Engineering Mechanics: Statics in SI Units, 12e

4 Force System Resultants

Chapter Objectives

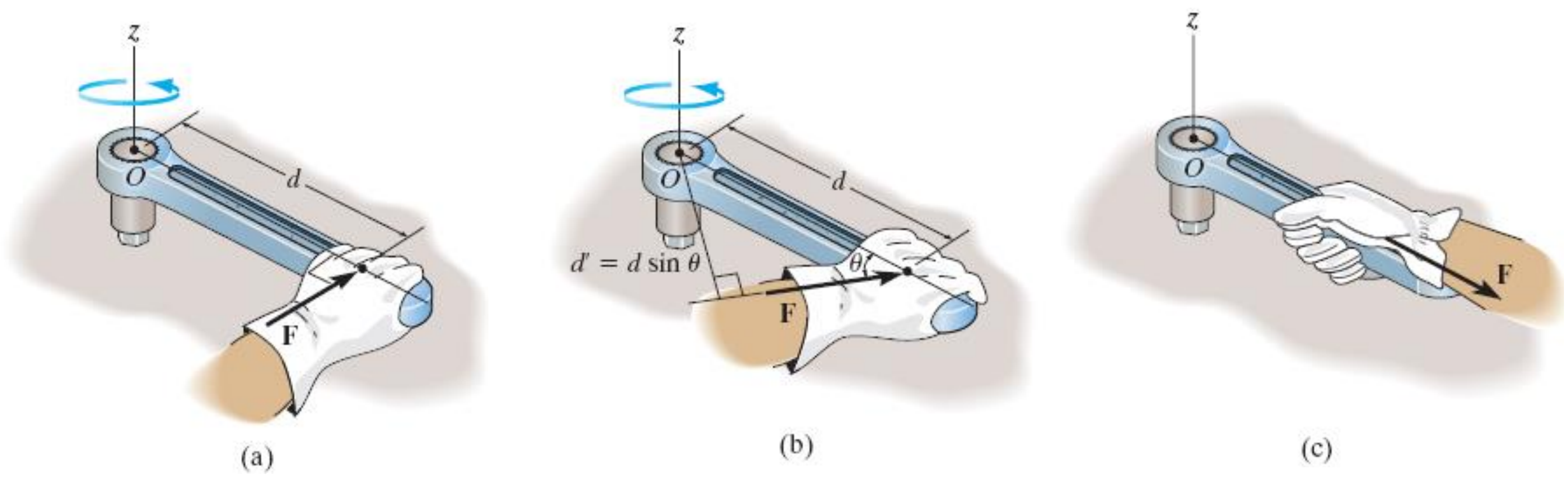
- To discuss the concept of moment of a force in two and three dimensions
- To define the moment of a couple.
- To present methods for determining the resultants of non-concurrent force systems
- To indicate how to reduce a simple distributed loading to a resultant force having a specified location

Chapter Outline

1. Moment of a Force – Scalar Formation
2. Cross Product
3. Moment of Force – Vector Formulation
4. Principle of Moments
5. Moment of a Couple
6. Simplification of a Force and Couple System
7. Reduction of a Simple Distributed Loading

4.1 Moment of a Force – Scalar Formation

- *Moment* of a force about a point or axis – a measure of the tendency of the force to cause a body to rotate about the point or axis
- Torque – tendency of rotation caused by $\mathbf{F}_x$ or simple moment $(\mathbf{M}_o)_z$



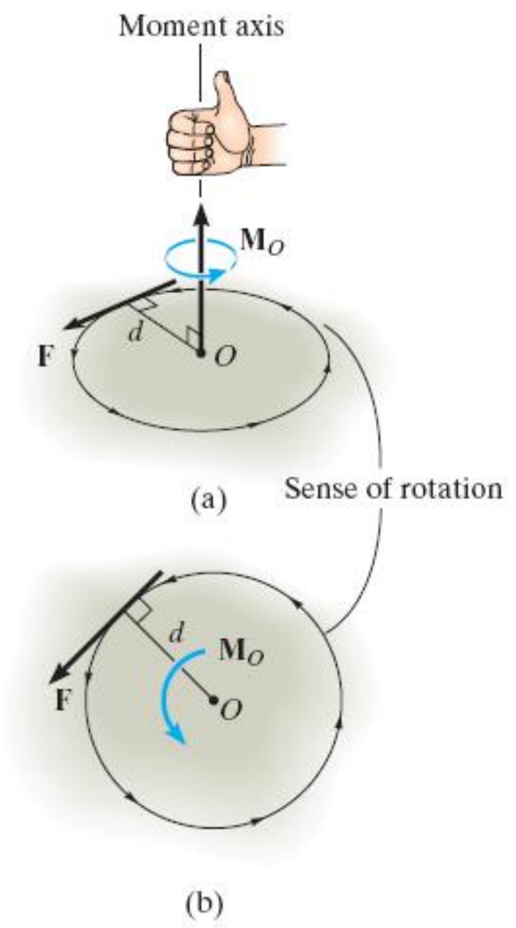
4.1 Moment of a Force – Scalar Formation

Magnitude

- For magnitude of $\mathbf{M}_O$,

$$\mathbf{M}_O = Fd \ (Nm)$$

where d = perpendicular distance from O to its line of action of force



Direction

- Direction using "right hand rule"

4.1 Moment of a Force – Scalar Formation

Sign Convection

If the direction of moment is **Clockwise** the magnitude of moment is **negative**

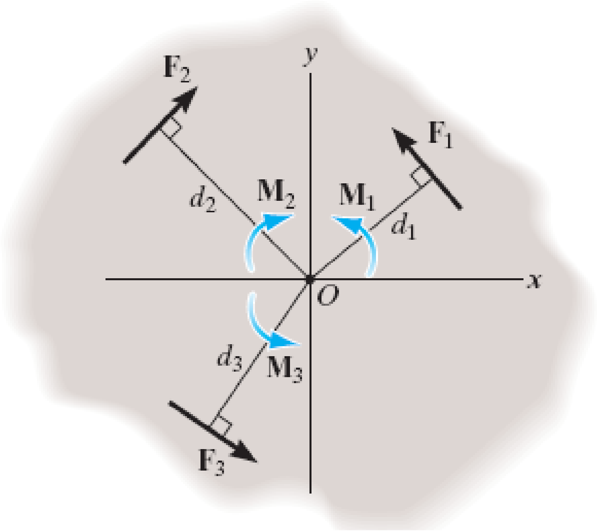
If the direction of moment is **Anti-Clockwise** the magnitude of moment is **positive**

