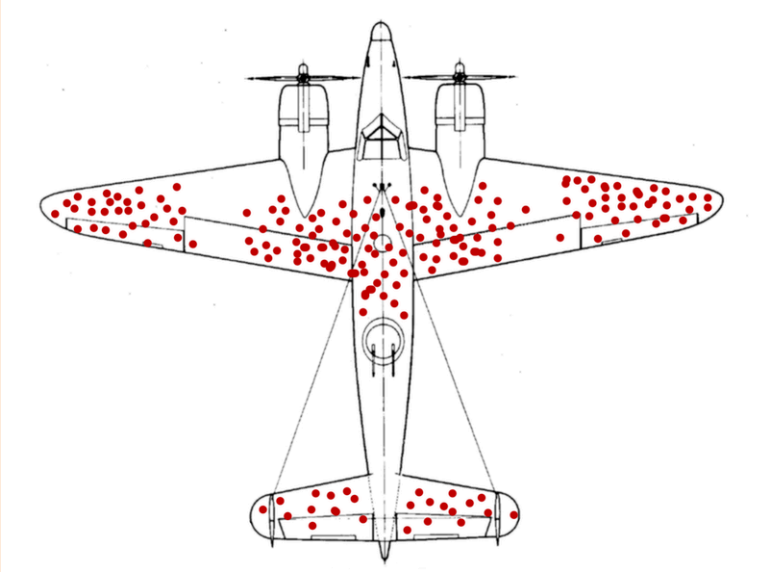
CPSC 340:
Machine Learning and Data Mining

L2Regularization

Bonus Slides

Related: Survivorship Bias

- Plotting location of bullet holes on planes returning from WW2:



- Where are the “relevant” parts of the plane to protect?
  - “Relevant” parts are actually where there are no bullets.
  - Planes shot in other places did not come back (armor was needed).

https://en.wikipedia.org/wiki/Survivorship_bias

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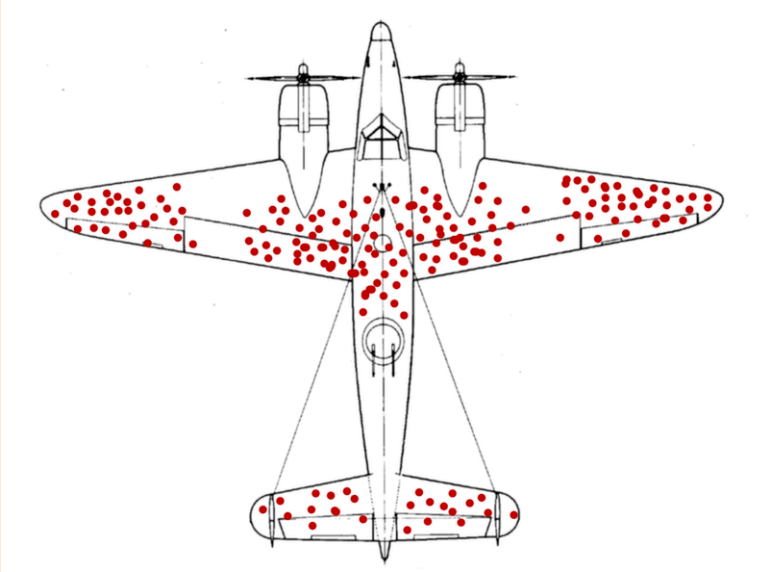

- This is an example of "survivorship bias":
  - Data is not IID because you only sample the "survivors".
  - Causes havoc for feature selection, and ML methods in general.

https://en.wikipedia.org/wiki/Survivorship_bias

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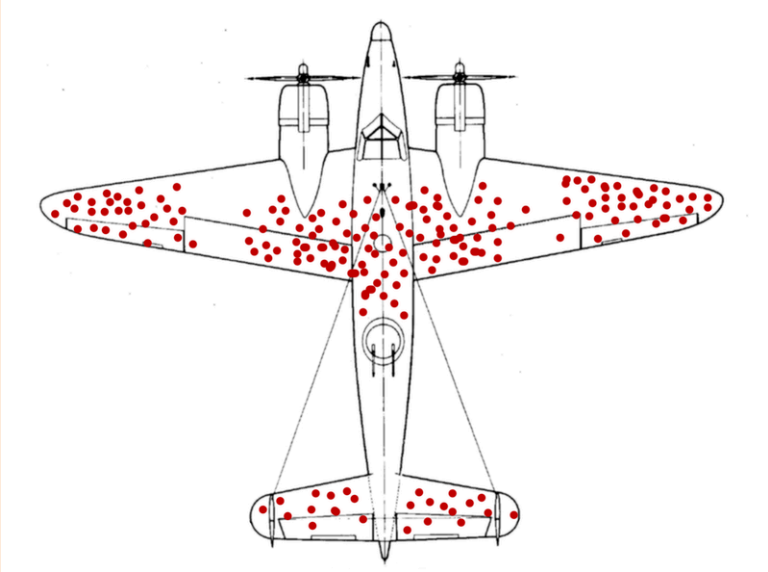

- People come to wrong conclusions due to survivor bias all the time.
  - Article on “secrets of success”, focusing on traits of successful people.
    - But ignoring the number of non-super-successful people with the same traits.
  - Article hypothesizing about various topics (allergies, mental illness, etc.).

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Why use L2-Regularization?

- It's a weird thing to do, but Mark says "always use regularization".
  - "Almost always decreases test error" should already convince you.

- But here are 6 more reasons:
  1. Solution 'w' is unique.
  2. X^TX does not need to be invertible (no collinearity issues).
  3. Less sensitive to changes in X or y.
  4. Gradient descent converges faster (bigger λ means fewer iterations).
  5. Stein's paradox: if $d \geq 3$, 'shrinking' moves us closer to 'true' w.
  6. Worst case: just set λ small and get the same performance.

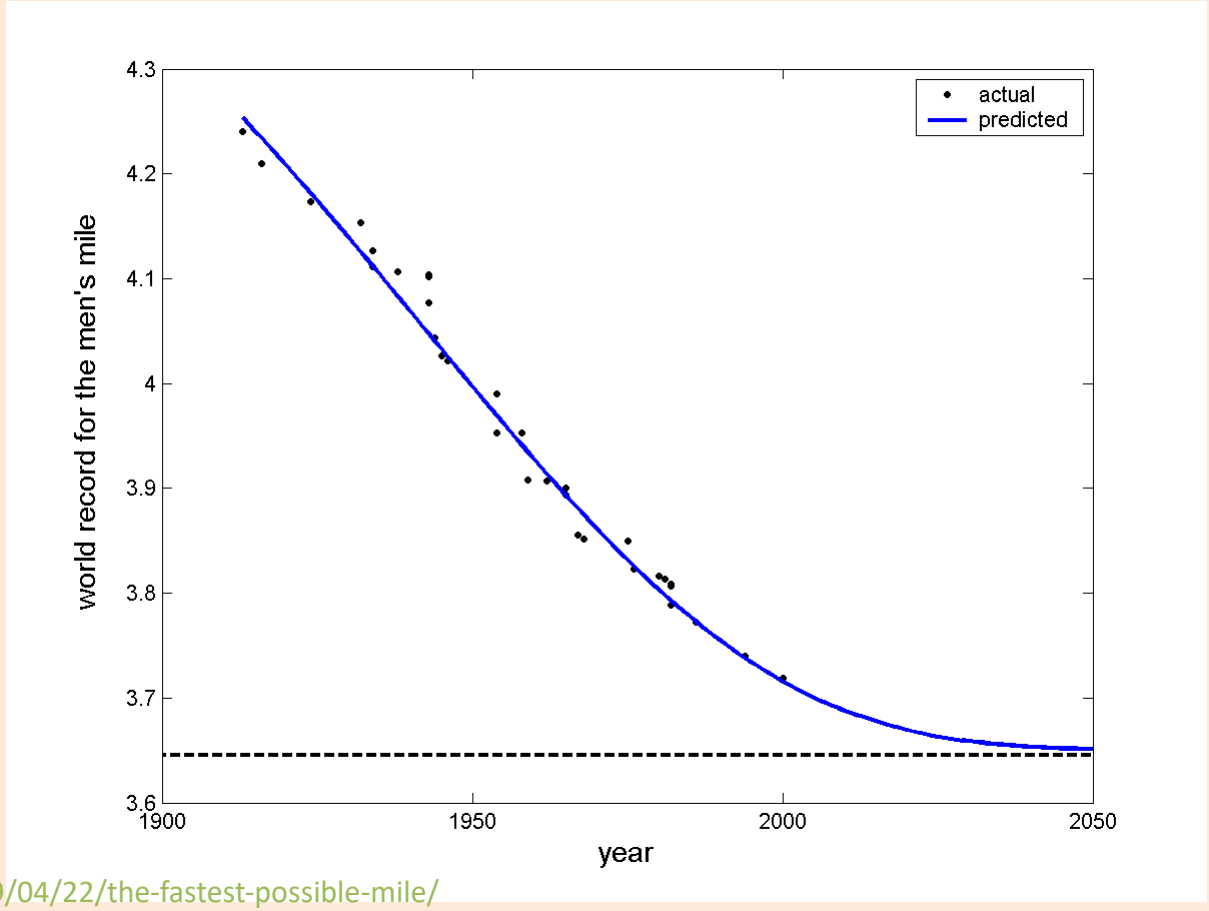
Regularizing the y-Intercept?

