-based schemes (variable-dimension PDA and IMM-PDA). We also describe some approaches that take advantage of signal amplitude distributions to improve the discrimination of target from clutter. These approaches can be integrated at low cost into most existing tracking filters. The discrimination of false alarms from target measurements can also be aided by consistency testing of velocity and/or and acceleration estimates in addition to the filter innovation.

5.1 Optimal Bayesian Filter for Manoeuvring Target in Clutter

We present a derivation of the optimal Bayesian filter (OBF) [72] in this part. The manoeuvring target model (3) presented earlier is modified for the case of cluttered measurements as

$$\begin{aligned} x(k+1) &= \begin{cases} F(k)x(k) + G(k)u_1(k) + v(k) & \text{for model 1} \\ F(k)x(k) + G(k)u_2(k) + v(k) & \text{for model 2} \\ \vdots & \vdots \\ F(k)x(k) + G(k)u_r(k) + v(k) & \text{for model r} \end{cases} \\ y(k) &= \begin{cases} H(k)x(k) + w(k) & \text{for the target} \\ \text{clutter} & \text{otherwise.} \end{cases} \end{aligned} \tag{31}$$

The target accelerations $u(k)$ are a finite-state Markov chain with known transition probabilities p_{ij}, with $u_i(k)$ denoting the event $\{u(k) = u_i\}$, $i = 1, \ldots, n$. The usual notation for cluttered measurement sets is adopted: $Y(k) = \{y_1(k), \ldots, y_{m_k}(k)\}$ (at scan k), $Y^k = \{Y(1), \ldots, Y(k)\}$ (measurement set up to time k). The *association event* $\theta_i(k)$ signifies the event that measurement $y_i(k)$ is due to the target ($i = 1, \ldots, m_k$), or no measurement is from the target ($i = 0$).

Seeking to evaluate the probability of a particular joint measurement association and manoeuvre sequence, we define the lth possible measurement history (recursively) as

$$\Theta_l^k = \{\Theta_s^{k-1}, \theta_{i_l}(k)\},\ l = 1, \ldots, L_k$$

where $L_k = (1 + m_1)(\cdots)(1 + m_k)$, and the pth possible manoeuvre history as

$$\Phi_p^k = \{\Phi_q^{k-1}, u_{i_p}(k)\},\ p = 1, \ldots, n^k.$$

The set of joint measurement and manoeuvre association events is $\chi_{lp}^k = \{\Theta_l^k, \Phi_l^k\}$ in which there are $L_k\, n^k$ elements!

Writing χ_{lp}^k as $\{\chi_{sq}^{k-1}, \chi_{lp}(k)\}$ where $\chi_{lp}(k) = \{\theta_{i_l}(k), u_{i_p}(k)\}$, we obtain the conditional mean estimator for $x(k)$ as

$$\hat{x}(k|k) \triangleq \mathrm{E}\{x(k)|Y^k\} = \sum_{p=1}^{n^k} \sum_{p=1}^{L_k} \beta_{lp}(k)\, \hat{x}_{lp}(k|k) \tag{32}$$

where the conditional state estimates are defined as $\hat{x}_{lp}(k|k) = \mathrm{E}\{x(k)|\chi_{lp}^k\}$. The association probabilities $\beta_{lp}(k) = \Pr(\chi_{lp}^k|Y^k)$ satisfy a recursion, namely

$$\beta_{lp}(k) = c_k^{-1} \mathrm{p}(Y(k)|\chi_{sq}^{k-1}, \chi_{lp}(k), Y^{k-1}) \Pr(\chi_{lp}(k)|\chi_{sq}^{k-1}, Y^{k-1}) \beta_{sq}(k-1).$$

in which c_k is a normalisation factor. Furthermore the terms on the right hand side are evaluated as

$$\mathrm{p}(Y(k)|\chi_{sq}^{k-1}, \chi_{lp}(k), Y^{k-1}) = \begin{cases} V_G^{-m_k} & i_l = 0 \\ V_G^{-m_k+1} P_G^{-1} N\{y_{i_l}(k); \hat{y}_{sq}(k|k-1), S_{sq}(k)\} & i_l > 0, \end{cases}$$

where $\hat{y}_{sq}(k|k-1) = \mathrm{E}\{y(k)|\chi_{sq}^{k-1}, Y^{k-1}\}$ and its covariance $S_{sq}(k)$ are computed by Kalman filtering the sth measurement sequence assuming the qth manoeuvre history, and V_G is the volume of the validation gate at time k. The computation of the association probabilities is completed with

$$\Pr(\chi_{lp}(k)|\chi_{sq}^{k-1}, Y^{k-1}) = \Pr(u(k) = u_{i_p}|u(k-1) = u_{i_q}) \ \Pr(\theta_{i_l}(k)|Y^{k-1})$$

and

$$\Pr(\theta_{i_l}(k)|Y^{k-1}) = \begin{cases} c^{-1}\frac{1}{m_k}\, P_D\, P_G\, g_c(m_k - 1) & i_l = 1, \dots, m_k \\ c^{-1}(1 - P_D P_G)\, g_c(m_k) & i_l = 0, \end{cases}$$

where P_D is the target detection probability, P_G the gate probability and it is customary to assume that the number of clutter points in the gate is given by a Poisson distribution

$$g_c(m) = \exp(-\lambda V_G)\frac{(\lambda V_G)^m}{m!}, \ m = 0, 1, \dots$$

The state error covariance is computed as

$$P(k|k) = \sum_{p=1}^{n^k}\sum_{p=1}^{L_k} \beta_{lp}(k)\, \hat{x}_{lp}(k|k)\, [P_{lp}(k|k) + \hat{x}_{lp}(k|k)\hat{x}_{lp}(k|k)'] - \hat{x}(k|k)\hat{x}(k|k)'.$$

The optimal Bayesian filter (32) is of course not realisable due to its increasing memory and computational requirements. There exist (N, M) scan approximations to it that lump measurement histories identical over the previous N scans and manoeuvre histories identical over the previous M scans (see [72]). These techniques amount to matching a single Gaussian density to the mixture (32) at the appropriate time. Note that the MAP state estimator would take only the estimate $\hat{x}_{lp}(k|k)$ corresponding to the maximum association probability $\beta_{lp}(k)$ at time k.

5.2 Single-Scan Pseudo-Bayesian Filter

In [57] a sub-optimal Bayesian adaptive filter for a manoeuvring target system of the following form is derived:

$$\begin{aligned} x(k+1) &= F(k)x(k) + v(k), \ v(k) \sim N\{0, Q(k)\} \\ y(k) &= \begin{cases} H(k)x(k) + w(k) \quad w(k) \sim N\{0, R(k)\} & \text{for the target} \\ \text{clutter} & \text{otherwise.} \end{cases} \end{aligned}$$

In this formulation the process and measurement noise covariances are *unknown* and take values in a discrete set

$$Q(k) \in \{Q_1, \dots, Q_p\}, \ R(k) \in \{R_1, \dots, R_l\}.$$

Let $\phi_{ij}(k)$ denote the event that at time k $\{Q(k) = Q_i, R(k) = R_j\}$, and define the association events $\theta_i(k)$ $i = 0, 1, \ldots, m_k$ as before. It follows that the conditional mean estimator of the state is expressible as

$$\begin{aligned}\hat{x}(k|k) &= \sum_{j=1}^{p}\sum_{l=1}^{q} \Pr(\phi_{lj}(k)|Y^k) \sum_{i=0}^{m_k} \mathrm{E}\{x(k)|\theta_i(k), \phi_{lj}(k), Y^k\} \\ &= \sum_{j=1}^{p}\sum_{l=1}^{q} \Pr(\phi_{lj}(k)|Y^k)\, \hat{x}_{jl}(k|k)\end{aligned}$$

where, following the usual pattern,

$$\hat{x}_{jl}(k|k) = \sum_{i=0}^{m_k} \beta_{ijl}(k)\, \mathrm{E}\{x(k)|\theta_i(k), \phi_{lj}(k), Y^k\},$$

and

$$\beta_{ijl}(k) \triangleq \Pr(\theta_i(k), \phi_{lj}(k)|Y^k).$$

This multiple-model PDA technique is also described in [11].

The filter is made practicable by summarising the prior state density based on Y^{k-1} by a single Gaussian with mean $\hat{x}(k-1|k-1)$ and covariance $P(k-1|k-1)$ (as in the GPB(1) or PDA approaches). Despite this single-scan approximation of the Gaussian mixture density, the computational requirements of this adaptive PDA filter remain high. In particular it is difficult in practice to determine a suitable grid size for discretisation of the covariance matrices. In a bearings-only tracking example for a ship travelling in a non-straight course with an average 10 clutter measurements per look, [57] used $p = 6$, $q = 6$ cells, or a total of 36 PDA filters.

