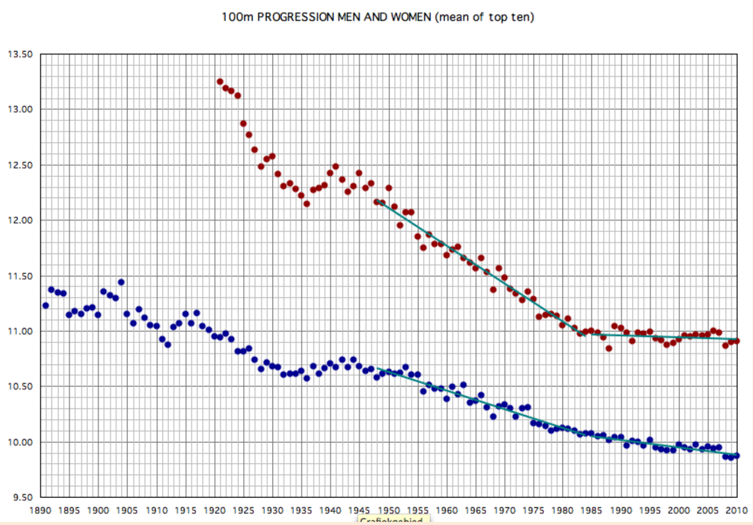
CPSC 340: Machine Learning and Data Mining

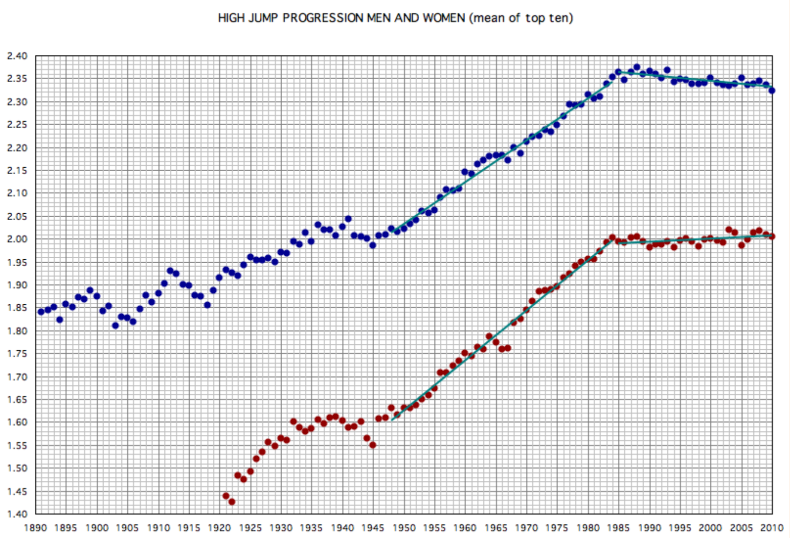
Nonlinear Regression

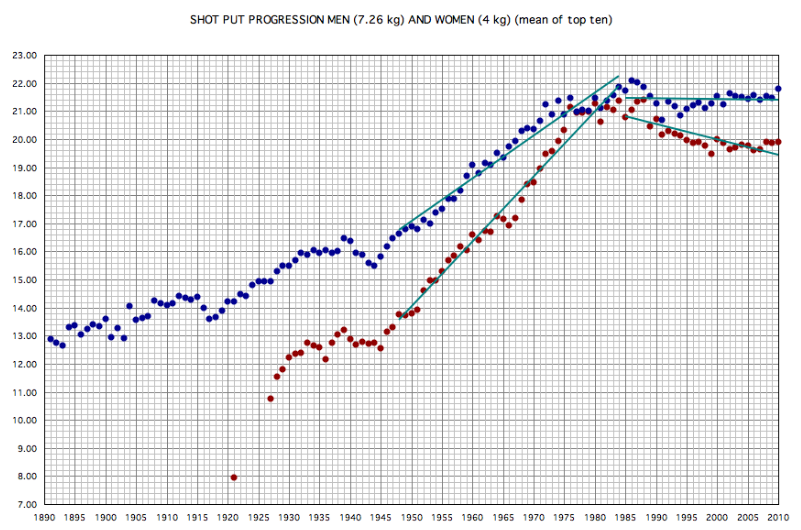
Bonus slides

Motivation: Non-Linear Progressions in Athletics

- Are top athletes going faster, higher, and farther?







