Exercise for the Lecture on Materials Science

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Exercise Sheet 3

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Exercise 1: Tensile Test

The following measurements were obtained from a tensile test performed on a cylindrical specimen made of an aluminum alloy. The applied tensile force and the corresponding specimen length were recorded. The initial length of the specimen is 50.800 mm, and the diameter is 12.8 mm.

Tensile Force (N)	**Length (mm)**
0	50.800
7330	50.813
15100	50.827
23100	50.841
30400	50.855
34400	51.054
38400	51.308
41300	51.816
44800	52.832
46200	53.848
47300	54.864
47500	55.880
46100	56.896
44800	57.658
42600	58.420
36400	59.182

Table 1: Measured tensile force and length of the aluminum alloy specimen

1. Calculate the engineering stress and engineering strain for all measurement points. Use the calculated values to plot a stress-strain diagram.

2. Why is the 0.2% proof stress used to determine the yield strength in certain materials, instead of directly obtaining the yield strength from the linear (Hookean) region?

3. Based on the generated stress-strain diagram, determine the following material properties of the aluminum alloy and mark them in the diagram:

   (a) Young's modulus

   (b) Yield strength (0.2% proof stress)

(c) Ultimate tensile strength

(d) Fracture strain (ductility)

4. Assume that the aluminum alloy is isotropic. A Poisson's ratio of $\nu = 0.33$ is obtained from reference tables. Estimate the volume change of the specimen under tensile loading. Then calculate the shear modulus G.

5. Calculate the resilience modulus of the aluminum alloy using the formula:

$$E_r = \frac{\sigma_y^2}{2E}$$

Exercise 2: Elasticity of a Rubber Strip

An elastic rubber strip has a length of 12 cm and a cross-sectional area of $1\,\mathrm{cm}^2$. At a temperature of 20 °C, the rubber is stretched to a length of 30 cm by applying a tensile stress of 2 MPa.

1. The Young's modulus E is a material-dependent property. In polymer physics, the number n (in moles per cubic centimeter) of elastically effective polymer chains is used as a measure of the crosslink density of an elastomer. Determine E and n from the given test parameters.

2. The rubber strip is intended for use as a flexible seal in a technical application, operating under varying temperatures and loads. Calculate the tensile stress required to stretch the rubber strip to a length of 20 cm at room temperature and to a length of 30 cm at 100 °C.

Exercise 3: Theory of Viscoelasticity

A viscoelastic material is subjected to an instantaneous stress jump of

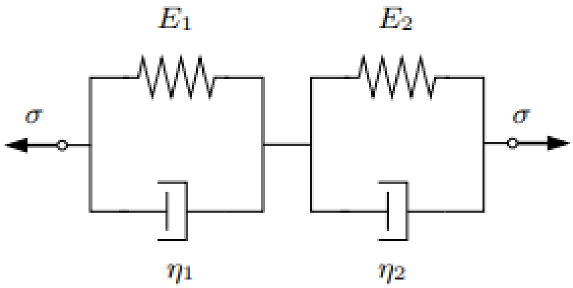
$$\sigma_0 = 1000\,\frac{N}{m^2}.$$

The relaxation time of the material is $\tau = 20\,s$.

1. Describe qualitatively how the strain $\varepsilon(t)$ of the material behaves over time.

2. Compare this behavior with that of an ideally elastic material and a purely viscous material.

Exercise 4: Modeling the Time-Dependent Deformation of a Viscoelastic Foam Using the Kelvin-Voigt Model



A viscoelastic polyurethane foam, as used in sports shoe midsoles, is to be mechanically characterized. The material can be idealized as a series connection of two Kelvin-Voigt elements [1].

The midsole consists of two material layers: a softer, more flexible upper layer with a spring stiffness of 5 MPa and a viscosity of $1.2 \times 10^5\,\mathrm{Pa \cdot s}$, and a stiffer lower layer with a spring stiffness of 20 MPa and a viscosity of $5 \times 10^5\,\mathrm{Pa \cdot s}$. The mechanical behavior of the midsole is modeled by a series connection of two Kelvin-Voigt elements.

In a load test, a runner applies a short-term constant compressive stress of 0.2 MPa to the sole. This load is maintained for 0.5 seconds before the foot is lifted and the load is removed.

Note: All necessary material parameters and loading data are contained in the text and must be extracted independently for the following calculations.

a) Derive the differential equation that relates stress σ and strain ε in the form:

$$p_0\sigma + p_1\dot{\sigma} = q_0\varepsilon + q_1\dot{\varepsilon} + q_2\ddot{\varepsilon}.$$

Determine the coefficients p_0, p_1, q_0, q_1, q_2.

b) Describe qualitatively the strain $\varepsilon(t)$ during loading ($0 \leq t \leq 0.5\,s$) and after unloading ($t > 0.5\,s$). Sketch this behavior and name the involved mechanical processes.

c) Discuss how an increase in η_1 would affect the functional performance of the shoe sole, particularly regarding shock absorption and energy return.

Source:

[1] Shen, Y., Golnaraghi, F., & Plumtree, A. (2001). Modelling compressive cyclic stress strain behaviour of structural foam. *International Journal of Fatigue*, **23**(6), 491–497. https://doi.org/10.1016/S0142-1123(01)00014-7