5.3 Joint Manoeuvre and Measurement Association

A similar approach to the preceding section was taken by [117, 78] for tracking multiple manoeuvring targets in clutter, with [82] covering in the single-target case. A multi-level white noise acceleration model was used with the input a finite Markov chain $u(k) \in \{u_1, \ldots, u_m\}$ with known transition probabilities p_{ij}. Gating was applied to validate measurements for tracking. The filter derivation, again based on a single-scan Gaussian approximation, is briefly summarised below.

Let $\theta_i(k)$ denote the ith association event and $u_j(k)$ the jth manoeuvre model. The association probabilities are computed by invoking the independence of the measurement and manoeuvre associations, *viz.*

$$\begin{aligned}\beta_{ij}(k) &= \Pr(\theta_i(k), u_j(k)|Y^k) \\ &= c^{-1}\mathrm{p}(Y(k)|\theta_i(k), u_j(k), Y^{k-1}) \Pr(\theta_i(k), u_j(k)|Y^{k-1}).\end{aligned}$$

The first term on the right is calculated as in standard PDA. The last term in the expression is

$$\begin{aligned}\Pr(\theta_i(k), u_j(k)|Y^{k-1}) &= \Pr(\theta_i(k)|u_j(k), Y^{k-1}) \Pr(u_i(k)|Y^{k-1}) \\ &= \Pr(\theta_i(k)|Y^{k-1}) \sum_{l=1}^{m} p_{lj} \Pr(u_l(k-1)|Y^{k-1})\end{aligned}$$

whence standard PDA-type calculations can be used to complete the derivation. The state estimate is given by

$$\hat{x}(k|k) = \sum_{i=0}^{m_k} \sum_{j=1}^{m} \beta_{ij}(k)\, \hat{x}_{ij}(k|k)$$

The simulation results in [117] assume a high probability of detection $P_D = 0.95$ and a very low clutter density of 0.1–0.2 returns in the gate on average. For this low clutter case, the technique appears to be able to track multiple manoeuvring targets that overlap, although the estimation accuracy is degraded during manoeuvres. The results in [78], which assume $P_D = 1$ and $P_{FA} = 0.1$ and 6 possible target accelerations, are hard to interpret. It is also unclear how the probability of false alarm indicated in the simulations translates to the usual clutter density.

5.4 Variable-Dimension PDA

Variable dimension PDA [23] is a direct extension of the variable-dimension filtering approach described in sections 3 and 4.3 to the case of cluttered measurements. Clutter is taken into account by replacing the usual Kalman filters for constant-velocity and constant-acceleration targets with PDA filters, with the idea that the constant-velocity PDA filter is maintained until a manoeuvre is detected at which time a constant-acceleration model takes over. The difficulty in this type of approach is the reliable detection of a manoeuvre in the presence of false measurements. A series of statistical tests based on the normalised filter innovations are needed to solve this problem. Since there is more than one measurement at each scan, there are many different sequences of innovations. The method is therefore to test all possible sequences of innovations $\{\rho_i(j)\}$, $j = k - N + 1, \ldots, k$ over a window of length N and initiate a new constant-acceleration PDA filter each time a sequence is found that satisfies the following tests.

1. Gating: $\{\rho_i(j)\}$ should be contained in a series of gates of increasing size.
2. Significance of the accelerations estimate: the innovations are white and zero-mean for a non-manoeuvring target, a target acceleration corresponds to correlation in this sequence. The presence of a non-zero acceleration is tested at a given significance level.
3. Goodness of fit test for the acceleration model: the MMSE of the residuals is computed allowing for the estimated acceleration.
4. If the previous tests are all satisfied, a measurement is expected at time $k + 1$ near the prediction based on the acceleration estimate. This is used to initiate a higher order Kalman filter which is then continued after time $k + 2$ using PDA to track the manoeuvre in clutter.

Since an innovation sequence corresponding to false alarms may pass all the above tests, the constant-velocity PDA filter is continued along with any manoeuvring PDA filters. This decreases the risk of track loss in clutter.

Track maintenance rules are provided in [23] for terminating and merging the extra tracks that are produced by this algorithm. The finish of the manoeuvre is detected by testing innovations in the gate of the PDA filter assuming a zero acceleration. Although the technique does not require retrospective initialisation, many PDA filters are needed to track a manoeuvring target in moderate clutter.

The performance of the variable-dimension PDA tracker is reported to be reasonable in moderate clutter densities (0.024 clutter detections on average in a $2\sigma_x \times 2\sigma_y$ measurement noise cell). In addition to the large number of filters that have to be maintained to minimise track loss, there are several thresholds that need to be set for the statistical tests described above.

5.5 IMM-PDA

A natural extension of the IMM algorithm for tracking a manoeuvring target in a cluttered environment appears in [30, 66]. This technique, called interacting multiple-model probabilistic data association (IMM-PDA), replaces the standard Kalman filters in the IMM algorithm (section 4.4) with PDA filters. The target models can be taken as constant-velocity and constant-acceleration for the manoeuvring target case, or constant-velocity and "no target" (non-existent target) models for the algorithm in [14, 15] which is designed for automatic track initiation and tracking in clutter. (The technique is clearly not restricted to just two models.) The transitions between the models are Markov with known transition probabilities. The major difference in the IMM computations is that the mode probabilities are formed using the PDA "likelihood" function

$$\begin{aligned} \Lambda_j(k) &= \mathrm{p}(Y(k)|M_j(k), Y^{k-1}) \\ &= V_G^{-m_k}(1 - P_D P_G) + V_G^{-m_k+1}\frac{P_D}{m_k}N\{y_i(k); \hat{y}_j(k|k-1), S_j(k)\} \\ \hat{y}_j(k|k-1) &= \mathrm{E}\{y(k)|M_j(k), Y^{k-1}\} \end{aligned}$$

where the symbols have usual meanings. The prediction $\hat{y}_j(k|k-1)$ is obtained from the IMM filter for model $M_j(k)$. The other computations are identical to the standard IMM algorithm. The performance of the IMM-PDA filter appears to be good for its level of complexity.

A further variation on this theme is provided in [64] which uses "integrated PDA" filters instead of PDA filters in the IMM-PDA algorithm, leading to the title IMM-IPDA. The integrated PDA filter [99] incorporates a target existence model and hence provides a track confidence measure. The target existence model is identical to the one in [44], although the association probability calculations are marginally different, replacing the usual m_k term in PDA by an expected number of measurements in the gate $\hat{m}_k$. The IMM-IPDA algorithm has essentially the same complexity as the IMM-PDA algorithm and the track confidence measure assists in automatic track formation and maintenance. Simulations evidenced in [64] point to acceptable performance against constant-velocity, constant-speed turning and linearly accelerating targets in moderate clutter.

In [143] the IMM-IPDA algorithm was put forward as a solution to the second benchmark problem for manoeuvring target tracking in the presence of clutter and countermeasures. The algorithm also monitors the predicted position error ϵ_x and automatically adjusts the radar revisit time to ensure that ϵ_x does not exceed a threshold.

An additional modification of the IMM-PDA idea appeared in [98] called integrated probabilistic data association with prediction-oriented multiple models (IPDA-PMM). The method uses a common prediction equation across all models to simplify the computations. Simulations for a slowly manoeuvring target in low density clutter show modest improvements over a constant-acceleration PDA filter.

5.6 Multi-hypothesis Techniques

Multi-hypothesis tracking (MHT) is a high performance, "measurement-oriented" multiple target tracking technique developed in [113]. Although not originally considered in the MHT framework, more recent work on MHT has focussed on the incorporation of manoeuvres. A detailed description of MHT is contained in [24] and we need only mention its essential features. MHT is a Bayesian algorithm that computes state estimates for all measurement hypotheses and their corresponding posterior probabilities. The hypothesis set allows for a measurement to be due to: (i) an existing target; (ii) a false alarm; or (iii) a new target. The inclusion of the third hypothesis allows MHT to initiate new target tracks automatically.

Due to the very high computational requirements of the MHT approach, efficient pruning of the decision tree is needed (see references in [76]). This is effected by clustering non-competing measurements and tracks, combining tracks with similar state estimates, deleting low-confidence tracks, pruning based on prior information (manoeuvrability constraints), and preventing initiation on measurements that have a high correlation to existing tracks.

