Engineering Mechanics: Statics in SI Units, 12e

3 Equilibrium of a Particle

Chapter Objectives

- To introduce the concept of the free-body diagram for a particle
- To show how to solve particle equilibrium problems using the equations of equilibrium

Chapter Outline

1. Condition for the Equilibrium of a Particle
2. The Free-Body Diagram
3. Coplanar Systems
4. Three-Dimensional Force Systems

3.1 Condition for the Equilibrium of a Particle

- Particle at *equilibrium* if

  - At rest

  - Moving at constant a constant velocity

- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle

3.1 Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

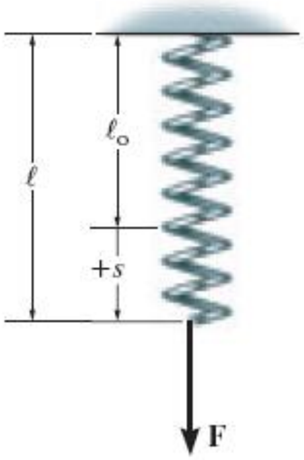
3.2 The Free-Body Diagram

- Best representation of all the unknown forces ($\sum \mathbf{F}$) which acts on a body
- A sketch showing the particle “free” from the surroundings with all the forces acting on it
- Consider three common connections in this subject –
  – Spring
  – Cables and Pulleys
  – Weight

3.2 The Free-Body Diagram

- Spring
  - Linear elastic spring: change in length is directly proportional to the force acting on it
  - *spring constant or stiffness k*: defines the elasticity of the spring
  - Magnitude of force when spring is elongated or compressed

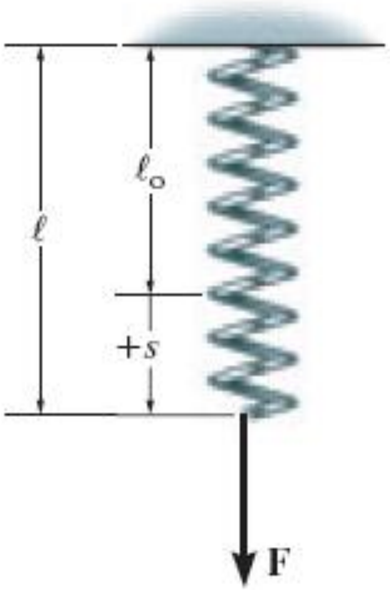
    ➔ $F = ks$



3.2 The Free-Body Diagram

- Spring

$$F = k\,s$$



F: spring force

k: stiffness constant

s: elongation of spring

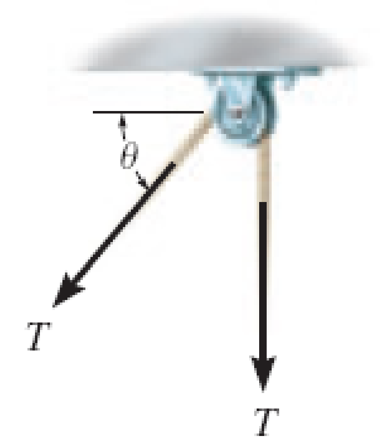
$s = L - L_0$

L: deformed length

L_0:undeformed length

3.2 The Free-Body Diagram

- Cables and Pulley
  - Cables (or cords) are assumed to have negligible weight and cannot stretch
  - **Tension** always acts in the direction of the cable
  - Tension force must have a constant magnitude for equilibrium
  - For any angle θ, the cable is subjected to a constant tension *T*



Cable is in tension

3.2 The Free-Body Diagram

- The bucket is held in equilibrium by the cable
- Force in the cable = weight of the bucket
- Isolate the bucket for FBD
- Two forces acting on the bucket, weight **W** and force **T** of the cable
- Resultant of forces = 0

$$\mathbf{W} = \mathbf{T}$$

3.2 The Free-Body Diagram

Procedure for Drawing a FBD

1. Draw outlined shape
2. Show all the forces
   - Active forces: particle in motion
   - Reactive forces: constraints that prevent motion
3. Identify each forces
   - Known forces with proper magnitude and direction
   - Letters used to represent magnitude and directions

