Scale free networks

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Learning objectives

- Define scale-free networks
- Explain the characteristics of scale-free networks
  - Power law distribution
  - Scale-free
  - Small world
  - Hubs and connectors
  - Growth
  - Competition
  - Robustness

How Everything Is Connected to Everything Else and What It Means for Business, Science, and Everyday Life

Linked

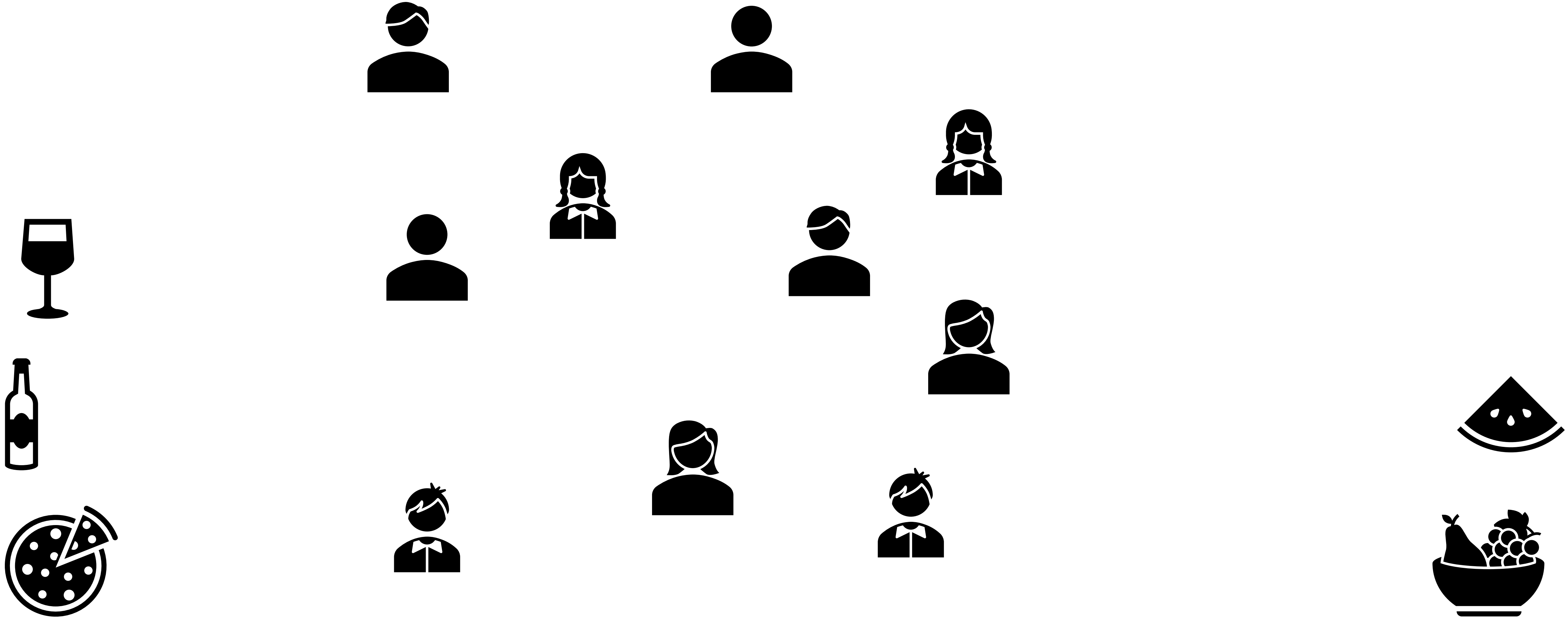
"Remarkable.... A sweeping look at a new and exciting science." —*Science*

