Engineering Mechanics: Statics in SI Units, 12e

5 Equilibrium of a Rigid Body

Chapter Objectives

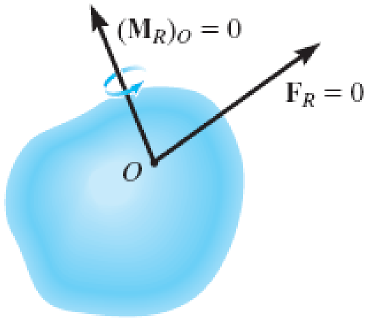
- To develop the equations of equilibrium for a rigid body
- To introduce the concept of the free-body diagram for a rigid body
- To show how to solve rigid-body equilibrium problems using the equations of equilibrium

Chapter Outline

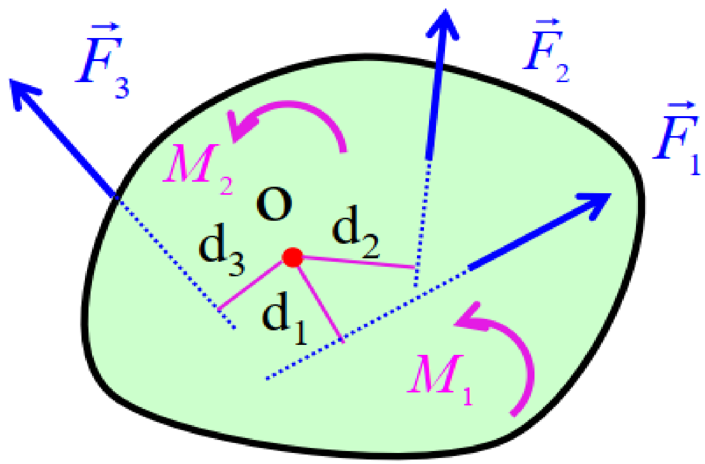
1. Conditions for Rigid Equilibrium
2. Support Reactions
3. Free-Body Diagrams
4. Equations of Equilibrium

5.1 Conditions for Rigid-Body Equilibrium

- *When an object acted upon by a system of forces & moments is in equilibrium, the following conditions are satisfied:*
- ***1. The sum of the forces is zero:*** $\sum \vec{F} = 0$
- ***2. The sum of the moments about any point is zero:*** $\sum \vec{M}_{any\ point} = 0$