http://www.at-a-lanta.nl/weia/Progressie.html
https://en.wikipedia.org/wiki/Usain_Bolt
http://www.britannica.com/biography/Florence-Griffith-Joyner

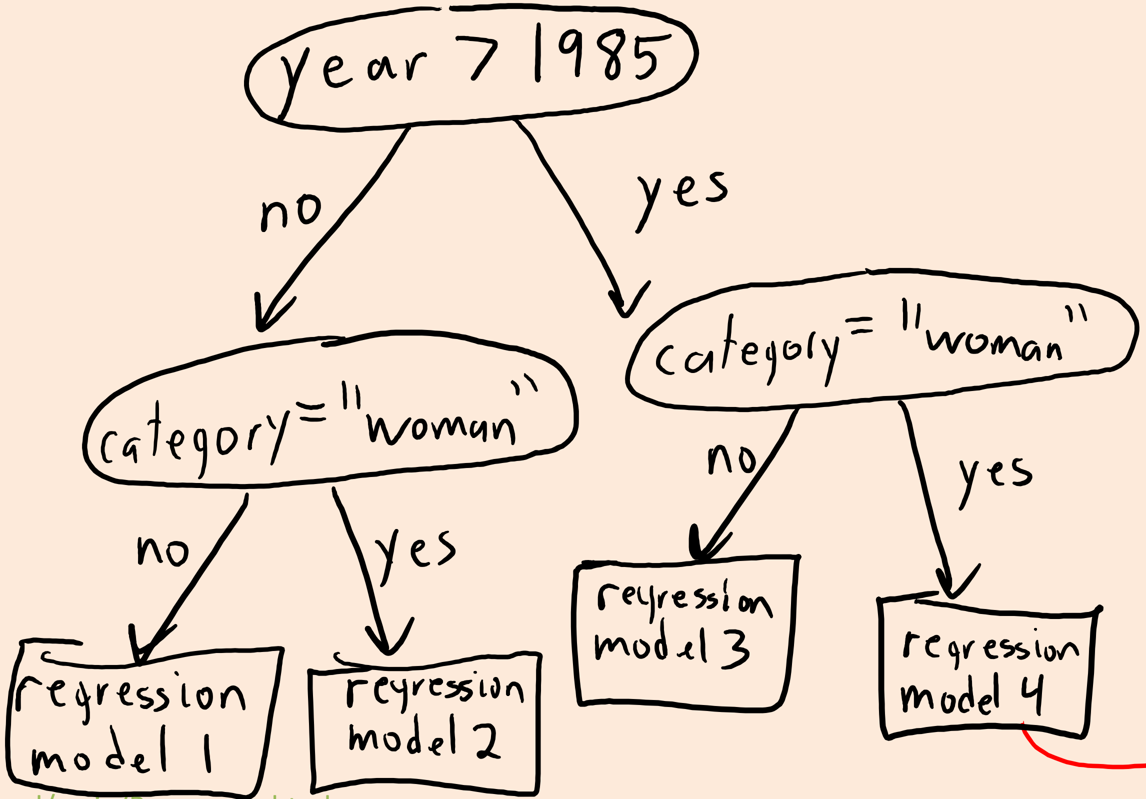
Adapting Counting/Distance-Based Methods