- Should we regularize the y-intercept?
- No! Why encourage it to be closer to zero? (It could be anywhere.)
  - You should be allowed to shift function up/down globally.
- Yes! It makes the solution unique and it easier to compute 'w'.
- Compromise: regularize by a smaller amount than other variables.

$$f(w, w_0) = \frac{1}{2}||Xw + w_0 - y||^2 + \frac{\lambda}{2}||w||^2 + \frac{\lambda_0}{2}w_0^2$$

Predicting the Future

- In principle, we can use any features x_i that we think are relevant.
- This makes it tempting to use time as a feature, and predict future.



https://gravityandlevity.wordpress.com/2009/04/22/the-fastest-possible-mile/

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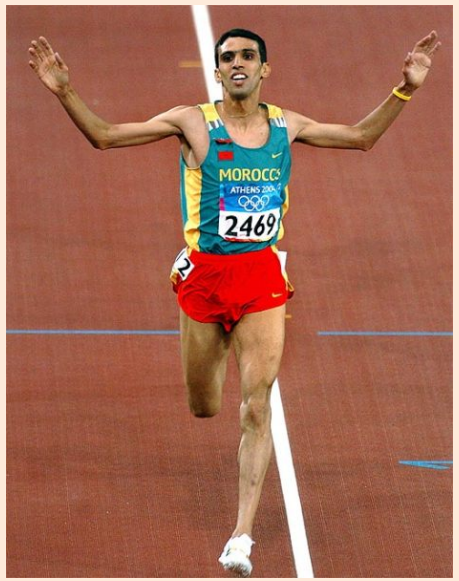
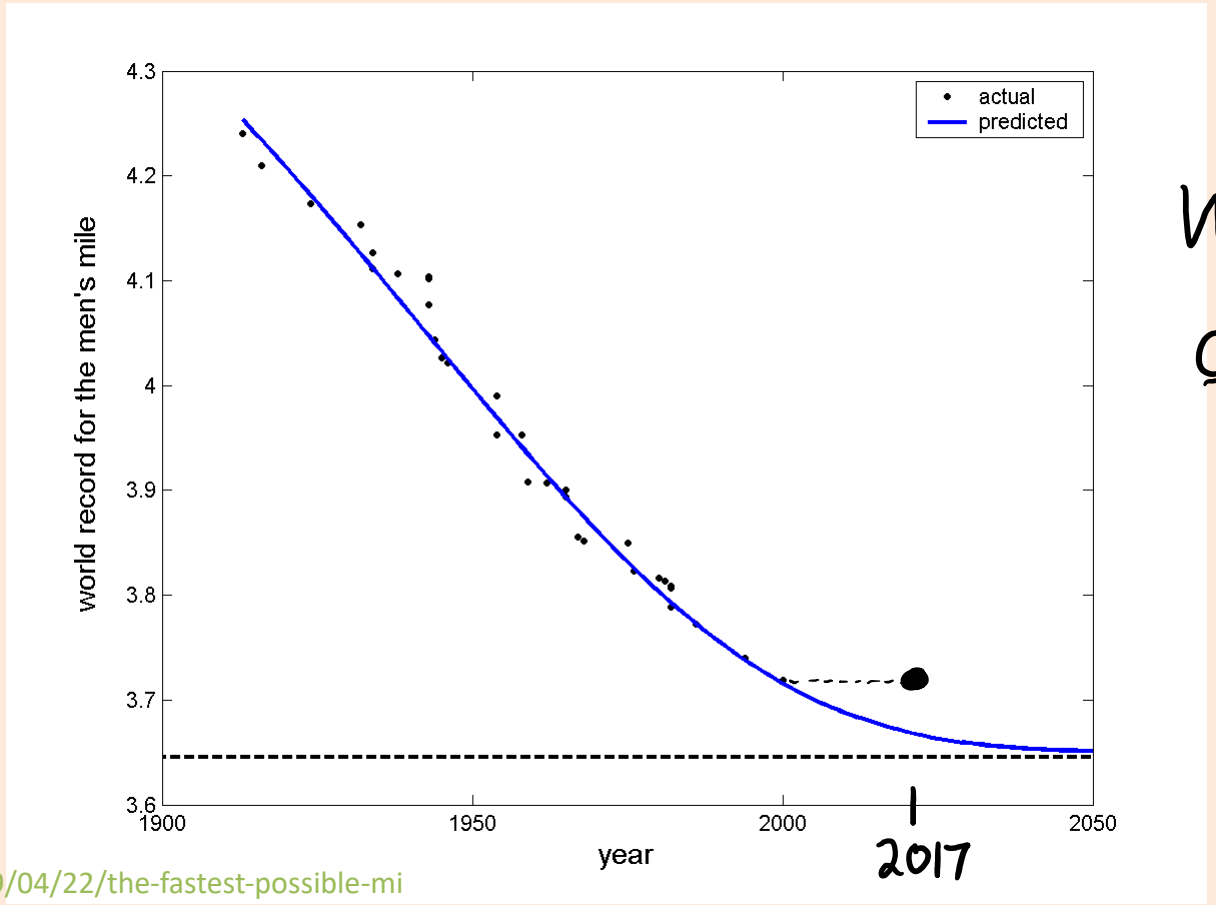
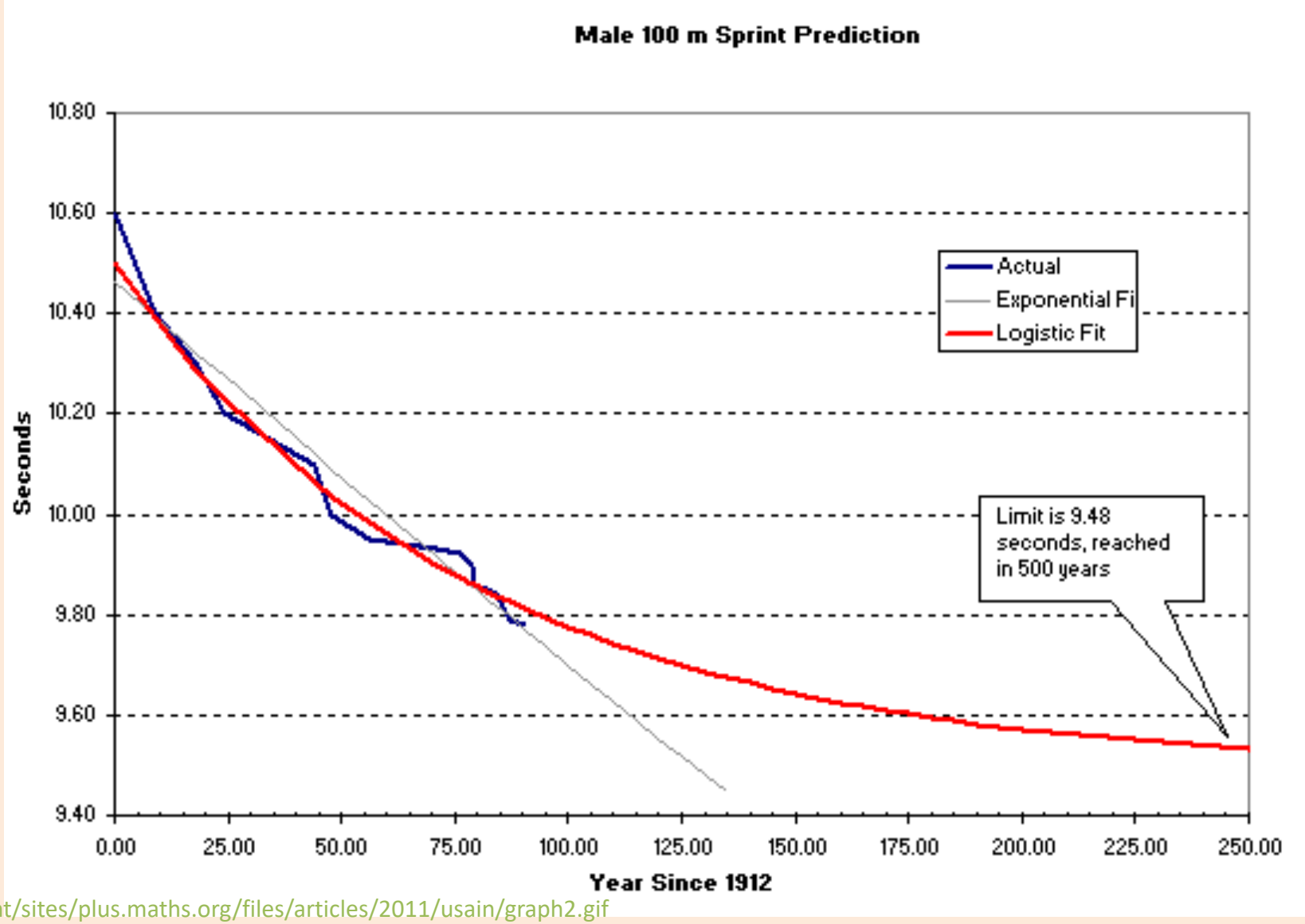




We need to be cautious about doing this.

https://gravityandlevity.wordpress.com/2009/04/22/the-fastest-possible-mi
https://overthehillsports.wordpress.com/tag/hicham-el-guerrouj/le/

Predicting 100m times 400 years in the future?



https://plus.maths.org/content/sites/plus.maths.org/files/articles/2011/usain/graph2.gif