5.1 Conditions for Rigid-Body Equilibrium



$$\vec{R} = \Sigma \vec{F} = 0$$
$$\vec{M}_o = 0$$

or

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_o = 0$$

Necessary

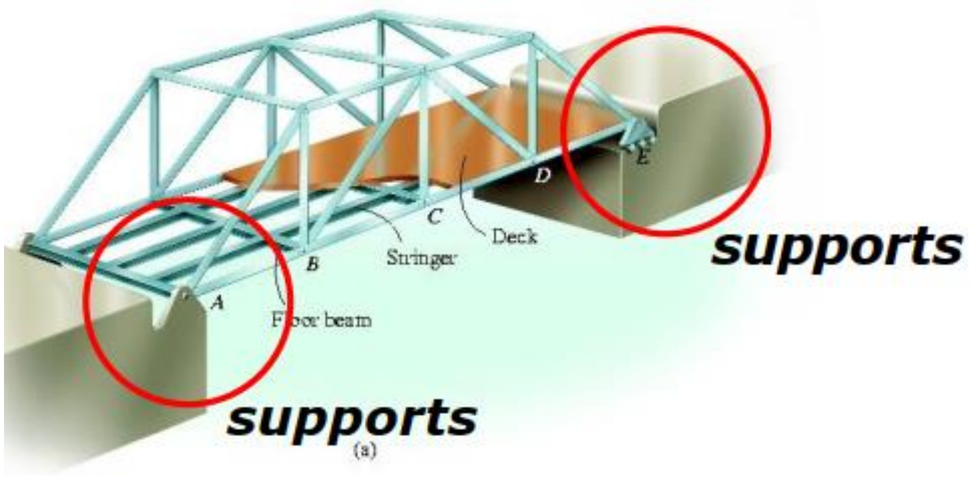
Body in Equilibrium

Sufficient

5.2 Support Reactions

Forces & couples exerted on an object by its supports are called reactions

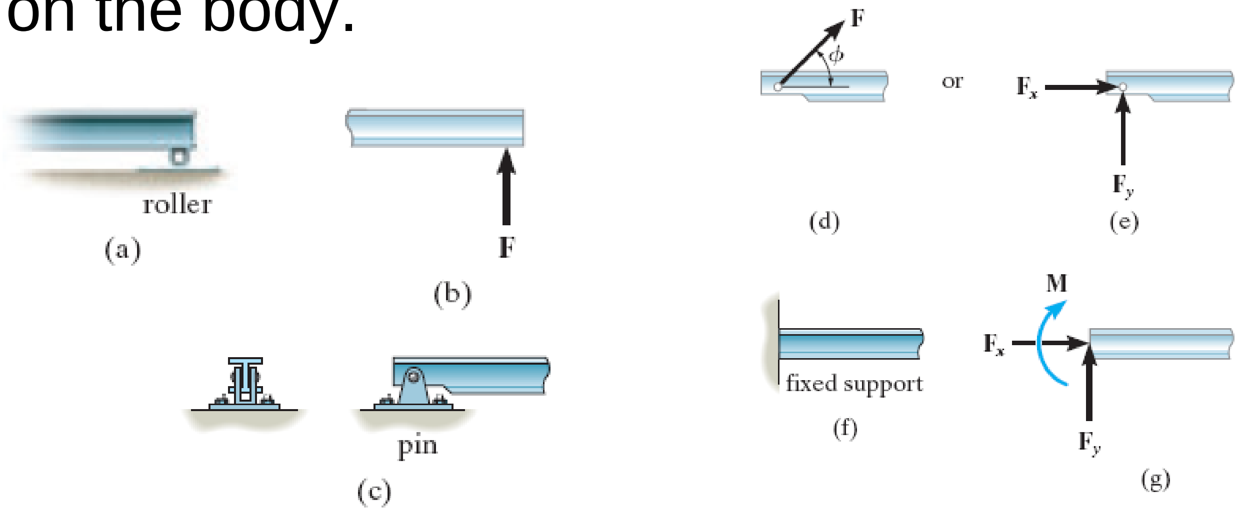
E.g. a bridge is held up by the reactions exerted by its supports.



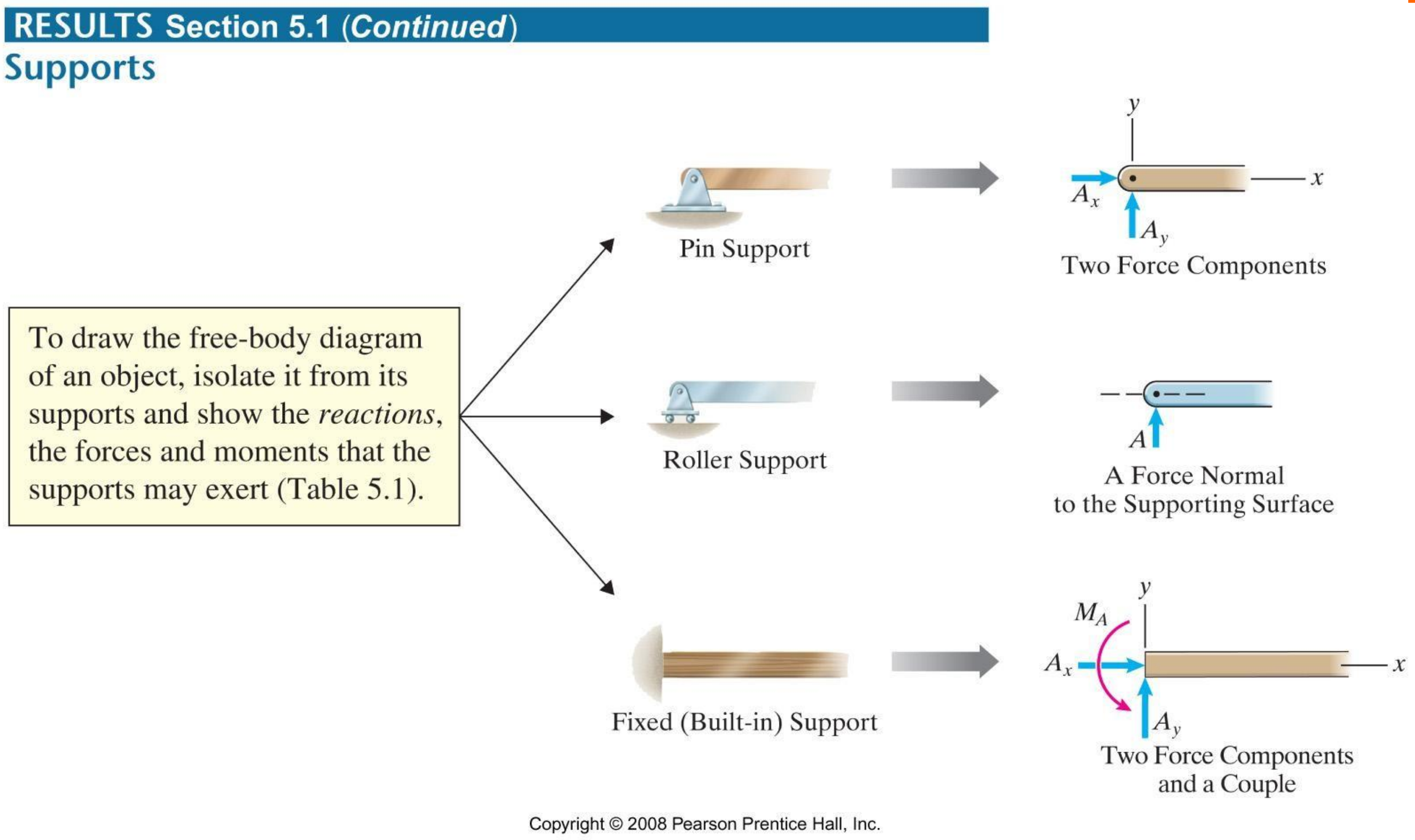
5.2 Support Reactions

General Rules

- If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.
- If rotation is prevented, a couple of moment is exerted on the body.

