Weather forecast

You want to spend the afternoon at the beach, but the sky is overcast at 10am. You know the following probabilities from experience, should you risk goint to the beach?

- On day where it rains in the afternoon, the sky is overcast at 10 am in 90% of the cases.
- The probability of an overcast sky at 10am is 10%.
- It is summer and the probability of having rain in the afternoon is 10%.

Discrete probabilities

Given are two sets of experimental results:

$$\begin{array}{lcccccccccccc} \textbf{Exp 1 (X):} & 1 & 2 & 1 & 2 & 3 & 3 & 3 & 2 & 2 & 2 & 1 & 1 \\ \textbf{Exp 2 (Y):} & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 & 1 & 1 & 1 \end{array} \tag{1}$$

Compute:
a) $\#(Y = 1)$, $\#(Y = 2)$, $\#(Y = 3)$
b) $\#(X = 1, Y = 1)$, $\#(X = 1, Y = 2)$, $\#(X = 1, Y = 3)$
c) $p(X = 1, Y = 1)$, $p(X = 1, Y = 2)$, $p(X = 1, Y = 3)$
d) $p(X = 1|Y = 1)$, $p(X = 1|Y = 2)$, $p(X = 1|Y = 3)$
e) $p(Y = 1|X = 1)$
f) $p(X = 1|Y \neq 1)$

Confusion matrix 1

Evaluating a classifier yield the following confusion matrix:

$$\begin{pmatrix} 50 & 10 & 20 \\ 10 & 70 & 0 \\ 20 & 0 & 60 \end{pmatrix} \tag{2}$$

Compute:
a) The number of samples in the test set
b) $p(\hat{y} = 1, \hat{t} = 1)$, $p(\hat{y} = 1, \hat{t} = 2)$, $p(\hat{y} = 3\hat{t} = 1)$
c) $p(\hat{y} = 1|\hat{t} = 1)$, $p(\hat{y} = 1|\hat{t} = 2)$, $p(\hat{y} = 3|\hat{t} = 1)$
d) $p(\hat{t} = 1)$, $p(\hat{t} = 2)$, $p(\hat{y} = 3)$
e) Express the probabilities a-d in words!
f) The probability of an incorrect classification under the restriction that the true class is 3.
g) The probability of an incorrect classification under the restriction that the classifier output is 2
h) The probability of an incorrect classification!

Confusion matrix 2

A three-class classifier is evaluated on a test set of 900 samples which contains all three classes in equal proportions.

- Classes 2 and 3 are always classified correctly
- Class is is confused with class 2 in 50% of the cases, and with class 3 in 20% of the cases.

Write down the confusion matrix!