The inclusion of target manoeuvres in MHT has been approached in [76] via a correlated-noise acceleration (Ornstein-Uhlenbeck) model in the context of formation tracking. The IMM and GPB algorithms have been applied within the MHT framework in [74] to represent the PDF sum over mode histories by finite Gaussian mixtures. A performance comparison is given for the IMM, GPB(1), GPB(2) and a MHT-IMM filter with truncation depth of 3 scans. It is concluded in [74] that the 3 scan-back approximation is near-optimal for the benchmark tracking problem [27]. The same author also applied fixed-lag Kalman smoothing or *retrodiction* to improve the state estimates obtained by the MHT-IMM algorithm and this is reported in [75]. An IMM-MHT algorithm called "IM3HT" has also been reported in [136]. The IM3HT algorithm is a natural embedding of IMM in the MHT framework using multiple models to cover possible target accelerations and multiple hypotheses to allow for false alarms and multiple targets. Comparisons of IM3HT with a MHT algorithm incorporating manoeuvre detection logic are given for target accelerations of up to 5 g for detection probabilities of 0.8 and 1.0.

5.7 Filters Using Amplitude Information

In the same way that Doppler frequency is used to enhance tracking and track initiation performance over techniques employing only position information, several authors have investigated the possibility of using signal amplitude information (or power, SNR, *etc.*) to increase the ability of the tracking algorithm to discriminate target measurements from clutter. More generally, any non-dynamical "feature" information (for instance target orientation, *etc.*) could be used for this purpose, if it is available. The paper [49] describes a distribution-free procedure for target detection in track-while-scan and phased-array radars. The detection procedure is distribution-free because it uses only the "rank" (or ordering) of the received measurement to determine whether a target signal is present or absent. Thus if the rank of the measurement is Z_{ij} in range bin i and beam position j (and at time k), a signal is declared present if $Z_{ij} \geq T$ or absent otherwise where T is the detection threshold.

The probability of false alarm for such a procedure depends only on the number of reference cells over which the ranking takes place, and not on the signal and noise distributions $\Pr(Z_{ij} = R|S + N)$ and $\Pr(Z_{ij} = R|N)$. The latter distributions are needed, however, for computing the detection probability.

The use of "rank" information, as described in [49] was applied to enhance the performance of the PDA filter for tracking a manoeuvring target in clutter [100]. They also consider an *ad hoc* procedure for adapting the detection threshold to trade off computation (due to excessive clutter) and detection performance. The only algebraic difference is in the calculation of the association probabilities for PDA, which in this case depend on a likelihood ratio (we will detail this modification shortly). The signal plus noise amplitude distribution was assumed to be Rayleigh, corresponding to a Swerling type-2 target [126]. In simulations at probabilities of detection between 0.6 and 0.9, performance of the amplitude-aided PDA filter was compared with that of a (constant-acceleration) PDA filter for tracking a target executing a constant-speed turn. Apart from the obvious advantage of good quality signal amplitude information,it is claimed that the method in [100] can track a manoeuvring target in clutter densities as high as 10 points (average) in the validation gate.

Similar work on the inclusion of amplitude information in PDA was carried out in [83], and we now outline the basic approach. The probability distributions $p_1(\cdot)$ and $p_0(\cdot)$ of the received signal envelope $a(k)$ under the signal present H_1 and signal absent H_0 hypotheses are assumed to be known. The system model uses an augmented measurement vector $y_a(k)$ defined by

$$y_a(k) = \begin{bmatrix} y(k) \\ a(k) \end{bmatrix}.$$

The state vector is not augmented, although this could be accomplished if filtering of the signal amplitude is required. The equations of the PDA filter are unchanged except for the computation of the conditional measurement density, which now must allow for the different probability densities of target and clutter measurements:

$$\mathrm{p}(y_i(k)|\theta_i(k), Y^k) = \begin{cases} P_G^{-1} N\{y_i(k); \hat{y}(k|k-1), S(k)\}\, p_1(a(k)) & i = 1, \ldots, m_k \\ V_G^{-1}\, p_0(a(k)) & i = 0 \end{cases}$$

This introduces a likelihood ratio into the calculation of the association probabilities (once they are normalised). Effectiveness of the technique relies on the accuracy with which the respective amplitude distributions are modelled.

Signal amplitude information was also included in the IMM-PDA algorithm in [84]. This uses the likelihood ratio arising from the signal-plus-noise and noise-only distributions to alter the PDA association probabilities as illustrated above. The modification is useful for enhancing track formation and tracking of manoeuvring targets in clutter through more effective signal discrimination. In the same paper a track acceptance test that uses a two-dimensional threshold, depending on both the target confidence and the probability distribution for false track duration, is described in detail. The target confidence measure is obtained by including a "no target" model in the IMM filter and the distribution of false track lifetimes is obtained via Monte Carlo simulation. The IMM filter is implemented with three other models: constant-velocity, left-turn and right-turn. The performance is claimed to be superior to that of the IMM-PDA filter (section 5.5) especially for weak targets and in heavy clutter.

Amplitude information has also been applied in the context of multi-hypothesis sonar tracking [112]. A feature of MHT algorithms is their ability to initiate tracks automatically in clutter, as well as allowing for multiple targets. The article [112] describes an adaptive re-initialisation procedure for MHT. The state variable includes SNR, bearing, frequency and their derivatives. A track confidence test (based on P_D and P_{FA}) is employed to detect track loss, for instance, due to a manoeuvre. The gate is then enlarged for a predetermined number of scans in order to re-acquire the target.

6 Discussion

We have given an extensive review of the existing literature on manoeuvring target models (section 2), manoeuvre detection (section 3), tracking of manoeuvring targets (section 4), and tracking of manoeuvring targets in clutter (section 5). Although the number of competing techniques is large, the number of comparative performance evaluations is small. A recent initiative [27] has been the definition of a benchmark manoeuvring target tracking problem for phased array radar.

Conventional manoeuvring target models we have covered include random input models, unknown deterministic input models, switching systems and Singer's correlated process noise model. Several extensions to these models have been investigated that use different stochastic representations or higher order approximations. Manoeuvre detection strategies are characterised by the selection of a test statistic based on the filter residuals using data windowing or exponential forgetting together with a likelihood ratio test. Several generalisations of these tests are available but do not appear to have found significant application. The use of additional sensors such as imaging sensors is a promising means of enhancing the manoeuvre detection capabilities of short-range radar tracking systems.

If false alarms are to be handled by a nearest-neighbour approach or other correlation test, the accepted techniques for manoeuvring target tracking can be classed as multi-level process noise, input estimation, variable-dimension filtering, two-stage Kalman filtering, and sub-optimal Bayesian filtering for switching systems including the IMM and GPB algorithms. The problem of tracking a manoeuvring target in clutter has been formulated within the framework of Bayesian probability theory using (i) a finite state Markov chain manoeuvre input and (ii) a Markov chain switching model. The optimal Bayesian filter for both these cases has combinatorially increasing complexity and therefore N-scan truncation is generally applied to simplify the filter implementation. The probabilistic data association filter, a single-scan Bayesian algorithm, can be employed to extend the variable dimension filter and IMM filter for tracking in clutter. The use of non-kinematic information such as measurement SNR, when this is reliable, is a promising avenue for measurement discrimination in clutter.

More recent approaches to the manoeuvring target tracking problem include the expectation maximisation algorithm and MHT framework and these deserve further comment. The EM technique, which is a batch algorithm, does not seem to be practicable in the cluttered case without substantial modification to avoid enumeration of joint manoeuvre and measurement associations. It is however possible to apply ideas from the recent probabilistic multi-hypothesis tracking (PMHT) algorithm [124] to simplify these computations, thus obtaining a practicable sub-optimal algorithm; the approaches presented in [149] and [88] are examples of this. The key simplifying assumption in applying the EM algorithm in this case is the supposition that the target may be associated with all measurements in a scan simultaneously [105]. This is exploited via mixing probabilities to condense an N-dimensional sum over all measurement-to-target associations over N scans to a single sum. This assumption has no physical basis and also modifies the tracking problem in a fundamental manner by assuming away the combinatorial explosion of association hypotheses. Numerical simulations of PHMT-based EM algorithms in [149] indicate some improvement over the IMM-PDA algorithm (section 5.5) in heavy clutter, although no comparisons with high performance techniques such as MHT or GPB approaches appear to have been effected as yet. Such comparisons would need to allow for the advantage of smoothing over filtering, as well as the operational consequences of increasing the latency in displaying tracks to the operator.

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References

[1] Ackerson, G. A., and Fu, K. S. "On State Estimation in Switching Environments", *IEEE Trans. AC*, vol. AC-28, pp. 10–17, Feb. 1970.

[2] Alouani, A.T., Blair, W.D. "Use of a kinematic constraint in tracking constant speed, maneuvering targets", *IEEE Transactions on Automatic Control*, vol. 38, no. 7, pp. 1107–11, July 1993.

