Exercise for the Lecture on Materials Science

Prof. Peter Müller-Buschbaum, Lea Westphal, Ziyan Zhang, Doan Duy Ky Le

Exercise Sheet 5

Solution

Exercise 1: Fracture Types

Image A – Moderately Ductile Fracture (SEM Image)

Mechanisms: Limited plastic deformation, typically initiated by microvoids at grain boundaries or inclusions. Dislocation motion is locally activated but restricted. Crack propagation occurs via void coalescence, without full necking.

Component Safety: Some ductility is present, allowing limited damage tolerance. However, risk of unstable crack propagation exists once local strength is exceeded.

Image B – Brittle Fracture (Chevron Pattern, Macroscopic)

Mechanisms: Crack growth occurs with no plastic deformation. Chevron markings indicate fast, unstable crack propagation. Minimal dislocation activity. No stress relaxation at the crack tip.

Component Safety: Very low. Fracture occurs suddenly without prior deformation. High notch sensitivity; applicable only in controlled loading environments.

Image C – Brittle Fracture (Facetted SEM)

Mechanisms: Facetted surface implies transgranular or intergranular fracture. No plastic deformation. Crack follows grain boundaries or crystallographic planes. Stress concentration at the crack tip remains unrelieved.

Component Safety: Very low. Sudden failure; sensitive to defects and notches. Critical under high strain rates or low temperatures.

Image D – Ductile Fracture (Cup-and-Cone, Macroscopic)

Mechanisms: Significant plastic deformation. Dislocation motion causes microvoid nucleation and coalescence. Pronounced necking typical of ductile failure.

Component Safety: Very high. Material shows visible deformation before failure. High energy absorption. Excellent damage and fault tolerance.

Exercise 2: Failure Analysis

Case 1: Brittle Plastic Housing (Polystyrene)

Given:

- Stress: $\sigma = 1.4\,\text{MPa}$
- Elastic modulus: $E = 3.2 \times 10^9\,\text{Pa}$
- Surface energy: $\gamma = 0.45\,\text{J}\,\text{m}^{-2}$

Objective: Determine the maximum allowable crack length using Griffith's equation:

$$a = \frac{2E\gamma}{\pi\sigma^2}$$

Solution:

$$a = \frac{2 \cdot 3.2 \cdot 10^9 \cdot 0.45}{\pi \cdot (1.4 \cdot 10^6)^2} = 0.47\,\text{mm}$$

Conclusion: A surface crack should not exceed 0.47 mm. Since no NDT is used, this poses a risk of brittle failure.

Case 2: Crack in an Aluminum Structure

Given:

- $K_{\text{IC}} = 38\,\text{MPa}\sqrt{\text{m}}$
- $\sigma_1 = 360\,\text{MPa},\ a_1 = 2.2\,\text{mm}$
- $a_2 = 4.5\,\text{mm}$

Solution:

$$\sigma_2 = \sigma_1 \cdot \sqrt{\frac{a_1}{a_2}} = 360 \cdot \sqrt{\frac{2.2}{4.5}} \approx 252\,\text{MPa}$$

Conclusion: The allowable stress for a longer crack is 252 MPa. Safety margin is reduced.

Case 3: Polymer in Medical Device

Given:

- $K_{\text{IC}} = 2.1\,\text{MPa}\sqrt{\text{m}}$
- $\sigma = 4.2\,\text{MPa}$
- $a = 0.4\,\text{mm}$