3.2.1 Summary of Free-body Diagram

1)Draw the outline of the shape

2)Show all forces:

a) Active forces
b) Reactive forces

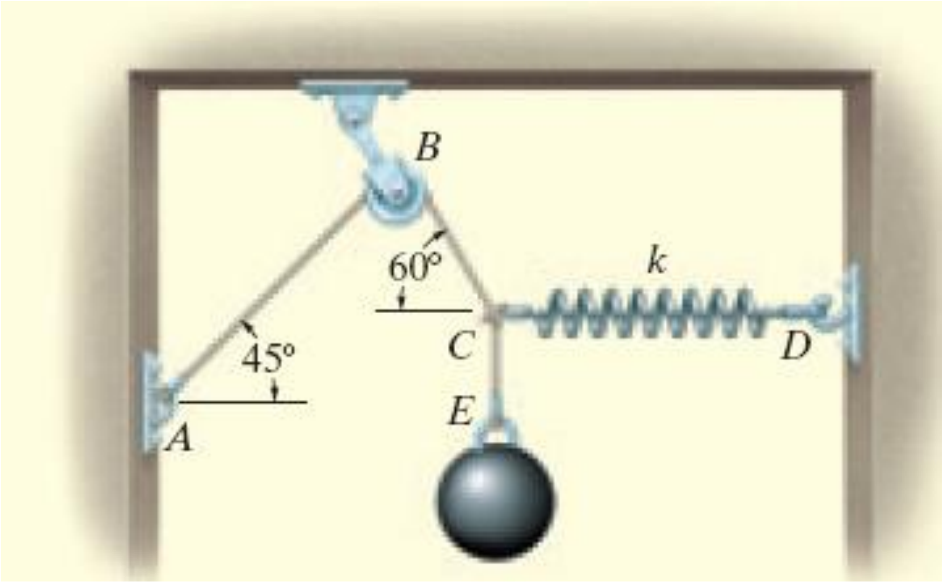
3) Apply the connections:

a) Spring
b) Cables and pulley
c) Weight

Replace them with forces

Example 3.1

The sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere at knot C.

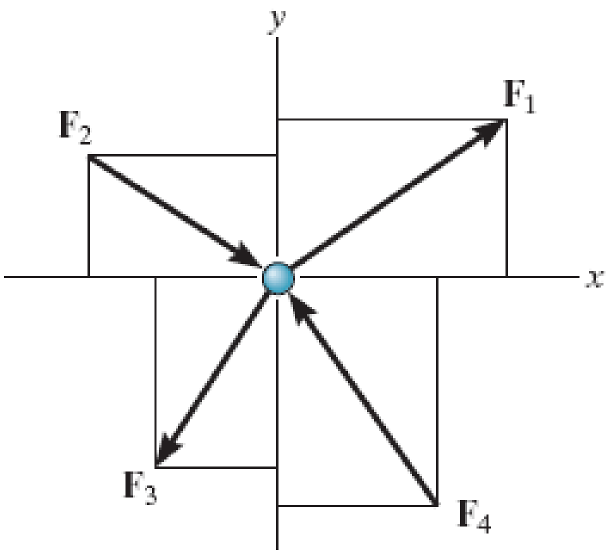


3.3 Coplanar Systems

- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

$$\sum \mathbf{F}_x = 0$$
$$\sum \mathbf{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero



3.3 Coplanar Systems

- Procedure for Analysis

1. Free-Body Diagram

   - Establish the x, y axes

   - Label all the unknown and known forces

2. Equations of Equilibrium

   - Apply $F = ks$ to find spring force

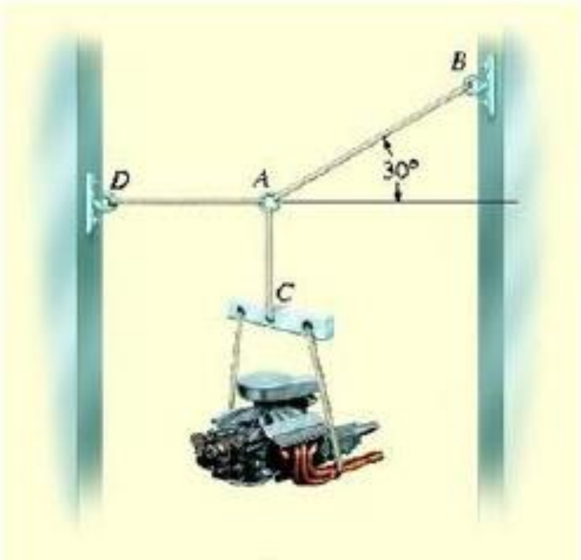
   - When negative result force is the reserve