[3] Alouani, A.T., Xia, P., Rice, T.R., Blair, W.D. "Two-stage Kalman estimator for tracking maneuvering targets", *Conference Proceedings 1991 IEEE International Conference on Systems, Man, and Cybernetics. 'Decision Aiding for Complex Systems*, pp. 761–6 vol.2, 1991, Charlottesville, VA, USA.

[4] Amoozegar, F., Sundareshan, M.K. "Target tracking by neural network maneuver modeling", *1994 IEEE International Conference on Neural Networks*, pp. 3932–7 vol.6, 1994, Orlando, FL, USA.

[5] Anderson, B. D. O., and Moore, J. B. *Optimal Filtering*, Prentice-Hall, NJ, 1979.

[6] Arnold, J., Shaw, S., and Pasternack, H. "Efficient Target Tracking Using Dynamic Programming", *IEEE Trans. AES*, vol. 29, pp. 44–56, Jan. 1993.

[7] Astrom, K. J., and Wittenmark, B. *Computer Controlled Systems: Theory and Design*, Prentice-Hall, N.J., 1984.

[8] Averbuch, A., Itzkowitz, S., and Kapon, T. "Radar Target Tracking—Viterbi versus IMM", *IEEE Trans. AES*, vol. 27, no. 3, pp. 550–563, 1991.

[9] Bar-Shalom, Y., and Fortmann, T. E. *Tracking and Data Association*, Academic Press, 1988.

[10] Bar-Shalom, Y., and Li, X.-R. *Estimation and Tracking: Principles, Techniques, and Software*, Artech House, MA, 1993. Reprinted by YBS Publishing 1998.

[11] Bar-Shalom, Y., and Li, X.-R. *Multitarget-Multisensor Tracking: Principles and Techniques*, ISSN 0895-9110, YBS Publishing, 1995.

[12] Bar-Shalom, Y., and Tse, E. "Tracking in a Cluttered Environment With Probabilistic Data Association", *Automatica*, vol. 11, pp. 451–460, 1975.

[13] Bar-Shalom, Y., Birmiwal, K. "Variable Dimension filter for Manoeuvring Target Tracking", *IEEE Trans. AES*, AES-18, Sept 1982, pp. 621–629.

[14] Bar-Shalom, Y., Chang, K. C., and Blom, H. A. P. "Automatic Track Formation in Clutter with a Recursive Algorithm", in *Multitarget-Multisensor Tracking: Advanced Applications*, pp. 25–42, Artech House, MA., 1990. Reprinted by YBS Publishing 1998.

[15] Bar-Shalom, Y., Chang, K. C., and Blom, H. A. P. "Automatic Track Formation in Clutter with a Recursive Algorithm", *Proc. 28th Conf. Dec. Control*, Florida, pp. 1402–1408, Dec. 1989.

[16] Bar-Shalom, Y., Chang, K. C., Blom, H. A. P. "Tracking a Manoeuvring Target Using Input Estimation Versus the Interacting Multiple Model Algorithm", *IEEE Trans. AES*, AES-25, Mar 1989, pp. 296–300.

[17] Barniv, Y. "Dynamic Programming Solution for Detecting Dim Moving Targets", *IEEE Trans. AES*, vol. AES-21, no. 1, pp. 144–156, Jan. 1985.

[18] Barniv, Y., and Kella, O. "Dynamic Programming Solution for Detecting Dim Moving Targets Part II: Analysis", *IEEE Trans. AES*, vol. 23, pp. 776–788, Nov. 1987.

[19] Baum, L. E., Petrie, T., Soules, G., and Weiss, N. "A Maximization Technique Occurring in the Statistical Analysis Probabilistic Functions of Markov Chains", *Ann. Math. Statistics*, vol. 41, pp. 164–171, 1970.

[20] Bauschlicher, J., Asher, R., Dayton, D. "A comparison of Markov and constant turn rate models in an adaptive Kalman filter tracker", *Proceedings of the IEEE 1989 National Aerospace and Electronics Conference NAECON 1989*, pp. 116–23 vol. 1, Dayton, OH, USA, 1989.

[21] Benes, V.E., Helmes, K.L., Rishel, R.W. "Estimating the terminal state of a maneuvering target", *Proceedings of the 33rd IEEE Conference on Decision and Control*, pp. 1663–5 vol.2, 1994, Lake Buena Vista, FL, USA.

[22] Bharucha-Reid, A. T. *Elements of the Theory of Markov Processes and Their Applications*, McGraw-Hill, N.Y., 1960.

[23] Birmiwal, K. Bar-Shalom, Y. "On Tracking a Maneuvering Target in Clutter", *IEEE Trans. AES*, vol. AES-20, no. 5, Sept. 1984, pp. 635–645.

[24] Blackman, S. *Multiple Target Tracking with Radar Applications*, Artech House, MA, 1986.

[25] Blair, W.D., Watson, G.A. "Interacting multiple bias model algorithm with application to tracking maneuvering targets", *Proceedings of the 31st IEEE Conference on Decision and Control*, pp. 3790–5 vol.4, 1992, Tucson, AZ, USA.

[26] Blair, W.D., Watson, G.A., Hoffman, S.A. "Second order interacting multiple model algorithm for tracking maneuvering targets", *Proceedings of the SPIE: Signal and Data Processing of Small Targets*, vol. 1954, pp. 518–29, 1993, Orlando, FL, USA.

[27] Blair, W.D., Watson, G.A., Hoffman, S.A. "Benchmark problem for beam pointing control of phased array radar against maneuvering targets", *Proceedings of the 1994 American Control Conference*, pp. 2071–5 vol.2, 1994, Baltimore, MD, USA.

[28] Blair, W.D., Watson, G.A., Rice, T.R. "Interacting multiple model filter for tracking maneuvering targets in spherical coordinates", *IEEE Proceedings of SOUTHEASTCON '91*, pp. 1055–9 vol.2, 1991, Williamsburg, VA, USA.

[29] Blair, W.D., Watson, G.A., Rice, T.R. "Tracking maneuvering targets with an interacting multiple model filter containing exponentially-correlated acceleration models", *Proceedings. The Twenty-Third Southeastern Symposium on System Theory*, pp. 224–8, 1991, Columbia, SC, USA.

[30] Blom, H. A. P. "A Sophisticated Tracking Algorithm for ATC surveillance Data" *Proc. Int. Radar Conf.*, Paris, pp. 393-398, May 1984.

[31] Blom, H. P., and Bar-Shalom, Y. "The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients", *IEEE Proc. AC*, vol. 33, no. 8, pp. 780–783, Aug. 1988.

[32] Blom, H. P., and Bar-Shalom, Y. "Time-Reversion of a Hybrid State Stochastic Difference System with a Jump-Linear Smoothing Application", *IEEE Trans. Info. Th.*, pp. 836–847, 1990.

[33] Bogler, P. L. "Tracking a Maneuvering Target Using Input Estimation", *IEEE Trans. AES*, vol. 23, no. 3, pp. 298–310, May 1987.

[34] Campo, L., Mookerjee, P., and Bar-Shalom, Y. "Failure Detection Via Recursive Estimation for a Class of Semi-Markov Switching Systems", *Proc. 27th Conf. Decision and Control*, pp. 1966–1971, Dec. 1988.

[35] Chan, Y. T., Hu, A. G. C., Plant, J. B. "A Kalman Filter Based Tracking Scheme with Input Estimation", *IEEE Trans. AES*, vol. AES-15, pp. 237–244, March, 1979.

[36] Chan, Y.T., Couture, F. "Manoeuvre detection and track correction by input estimation", *IEE Proceedings F [Radar and Signal Processing]*, vol. 140, no. 1, pp. 21–8, Feb. 1993.

[37] Chang, C. B., and Athans, M. "State Estimation for Discrete Systems with Switching Parameters", *IEEE Trans. AES*, vol. AES-14, pp. 418–425, May 1978.

[38] Chang, C. B., Whiting, R. H., and Athans, M. “On the State and Parameter Estimation for Maneuvering Reentry Vehicles”, *IEEE Trans. AC*, vol. AC-22, pp. 99–105, Feb. 1977.

[39] Chang, W.-T., and Lin, S.-A. “Incremental Maneuver Estimation Model for Target Tracking”, *IEEE Trans. AES*, vol. AES-28, no. 2, pp. 439–451, Apr. 1992.

[40] Chung, Y.-N., Gustafson, D.L., Emre, E. “Extended solution to multiple maneuvering target tracking”, *IEEE Transactions AES*, vol. 26 Iss: 5 , pp. 876–87, Sept. 1990.