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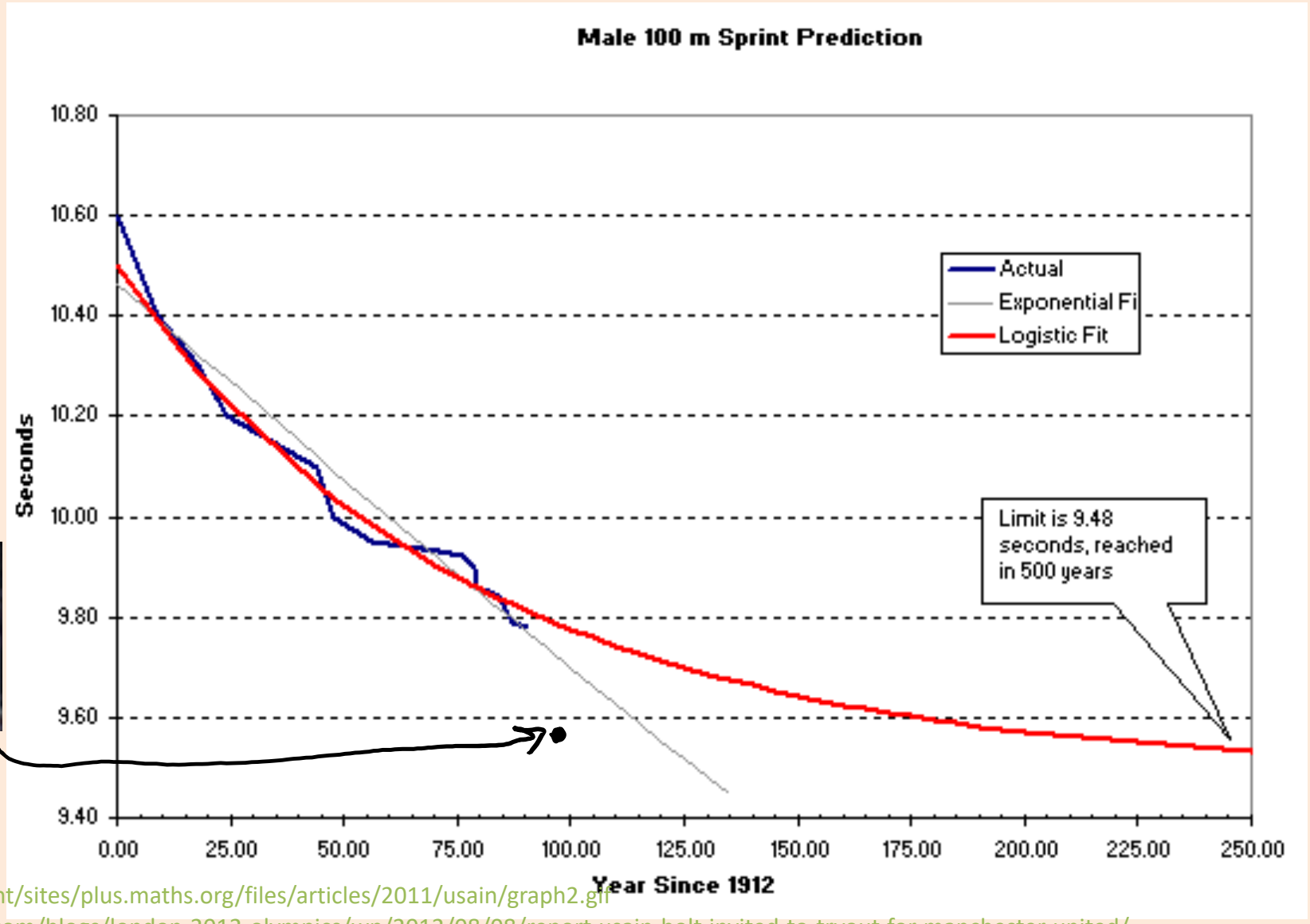

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https://plus.maths.org/content/sites/plus.maths.org/files/articles/2011/usain/graph2.gif

http://www.washingtonpost.com/blogs/london-2012-olympics/wp/2012/08/08/report-usain-bolt-invited-to-tryout-for-manchester-united/

Interpolation vs Extrapolation

- Interpolation is task of predicting “between the data points”.
  - Regression models are good at this if you have enough data and function is continuous.
- Extrapolation is task of prediction outside the range of the data points.
  - Without assumptions, regression models can be embarrassingly-bad at this.

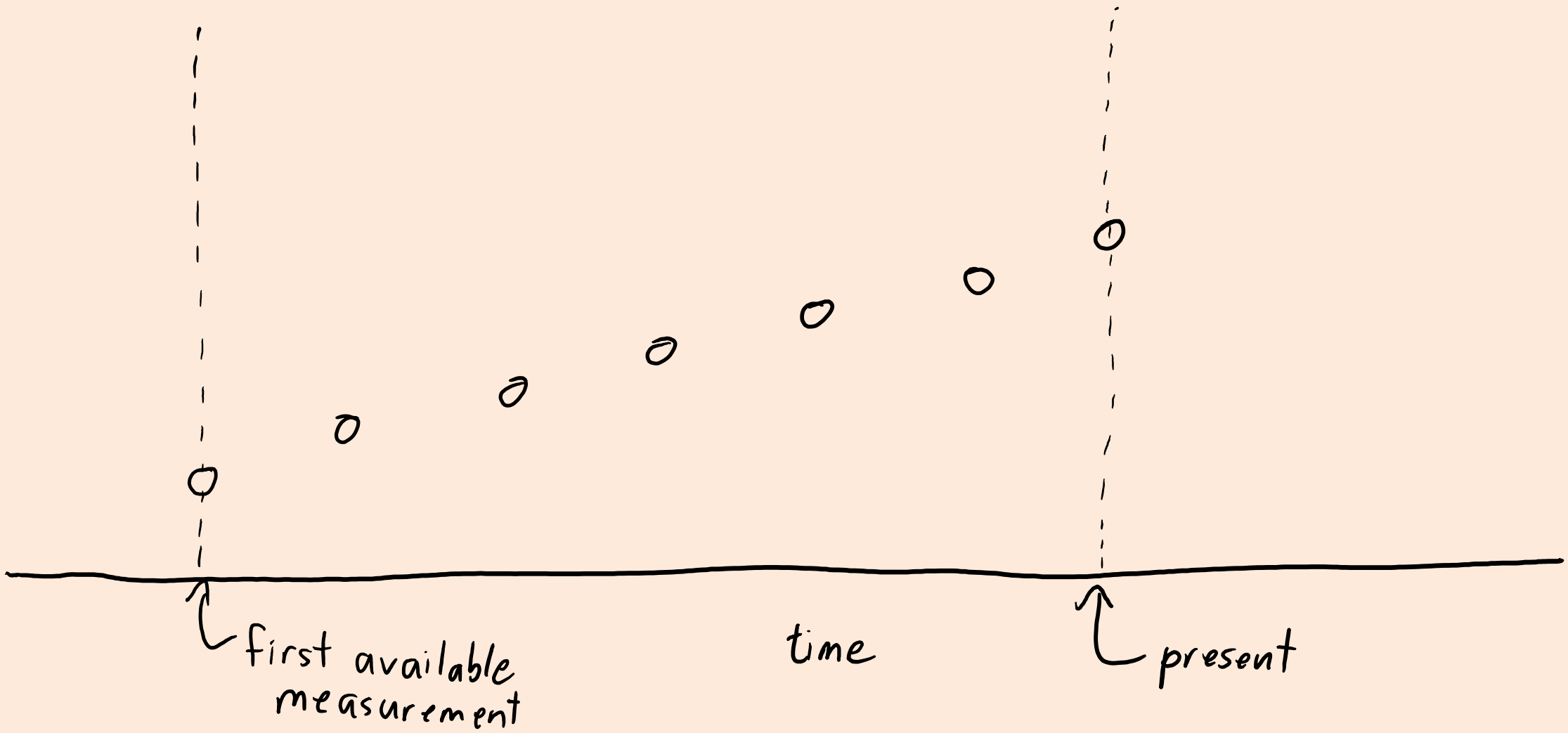
- If you run the 100m regression models backwards in time:
  - They predict that humans used to be really really slow!

- If you run the 100m regression models forwards in time:
  - They might eventually predict arbitrarily-small 100m times.
  - The linear model actually predicts negative times in the future.
    - These time traveling races in 2060 should be pretty exciting!

