Improving Cognitive Flexibility in Calculus: The Role of Interleaving Approach in Higher Education

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Abstract

The study focuses on improving calculus education through the interleaving approach, aligning with CFT's emphasis on adaptability and problem-solving. This study is anchored in the Cognitive Flexibility Theory (CFT), which posits that students must engage with new material from multiple perspectives and with flexible thought processes to develop higher-order thinking skills and beneficial affective changes. The research investigates the effectiveness of interleaving in improving calculus learning outcomes among 46 students of BSED 2 Mathematics students at Central Philippines State University during the 2023-2024 academic year. Using a quasi-experimental design, the study compares the learning outcomes of the traditional approach and interleaving approach through pre-test and post-test assessments using the Mann-Whitney U-test as a tool. The results revealed that before the intervention, participants from both interleave and blocked approaches had low levels of performance. After the intervention was given, it was found that the blocked approach improves to high level while interleaved improved to a very high level of performance. Interleaved practice sessions into educational strategies were essential for maximizing learning outcomes and providing practical recommendations for educators to enhance calculus instruction.

Keywords: Interleaving Approach, Calculus, Calculus Education

Introduction

Improving calculus education through the interleaving approach is a pedagogical method that can significantly enhance students' understanding and retention of calculus concepts. Interleaving involves mixing different but related topics or problem types within a single study session, rather than focusing on one topic at a time—a strategy known as blocked practice (Rohrer, 2018). By interleaving topics, students can strengthen long-term memory and develop deeper comprehension of the material (Rohrer, Dedrick, & Stershic, 2019).

Calculus is traditionally taught sequentially and linearly, where students learn one concept at a time and practice it thoroughly before moving to the next. While this approach may give the impression of mastery, it often leads to reliance on pattern recognition or short-term memory rather than genuine understanding, creating "learning illusions"—an inaccurate sense of comprehension. Traditional calculus instruction has also been criticized as a "litany of procedures and template problems," where students primarily practice routine algebraic manipulations. This can be especially discouraging for students from underprivileged backgrounds, who, despite understanding fundamental concepts, may struggle with complex manipulations and ultimately disengage (Zeytuncu, Cavlazoglu, & Sahin, 2015).

Additionally, traditional lecture-based teaching in large college classrooms often limits student engagement. Many students find it difficult to remain focused or actively participate in the learning process, which negatively impacts their overall learning outcomes. As experienced researchers and educators have observed, innovative approaches like interleaving may offer a solution by fostering deeper comprehension, active learning, and sustained retention of calculus concepts.

Objective of the study

The primary purpose of this study is to evaluate the effectiveness of the interleaving approach in enhancing calculus education. Specifically, it seeks to determine whether this pedagogical method improves students' learning outcomes, fosters a deeper understanding of calculus concepts, and enhances knowledge retention and problem-solving skills compared to traditional teaching methods.

Theoretical and Conceptual Framework

This study is grounded in the **Cognitive Flexibility Theory (CFT),** as proposed by Cheng and Koszalka (2016). **Cognitive Flexibility Theory** posits that for students to engage deeply with new material, they must approach it from multiple perspectives and employ flexible thought processes. Research indicates that such flexible thinking fosters higher-order cognitive skills, such as problem-solving, and initiates positive changes in the learner's affective domain. Incorporating CFT principles into educational resources is thus expected to encourage meaningful engagement with learning materials.

Cognitive Flexibility Theory emphasizes the application of knowledge and skills beyond their initial learning context, advocating for diverse perspectives and the use of multiple case studies to illustrate various scenarios. The theory underscores the importance of situationally targeted education, asserting that effective learning depends on context. By focusing on cognitive flexibility, CFT highlights the learner's ability to adapt knowledge and strategies across different situations, reinforcing the idea that successful learning involves both knowledge acquisition and its versatile application.

The relevance of CFT to enhancing calculus education through the interleaving approach lies in its shared emphasis on adaptability, problem-solving, and transitioning between mental frameworks. CFT's principles align closely with the goals and strategies of interleaved learning, making it an ideal theoretical foundation for this study. The conceptual framework integrates key theoretical constructs, research hypotheses, and variables to guide the research design and analysis, providing a structured approach to investigating the effectiveness of the interleaving method in calculus education.