[41] Cloutier, J.R., Lin, C.-F., Yang, C. “Maneuvering target tracking via smoothing and filtering through measurement concatenation”, *Journal of Guidance, Control, and Dynamics*, vol. 16, no. 2, pp. 377–84, March 1993.

[42] Cloutier, J.R., Lin, C.-F., Yang, C. “Enhanced variable dimension filter for maneuvering target tracking”, *IEEE Transactions AES*, vol. 29, no. 3, pp. 786–97, July 1993.

[43] Cloutier, J.R., Yang, C., Lin, C.-F. “Sequential smoothing and filtering for maneuvering target tracking”, *Proceedings of the 1992 American Control Conference*, pp. 3236–41 vol.4, 1992, Chicago, IL, USA.

[44] Colegrove, S. B., Davis, A. W., and Ayliffe, J. K. “Track Initiation and Nearest Neighbours Incorporated into Probabilistic Data Association”, *J. Elec. and Electronic Eng., Australia*, vol. 6, no. 3, pp. 191–198, Sept. 1986.

[45] Cox, D. R. *Renewal Theory*, Menthuen, London, 1962.

[46] Demirbaş, K. “Manoeuvring-Target Tracking With the Viterbi Algorithm in the Presence of Interference”, *IEE Proc. F*, vol. 136, pp. 262–268, Dec. 1989.

[47] Demirbaş, K. “Manoeuvring Target Tracking With Hypothesis Testing” *IEEE Trans. AES*, vol. 23, no. 6, pp. 757–766, Nov. 1987.

[48] Dempster, A. P., Laird, N. M., and Rubin, D. B. “Maximum Likelihood from Incomplete Data via the *EM* Algorithm”, *J. Royal Statistical Soc.*, vol. 39, no. 1, pp. 1–38, 1977.

[49] Dillard, G. M., and Antoniak, C. E. “A Practical Distribution-Free Detection Procedure for Multiple Range-Bin Radars”, *IEEE Trans. AES*, vol. AES-6, no. 5, pp. 629–635, Sept. 1970.

[50] Dufour, F., and Mariton, M. “Tracking a 3-D Maneuvring Target with Passive Sensors”, *IEEE Trans. AES*, vol. 27, no. 4, pp. 725–739, Jul. 1991.

[51] Farina, A., and Studer, F. A. *Radar Data Processing, vol. I: Introduction and Tracking*, J. Wiley, 1985.

[52] Farooq, M., Bruder, S. “Comments on ’Tracking a Manoeuvring Target Using Input Estimation”, *IEEE Trans. AES*, AES-25, Mar 1989, pp. 300–302.

[53] Farooq, M., Bruder, S. “Information type filters for tracking a maneuvering target”, *IEEE Transactions AES*, vol. 26, no. 3 , pp. 441–54, May 1990.

[54] Farooq, M., Bruder, S., Quach, T., Lim, S.S. “Adaptive filtering techniques for manoeuvring targets”, *Proceedings of the 34th Midwest Symposium on Circuits and Systems*, pp. 31–4, vol.1, 1992, Monterey, CA, USA.

[55] Forney Jr., G. D. “The Viterbi Algorithm”, *Proc. IEEE*, vol. 61, no. 3, pp. 268–278, Mar. 1973.

[56] Fraser, D. C. and Potter, J. E. “The Optimal Linear Smoother as a Combination of Two Optimal Linear Filters”, *IEEE Trans. AC*, pp. 387–390, 1969.

[57] Gauvrit, M. “Bayesian Adaptive Filter for Tracking with Measurements of Uncertain Origin”, *Automatica*, vol. 20, no. 2, pp. 217–224, 1984.

[58] Gholson, N. H., Moose, R. L. “Manoeuvring Target Tracking Using Adaptive State Estimation”, *IEEE Trans. AES*, AES-13, May 1977, pp. 310–317.

[59] Gustafson, F. “The Marginalized Likelihood Ratio Test for Detecting Abrupt Changes”, *IEEE Trans. AC*, vol. AC-41, pp. 66–78, Jan. 1996.

[60] Guu, J.-A., Wei, C.-H. “Tracking technique for manoeuvring target with correlated measurement noises and unknown parameters”, *IEE Proceedings F [Radar and Signal Processing]*, vol. 138, no. 3, pp. 278–88, June 1991.

[61] Hampton, R. L. T., and Cooke, J. R. “Unsupervised Tracking of Maneuvering Vehicles”, *IEEE Trans. AES*, vol. AES-9, pp. 197–207, March 1973.

[62] Helferty, J.P. “Improved tracking of maneuvering targets: the use of turn-rate distributions for acceleration modeling”, *IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems*, pp. 515–20 , 1994, Las Vegas, NV, USA.

[63] Helmick, R.E., Blair, W.D., Hoffman, S.A. “Interacting multiple-model approach to fixed-interval smoothing”, *Proceedings of the 32nd IEEE Conference on Decision and Control*, pp. 3052–7 vol.4, 1993, San Antonio, TX, USA.

[64] Helmick, R.E., Watson, G.A. “IMM-IPDAF for track formation on maneuvering targets in cluttered environments”, *Proceedings of the SPIE: Signal and Data Processing of Small Targets*, vol. 2235, pp. 460–71, 1994, Orlando, FL, USA.

[65] Hou Ming. “Comments on 'Tracking a Manoeuvring Target Using Input Estimation' ”, *IEEE Trans. AES*, AES-25, Mar 1989, p280.

[66] Houles, A., and Bar-Shalom, Y. “Multisensor Tracking of a Maneuvring Target in Clutter”, *IEEE Trans. AES*, vol. AES-25, no. 2, pp. 176–189, Mar. 1989.

[67] Howard, R. A. *Dynamic Probabilistic Systems: Markov Models*, John Wiley, N.Y., 1971.

[68] Jaffer, A. G., and Gupta, S. C. “Recursive Bayesian Estimation with Uncertain Obsevation”, *IEEE Trans. Info. Th.*, vol. IT-17, pp. 614–616, Sept. 1971.

[69] Jazwinski, A. H. *Stochastic Processes and Filtering Theory*, Academic Press, 1970.

[70] Kailath, T. *Linear Systems*, Prentice-Hall, Englewood Cliffs, N.J., 1980.

[71] Katsikas, S.K., Lainiotis, D.G. “Partitioning algorithms in underwater passive target tracking”, *OCEANS '93. Engineering in Harmony with Ocean Proceedings*, pp. I463–8 vol.1, 1993, Victoria, BC, Canada.

[72] Kenefic, R. “Optimum Tracking of Maneuvering Target in Clutter”, *IEEE Trans. AC*, vol. AC-26, pp. 750–753, June 1981.

[73] Kim, K., Hansen, J. “Development of fuzzy algorithm for tracking of maneuvering targets”, *1992 IEEE International Conference on Systems, Man and Cybernetics*, pp. 803–8 vol.1, 1992, Chicago, IL, USA.

[74] Koch, W. “On Bayesian IMM-Tracking for Phased Array Radar” *Proc. Int. Radar Symposium*, Munich, pp. 715–723, Sept. 1998.

[75] Koch, W. “Retrodiction for Bayesian Multiple Hypothesis/Multiple Target Tracking in Densely Cluttered Environment”, *Proc. SPIE*, Orlando, vol. 2759, pp. 429–440, Apr. 1996.

[76] Koch, W., and Van Keuk, G. “Multiple Hypothesis Track Maintenance with Possibly Unresolved Measurements”, *IEEE Trans. AES*, vol. 33, no. 3, pp. 883–892, July 1997.

[77] Korn, J., Gully, S. W., and Willsky, A. S. “Application of Generalized Likelihood Ratio Algorithm to Manoeuvre Detection and Estimation”, *Proc. Amer. Control Conf.*, pp. 792–798, June 1982.

[78] Kosuge, Y. “Multiple maneuver model joint probabilistic data association for multi-target tracking”, *ISNCR-89. Noise and Clutter Rejection in Radars and Imaging Sensors.*, pp. 602–7, 1990, Kyoto, Japan.

[79] La Scala, B. F. and Pulford, G. W. "Viterbi Data Association Tracking for Over-the-Horizon Radar", *Proc. International Radar Symposium*, Vol. 3, pp. 1155–1164, Munich, Sept. 1998.

[80] La Scala, B. F. and Pulford, G. W. "An Analysis of Manoeuvring Target Detectors and Trackers for Over-The-Horizon Radar", CSSIP Report No. 29/96 to High Frequency Radar Division, Nov. 1996, `http:// www.ee.mu.oz.au/ research/ cssip/ publications/ 9798/ vda-man1.pdf`.