- Some discussion here:
  - http://callingbullshit.org/case_studies/case_study_gender_gap_running.html

https://www.smbc-comics.com/comic/rise-of-the-machines

No Free Lunch, Consistency, and the Future



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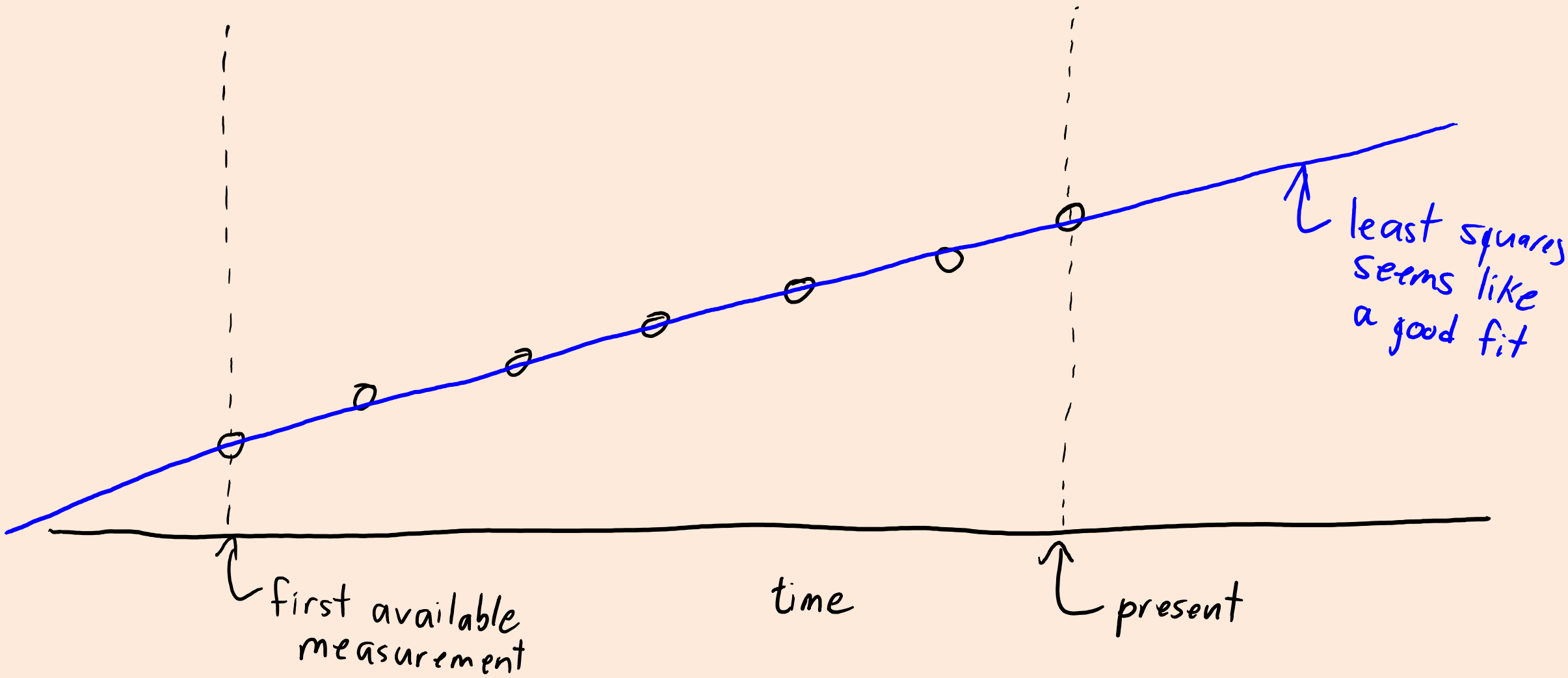

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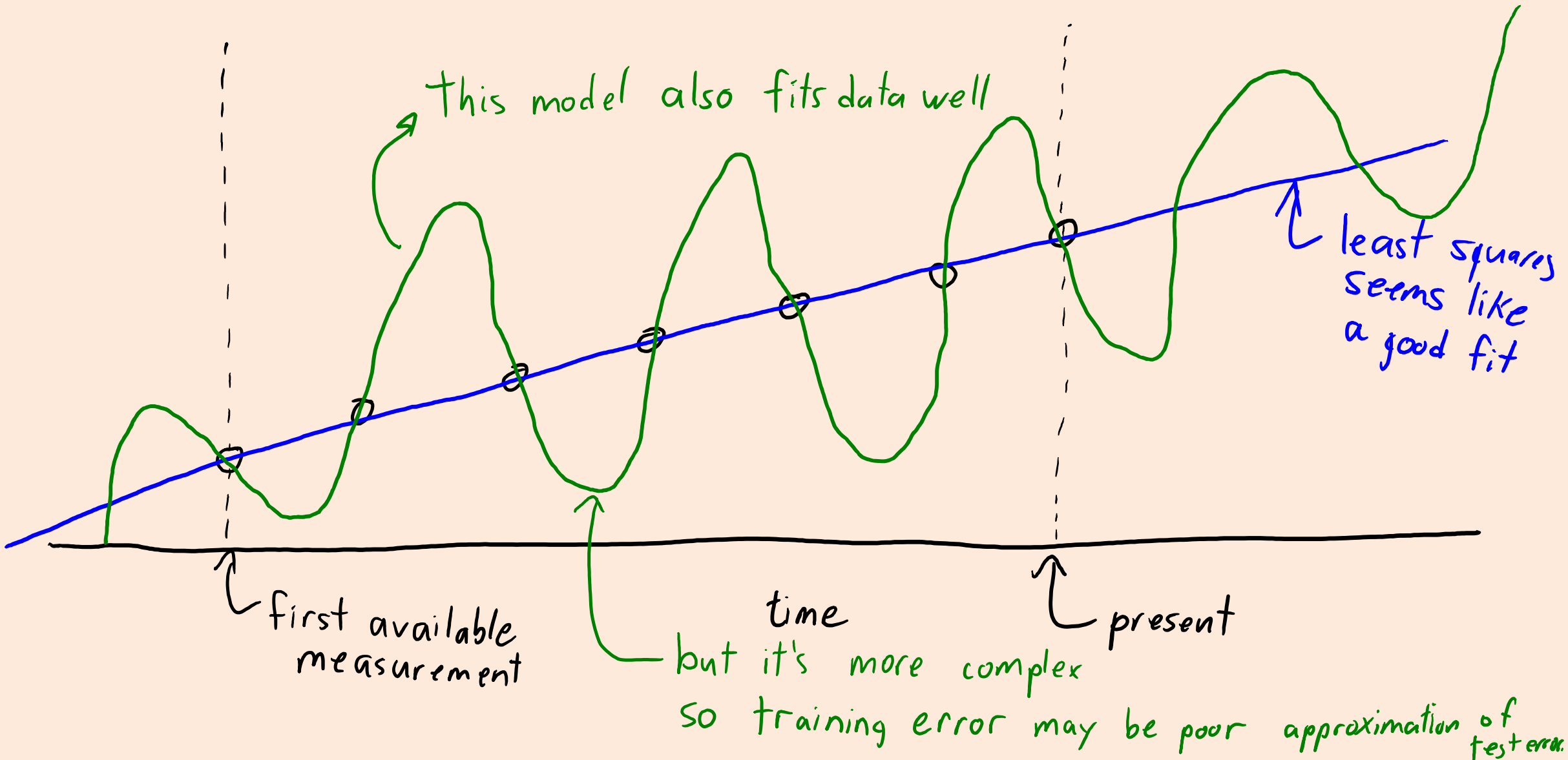
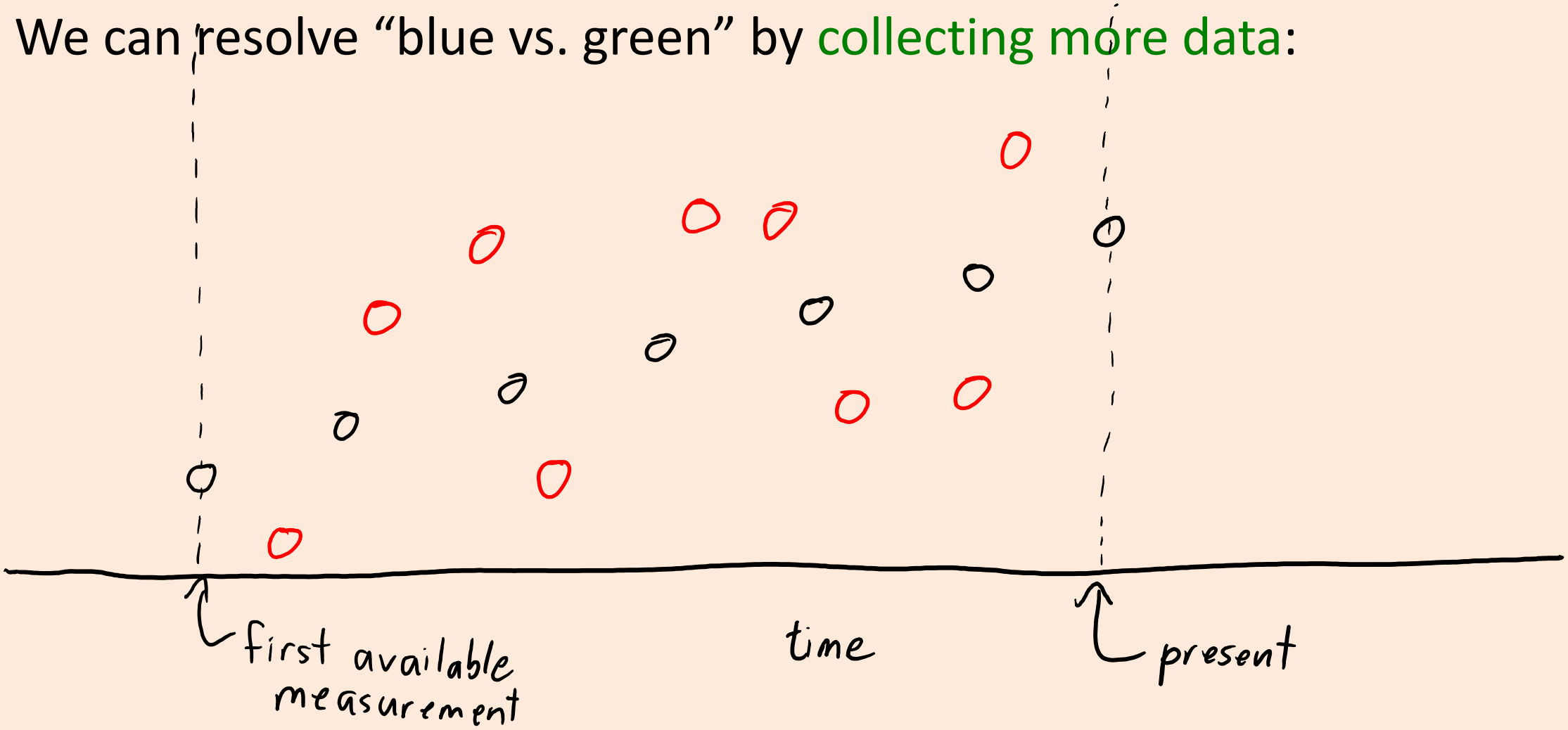