4.1 Moment of a Force – Scalar Formation

Resultant Moment

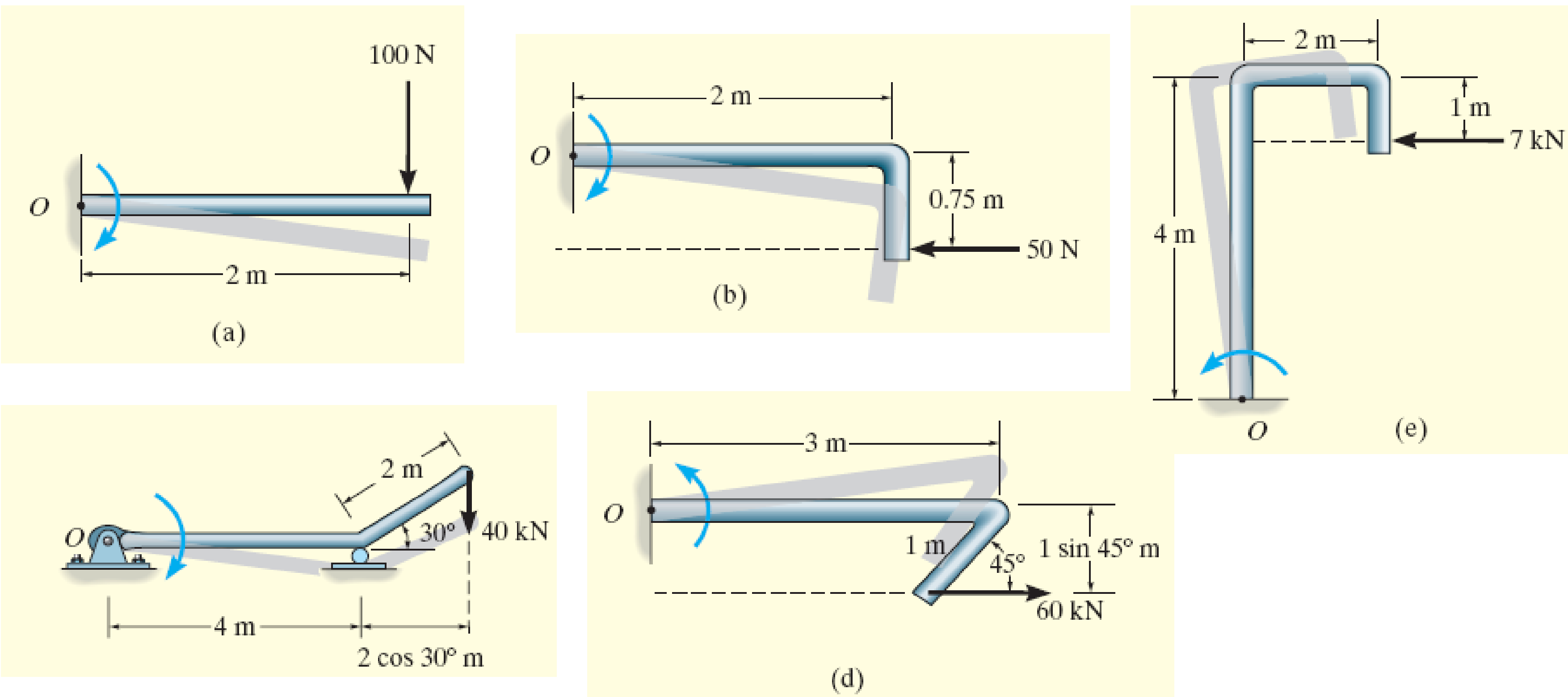
- Resultant moment, $\mathbf{M}_{Ro}$ = moments of all the forces

$$\mathbf{M}_{Ro} = \sum Fd$$



Example 4.1

For each case, determine the moment of the force about point **O**.



Solution

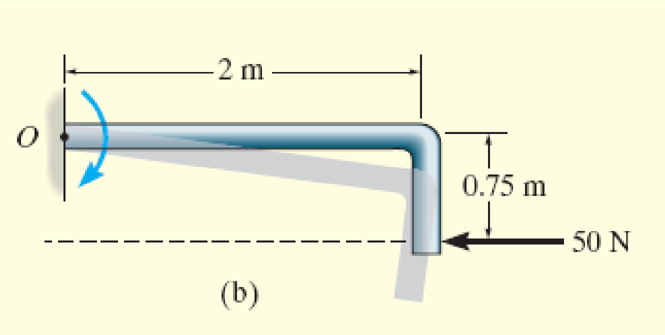
Line of action is extended as a dashed line to establish moment arm **d**.

Tendency to rotate is indicated and the orbit is shown as a colored curl.

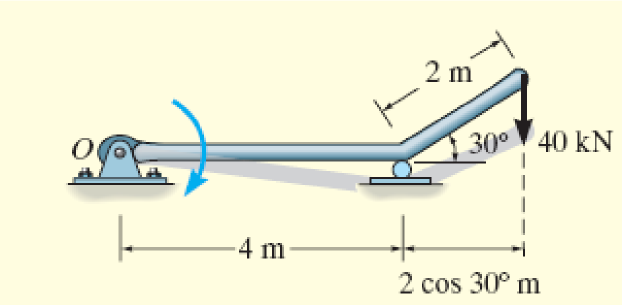
$$(a) M_o = (100N)(2m) = 200N.m(CW)$$

Solution

$$(b) M_o = (50N)(0.75m) = 37.5 N.m (CW)$$

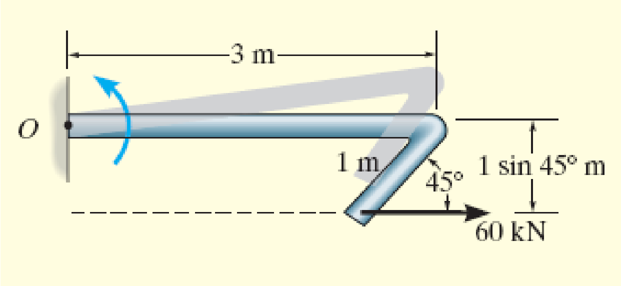


$$(c) M_o = (40N)(4m + 2\cos 30^\circ m) = 229 N.m (CW)$$



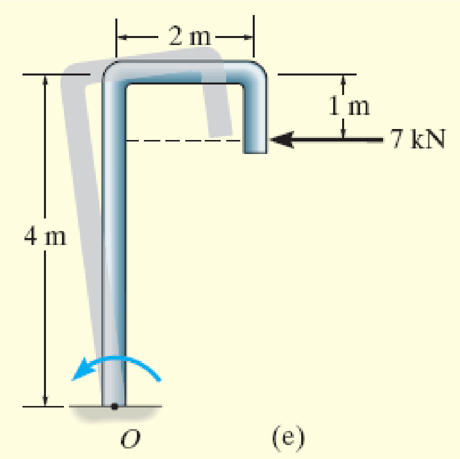
Solution

$$(d)M_o = (60N)(1\sin 45^{\circ} m) = 42.4N.m(CCW)$$



(d)

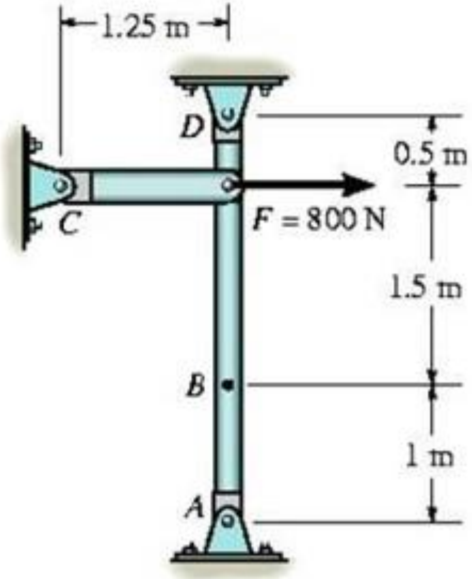
$$(e)M_o = (7kN)(4m - 1m) = 21.0kN.m(CCW)$$



(e)

Example 4.2

Determine the moments of the 800 N force acting on the frame about points A, B, C and D.