Albert-László Barabási

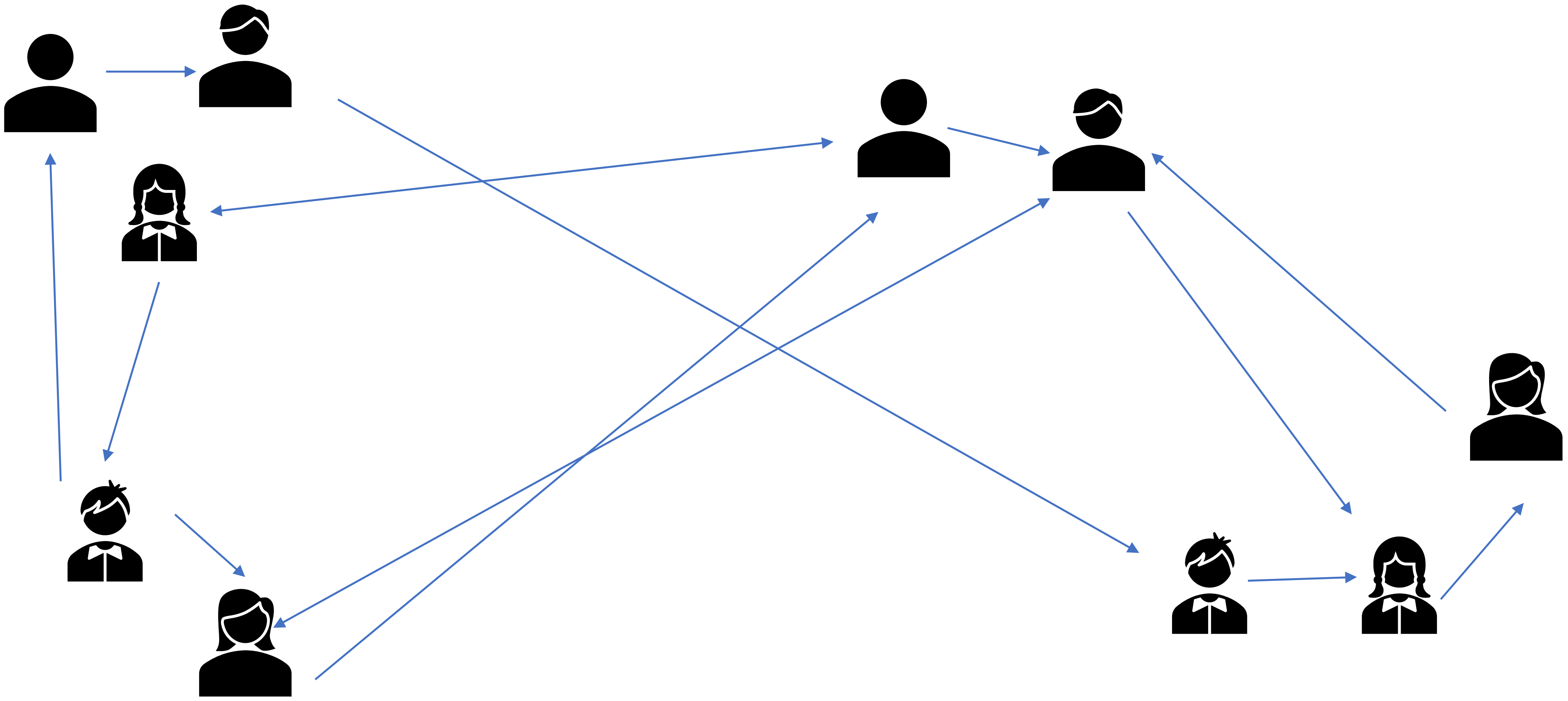
Networks

- Networks are present everywhere
  - The internet, power grids and transportation systems
- Networks have existed before the internet
- "How networks form" is an area of ongoing research
  - How do the interactions of several malfunctioning nodes in a complex genetic network result in cancer?
  - How does diffusion occur so rapidly in certain social and communications systems, leading to epidemics of diseases and computer viruses?
  - How do some networks continue to function even after the vast majority of their nodes have failed?
  - How does misinformation spread so quickly and have a wider reach compared to *correct* information?