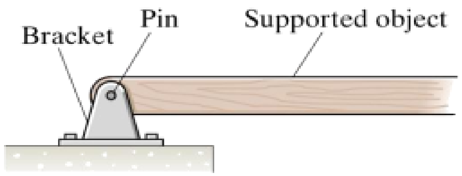


5.2 Support Reactions



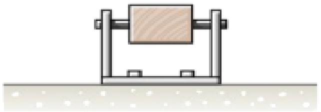
5.2 Support Reactions

- ***The Pin Support:***
  - Figure a: a pin support
    - a bracket to which an object is attached by a smooth pin that passes through the bracket & the object
  - Figure b: side view



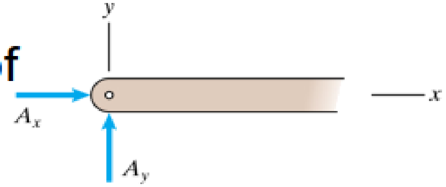
(a)

SYMBOL USED IN NOTEBOOK



(b)

•The arrows indicate the directions of the reactions A_x and A_y



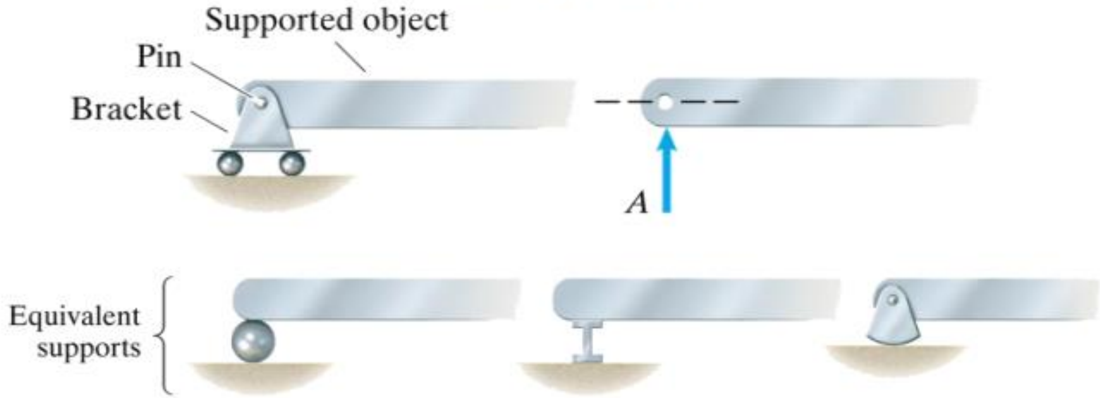
5.2 Support Reactions

Examples of **Pin Support** in real life

5.2 Support Reactions

The Roller Support

It can move freely in the direction parallel to the surface on which it rolls, it can't exert a force parallel to the surface but can exert a force normal (perpendicular) to this surface



•The arrow indicate the directions of the reaction ***A***

SYMBOL USED IN NOTEBOOK 

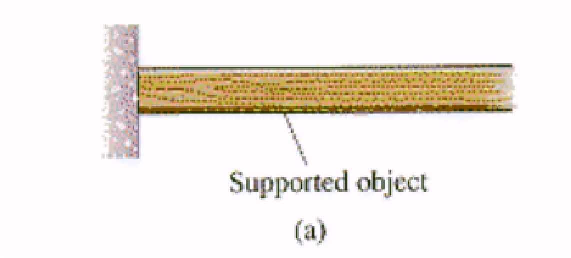
5.2 Support Reactions

Examples of **Roller Support** in real life

5.2 Support Reactions

The Fixed Support

The fixed support shows the supported object literally built into a wall (built-in)



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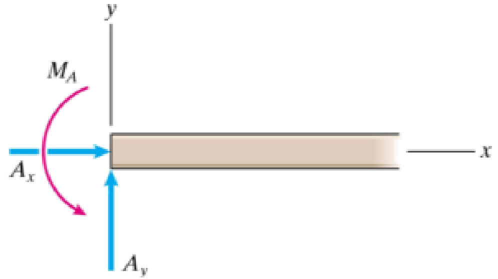