[81] La Scala, B. F. and Pulford, G. W. "Implementation of a Viterbi Data Association Tracker for Manoeuvring Targets in Over-The-Horizon Radar", CSSIP Report No. 12/97 to High Frequency Radar Division, May 1997, `http:// www.ee.mu.oz.au/ research/ cssip/ publications/ 9798/ vda-man2.pdf`.

[82] Lee Hsi-Jian, Chyan-Goei Chung, Ker-Chang Chang. "Manoeuvring target tracking in a cluttered environment from noisy images by an adaptive probabilistic data association filter", *International Journal of Systems Science*, vol. 22, no. 1 , pp. 133–49, Jan. 1991.

[83] Lerro, D., and Bar-Shalom, Y. "Automated Tracking with Target Amplitude Information", *Proc. Amer. Contr. Conf.*, pp. 2875–2880, 1990.

[84] Lerro, D., and Bar-Shalom, Y. "Interactive Multiple Model Tracking with Target Amplitude Feature", *IEEE Trans. Aerosp & Electronic Sys.*, pp. 494–509, Apr. 1993.

[85] Li, X. R. and Bar-Shalom, Y. "Multiple-Model Estimation with Variable Structure", *IEEE Trans. AC*, vol. 41, pp. 478–493, April 1996.

[86] Lim, S.S., Farooq, M. "Maneuvering target tracking using jump processes", *Proceedings of the 30th IEEE Conference on Decision and Control*, pp. 2049–54 vol.2, 1991, Brighton, UK.

[87] Ljung, L. *System Identification: Theory for the User*, Prentice Hall, N. J., 1987.

[88] Logothetis, A., Krishnamurthy, V., Holst, J., and Isaksson, A. " Modal State Estimation of a Maneuvering Target in Clutter", *Proc. 36th IEEE Conf. on Decision and Control*, San Diego, pp.5024–5029, Dec.1997.

[89] Martinerie, F., and Forster, P. "Data Association and Tracking Using Hidden Markov Models and Dynamic Programming", *IEEE ICASSP*, vol. II, pp. 449–452, 1992.

[90] Maybeck, P. S., and Hentz, K. P. "Investigation of Moving Bank Multiple Model Adaptive Algorithms", *AIAA J. Guidance, Control, Dynamics*, vol. 10, pp. 90–96, Jan. 1987.

[91] McAulay, R. J., Denlinger, E. "A Decision-Directed Adaptive Tracker", *IEEE Trans. AES*, AES-9, Mar 1973, pp. 229–236.

[92] Mehrotra, K., and Mahapatra, P. R. "A jerk Model for Tracking Highly Maneuvering Targets", *IEEE Trans. AES*, vol. 33, no. 4, pp. 1094–1105, Oct. 1997.

[93] Moon, T. K. "The Expectation-Maximisation Algorithm", *IEEE Signal Processing Magazine*, pp. 47–60, Nov. 1996.

[94] Moose, R. L. "An Adaptive State Estimation Solution to the Manoeuvring Target Problem", *IEEE Trans. Auto Control*, AC-20, June 1975, pp. 359–362.

[95] Moose, R. L., Vanlandingham, H. F., McCabe, D. H. "Modeling and Estimation for Tracking Manoeuvring Targets", *IEEE Trans. AES*, AES-13, May 1979, pp. 448–456.

[96] Munir, A., Atherton, D.P. "Adaptive interacting multiple model algorithm for tracking a manoeuvring target", *IEE Proceedings - Radar, Sonar and Navigation*, vol. 142, no. 1, pp. 11–17, Feb. 1995.

[97] Munir, A., Atherton, D.P. "Maneuvering target tracking using an adaptive interacting multiple model algorithm", *Proceedings of the 1994 American Control Conference*, pp. 1324–8 vol.2, 1994, Baltimore, MD, USA.

[98] Musicki, D., Evans, R. "IPDA-PMM algorithm for tracking maneuvering targets in clutter", *Proceedings of the 33rd IEEE Conference on Decision and Control*, pp. 3308–9 vol.4, 1994, Lake Buena Vista, FL, USA.

[99] Mušicki, D., Evans, R. J., and Stanković, S. "Integrated Probabilistic Data Association", *IEEE Trans. Auto. Control*, Vol. 39, no. 6, pp. 1237–1241, June 1994.

[100] Nagarajan, V., Sharma, R. N., Chidambara, M. R. "An Algorithm for Tracking a Maneuvering Target in Clutter", *IEEE Trans. AES*, vol. AES-20, pp. 560–573, Sept. 1984.

[101] Ott, P., Chan, K.C.C., Leung, H. "A study on fuzzy tracking of maneuvering aircraft using real data", *Proceedings of the Third IEEE Conference on Fuzzy Systems*, pp. 1671–5 vol.3, 1994, Orlando, FL, USA.

[102] Papoulis, A. *Probability, Random Variables, and Stochastic Processes*, Mc Graw-Hill, NY, 1991.

[103] Park Yong Hwan, Seo, J.H., Lee, J.G. "Tracking using the variable-dimension filter with input estimation", *IEEE Transactions AES*, vol. 31, no. 1, pp. 399–408, Jan. 1995.

[104] Pattipati, K. R., and Sandell Jr., N. R. "A Unified View of State Estimation in Switching Environments", *Proc. Amer. Control Conf.*, June, 1983.

[105] Pulford, G. W. "Is the Probabilistic Multihypothesis Tracking Algorithm Optimal?", University of Melbourne Technical Note, March 1998, `http:// www.ee.mu.oz.au/ research/ cssip/ staff/ gpulford/ pmht_note.pdf`.

[106] Pulford, G. W. and B. F. La Scala. "Manoeuvring Target Tracking using the Expectation-Maximisation Algorithm", *Proc. 4th Int Conf. on Control, Automation, Robotics and Vision*, Singapore, pp. 2340–2344, Dec. 1996.

[107] Pulford, G. W., and B. F. La Scala. "Over-The-Horizon Radar Tracking Using The Viterbi Algorithm - Second Report to High Frequency Radar Division", CSSIP Report No. 16/95, Adelaide, August 1995, `http:// www.ee.mu.oz.au/ research/ cssip/ publications/ 9798/ vda-rep2.pdf`.

[108] Pulford, G. W., and B. F. La Scala. "Over-The-Horizon Radar Tracking Using The Viterbi Algorithm - Third Report to High Frequency Radar Division", CSSIP Report No. 27/95, Adelaide, December 1995, `http:// www.ee.mu.oz.au/ research/ cssip/ publications/ 9798/ vda-rep3.pdf`.

[109] Pulford, G. W., and La Scala, B. F. "MAP Estimation of Target Manoeuvre Sequence with the Expectation-Maximisation Algorithm", *Proc. 4th Int Conf. on Control, Automation, Robotics and Vision*, Singapore, pp. 2340–2344, Dec. 1996.

[110] Pulford, G. W., and La Scala, B. F. "A Survey of Manoeuvring Target Tracking Methods and their Applicability to Over-The-Horizon Radar", CSSIP Report No. 14/96 to High Frequency Radar Division, July 1996, `http:// www.ee.mu.oz.au/ research/ cssip/ publications/ 9798/ manoeuvre_survey.pdf`.

[111] Quach, T. and Farooq, M. "Maximum Likelihood Track Formation with the Viterbi Algorithm", *Proc. 33rd IEEE Conf. on Decision and Control*, Orlando, FL, pp. 271–276, Dec. 1994.

[112] Quach, T., Farooq, M. "Application of the MHT to track formation of a single maneuvering acoustic source in clutter", *Proceedings of the 36th Midwest Symposium on Circuits and Systems*, pp. 684–7 vol.1, 1993, Detroit, MI, USA.

[113] Reid, D. B. "An Algorithm for Tracking Multiple Targets", *IEEE Trans. AC*, pp. 843–854, Dec. 1979.

[114] Ricker, G. G., Williams, J. R. "Adaptive Tracking Filter Manoeuvring Targets", *IEEE Trans. AES*, AES-14, Jan. 1978, pp. 185–193.

[115] Rouhi, A., Farooq, M., Bruder, S. "Multiple structure adaptive target tracking", *Proceedings of the 32nd Midwest Symposium on Circuits and Systems*, pp. 613–16 vol.1, 1990, Champaign, IL, USA.

[116] Sastry, C.R., Slocumb, B.J., West, P.D., Kamen, E.W. "Tracking a maneuvering target using jump filters", *Proceedings of the 1994 American Control Conference*, pp. 2081–7 vol.2, 1994, Baltimore, MD, USA.