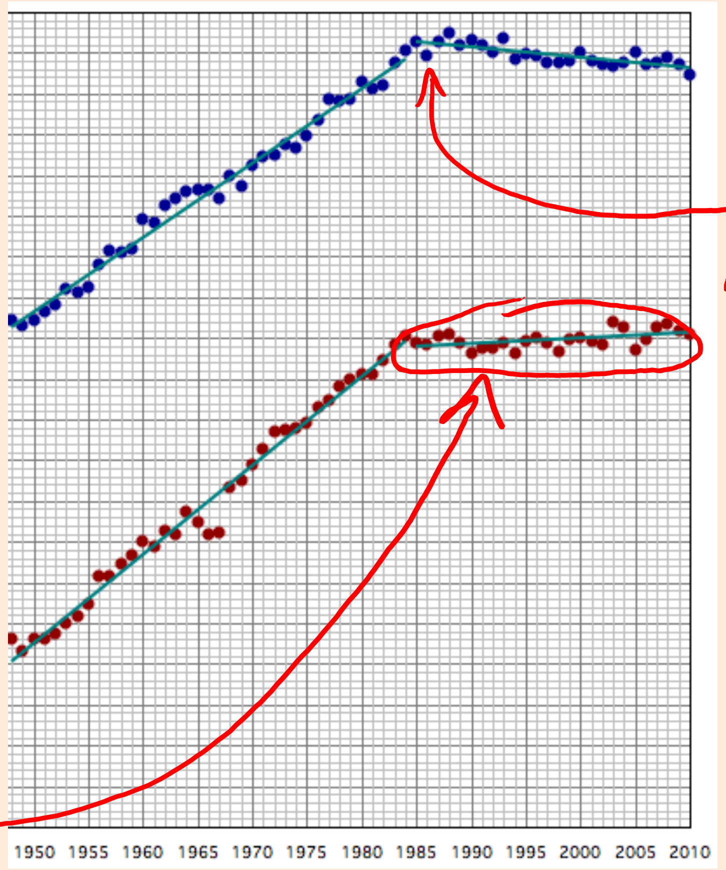
- We can adapt our classification methods to perform regression:

http://www.at-a-lanta.nl/weia/Progressie.html

Adapting Counting/Distance-Based Methods

- We can adapt our classification methods to perform regression:
  - Regression tree: tree with mean value or linear regression at leaves.





Not necessarily continuous.

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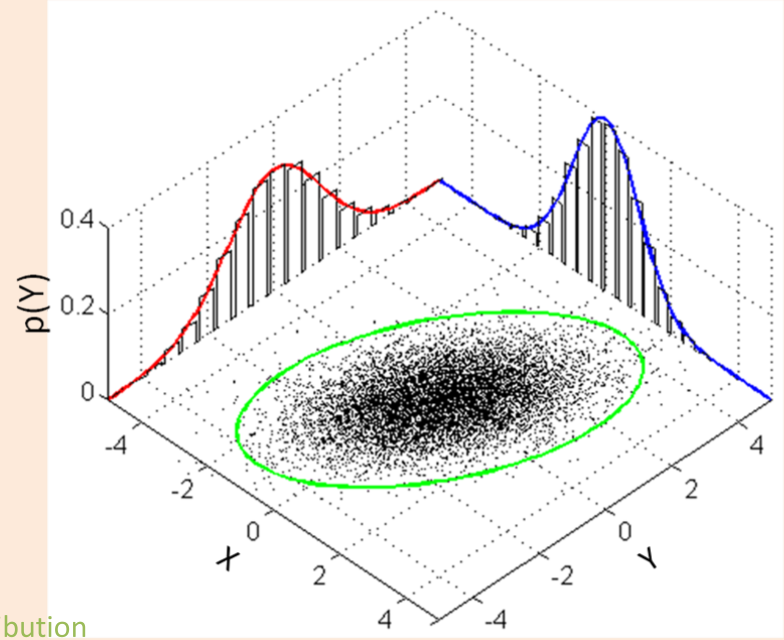
Adapting Counting/Distance-Based Methods

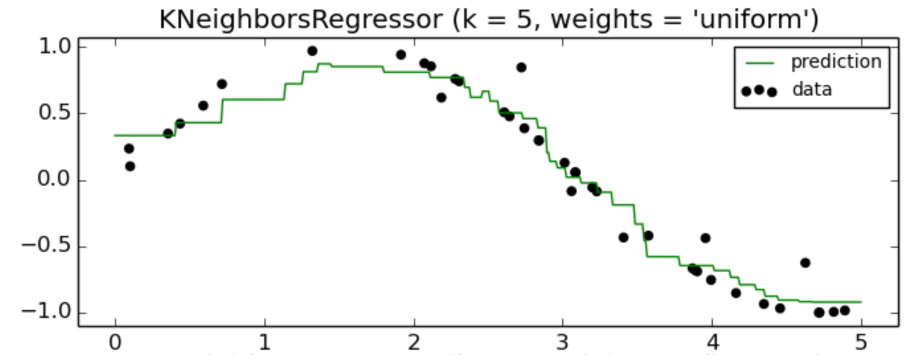
- We can adapt our classification methods to perform regression:
  - Regression tree: tree with mean value or linear regression at leaves.
  - Probabilistic models: fit p(x_i | y_i) and p(y_i) with Gaussian or other model.
    - CPSC 540.