   - Apply the equations of equilibrium

$$\sum \mathbf{F}_x = 0 \qquad \sum \mathbf{F}_y = 0$$

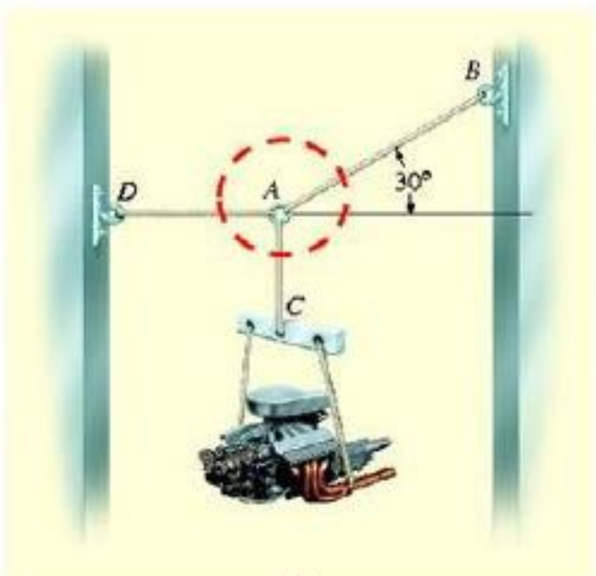
Example 3.2

Determine the tension in cables AB and AD for equilibrium of the 250kg engine.

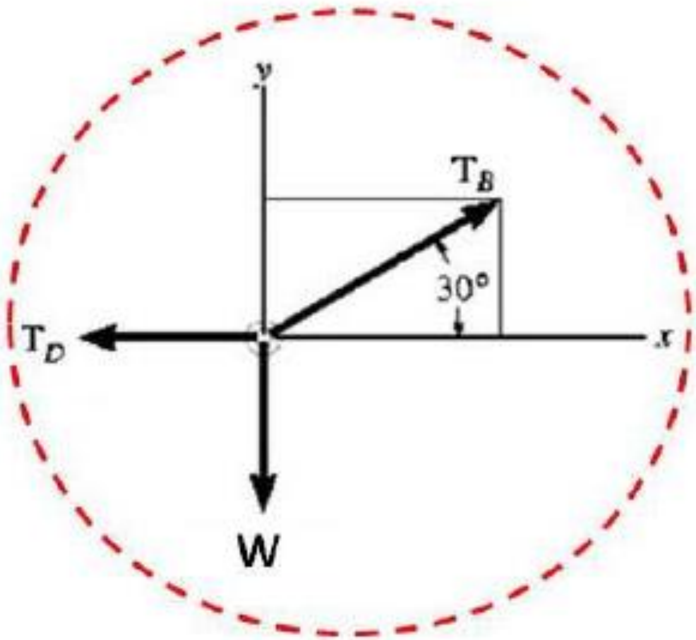


Solution

Free-body diagram (FBD)

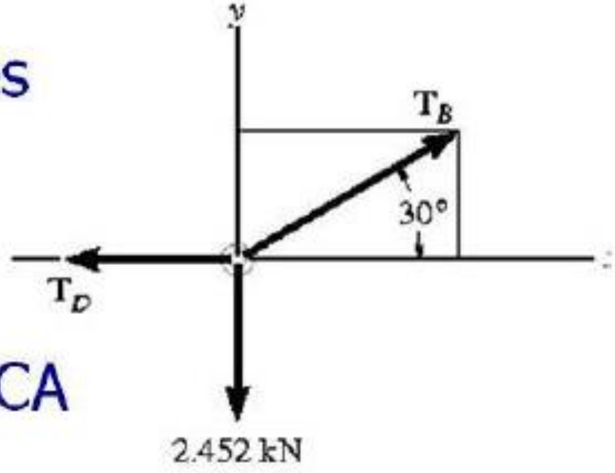


(a)



FBD at Point A

- Initially, two forces acting, forces of cables AB and AD
- Engine Weight [W=m.g]
  $= (250kg)(9.81m/s^2)$
  $= 2.452$ kN supported by cable CA



FBD of the ring A

- Finally, three forces acting, forces $\mathbf{T}_B$ and $\mathbf{T}_D$ and engine weight on cable CA

Solution

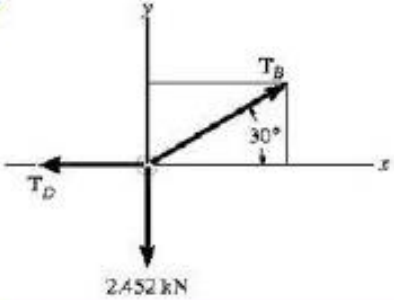
$+\rightarrow \quad \Sigma \mathbf{F}_x = 0;$ $T_B \cos 30^\circ - T_D = 0$