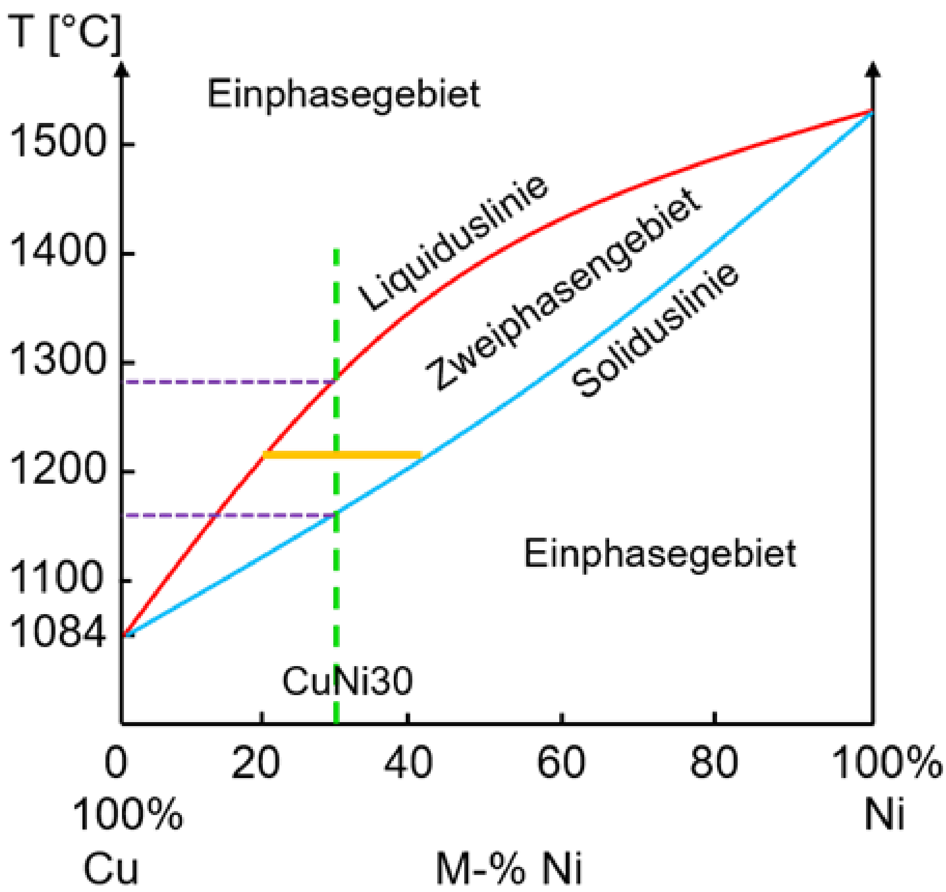
Solution:

$$\sigma_c = \frac{K_{\text{IC}}}{\sqrt{\pi a}} = \frac{2.1}{\sqrt{\pi \cdot 4.0 \cdot 10^{-4}}} \approx 59.3\,\text{MPa}$$

Conclusion: Applied stress is much lower than critical stress. Crack is non-critical.

Exercise 3: Binary Phase Diagrams (Cu–Ni)

c) Solidification range: 1280 °C to 1160 °C
d) Equal phase amounts at approx. 1210 °C
e) At this temperature: Liquid = 20 % Ni, Solid = 40 % Ni



Exercise 4: Eutectic Phase Diagrams

4.1 Phase Mass Fractions

From the lever rule:

$$w_\alpha = \frac{4}{7} \approx 57.1\,\%, \quad w_\beta = 42.9\,\%$$

4.2 Phase Compositions

- α: 80 % A, 20 % B
- β: 10 % A, 90 % B

4.3 Shift of Point P to the Right

- w_β increases, w_α decreases.
- Compositions C_α and C_β remain constant.

4.4 Eutectic vs. Non-Eutectic Solidification

Eutectic:

$$L \to \alpha + \beta \quad \text{(at constant temperature)}$$

Fine lamellar structure forms directly.

Non-Eutectic:

1. Primary phase α or β solidifies first.
2. Remaining liquid solidifies as eutectic mixture.

Exercise 5: Determining Phase Boundary Compositions (Pb–Sn)

Given:

Composition C_0 [wt.-% Sn]	w_α	w_β
15	0.20	0.80
60	0.78	0.22

Calculation:

Lever rule:

$$w_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha}$$

From the two equations:

$$0.20 = \frac{C_\beta - 15}{C_\beta - C_\alpha}, \quad 0.78 = \frac{C_\beta - 60}{C_\beta - C_\alpha}$$

Solving:

$$C_\alpha \approx 77.6\,\%, \quad C_\beta \approx 30.5\,\%$$

Interpretation:

At 150 °C, the α-phase is Sn-rich, the β-phase is Pb-rich. The calculated compositions match the solubility limits.