Solution

Scalar Analysis

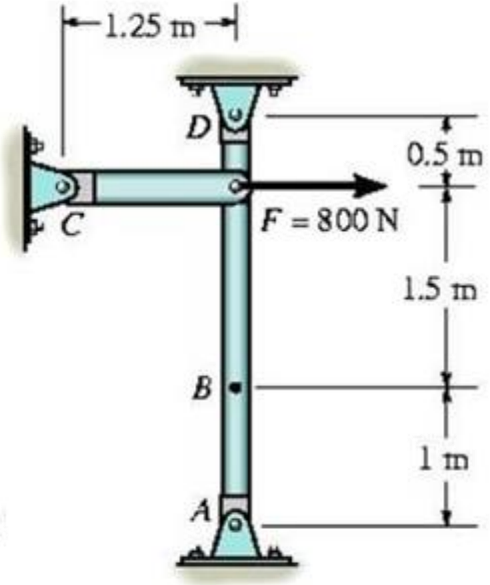
$M_A = (2.5m)(800N) = 2000\ N.m\ (CW)$

$M_B = (1.5m)(800N) = 1200\ N.m\ (CW)$

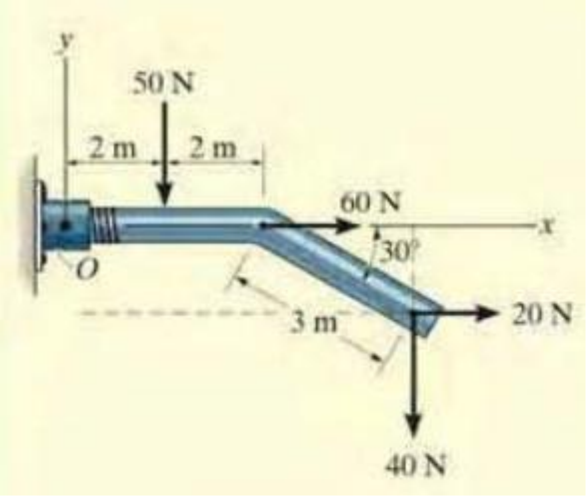
$M_C = (0m)(800N) = 0\ N.m$

Line of action of **F** passes through C

$M_D = (0.5m)(800N) = 400\ N.m\ (CCW)$



Classwork

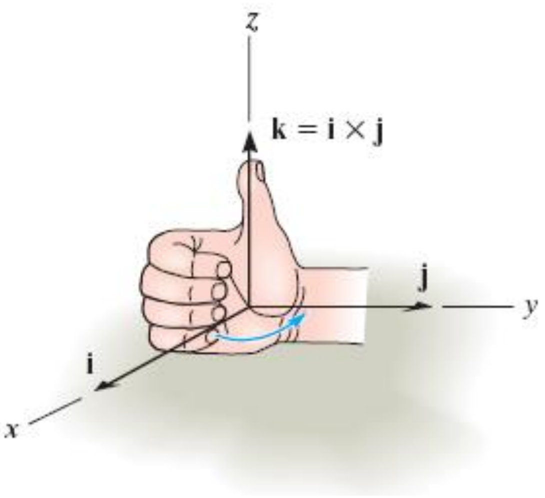


Determine the resultant moment of the four forces acting on the rod shown about point O.

4.2 Cross Product

Cartesian Vector Formulation

$$\vec{A} X \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



4.2 Cross Product

the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

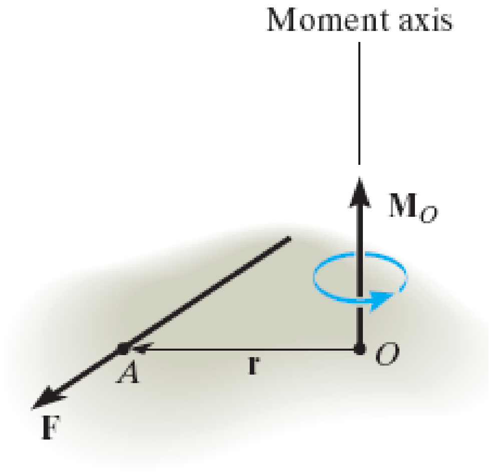
For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

4.3 Moment of Force - Vector Formulation

- Moment of force **F** about point O can be expressed using cross product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$



Magnitude

- For magnitude of cross product,

$$M_O = rF \sin\theta$$

- Treat **r** as a sliding vector. Since $d = r \sin\theta$,

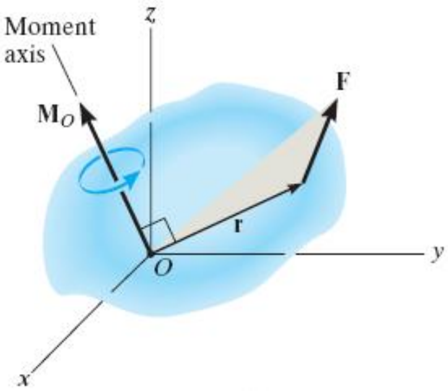
$$M_O = rF \sin\theta = F\,(r\sin\theta) = Fd$$

4.3 Moment of Force - Vector Formulation

Cartesian Vector Formulation

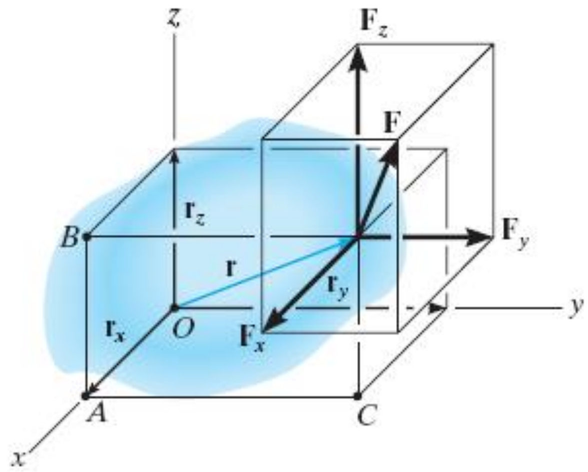
- For force expressed in Cartesian form,