SYMBOL USED IN NOTEBOOK

A fixed support can exert

2 components of force

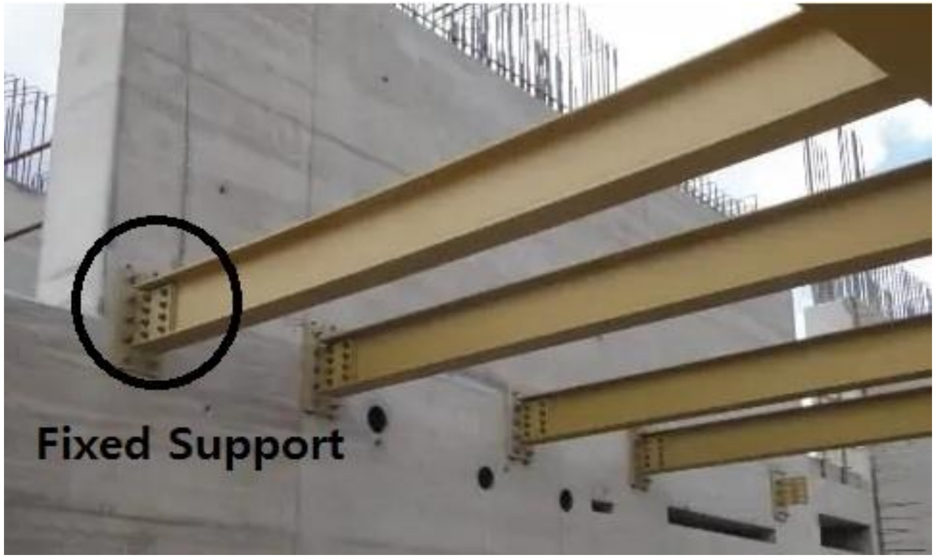
A_X A_Y

and a **couple**

M_A

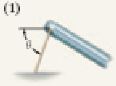
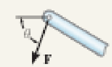
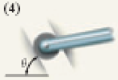
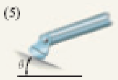
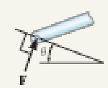
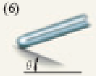
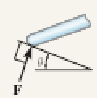
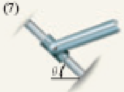
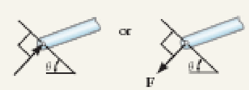
5.2 Support Reactions

Examples of **Fixed Support** in real life

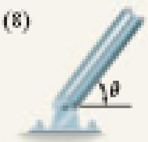
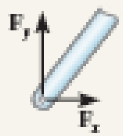
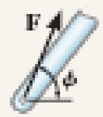
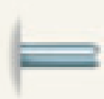
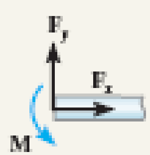
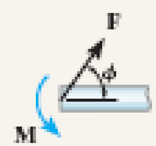


5.2 Support Reactions

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot		One unknown. The reaction is a force which acts perpendicular to the slot.
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod		One unknown. The reaction is a force which acts perpendicular to the rod.

5.2 Support Reactions

TABLE 5–1 Continued		
Types of Connection	**Reaction**	**Number of Unknowns**
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

5.3 Free Body Diagrams

The Free-Body Diagram ≡ FBD

To apply equilibrium equations we must account for all known and unknown forces acting on the object.

The best way to do this is to draw a *free-body diagram* FBD of the body.

This is a sketch that shows the rigid body "free" from its surroundings with all the forces acting on it.

Force Types

Active Forces : **tend to set the body in motion.**

Reactive Forces : **result from constraints or supports and tend to prevent motion**

5.3 Free Body Diagrams

Procedure for Drawing a FBD

1. Draw Outlined Shape

- Imagine body to be isolated or cut free from its constraints
- Draw outline shape

