

Sigma Notation: Mean and Variance

Video companion

1 Introduction

Important equations for this video:

$$\begin{aligned} X &= \{x_1, \dots, x_n\} \\ \mu_x &= \frac{1}{n} \sum_{i=1}^n x_i \\ \sigma_x^2 &= \frac{1}{n} \left[\sum_{i=1}^n (x_i - \mu_x)^2 \right] \end{aligned}$$

The symbol μ_x is the “mean of x ,” and σ_x^2 is the “variance of x .” The standard deviation is denoted σ_x .

2 Mean

Example:

$$\begin{aligned} Z &= \{1, 5, 12\} \\ |Z| &= 3 \\ \mu_z &= \frac{1 + 5 + 12}{3} = \frac{18}{3} = 6 \end{aligned}$$

The mean μ_z is also denoted $\mu(z)$ or simply μ .

Symbolic example:

$$\begin{aligned} Y &= \{y_1, y_2, y_3, y_4\} \\ \mu_y &= \frac{1}{4}(y_1 + y_2 + y_3 + y_4) \\ &= \frac{1}{4} \left(\sum_{i=1}^4 y_i \right) \end{aligned}$$

In general, suppose you have a set

$$X = \{x_1, x_2, \dots, x_n\},$$

then the mean of X is

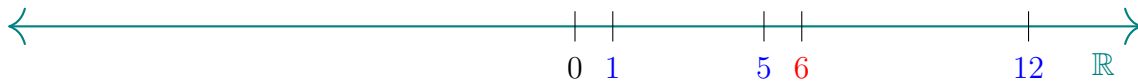
$$\mu_x = \frac{1}{n} \left(\sum_{i=1}^n x_i \right).$$

The variable i is a counter. The variable n is a number, which tells you when to stop counting.

3 Mean centering

$$Z = \{1, 5, 12\}$$

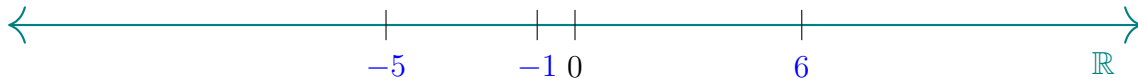
$$\mu_z = 6$$



$$Z' = \{1 - 6, 5 - 6, 12 - 6\}$$

$$= \{-5, -1, 6\}$$

$$\mu_{z'} = 0$$



Mean centering data produces a new data set, which has the same relationships, but the mean is zero.

4 Variance

$$Z = \{1, 5, 12\}$$

$$\mu_z = 6$$

$$W = \{5, 6, 7\}$$

$$\mu_w = 6$$



Set Z (blue) is more “spread out” than set W (olive).

If $X = \{x_1, \dots, x_n\}$, the variance of X is

$$\sigma_x^2 = \frac{1}{n} \left[\sum_{i=1}^n (x_i - \mu_x)^2 \right].$$

The standard deviation is given by

$$\sigma_x = \sqrt{\sigma_x^2}.$$

Z and W have the same mean, but Z is more spread out, so σ_z should be greater than σ_w .

$$\begin{aligned} \sigma_w^2 &= \frac{1}{3} \left[\sum_{i=1}^3 (w_i - \mu_w)^2 \right] \\ &= \frac{1}{3} [(5 - 6)^2 + (6 - 6)^2 + (7 - 6)^2] \\ &= \frac{1}{3} [(-1)^2 + 0^2 + 1^2] \\ &= \frac{2}{3} \\ \sigma_w &= \sqrt{\frac{2}{3}} \end{aligned}$$

$$\begin{aligned}\sigma_z^2 &= \frac{1}{3} \left[\sum_{i=1}^3 (z_i - \mu_z)^2 \right] \\ &= \frac{1}{3} [(1 - 6)^2 + (5 - 6)^2 + (12 - 6)^2] \\ &= \frac{1}{3} [(-5)^2 + (-1)^2 + 6^2] \\ &= \frac{62}{3} \\ \sigma_z &= \sqrt{\frac{62}{3}}\end{aligned}$$

$\sigma_z^2 \gg \sigma_w^2$, so Z is much more spread out than W .