$$\vec{M}_O = \vec{r} X \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



- With the determinant expended,

$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

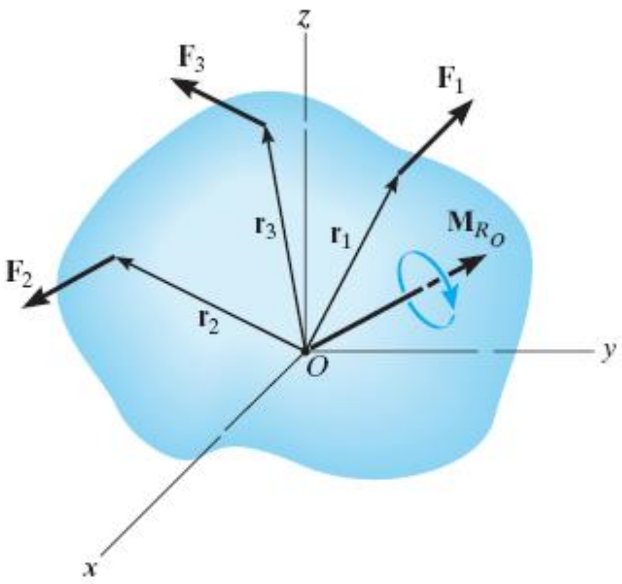


4.3 Moment of Force - Vector Formulation

Resultant Moment of a System of Forces

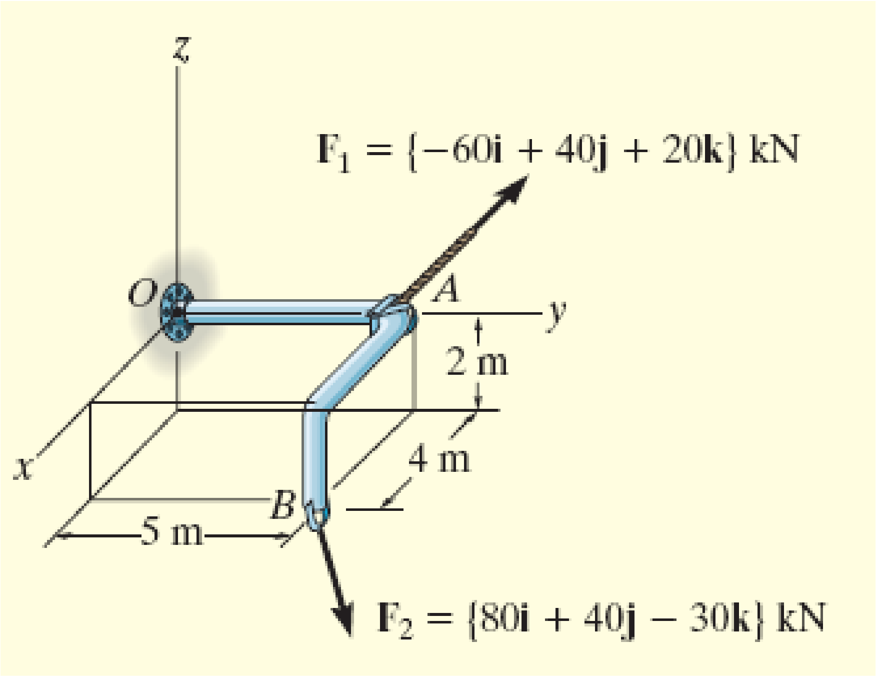
- Resultant moment of forces about point O can be determined by vector addition

$$\mathbf{M}_{Ro} = \sum(\mathbf{r} \times \mathbf{F})$$



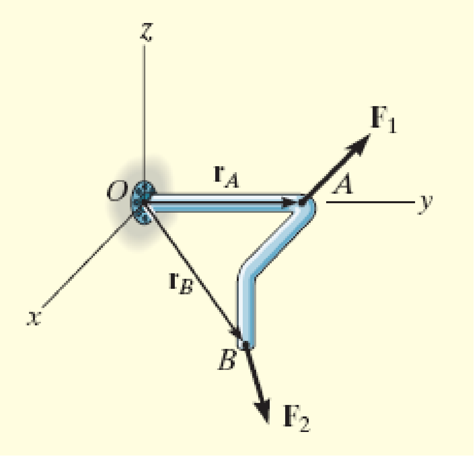
Example 4.4

Two forces act on the rod. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



Solution

Position vectors are directed from point O to each force as shown.



These vectors are

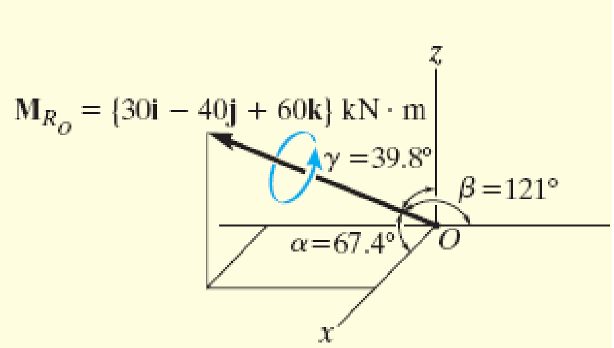
$$r_A = \{5j\}\,\text{m}$$
$$r_B = \{4i + 5j - 2k\}\,\text{m}$$

The resultant moment about O is

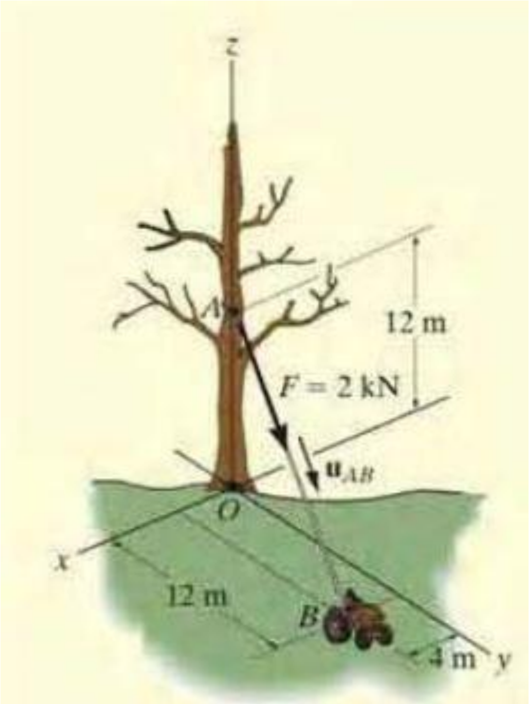
$$\vec{M}_O = \sum(r \times F) = r_A \times F + r_B \times F$$

$$= \begin{vmatrix} i & j & k \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= \{30i - 40j + 60k\}\,\text{kN}\cdot\text{m}$$



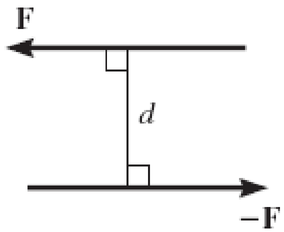
Classwork



Determine the moment produced by the force F about point O. Express the result as a Cartesian vector.

4.5 Moment of a Couple

- Couple
  - two parallel forces
  - same magnitude but opposite direction
  - separated by perpendicular distance d
- Resultant force = 0
- Tendency to rotate in specified direction
- Couple moment = sum of moments of both couple forces about any arbitrary point



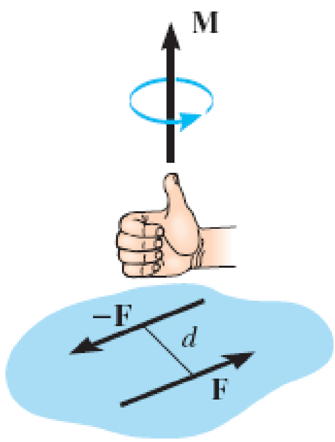
4.6 Moment of a Couple

Scalar Formulation

- Magnitude of couple moment

$$M = Fd$$

- Direction and sense are determined by right hand rule
- **M** acts perpendicular to plane containing the forces