$+\uparrow \quad \Sigma \mathbf{F}_y = 0;$ $T_B \sin 30^\circ - 2.452 = 0$

Solving,

$T_B = 4.904$ kN

$T_D = 4.247$ kN

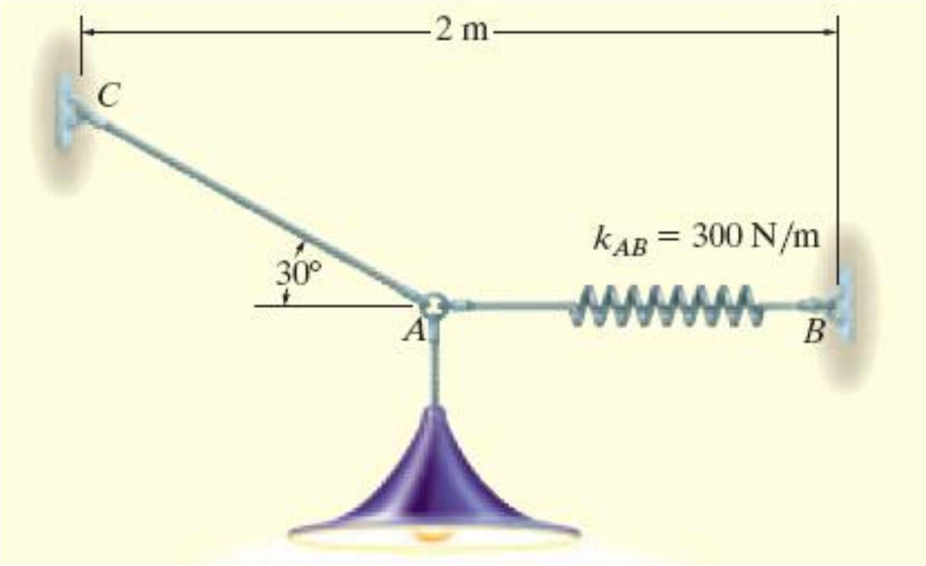


FBD of the ring A

*Note: Neglect the weights of the cables since they are small compared to the weight of the engine

Example 3.3

Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is $l'_{AB} = 0.4$m, and the spring has a stiffness of $k_{AB} = 300$N/m.



Solution

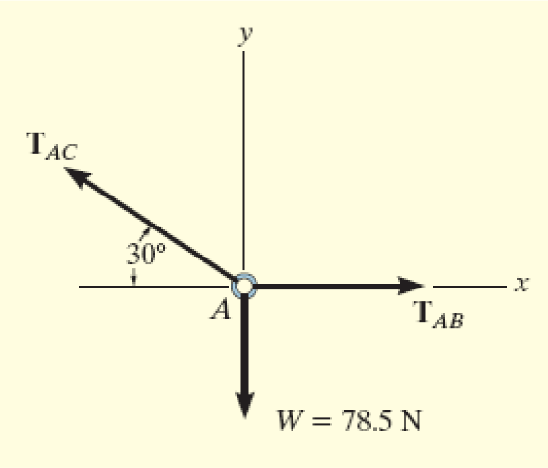
<u>FBD at Point A</u>

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by $F = ks$.

$+\rightarrow \quad \sum \mathbf{F}_x = 0; \quad T_{AB} - T_{AC}\cos 30^{\circ} = 0$

$+\uparrow \quad \sum \mathbf{F}_y = 0; \quad T_{AC}\sin 30^{\circ} - 78.5N = 0$



Solving,

$T_{AC} = 157.0\text{kN}$

$T_{AB} = 136.0\text{kN}$

Solution

$T_{AB} = k_{AB}s_{AB}$; $136.0N = 300\text{N/m}(s_{AB})$

$s_{AB} = 0.453\text{m}$ (stretch of the spring)

