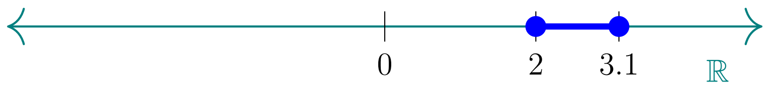
Numbers: Intervals and Interval Notation

Video companion

1 Closed intervals

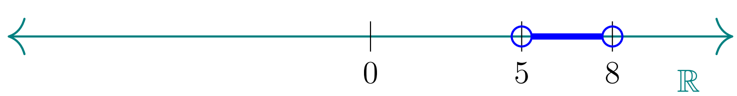


Real number line is an infinite set. There are also infinite subsets.

$[2, 3.1] = \{x \in \mathbb{R} : 2 \leq x \leq 3.1\}$

$$\begin{aligned} 2.3 &\in [2, 3.1] && \text{because } 2 \leq 2.3 \leq 3.1 \\ 3 &\in [2, 3.1] \\ 3.1 &\in [2, 3.1] \\ 1 &\notin [2, 3.1] && \text{because } 2 \not\leq 1 \leq 3.1 \end{aligned}$$

2 Open intervals

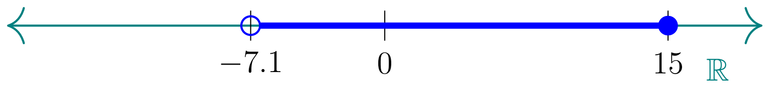


$(5, 8) = \{x \in \mathbb{R} : 5 < x < 8\}$

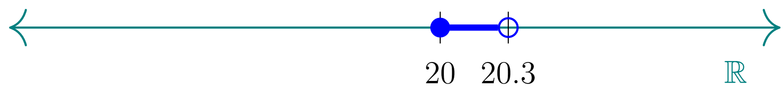
$$\begin{aligned} 5.5 &\in (5, 8) && \text{because } 5 < 5.5 < 8 \\ 5.0001 &\in (5, 8) \\ 5 &\notin (5, 8) && \text{because } 5 \not< 5 < 8 \end{aligned}$$

The intervals $[5, 8]$ and $(5, 8)$ differ at exactly two numbers: 5 and 8.

3 Half-open intervals



$(-7.1, 15] = \{x \in \mathbb{R} : -7.1 < x \leq 15\}$

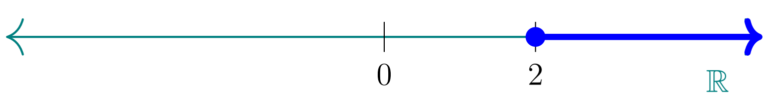


$[20, 20.3) = \{x \in \mathbb{R} : 20 \leq x < 20.3\}$

4 Recap vocabulary

- Closed intervals $[2, 3.1]$
- Open intervals $(5, 8)$
- Half-open intervals $(2, 3]$, $[20, 20.3)$

5 Rays



$[2, \infty) = \{x \in \mathbb{R} : x \geq 2\}$

Another example:
$(-\infty, 7.1) = \{x \in \mathbb{R} : x < 7.1\}$

6 What does an "answer" mean?

Solving an equality gives you a number:

$$
\begin{aligned}
x + 5 &= 10 \\
x &= 5
\end{aligned}
$$

Solving an inequality give you an interval:

$$
\begin{aligned}
1 &\leq x + 5 < 10 \\
-4 &\leq x < 5 \\
x &\in [-4, 5)
\end{aligned}
$$