4.6 Moment of a Couple

Vector Formulation

- For couple moment,

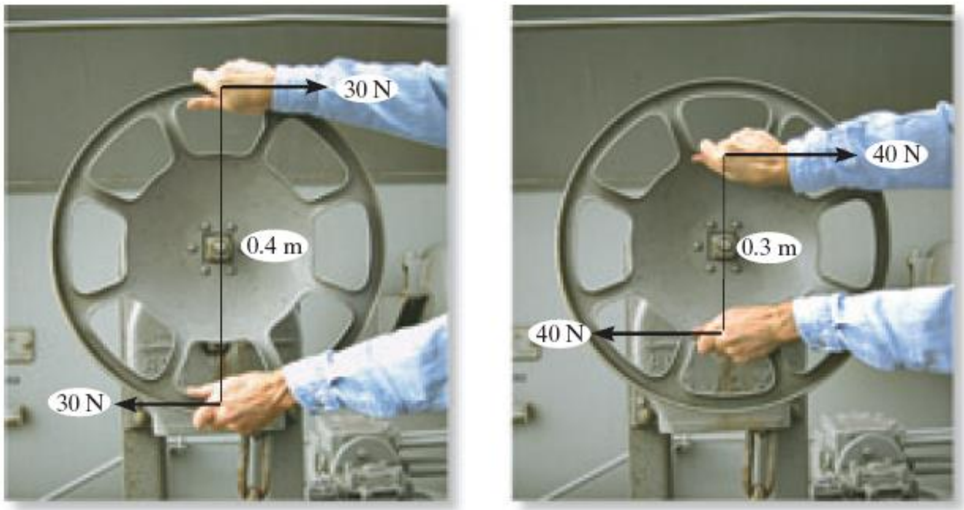
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- If moments are taken about point A, moment of –**F** is zero about this point
- **r** is crossed with the force to which it is directed

4.6 Moment of a Couple

Equivalent Couples

- 2 couples are equivalent if they produce the same moment
- Forces of equal couples lie on the same plane or plane parallel to one another



4.6 Moment of a Couple

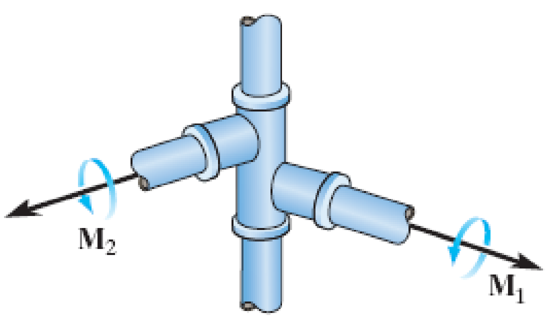
Resultant Couple Moment

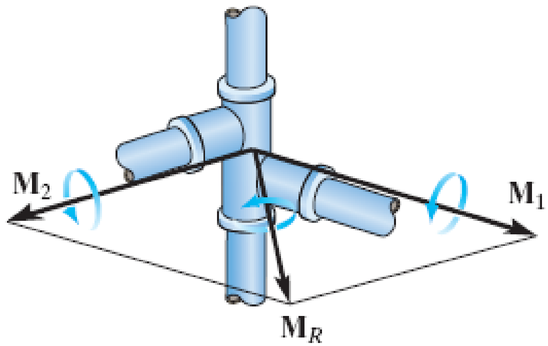
- Couple moments are free vectors and may be applied to any point P and added vectorially
- For resultant moment of two couples at point P,

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

- For more than 2 moments,

$$\mathbf{M}_R = \sum(\mathbf{r} \times \mathbf{F})$$





4.7 Simplification of a Force and Couple System

- An equivalent system is when the *external effects* are the same as those caused by the original force and couple moment system
- External effects of a system is the *translating and rotating motion* of the body
- Or refers to the *reactive forces* at the supports if the body is held fixed

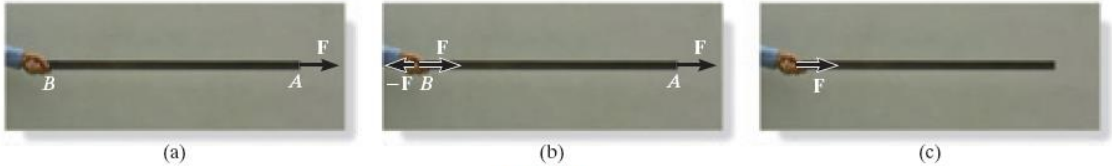
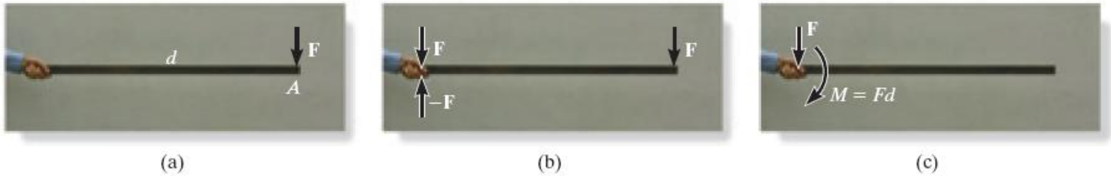


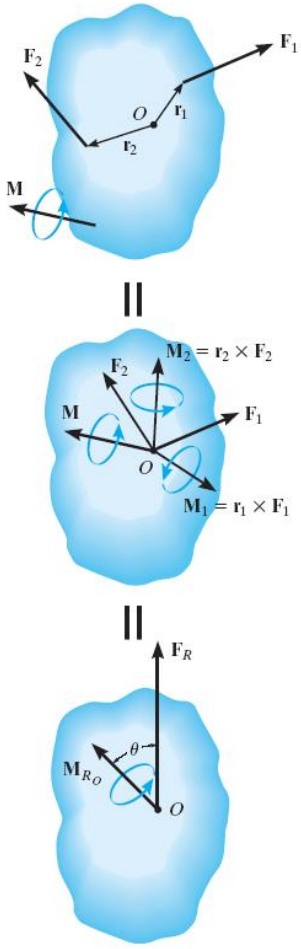
Fig. 4–34



4.7 Simplification of a Force and Couple System

- Equivalent resultant force acting at point *O* and a resultant couple moment is expressed as

$$F_R = \sum F$$
$$(M_R)_O = \sum M_O + \sum M$$



- If force system lies in the *x*–*y* plane and couple moments are perpendicular to this plane,

$$(F_R)_x = \sum F_x$$
$$(F_R)_y = \sum F_y$$
$$(M_R)_O = \sum M_O + \sum M$$

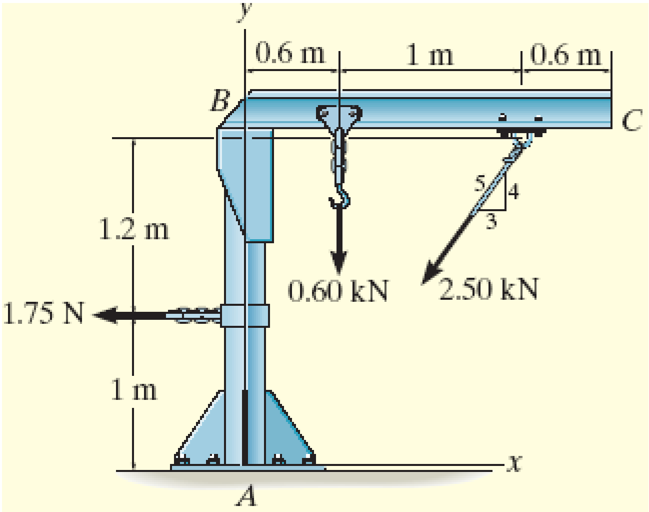
4.7 Simplification of a Force and Couple System

Procedure for Analysis

1. Establish the coordinate axes with the origin located at point *O* and the axes having a selected orientation
2. Force Summation
3. Moment Summation

Example 4.18

The jib crane is subjected to three coplanar forces. Replace this loading with an *equivalent resultant force from* column AB and boom BC.