2. Show All Forces and Couple Moments

- Identify all external forces and couple moments that act on the body

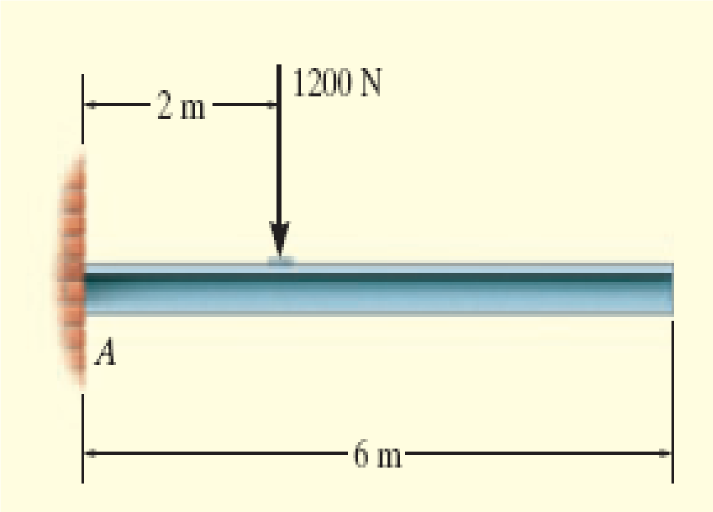
5.3 Free Body Diagrams

3. Identify Each Loading and Give Dimensions

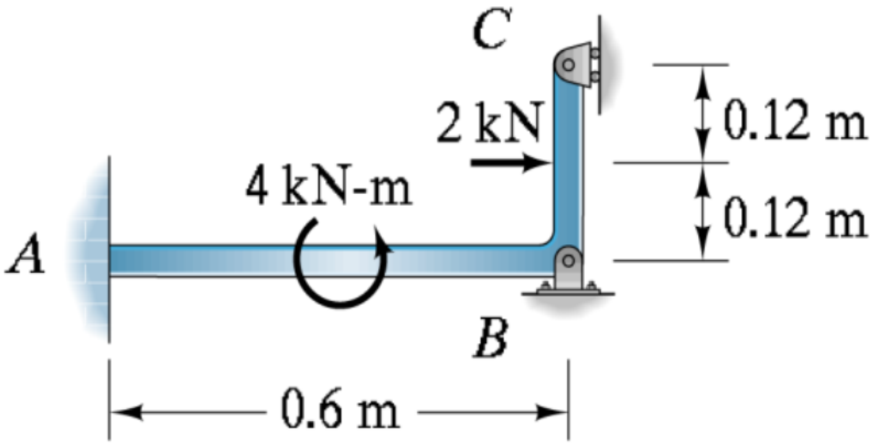
- Indicate dimensions for calculation of forces
- Known forces and couple moments should be properly labeled with their magnitudes and directions

Example 5.1

Draw the free-body diagram for shapes, a and b



a)



b)

5.4 Equations of Equilibrium

- For equilibrium of a rigid body in 2D,

  $$\sum F_x = 0; \sum F_y = 0; \sum M_O = 0$$

- $\sum F_x$ and $\sum F_y$ represent sums of x and y components of all the forces

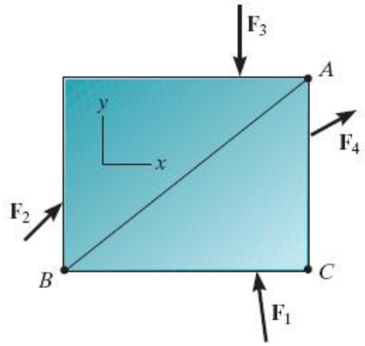
- $\sum M_O$ represents the sum of the couple moments and moments of the force components

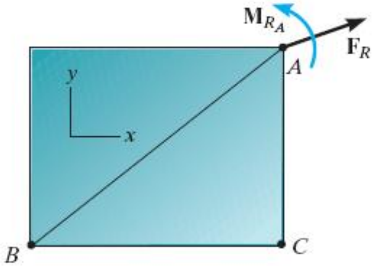
5.4 Equations of Equilibrium

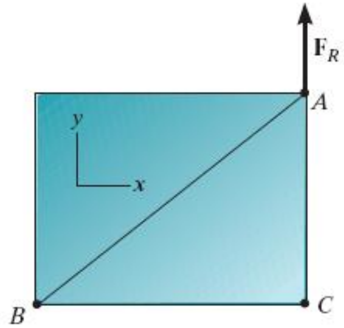
Alternative Sets of Equilibrium Equations

- For coplanar equilibrium problems,
  $\sum F_x = 0$; $\sum F_y = 0$; $\sum M_O = 0$

- 2 alternative sets of 3 independent equilibrium equations,
  $\sum F_x = 0$; $\sum M_A = 0$; $\sum M_B = 0$







5.4 Equations of Equilibrium

Procedure for Analysis

<u>Free-Body Diagram</u>

- Force or couple moment having an unknown magnitude but known line of action can be assumed
- Indicate the dimensions of the body necessary for computing the moments of forces