https://en.wikipedia.org/wiki/Multivariate_normal_distribution

Adapting Counting/Distance-Based Methods

- We can adapt our classification methods to perform regression:
  - Regression tree: tree with mean value or linear regression at leaves.
  - Probabilistic models: fit $p(x_i \mid y_i)$ and $p(y_i)$ with Gaussian or other model.
  - Non-parametric models:
    - KNN regression:
      - Find ‘k’ nearest neighbours of $\tilde{x}_i$.
      - Return the mean of the corresponding y_i.



http://scikit-learn.org/stable/modules/neighbors.html

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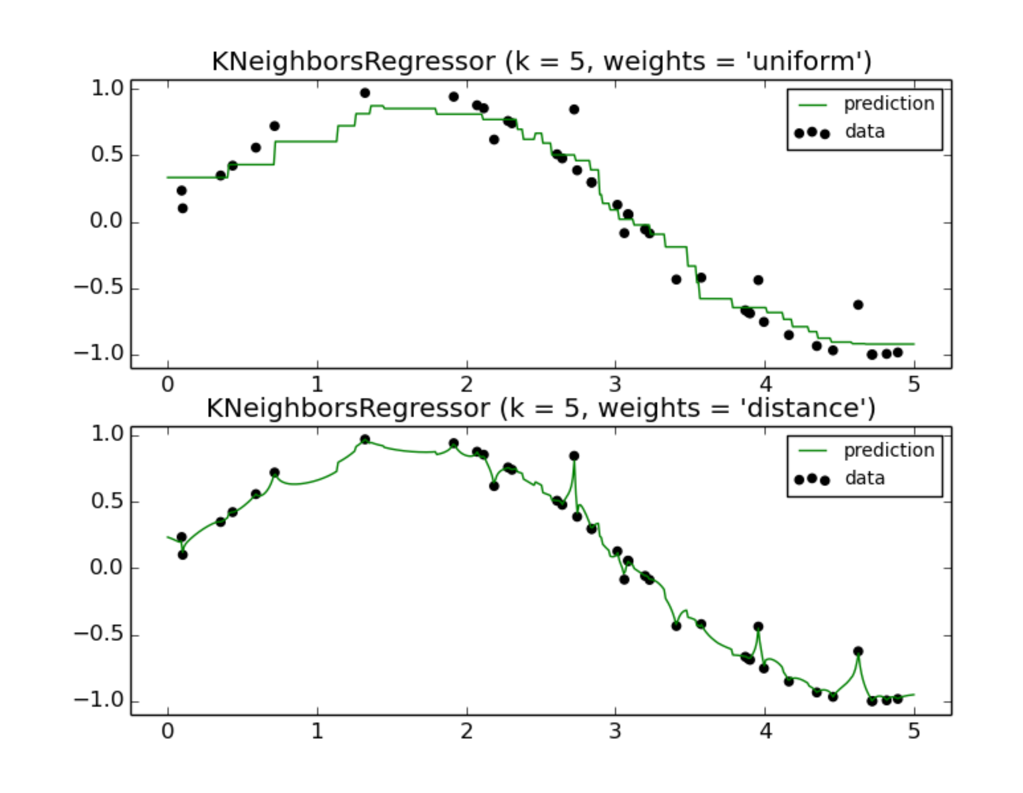
  - Non-parametric models:
    - KNN regression.
    - Could be weighted by distance.
      - Close points 'j' get more "weight" w_{ij}.