No Free Lunch, Consistency, and the Future

- We can resolve “blue vs. green” by collecting more data:



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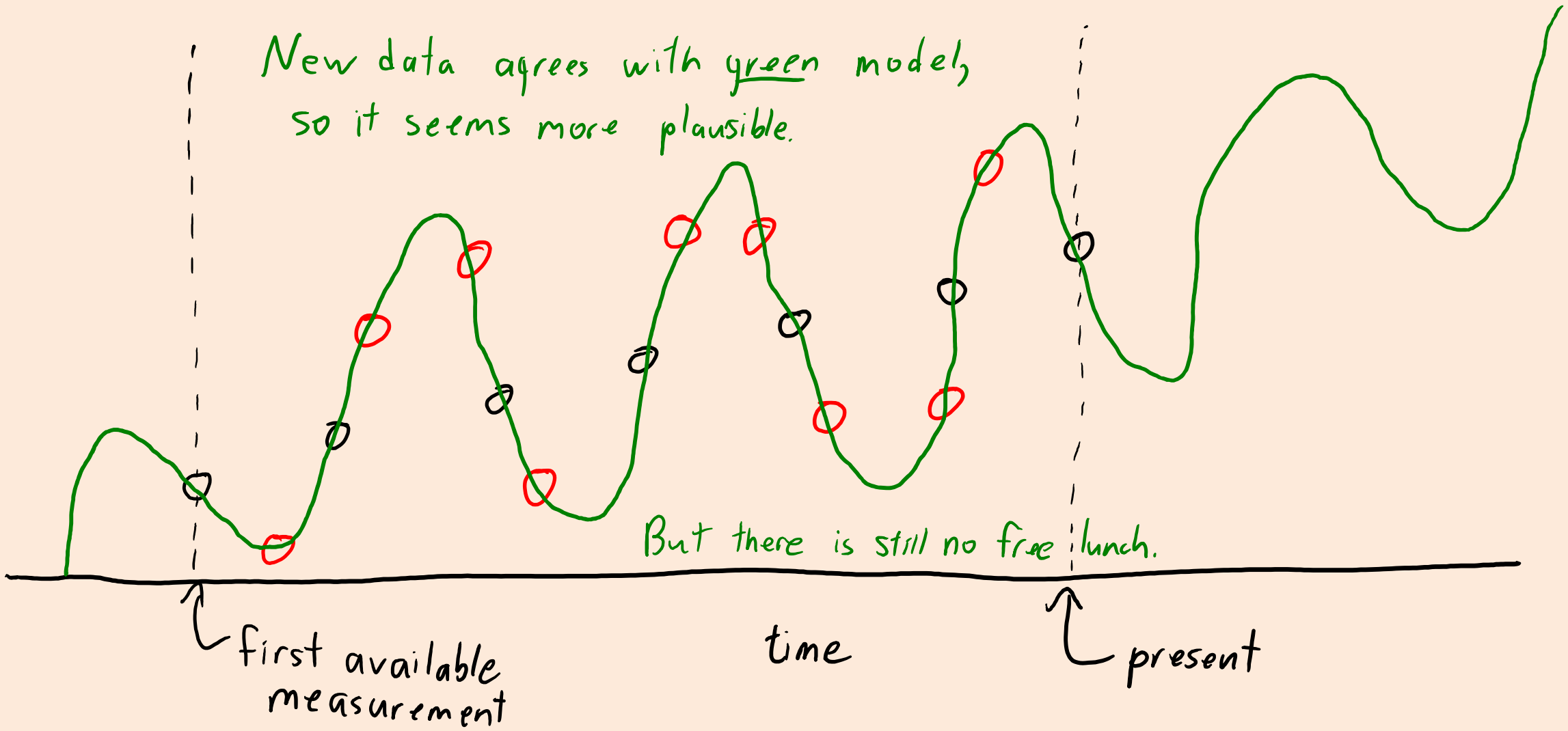

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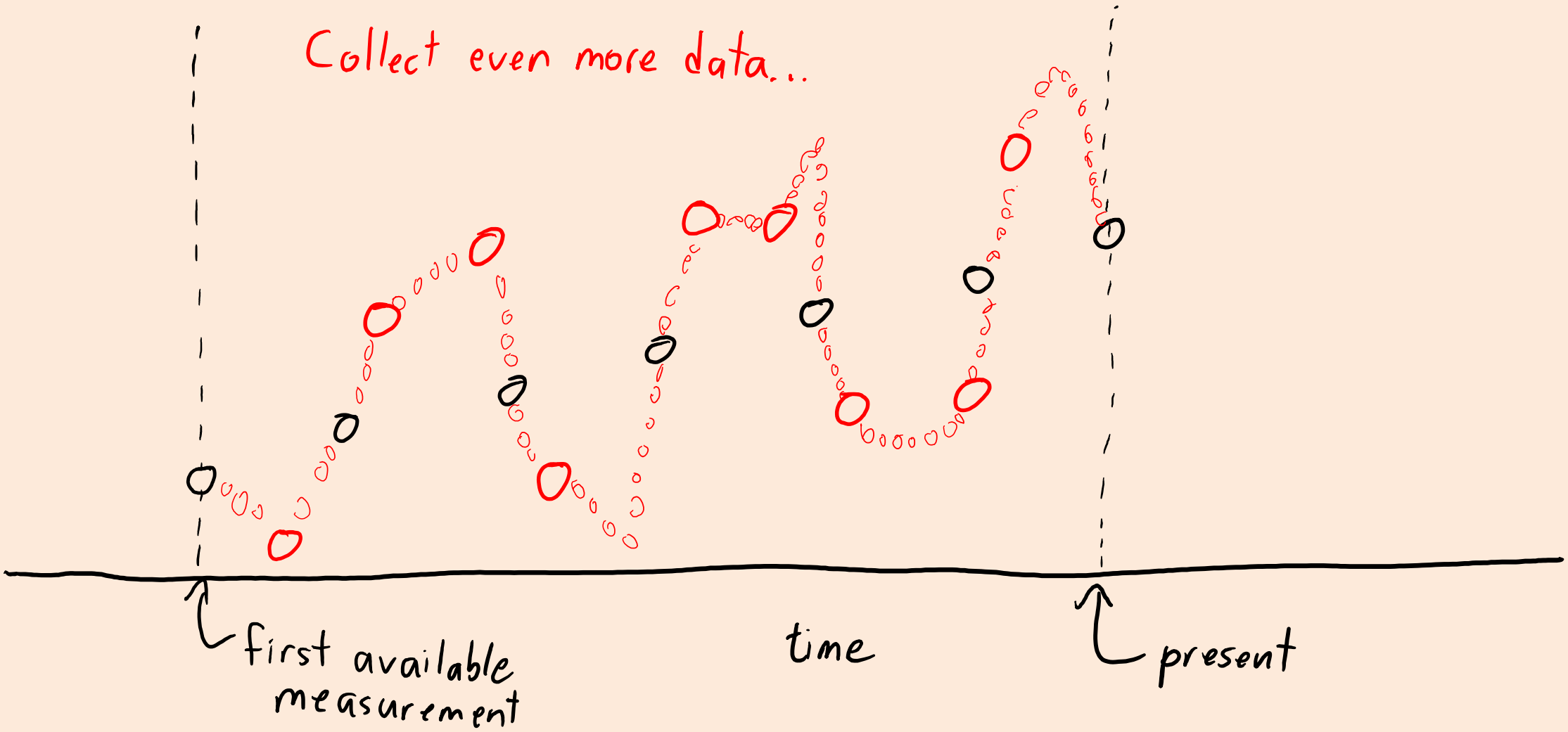