Solution

Force Summation

$$+\rightarrow F_{Rx} = \Sigma F_x;$$

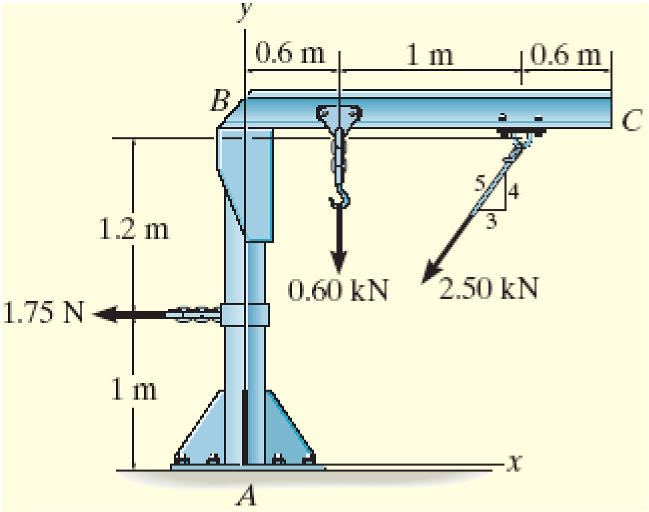
$$F_{Rx} = -2.5kN\left(\frac{3}{5}\right) - 1.75kN$$

$$= -3.25kN = 3.25kN \leftarrow$$

$$+\rightarrow F_{Ry} = \Sigma F_y;$$

$$F_{Ry} = -2.5N\left(\frac{4}{5}\right) - 0.6kN$$

$$= -2.60kN = 2.60N \downarrow$$

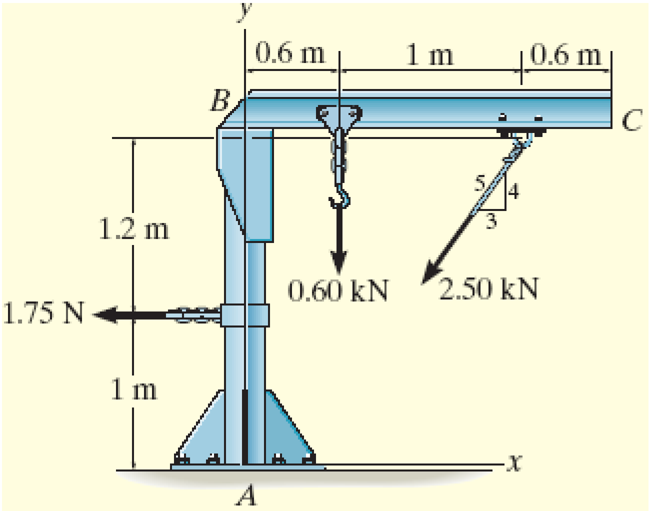


Solution

For magnitude of resultant force,

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(3.25)^2 + (2.60)^2}$$

$$= 4.16kN$$



For the direction of the resultant force,

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{2.60}{3.25}\right)$$

$$= 38.7^\circ$$

Solution

Moment Summation

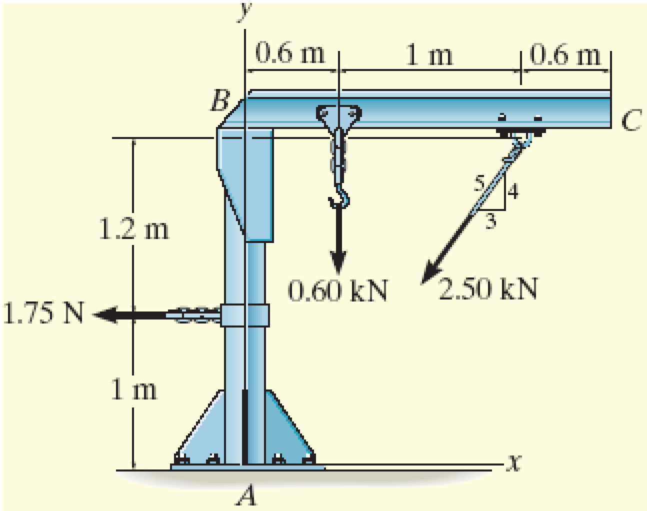
➔ Summation of moments about point A,

$$M_{RA} = \Sigma M_A;$$

$$= 1.75kn(1m) - 0.6kN(0.6m)$$

$$+ 2.50kN\left(\frac{3}{5}\right)(2.2m) - 2.50kN\left(\frac{4}{5}\right)(1.6m)$$

$$= -1.8 \text{ kN.m}$$

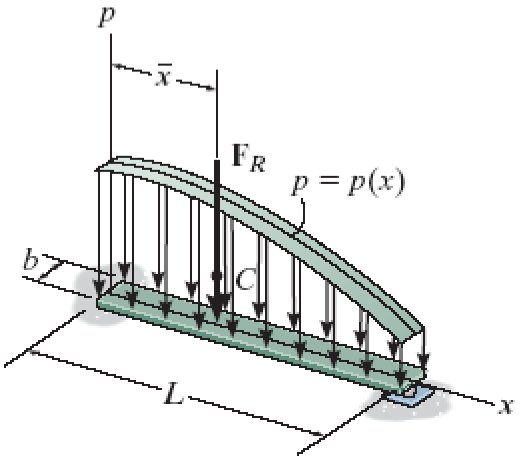


4.8 Reduction of a Simple Distributed Loading

- Large surface area of a body may be subjected to distributed loadings
- Loadings on the surface is defined as pressure
- Pressure is measured in Pascal (Pa): 1 Pa = 1N/m^2

Uniform Loading Along a Single Axis

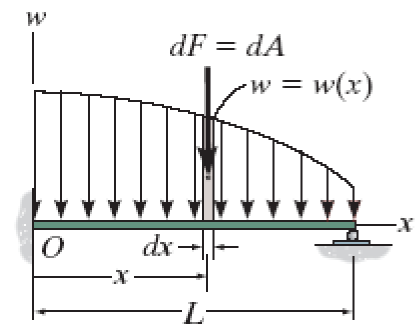
- Most common type of distributed loading is uniform along a single axis



4.8 Reduction of a Simple Distributed Loading

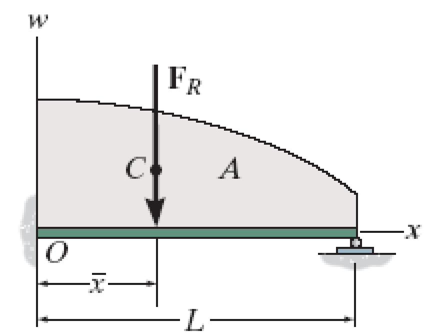
Magnitude of Resultant Force

- Magnitude of $d\mathbf{F}$ is determined from differential *area dA* under the loading curve.
- For length *L*,



$$F_R = \int_L w(x)\,dx = \int_A dA = A$$

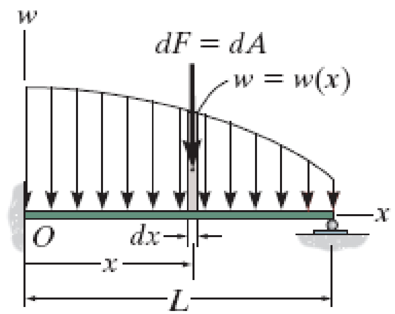
- *Magnitude of the resultant force is equal to the total area A under the loading diagram*.