5.4 Equations of Equilibrium

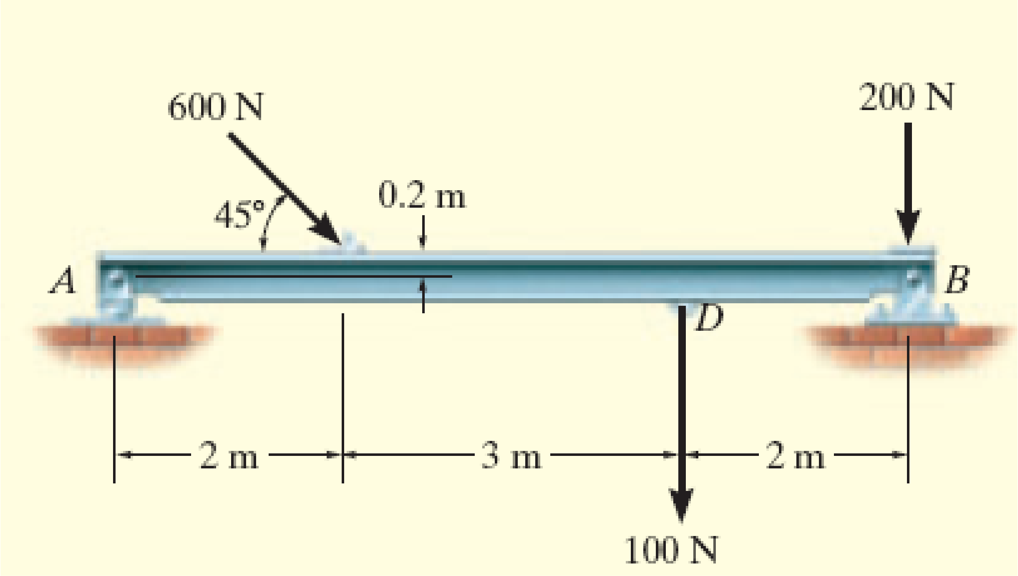
Procedure for Analysis

Equations of Equilibrium

- Apply $\sum M_O = 0$ about a point O
- Unknowns moments of forces are zero about O and a direct solution for the third unknowns can be determined
- Orient the x and y axes along the lines that will provide the simplest resolution of the forces into their x and y components
- Negative result scalar is opposite to that was assumed on the FBD

Example 5.2

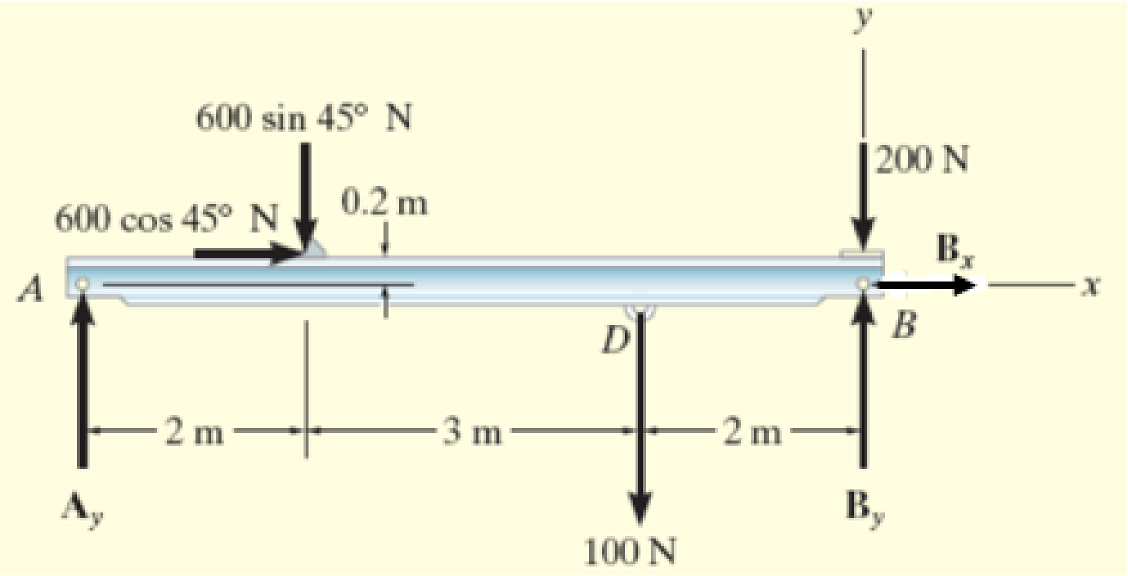
Determine the horizontal and vertical components of the reaction for support A (roller) and Support B(pin).



Solution

Free Body Diagrams

- 600N represented by x and y components
- 200N force acts on the beam at B



Solution

Equations of Equilibrium

$$+\rightarrow \sum M_B = 0; \quad 600\cos 45^\circ N + B_x = 0 \Rightarrow B_x = -424\ \mathrm{N}$$

$$\sum M_B = 0;$$

$$100N(2m) + (600\sin 45^\circ N)(5m) - (600\cos 45^\circ N)(0.2m) - A_y(7m) = 0$$

