Exercise for the Lecture on Materials Science

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Exercise Sheet 4

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Exercise 1 – Elasticity, Plastic Deformation, and Necking

A cylindrical rod made of a high-strength aluminum alloy with $E = 70\,\text{GPa}$ and yield strength $R_{P0.2} = 220\,\text{MPa}$ has an initial length of $L_0 = 100\,\text{mm}$ and a diameter of $d = 10\,\text{mm}$. The rod is subjected to an axial tensile force. At a stress of 140 MPa, no permanent deformation remains after unloading. At a stress of 220 MPa, a permanent elongation of 0.8 mm remains after unloading.

a) Describe the elastic and plastic behavior of the rod during deformation. Discuss the return to the initial state, the material deformation, and the underlying physical mechanisms.

b) Calculate the elastic strain and absolute elongation of the rod at a stress of 140 MPa.

c) Calculate the total strain at a stress of 220 MPa, given that 0.8 mm permanent elongation remains after unloading. How large is the elastic recovery?

d) For case c), also calculate the engineering and true stress and strain. Explain the difference between engineering and true quantities and why this difference becomes important in the case of large plastic deformation.

e) The test is conducted before necking occurs. Explain what is meant by necking, how it affects the stress-strain curve, and why the distinction between uniform deformation and necking is important for describing material behavior.

Exercise 2 – Hardness Testing

A brass workpiece is tested using a ball with diameter $D = 10\,\text{mm}$ under a test force of $F = 3000\,\text{N}$. The indentation diameter is $d = 3.2\,\text{mm}$.

a) Calculate the Brinell hardness number (HB).

b) Estimate the tensile strength of the material based on the determined hardness value.

c) Discuss the relationship between hardness and ductility.

Exercise 3 – Mechanical Properties of Polymers

At room temperature, a PE-HD specimen with an initial length of $L_0 = 80\,\text{mm}$ is tested in a tensile experiment. The material has a Young's modulus of $E = 1000\,\text{MPa}$, reaches the yield strength at approximately $\sigma_\text{P} = 20\,\text{MPa}$, and fails at a tensile strength of around $\sigma_\text{M} = 30\,\text{MPa}$. The maximum strain at fracture is approximately $A = 600\,\%$.

a) Sketch the stress-strain diagram for the given material. Describe the mechanical behavior and discuss differences in modulus, fracture behavior, and temperature dependence.

b) Calculate the strain and absolute elongation of the sample when a stress of 15 MPa is applied and the deformation remains elastic.

c) If the sample is stretched up to the yield strength (20 MPa) and then unloaded, it only partially returns to its original length. Explain why and how this is mechanically understood.

d) The glass transition temperature (T_g) of the polymer is $-80\,°\text{C}$. Explain how the mechanical behavior of the material differs at $-100\,°\text{C}$ and $60\,°\text{C}$, based on molecular mobility.

e) Discuss why PE-HD has a higher elastic modulus and tensile strength compared to PE-LD, although both are made from ethylene.

f) Sketch the expected stress-strain curves of the following polymer materials in the same diagram. Briefly justify each assignment based on the mechanical behavior.

   i) Lightly crosslinked polyisoprene

   ii) Heavily crosslinked polyisoprene

   iii) Linear polyethylene

Exercise 4 – Polymer Chains

A linear polyethylene terephthalate (PET) sample has a molar mass of $M = 150\,\mathrm{kg\,mol^{-1}}$.

a) Calculate the degree of polymerization and the maximum stretched length of the PET chain. Use an approximate value of $M_0 = 192\,\mathrm{g\,mol^{-1}}$ for the molar mass of the repeating unit. The C–C bond length is 0.154 nm, and assume two C–C bonds per repeating unit along the chain.

b) For linear polymers, the average end-to-end distance in the disordered state is $r = d \cdot \sqrt{n}$, where d is the bond length and n the number of atoms along the main chain. Estimate r assuming about 10 atoms per repeating unit.