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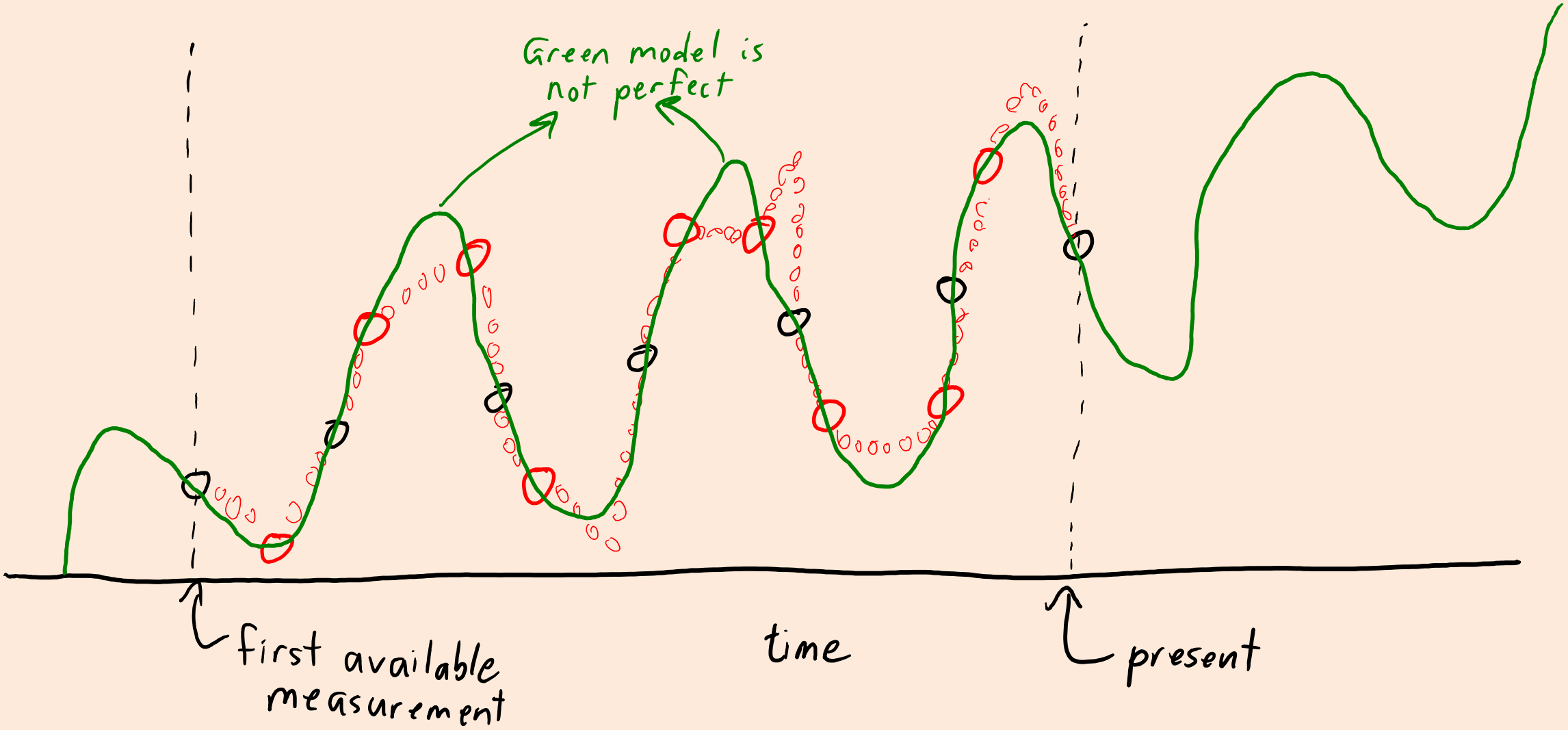

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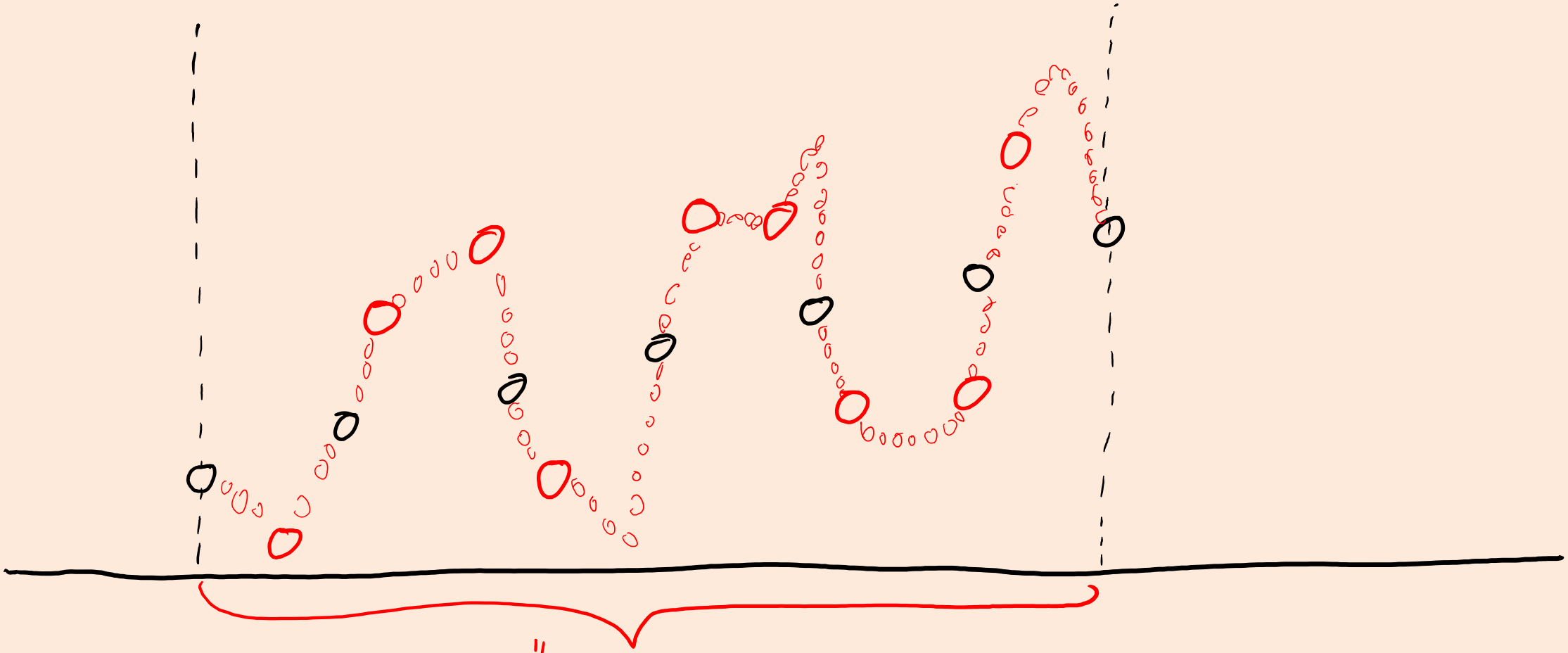

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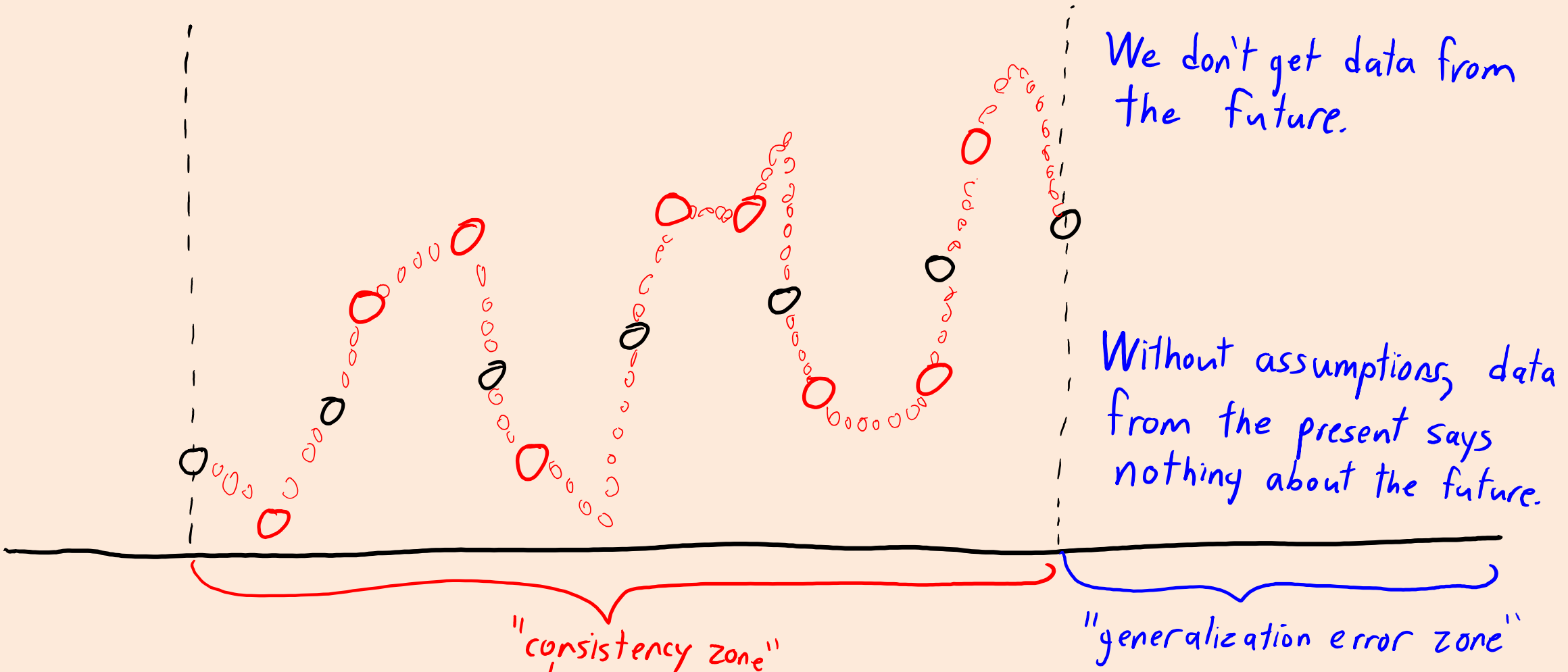