Methodology

This study employed a non-equivalent controlled group design without randomization. Participants included 46 second-year Bachelor of Secondary Education students majoring in Mathematics at Central Philippines State University, San Carlos Campus, enrolled in Calculus 1 during the Academic Year 2023-2024. Section A comprised 22 students, while Section B had 24. A researcher-developed 15-item multiple-choice questionnaire aligned with the syllabus learning objectives served as the instrument. Five subject matter experts, all holding at least a master's degree, validated the questionnaire for clarity, relevance, and alignment, achieving a perfect Content Validity Index (CVI) of 1. Reliability was assessed through a test-retest method, yielding a correlation coefficient of 0.826, indicating acceptable stability. Participants were fully informed about the study's nature, assured of confidentiality, and informed that data would be used exclusively for research. Data normality tests revealed a non-normal distribution, prompting the use of non-parametric methods. Descriptive statistics, including mean and standard deviation, summarized the data, while the Wilcoxon rank-sum test and Mann-Whitney U test compared group responses and identified significant differences. This rigorous methodology, encompassing expert validation, reliability testing, and robust statistical analysis, ensures the reliability, validity, and credibility of the study's findings.

Results and Discussion

A. Level of Performance of Students in Blocked Approach

Table 1. Level Performance of Participants in Controlled Group

Blocked Approach	Mean	Standard Deviation	Descriptive Interpretation
Pre-test	4.86	1.49	Low
Post-test	10.83	1.34	High

Scale: *15.00-12.01 – Very High, 12.00-9.01 – High, 9.00-6.01 – Moderate 6.00-3.01 – Low, 3.00-0.00 – Very Low*
Table 1 shows the participants' performance levels using the blocked approach. The pre-test mean score was 4.86, with a standard deviation of 1.49, indicating that students, on average, performed poorly on the initial assessment before the instructional intervention. Following the intervention, the post-test mean score significantly increased to 10.83, with a

reduced standard deviation of 1.34. This substantial improvement suggests that the blocked approach effectively enhanced students' understanding of calculus concepts across the cohort.

Rohrer, Dedrick, and Stershic (2015) argue that while a small block of problems can be beneficial initially—particularly when students are introduced to a new problem type—extended practice using the blocked approach quickly yields diminishing returns. This suggests that while an initial focus helps students understand and apply strategies effectively, continued repetition within a single session may hinder overall learning effectiveness.

B. Level of Performance of Students in Interleaving Approach

Table 2. Level Performance of Participants in the Experimental Group

Interleaving Approach	Mean	Standard Deviation	Descriptive Interpretation
Pre-test	4.42	1.501	Low
Post-test	12.21	1.285	Very High

Scale: 15.00-12.01 – Very High, 12.00-9.01 – High, 9.00-6.01 – Moderate 6.00-3.01 – Low, 3.00-0.00 – Very Low
Table 2 presents the performance levels of participants using the interleaving approach. The pre-test mean score was 4.42, indicating a low initial understanding of calculus concepts. Following the instructional intervention, the post-test mean score increased significantly to 12.21, demonstrating a marked improvement in students' understanding.

In mathematics education, interleaved practice has proven to be a highly effective strategy. Rohrer, Dedrick, and Stershic (2015) emphasize that interleaved practice enhances students' performance by simply rearranging practice problems, without altering the core content or lesson structure.

C. Difference Analysis between Student's Pre-tests
Table 3. Difference analysis between students' pre-tests

Indicator	Mean Rank	U-test	p-value	Conclusion
Blocked Approach	24.84			
Interleaved Approach	22.27	234.5	.506	Not Sig.

Legend: Not sig. – Not significant if p-value is greater than 0.05
Table 3 compares the pre-test results between the control and experimental groups using the Mann-Whitney U-test. The interleaved approach had a mean rank of 22.27, while the blocked approach had a mean rank of 24.84. The U-test value was 234.5, with a p-value of 0.506. As the null hypothesis could not be rejected, there was no significant difference between the control and experimental groups in the pre-test. This suggests that, prior to any intervention or treatment, the groups were statistically equivalent in the characteristics being examined.
This finding strengthens the study's internal validity, as it indicates that factors other than the intervention are less likely to influence the results. Ensuring no substantial disparities in pre-test scores between control and experimental groups is critical for experimental design validity. As Demirer and Aydın (2020) emphasize, achieving this equivalence ensures that any observed post-test differences can be attributed to the intervention rather than pre-existing group differences.

D. Difference Analysis on Performance of Controlled Group
Table 4. Difference analysis between pre-test and post-test on block approach

Blocked Approach		Mean rank	p-value	Conclusion
Pre-test	Negative Rank	.00		
Post-test	Positive Rank	11.5	.000	Sig.

