AI5031: Machine Learning, exercise sheet 1

1 Notation and math primitives

Compute the resulting value of the following mathematical expressions:

1. $\sum_{i=0}^{10} i$
2. $\prod_{i=1}^{5} i$
3. $\sum_{i=0}^{4} i^2$
4. $\sum_{i=1}^{3} x_i y_i$ if $\vec{x} = (1,2,3)^T$ and $\vec{y} = (2,1,0)^T$
5. $\sum_{i=1}^{3} (x_i + y_i)$ if $\vec{x} = (1,2,3)^T$ and $\vec{y} = (2,1,0)^T$
6. $\frac{1}{3}\sum_{i=1}^{3} x_i^2$ if $\vec{x} = (1,2,3)^T$

2 Matrix multiplication

We have column vectors $\vec{x} = (1,2,3,0)^T$, $\vec{y} = (2,3,4,5)^T$, $\vec{z} = (2,3,0)^T$ and a matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$. Give the result of the following matrix multiplications if they are legally possible:

1. $A\vec{y}$
2. $A\vec{x}$
3. $\vec{x}^T\vec{y}$ (note: this is another way of writing the scalar product)
4. $\vec{x}A$
5. $A\vec{y}^T$
6. $\vec{x}^T A\vec{y}$ (note: this is called the *Mahalanobis distance* of two vectors given a matrix A)
7. $\vec{x}\vec{y}^T$ (note: this is called the *outer product* of two vectors)
8. $A\vec{z}$
9. $\vec{z}\vec{x}^T$
10. AA
11. AA^T

3 Functions

The Rectified Linear Unit (ReLU) function $f : x \in \mathbb{R} \mapsto y \in \mathbb{R}$ is defined as $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{else} \end{cases}$ Assuming the quantities from the previous exercise and the matrix $B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, compute:

1. $f(\vec{x})$
2. $f(B\vec{x})$
3. $f(B)$
4. $f(\vec{x}^T\vec{y})$

4 Vector-scalar and vector-vector functions

Let $f(x) =< \vec{x}, (1,1,0,0) >$ be a vector-scalar function and $g_i(\vec{x}) = x_i^2$ a vector-vector function. Please compute:

1. $f\left((1,1,1,1)^T\right)$
2. $f(B)$ using the matrix B from the previous exercise
3. $f(B^T)$ using the matrix B from the previous exercise
4. $g\left((1,2,3)^T\right)$
5. $g(B)$ using B from the previous exercise

5 Matrix contractions and slices

Using the matrix A from exercise 2, compute the following expressions:

1. $\sum_{i=1}^{4} A_{2i}$
2. $\sum_{i=1}^{4} A_{i1}$
3. $\sum_{i=1}^{4} A_{ii}$ (note: this is called the *trace* of A. What elements are being summed here?)
4. $A_{1,2:3}$
5. $A_{2:3,1}$
6. $A_{:,1}$
7. $A_{1:2,2:3}$

6 Tensor contractions and slicing

Assuming a tensor $T \in \mathbb{R}^{3,2,4}$ whose values are all 1.0. What is the shape of the following tensor contractions?

1. $\tilde{T}_{ab} = \sum_{i=1}^{3} T_{iab}$
2. $\tilde{T}_{ab} = \sum_{i=1}^{2} T_{aib}$
3. $\tilde{T}_{ab} = \sum_{i=1}^{4} T_{abi}$
4. $\tilde{T}_{ab} = \sum_{i=1}^{2} \sum_{j=1}^{4} T_{aij}$