4.8 Reduction of a Simple Distributed Loading

Location of Resultant Force

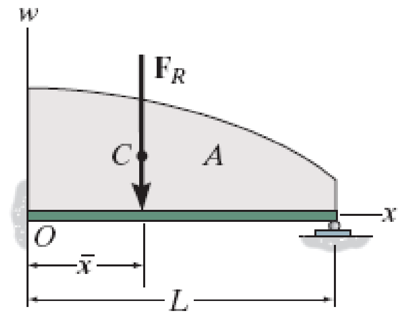
- $M_R = \sum M_O$
- $d\mathbf{F}$ produces a moment of $xdF = x\,w(x)\,dx$ about O
- For the entire plate,



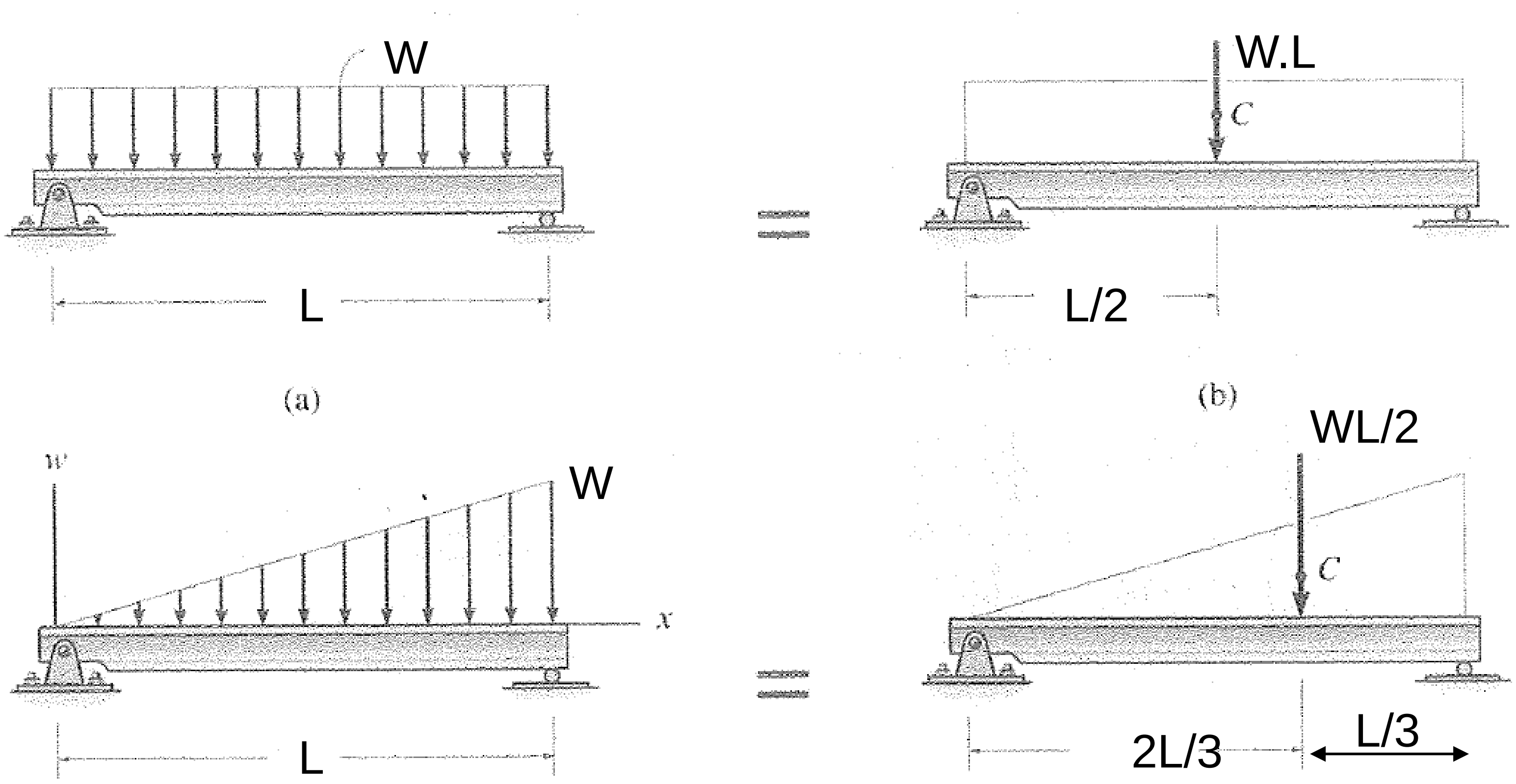
$$M_{Ro} = \Sigma M_O \qquad \bar{x}F_R = \int_L x w(x) dx$$

- Solving for $\bar{x}$

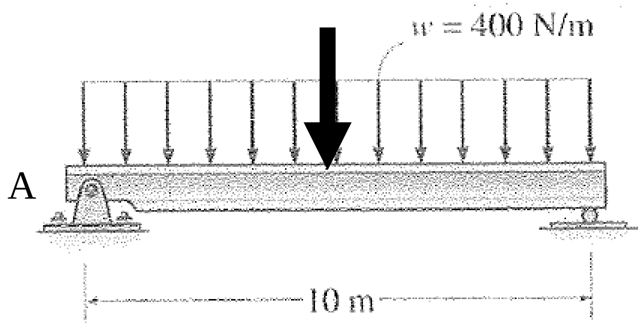
$$\bar{x} = \frac{\int_L x w(x) dx}{\int_L w(x) dx} = \frac{\int_A x dA}{\int_A dA}$$