[117] Sengupta, D., Iltis, R.A. "Tracking of multiple maneuvering targets in clutter by joint probabilistic data and maneuver association", *Proceedings of the 1989 American Control Conference*, pp. 2696–701 vol.3, 1989, Pittsburgh, PA, USA.

[118] Sheldon, S. N., and Maybeck, P. S. "An Optimizing Design Strategy for Multiple Model Adaptive Estimation and Control", *IEEE Trans. AC*, vol. AC-38, pp. 651–654, April 1993.

[119] Singer, R. A. "Estimating Optimal Tracking Filter Performance for Manned Maneuvering Targets", *IEEE Trans. AES*, vol. 6, pp. 473–483, July 1970.

[120] Singer, R. A., Sea, R. G., and Housewright, K. B. "Derivation and Evaluation of Improved Tracking Filters for Use in Dense Multitarget Environments", *IEEE Trans. IT*, vol. IT-20, no. 4, pp. 423–432, Jul. 1974.

[121] Slocumb, B.J., West, P.D., Shirey, T.N., Kamen, E.W. "Tracking a maneuvering target in the presence of false returns and ECM using a variable state dimension Kalman filter", *Proceedings of the 1995 American Control Conference*, pp. 2611–15 vol.4, 1995, Seattle, WA, USA.

[122] Smith, P. L., and Buechler, G. "A Branching Algorithm for Discriminating and Tracking Multiple Objects", *IEEE Trans. AC*, vol. AC-20, pp. 101–104, Feb. 1975.

[123] Streit, R. L., and Barrett, R. F. "Frequency Line Tracking Using Hidden Markov Models", *IEEE Trans. ASSP*, pp. 586–598, 1990.

[124] Streit, R. L., and Luginbuhl, T. E. "Maximum Likelihood Method for Probabilistic Multi-Hypothesis Tracking", *Proc. SPIE Symp. Signal & Data Proces. Small Targets*, vol. 2335–24, April 1994.

[125] Sun, H.M., Chiang, S.M. "Manoeuvring target tracking algorithm for a radar system", *International Journal of Systems Science*, vol. 20, no. 10 , pp. 1801–11, Oct. 1989.

[126] Swerling, P. "Probability of Detection for Fluctuating Targets", *IRE Trans. Info. Th.*, vol. IT-6, pp. 269–308, April 1960.

[127] Sworder, D. D., Singer, P. F., Doria, D., and Hutchins, R. G. "Image-Enhanced Estimation Methods", *Proc. IEEE*, vol. 81, no. 6, pp. 797–812, June 1993.

[128] Sworder, D.D., Kent, M., Vojak, R., Hutchins, R.G. "Renewal models for maneuvering targets", *IEEE Transactions AES*, vol. 31, no. 1, pp. 138–50, Jan. 1995.

[129] Tanner, M. A. *Tools for Statistical Inference: Methods for Exploration of Posterior Distributions and Likelihood Functions*, 2nd ed., Springer-Verlag, N.Y., 1993.

[130] Tao Tao, Xiong Youleng, Yang Suzi. "Target grey tracking in a multitarget environment", *Journal of Grey Systems*, vol. 4, no. 2 , pp. 153–61, 1992.

[131] Thorp, J. S. "Optimal Tracking of Manoeuvring Targets", *IEEE Trans. AES*, AES-9, July 1973, pp. 512–519.

[132] Titterington, D. M., Smith, A. F. M., and Makov, U. E. *Statistical Analysis of Finite Mixture Distributions*, John Wiley, NY, 1985.

[133] Tomasini, B., Gauvrit, M., and Siffredi, B. "Bayesian Adaptive Commands Filter for Maneuvring Target Tracking with Measurements of Uncertain Origin", *Proc. ISSPA'87*, Brisbane, pp. 473–479, Aug. 1987.

[134] Tomasini, B., Gauvrit, M., Siffredi, B. "Bayesian adaptive filters for multiple maneuvering target tracking with measurements of uncertain origin", *Proceedings of the 28th IEEE Conference on Decision and Control*, pp. 1397–9 vol.2, 1989, Tampa, FL, USA.

[135] Tomasini, B., Gauvrit, M., Siffredi, B. "Bayesian adaptative filter for multiple maneuvering target tracking with measurements of uncertain origin", *Proceedings of the SPIE: Signal and Data Processing of Small Targets*, vol. 1096 , pp. 209–14, 1989, Orlando, FL, USA.

[136] Torelli, R., Graziano, A., and Farina, A. "IM3HT Algorithm: A Joint Formulation of IMM and MHT for Multitarget Tracking", *Proc. Euro. Control Conf.*, Bruxelles, 1997.

[137] Tsaknakis, H., Athans, M. "Tracking maneuvering targets using H-infinity filters", *Proceedings of the 1994 American Control Conference*, pp. 1796–803 vol.2, 1994, Baltimore, MD, USA.

[138] Tugnait, J. K. "Detection and Estimation for Abruptly Changing Systems", *Automatica*, vol. 18, no. 5, pp. 607–615, 1982.

[139] Van Trees, H. L. *Detection, Estimation, and Modulation Theory Part 1*, John Wiley, New York, 1968.

[140] Walton, A.M. "Tracking manoeuvring targets with a low probability of detection in clutter", *IEE Proceedings F [Radar and Signal Processing]*, vol. 137, no. 3 , pp. 183–6, June 1990.

[141] Wang, H.P., Kung, M.C., Lin, T.Y. "Multi-model adaptive Kalman filter design for manoeuvring target tracking", *International Journal of Systems Science*, vol. 25, no. 11, pp. 2039–46, Nov. 1994.

[142] Wang, T.C., Varshney, P.K. "A tracking algorithm for maneuvering targets", *IEEE Transactions AES*, vol. 29, no. 3, pp. 910–25, July 1993.

[143] Watson, G. A., and Blair, W. D. "Solution to Second Benchmark Problems for Tracking Manoeuvring Targets in the Presence of False Alarms and ECM", *Proc. SPIE*, San Diego, vol. 2561, pp. 263–274, July 1995.

[144] Watson, G.A., and Blair, W.D. "Interacting acceleration compensation algorithm for tracking maneuvering targets", *Record of the 1993 IEEE National Radar Conference*, pp. 281–5, 1993, Lynnfield, MA, USA.

[145] Watson, G.A., and Blair, W.D. "IMM algorithm for tracking targets that maneuver through coordinated turns", *Proceedings of the SPIE: Signal and Data Processing of Small Targets*, vol. 1698, pp. 236–47, 1992, Orlando, FL, USA.

[146] Watson, G.D., Blair, W.D. "Constant speed prediction for maneuvering targets using a three dimensional turning rate", *Proceedings. The Twenty-Third Southeastern Symposium on System Theory*, pp. 239–43, 1991, Columbia, SC, USA.

[147] West, P, Haddad, A.H. "Switched-Markov filtering for tracking maneuvering targets", *Proceedings of the 1991 American Control Conference*, pp. 2299–300 vol.3, 1991, Boston, MA, USA.

[148] White, L. B. "Robust Approximate Likelihood Ratio Tests for Nonlinear Dynamic Systems", *Proc. ICASSP*, vol. 4, pp. 357–360, Adelaide, 1994.

[149] Willett, P., Ruan, Y., and Streit, R. "The PMHT for Maneuvering Targets", *Proc SPIE*, Orlando, vol. 3373, April 1998.

[150] Xie Xianya, and Evans, R. J. "Multiple Target Tracking and Multiple Frequency Line Tracking Using Hidden Markov Models", *IEEE Trans. Signal Proc.*, vol. 39, no. 12, pp. 2659–2676, Dec. 1991.

[151] Xie Xianya, and Evans, R. J. "Multiple Frequency Line Tracking with Hidden Markov Models - Further Results", *IEEE Trans. Signal Proc.*, vol. 41, no. 1, pp. 334–343, 1993.

[152] Zhongliang Jing, Zhang Guowei, Zhou Hongren. “A fast adaptive neural network scheme for multi-maneuvering target tracking”, *Proceedings of the 32nd IEEE Conference on Decision and Control*, pp. 3253–8 vol.4, 1993, San Antonio, TX, USA.

[153] Zhongliang Jing, Zhou Hongren, Wang Peide. “Tracking initiation and termination of multiple maneuvering targets in a dense multi-return environment”, *Proceedings of the 29th IEEE Conference on Decision and Control*, pp. 2270–5 vol.4, 1990, Honolulu, HI, USA.