For stretched length,

$l_{AB} = l'_{AB} + s_{AB}$

$l_{AB} = 0.4\text{m} + 0.453\text{m}$

$= 0.853\text{m}$

For horizontal distance BC,

$2\text{m} = l_{AC}\cos 30^\circ + 0.853\text{m}$

$l_{AC} = 1.32\text{m}$

3.4 Three-Dimensional Force Systems

- For particle equilibrium

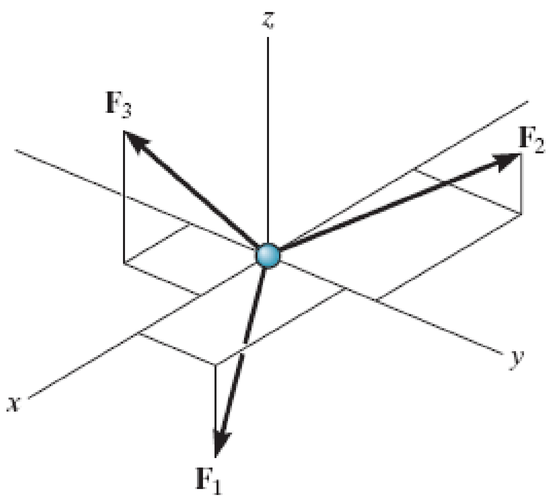
$$\sum \mathbf{F} = 0$$

- Resolving into **i**, **j**, **k** components

$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0$$

- Three scalar equations representing algebraic sums of the x, y, z forces

$$\sum F_x \mathbf{i} = 0$$
$$\sum F_y \mathbf{j} = 0$$
$$\sum F_z \mathbf{k} = 0$$



3.4 Three-Dimensional Force Systems

- Procedure for Analysis

Free-body Diagram

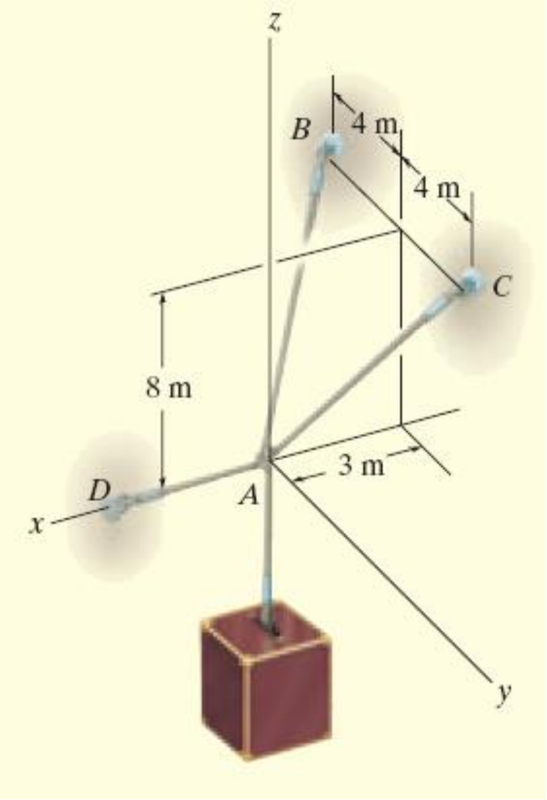
- Establish the x, y, z axes
- Label all known and unknown force

Equations of Equilibrium

- Apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$
- Substitute vectors into $\sum \mathbf{F} = 0$ and set **i**, **j**, **k** components = 0
- Negative results indicate that the sense of the force is opposite to that shown in the FBD.

Example 3.4

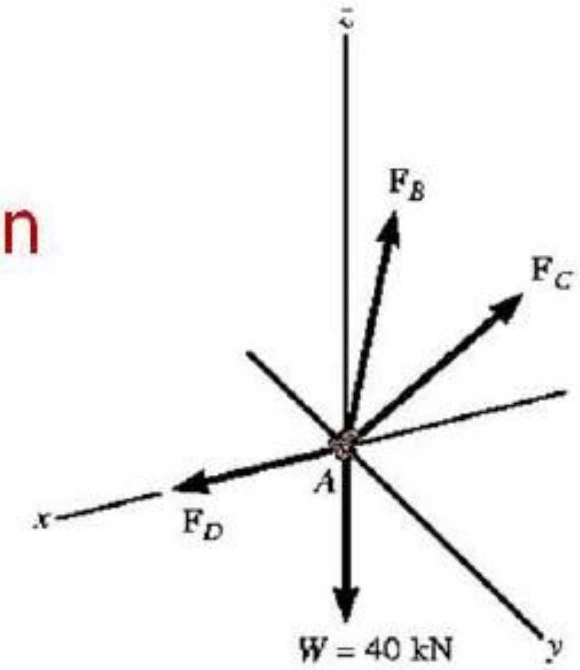
Determine the force developed in each cable used to support the 40kN crate.