How networks form

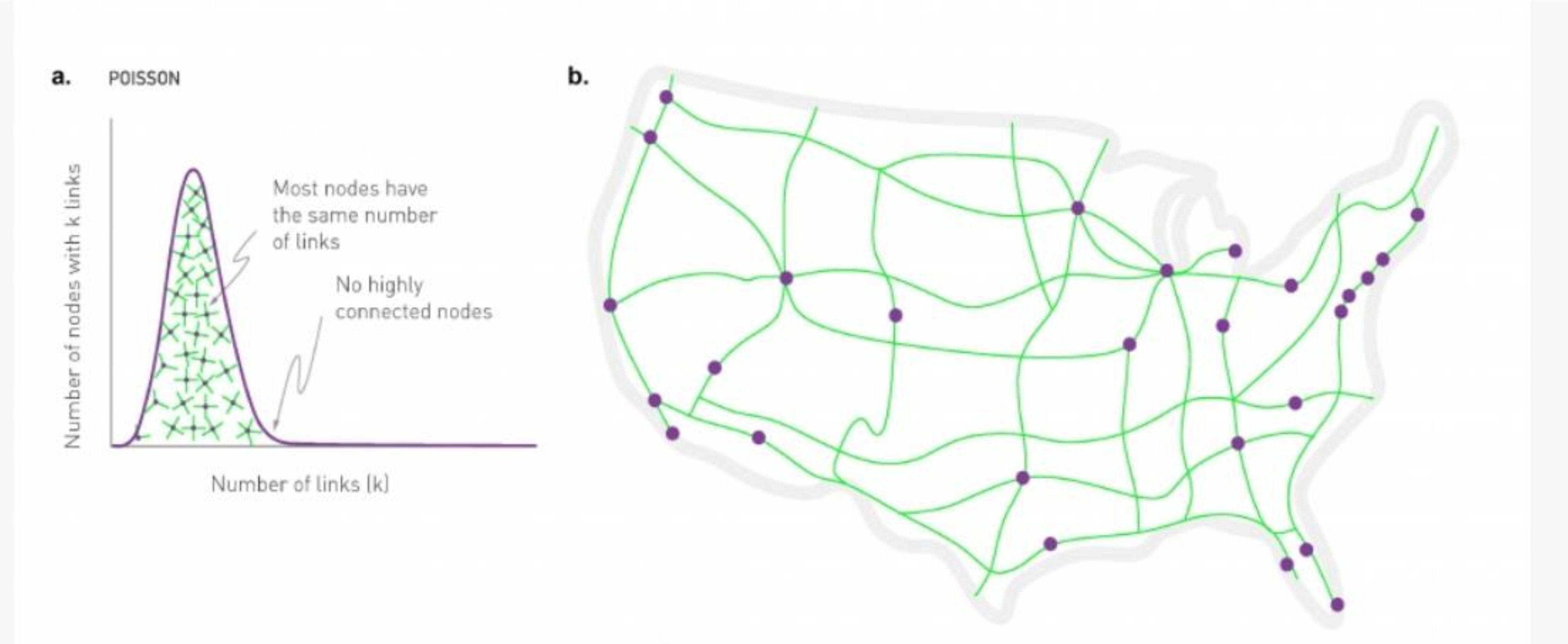


How networks form



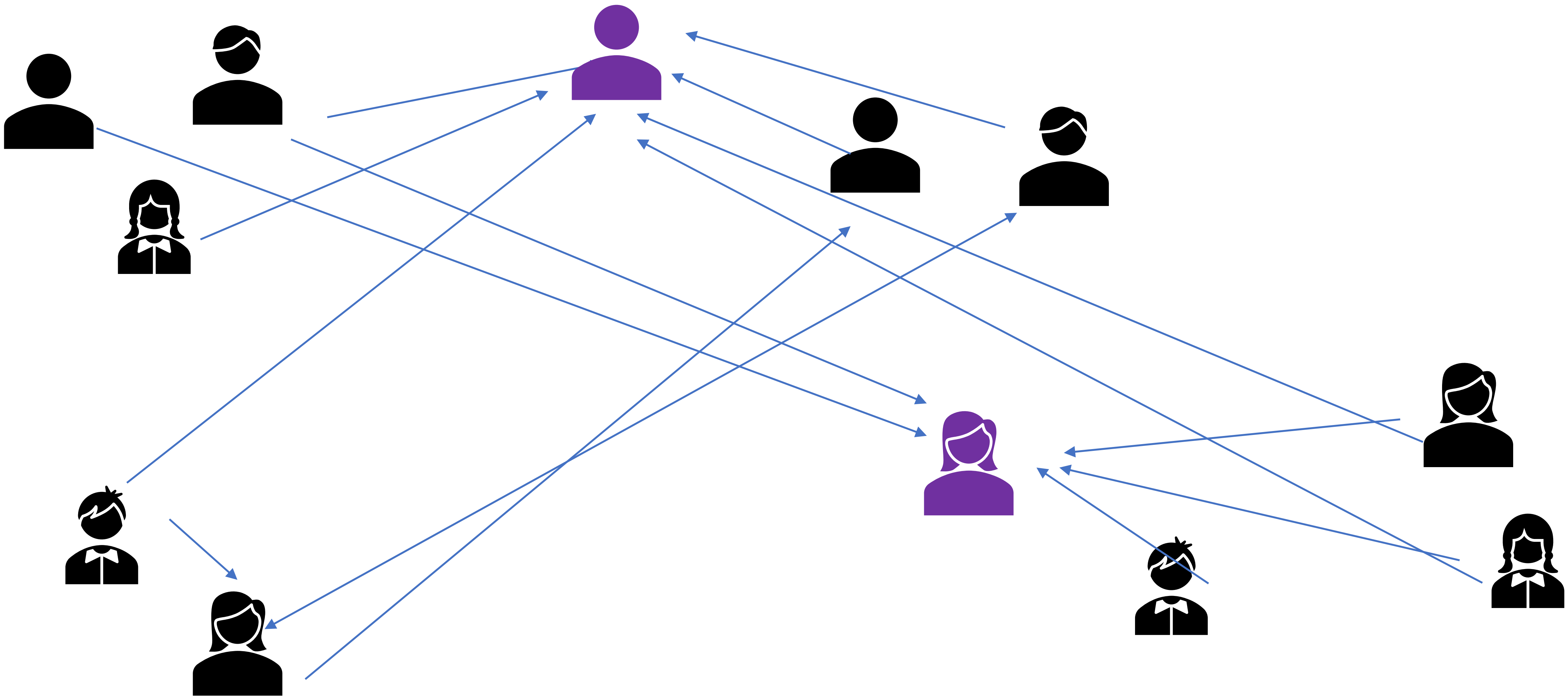
Random networks

- Connections are random
- Usually no outliers
- No highly popular individuals/nodes
- Most nodes have comparable degrees
- Typically have a Poisson distribution
- Accidental failure of a node in the network can fracture the system into non-communicating islands
- Number of nodes is constant over time
- All nodes have similar degree, thus no competition
- For many years, researchers treated networks as being random



Source: Network Science. Barabasi. 2016

How networks form



Scale-free networks

- A scale-free network is one where the nodes are not randomly connected
- The degree distribution follows the power law
  - Few nodes have very high degree
  - Few nodes are connected to most of the other nodes in the network
- Scale-free networks include many "very connected" nodes, called hubs, that shape the way the network operates

Some characteristics of scale-free networks

- They are scale-free
- Follow the power law distribution
- Form a small world
- Have hubs and connectors
- Growth
- Competition
- Robustness

Scale-free

- They lack a scale
  - If we randomly choose a node, the node’s degree could be small or arbitrarily large, thus, they have no meaningful internal scale but are “scale-free”
  - Nodes with widely different degrees coexist in the network
- Some nodes have an unlimited number of links