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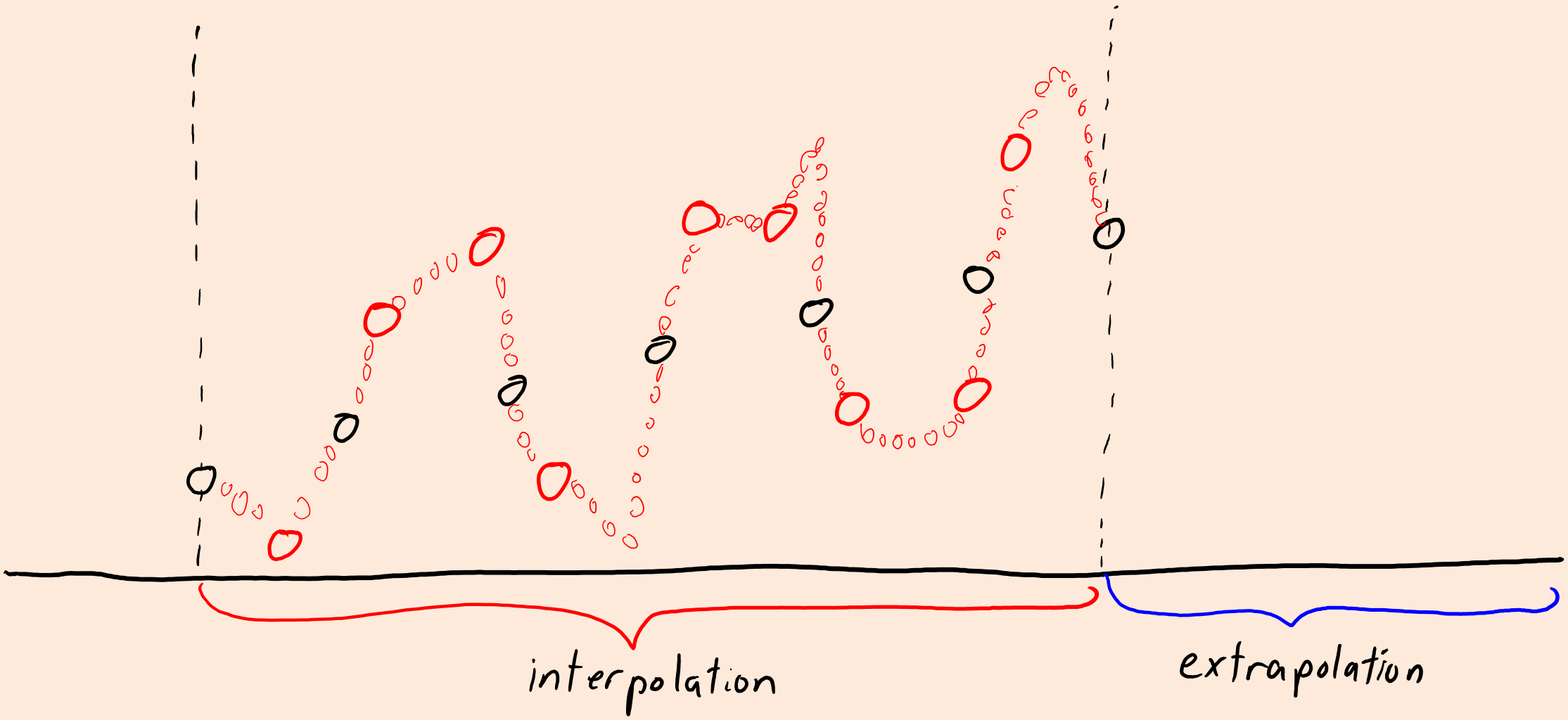
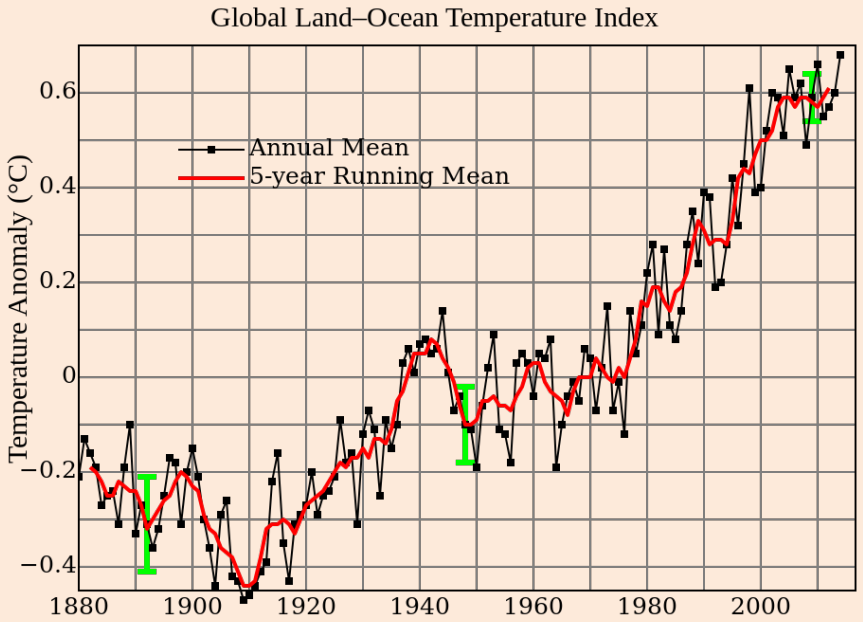


Discussion: Climate Models

- Has Earth warmed up over last 100 years? (Consistency zone)
  - Data clearly says “yes”.

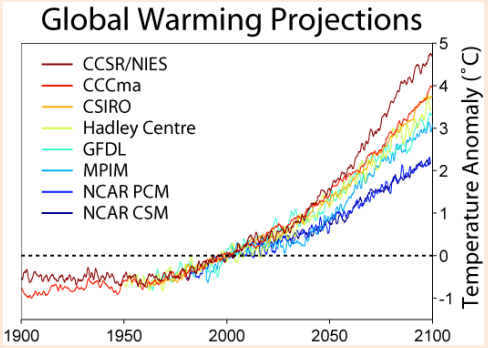


- Will Earth continue to warm over next 100 years? (generalization error)
  - We should be more skeptical about models that predict future events.

https://en.wikipedia.org/wiki/Global_warming

Discussion: Climate Models

- So should we all become global warming skeptics?
- If we average over models that overfit in *independent* ways, we expect the test error to be lower, so this gives more confidence:



  - We should be skeptical of individual models, but agreeing predictions made by models with different data/assumptions are more likely be true.
- All the near-future predictions agree, so they are likely to be accurate.
  - And it's probably reasonable to assume fairly continuous change (no big "jumps").
- Variance is higher further into future, so predictions are less reliable.
  - Relying more on assumptions and less on data.

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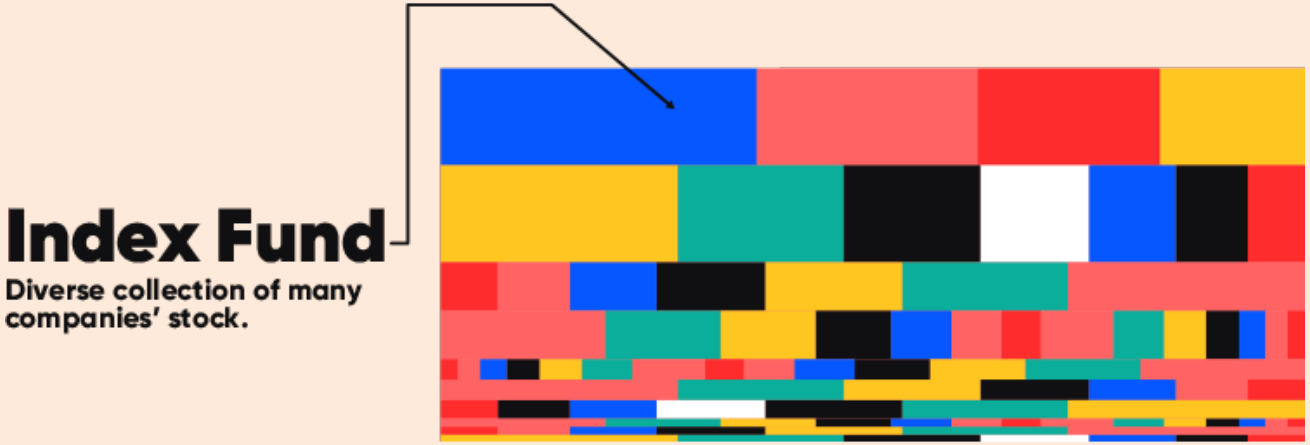
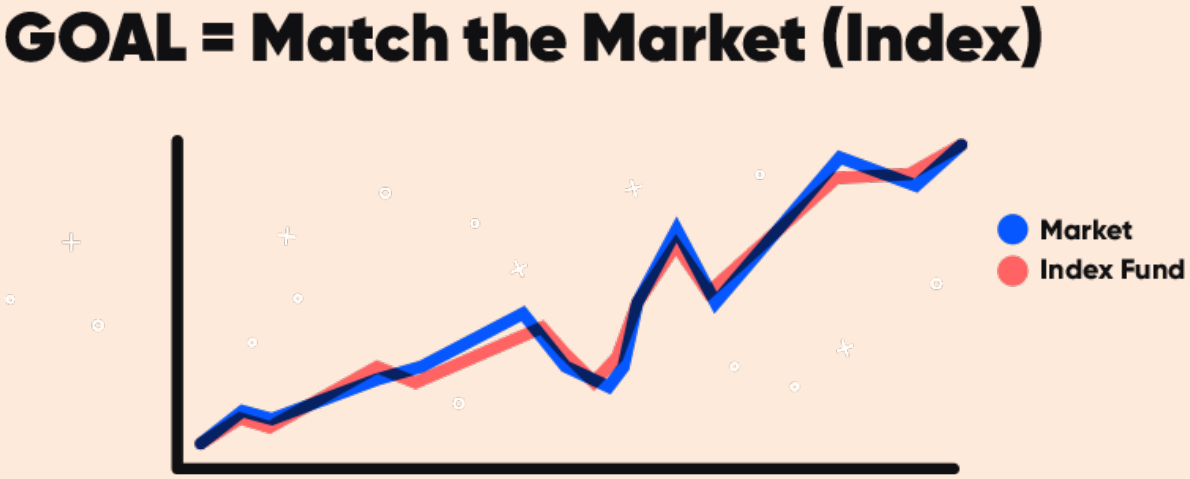
Index Funds: Ensemble Extrapolation for Investing

- Want to do extrapolation when investing money.
  - What will this be worth in the future?
- Index funds can be viewed as an ensemble method for investing.
  - For example, buy stock in top 500 companies proportional to value.
  - Tries to follow average price increase/decrease.