Legend: Sig. – significant if the p-value is lesser than or equal to .05
Table 4 presents a comparison of the pre-test and post-test results for the control group, analyzed using the Wilcoxon test. The analysis reveals a negative mean rank of 0.00 and a positive mean rank of 11.50 for the students' scores, with a p-value of 0.000. Based on this result, the null hypothesis was rejected, indicating a statistically significant difference between the pre-test and post-test scores for the control group. This finding suggests that the current method of teaching calculus had a measurable impact on the assessed variable, reflecting a change in students' performance, behavior, or attributes.

Similarly, Demirer and Aydın (2020) investigated the use of a blocked teaching method to manage unpredictability in calculus instruction. Their study demonstrated significant improvements in students' post-test scores compared to their pre-test scores, underscoring the effectiveness of structured strategies in educational settings.

E. Difference Analysis on Performance of Experimental Group

Table 5. Difference analysis between pre-test and post-test on interleaving approach

Interleaving Approach		Mean Rank	p-value	Conclusion
Pre-test	Negative Rank	.00	.000	Sig.
Post-test	Positive Rank	12.50		

Legend: Sig.–significant if the p-value is lesser than or equal to .05

Table 5. compares the pre-test and post-test results of the experimental group, analyzed using the Wilcoxon test. The analysis indicates a negative mean rank of 0.00 and a positive mean rank of 12.50. With a p-value of 0.000, the null hypothesis was rejected, signifying a statistically significant difference between the pre-test and post-test results for the group utilizing the interleaved learning method. This finding implies that the interleaved learning approach had a notable impact on participants' performance and understanding of the target topic.

The results suggest that interleaving—where different topics or skills are alternated during the learning process—fosters a more robust and enduring comprehension of the subject matter. Supporting this, Saavedra and Moreno-Guerrero (2021) examined the effectiveness of the interleaving strategy in teaching calculus. Their study reported significant improvements in students' post-test scores compared to their pre-test scores, further highlighting the strategy's ability to enhance calculus learning outcomes.

F. Difference Analysis between Student's Post-tests

Table 6. Difference analysis on the post-test between the controlled and experimental groups.

Experimental	Mean Rank	U-test	p-value	Conclusion
Blocked Approach	18.36	151.00	.010	Sig.
Experimental Approach	28.21			

Legend: Sig. – significant if the p-value is lesser than or equal to .05

Table 6 compares the pre-test results of the control and experimental groups using the Mann-Whitney U-test. The control group achieved an average rank of 18.36, while the experimental group recorded a higher average rank of 28.21. With a U-test value of 151.00 and a p-value of 0.00, the results indicate a significant difference in performance between the two groups. Students employing the interleaved learning strategy outperformed those using the traditional blocked approach, suggesting that the interleaved method is more effective in enhancing long-term retention and comprehension of the material.

This finding highlights the importance of instructional designs that incorporate varied and interleaved practice sessions rather than repeated, blocked sessions focusing on a single topic. While the blocked approach often created an initial sense of mastery for specific types of problems, it frequently hindered the ability to transfer knowledge to novel situations. Conversely, the interleaved method encouraged a deeper and more adaptable understanding of calculus concepts, leading to better performance on tests and in subsequent applications.

Following the intervention, students participated in a focus group discussion, where they reported that learning calculus through the interleaved approach significantly improved their long-term memory retention compared to the blocked approach. Students noted that practicing with varied problem types helped them identify underlying patterns and principles, resulting in a deeper understanding and improved recall over time. While the blocked method initially provided a sense of confidence in solving specific problems, it often fell short when applying knowledge to new contexts. In contrast, the interleaved approach fostered a more durable and flexible grasp of calculus concepts, enhancing their ability to succeed in assessments and real-world applications.

Conclusion

This study provides valuable insights into the effectiveness of interleaved practice as a superior teaching strategy for enhancing calculus comprehension and retention compared to traditional blocked methods. While blocked practice provides short-term benefits by fostering problem-specific mastery, interleaving promotes a deeper understanding of concepts and improves the ability to transfer knowledge to new contexts. The findings underscore the potential of interleaved practice to cultivate robust and lasting mastery of mathematical concepts, making it a more effective

instructional approach. Educators are encouraged to incorporate interleaved practice into their teaching strategies to improve student learning outcomes and foster a more adaptable understanding of calculus.

Recommendation

Educational institutions should integrate interleaved practice into their calculus curricula to enhance student comprehension and retention. Teachers should incorporate diverse problem types into lesson plans, promoting a variety of problem-solving experiences. Curriculum developers are advised to update instructional materials to align with the interleaved approach. Additionally, students should engage in collaborative discussions to share strategies and use feedback to refine their learning methods. Future research should investigate the effectiveness of interleaved practice across diverse educational settings and among different student demographics, providing further insights into its broader applicability and benefits.

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