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  - Non-parametric models:
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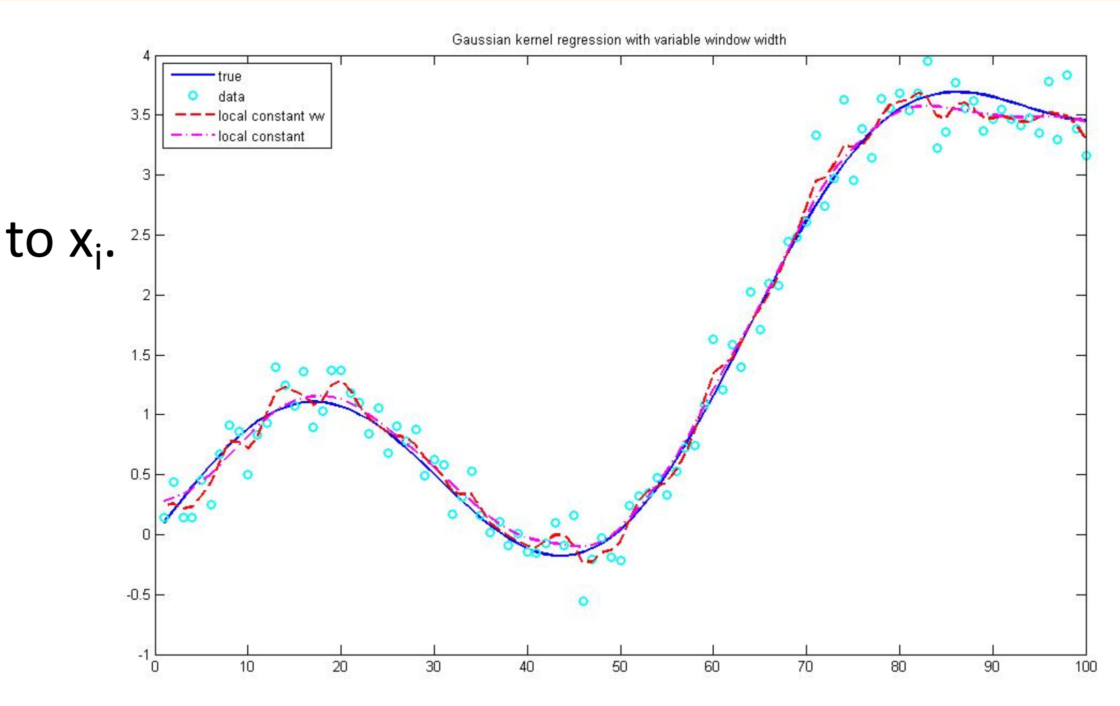
    - Could be weighted by distance.
    - ‘Nadaraya-Waston’: weight *all* y_i by distance to x_i.

$$\hat{y}_i = \frac{\sum_{j=1}^{n} v_{ij} y_j}{\sum_{j=1}^{n} v_{ij}}$$



http://www.mathworks.com/matlabcentral/fileexchange/35316-kernel-regression-with-variable-window-width/content/ksr_vw.m