Power law distribution

- Scale-free networks follow the power law distribution
  - Most nodes have only a few links. These numerous small nodes are held together by a few highly connected nodes.
  - Few nodes have many links
- Also known as the 80/20 rule, Pareto principle
- Vilfredo Pareto, 1910 observed:
  - 80% of the peas are produced by 20% of the plants
  - 80% of the wealth in Italy was owned by 20% of the population
  - 80% of profits are generated by 20% of the employees (Murphy’s law)
  - 80% of the complaints are generated by 20% of the customers
  - 80% of the decisions are made in 20% of the meeting time…
  - 4/5 of our efforts are largely irrelevant
  - 80% of the links point to 15% of the websites
  - 80% of the citations go to 38% of the scientists…

Power law distribution

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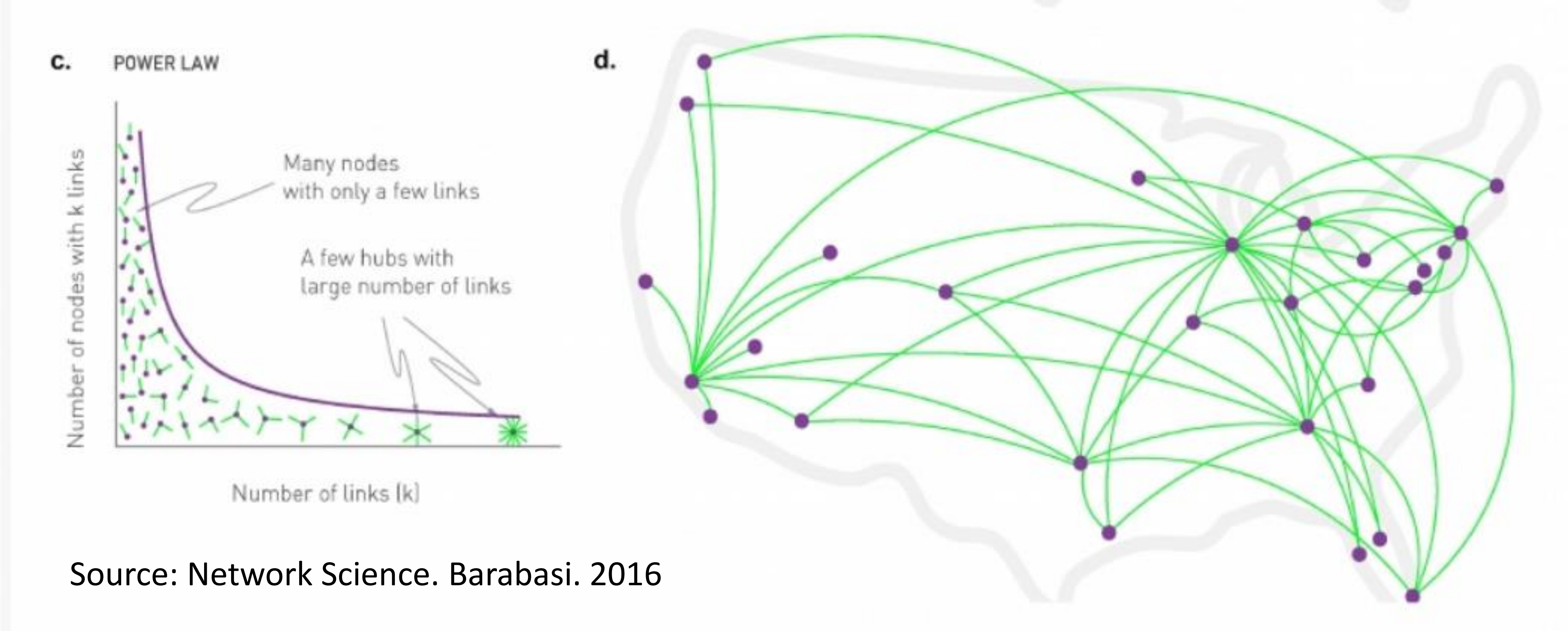


Source: Network Science. Barabasi. 2016

Small world

- The Stanley Milgram experiment, 1967
  - find the average “distance” between any two people in the US
  - 42 of the 160 letter made it to the destination
  - Average path length: 5.5
- Experiment repeated with email by Watts and Strogatz
  - resulting in average path length about 6.8
- So human networks (social networks) form a **small world**…
- How about the Web?
  - Analysis of 800,000,000 nodes (documents) found that the average path length is 18.59…. Still a small world.
- Does this knowledge make it easy to find what you look for?
  - No, we need cues to search, some extra information