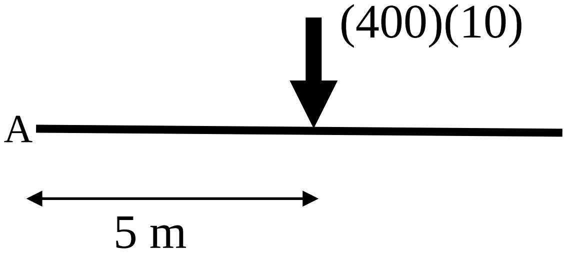


4.8 Reduction of a Simple Distributed Loading



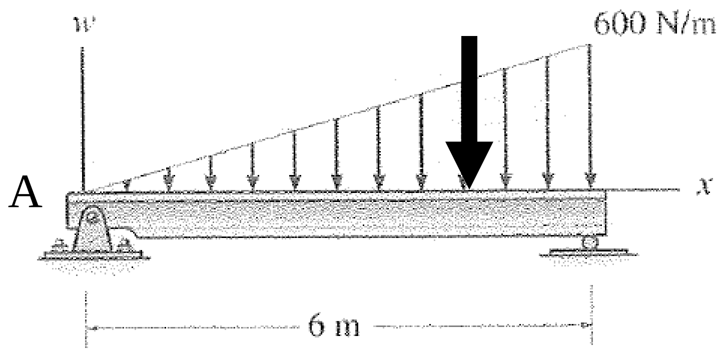
Find the resultant moment at point A

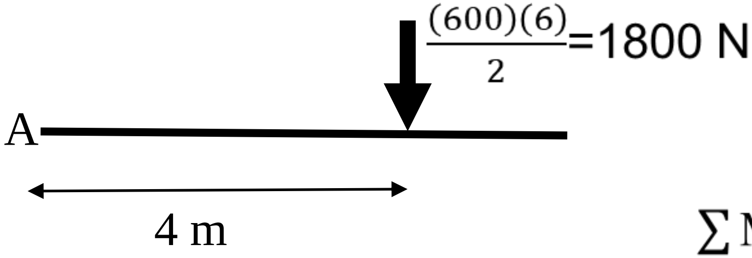




$\sum MA = -(400)(10)(5) = -20000$ N or -20 kN

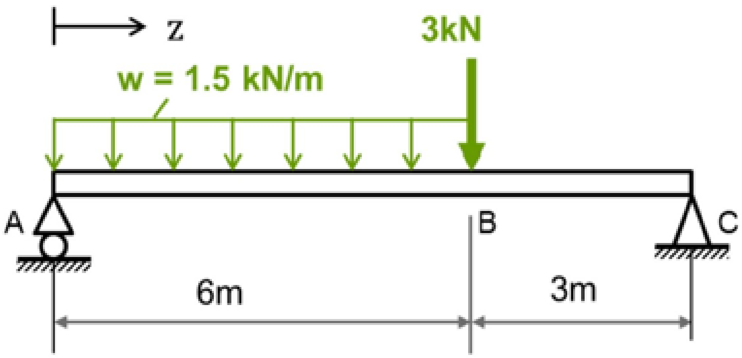
Find the resultant moment at point A



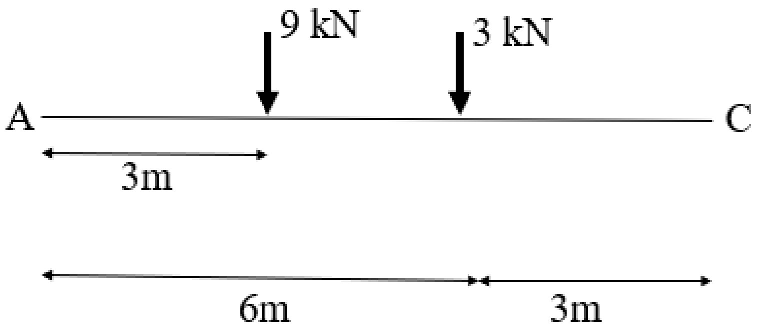


$\sum M_A$=(1800) (4) = 7200 N.m

Find the resultant moment at points A and C

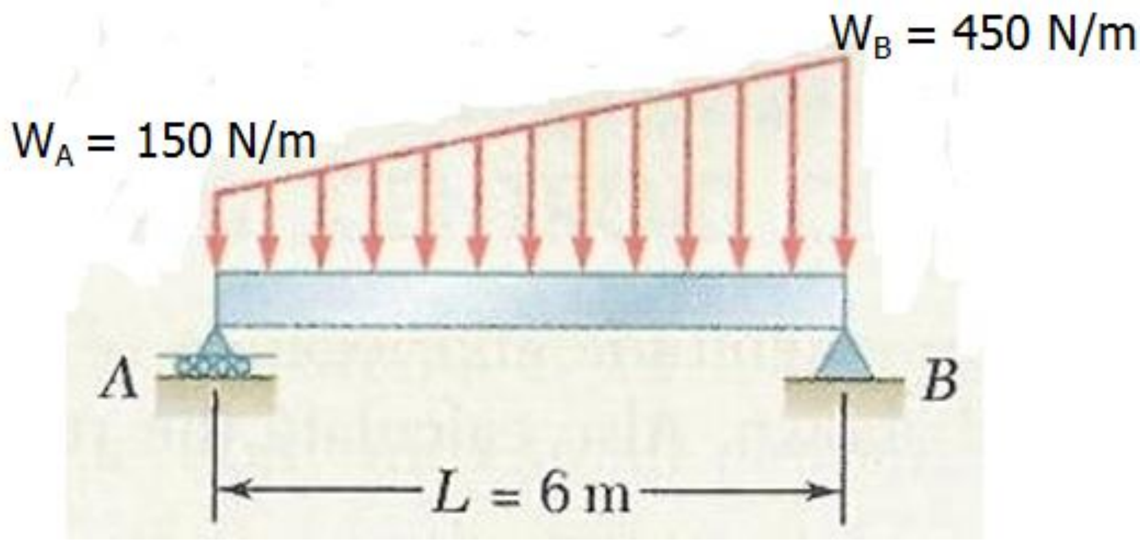


$\sum M_A = -(9)(3) - (3)(6) = -45$ kN.m CW



$\sum M_C = (9)(6) + (3)(3) = 63$ kN.m CCW

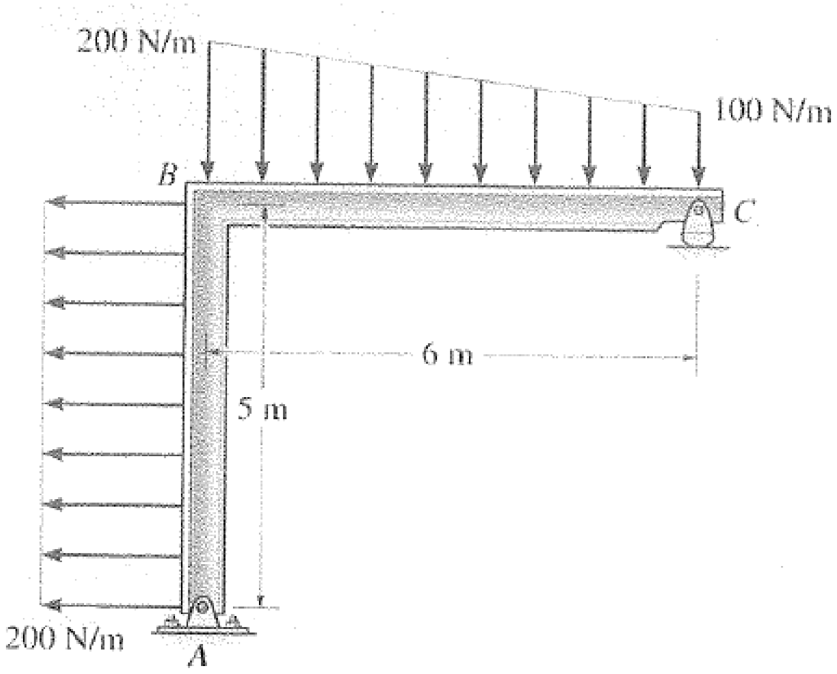
Find the resultant moment at point A



$\sum M_A$= - (150)(6)(3) – ((300)(6)/2) (4)

= -2700 – 3600 = -6300 N or 6.3 kN.m

Find the resultant moment at point A



$\sum M_A = (200)(5)(2.5) - (100)(6)(3)$

$- ((100)(6)/2)\,(2)$

$= 2500 - 1800 - 600 = 100$ N.m