$$A_y = 319N$$

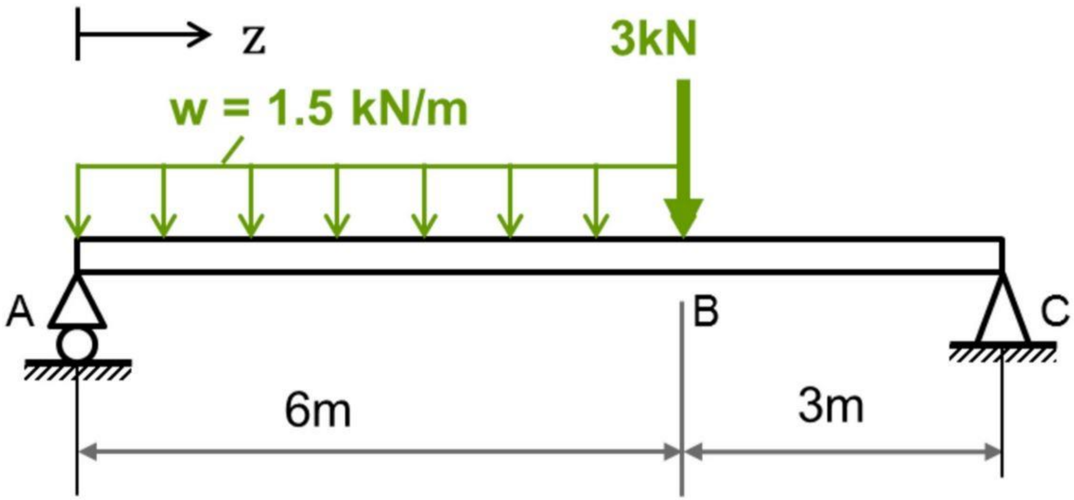
$$+\uparrow \sum F_y = 0;$$

$$319N - 600\sin 45^\circ N - 100N - 200N + B_y = 0$$

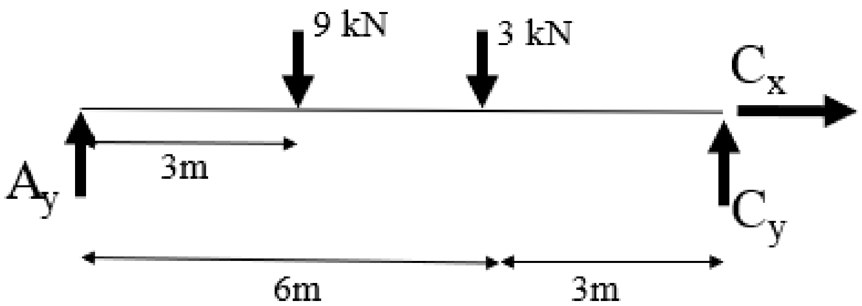
$$B_y = 405N$$

Example 5.3

Determine the Support Reactions at A and C



Solution



$\sum F_x = 0 \quad A_x = 0$

$\sum Fy = 0 \quad A_y + C_y - 3 - (1.5)(6) = 0 \qquad A_y + C_y = 12$

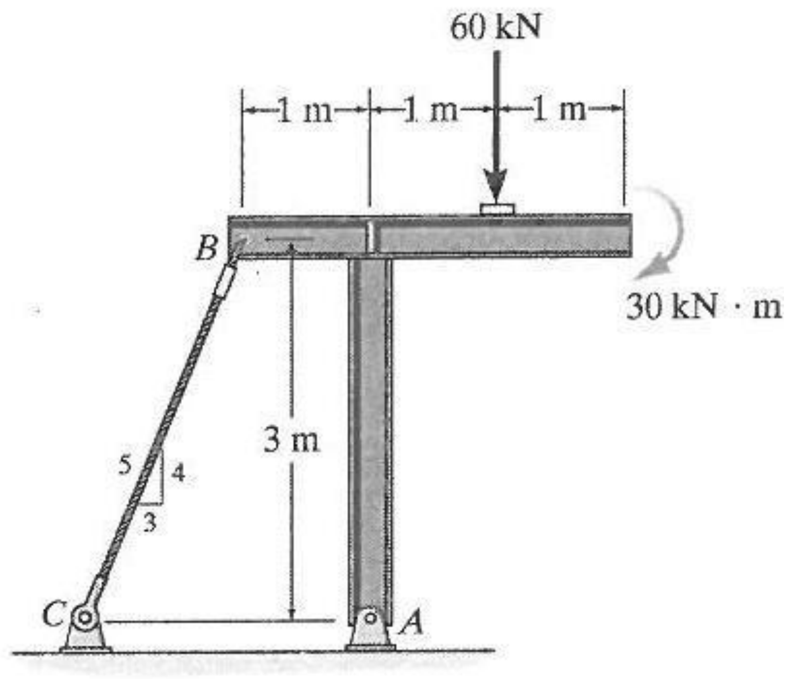
$\sum M_C = 0 \qquad (-A_y)(9) + (9)(6) + (3)(3) = 0$