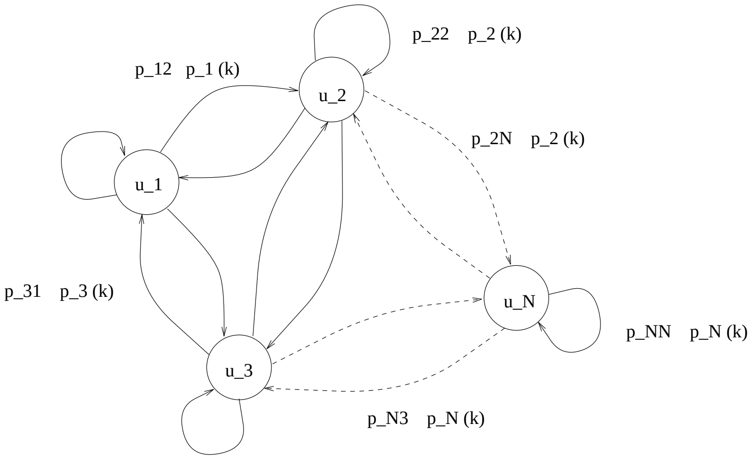

Figure 1: Finite-state, discrete-time semi-Markov model of manoeuvring target acceleration. The model parameters are the transition probabilities p_{ij} and the holding-time distributions $p_i(k)$ which determine the probability of switching as a function of the time spent in the current manoeuvre state.

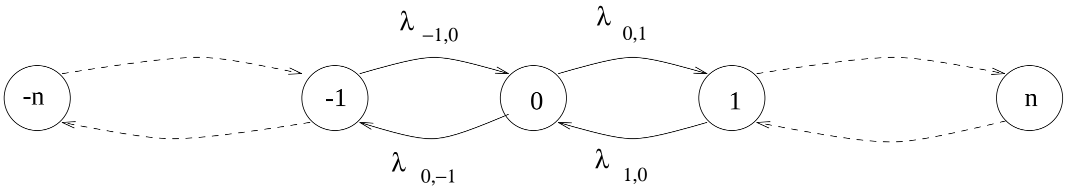

Figure 2: Discrete-state Poisson process manoeuvre model.

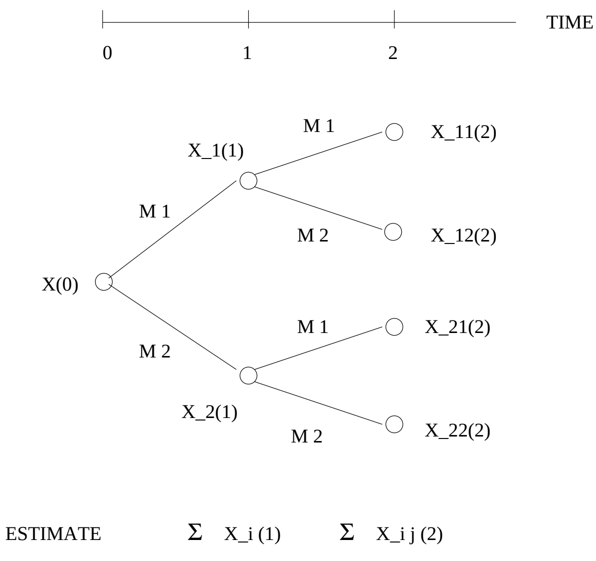

Figure 3: Idea behind branching filters (*e.g.*, MHT) and optimal Bayesian algorithms with two possible manoeuvre models $M1$ or $M2$ at each time instant. The MAP estimate is obtained by taking the most probable manoeuvre sequence at time k, while the MMSE estimate corresponds to a weighted sum of all conditional state estimates at a given time. Sub-optimal approximations are obtained by merging common path histories and deleting unlikely branches.

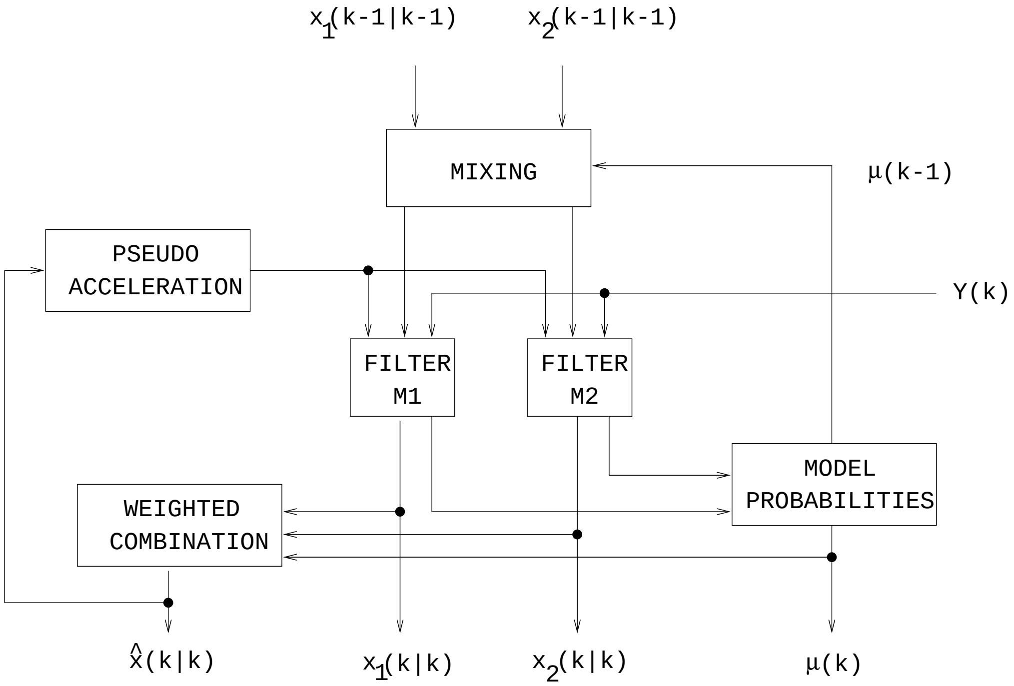

Figure 4: Block diagram of the interacting multiple model algorithm with acceleration compensation.

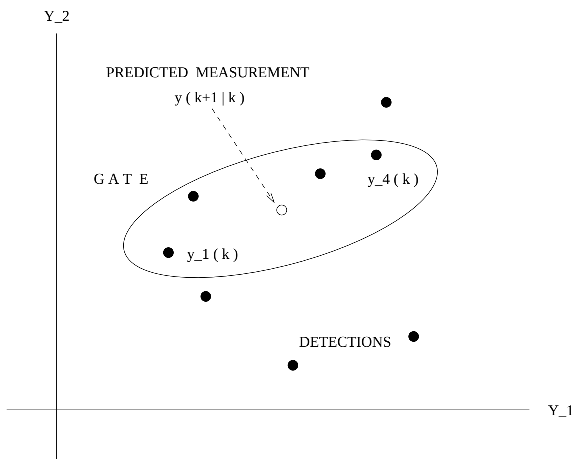

Figure 5: Gating is applied to measurements to reduce the processing required for tracking in a cluttered environment. The gate is centred on the predicted measurement.

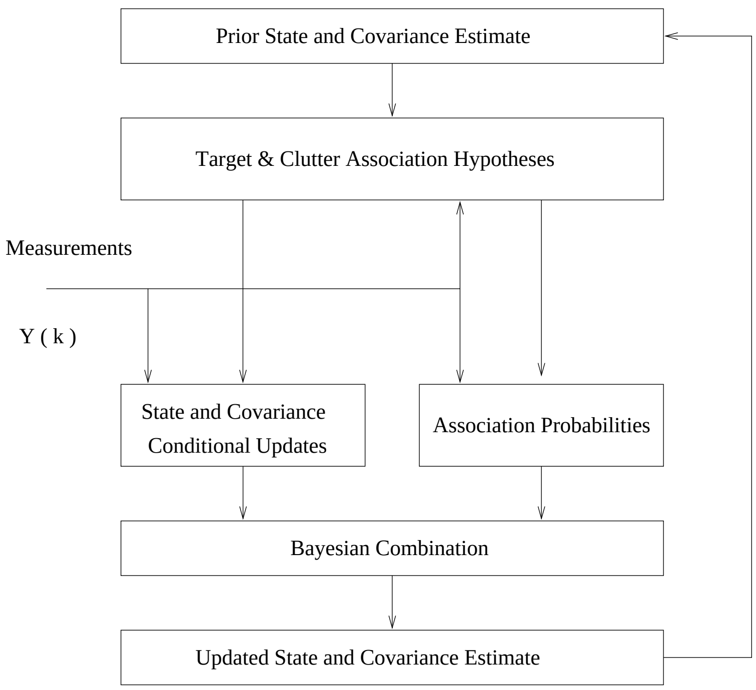

Figure 6: Block diagram of a probabilistic data association filter. The same processing applies to any algorithm that approximates the measurement density by a single Gaussian at each scan.