Small world

- Short path between any 2 nodes in scale-free networks
  - Concept of a small world
- https://www.youtube.com/watch?v=Rmn-amJ9UA4
  - Kevin Bacon
- https://oracleofbacon.org/

Hubs and connectors

- Nodes with very high degree
- Can connect other nodes in the network
- Typically create the shortest path to everyone else in the network
- The unpopular nodes are held together by the connectors/hubs
- Few connectors know an unusually large number of people
- In a random network, everyone has an equal chance of being noticed
- In a scale-free network, the hubs/connectors get noticed more
- Examples –huge variances between:
    - Number of acquaintances people have (some have 2000+ FB friends!)
    - Number of links pointing to websites varies hugely
    - Number of daily visits of websites
    - Number of phone calls placed by different telephones in the network
    - Number of reactions done by different molecules in a cell
- Hubs/connectors are important because they
    - Create trends
    - Make important deals
    - Spread information
    - Help launch a product

Growth

- Scale-free networks grow incrementally with the addition of new nodes
  - Unlike random networks where the number of nodes is constant over time
- How do networks grow?
  - In a random network, new nodes connect to randomly picked existing nodes
  - But in reality, new nodes are more likely to connect to nodes which already have a lot of connections – *rich-get-richer* phenomenon
  - Preferential attachment; popularity attracts
- Growth and preferential attachment explain the hubs and power laws in complex networks.
- But this model predicts that the oldest nodes would have most links
- Newcomers will have no chance.
- In random networks, all nodes stand the same chance. Not the same in real world networks

Competition

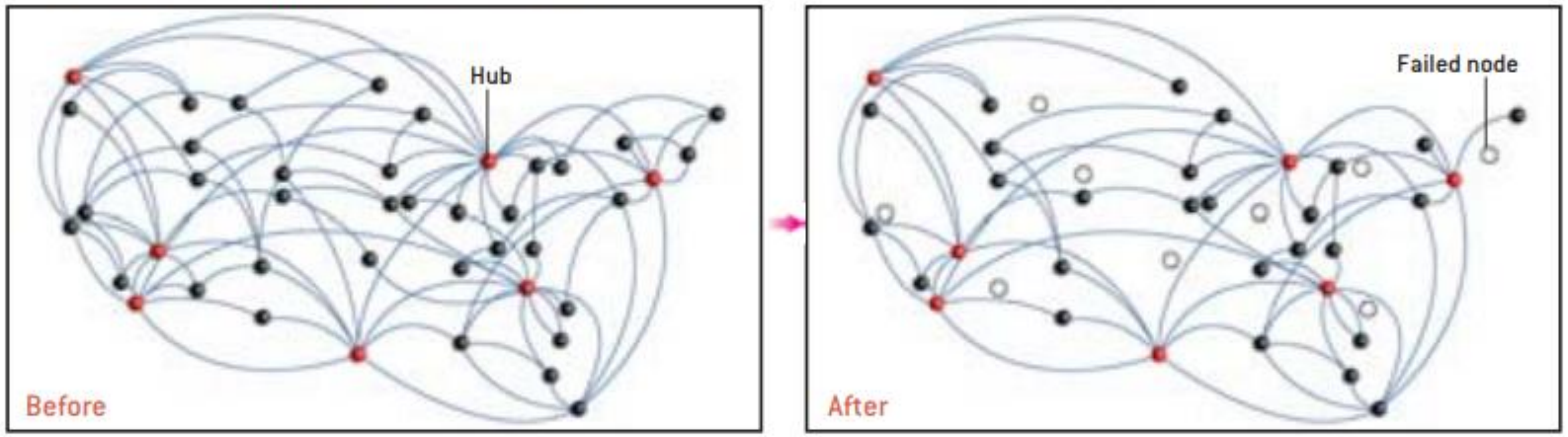
- How do we explain successful newcomers?
  - For example, Google?
- Scale-free networks are competitive systems in which nodes fight for links
- Each node has unique characteristics that are apparent; intrinsic qualities that influence the rate at which they acquire links
- All nodes are not equal, some are “better”, more attractive, offer a better quality of service, or are better at making friends, or have better looks (younger actors/actresses)
- Each node has a certain fitness
  - Ability to make friends relative to others, company’s competence in luring and keeping customers, actor’s aptitude to be likes and recommended compared to other aspiring actors
- Fitness of a node is a measure of its ability to stay in front of the competition
- The “Fit get rich” model leads to a scale-free topology