Solution

FBD at Point A

To expose all three unknown forces in the cables



FBD at Point A

Solution

- To determine the Cartesian vectors first the unit vectors of the two forces T_B and T_C should be calculated. Hence the coordinates of A = {0, 0, 0}

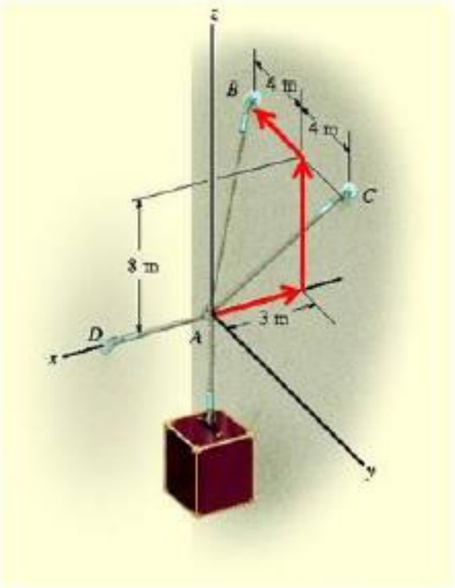
  B = {-3, -4, 8}

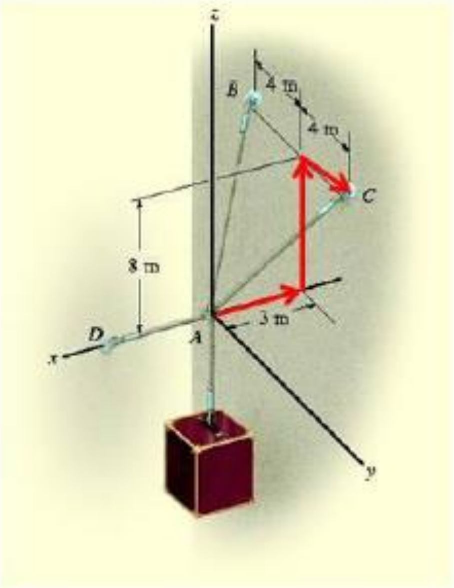
  C = {-3, 4, 8} were

  determined.

Solution

Coordinates A = {0, 0, 0} B = {-3, -4, 8} C = {-3, 4, 8}





$$\vec{r}_B = -3\vec{i} - 4\vec{j} + 8\vec{k}$$

$$\vec{r}_C = -3\vec{i} + 4\vec{j} + 8\vec{k}$$

Solution

- The unit vectors are:
- $u_{AB} = (\mathbf{r}_B / r_B) = \dfrac{-3i - 4j + 8k}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}} = -\dfrac{3}{9.434}i - \dfrac{4}{9.434}j + \dfrac{8}{9.434}k$
- $u_{AC} = (\mathbf{r}_C / r_C) = \dfrac{-3i + 4j + 8k}{\sqrt{(-3)^2 + (4)^2 + (8)^2}} = -\dfrac{3}{9.434}i + \dfrac{4}{9.434}j + \dfrac{8}{9.434}k$

Solution

Equations of Equilibrium

Expressing each forces in Cartesian vectors,

$$\mathbf{F}_B = F_B(\mathbf{r}_B / r_B)$$
$$= -0.318F_B\,\mathbf{i} - 0.424F_B\,\mathbf{j} + 0.848F_B\,\mathbf{k}$$
$$\mathbf{F}_C = F_C(\mathbf{r}_C / r_C)$$
$$= -0.318F_C\,\mathbf{i} + 0.424F_C\,\mathbf{j} + 0.848F_C\,\mathbf{k}$$
$$\mathbf{F}_D = F_D\,\mathbf{i}$$
$$\mathbf{W} = -40\,\mathbf{k}$$

Solution

For equilibrium,

$$\Sigma \mathbf{F} = 0; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0$$

$$-0.318F_B\,\mathbf{i} - 0.424F_B\,\mathbf{j} + 0.848F_B\,\mathbf{k} - 0.318F_C\,\mathbf{i}$$
$$+ 0.424F_C\,\mathbf{j} + 0.848F_C\,\mathbf{k} + F_D\,\mathbf{i} - 40\,\mathbf{k} = 0$$

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0$$

$$\Sigma F_y = 0; \qquad -0.424F_B \; - 0.424F_C = 0$$

$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0$$

Solution

Solving,

$F_B = F_C = 23.565 \text{ kN}$

$F_D = 15.0 \text{ kN}$