  - This simple investing strategy outperforms most fund managers.

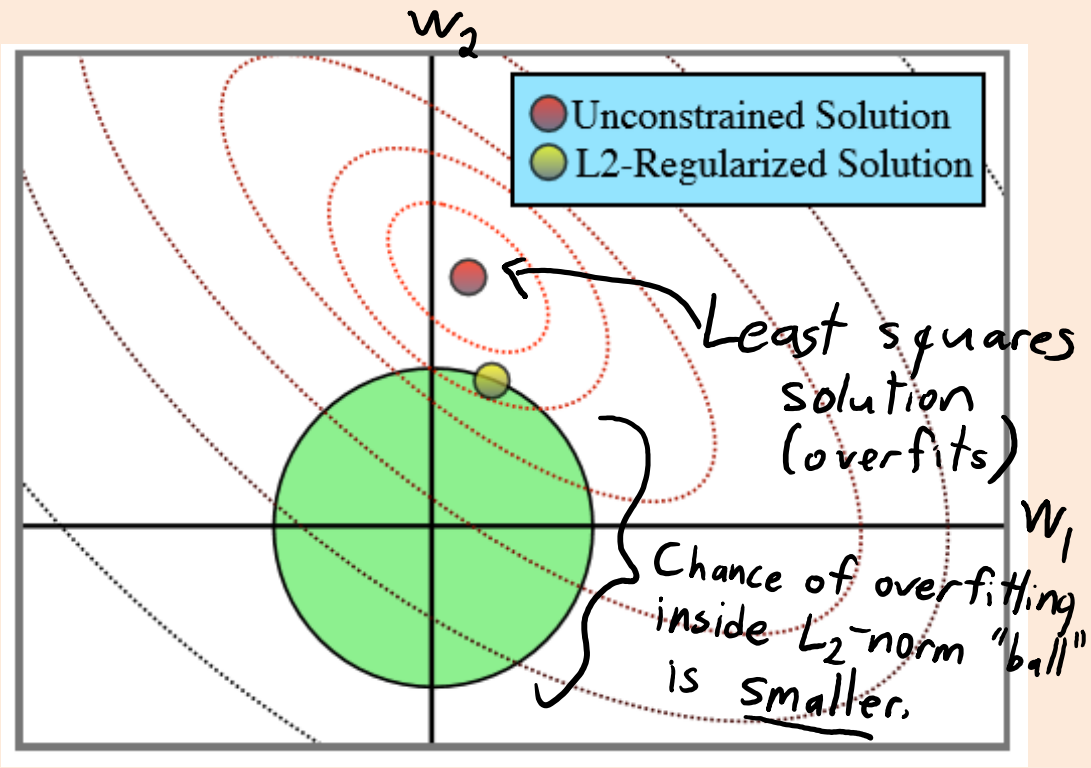
http://fibydesign.com/005-introduction-to-index-investing-stocks-index-funds-vtsax/

L2-Regularization

- Standard regularization strategy is L2-regularization:

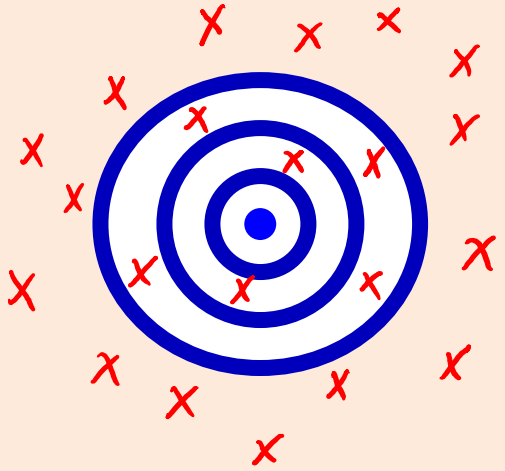
$$f(w) = \frac{1}{2}\sum_{i=1}^{n}(w^T x_i - y_i)^2 + \frac{\lambda}{2}\sum_{j=1}^{d} w_j^2 \quad \text{or} \quad f(w) = \frac{1}{2}\|Xw - y\|^2 + \frac{\lambda}{2}\|w\|^2$$

- Equivalent to minimizing squared error but keeping L2-norm small.



Regularization/Shrinking Paradox

- We throw darts at a target:
  - Assume we don't always hit the exact center.
  - Assume the darts follow a symmetric pattern around center.



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  1. Choose some arbitrary location '0'.
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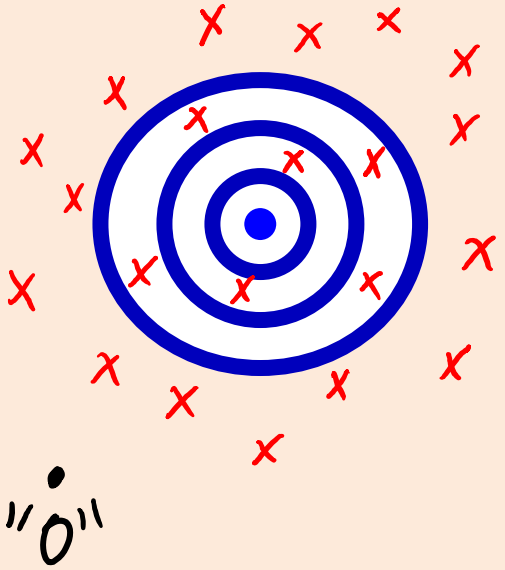

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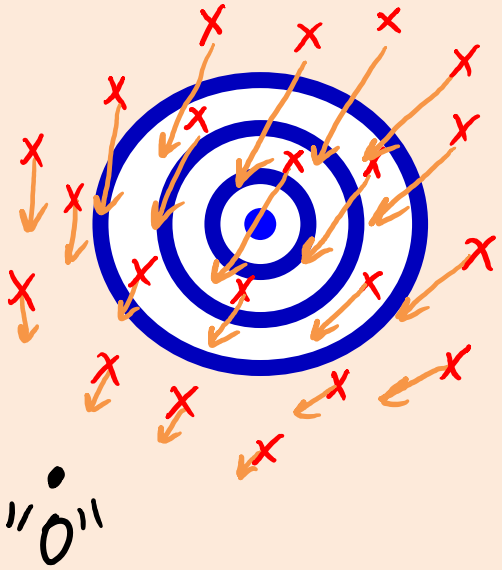
- If small enough, darts will be closer to center on average.



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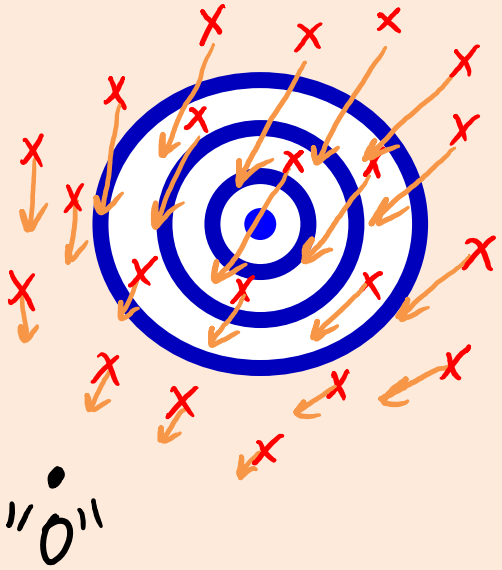
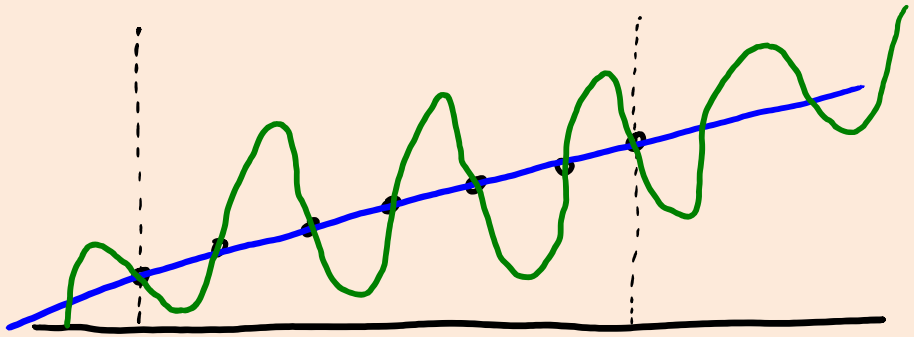


Visualization of the related higher-dimensional paradox that the mean of data coming from a Gaussian is not the best estimate of the mean of the Gaussian in 3-dimensions or higher: https://www.naftaliharris.com/blog/steinviz

Ockham's Razor vs. No Free Lunch

- Ockham's razor is a problem-solving principle:
  - "Among competing hypotheses, the one with the fewest assumptions should be selected."
  - Suggests we should select linear model.



- Fundamental trade-off:
  - If same training error, pick model less likely to overfit.
  - Formal version of Occam's problem-solving principle.
  - Also suggests we should select linear model.
- No free lunch theorem:
  - There *exists possible datasets* where you should select the green model.