Robustness

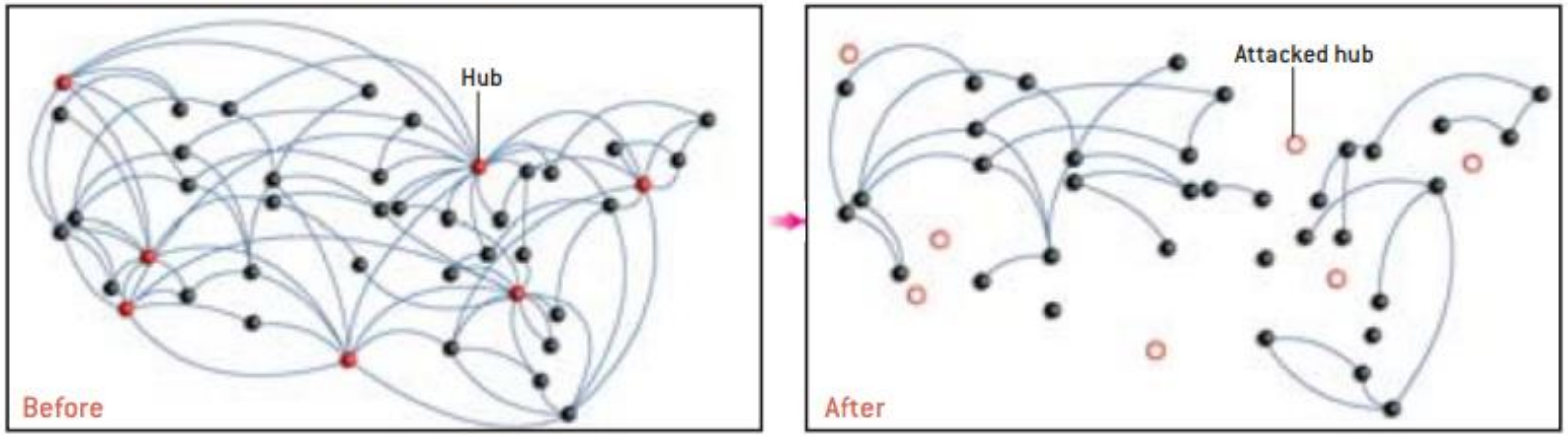
- Scale-free networks are extremely robust in case of random failures
- Robustness. The ability of complex systems to still function ehn their structure changes significantly
- Studying network resilience
  - In random networks, some node failures can easily break a network into isolated, non-communicating frameworks –e.g. closing simultaneously highways in and out of Jacksonville and Lake City, FL, will close Florida to the rest of the US.
  - Yet, a study of the Internet resilience showed that we can remove 80% of all nodes, and the remaining 20% will still remain connected
  - The key to this is the presence of hubs, removing nodes randomly is not likely to affect them, and they hold the NW
  - Since there are more small nodes than hubs (nodes with high degree), if failures in networks affect each node with an equal chance, small nodes will far more likely be affected than hubs
  - Accidental removal of a single hub will also not be fatal since the other large hubs can maintain the network?
    - Does this apply in social networks? What of an attack on all hubs?

Robustness

Scale-Free Network, Accidental Node Failure



Scale-Free Network, Attack on Hubs



Differences between random networks and scale-free networks

Random	Scale free

References

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