Adapting Counting/

- We can adapt our classificatio
  - Regression tree: tree with mea
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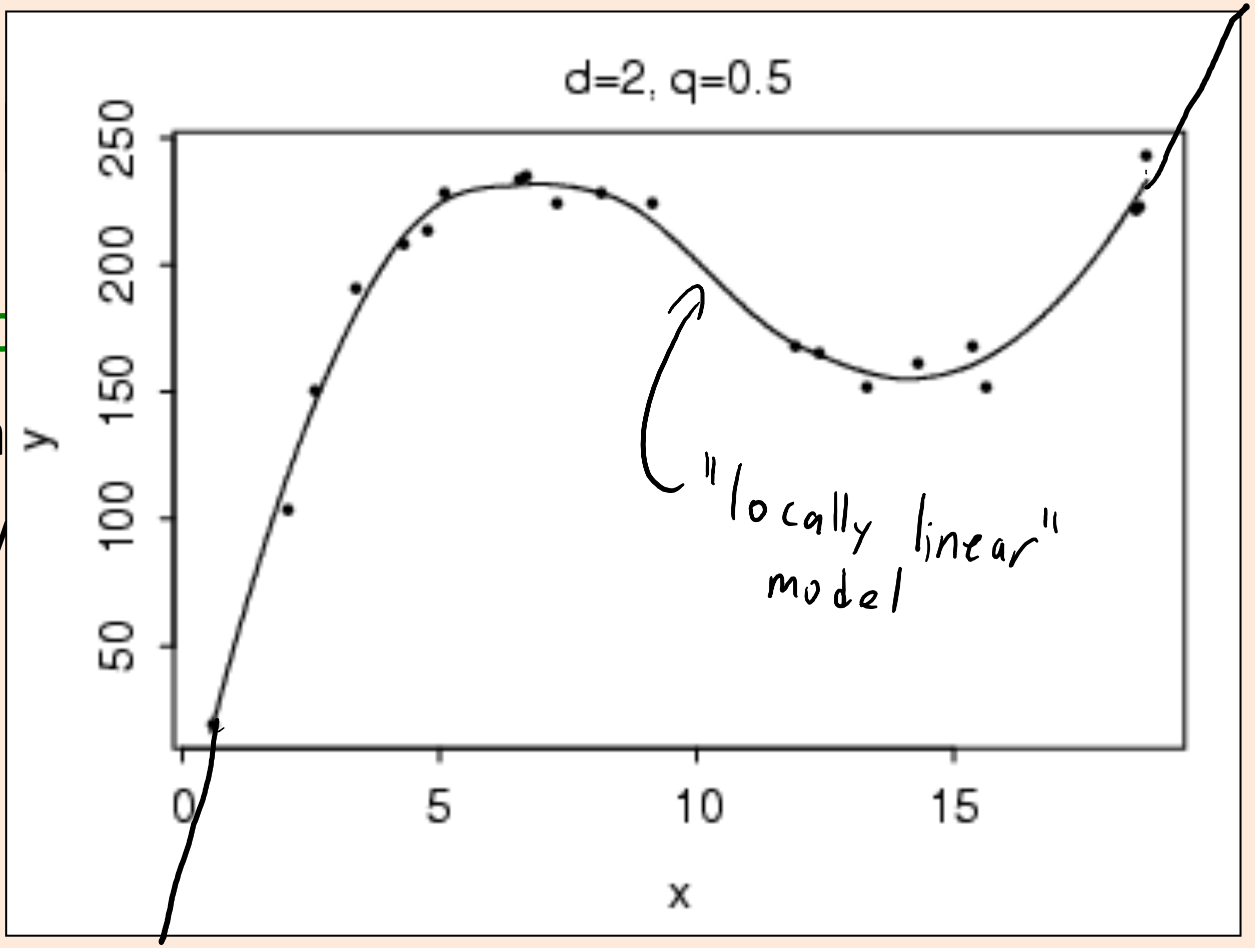
  - Non-parametric models:
    - KNN regression.
    - Could be weighted by distance.
    - ‘Nadaraya-Waston’: weight *all* y_i
    - ‘Locally linear regression’: for each x_i, fit a linear model weighted by distance.

(Better than KNN and NW at boundaries.)



"locally linear" model

http://www.itl.nist.gov/div898/handbook/pmd/section4/pmd423.htm

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    - 'Locally linear regression': for each x_i, fit a linear model weighted by distance.
      (Better than KNN and NW at boundaries.)
  - Ensemble methods:
    - Can improve performance by averaging predictions across regression models.

Adapting Counting/Distance-Based Methods

- We can adapt our classification methods to perform regression.

- Applications:
  - Regression forests for fluid simulation:
    - https://www.youtube.com/watch?v=kGB7Wd9CudA
  - KNN for image completion:
    - http://graphics.cs.cmu.edu/projects/scene-completion
    - Combined with “graph cuts” and “Poisson blending”.
    - See also “PatchMatch”: https://vimeo.com/5024379
  - KNN regression for “voice photoshop”:
    - https://www.youtube.com/watch?v=I3l4XLZ59iw
    - Combined with “dynamic time warping” and “Poisson blending”.

- But we’ll focus on linear models with non-linear transforms.
  - These are the building blocks for more advanced methods.

http://www.itl.nist.gov/div898/handbook/pmd/section4/pmd423.htm

“Change of Basis” Terminology

- Instead of “nonlinear feature transform”, in machine learning it is common to use the expression “change of basis”.
  - The z_i are the “coordinates in the new basis” of the training example.
- “Change of basis” means something different in math:
  - Math: basis vectors must be linearly independent (in ML we don’t care).
  - Math: change of basis must span the same space (in ML we change space).
- Unfortunately, saying “change of basis” in ML is common.
  - When I say “change of basis”, just think “nonlinear feature transform”.