$A_y = 7\ \text{kN}$ $\qquad C_y = 5\ \text{kN}$

Example 5.4

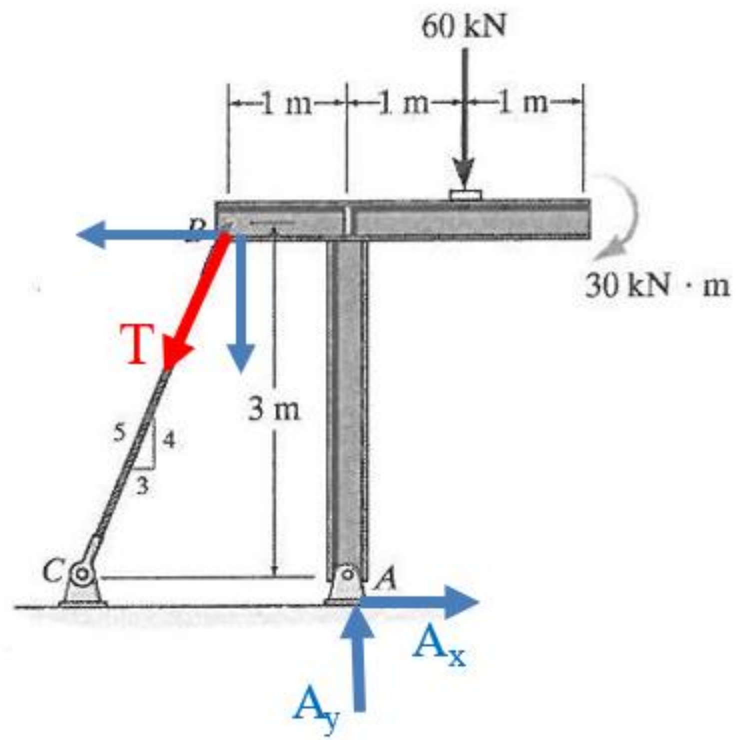
There is a Cable CB, determine:

a)The tension force at CB
b) the support reaction at A (pin)



Solution

Free-body Diagram



Solution

$$\circlearrowleft + \sum M_A = 0;$$

$$(T)(3/5)(1) + (T)(4/5)(3) - (60)(1) - 30 = 0$$

$$\boxed{T = 30\ \text{kN}}$$

$$\xrightarrow{+} \sum F_x = 0;$$

$$A_x - (30)(3/5) = 0 \qquad \boxed{A_x = 18\ \text{kN}}$$

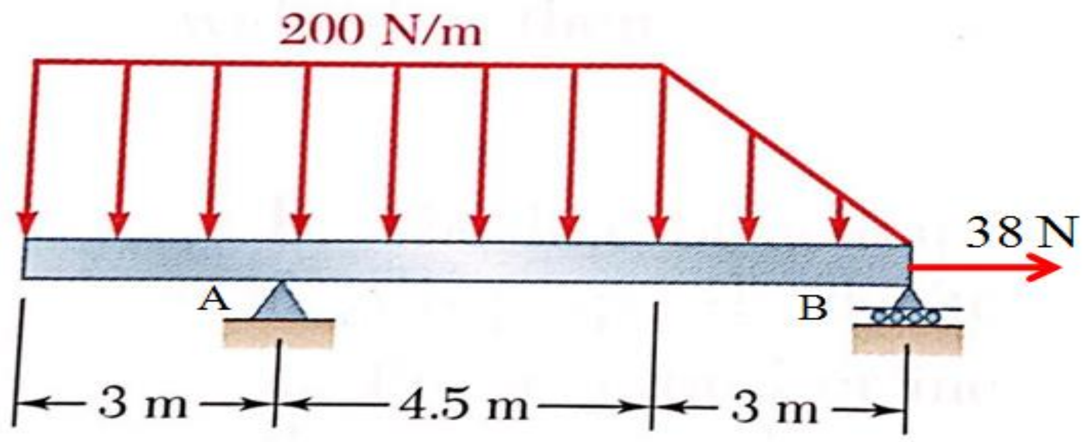
$$+\uparrow \sum F_y = 0;$$

$$A_y - (30)(4/5) - 60 = 0$$

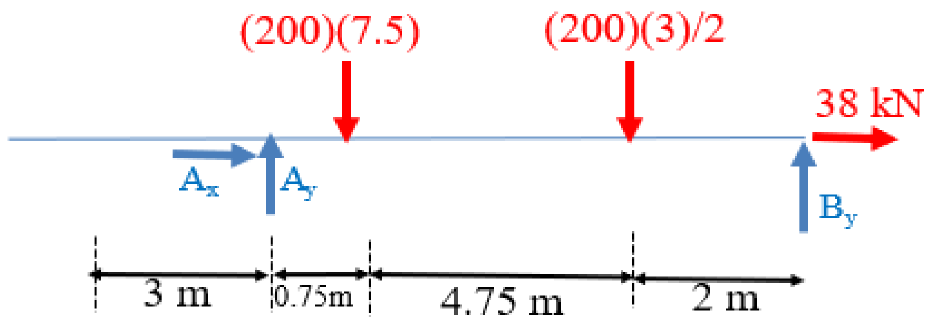
$$\boxed{A_y = 84\ \text{kN}}$$

Example 5.5

determine the reactions at pin support **A** and roller support **B**



Solution



$\xrightarrow{+} \sum F_x = 0;$

$A_x + 38 = 0$

$\mathbf{A_x = -38\ N}$

$\circlearrowleft + \sum M_B = 0;$

$-(A_y)(7.5) + (1500)(6.75) + (300)(2) = 0$

$\mathbf{A_y = 1430\ N}$

$+\uparrow \sum F_y = 0;$

$B_y + 1430 - 1500 - 300 = 0$

$\mathbf{B_y = 370\ N}$